ECE 340 Lectures 6-7
Semiconductor Electronics

Spring 2022
10:00-10:50am
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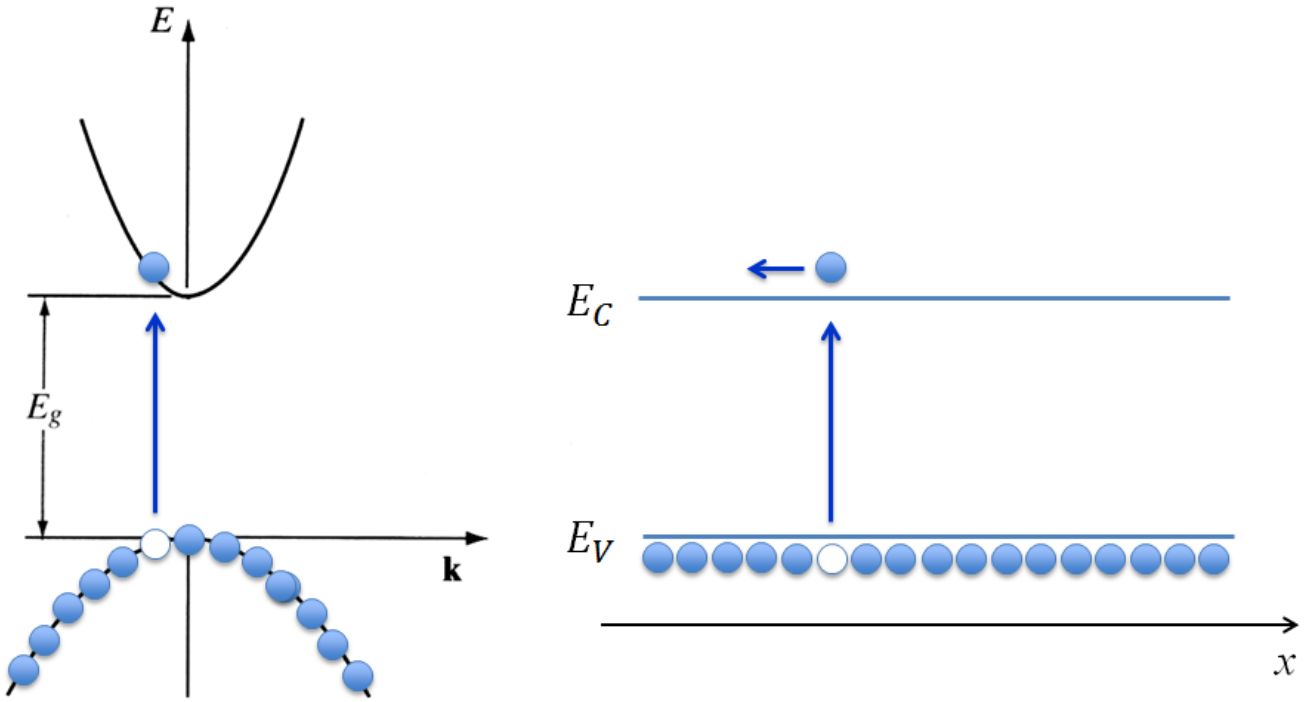
Today's Discussion

- Intrinsic and Extrinsic Semiconductors
- Statistics of electron distributions
- Fermi-Dirac statistics
- Maxwell-Boltzmann statistics
- The Fermi level
- Electron & hole concentrations in equilibrium

Intrinsic Semiconductor

- A **pure** semiconductor is called “intrinsic”



- With “**bulk semiconductor**” we refer from now on to a large piece of uniform material.

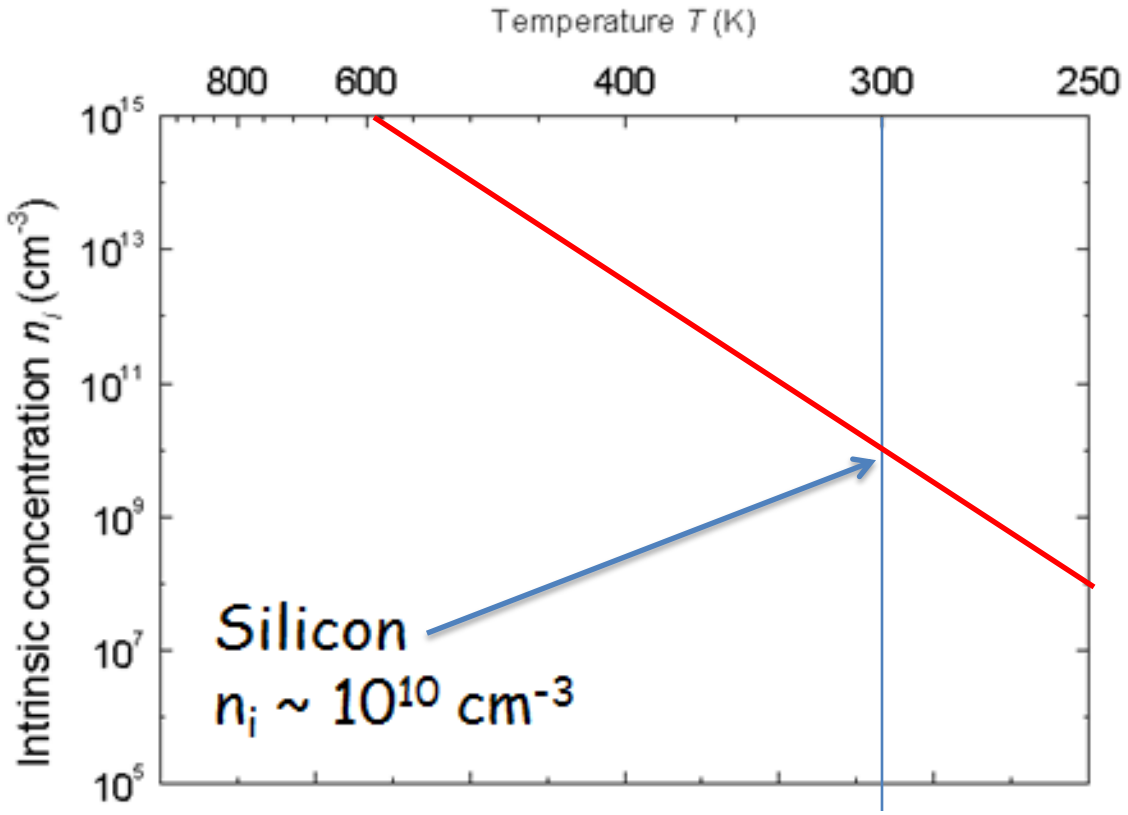
Intrinsic Semiconductor

- The concentration of electrons and holes in intrinsic bulk semiconductor is the balance of continuous **generation** and **recombination** of electron-hole pairs.

- There are always as many electrons in the conduction band as holes in the valence band

Intrinsic Semiconductor

- The **intrinsic concentration** depends on temperature



Electron-hole generation rate G_0 always balances the recombination rate R_0

$$G_0 = R_0$$

Textbook uses $n_i = 1.5 \times 10^{10}$ cm^{-3}

$$n_0 p_0 = {n_i}^2 \qquad n_0 = p_0 = n_i$$

Extrinsic Semiconductor

- **Impurities are introduced to force the material to have more electrons than holes or *vice versa***
  - An atom with more than 4 valence electrons can donate electrons to the conduction band while becoming a fixed positively charged ion. This is called a **donor**.
  - An atom with less than 4 valence electrons can attract electrons from the valence band, creating holes and becoming a fixed negatively charged ion. This is called an **acceptor**.

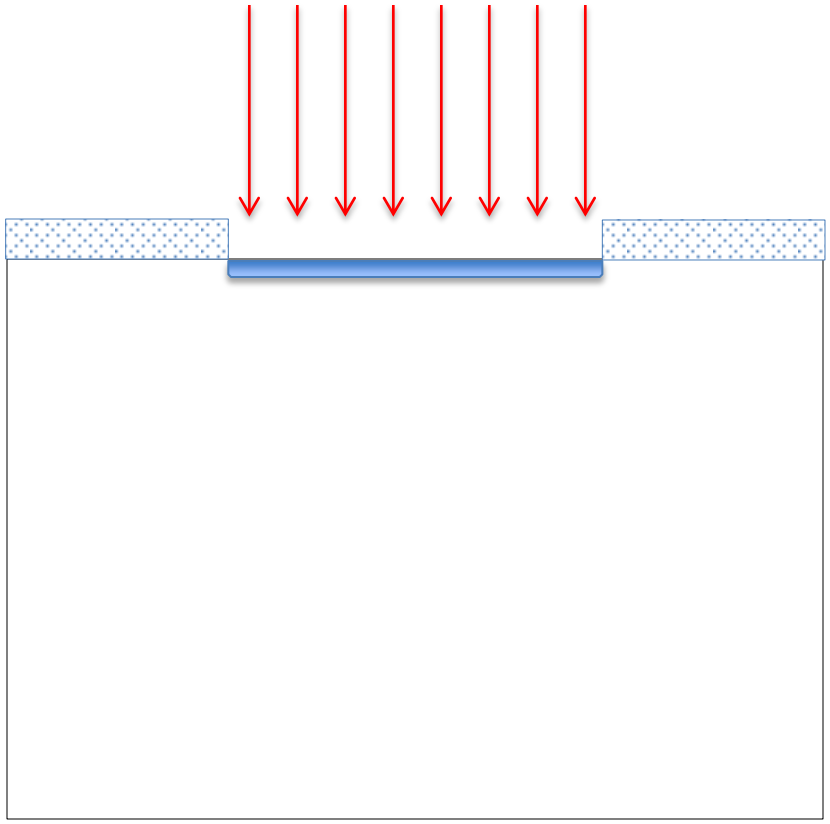
Extrinsic Semiconductor

- **The process of introducing impurities in the semiconductor for the purpose of changing material properties is called "doping".**
- **The impurities introduced with this process are called "dopants".**

- **Dopants are more commonly added:**
  - **During growth of the crystal**
  - **By diffusion at high temperature**
  - **By ion implantation**

Diffusion (pre-deposition + drive-in)

