

ECE 340 Lecture 5

Semiconductor Electronics

Spring 2022

10:00-10:50am

Professor Umberto Ravaioli

Department of Electrical and Computer Engineering

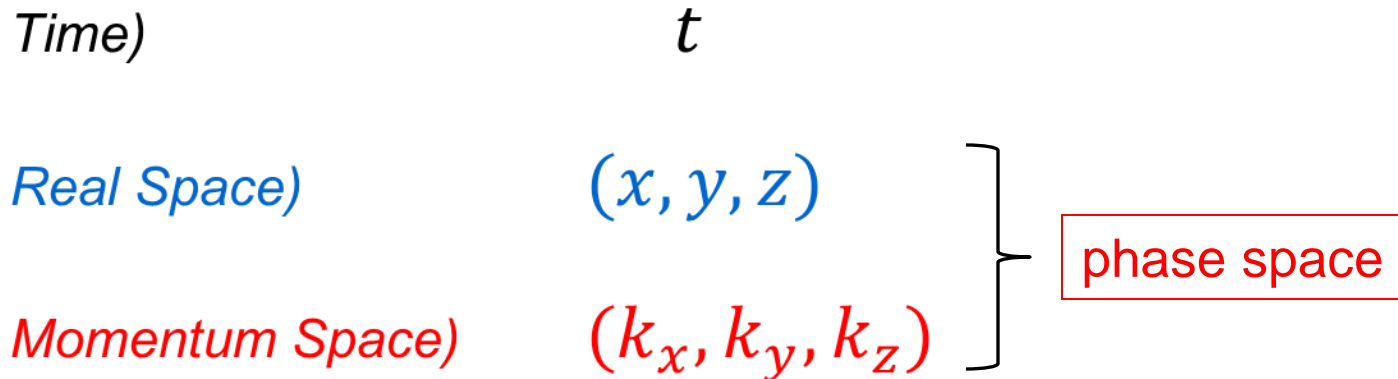
2062 ECE Building

Today's Discussion

- Electrons and Holes
- Effective Mass
- Intrinsic Material
- Extrinsic Material

Phase Space

Dynamics of electrons is studied in a generalized system of coordinates:



The kinetic energy state of the particle is given by a function

$$E = f(k_x, k_y, k_z)$$

This is usually known as “band structure”.

As we have seen, energy in a crystal is a multi-valued function, so the band structure has many branches.

Classical particle

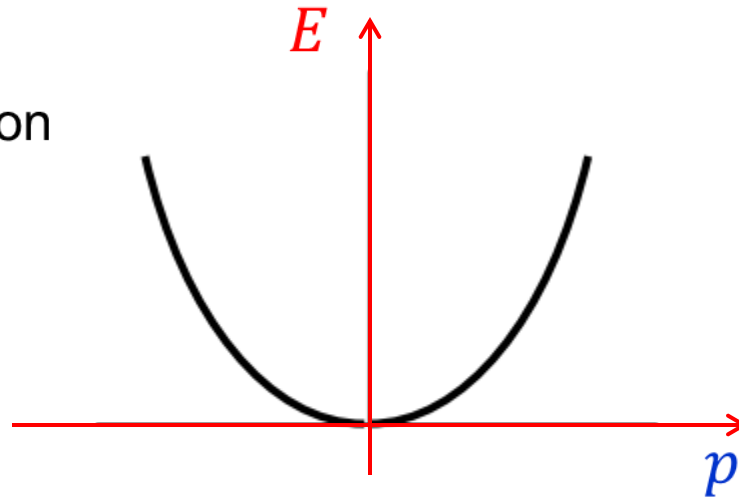
For a classical particle in **free space** there is only a single value of energy for each set of momentum coordinates.

$$E = \frac{1}{2}mv^2$$

Momentum is defined as

$$p = mv \rightarrow E = \frac{p^2}{2m}$$

This is a simple parabolic relation



Quantum mechanical particle

For a quantum mechanical particle described by the Schrödinger equation inside a crystal, we define a new “crystal” momentum

$$\langle p \rangle = \hbar k$$

Here k is the wave number of the wave describing the particle quantum-mechanically and for convenience it is used most often as momentum coordinate.

Quantum mechanical particle

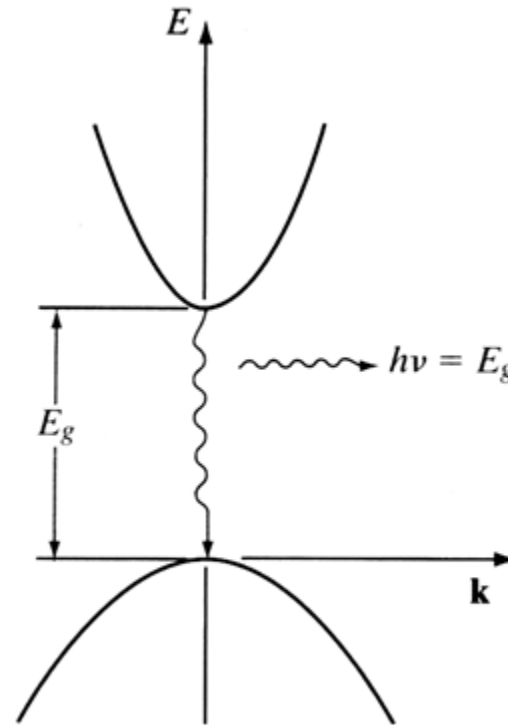
We model the crystal as a new kind of “free space” where the particles obey a new energy-momentum relationship given by the band structure, containing the effects of the periodic atomic potentials.

Therefore, what we study is not an electron “particle” as in an isolated atom or in vacuum, but an electron “quasi-particle” which obeys special dynamic laws as dictated by the specific crystal in which it moves.

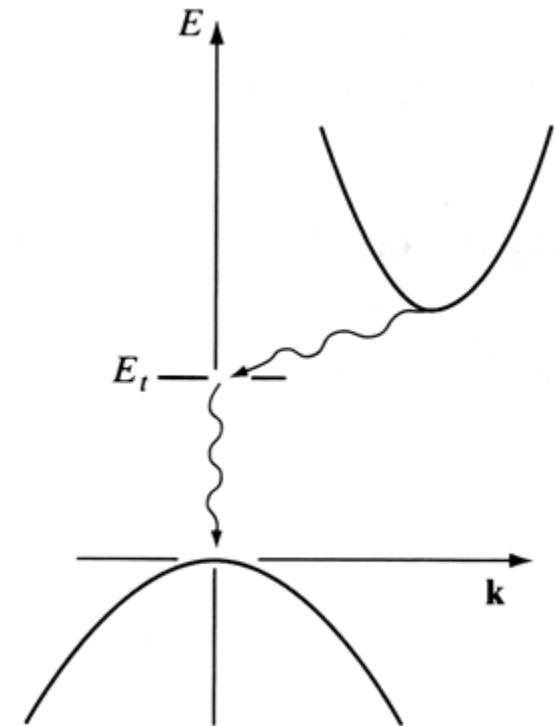
Simple examples of band structure

Lowest energy solution in the conduction band

Highest energy solution in the valence band



(a) Direct

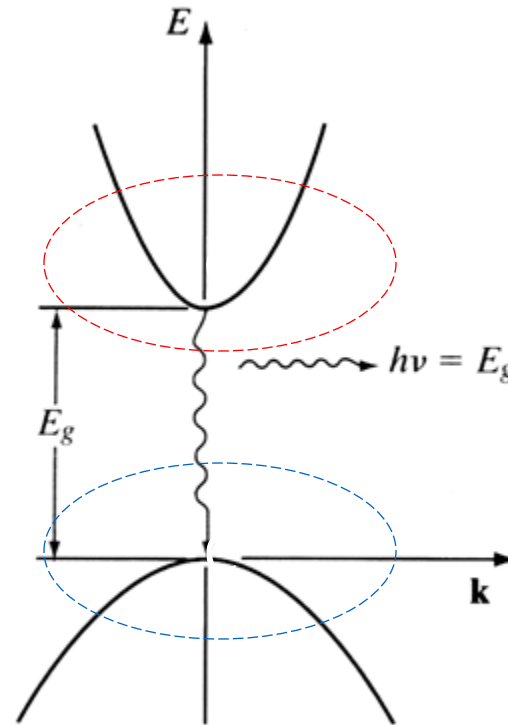


(b) Indirect

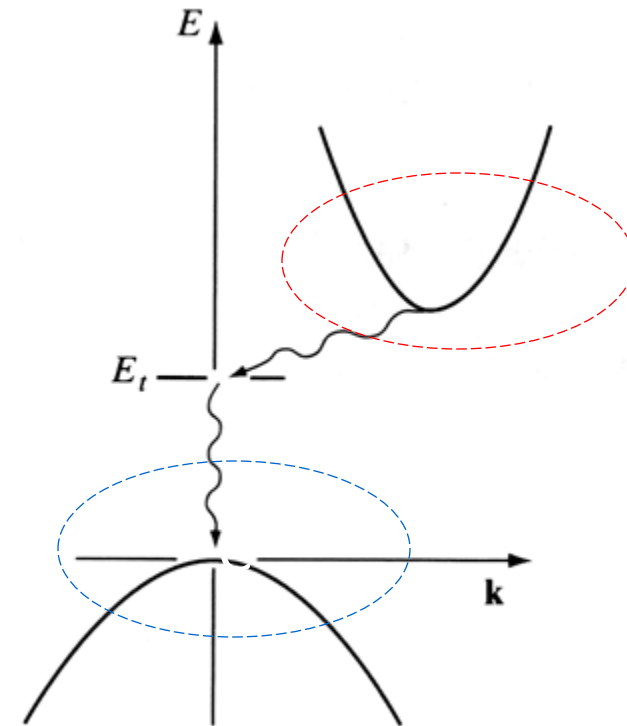
Simple examples of band structure

Often it is sufficient to analyze conduction band behavior only in the region close to the minimum of the lowest energy branch, called a “valley”.

In many cases the $E(k)$ relations in the valleys are with good approximation parabolic



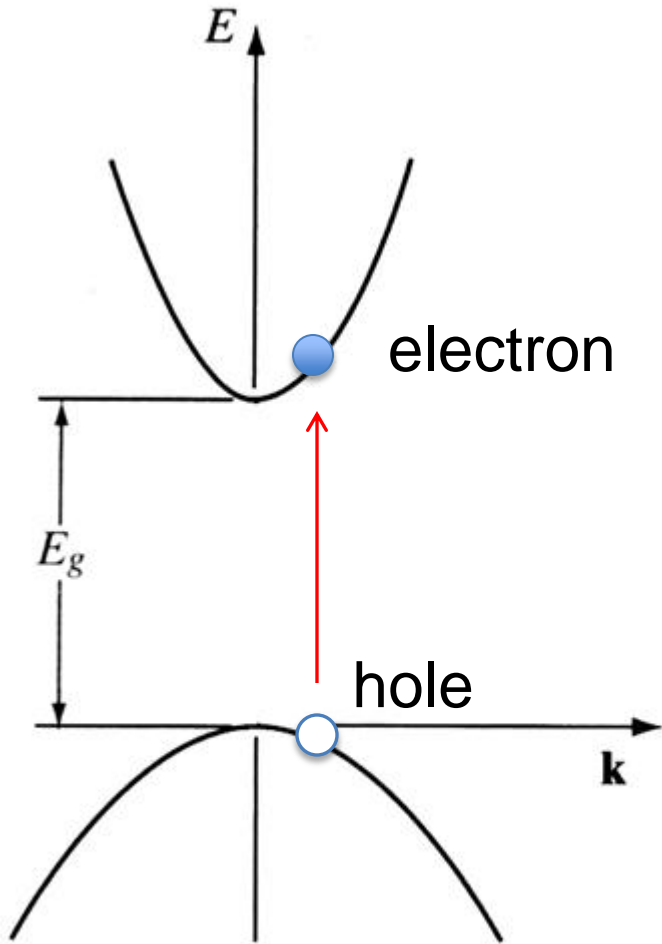
(a) Direct



(b) Indirect

Often it is sufficient to analyze valence band behavior only in the region close to the maximum of the highest energy branch, also called a “valley”.

