

# **ECE 340 Lectures 6-7**

# **Semiconductor Electronics**

Spring 2022

10:00-10:50am

Professor Umberto Ravaioli

Department of Electrical and Computer Engineering

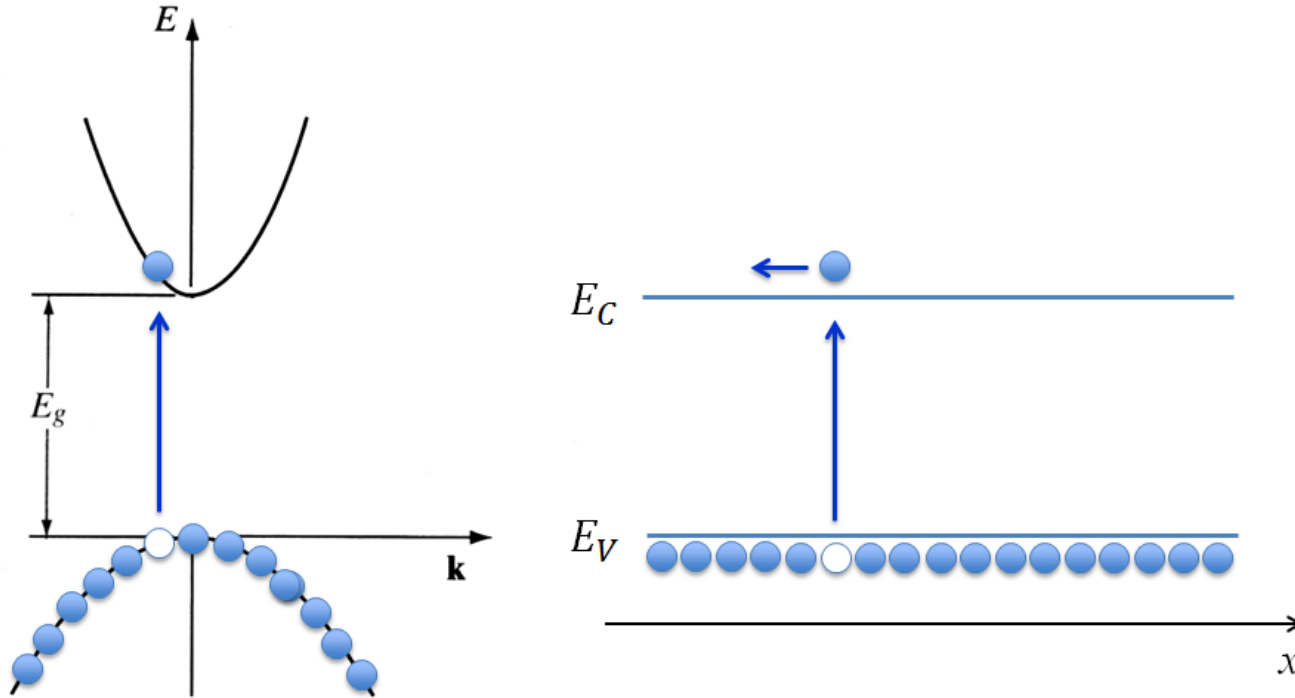
2062 ECE Building

# Today's Discussion

- Intrinsic and Extrinsic Semiconductors
- Statistics of electron distributions
- Fermi-Dirac statistics
- Maxwell-Boltzmann statistics
- The Fermi level
- Electron & hole concentrations in equilibrium

# Intrinsic Semiconductor

- A **pure** semiconductor is called “intrinsic”



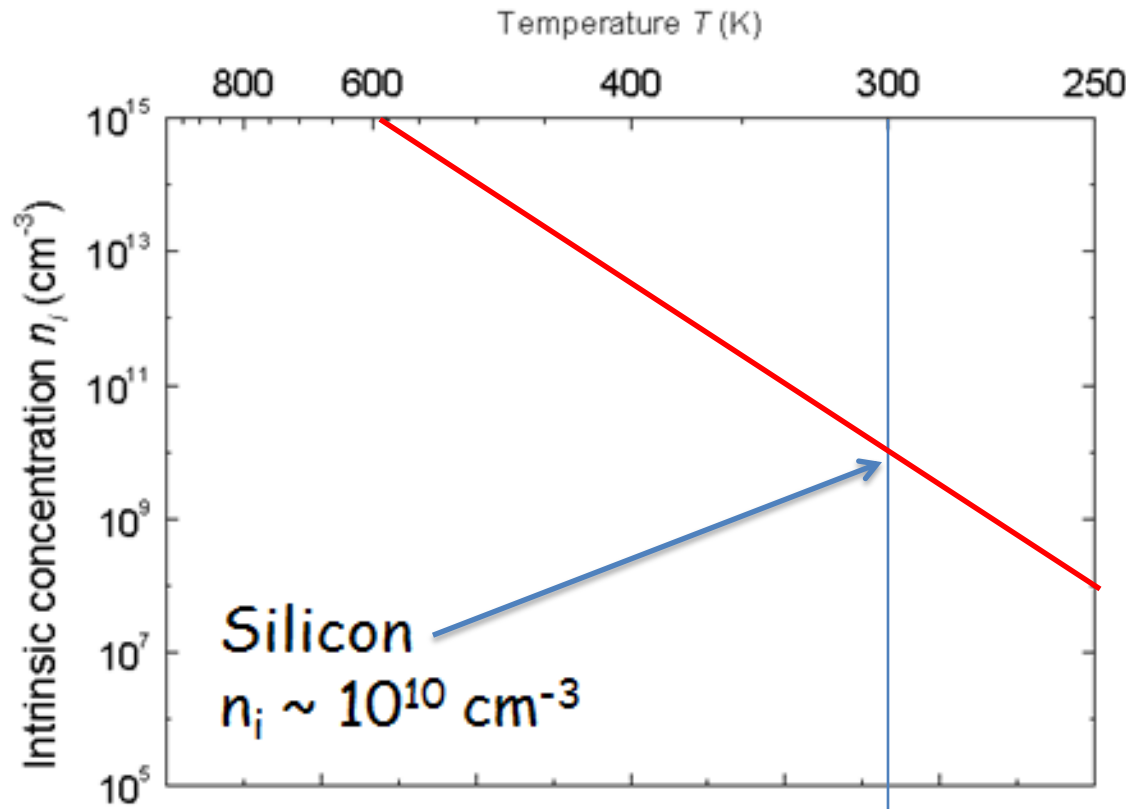
- With “**bulk semiconductor**” we refer from now on to a large piece of uniform material.

# Intrinsic Semiconductor

- The concentration of electrons and holes in intrinsic bulk semiconductor is the balance of continuous **generation** and **recombination** of electron-hole pairs.
- There are always as many electrons in the conduction band as holes in the valence band

# Intrinsic Semiconductor

- The **intrinsic concentration** depends on temperature



Electron-hole generation rate  $G_0$  always balances the recombination rate  $R_0$

$$G_0 = R_0$$

Textbook uses  
 $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$

$$n_0 p_0 = n_i^2 \quad n_0 = p_0 = n_i$$

# Extrinsic Semiconductor

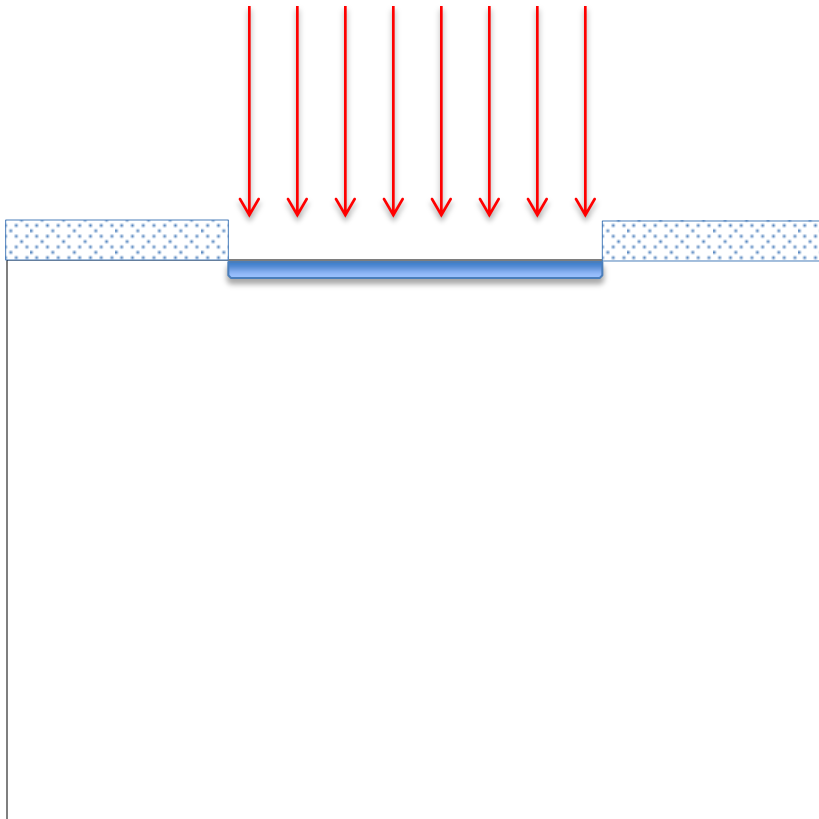
- **Impurities are introduced to force the material to have more electrons than holes or *vice versa***
  - An atom with more than 4 valence electrons can donate electrons to the conduction band while becoming a fixed positively charged ion. This is called a **donor**.
  - An atom with less than 4 valence electrons can attract electrons from the valence band, creating holes and becoming a fixed negatively charged ion. This is called an **acceptor**.