Pre-deposition



Drive-in

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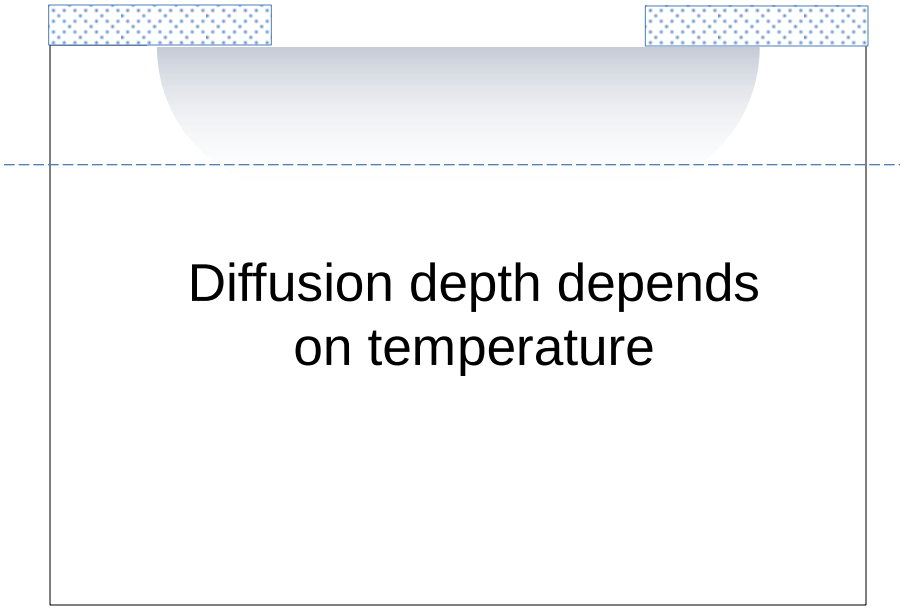
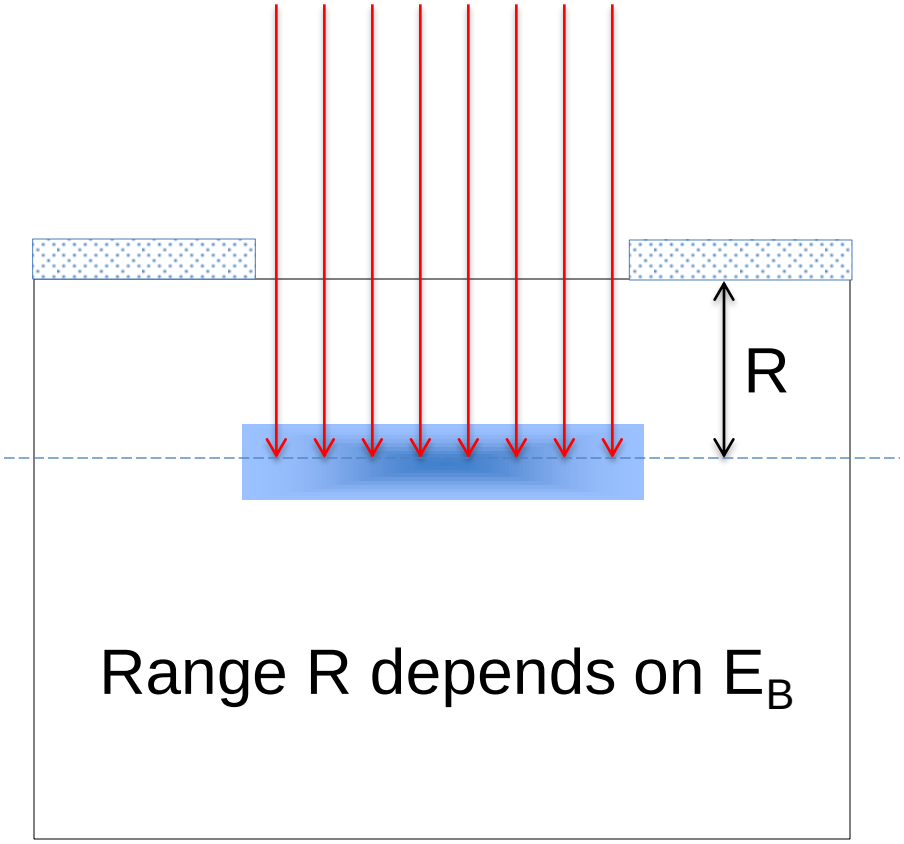


