ECE 340 Lecture 5
Semiconductor Electronics

Spring 2022
10:00-10:50am
Professor Umberto Ravaioli
Department of Electrical and Computer Engineering
2062 ECE Building
Today’s Discussion

• Electrons and Holes
• Effective Mass
• Intrinsic Material
• Extrinsic Material
Dynamics of electrons is studied in a generalized system of coordinates:

\[
\begin{align*}
\text{Time) } & \quad t \\
\text{Real Space) } & \quad (x, y, z) \\
\text{Momentum Space) } & \quad (k_x, k_y, k_z)
\end{align*}
\]

The kinetic energy state of the particle is given by a function

\[ E = f(k_x, k_y, k_z) \]

This is usually known as “band structure”. As we have seen, energy in a crystal is a multi-valued function, so the band structure has many branches.
For a classical particle in \textbf{free space} there is only a single value of energy for each set of momentum coordinates.

\[ E = \frac{1}{2}mv^2 \]

Momentum is defined as \[ p = mv \rightarrow E = \frac{p^2}{2m} \]

This is a simple parabolic relation.
For a quantum mechanical particle described by the Schrödinger equation inside a crystal, we define a new “crystal” momentum

\[ \langle p \rangle = \hbar k \]

Here \( k \) is the wave number of the wave describing the particle quantum-mechanically and for convenience it is used most often as momentum coordinate.
We model the crystal as a new kind of “free space” where the particles obey a new energy-momentum relationship given by the band structure, containing the effects of the periodic atomic potentials.

Therefore, what we study is not an electron “particle” as in an isolated atom or in vacuum, but an electron “quasi-particle” which obeys special dynamic laws as dictated by the specific crystal in which it moves.
Simple examples of band structure

Lowest energy solution in the conduction band

Highest energy solution in the valence band

(a) Direct

(b) Indirect
Often it is sufficient to analyze conduction band behavior only in the region close to the minimum of the lowest energy branch, called a “valley”.

In many cases the $E(k)$ relations in the valleys are with good approximation parabolic.

Often it is sufficient to analyze valence band behavior only in the region close to the maximum of the highest energy branch, also called a “valley”.
At finite temperature, electrons have the probability to acquire sufficient thermal energy to be excited to the conduction band.

An empty state is left behind, which essentially behaves like a positive “quasi-particle” (hole) moving when neighboring valence electron jump into that empty space leaving another one behind.
Direct or indirect?

Si

Energy [eV]

[111] [000] [100]
Direct or indirect?
Direct or indirect?

GaAs

Energy [eV]

[111]  [000]  [100]
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<thead>
<tr>
<th>Material</th>
<th>$E_g$ (eV)</th>
<th>$\mu_n$ (cm$^2$/V·s)</th>
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All values at 300 K.

*Vaporizes

C (diamond) $E_g = 5.5eV$

SiO$_2$ $E_g = 9.0eV$
Vacuum tube

Newton’s Law

\[ F = -qE = m_0 \frac{dv}{dt} \]

Kinetic Energy

\[ E = \frac{p^2}{2m_0} = \frac{1}{2} m_0 v^2 \]

Current Density

\[ J = (-q) \sum_i^N v_i > 0 \]

Apply Electric Field

Momentum space

Real space
Semiconductor at $T = 0K$

Current Density $J = (-q) \sum_{i}^{N} v_i = 0$

Apply Electric Field

$E_C$

$E_V$

Momentum space

Real space
Semiconductor at $T > 0K$

Momentum space

Real space

$E_g$

$E_V$

$E_C$

empty state

$x$
Semiconductor at $T > 0K$

Current Density  \[ J = (-q) \sum_{i}^{N} v_i > 0 \]

Apply Electric Field

$E_C$  

$E_V$  

empty state

Momentum space  

Real space
In the valence band:

Current Density \[ J = (-q) \sum_{i}^{N} v_i - (-q) v_j \]
\[ = 0 + q v_j \]

The electrons in the valence band fill the empty state leaving another one behind as if the state is moving like a positive particle.

