ECE 340 Lecture 8
Semiconductor Devices

Spring 2022
10:00-10:50am

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Today's Discussion

- Electron & hole concentrations in equilibrium
- Temperature Dependence of carrier concentration
- Compensation and Space Charge Neutrality

Major points from Lecture 7

Density of electrons – Conduction Band

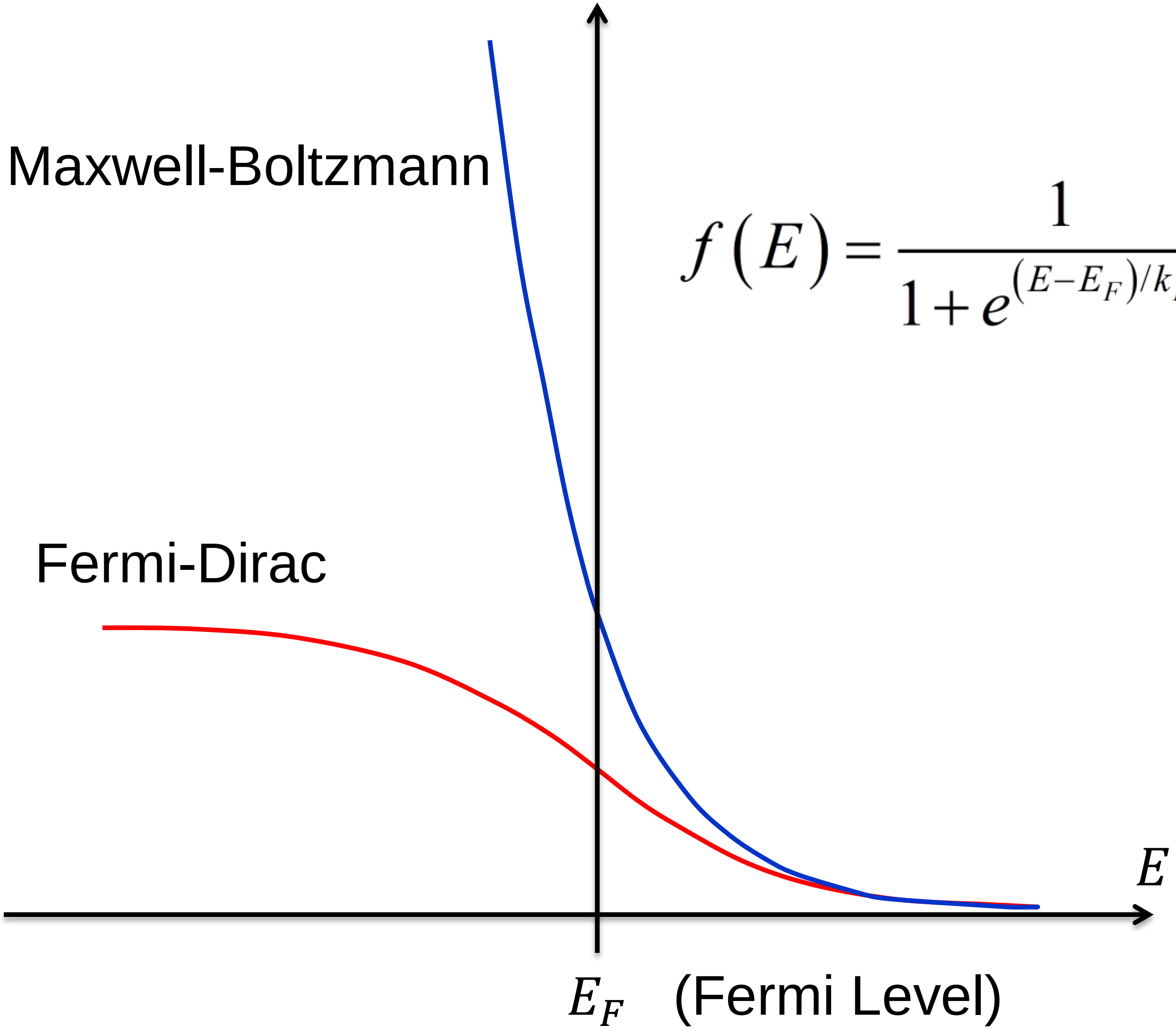
$$n_0 = \int_{E_c}^{\infty} \underbrace{f(E)}_{\text{probability of occupation}} \underbrace{N(E)dE}_{\text{density of states}}$$