# Extrinsic Semiconductor

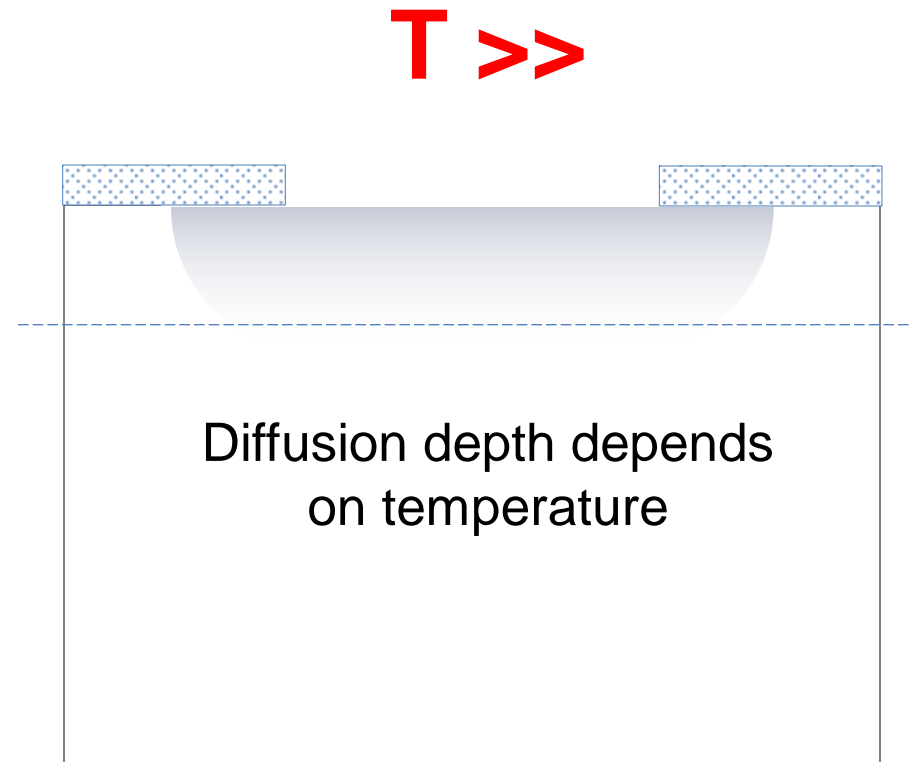
- The process of introducing impurities in the semiconductor for the purpose of changing material properties is called **“doping”**.
- The impurities introduced with this process are called **“dopants”**.
- **Dopants are more commonly added:**
  - During growth of the crystal
  - By diffusion at high temperature
  - By ion implantation

# Diffusion (pre-deposition + drive-in)

Pre-deposition



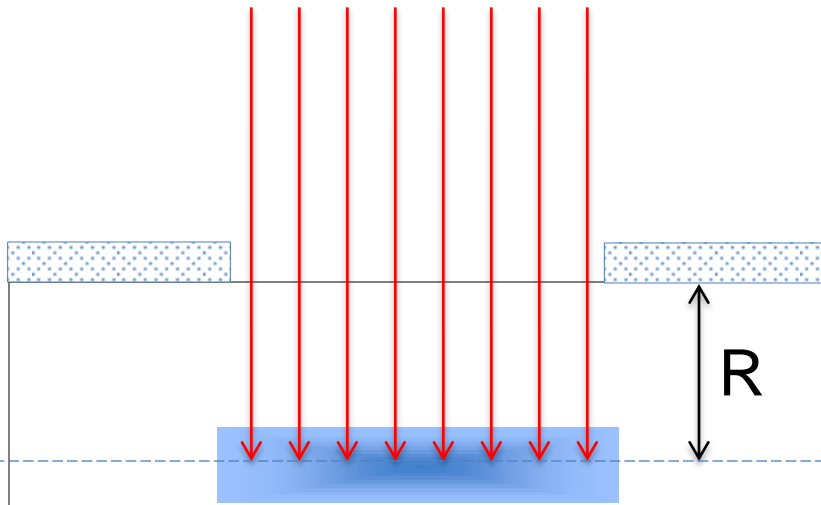
Drive-in





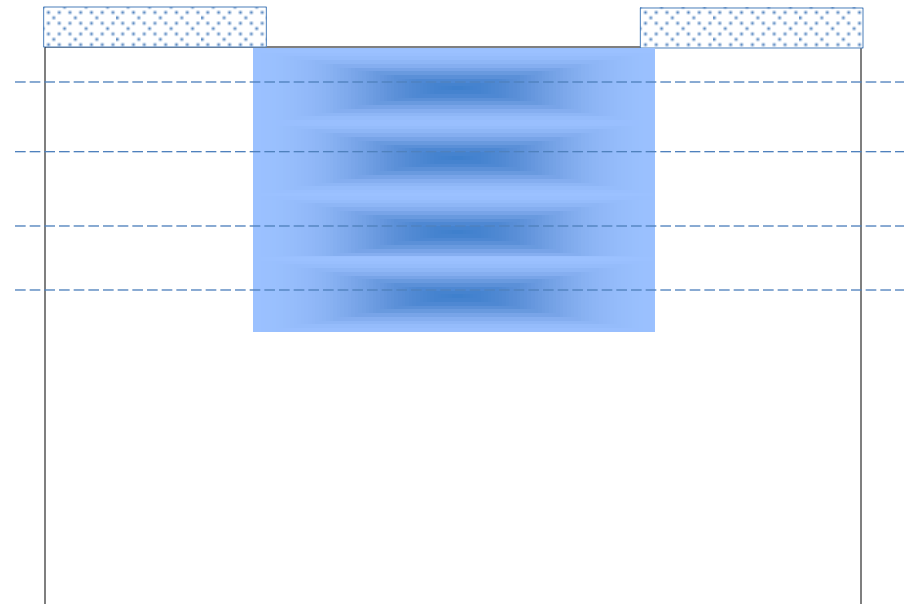
# Ion implantation

ion beam with energy  $E_B$



Range  $R$  depends on  $E_B$

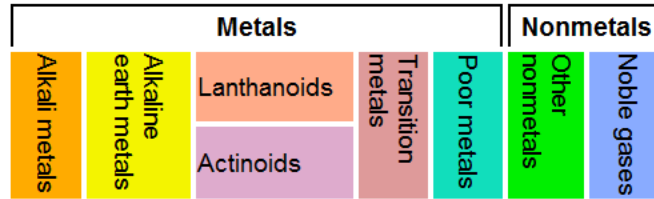
Doping profiles are obtained by implantations of many layers at different energies. A diffusion process follows implantation to smooth out the dopant profile.



# Common Dopants for Silicon

1	2	3	4	5	6	7	8	9	10	11	12	III	IV	V	VI	17	18				
1 <b>H</b> Hydrogen 1.00794	2 <b>He</b> Helium 4.002602															17 <b>Cl</b> Chlorine 35.453	18 <b>Ar</b> Argon 39.948				
3 <b>Li</b> Lithium 6.941	4 <b>Be</b> Beryllium 9.012182															7 <b>N</b> Nitrogen 14.0067	8 <b>O</b> Oxygen 15.9994	9 <b>F</b> Fluorine 18.9984032	10 <b>Ne</b> Neon 20.1797		
11 <b>Na</b> Sodium 22.98976928	12 <b>Mg</b> Magnesium 24.3050															13 <b>Al</b> Aluminium 26.9815386	14 <b>Si</b> Silicon 28.0855	15 <b>P</b> Phosphorus 30.973762	16 <b>S</b> Sulfur 32.065	17 <b>Cl</b> Chlorine 35.453	18 <b>Ar</b> Argon 39.948
19 <b>K</b> Potassium 39.0983	20 <b>Ca</b> Calcium 40.078	21 <b>Sc</b> Scandium 44.955912	22 <b>Ti</b> Titanium 47.887	23 <b>V</b> Vanadium 50.9415	24 <b>Cr</b> Chromium 51.9961	25 <b>Mn</b> Manganese 54.938045	26 <b>Fe</b> Iron 55.845	27 <b>Co</b> Cobalt 58.933195	28 <b>Ni</b> Nickel 58.6934	29 <b>Cu</b> Copper 63.546	30 <b>Zn</b> Zinc 65.38	31 <b>Ga</b> Gallium 69.723	32 <b>Ge</b> Germanium 72.64	33 <b>As</b> Arsenic 74.92160	34 <b>Se</b> Selenium 78.96	35 <b>Br</b> Bromine 79.904	36 <b>Kr</b> Krypton 83.798				
37 <b>Rb</b> Rubidium 85.4678	38 <b>Sr</b> Strontium 87.62	39 <b>Y</b> Yttrium 88.90585	40 <b>Zr</b> Zirconium 91.224	41 <b>Nb</b> Niobium 92.90638	42 <b>Mo</b> Molybdenum 95.96	43 <b>Tc</b> Technetium (97.9072)	44 <b>Ru</b> Ruthenium 101.07	45 <b>Rh</b> Rhodium 102.90550	46 <b>Pd</b> Palladium 106.42	47 <b>Ag</b> Silver 107.8682	48 <b>Cd</b> Cadmium 112.411	49 <b>In</b> Indium 114.818	50 <b>Sn</b> Tin 118.710	51 <b>Sb</b> Antimony 121.760	52 <b>Te</b> Tellurium 127.60	53 <b>I</b> Iodine 126.90447	54 <b>Xe</b> Xenon 131.293				
55 <b>Cs</b> Caesium 132.9054519	56 <b>Ba</b> Barium 137.327	57-71 Lanthanoids	72 <b>Hf</b> Hafnium 178.49	73 <b>Ta</b> Tantalum 180.94788	74 <b>W</b> Tungsten 183.84	75 <b>Re</b> Rhenium 186.207	76 <b>Os</b> Osmium 190.23	77 <b>Ir</b> Iridium 192.217	78 <b>Pt</b> Platinum 195.084	79 <b>Au</b> Gold 196.966569	80 <b>Hg</b> Mercury 200.59	81 <b>Tl</b> Thallium 204.3833	82 <b>Pb</b> Lead 207.2	83 <b>Bi</b> Bismuth 208.98040	84 <b>Po</b> Polonium (209)	85 <b>At</b> Astatine (210)	86 <b>Rn</b> Radon (222)				
87 <b>Fr</b> Francium (223)	88 <b>Ra</b> Radium (226)	89-103 Actinoids	104 <b>Rf</b> Rutherfordium (261)	105 <b>Db</b> Dubnium (262)	106 <b>Sg</b> Seaborgium (266)	107 <b>Bh</b> Bohrium (264)	108 <b>Hs</b> Hassium (277)	109 <b>Mt</b> Meitnerium (268)	110 <b>Ds</b> Darmstadtium (271)	111 <b>Rg</b> Roentgenium (272)	112 <b>Uub</b> Ununbium (285)	113 <b>Uut</b> Ununtrium (284)	114 <b>Uuq</b> Ununquadium (289)	115 <b>Uup</b> Ununpentium (288)	116 <b>Uuh</b> Ununhexium (292)	117 <b>Uus</b> Ununseptium	118 <b>Uuo</b> Ununoctium (294)				