Diffusion depth depends on temperature

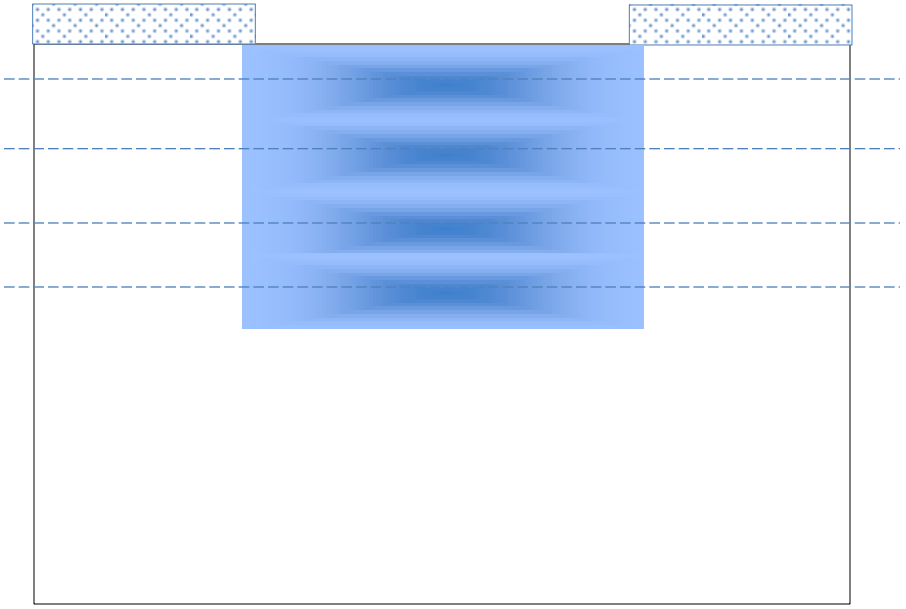
Ion implantation

ion beam with energy E_B

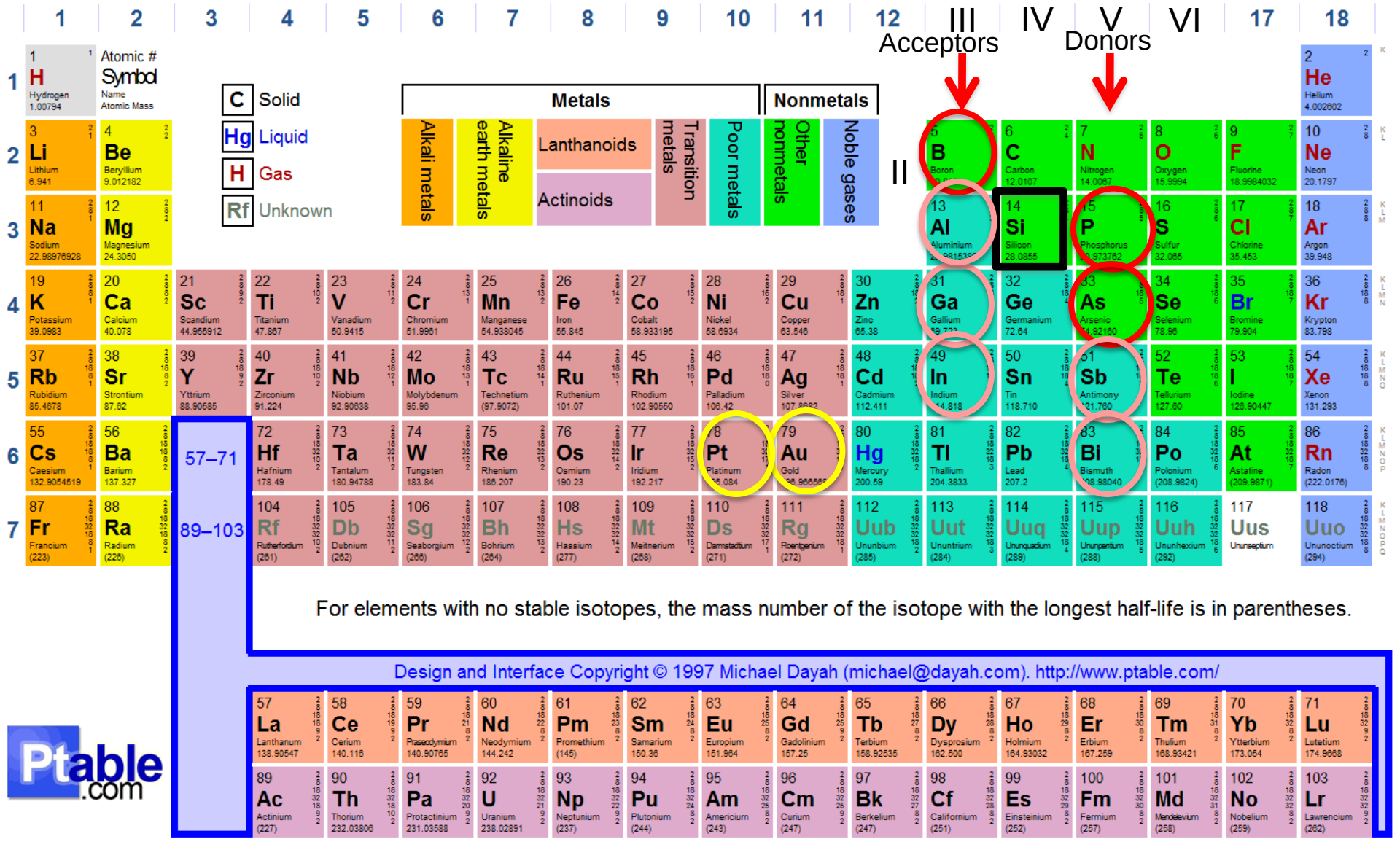


Range R depends on E_B

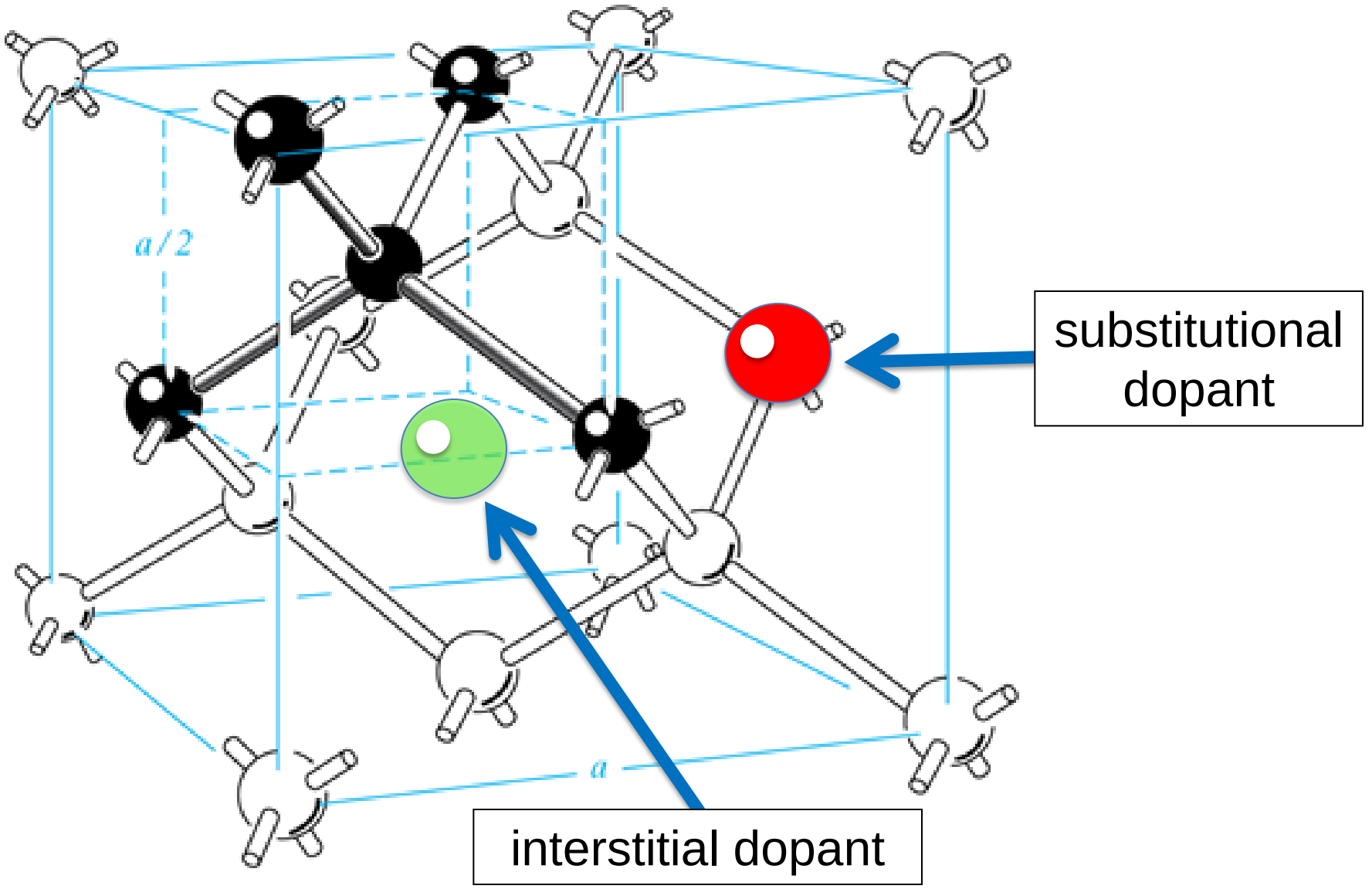
Doping profiles are obtained by implantations of many layers at different energies. A diffusion process follows implantation to smooth out the dopant profile.



Common Dopants for Silicon



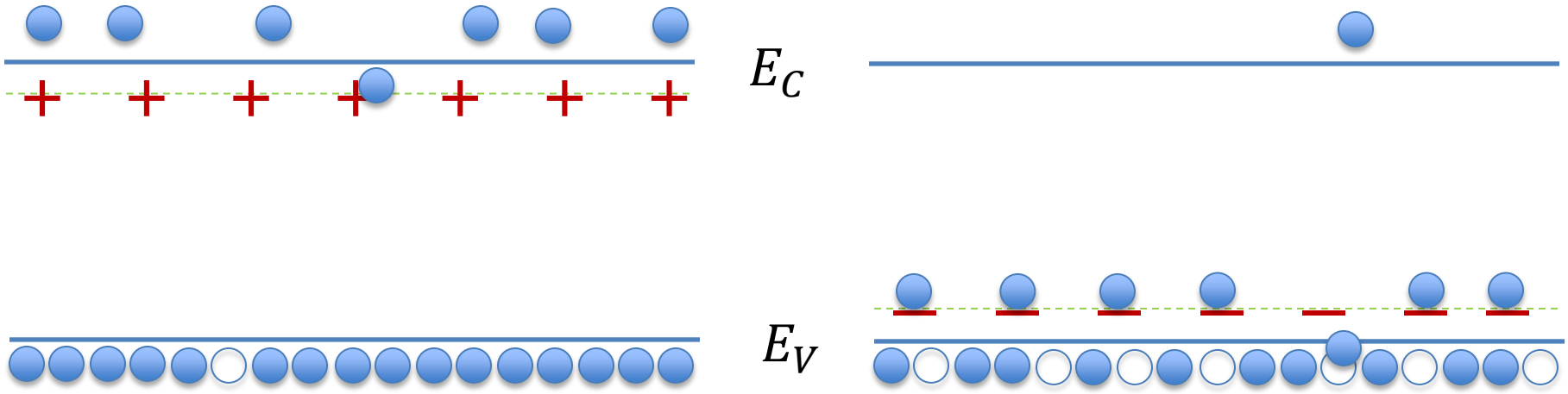
Doping atoms



Donors and Acceptors

Donors create energy levels close to the conduction band

Acceptors create energy levels close to the valence band



n-type *p-type*

Practical doping concentrations in Si are in the range

10^{14} to 10^{20} cm^{-3}

Evaluate Approximate Dopant Binding Energy

Simple Bohr's atom model for column V (e.g. As)

Ground state energy for outermost electron →

$$E = \frac{m_0 q^4}{2(4\pi\varepsilon_0)^2 \hbar^2}$$

(vacuum)

Hydrogen in vacuum $E = 13.6$ eV

Adapt for Si crystal environment

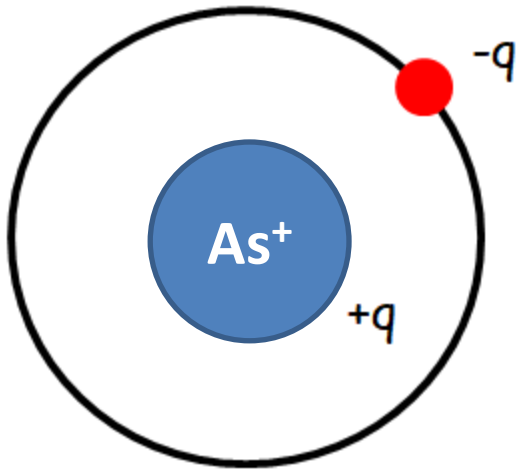
$$E = \frac{m_n^* q^4}{2(4\pi\varepsilon_r\varepsilon_0)^2 \hbar^2}$$

$$m_n^* \text{ (Si)} = 0.26 m_0 = \text{conductivity mass}$$

$$\varepsilon_r \text{ (Si)} = 11.8$$

Assume the 4 covalent bonding electrons are tightly bound and extra electron has hydrogen-like orbit

$E \approx 0.0254$ eV



Actual measured values

Donor in Si	P	As	Sb
Binding energy (eV)	0.045	0.054	0.039

Acceptor in Si	B	Al	Ga	In
Binding energy (eV)	0.045	0.067	0.072	0.16

Radius of Binding Electron Orbit

Simple Bohr's atom model for column V (e.g. As)

Radius of orbit

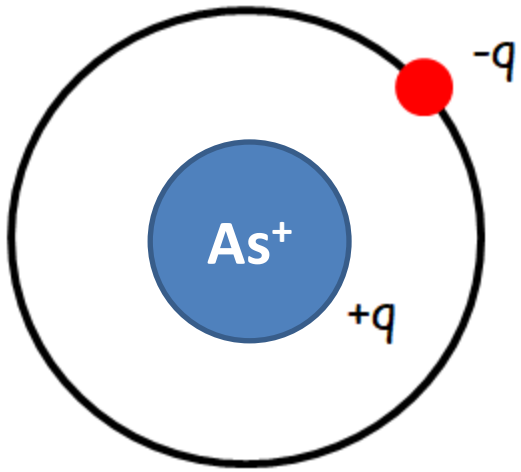
$$r = \frac{4\pi\varepsilon_0\hbar^2}{m_0 q^2}$$

(vacuum)

Adapt for Si crystal environment

$$r = \frac{4\pi\varepsilon_r\varepsilon_0\hbar^2}{m_n^* q^2} = \frac{4\times 3.14\times 11.8\times(8.85\times10^{-12})\times(1.054\times10^{-34})^2}{0.26\times(9.11\times10^{-31})\times(1.602\times10^{-19})^2} =$$

$$r = 2.41\times10^{-9}\,\text{m} = 2.41\,\text{nm}$$



Use **conductivity mass**. For ellipsoidal band in Si:

$$m_n^* = \left[\frac{1}{3}\left(\frac{1}{m_l}+\frac{2}{m_t}\right)\right]^{-1} = \left[\frac{1}{3}\left(\frac{1}{0.98\,m_0}+\frac{2}{0.19\,m_0}\right)\right]^{-1} = 0.26\,m_0$$

Conductivity mass & Density of States mass

For density of states calculation, use the density of states mass

$$m_n^* = \left[m_1\, m_2\, m_3\right]^{1/3} \underbrace{M_C^{2/3}}_{\substack{\#\text{equivalent} \\ \text{valleys}}} = \underbrace{\left[m_l\, m_t^2\right]^{1/3} 6^{2/3}}_{Si}$$

$$m_p^* = \left[m_{lh}^{3/2} + m_{hh}^{3/2}\right]^{2/3}$$

For transport calculation (e.g., current), use the conductivity mass

$$m_n^* = \left[\frac{1}{3}\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\right)\right]^{-1} = \underbrace{\left[\frac{1}{3}\left(\frac{1}{m_t} + \frac{2}{m_t}\right)\right]^{-1}}_{Si}$$