We arrived at this integrated form

$$n_0 = \underbrace{N_C}_{\text{effective density of states}} \times \underbrace{f(E_C)}_{\text{probability of occupation for band edge energy}}$$

Problem: Fermi function cannot be integrated analytically in 3D

Probability of occupation (statistics)



$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \simeq e^{-(E-E_F)/k_B T}$$

From Lecture 7: Intrinsic case formulas

$$n_i = N_C \, e^{-(E_C - E_i)/k_B T}$$

$$p_i = N_V \, e^{-(E_i - E_V)/k_B T}$$

$$\boxed{n_i = p_i}$$

Intrinsic Fermi level

$$E_i = \frac{E_C + E_V}{2} + \frac{3k_B T}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Effective density of states

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2} \qquad N_V = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}$$

From Lecture 7: Extrinsic case formulas

Using effective density of states

$$n_0 = N_C \, e^{-(E_C - E_F)/k_B T}$$

$$p_0 = N_V \, e^{-(E_F - E_V)/k_B T}$$

Using intrinsic reference concentration

$$n_0 = n_i \, e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i \, e^{(E_i - E_F)/k_B T}$$

mass action law

$$\boxed{n_0 p_0 = n_i^2}$$

Example

- Si doped with 10^{16} B atoms/cm^3 at T=300K in equilibrium.
    1. Find concentration of electrons and holes
    2. Determine Fermi level

1. Boron is an acceptor. We can start by assuming that at 300K all acceptors are ionized (this is an approximation which is not correct at much lower or much higher T)

$$p_0 \approx N_A = 10^{16}\text{cm}^{-3}$$

- valid at high T as long as $n_i \ll N_A$
- at much lower T, decreasing $p_0 < N_A$

$$n_0 = \frac{(n_i)^2}{p_0} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = \frac{2.25\times10^{20}}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Example

2. Determine Fermi level

$$E_{gap}(\mathrm{Si}) = 1.12\ \mathrm{eV}$$

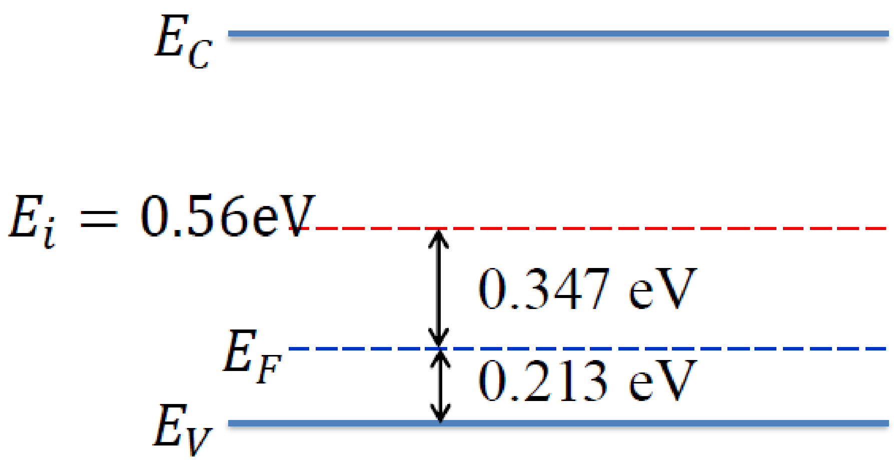
$$E_i \approx 0.5E_{gap} = 0.56\ \mathrm{eV}$$

$$p_0 = n_i\, e^{(E_i - E_F)/k_B T}$$

$$\frac{E_i - E_F}{k_B T} = \ln\left(\frac{p_0}{n_i}\right)$$

$$E_i - E_F = k_B T \times \ln\left(\frac{p_0}{n_i}\right) =$$

$$= 0.0259 \ln\left(\frac{10^{16}}{1.5\times 10^{10}}\right) = 0.34732\,\mathrm{eV}$$