Simple examples of band structure

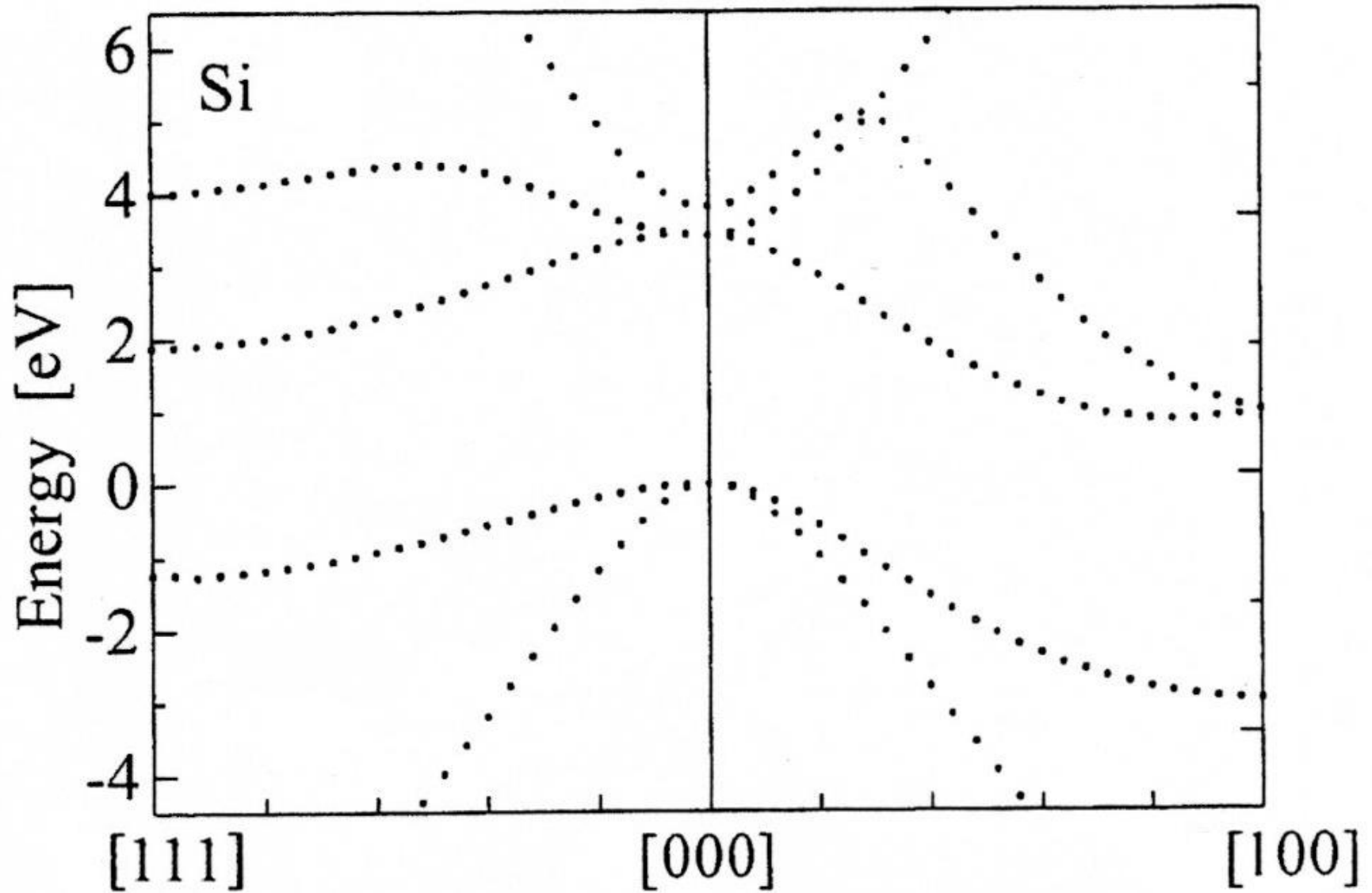


(a) Direct

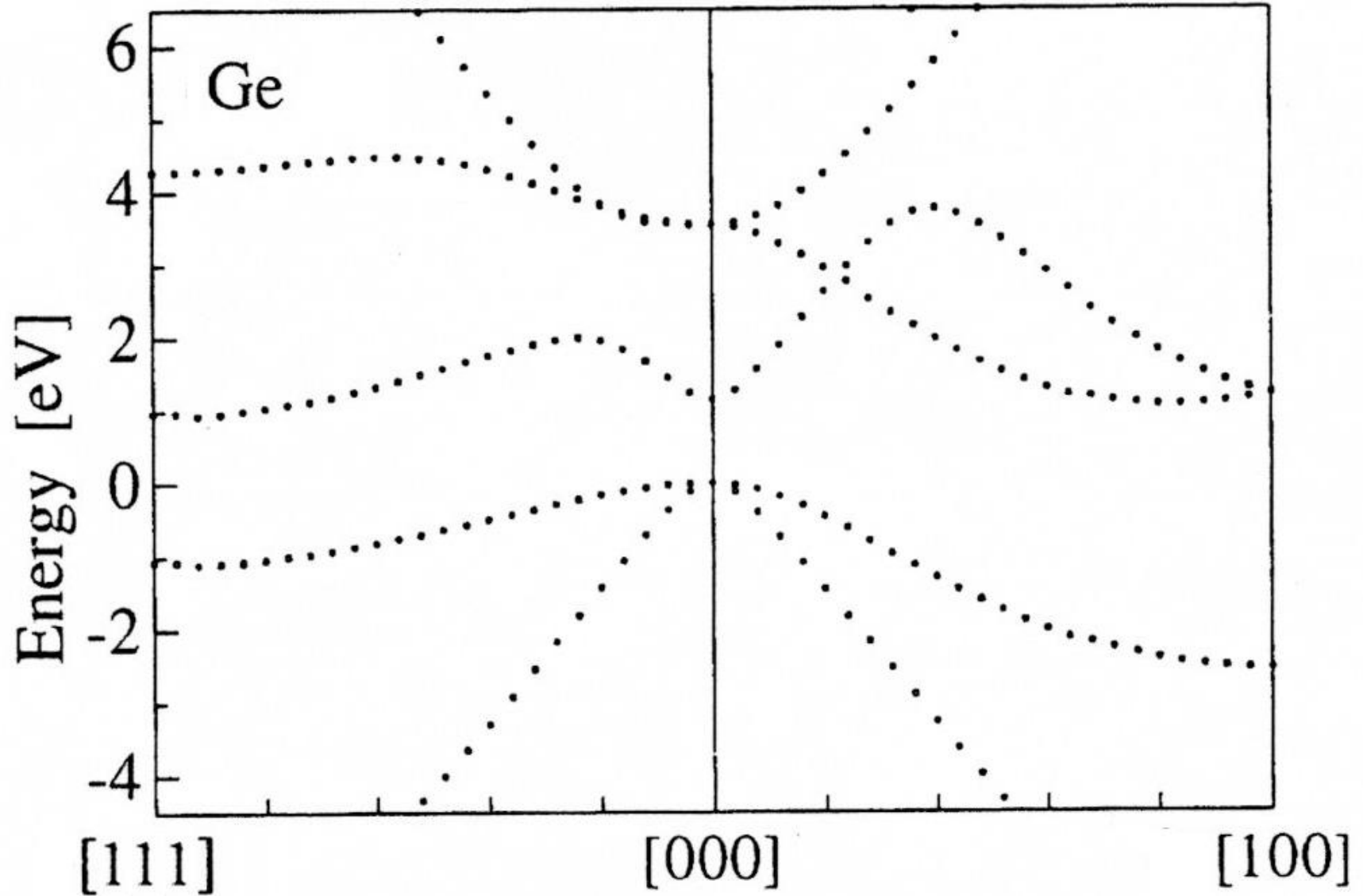
At finite temperature, electrons have the probability to acquire sufficient thermal energy to be excited to the conduction band.

An empty state is left behind, which essentially behaves like a positive “quasi-particle” (hole) moving when neighboring valence electron jump into that empty space leaving another one behind

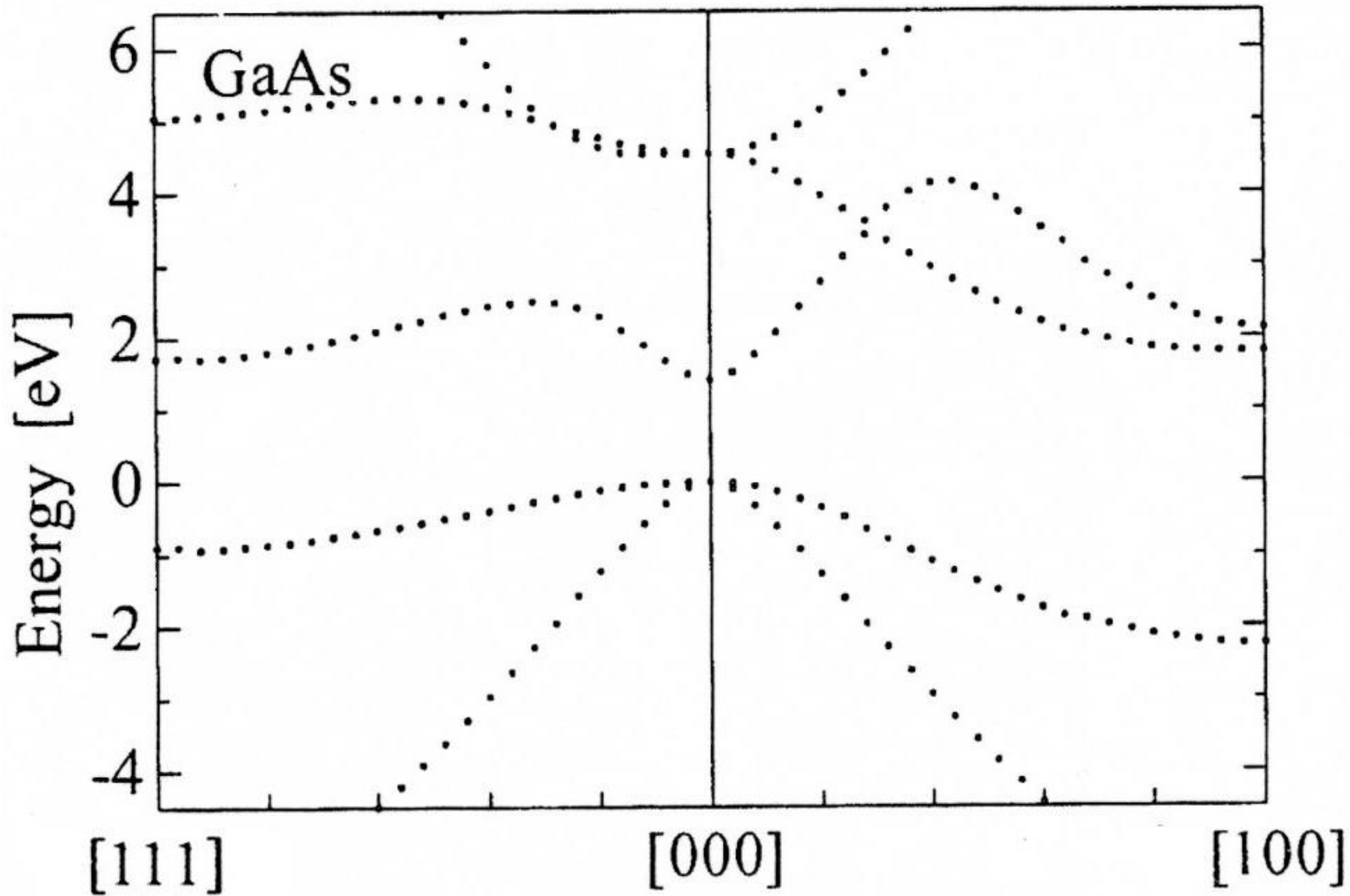
Direct or indirect?



Direct or indirect?



Direct or indirect?



Semiconductor Material Properties

| | | E_g (eV) | μ_n ($\text{cm}^2/\text{V-s}$) | μ_p ($\text{cm}^2/\text{V-s}$) | m_n^*/m_0 (m_l, m_t) | m_p^*/m_0 (m_{lh}, m_{hh}) | a (Å) | ϵ_r | Density (g/cm^3) | Melting point (°C) |
|------------------|----------|---------------|---|---|-------------------------------|-------------------------------------|---------|--------------|---------------------------------------|--------------------------|
| Si | (i/D) | 1.11 | 1350 | 480 | 0.98, 0.19 | 0.16, 0.49 | 5.43 | 11.8 | 2.33 | 1415 |
| Ge | (i/D) | 0.67 | 3900 | 1900 | 1.64, 0.082 | 0.04, 0.28 | 5.65 | 16 | 5.32 | 936 |
| SiC (α) | (i/W) | 2.86 | 500 | — | 0.6 | 1.0 | 3.08 | 10.2 | 3.21 | 2830 |
| AlP | (i/Z) | 2.45 | 80 | — | — | 0.2, 0.63 | 5.46 | 9.8 | 2.40 | 2000 |
| AlAs | (i/Z) | 2.16 | 1200 | 420 | 2.0 | 0.15, 0.76 | 5.66 | 10.9 | 3.60 | 1740 |
| AlSb | (i/Z) | 1.6 | 200 | 300 | 0.12 | 0.98 | 6.14 | 11 | 4.26 | 1080 |
| GaP | (i/Z) | 2.26 | 300 | 150 | 1.12, 0.22 | 0.14, 0.79 | 5.45 | 11.1 | 4.13 | 1467 |
| GaAs | (d/Z) | 1.43 | 8500 | 400 | 0.067 | 0.074, 0.50 | 5.65 | 13.2 | 5.31 | 1238 |
| GaN | (d/Z, W) | 3.4 | 380 | — | 0.19 | 0.60 | 4.5 | 12.2 | 6.1 | 2530 |
| GaSb | (d/Z) | 0.7 | 5000 | 1000 | 0.042 | 0.06, 0.23 | 6.09 | 15.7 | 5.61 | 712 |
| InP | (d/Z) | 1.35 | 4000 | 100 | 0.077 | 0.089, 0.85 | 5.87 | 12.4 | 4.79 | 1070 |
| InAs | (d/Z) | 0.36 | 22600 | 200 | 0.023 | 0.025, 0.41 | 6.06 | 14.6 | 5.67 | 943 |
| InSb | (d/Z) | 0.18 | 10^5 | 1700 | 0.014 | 0.015, 0.40 | 6.48 | 17.7 | 5.78 | 525 |
| ZnS | (d/Z, W) | 3.6 | 180 | 10 | 0.28 | — | 5.409 | 8.9 | 4.09 | 1650* |
| ZnSe | (d/Z) | 2.7 | 600 | 28 | 0.14 | 0.60 | 5.671 | 9.2 | 5.65 | 1100* |
| ZnTe | (d/Z) | 2.25 | 530 | 100 | 0.18 | 0.65 | 6.101 | 10.4 | 5.51 | 1238* |
| CdS | (d/W, Z) | 2.42 | 250 | 15 | 0.21 | 0.80 | 4.137 | 8.9 | 4.82 | 1475 |
| CdSe | (d/W) | 1.73 | 800 | — | 0.13 | 0.45 | 4.30 | 10.2 | 5.81 | 1258 |
| CdTe | (d/Z) | 1.58 | 1050 | 100 | 0.10 | 0.37 | 6.482 | 10.2 | 6.20 | 1098 |
| PbS | (i/H) | 0.37 | 575 | 200 | 0.22 | 0.29 | 5.936 | 17.0 | 7.6 | 1119 |
| PbSe | (i/H) | 0.27 | 1500 | 1500 | — | — | 6.147 | 23.6 | 8.73 | 1081 |
| PbTe | (i/H) | 0.29 | 6000 | 4000 | 0.17 | 0.20 | 6.452 | 30 | 8.16 | 925 |

C (diamond)
 $E_g = 5.5\text{eV}$

SiO₂
 $E_g = 9.0\text{eV}$

All values at 300 K.