- C** Solid
- Hg** Liquid
- H** Gas
- Rf** Unknown



III Acceptors  
V Donors



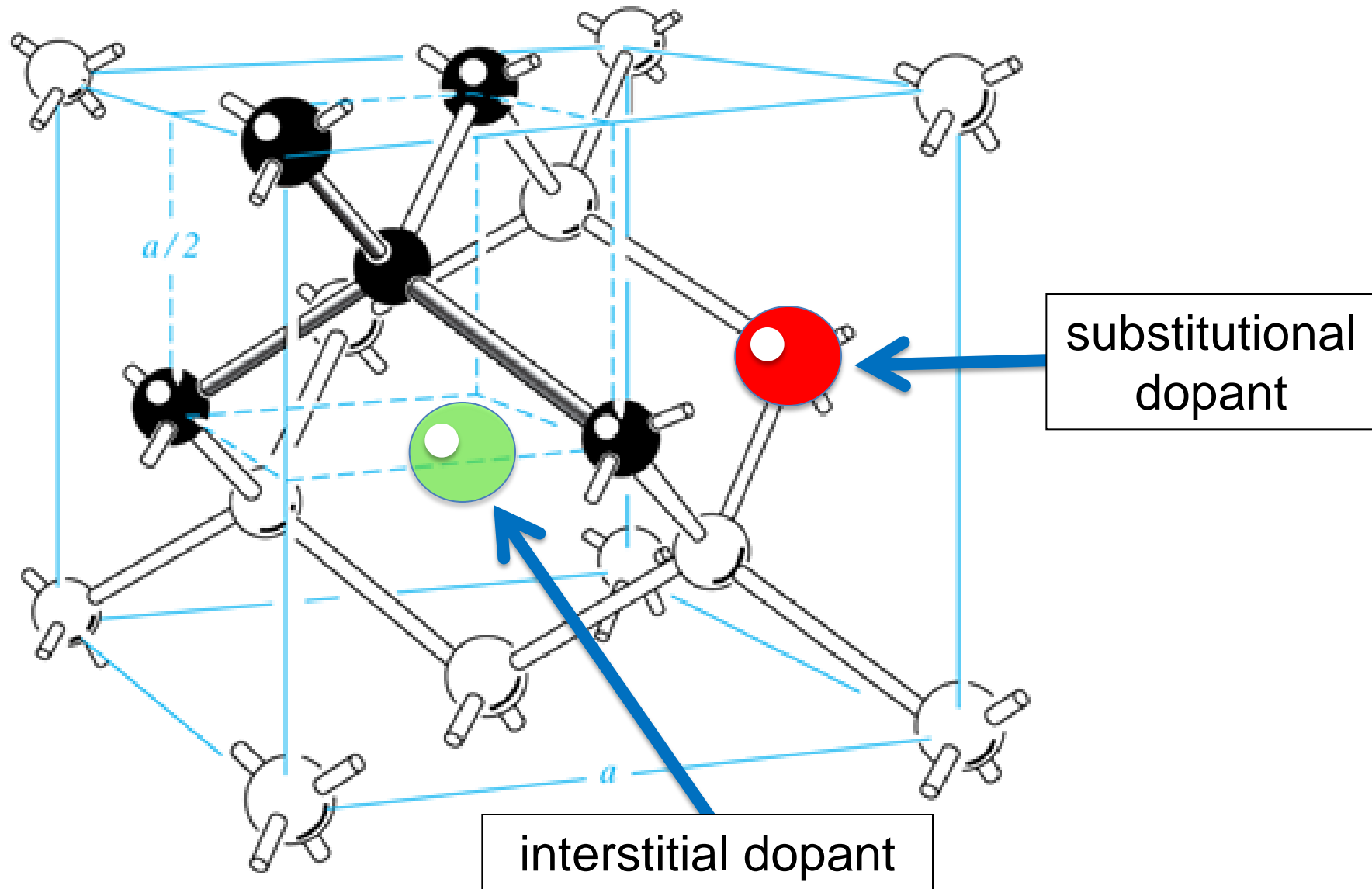
For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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57 <b>La</b> Lanthanum 138.90547	58 <b>Ce</b> Cerium 140.116	59 <b>Pr</b> Praseodymium 140.90765	60 <b>Nd</b> Neodymium 144.242	61 <b>Pm</b> Promethium (145)	62 <b>Sm</b> Samarium 150.36	63 <b>Eu</b> Europium 151.964	64 <b>Gd</b> Gadolinium 157.25	65 <b>Tb</b> Terbium 158.92535	66 <b>Dy</b> Dysprosium 162.500	67 <b>Ho</b> Holmium 164.93032	68 <b>Er</b> Erbium 167.259	69 <b>Tm</b> Thulium 168.93421	70 <b>Yb</b> Ytterbium 173.054	71 <b>Lu</b> Lutetium 174.9668
89 <b>Ac</b> Actinium (227)	90 <b>Th</b> Thorium 232.03806	91 <b>Pa</b> Protactinium 231.03588	92 <b>U</b> Uranium 238.02891	93 <b>Np</b> Neptunium (237)	94 <b>Pu</b> Plutonium (244)	95 <b>Am</b> Americium (243)	96 <b>Cm</b> Curium (247)	97 <b>Bk</b> Berkelium (247)	98 <b>Cf</b> Californium (251)	99 <b>Es</b> Einsteinium (252)	100 <b>Fm</b> Fermium (257)	101 <b>Md</b> Mendelevium (258)	102 <b>No</b> Nobelium (259)	103 <b>Lr</b> Lawrencium (262)

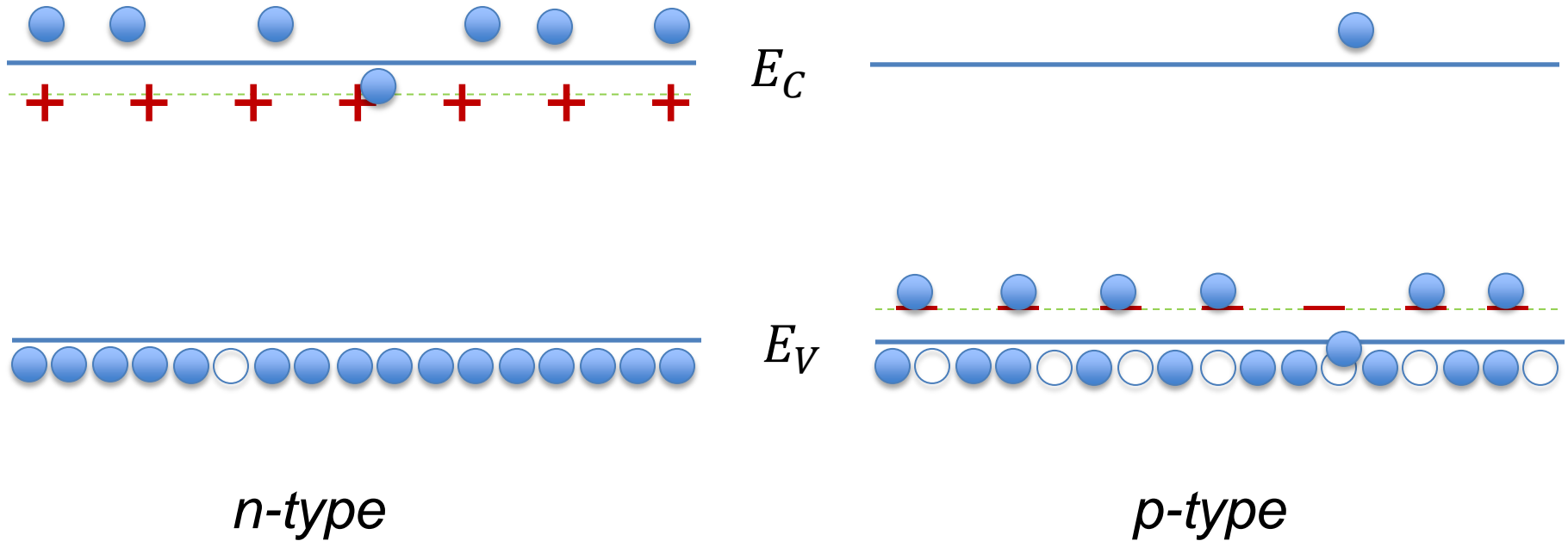


# Doping atoms



# Donors and Acceptors

Donors create energy levels close to the conduction band  
Acceptors create energy levels close to the valence band



Practical doping concentrations in Si are in the range  
 $10^{14}$  to  $10^{20}$   $\text{cm}^{-3}$

# Evaluate Approximate Dopant Binding Energy

Simple Bohr's atom model for column V (e.g. As)

Ground state energy for outermost electron

Hydrogen in vacuum  $E = 13.6 \text{ eV}$

$$E = \frac{m_0 q^4}{2(4\pi\epsilon_0)^2 \hbar^2}$$

(vacuum)

Adapt for Si crystal environment

$$E = \frac{m_n^* q^4}{2(4\pi\epsilon_r \epsilon_0)^2 \hbar^2}$$

$m_n^*$  (Si) =  $0.26 m_0$  = conductivity mass

$\epsilon_r$  (Si) = 11.8

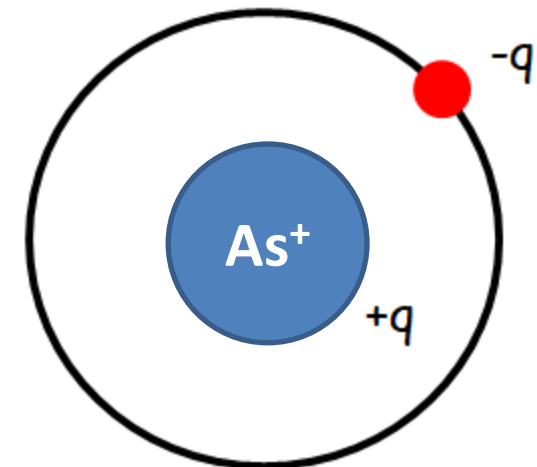
Assume the 4 covalent bonding electrons are tightly bound and extra electron has hydrogen-like orbit

$E \approx 0.0254 \text{ eV}$

Actual measured values

Donor in Si	P	As	Sb
Binding energy (eV)	0.045	0.054	0.039

Acceptor in Si	B	Al	Ga	In
Binding energy (eV)	0.045	0.067	0.072	0.16



# Radius of Binding Electron Orbit

Simple Bohr's atom model for column V (e.g. As)

Radius of orbit



$$r = \frac{4\pi\epsilon_0\hbar^2}{m_0q^2}$$

(vacuum)

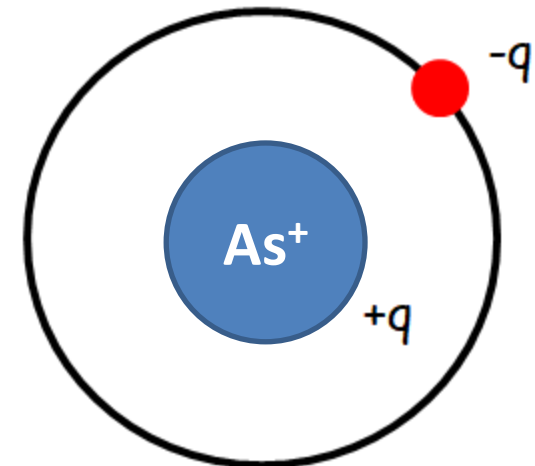
Adapt for Si crystal environment

$$r = \frac{4\pi\epsilon_r\epsilon_0\hbar^2}{m_n^*q^2} = \frac{4 \times 3.14 \times 11.8 \times (8.85 \times 10^{-12}) \times (1.054 \times 10^{-34})^2}{0.26 \times (9.11 \times 10^{-31}) \times (1.602 \times 10^{-19})^2} =$$

$$r = 2.41 \times 10^{-9} \text{ m} = 2.41 \text{ nm}$$

Use **conductivity mass**. For ellipsoidal band in Si:

$$m_n^* = \left[ \frac{1}{3} \left( \frac{1}{m_l} + \frac{2}{m_t} \right) \right]^{-1} = \left[ \frac{1}{3} \left( \frac{1}{0.98 m_0} + \frac{2}{0.19 m_0} \right) \right]^{-1} = 0.26 m_0$$