Effective Density of States and Masses

The next slide presents a summary of values of effective density of states and effective masses, assembled for representative semiconductor materials (Si, Ge, GaAs).

Keep in mind that these are approximate values and are not meant to represent "absolutely correct" ones. There is a always certain variance in the available measurements of related physical constants. Values may also depend on the model used to calculate them.

Take the following table as a reasonable set of data which gives a good guideline for practical applications.

Effective Density of States and Masses

T = 300K

Material	Valence Band	Conduction Band
Si	$N_V = 1.08 \times 10^{19}\text{cm}^{-3}$ $[= 1.83 \times 10^{19}\text{cm}^{-3}]^{\dagger}$ $m^*_{p(d)} = 0.57\, m_0\ [0.81\, m_0]^{\dagger}$ $m^*_{p(c)} = 0.36\, m_0\ [0.386\, m_0]^{\dagger}$	$N_C = 2.82 \times 10^{19}\text{cm}^{-3}$ $m^*_{n(d)} = 1.08\, m_0$ $m^*_{n(c)} = 0.26\, m_0$
Ge	$N_V = 3.92 \times 10^{18}\text{cm}^{-3}$ $m^*_{p(d)} = 0.29\, m_0$ $m^*_{p(c)} = 0.21 m_0$	$N_C = 1.05 \times 10^{19}\text{cm}^{-3}$ $m^*_{n(d)} = 0.56\, m_0$ $m^*_{n(c)} = 0.12\, m_0$
GaAs	$N_V = 8.68 \times 10^{18}\text{cm}^{-3}$ $m^*_{p(d)} = 0.47 m_0$ $m^*_{p(h)} = 0.34\, m_0$	$N_C = 4.37 \times 10^{17}\text{cm}^{-3}$ $m^*_{n(d)} = 0.067\, m_0$ $m^*_{n(c)} = 0.067\, m_0$

$m_0 = 9.10938356 \times 10^{-31}\text{kg}$

†corrected for realistic warped symmetry of heavy holes valence band in Si

(d) = indicates "density of states mass" (c) = indicates "conductivity mass"

Effective Density of States and Masses

Material	Valence Band	Conduction Band
Si	$N_V = 1.08 \times 10^{19}\text{cm}^{-3}$ $[= 1.83 \times 10^{19}\text{cm}^{-3}]^\dagger$ $m^*_{p(d)} = 0.57\, m_0$ $[0.81\, m_0]^\dagger$ $m^*_{p(c)} = 0.36\, m_0$ $[0.386\, m_0]^\dagger$	$N_C = 2.82 \times 10^{19}\text{cm}^{-3}$ $m^*_{n(d)} = 1.08\, m_0$ $m^*_{n(c)} = 0.26\, m_0$

The calculation of n_i is sensitive to the value of E_g (besides the model for N_V) used. If you fix a conventional value of n_i :

$N_V = 1.83 \times 10^{19}\text{cm}^{-3}$
$E_g = 1.095$ eV
$n_i = 1.5 \times 10^{10}\text{cm}^{-3}$

↔

$N_V = 1.08 \times 10^{19}\text{cm}^{-3}$
$E_g = 1.081$ eV
$n_i = 1.5 \times 10^{10}\text{cm}^{-3}$

There is always some built-in "uncertainty" in any set of physical parameters obtained from a combination of measured data and theoretical models.