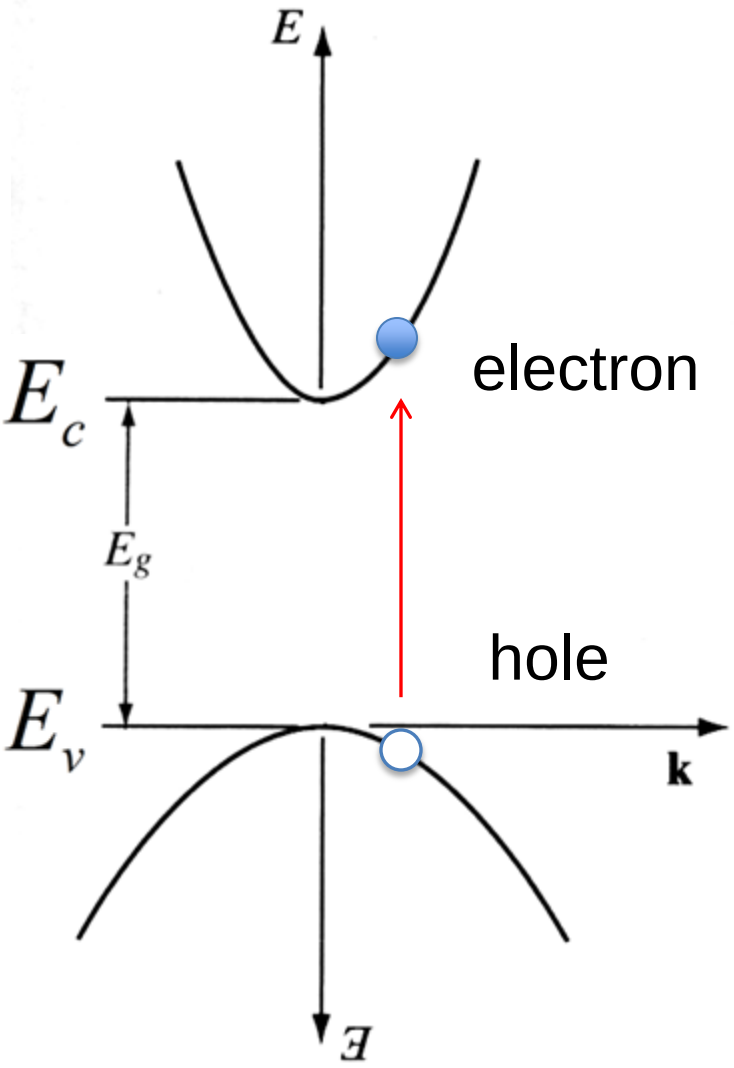
Conductivity mass for holes is complicated!

Holes have *light hole* and *heavy hole* bands which overlap, therefore, there are two mass components proportional to the band occupation (correlated to density of states, which in turn depends on $(m_h^*)^{3/2}$)

$$m_h^* \approx \left(\frac{\%\ occup.heavy\ band}{m_{hh}^*} + \frac{\%\ occup.light\ band}{m_{lh}^*} \right)^{-1}$$

In silicon, further complication is due to strong anisotropy of heavy hole band ("warping" of band) which requires special corrections.

So far we have an energy model



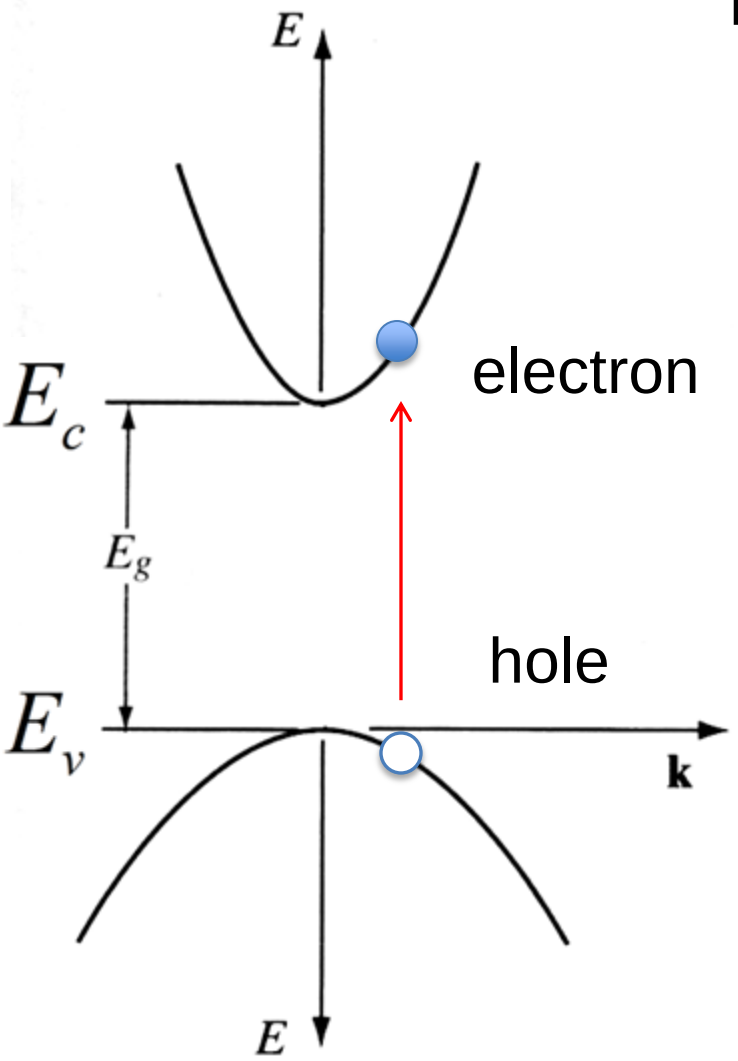
$$E_c(k) = \frac{\hbar^2 k^2}{2 m_n^* m_o}$$

Parabolic valley

$$E_v(k) = \frac{\hbar^2 k^2}{2 m_p^* m_o}$$

Parabolic valley

We have an energy model, so far



But we still need more information

$$E_c(k) = \frac{\hbar^2 k^2}{2 m_n^* m_o}$$

Parabolic valley

We will build devices with different semiconductor "parts". We need to find out more about mobile carrier statistics under different conditions.

$$E_v(k) = \frac{\hbar^2 k^2}{2 m_p^* m_o}$$

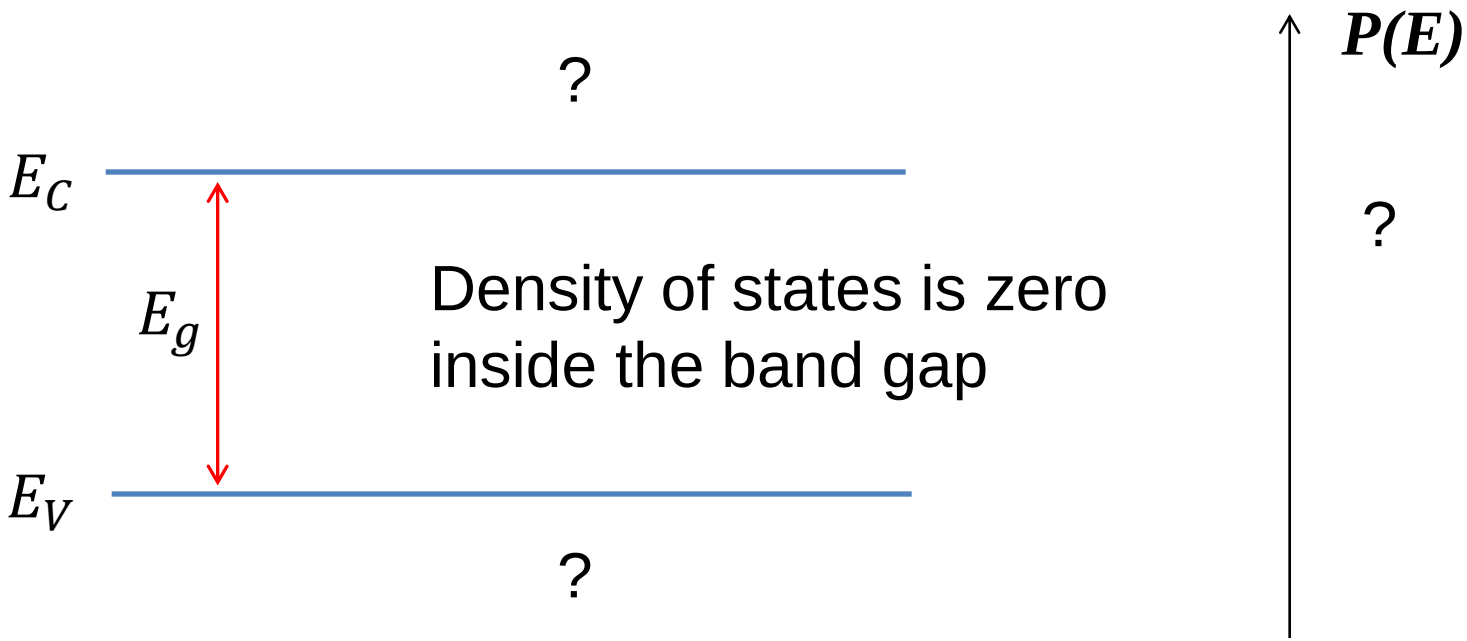
Parabolic valley

Behavior of mobile carriers - Questions

Given a semiconductor with given dopant density, what is the density of electrons and holes in the respective bands?

The band structure is based on information about available energy states which carriers can occupy. What are those states, and what is the probability of occupation?

How do temperature and other parameters affect these answers?



Probability distribution

Electrons in the “virtual free space” of the conduction band, can be studied as a “**gas of particles**” if we assume that they do not interact very strongly with each other (i.e. they are “diluted”).

Classical particles obey the Maxwell-Boltzmann distribution.

In some conditions (but not always) this is adequate for semiconductors.

Electrons are Fermions

In general electrons are quantum particles that obey the Pauli Exclusion Principle:

"At most one particle with a given set of quantum numbers can occupy an energy state".

These particles are called "Fermions".

Two electrons occupying the same orbital must have opposite spin (1/2,-1/2).