*Vaporizes

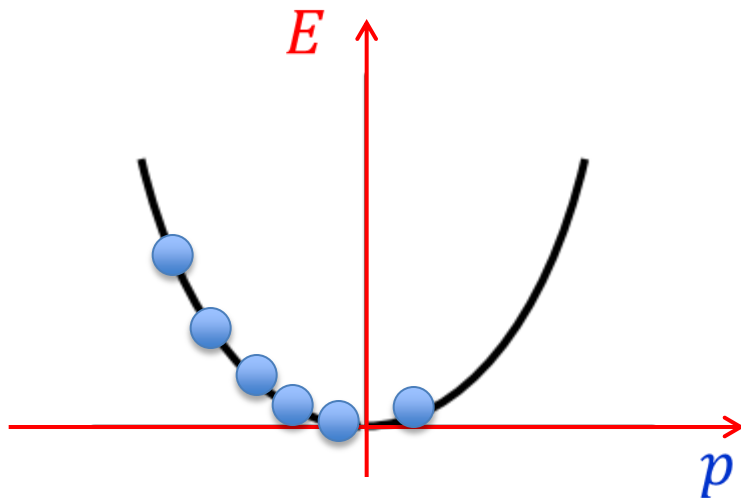
Vacuum tube

Newton's Law

$$F = -qE = m_0 \frac{dv}{dt}$$

Kinetic Energy

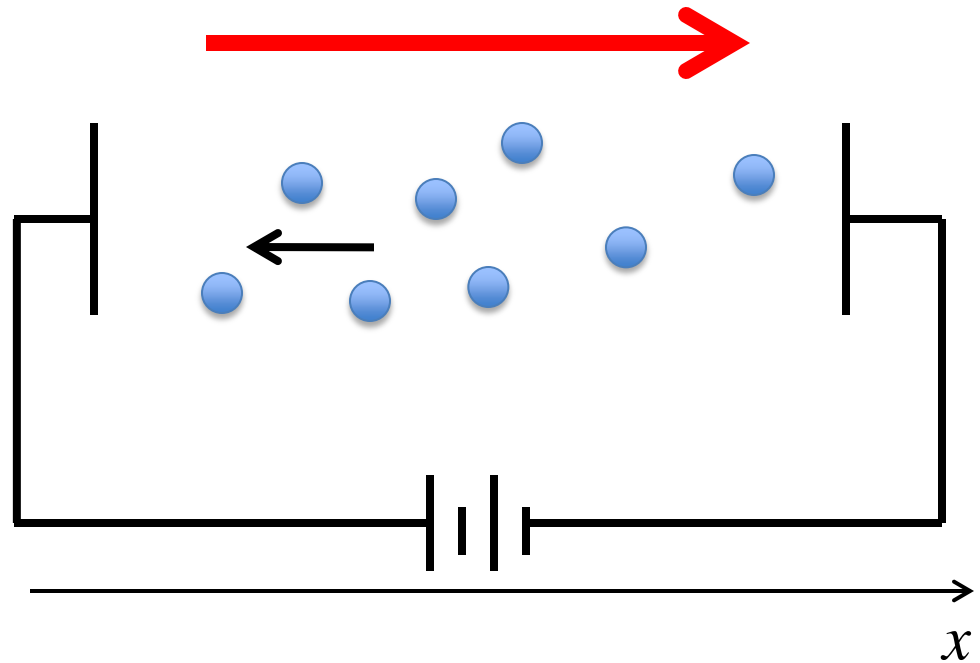
$$E = \frac{p^2}{2m_0} = \frac{1}{2} m_0 v^2$$