# Conductivity mass & Density of States mass

For density of states calculation, use the **density of states mass**

$$m_n^* = [m_1 m_2 m_3]^{1/3} \underbrace{M_C^{2/3}}_{\substack{\text{\#equivalent} \\ \text{valleys}}} = \underbrace{[m_l m_t^2]^{1/3}}_{Si} 6^{2/3}$$

$$m_p^* = [m_{lh}^{3/2} + m_{hh}^{3/2}]^{2/3}$$

For transport calculation (e.g., current), use the **conductivity mass**

$$m_n^* = \left[ \frac{1}{3} \left( \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} \right) \right]^{-1} = \underbrace{\left[ \frac{1}{3} \left( \frac{1}{m_t} + \frac{2}{m_t} \right) \right]^{-1}}_{Si}$$

# Conductivity mass for holes is complicated!

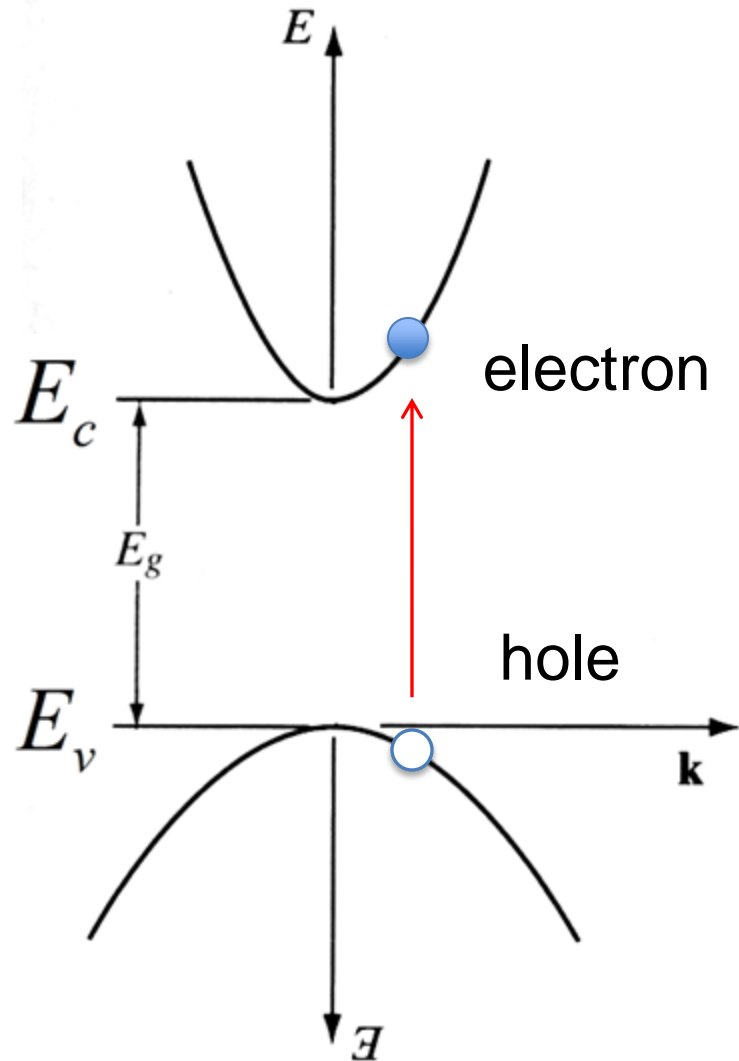
Holes have *light hole* and *heavy hole* bands which overlap, therefore, there are two mass components proportional to the band occupation (correlated to density of states, which in turn depends on  $(m_h^*)^{3/2}$ )

$$m_h^* \approx \left( \frac{\% \text{ occup. heavy band}}{m_{hh}^*} + \frac{\% \text{ occup. light band}}{m_{lh}^*} \right)^{-1}$$

In silicon, further complication is due to strong anisotropy of heavy hole band (“warping” of band) which requires special corrections.



# So far we have an energy model



$$E_c(k) = \frac{\hbar^2 k^2}{2m_n^* m_o}$$

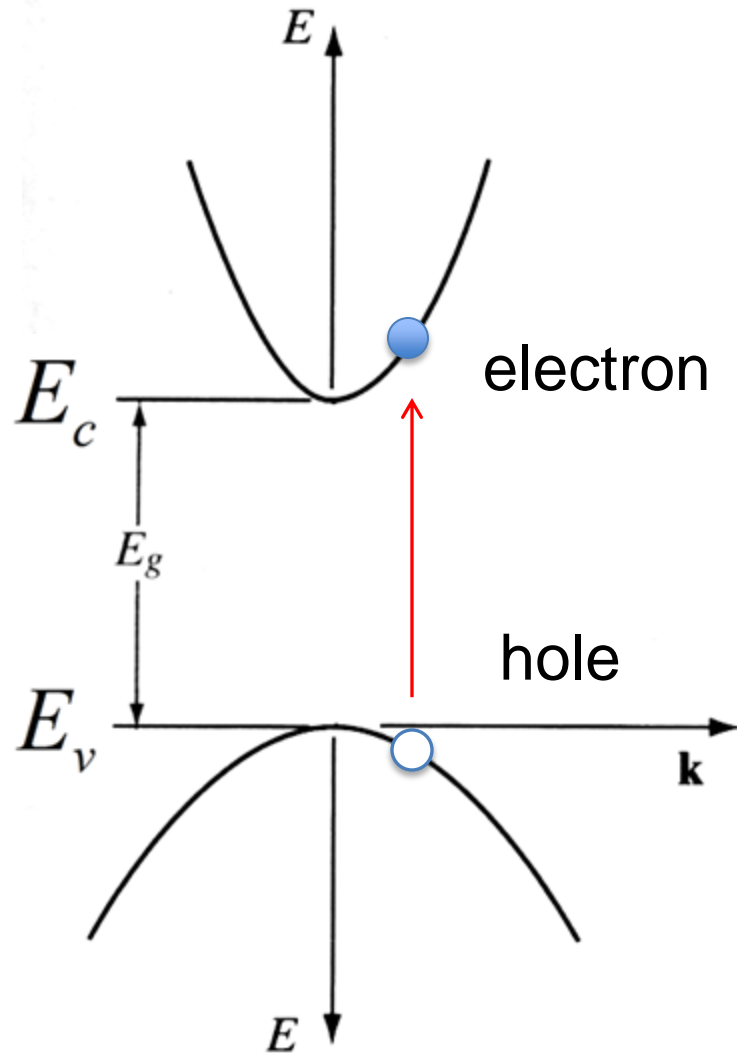
Parabolic valley

$$E_v(k) = E_v - \frac{\hbar^2 k^2}{2m_p^* m_o}$$

Parabolic valley

# We have an energy model, so far

But we still need more information



$$E_c(k) = \frac{\hbar^2 k^2}{2m_n^* m_o}$$

Parabolic valley

We will build devices with different semiconductor “parts”. We need to find out more about mobile carrier statistics under different conditions.

$$E_v(k) = \frac{\hbar^2 k^2}{2m_p^* m_o}$$

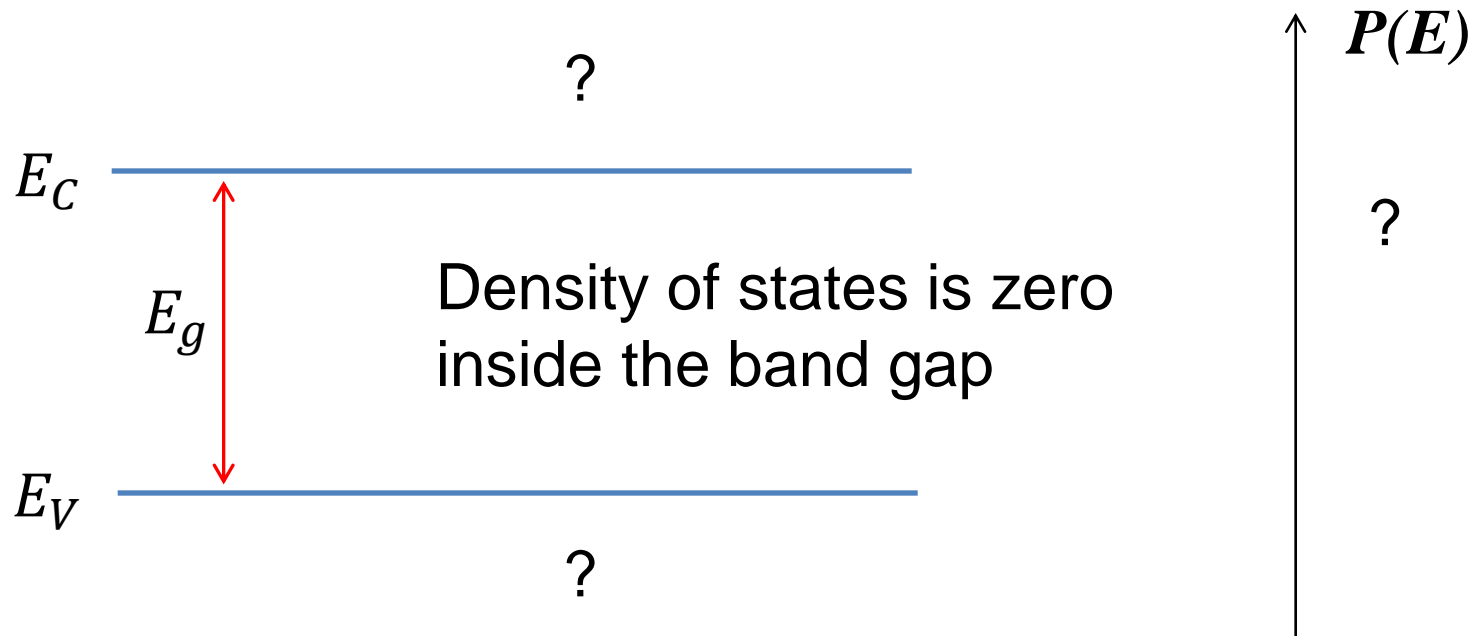
Parabolic valley

# Behavior of mobile carriers - Questions

Given a semiconductor with given dopant density, what is the density of electrons and holes in the respective bands?

The band structure is based on information about available energy states which carriers can occupy. What are those states, and what is the **probability of occupation**?

**How do temperature and other parameters affect these answers?**



# Probability distribution

Electrons in the “virtual free space” of the conduction band, can be studied as a “**gas of particles**” if we assume that they do not interact very strongly with each other (i.e. they are “diluted”).