Fermi-Dirac Statistics

Electrons in solids obey the Fermi-Dirac statistics:

$$f(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

E_F = Fermi Level

k_B = Boltzmann constant

$k_B = 8.62 \times 10^{-5}$ eV/K

Fermi-Dirac Statistics

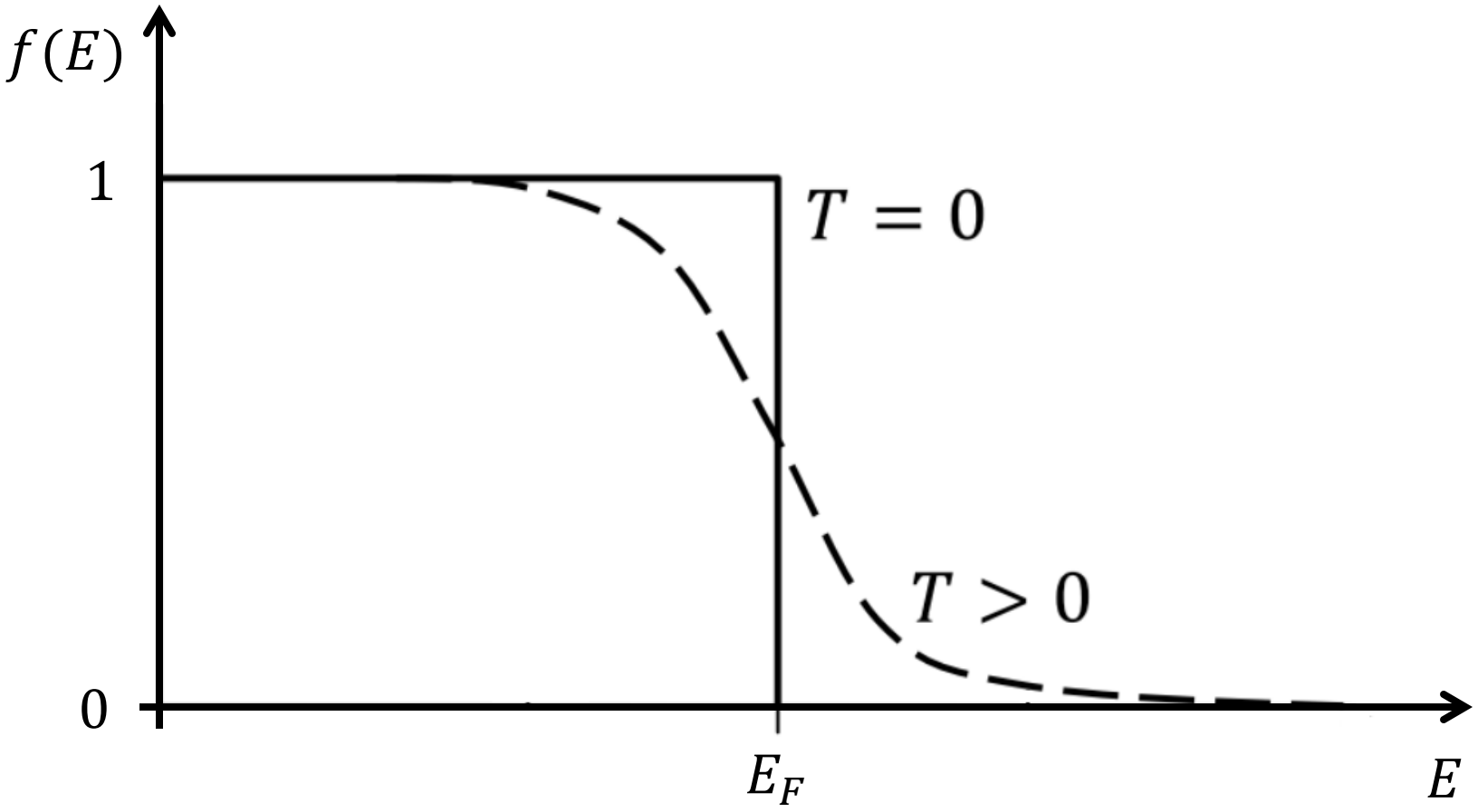
What about holes? Use: probability of non-occupation

$$1-f(E_F)=1-\frac{1}{1+e^{(E-E_F)/k_BT}}=\frac{1+e^{(E-E_F)/k_BT}-1}{1+e^{(E-E_F)/k_BT}}$$

$$=\frac{e^{(E-E_F)/k_BT}}{1+e^{(E-E_F)/k_BT}}=\frac{1}{\frac{1}{e^{(E-E_F)/k_BT}}+\frac{e^{(E-E_F)/k_BT}}{e^{(E-E_F)/k_BT}}}$$