Therefore, a “quasi-particle” called “hole” can be used to model this kind of current.

The electron adds to the current density by moving in the opposite direction (same current direction as the hole since it has negative charge).
Dynamics of the particles does not take place according to the classical mechanical laws as in free space, but it depends on the band structure $E(k)$.

However, the bottom of the conduction band and the top of the valence band are approximately parabolic in a range of energies.

In these conditions, particles still move according to a Netwon-like dynamics, as if possessing a different mass which depends on the curvature of the parabola.

$$F = -qE = m_n^* \frac{dv}{dt}$$

electron effective mass

$$m_p^*$$

hole effective mass is typically different
Rules of thumb for band curvature

\[ \frac{d^2 E}{dk^2} = \text{Curvature of the band} \]

\[ m^* = \hbar^2 \left( \frac{d^2 E}{dk^2} \right)^{-1} \]

Stronger curvature:
- Less energy states available
- Lighter effective mass

Gentler curvature:
- More energy states available
- Heavier effective mass
Silicon band structure

Energy [eV]

Si

[111] [000] [100]
Valleys

heavy holes
light holes
main electron valley
heavy holes
light holes
Germanium band structure

main electron valley
Germanium Equivalent Valleys
GaAs band structure
GaAs band structure

- Satellite valley (L)
- Main electron valley
- Satellite valley (X)
GaAs Equivalent Valleys ($\Gamma$ & L)
The effective mass depends on the band curvature, therefore, it depends on spatial direction for a valley which is not spherical.

For some applications to evaluate material properties, one may use a mass averaged over the valleys and the directions (density of states mass).

In charge transport calculations one may use instead the appropriate average mass for a given direction (conductivity mass).
# Semiconductor Material Properties

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All values at 300 K.

*C (diamond) $E_g = 5.5eV$

SiO$_2$

$E_g = 9.0eV$
Intrinsic Semiconductor

- A pure semiconductor is called “intrinsic”

- There are always as many electrons in the conduction band as holes in the valence band.
Intrinsic Semiconductor

- The intrinsic concentration depends on temperature

\[ n_0 p_0 = n_i^2 \quad \Rightarrow \quad n_0 = p_0 = n_i \]

Electron-hole generation rate \( G_0 \) always balances the recombination rate \( R_0 \)

\[ G_0 = R_0 \]

Textbook uses

\[ n_i = 1.5 \times 10^{10} \, \text{cm}^{-3} \]
Extrinsic Semiconductor

• Impurities are introduced to force the material to have more electrons than holes or *vice versa*
  
  – An atom with more than 4 valence electrons can donate electrons to the conduction band while becoming a fixed positively charged ion. This is called a *donor*.
  
  – An atom with less than 4 valence electrons can attract electrons from the valence band, creating holes and becoming a fixed negatively charged ion. This is called an *acceptor*. 
Extrinsic Semiconductor

- The process of introducing impurities in the semiconductor for the purpose of changing material properties is called “doping”.
- The impurities introduced with this process are called “dopants”.

- Dopants are more commonly added:
  - During growth of the crystal
  - By diffusion at high temperature
  - By ion implantation
Diffusion (pre-deposition + drive-in)

Pre-deposition

Drive-in

\[ T \gg \]

Diffusion depth depends on temperature
Ion implantation

Ion beam with energy $E_B$

Range $R$ depends on $E_B$

Doping profile obtained by implantation at different energies

Diffusion process can follow to smooth out profile
Common Dopants for Silicon

<table>
<thead>
<tr>
<th>Dopant</th>
<th>Symbol</th>
<th>Mass Number</th>
<th>Half-Life</th>
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<tr>
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<td>B</td>
<td>57-71</td>
<td>longest</td>
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<tr>
<td>Arsenic</td>
<td>As</td>
<td>73-77</td>
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For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
Doping atoms

substitutional dopant

interstitial dopant
Donors and Acceptors

Donors should create energy levels close to the conduction band
Acceptors should create energy levels close to the valence band