**Classical particles obey the Maxwell-Boltzmann distribution.**

**In some conditions (but not always) this is adequate for semiconductors.**

# Electrons are Fermions

In general electrons are quantum particles that obey the Pauli Exclusion Principle:

**“At most one particle with a given set of quantum numbers can occupy an energy state”.**

These particles are called “Fermions”.

**Two electrons occupying the same orbital must have opposite spin ( $1/2, -1/2$ ).**

# Fermi-Dirac Statistics

Electrons in solids obey the Fermi-Dirac statistics:

$$f(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$


$E_F$  = Fermi Level

$k_B$  = Boltzmann constant

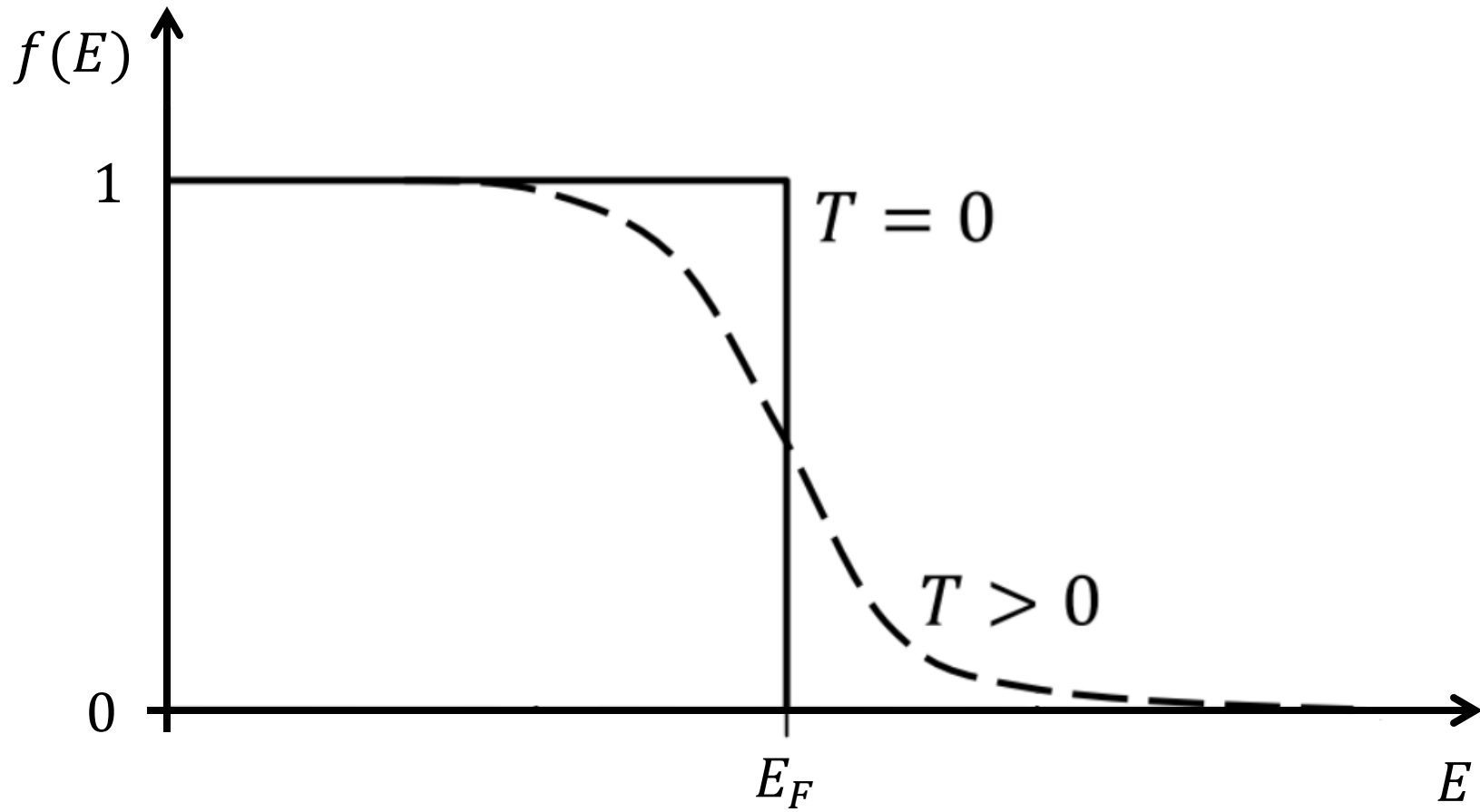
$$k_B = 8.62 \times 10^{-5} \text{ eV/K}$$

# Fermi-Dirac Statistics

What about holes? Use: **probability of non-occupation**

$$\begin{aligned}1 - f(E_F) &= 1 - \frac{1}{1 + e^{(E-E_F)/k_B T}} = \frac{1 + e^{(E-E_F)/k_B T} - 1}{1 + e^{(E-E_F)/k_B T}} \\&= \frac{e^{(E-E_F)/k_B T}}{1 + e^{(E-E_F)/k_B T}} = \frac{1}{\frac{1}{e^{(E-E_F)/k_B T}} + \frac{e^{(E-E_F)/k_B T}}{e^{(E-E_F)/k_B T}}} \\&= \frac{1}{1 + e^{(E_F-E)/k_B T}}\end{aligned}$$