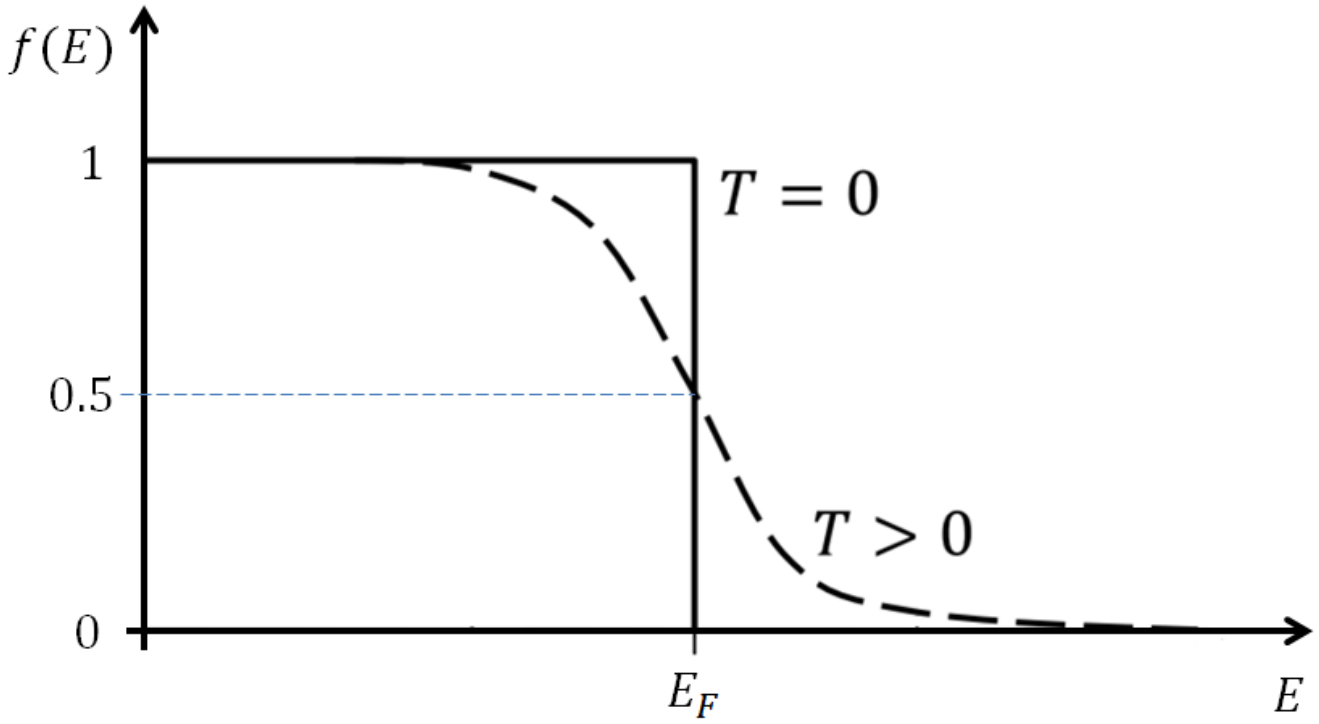
$$=\frac{1}{1+e^{(E_F-E)/k_BT}}$$

Fermi-Dirac Statistics

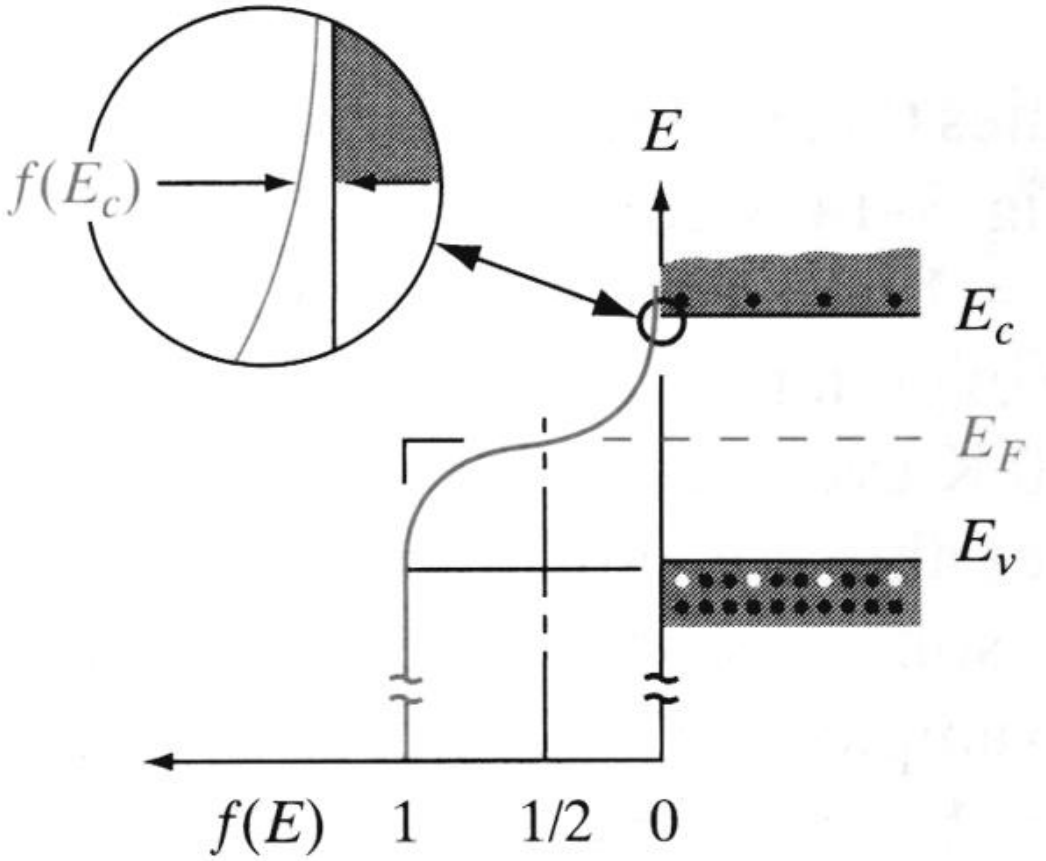


Probability of Fermi Level

$$f(E_F) = \frac{1}{1+e^{(E_F - E_F)/k_B T}} = \frac{1}{1+1} = \frac{1}{2}$$

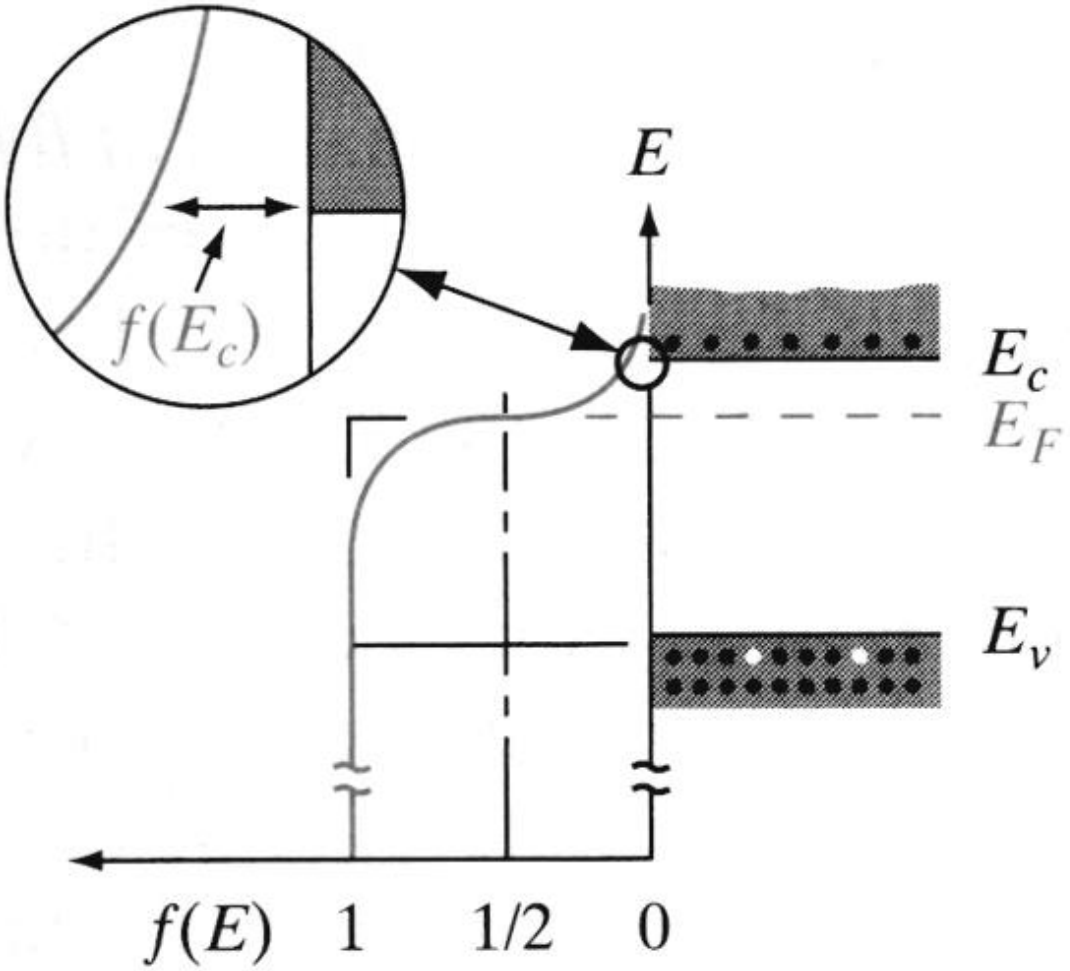


Fermi-Dirac Statistics – Intrinsic Case



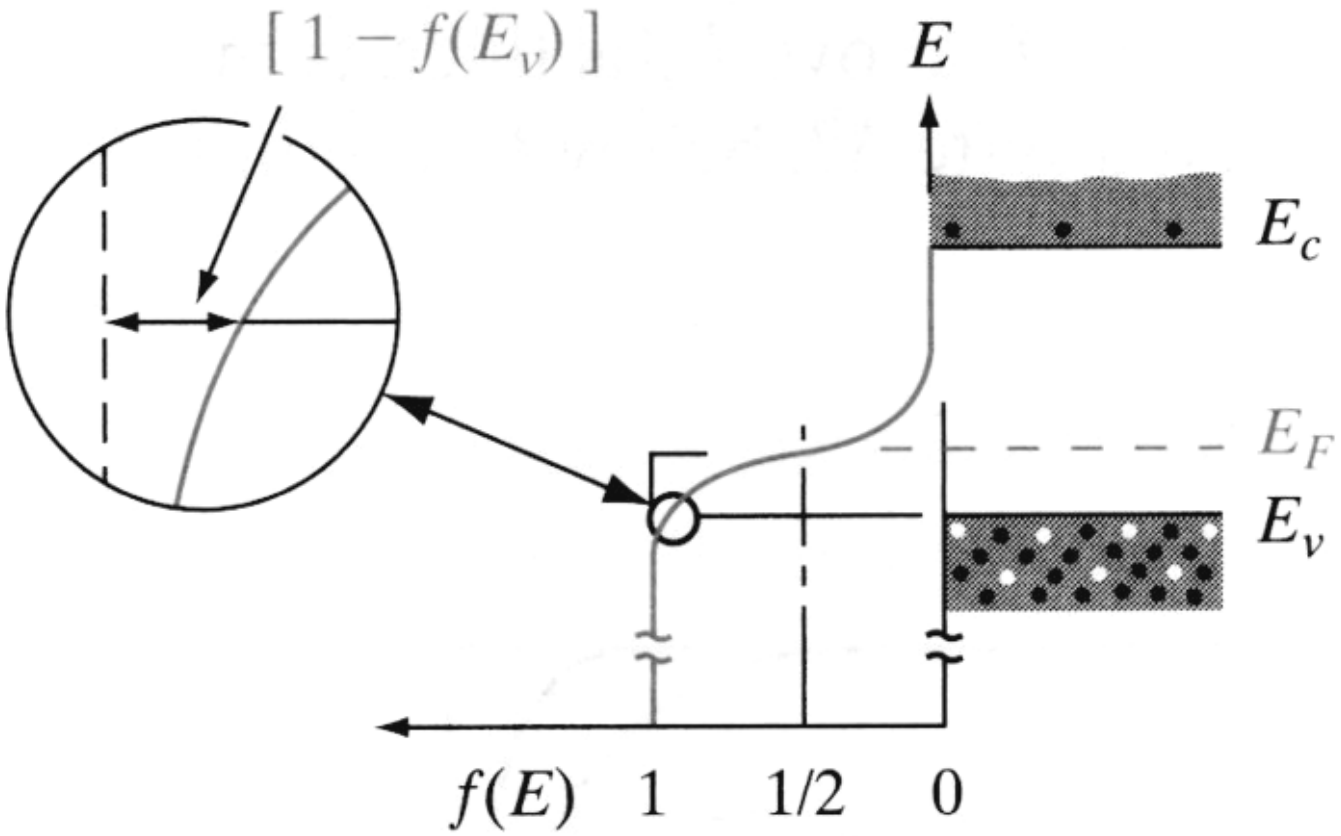
(a) Intrinsic

Fermi-Dirac Statistics – n-type Case



(b) n-type

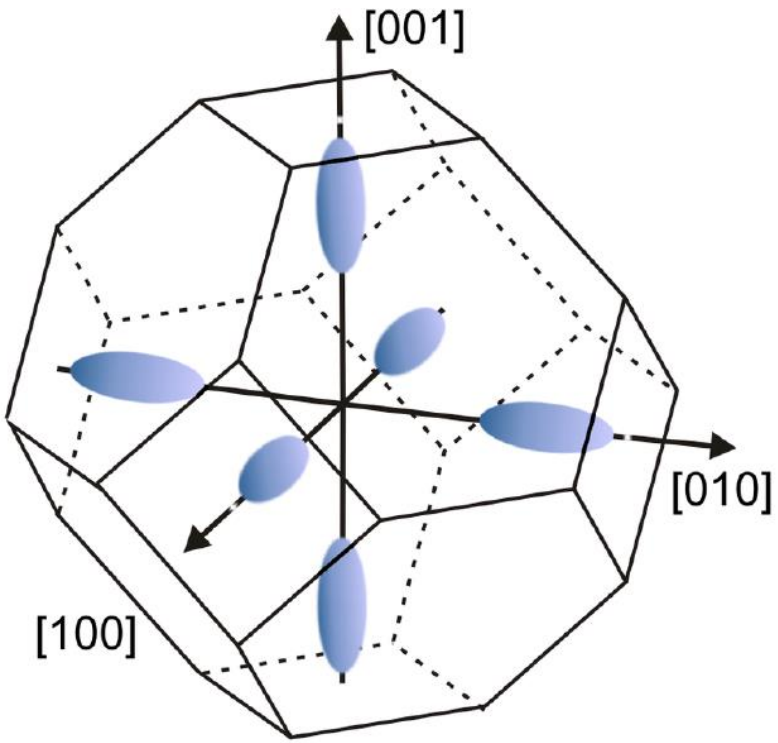
Fermi-Dirac Statistics – p-type Case



(c) p-type

Outline of Density of States Calculation

NOTE: This topic is treated in Appendix IV of the textbook and it will only be invoked here to determine the “effective density of states”.



Unit cell of momentum space

Density of States in 3-D Result

Density of state as a function of kinetic energy in a parabolic band.

(proof outlined in Appendix IV of the textbook)

$$N(E)\,dE = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}\sqrt{E}\;dE$$

Density of electrons – Conduction Band

$$n_0 = \int_{E_c}^{\infty} \underbrace{f(E)}_{\text{probability of occupation}} \underbrace{N(E)\,dE}_{\text{density of states}}$$

We wish to get to this integrated form

$$n_0 = \underbrace{N_C}_{\text{effective density of states}} \times \underbrace{f(E_C)}_{\text{probability of occupation for band edge energy}}$$

Problem: Fermi function cannot be integrated analytically in 3D

Assume $(E_c - E_F) \gg k_B T$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \simeq e^{-(E-E_F)/k_B T}$$

Use the previous result for density of states

$$N(E)\,dE = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}\sqrt{E}\;dE$$

Now we can integrate analytically

$$n_0 = \int_{E_c}^{\infty} f(E)\,N(E)\,dE$$

Electron density in the conduction band

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$$

After a few manipulations

$$n_0 = N_C \, e^{-(E_C - E_F)/k_B T}$$

Effective density of state

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2}$$

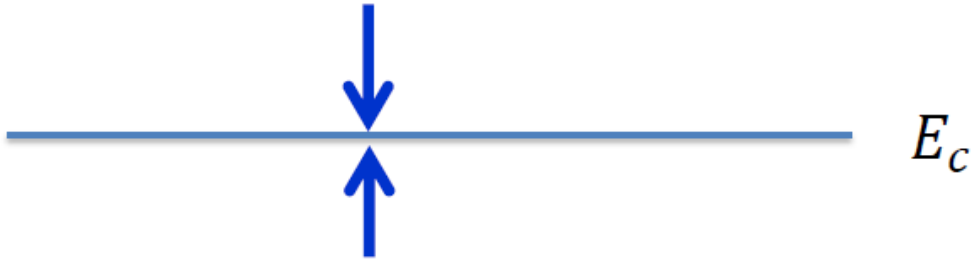
"Density-of-states" effective mass.
For Si 6 valleys contribute.

$$\sqrt{\left(m_n^*\right)^3} = 6\sqrt{\left(m_l m_t^2\right)}$$

Electron density in the conduction band

$$n_0 = N_C \, e^{-(E_C - E_F)/k_B T}$$

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2}$$



Final result gives an equivalent averaged model where it is as if all the available states are concentrated at E_c with a single probability of occupation which depends on the Fermi Level.