Momentum space

Current Density $J = (-q) \sum_i^N v_i > 0$

Apply Electric Field

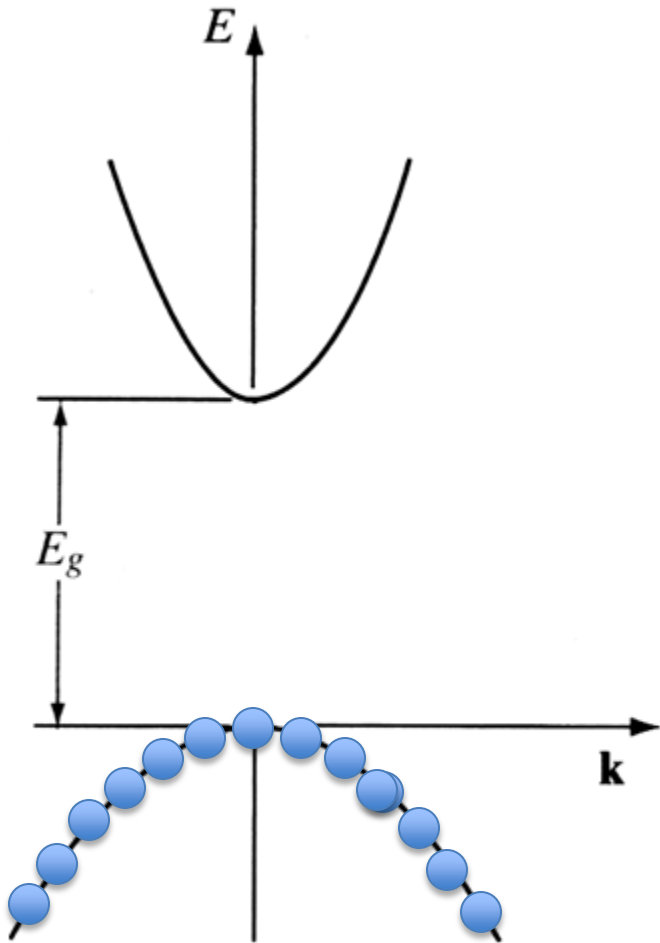


Real space

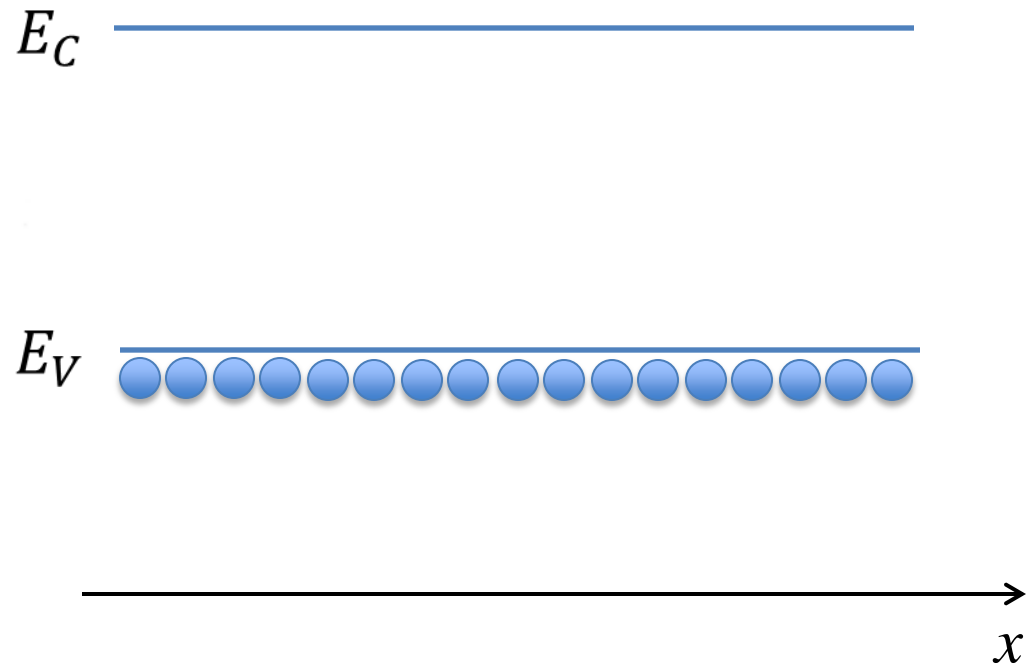
Semiconductor at $T = 0\text{K}$

Current Density $J = (-q) \sum_i^N v_i = 0$

Apply Electric Field

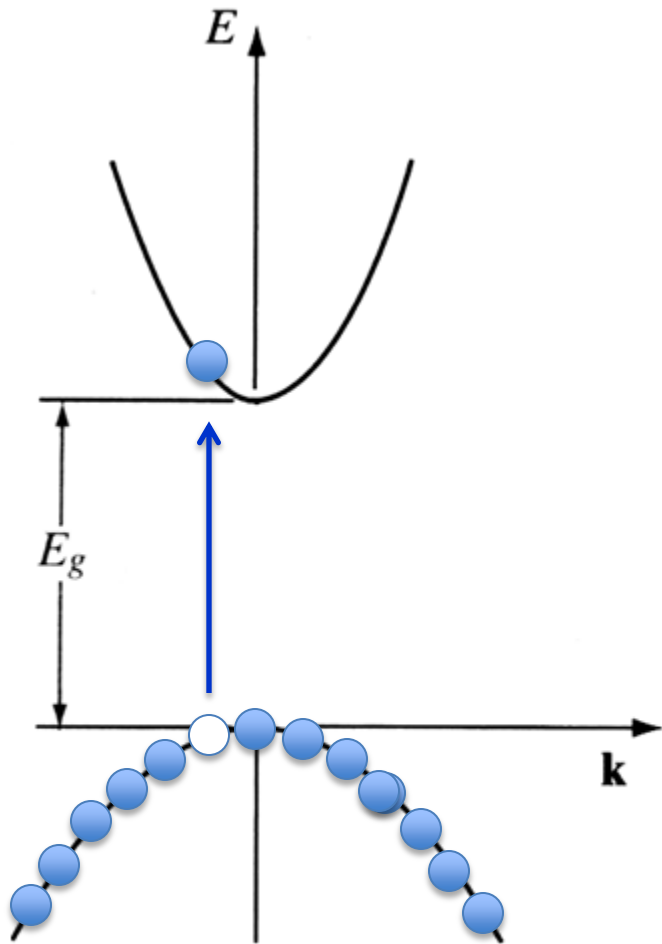


Momentum space

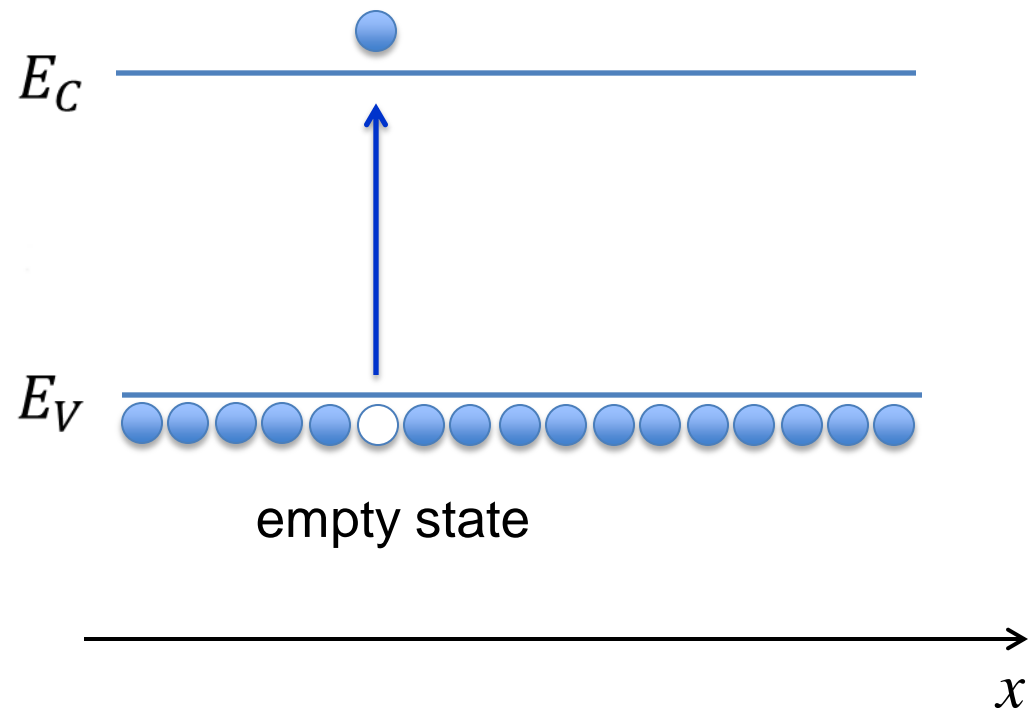


Real space

Semiconductor at $T > 0\text{K}$



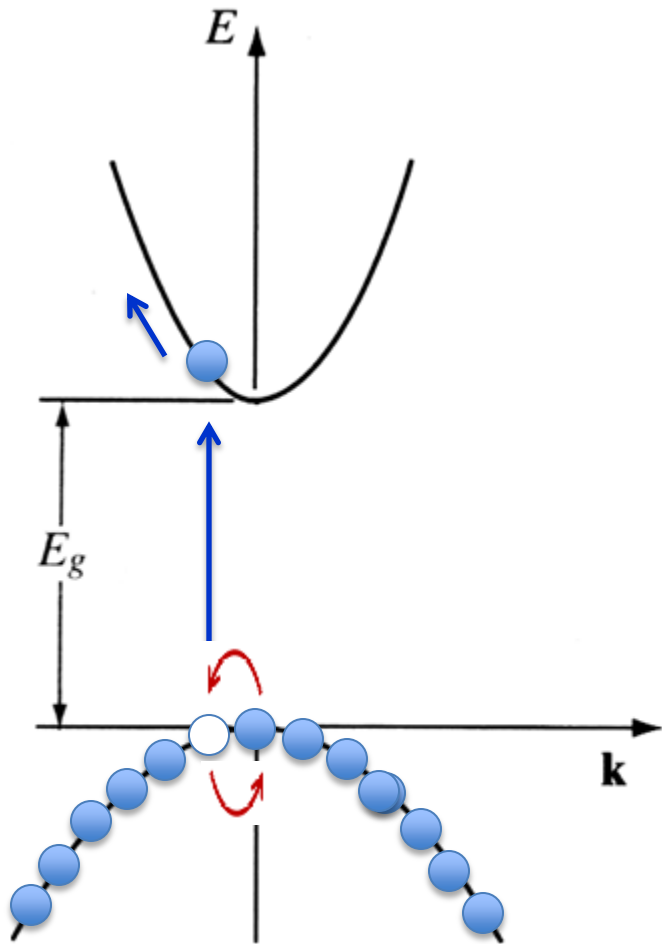
Momentum space



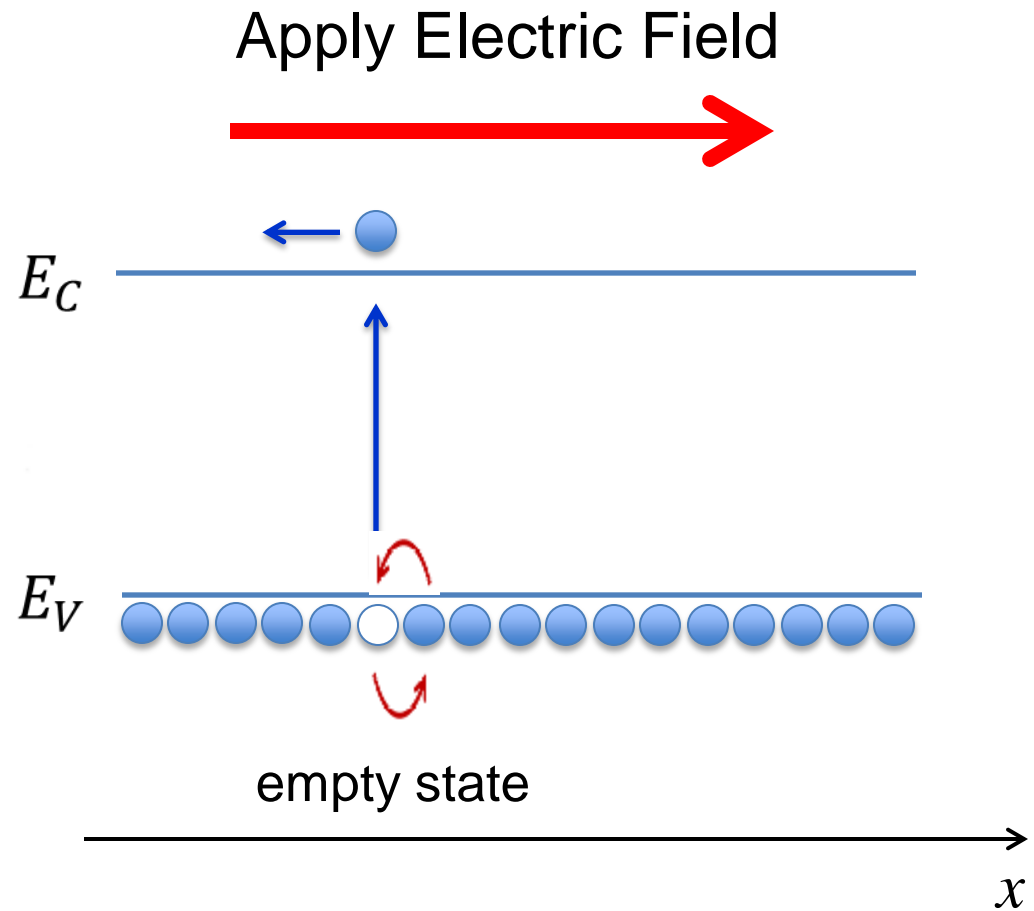
Real space

Semiconductor at $T > 0K$

$$\text{Current Density } J = (-q) \sum_i^N v_i > 0$$



Momentum space



Real space

Current takes place in two bands

In the valence band:

$$\begin{aligned} \text{Current Density } J &= (-q) \sum_i^N v_i - (-q) v_j \\ &= 0 + qv_j \end{aligned}$$

The electrons in the valence band fill the empty state leaving another one behind as if the state is moving like a positive particle.

Therefore, a “quasi-particle” called “hole” can be used to model this kind of current.

The electron adds to the current density by moving in the opposite direction (same current direction as the hole since it has negative charge).

Effective mass

Dynamics of the particles does not take place according to the classical mechanical laws as in free space, but it depends on the band structure $E(\mathbf{k})$.

However, the *bottom of the conduction band* and the *top of the valence band* are approximately parabolic in a range of energies.

In these conditions, particles still move according to a Newton-like dynamics, as if possessing a different *mass* which depends on the curvature of the parabola.

$$F = -qE = m_n^* \frac{dv}{dt}$$

electron effective mass

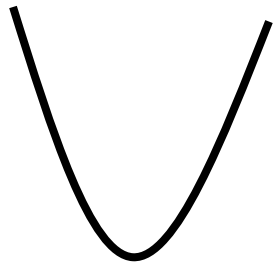
$$m_p^*$$

hole effective mass is typically different

Rules of thumb for band curvature

$$\frac{d^2 E}{dk^2} = \text{Curvature of the band}$$

$$m^* = \hbar^2 \left(\frac{d^2 E}{dk^2} \right)^{-1}$$



Stronger curvature:

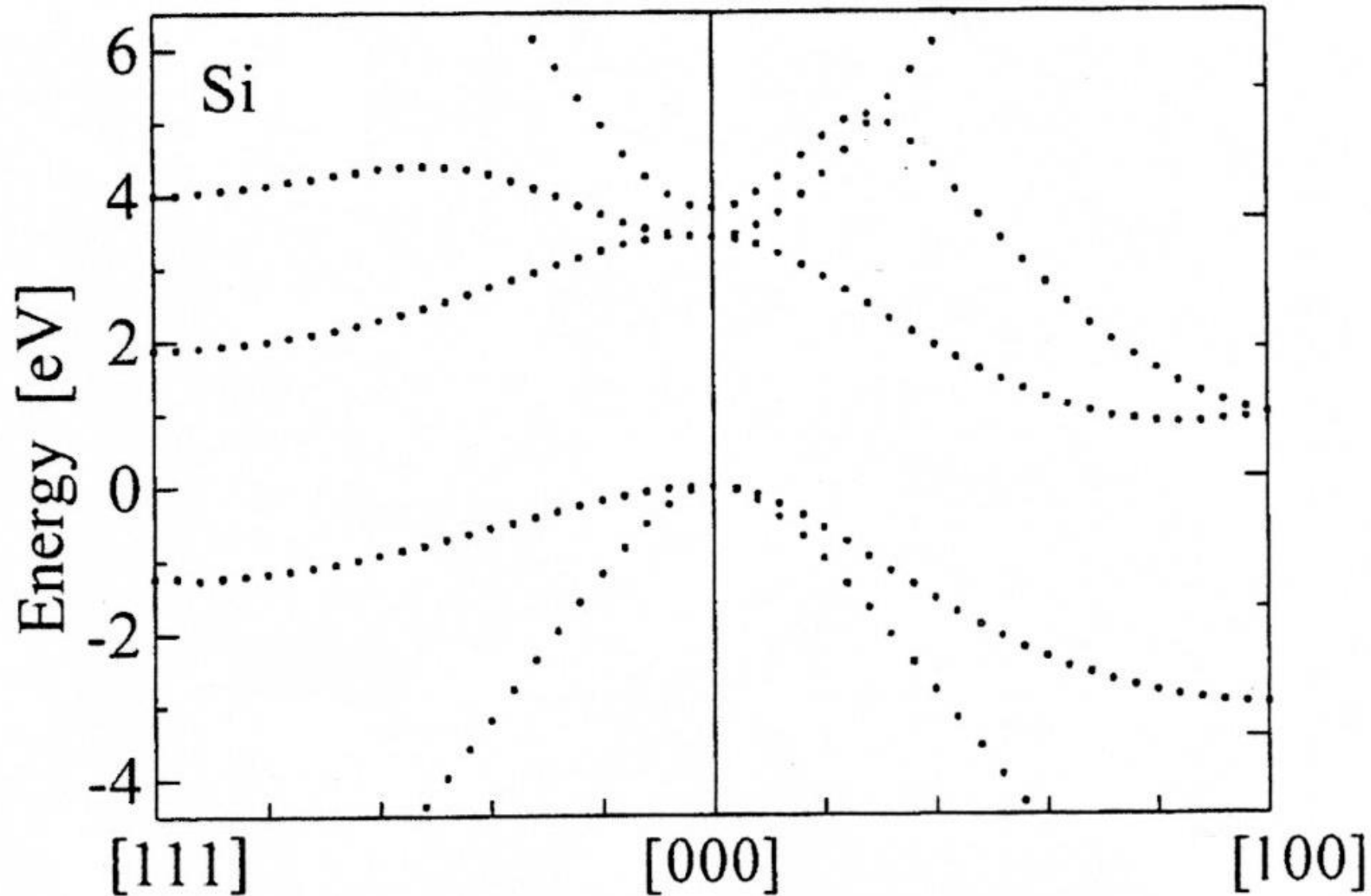
- Less energy states available
- Lighter effective mass



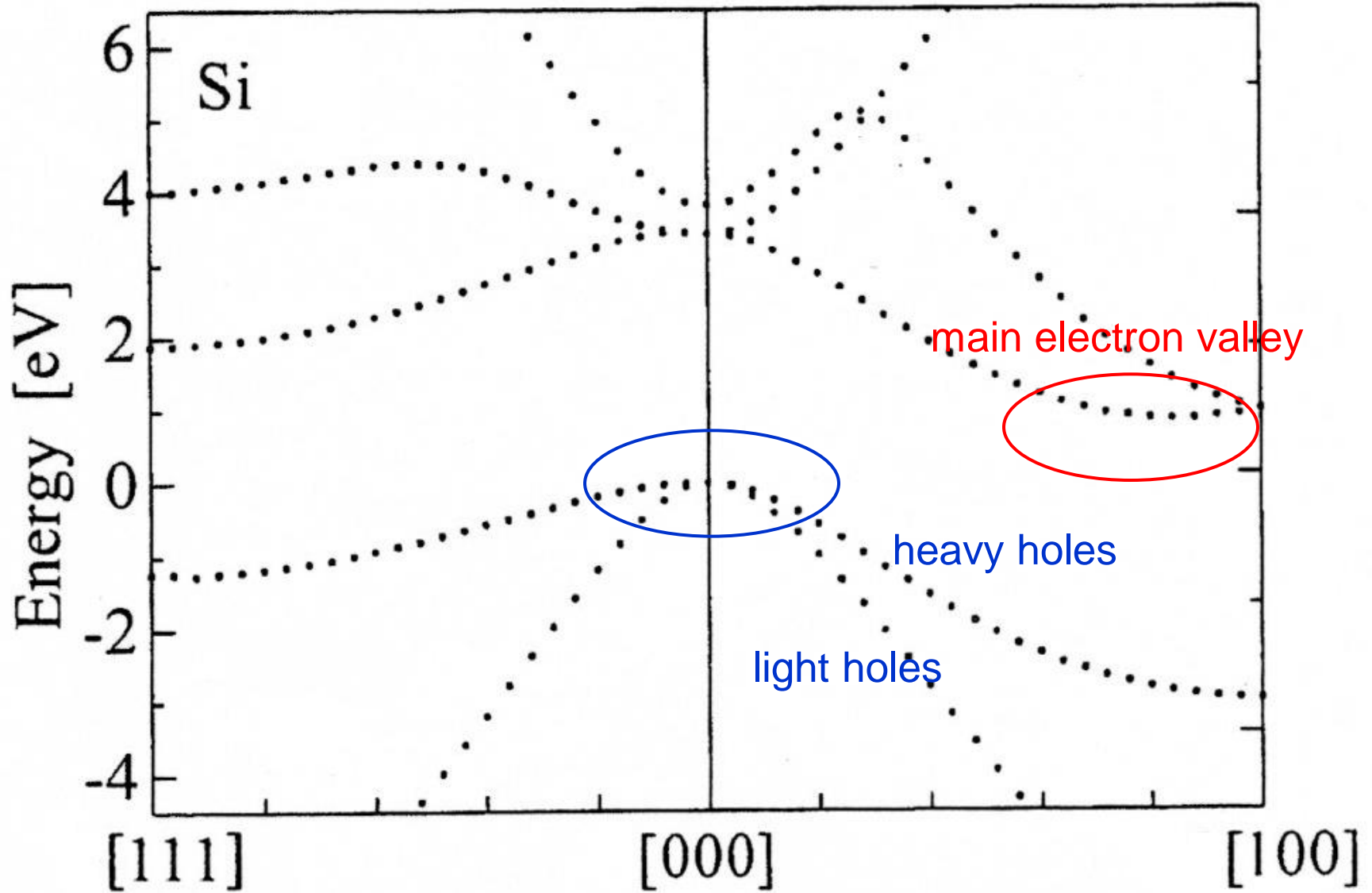
Gentler curvature:

- More energy states available
- Heavier effective mass

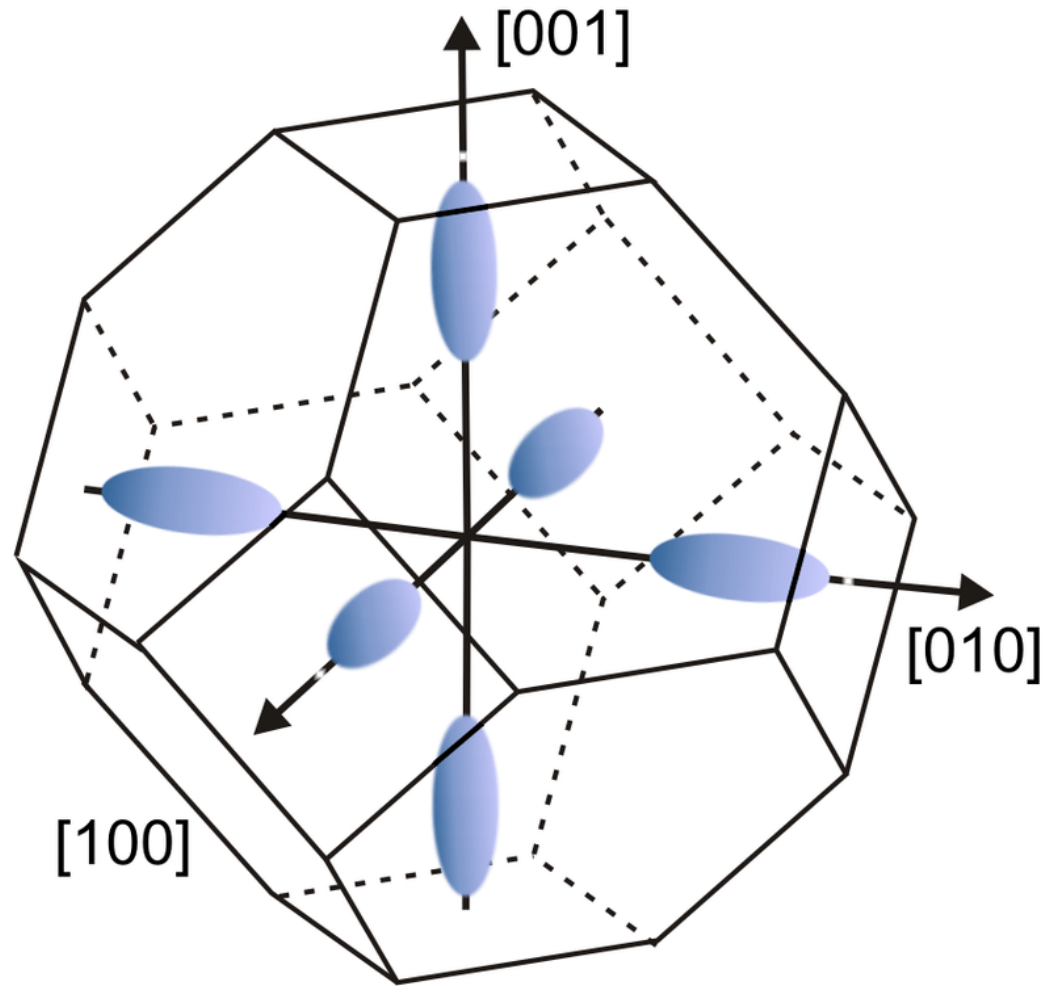
Silicon band structure



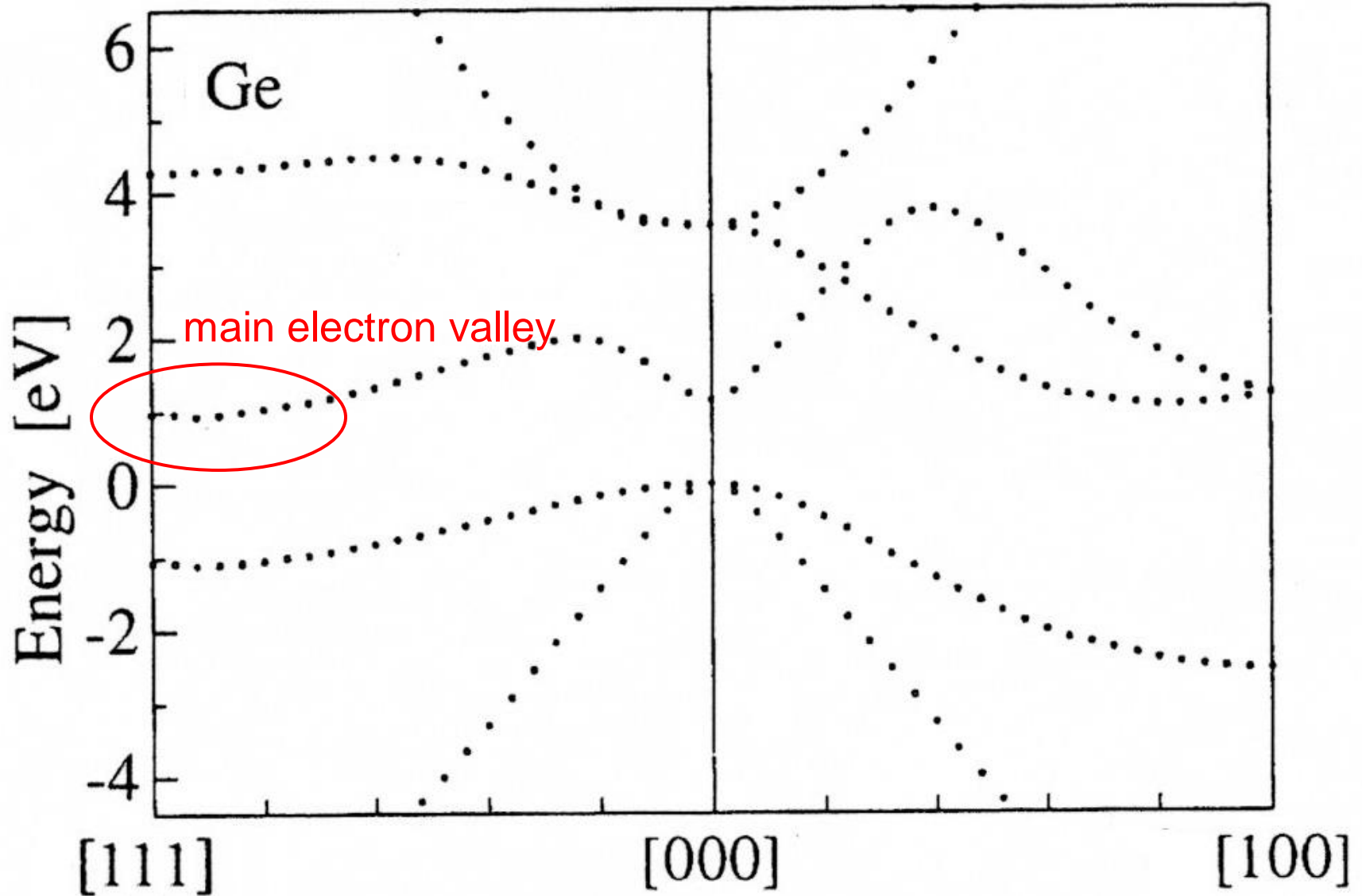
Valleys



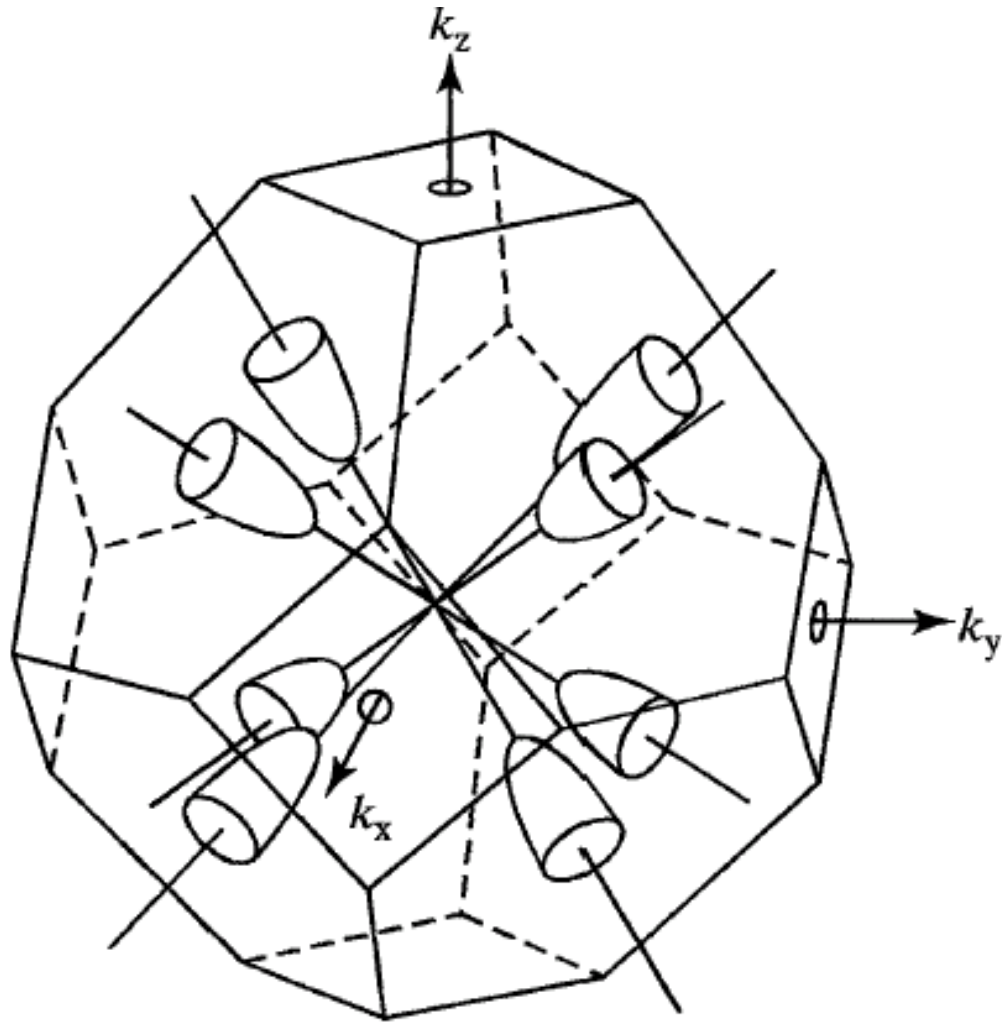
Silicon Equivalent Valleys



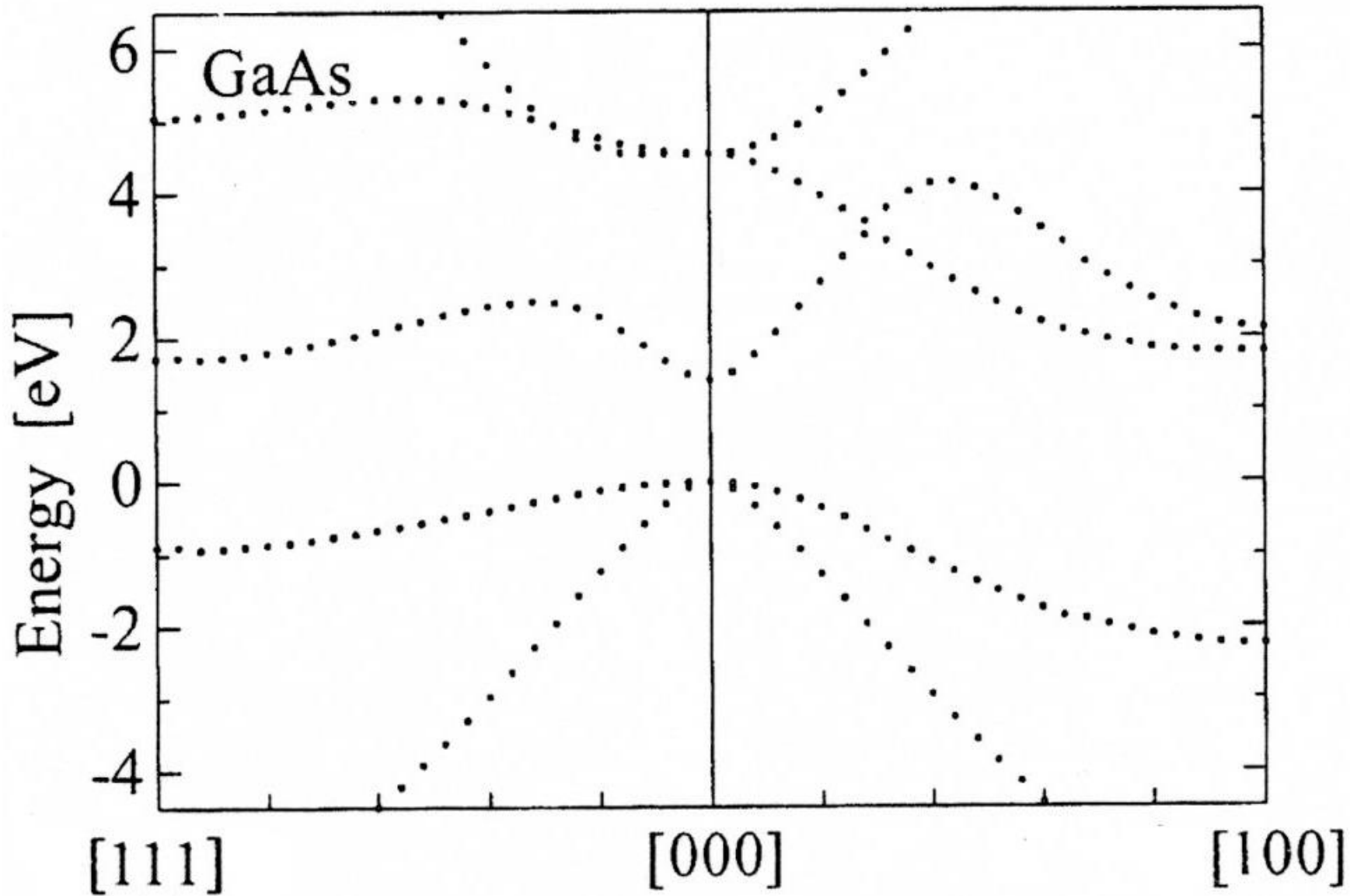
Germanium band structure



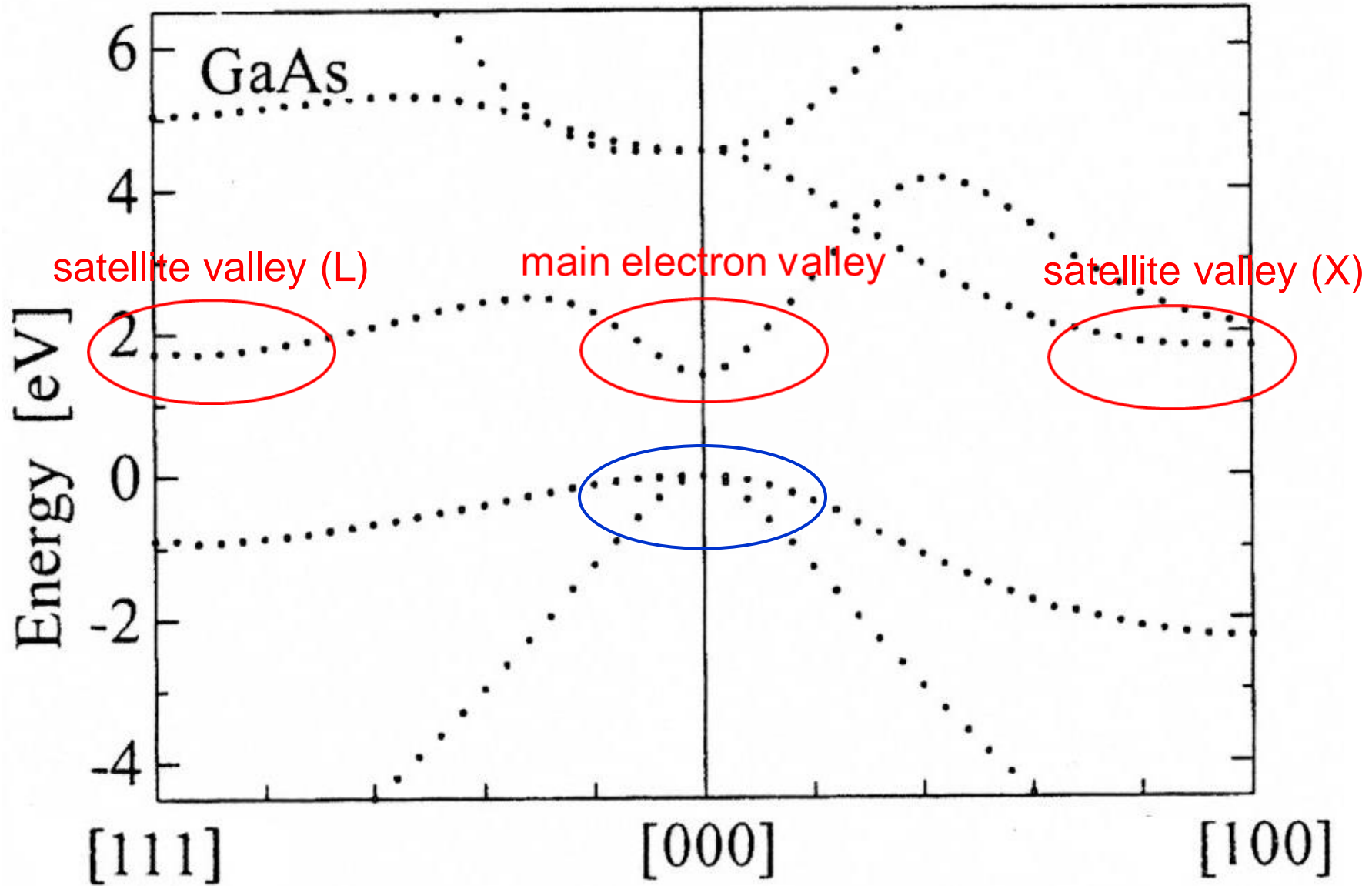
Germanium Equivalent Valleys



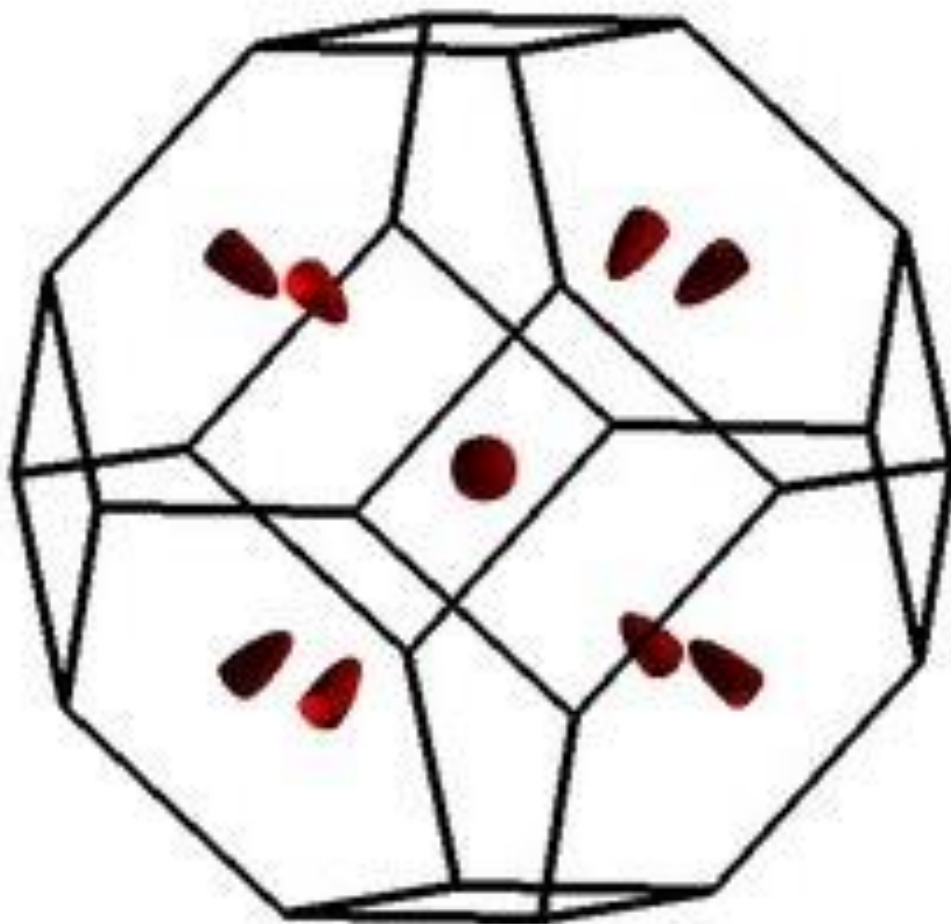
GaAs band structure



GaAs band structure



GaAs Equivalent Valleys (Γ & L)



Effective Mass is not “unique”

The effective mass depends on the band curvature, therefore, it depends on spatial direction for a valley which is not spherical.

For some applications to evaluate material properties, one may use a mass averaged over the valleys and the directions (density of states mass).

In charge transport calculations one may use instead the appropriate average mass for a given direction (conductivity mass)

Semiconductor Material Properties

| | | E_g (eV) | μ_n ($\text{cm}^2/\text{V}\cdot\text{s}$) | μ_p ($\text{cm}^2/\text{V}\cdot\text{s}$) | m_n^*/m_0 (m_l, m_t) | m_p^*/m_0 (m_{lh}, m_{hh}) | a (Å) | ϵ_r | Density (g/cm^3) | Melting point (°C) |
|------------------|----------|---------------|--|--|-------------------------------|-------------------------------------|---------|--------------|---------------------------------------|--------------------------|
| Si | (i/D) | 1.11 | 1350 | 480 | 0.98, 0.19 | 0.16, 0.49 | 5.43 | 11.8 | 2.33 | 1415 |
| Ge | (i/D) | 0.67 | 3900 | 1900 | 1.64, 0.082 | 0.04, 0.28 | 5.65 | 16 | 5.32 | 936 |