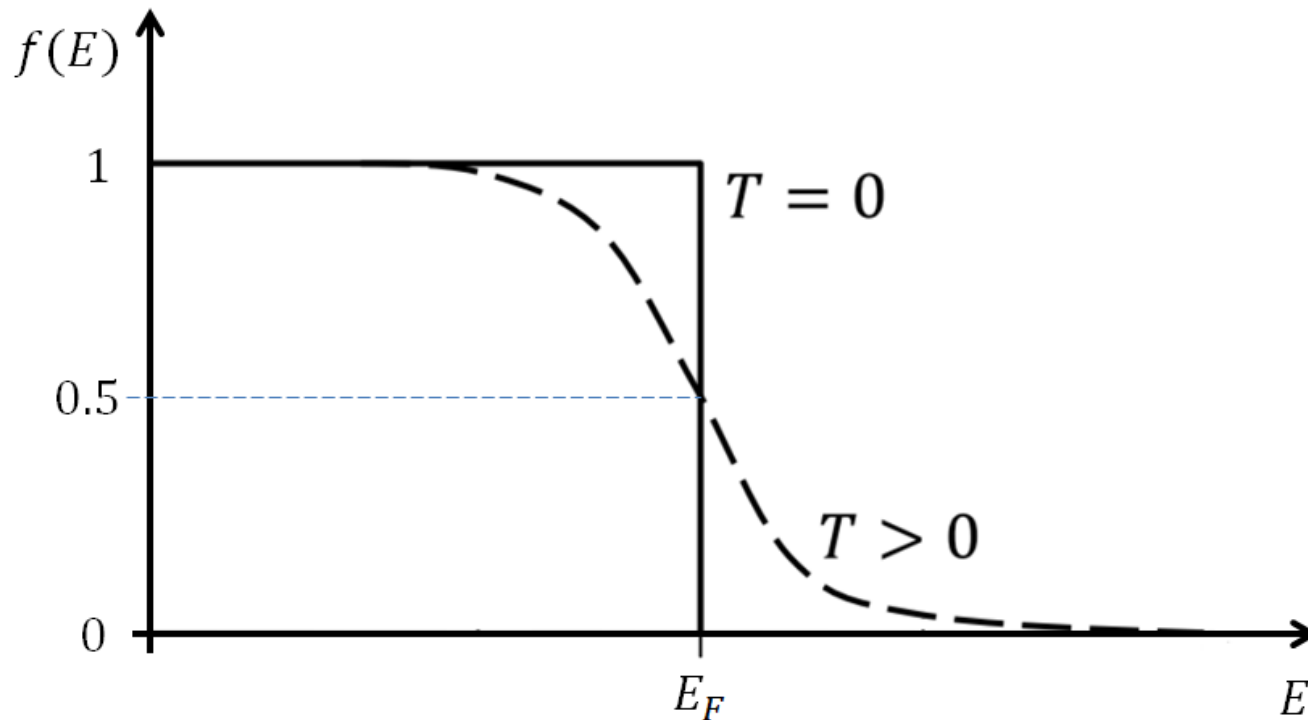
# Fermi-Dirac Statistics



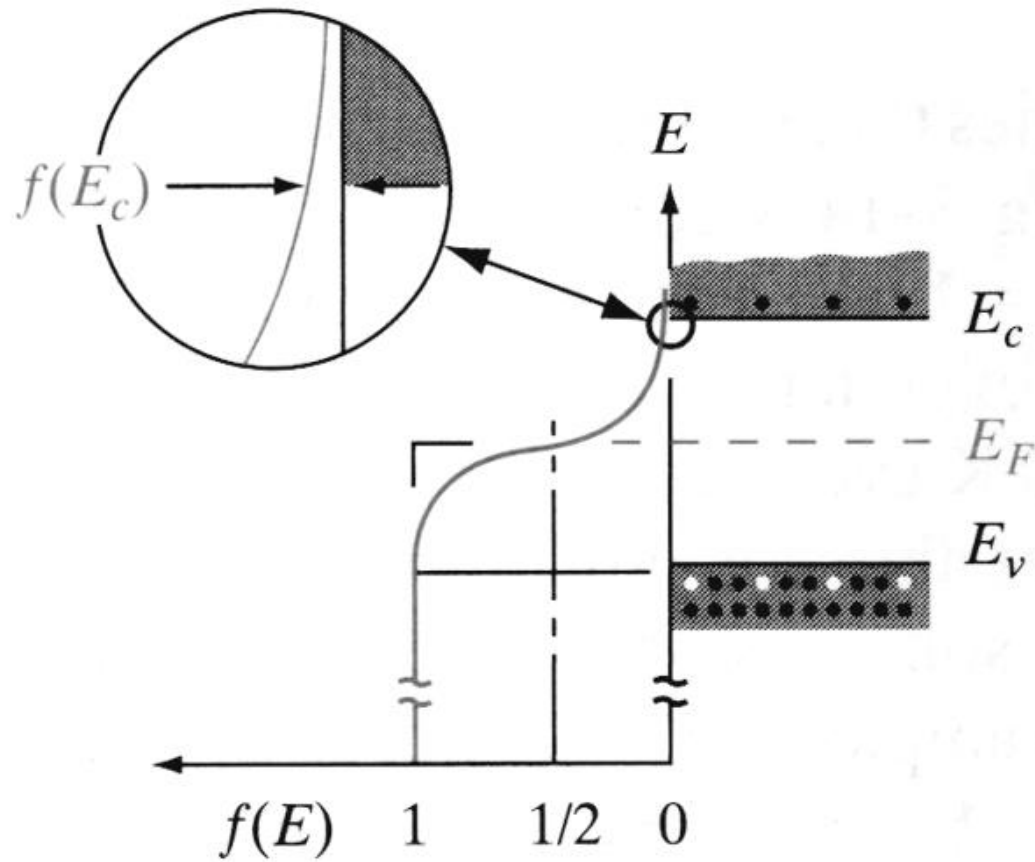


# Probability of Fermi Level

$$f(E_F) = \frac{1}{1 + e^{(E_F - E_F)/k_B T}} = \frac{1}{1 + 1} = \frac{1}{2}$$

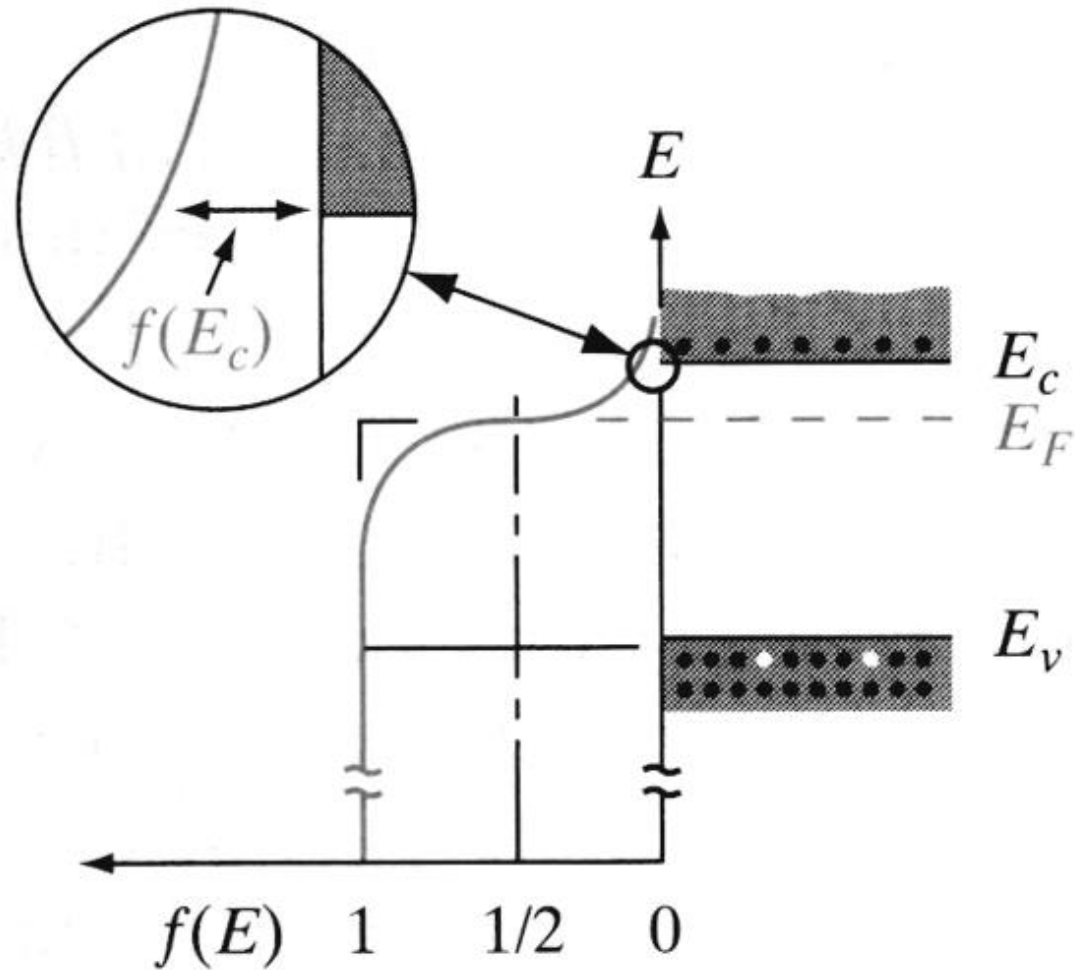


# Fermi-Dirac Statistics – Intrinsic Case



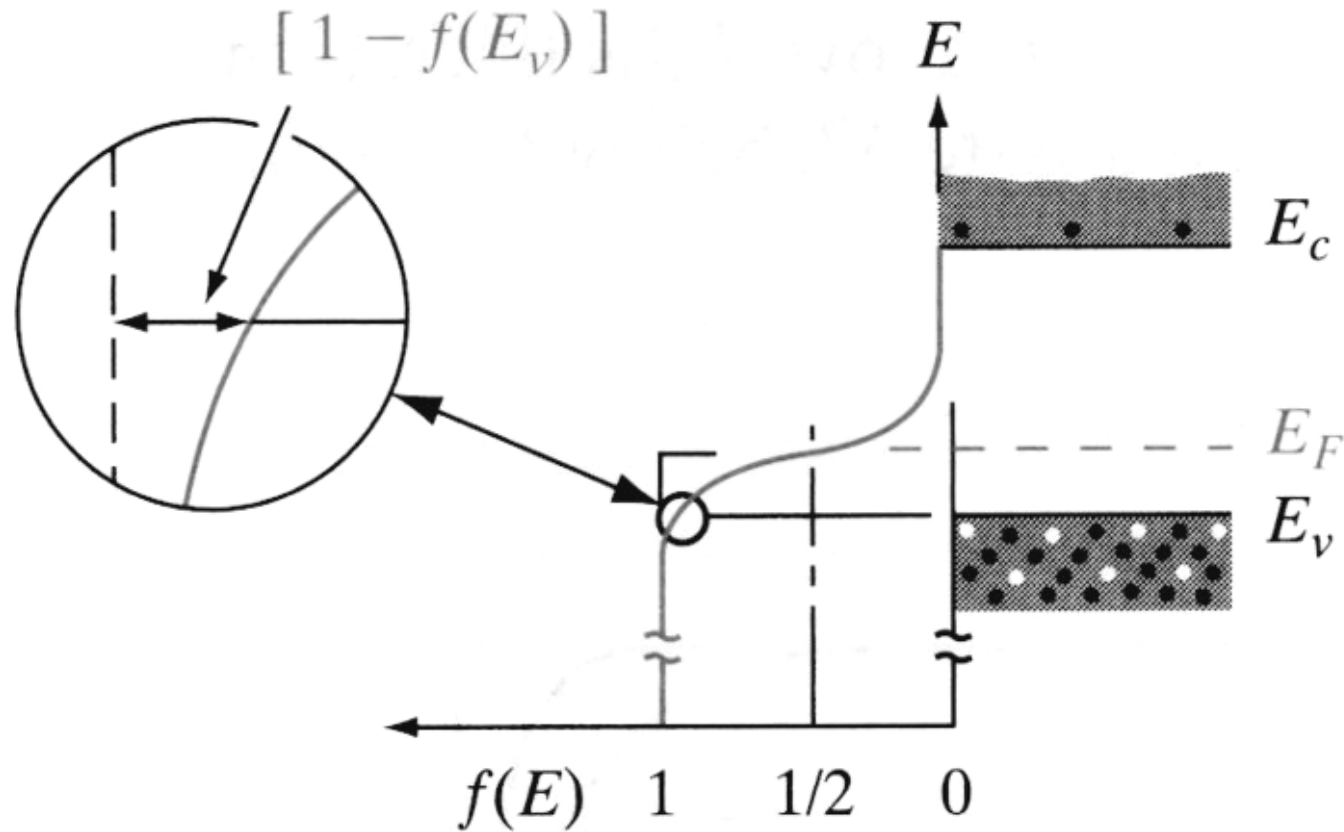
(a) Intrinsic

# Fermi-Dirac Statistics – n-type Case



(b) n-type

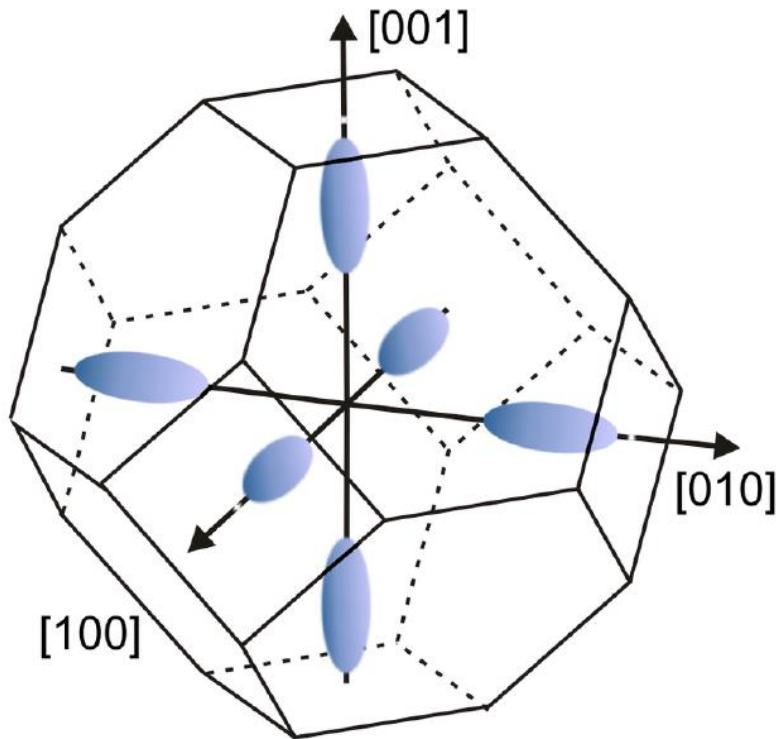
# Fermi-Dirac Statistics – p-type Case



(c) p-type

# Outline of Density of States Calculation

NOTE: This topic is treated in Appendix IV of the textbook and it will only be invoked here to determine the “effective density of states”.



Unit cell of momentum space

# Density of States in 3-D Result

**Density of state as a function of kinetic energy in a parabolic band.**

**(proof outlined in Appendix IV of the textbook)**

$$N(E) dE = \frac{\sqrt{2}}{\pi^2} \left( \frac{m^*}{\hbar^2} \right)^{3/2} \sqrt{E} dE$$

# Density of electrons – Conduction Band

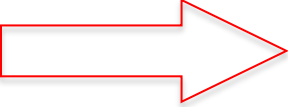
$$n_0 = \int_{E_c}^{\infty} \underbrace{f(E)}_{\text{probability of occupation}} \underbrace{N(E) dE}_{\text{density of states}}$$

We wish to get to this integrated form

$$n_0 = \underbrace{N_c}_{\text{effective density of states}} \times \underbrace{f(E_c)}_{\text{probability of occupation for band edge energy}}$$

**Problem: Fermi function cannot be integrated analytically in 3D**

Assume  $(E_c - E_F) \gg k_B T$

  $f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \simeq e^{-(E-E_F)/k_B T}$

Use the previous result for density of states

$$N(E) dE = \frac{\sqrt{2}}{\pi^2} \left( \frac{m^*}{\hbar^2} \right)^{3/2} \sqrt{E} dE$$

Now we can integrate analytically

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$$



# Electron density in the conduction band

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$$

After a few manipulations

$$n_0 = N_C e^{-(E_C - E_F)/k_B T}$$

Effective density of state

$$N_C = 2 \left( \frac{2\pi m_n^* k_B T}{h^2} \right)^{3/2}$$

“Density-of-states” effective mass.