When $(E_c - E_F) \gg k_B T$ is not true, model breaks down!

Hole density in the valence band

Similar process for holes

$$p_0 = N_V \underbrace{\left[1 - f\left(E_V\right)\right]}_{\text{probability of non-occupation}}$$

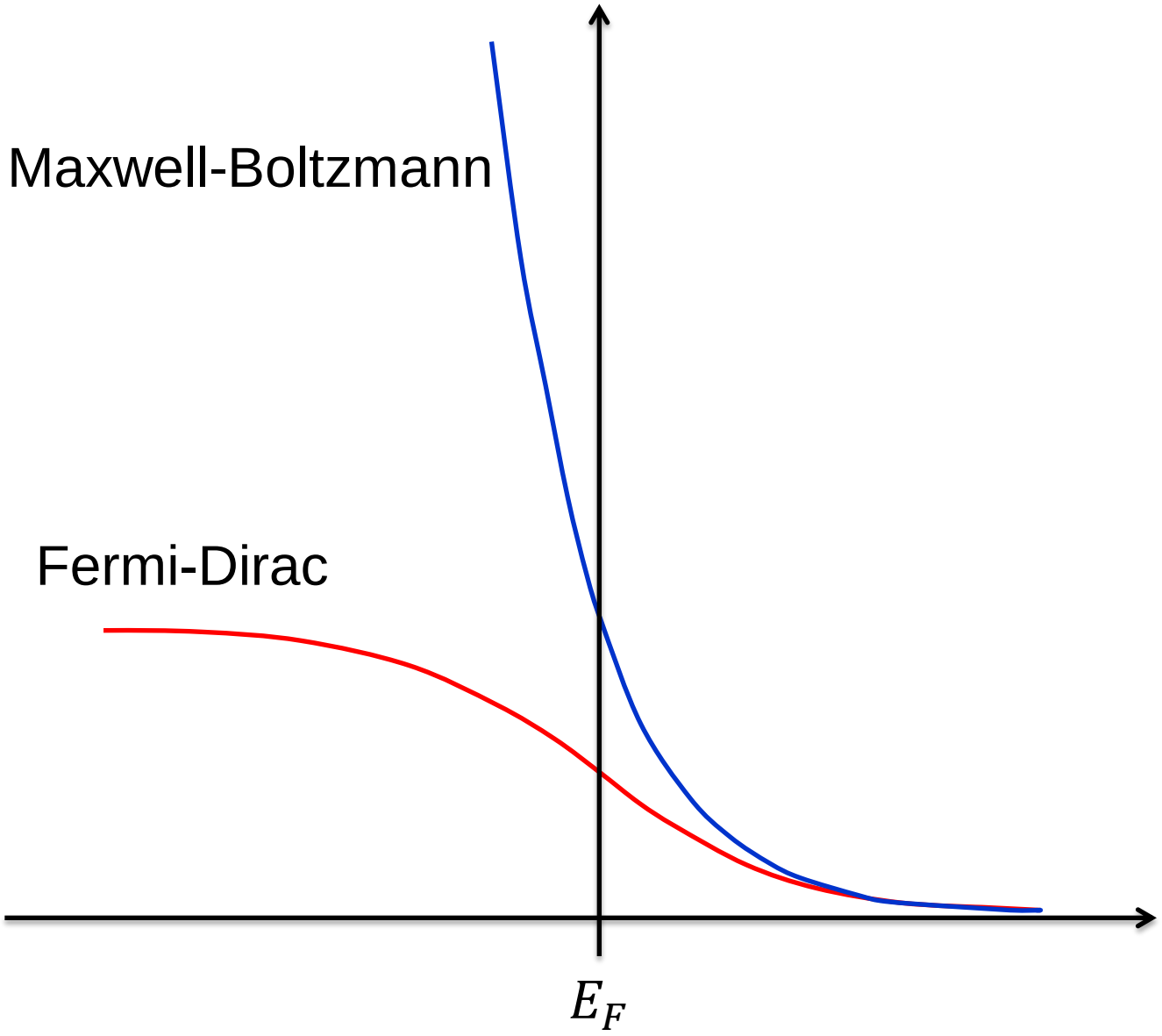
$$1 - f\left(E_V\right) = 1 - \frac{1}{1 + e^{\left(E_V - E_F\right)/k_B T}} \simeq e^{-\left(E_F - E_V\right)/k_B T}$$

$$\boxed{p_0 = N_V \, e^{-\left(E_F - E_V\right)/k_B T}}$$

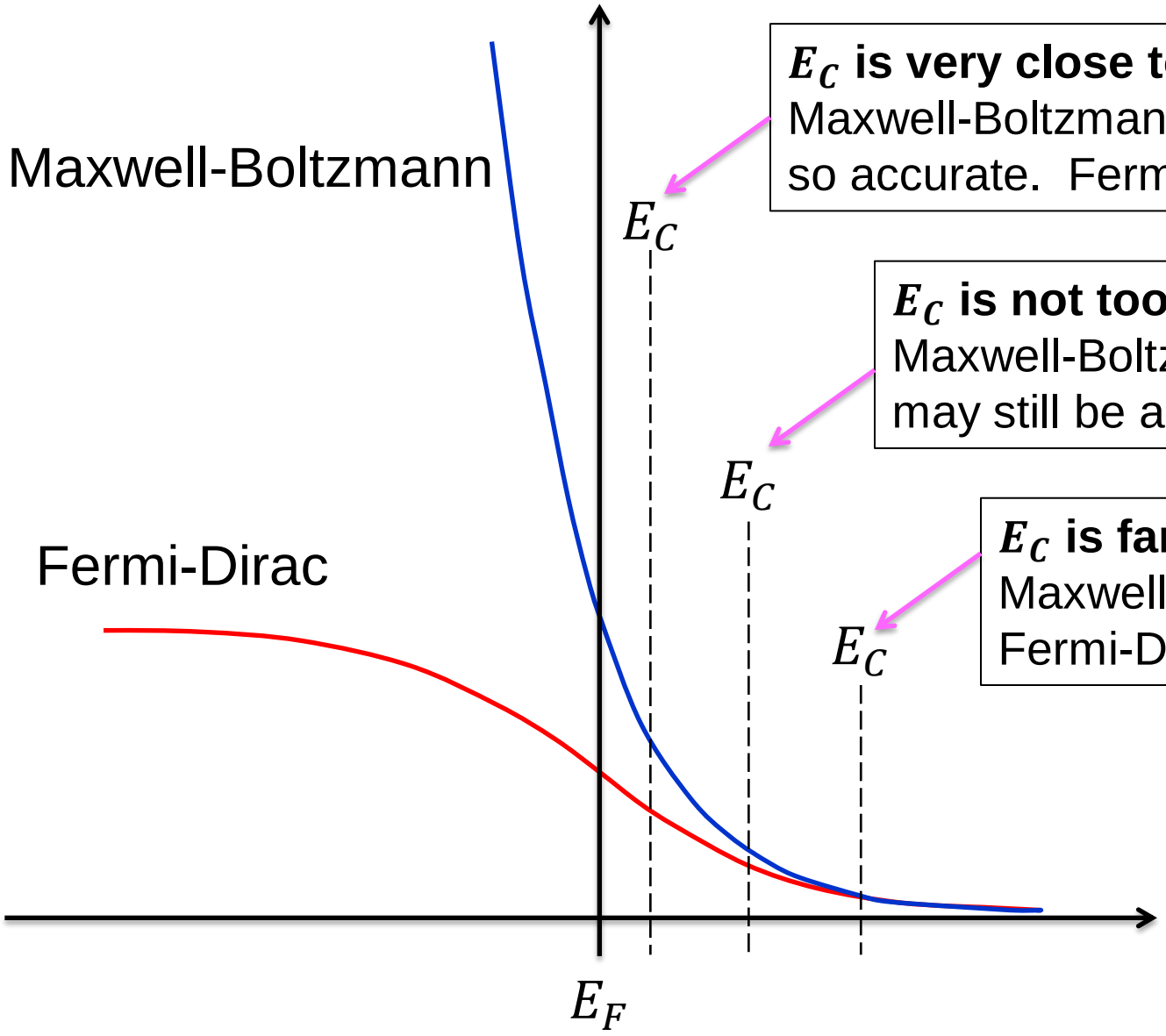
Effective density of states

$$N_V = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}$$

Statistics



Statistics

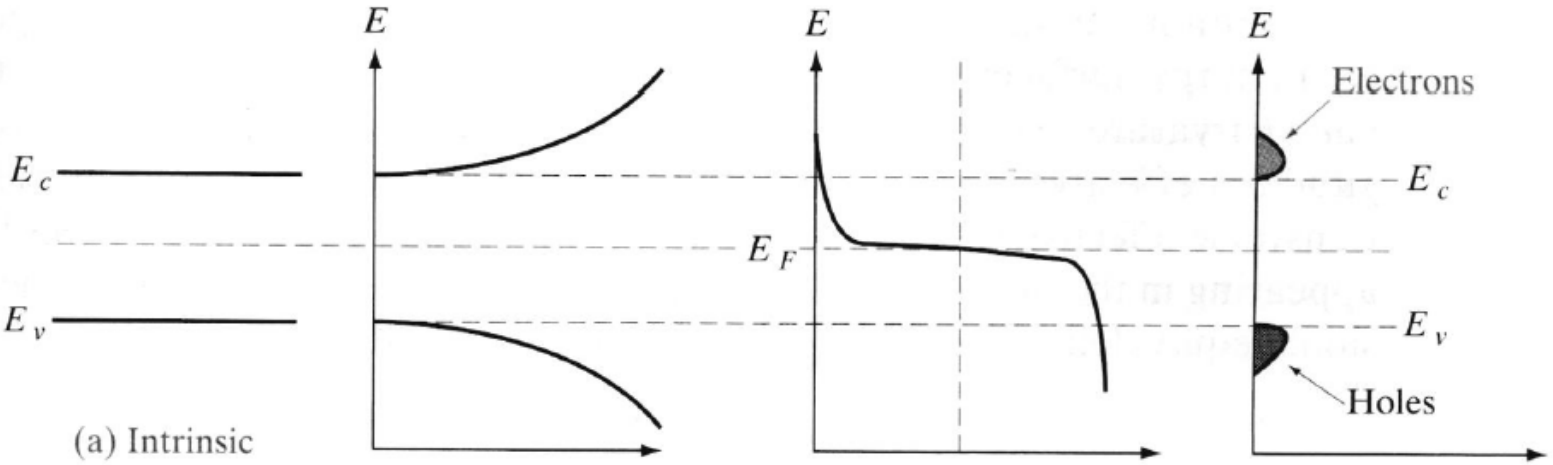


E_C is very close to E_F
Maxwell-Boltzmann distribution is not so accurate. Fermi-Dirac is needed