| SiC (α) | (i/W) | 2.86 | 500 | — | 0.6 | 1.0 | 3.08 | 10.2 | 3.21 | 2830 |
| AlP | (i/Z) | 2.45 | 80 | — | — | 0.2, 0.63 | 5.46 | 9.8 | 2.40 | 2000 |
| AlAs | (i/Z) | 2.16 | 1200 | 420 | 2.0 | 0.15, 0.76 | 5.66 | 10.9 | 3.60 | 1740 |
| AlSb | (i/Z) | 1.6 | 200 | 300 | 0.12 | 0.98 | 6.14 | 11 | 4.26 | 1080 |
| GaP | (i/Z) | 2.26 | 300 | 150 | 1.12, 0.22 | 0.14, 0.79 | 5.45 | 11.1 | 4.13 | 1467 |
| GaAs | (d/Z) | 1.43 | 8500 | 400 | 0.067 | 0.074, 0.50 | 5.65 | 13.2 | 5.31 | 1238 |
| GaN | (d/Z, W) | 3.4 | 380 | — | 0.19 | 0.60 | 4.5 | 12.2 | 6.1 | 2530 |
| GaSb | (d/Z) | 0.7 | 5000 | 1000 | 0.042 | 0.06, 0.23 | 6.09 | 15.7 | 5.61 | 712 |
| InP | (d/Z) | 1.35 | 4000 | 100 | 0.077 | 0.089, 0.85 | 5.87 | 12.4 | 4.79 | 1070 |
| InAs | (d/Z) | 0.36 | 22600 | 200 | 0.023 | 0.025, 0.41 | 6.06 | 14.6 | 5.67 | 943 |
| InSb | (d/Z) | 0.18 | 10^5 | 1700 | 0.014 | 0.015, 0.40 | 6.48 | 17.7 | 5.78 | 525 |
| ZnS | (d/Z, W) | 3.6 | 180 | 10 | 0.28 | — | 5.409 | 8.9 | 4.09 | 1650* |
| ZnSe | (d/Z) | 2.7 | 600 | 28 | 0.14 | 0.60 | 5.671 | 9.2 | 5.65 | 1100* |
| ZnTe | (d/Z) | 2.25 | 530 | 100 | 0.18 | 0.65 | 6.101 | 10.4 | 5.51 | 1238* |
| CdS | (d/W, Z) | 2.42 | 250 | 15 | 0.21 | 0.80 | 4.137 | 8.9 | 4.82 | 1475 |
| CdSe | (d/W) | 1.73 | 800 | — | 0.13 | 0.45 | 4.30 | 10.2 | 5.81 | 1258 |
| CdTe | (d/Z) | 1.58 | 1050 | 100 | 0.10 | 0.37 | 6.482 | 10.2 | 6.20 | 1098 |
| PbS | (i/H) | 0.37 | 575 | 200 | 0.22 | 0.29 | 5.936 | 17.0 | 7.6 | 1119 |
| PbSe | (i/H) | 0.27 | 1500 | 1500 | — | — | 6.147 | 23.6 | 8.73 | 1081 |
| PbTe | (i/H) | 0.29 | 6000 | 4000 | 0.17 | 0.20 | 6.452 | 30 | 8.16 | 925 |