For Si 6 valleys contribute.

$$\sqrt{(m_n^*)^3} = 6\sqrt{(m_l m_t^2)}$$

# Electron density in the conduction band

$$n_0 = N_C e^{-(E_C - E_F)/k_B T}$$

$$N_C = 2 \left( \frac{2\pi m_n^* k_B T}{h^2} \right)^{3/2} \rightarrow \text{---} E_C$$


Final result gives an equivalent averaged model where it is as if all the available states are concentrated at  $E_C$  with a single probability of occupation which depends on the Fermi Level.

When  $(E_C - E_F) \gg k_B T$  is not true, model breaks down!

# Hole density in the valence band

Similar process  
for holes

$$p_0 = N_V \underbrace{\left[ 1 - f(E_V) \right]}_{\text{probability of non-occupation}}$$

$$1 - f(E_V) = 1 - \frac{1}{1 + e^{(E_V - E_F)/k_B T}} \simeq e^{-(E_F - E_V)/k_B T}$$

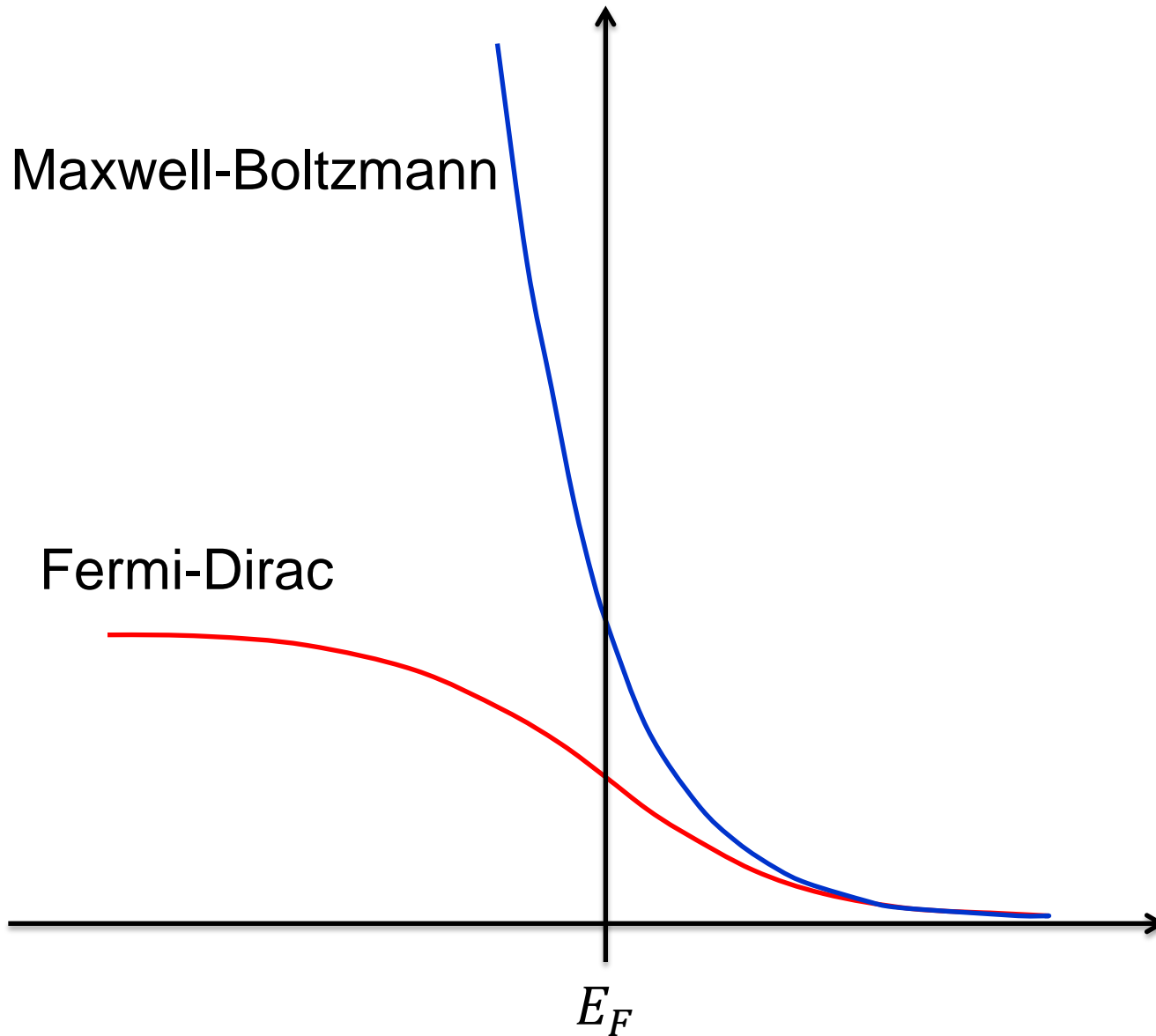


$$p_0 = N_V e^{-(E_F - E_V)/k_B T}$$

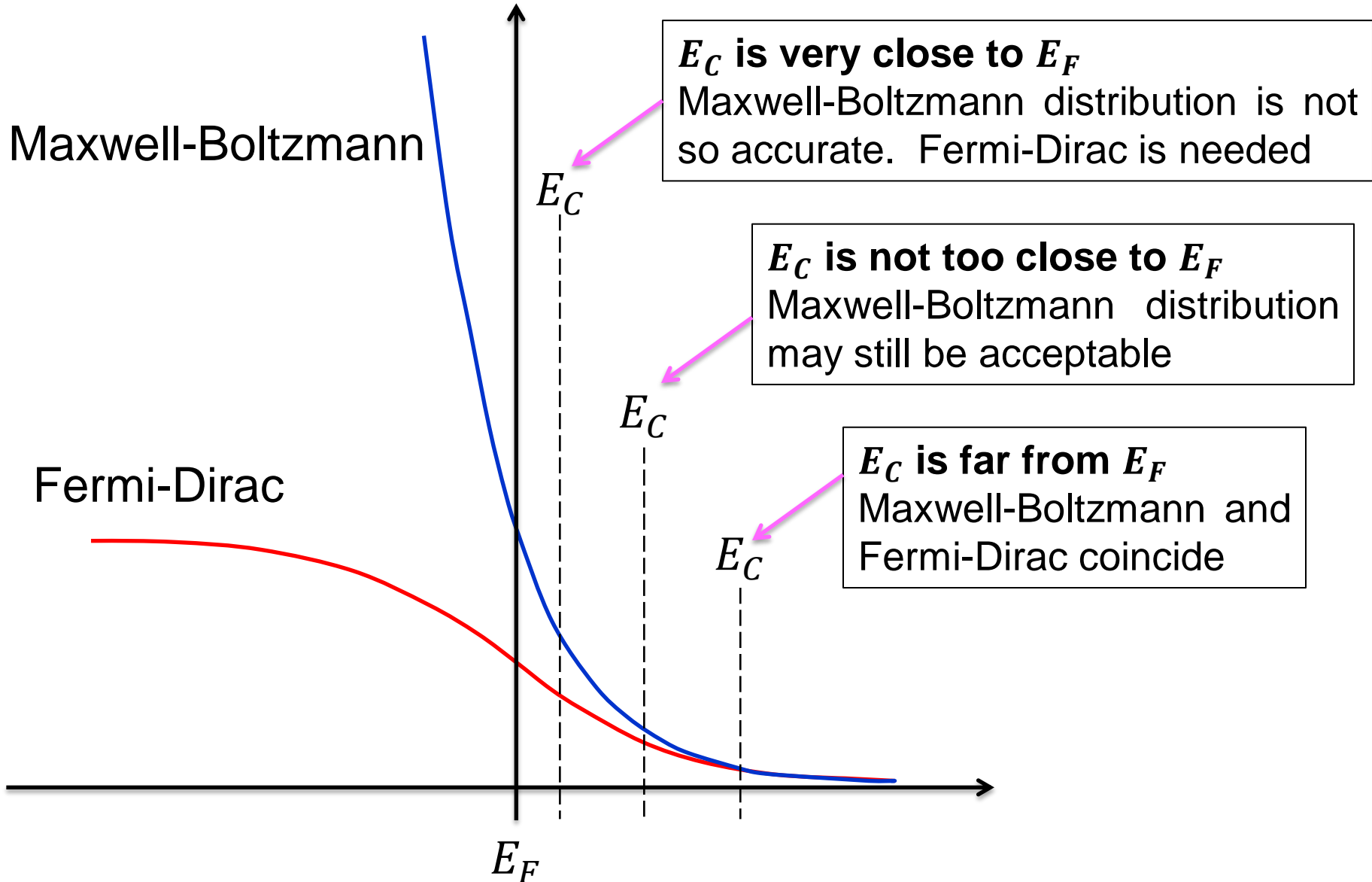
Effective density  
of states

$$N_V = 2 \left( \frac{2\pi m_p^* k_B T}{h^2} \right)^{3/2}$$

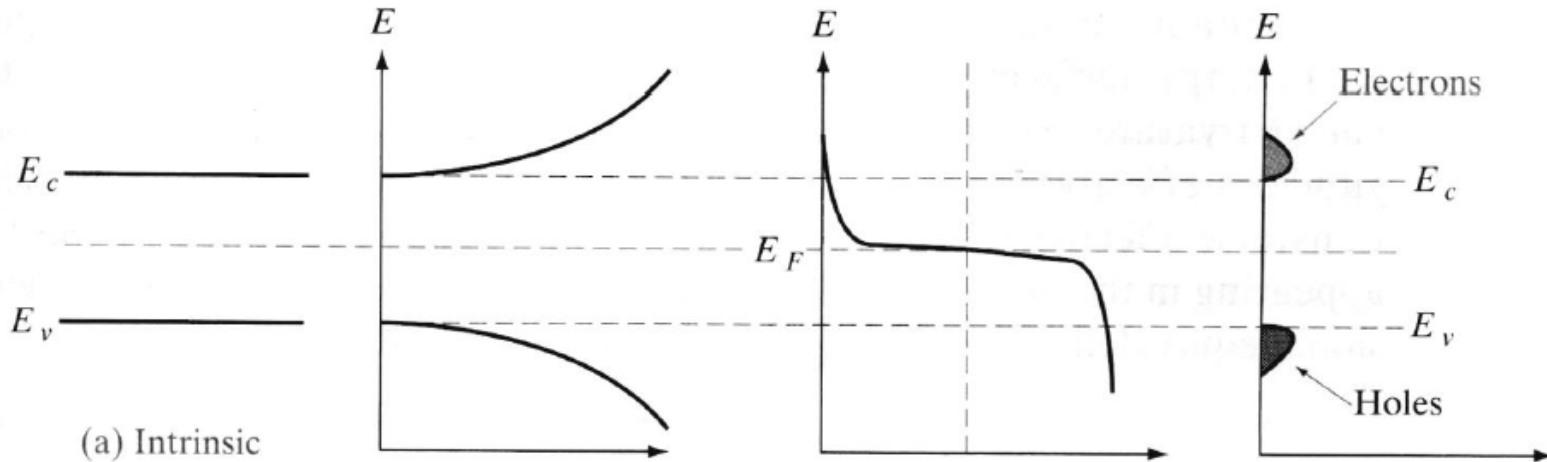
# Statistics



# Statistics



# Intrinsic case



$$n_i = N_C e^{-(E_C - E_i)/k_B T}$$

$$p_i = N_V e^{-(E_i - E_V)/k_B T}$$

$$n_i = p_i$$

# Intrinsic Fermi Level

$$N_C e^{-(E_C - E_i)/k_B T} = N_V e^{-(E_i - E_V)/k_B T}$$

$$\exp\left(\frac{2E_i - E_C - E_V}{k_B T}\right) = \frac{N_V}{N_C}$$

$$\frac{2E_i - E_C - E_V}{k_B T} = \ln\left(\frac{N_V}{N_C}\right)$$

$$E_i = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right) = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$$= \frac{E_C + E_V}{2} + \frac{3k_B T}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