E_C is not too close to E_F
Maxwell-Boltzmann distribution may still be acceptable

E_C is far from E_F
Maxwell-Boltzmann and Fermi-Dirac coincide

Intrinsic case



$$n_i = N_C\, e^{-(E_C - E_i)/k_B T}$$

$$p_i = N_V\, e^{-(E_i - E_V)/k_B T}$$

$$n_i = p_i$$

Intrinsic Fermi Level

$$N_C\, e^{-(E_C - E_i)/k_B T} = N_V\, e^{-(E_i - E_V)/k_B T}$$

$$\exp\left(\frac{2E_i - E_C - E_V}{k_B T}\right) = \frac{N_V}{N_C}$$

$$\frac{2E_i - E_C - E_V}{k_B T} = \ln\left(\frac{N_V}{N_C}\right)$$

$$E_i = \frac{E_C + E_V}{2} + \frac{k_B T}{2}\ln\left(\frac{N_V}{N_C}\right) = \frac{E_C + E_V}{2} + \frac{k_B T}{2}\ln\left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$$= \frac{E_C + E_V}{2} + \frac{3k_B T}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Intrinsic concentration

$$n_i p_i = (n_i)^2 =$$

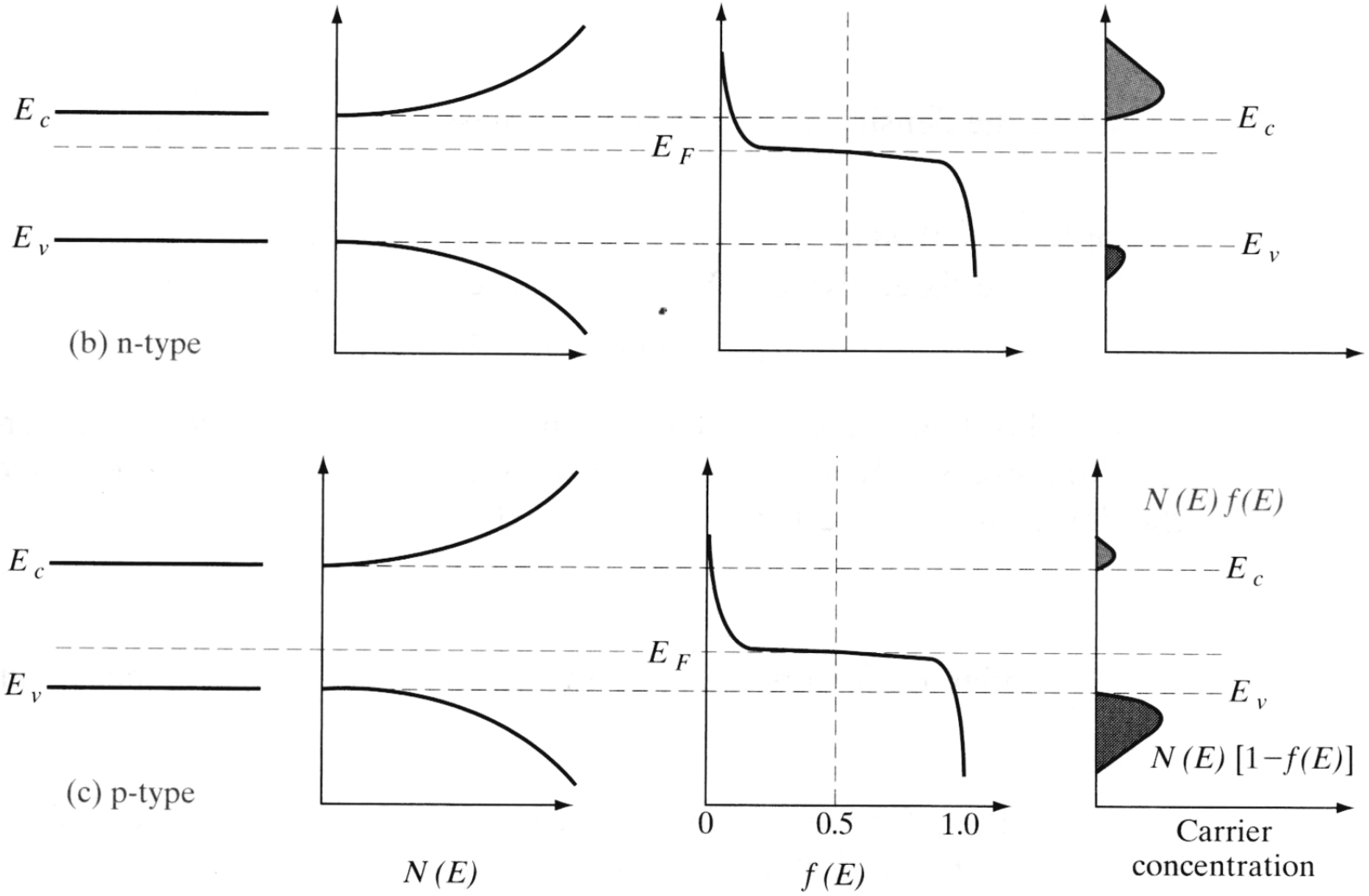
$$= N_C\, e^{-(E_C - E_i)/k_B T} \times N_V\, e^{-(E_i - E_V)/k_B T}$$

$$= N_C\, N_V e^{-(E_C - E_i + E_i - E_V)/k_B T}$$

$$= N_C\, N_V e^{-(E_C - E_V)/k_B T} = N_C\, N_V e^{-E_g/k_B T}$$

$$n_i = \sqrt{N_C\, N_V}\, e^{-E_g/2k_B T}$$

Extrinsic case



Extrinsic case

$$n_0 p_0 = N_C\, e^{-(E_C - E_F)/k_B T} \times N_V\, e^{-(E_F - E_V)/k_B T}$$

$$= N_C\, N_V e^{-(E_C - E_V)/k_B T} =$$

$$= N_C\, N_V e^{-E_g/k_B T}$$

Same as intrinsic case!

→ $n_0 p_0 = n_i^2$

Extrinsic case – useful alternative formulas

$$n_0 = N_C\, e^{-\left(E_C - E_i + E_i - E_F\right)/k_B T} =$$

$$= \underbrace{N_C\, e^{-\left(E_C - E_i\right)/k_B T}}_{n_i}\, e^{-\left(E_i - E_F\right)/k_B T} =$$

$$= n_i\, e^{\left(E_F - E_i\right)/k_B T}$$

Extrinsic case – useful alternative formulas

$$p_0 = N_V\, e^{-\left(E_F - E_i + E_i - E_V\right)/k_B T} =$$

$$= \underbrace{N_C\, e^{-\left(E_i - E_V\right)/k_B T}}_{p_i = n_i} e^{-\left(E_F - E_i\right)/k_B T} =$$

$$= n_i\, e^{\left(E_i - E_F\right)/k_B T}$$

Extrinsic case – useful alternative formulas

$$n_0 = n_i \, e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i \, e^{(E_i - E_F)/k_B T}$$

Only using Fermi level and the reference intrinsic Fermi level.

These forms are very useful for computation

Example

- Si doped with 10^{16} B atoms/cm^3 at T=300K in equilibrium.
  1. Find concentration of electrons and holes
  2. Determine Fermi level

1. Boron is an acceptor. We can start by assuming that at 300K all acceptors are ionized (this is an approximation which is not correct at much lower or much higher T)

$$p_0 \approx N_A = 10^{16}\text{cm}^{-3}$$

$$n_0 = \frac{(n_i)^2}{p_0} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = \frac{2.25\times10^{20}}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$$

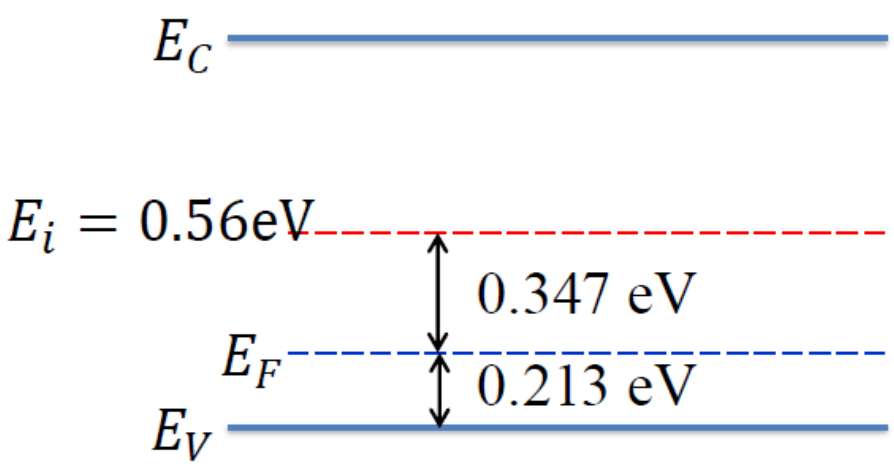
Example

2. Determine Fermi level

$$E_{gap}(\text{Si}) = 1.12\ \text{eV}$$
$$E_i \approx 0.5E_{gap} = 0.56\ \text{eV}$$

$$p_0 = n_i\, e^{(E_i - E_F)/k_B T}$$

$$\frac{E_i - E_F}{k_B T} = \ln\left(\frac{p_0}{n_i}\right)$$



$$E_i - E_F = k_B T \times \ln\left(\frac{p_0}{n_i}\right) =$$

$$= 0.0259 \ln\left(\frac{10^{16}}{1.5\times 10^{10}}\right) = 0.34732\,\text{eV}$$

Extras

- If you would like to consult the first ever book on solid state devices, a pdf scan of Shockley's "Electrons and Holes in Semiconductors (with application to transistor electronics)" of 1950, is available at

https://archive.org/details/ElectronsAndHolesInSemiconductors/mode/2up