C (diamond)
 $E_g = 5.5\text{eV}$

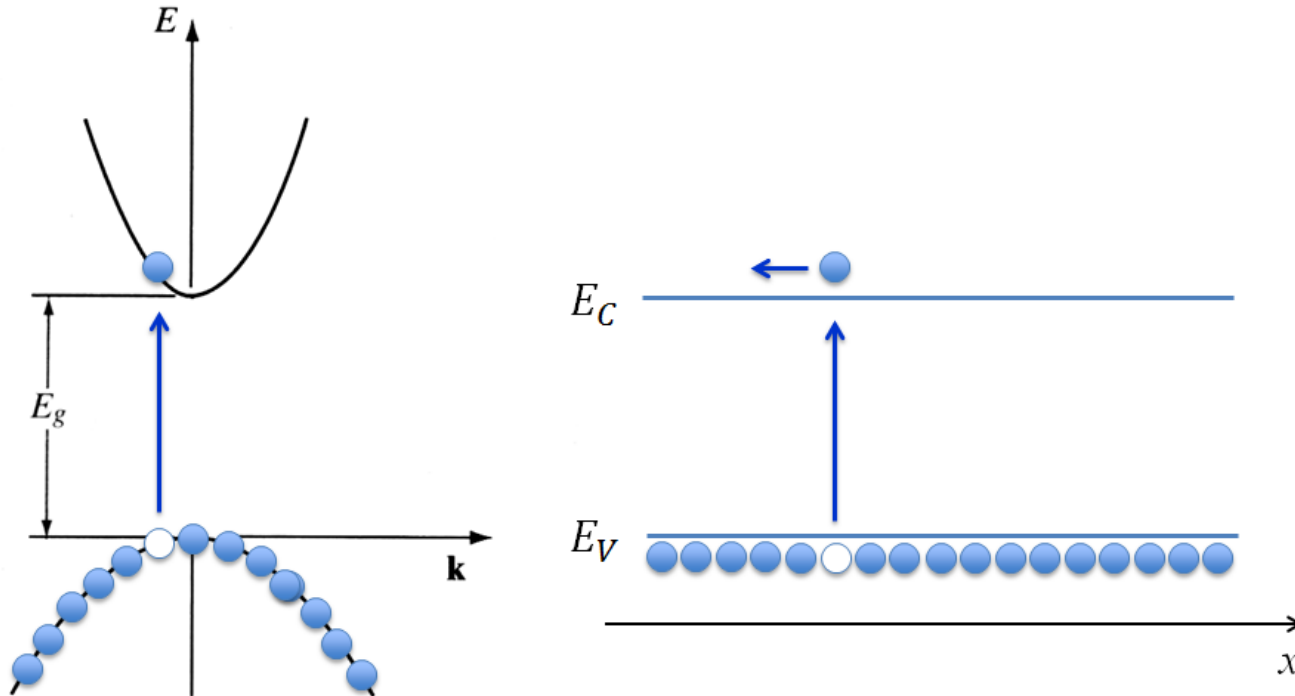
SiO_2
 $E_g = 9.0\text{eV}$

All values at 300 K.

*Vaporizes

Intrinsic Semiconductor

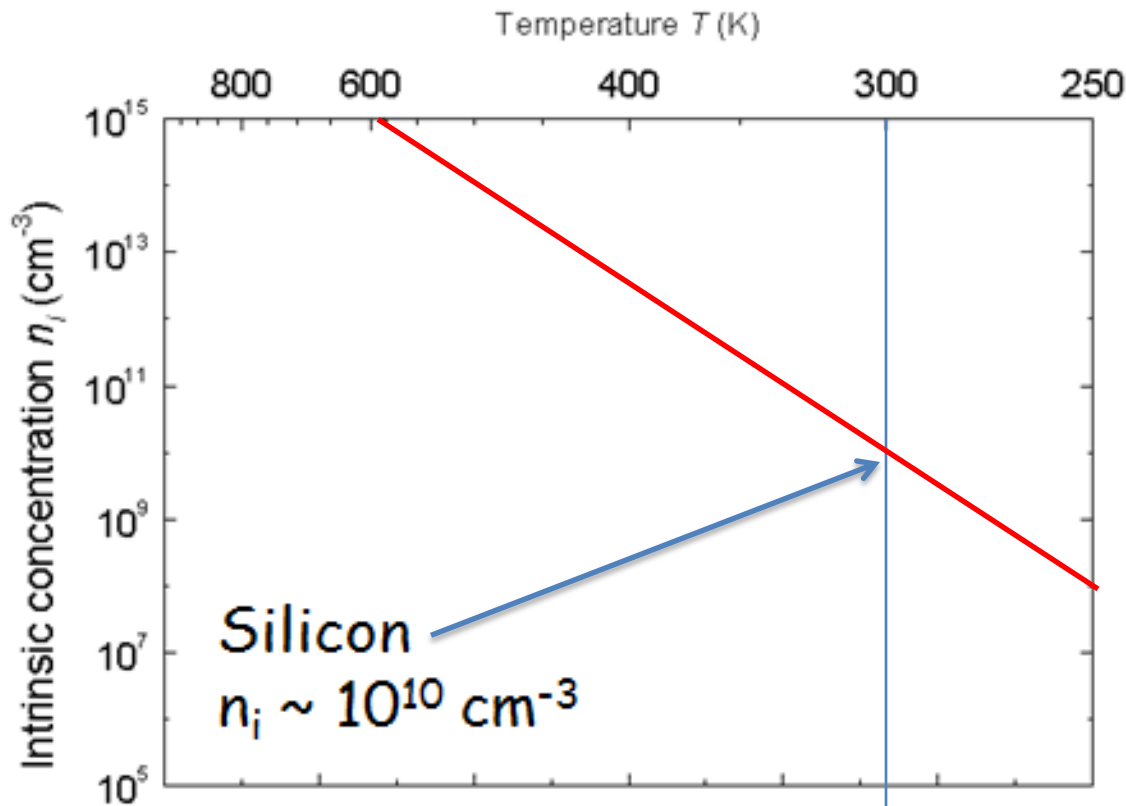
- A pure semiconductor is called “intrinsic”



- There are always as many electrons in the conduction band as holes in the valence band

Intrinsic Semiconductor

- The intrinsic concentration depends on temperature



Electron-hole generation rate G_0 always balances the recombination rate R_0

$$G_0 = R_0$$

$$n_0 p_0 = n_i^2 \quad n_0 = p_0 = n_i$$

Textbook uses

$$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

Extrinsic Semiconductor

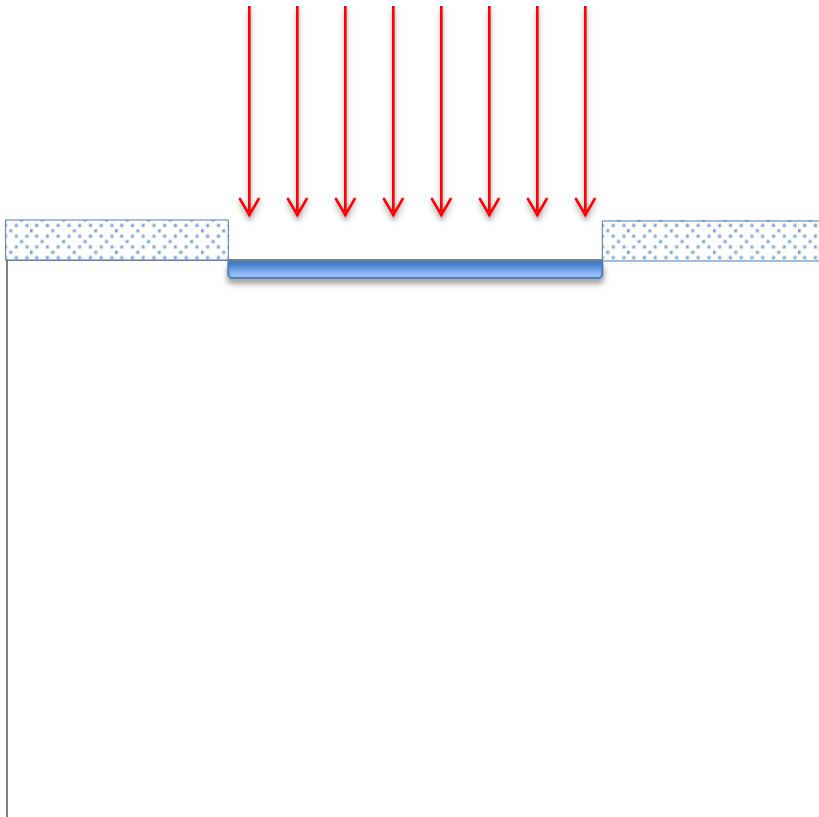
- **Impurities are introduced to force the material to have more electrons than holes or *vice versa***
 - An atom with more than 4 valence electrons can donate electrons to the conduction band while becoming a fixed positively charged ion. This is called a **donor**.
 - An atom with less than 4 valence electrons can attract electrons from the valence band, creating holes and becoming a fixed negatively charged ion. This is called an **acceptor**.

Extrinsic Semiconductor

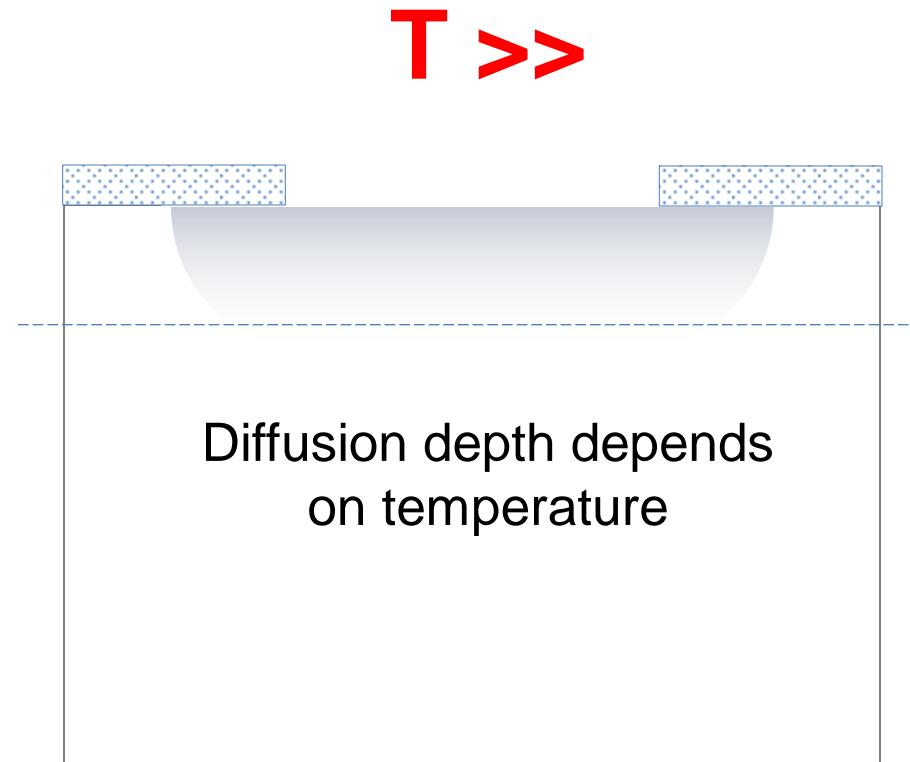
- The process of introducing impurities in the semiconductor for the purpose of changing material properties is called **“doping”**.
- The impurities introduced with this process are called **“dopants”**.
- **Dopants are more commonly added:**
 - During growth of the crystal
 - By diffusion at high temperature
 - By ion implantation

Diffusion (pre-deposition + drive-in)

Pre-deposition

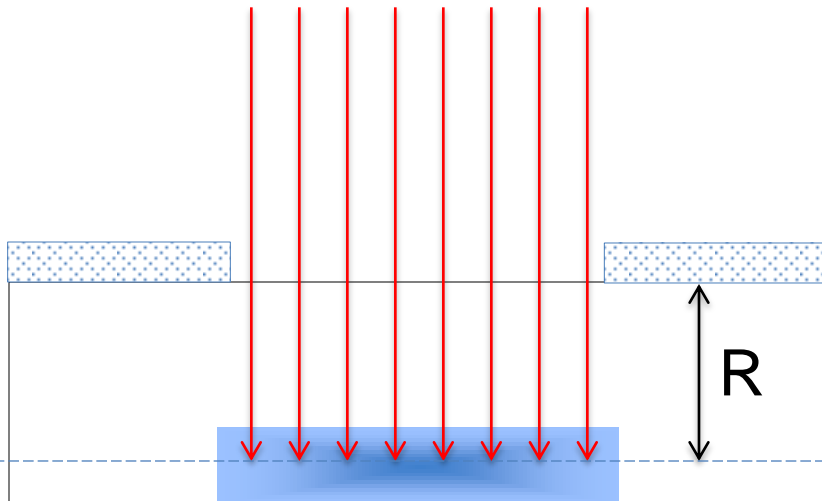


Drive-in



Ion implantation

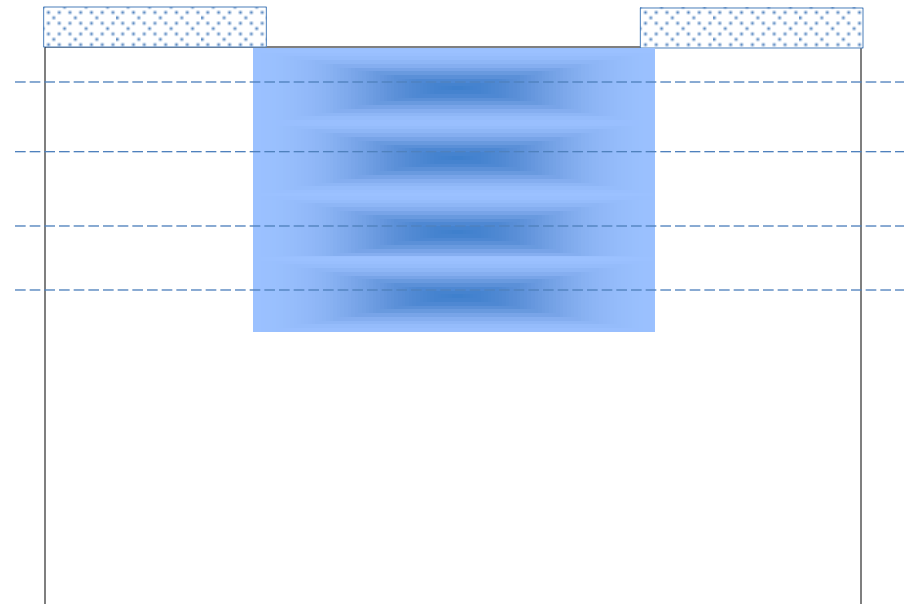
ion beam with energy E_B



Range R depends on E_B

Doping profile obtained by implantation at different energies

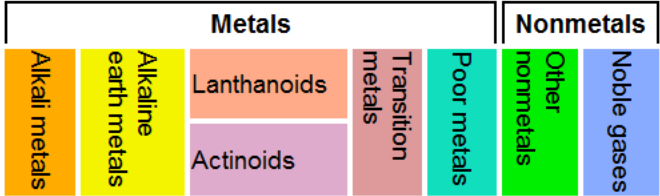
Diffusion process can follow to smooth out profile