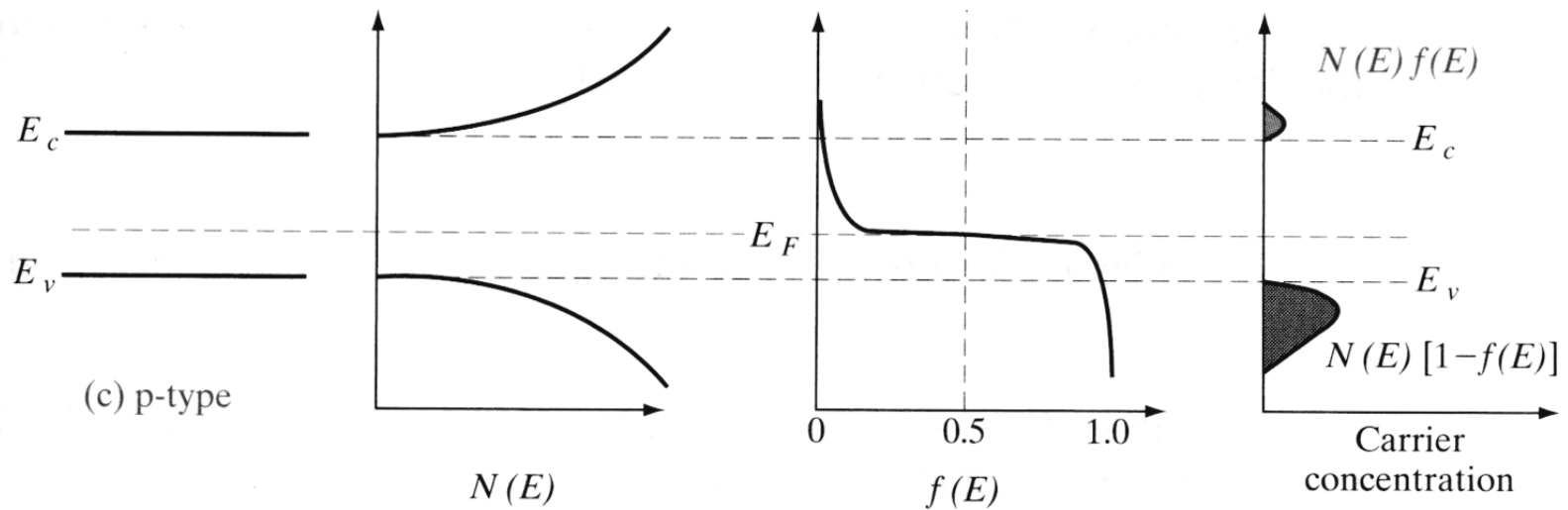
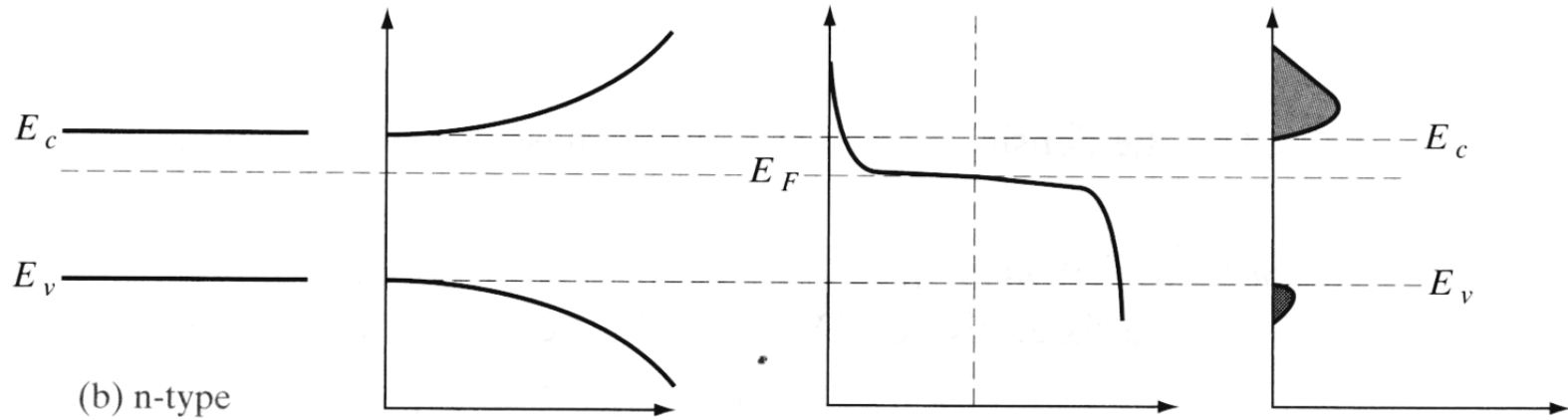
# Intrinsic concentration

$$\begin{aligned}n_i p_i &= (n_i)^2 = \\&= N_C e^{-(E_C - E_i)/k_B T} \times N_V e^{-(E_i - E_V)/k_B T} \\&= N_C N_V e^{-(E_C - E_i + E_i - E_V)/k_B T} \\&= N_C N_V e^{-(E_C - E_V)/k_B T} = N_C N_V e^{-E_g/k_B T}\end{aligned}$$

$$n_i = \sqrt{N_C N_V} e^{-E_g/2k_B T}$$



# Extrinsic case



# Extrinsic case

$$\begin{aligned}n_0 p_0 &= N_C e^{-(E_C - E_F)/k_B T} \times N_V e^{-(E_F - E_V)/k_B T} \\&= N_C N_V e^{-(E_C - E_V)/k_B T} = \\&= N_C N_V e^{-E_g/k_B T}\end{aligned}$$

Same as intrinsic case!

  $n_0 p_0 = n_i^2$

# Extrinsic case – useful alternative formulas

$$\begin{aligned}n_0 &= N_C e^{-(E_C - E_i + E_i - E_F)/k_B T} = \\&= \underbrace{N_C e^{-(E_C - E_i)/k_B T}}_{n_i} e^{-(E_i - E_F)/k_B T} = \\&= n_i e^{(E_F - E_i)/k_B T}\end{aligned}$$

# Extrinsic case – useful alternative formulas

$$\begin{aligned} p_0 &= N_V e^{-(E_F - E_i + E_i - E_V)/k_B T} = \\ &= \underbrace{N_C e^{-(E_i - E_V)/k_B T}}_{p_i = n_i} e^{-(E_F - E_i)/k_B T} = \\ &= n_i e^{(E_i - E_F)/k_B T} \end{aligned}$$

# Extrinsic case – useful alternative formulas

$$n_0 = n_i e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

Only using Fermi level and the reference intrinsic Fermi level.

These forms are very useful for computation

# Example

- Si doped with  $10^{16}$  B atoms/cm<sup>3</sup> at T=300K in equilibrium.
  1. Find concentration of electrons and holes
  2. Determine Fermi level
- 1. Boron is an acceptor. We can start by assuming that at 300K all acceptors are ionized (this is an approximation which is not correct at much lower or much higher T)

$$p_0 \approx N_A = 10^{16} \text{ cm}^{-3}$$

$$n_0 = \frac{(n_i)^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = \frac{2.25 \times 10^{20}}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

# Example

## 2. Determine Fermi level

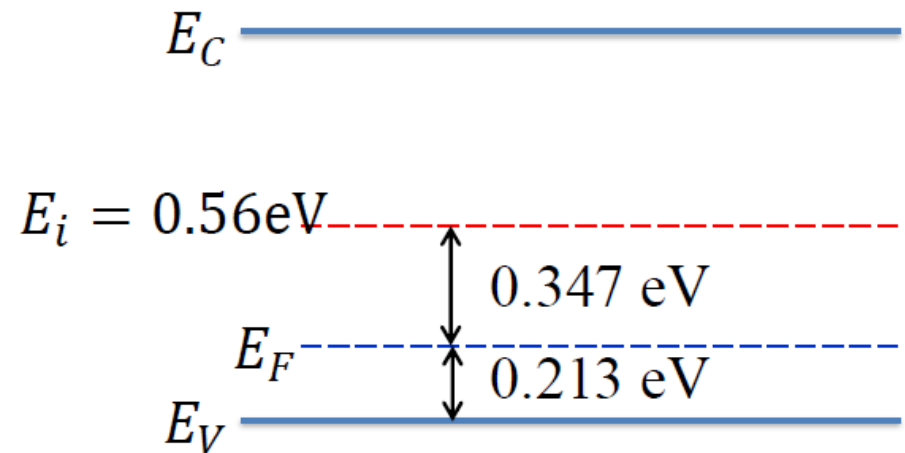
$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

$$\frac{E_i - E_F}{k_B T} = \ln\left(\frac{p_0}{n_i}\right)$$

$$E_i - E_F = k_B T \times \ln\left(\frac{p_0}{n_i}\right) =$$

$$= 0.0259 \ln\left(\frac{10^{16}}{1.5 \times 10^{10}}\right) = 0.34732 \text{ eV}$$

$$E_{gap}(\text{Si}) = 1.12 \text{ eV}$$
$$E_i \approx 0.5 E_{gap} = 0.56 \text{ eV}$$



# Extras

- If you would like to consult the first ever book on solid state devices, a pdf scan of Shockley's "Electrons and Holes in Semiconductors (with application to transistor electronics)" of 1950, is available at

<https://archive.org/details/ElectronsAndHolesInSemiconductors/mode/2up>