Common Dopants for Silicon

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | III | IV | V | VI | 17 | 18 |
|---|---|--|--|--|---|--|--|---|---|--|--|---|---|---|---|---|--|
| 1 H Hydrogen 1.00794 | 2 He Helium 4.002602 | | | | | | | | | | | | | | | | |
| 3 Li Lithium 6.941 | 4 Be Beryllium 9.012182 | | | | | | | | | | | | | | | | |
| 11 Na Sodium 22.98976928 | 12 Mg Magnesium 24.3050 | | | | | | | | | | | | | | | | |
| 19 K Potassium 39.0983 | 20 Ca Calcium 40.078 | 21 Sc Scandium 44.955912 | 22 Ti Titanium 47.887 | 23 V Vanadium 50.9415 | 24 Cr Chromium 51.9961 | 25 Mn Manganese 54.938045 | 26 Fe Iron 55.845 | 27 Co Cobalt 58.933195 | 28 Ni Nickel 58.6934 | 29 Cu Copper 63.546 | 30 Zn Zinc 65.38 | 31 Ga Gallium 69.723 | 32 Ge Germanium 72.64 | 33 As Arsenic 74.92160 | 34 Se Selenium 78.96 | 35 Br Bromine 79.904 | 36 Kr Krypton 83.798 |
| 37 Rb Rubidium 85.4678 | 38 Sr Strontium 87.62 | 39 Y Yttrium 88.90585 | 40 Zr Zirconium 91.224 | 41 Nb Niobium 92.90638 | 42 Mo Molybdenum 95.96 | 43 Tc Technetium (97.9072) | 44 Ru Ruthenium 101.07 | 45 Rh Rhodium 102.90550 | 46 Pd Palladium 106.42 | 47 Ag Silver 107.8682 | 48 Cd Cadmium 112.411 | 49 In Indium 114.818 | 50 Sn Tin 118.710 | 51 Sb Antimony 121.760 | 52 Te Tellurium 127.60 | 53 I Iodine 126.90447 | 54 Xe Xenon 131.293 |
| 55 Cs Caesium 132.9054519 | 56 Ba Barium 137.327 | 57-71 | 72 Hf Hafnium 178.49 | 73 Ta Tantalum 180.94788 | 74 W Tungsten 183.84 | 75 Re Rhenium 186.207 | 76 Os Osmium 190.23 | 77 Ir Iridium 192.217 | 78 Pt Platinum 195.084 | 79 Au Gold 196.966569 | 80 Hg Mercury 200.59 | 81 Tl Thallium 204.3833 | 82 Pb Lead 207.2 | 83 Bi Bismuth 208.98040 | 84 Po Polonium (208.9824) | 85 At Astatine (209.9871) | 86 Rn Radon (222.0176) |
| 87 Fr Francium (223) | 88 Ra Radium (226) | 89-103 | 104 Rf Rutherfordium (261) | 105 Db Dubnium (262) | 106 Sg Seaborgium (266) | 107 Bh Bohrium (264) | 108 Hs Hassium (277) | 109 Mt Meitnerium (268) | 110 Ds Darmstadtium (271) | 111 Rg Roentgenium (272) | 112 Uub Ununbium (285) | 113 Uut Ununtrium (284) | 114 Uuq Ununquadium (289) | 115 Uup Ununpentium (288) | 116 Uuh Ununhexium (292) | 117 Uus Ununseptium | 118 Uuo Ununoctium (294) |

- C** Solid
- Hg** Liquid
- H** Gas
- Rf** Unknown



III Acceptors
IV
V Donors
VI



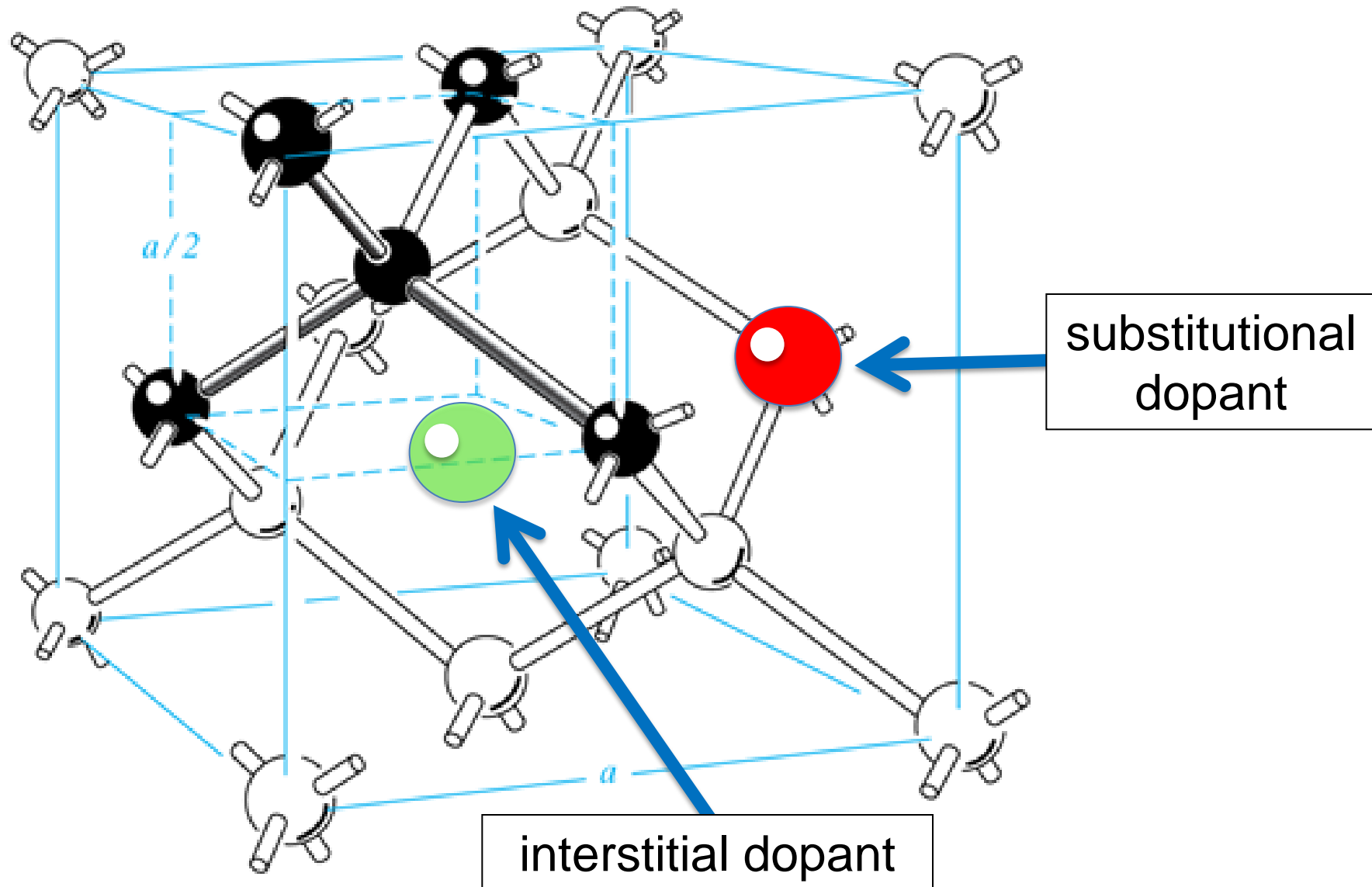
For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

Design and Interface Copyright © 1997 Michael Dayah (michael@dayah.com). <http://www.ptable.com/>

| | | | | | | | | | | | | | | |
|---|---|--|---|--|---------------------------------------|--|---|---|--|---|--------------------------------------|--|---|---|
| 57 La Lanthanum 138.90547 | 58 Ce Cerium 140.116 | 59 Pr Praseodymium 140.90765 | 60 Nd Neodymium 144.242 | 61 Pm Promethium (145) | 62 Sm Samarium 150.36 | 63 Eu Europium 151.964 | 64 Gd Gadolinium 157.25 | 65 Tb Terbium 158.92535 | 66 Dy Dysprosium 162.500 | 67 Ho Holmium 164.93032 | 68 Er Erbium 167.259 | 69 Tm Thulium 168.93421 | 70 Yb Ytterbium 173.054 | 71 Lu Lutetium 174.9668 |
| 89 Ac Actinium (227) | 90 Th Thorium 232.03806 | 91 Pa Protactinium 231.03588 | 92 U Uranium 238.02891 | 93 Np Neptunium (237) | 94 Pu Plutonium (244) | 95 Am Americium (243) | 96 Cm Curium (247) | 97 Bk Berkelium (247) | 98 Cf Californium (251) | 99 Es Einsteinium (252) | 100 Fm Fermium (257) | 101 Md Mendelevium (258) | 102 No Nobelium (259) | 103 Lr Lawrencium (262) |

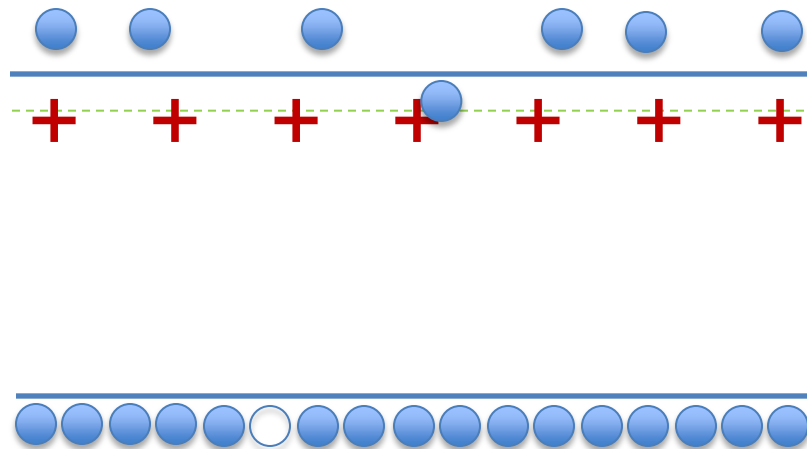


Doping atoms

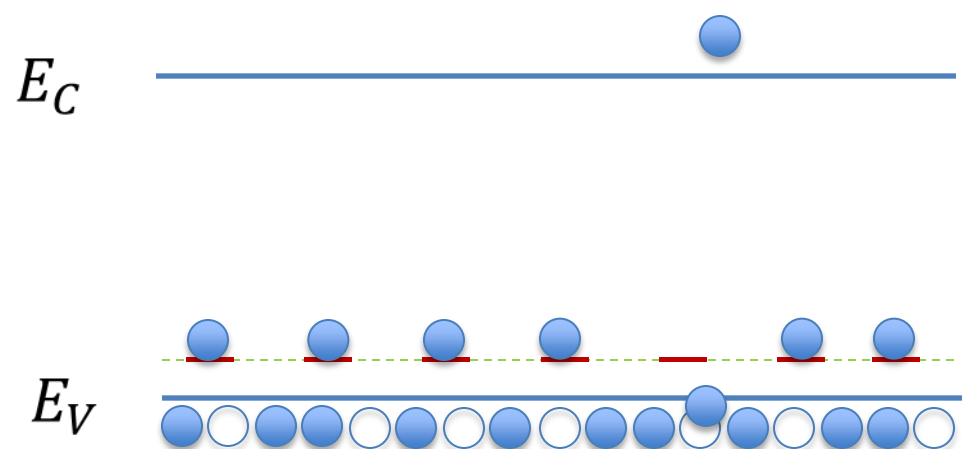


Donors and Acceptors

Donors should create energy levels close to the conduction band
Acceptors should create energy levels close to the valence band



n-type



p-type