†corrected for realistic warped symmetry of valence band in Si

Intrinsic concentration

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2}$$

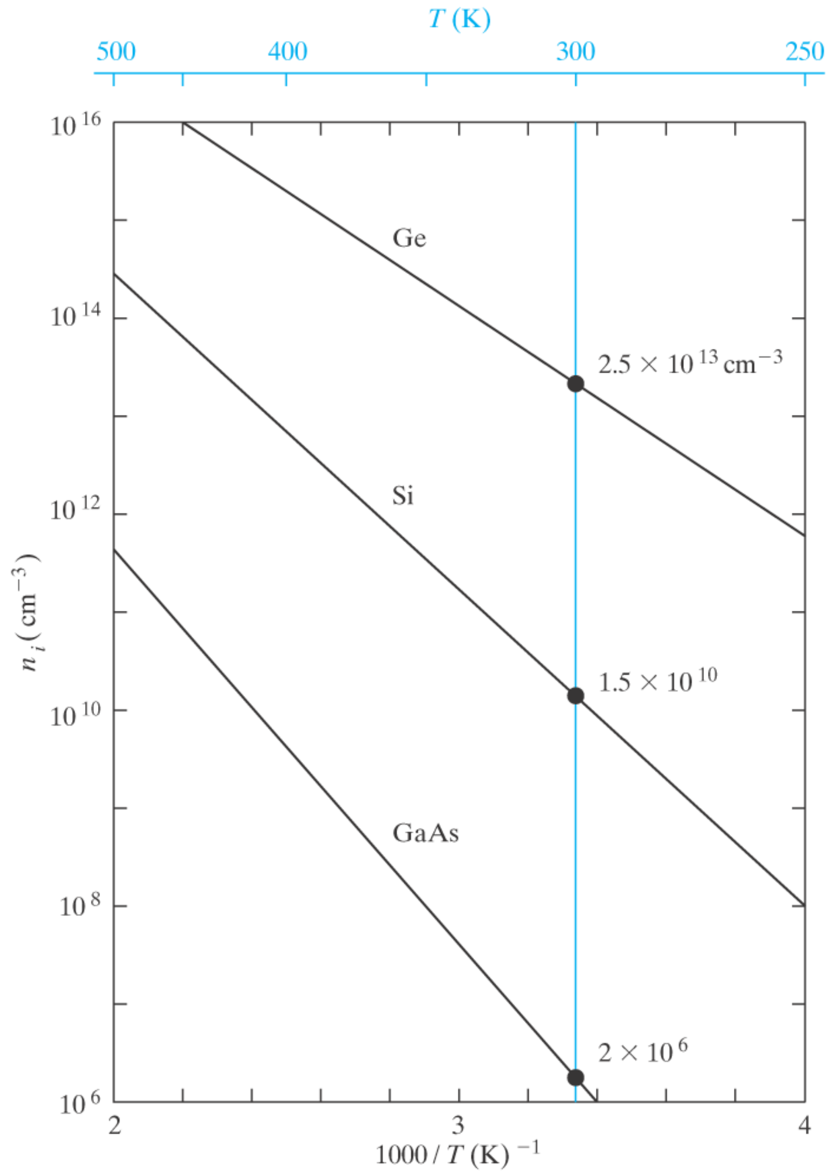
$$N_V = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}$$

$$n_i = \sqrt{N_C \; N_V} \; e^{-E_g/2k_BT} =$$

$$= 2\left(\frac{2\pi \, m_0 \, k_B}{h^2}\right)^{3/2}\left(\frac{m_n^* \, m_p^*}{m_0}\right)^{3/4} T^{3/2} \; e^{-E_g/2k_BT}$$

$$= 4.83\times 10^{15}\left(\frac{m_n^* \, m_p^*}{m_0}\right)^{3/4} T^{3/2} \; e^{-E_g/2k_BT} \quad \text{cm}^{-3}$$

Temperature dependence of n_i



Extrinsic semiconductor case

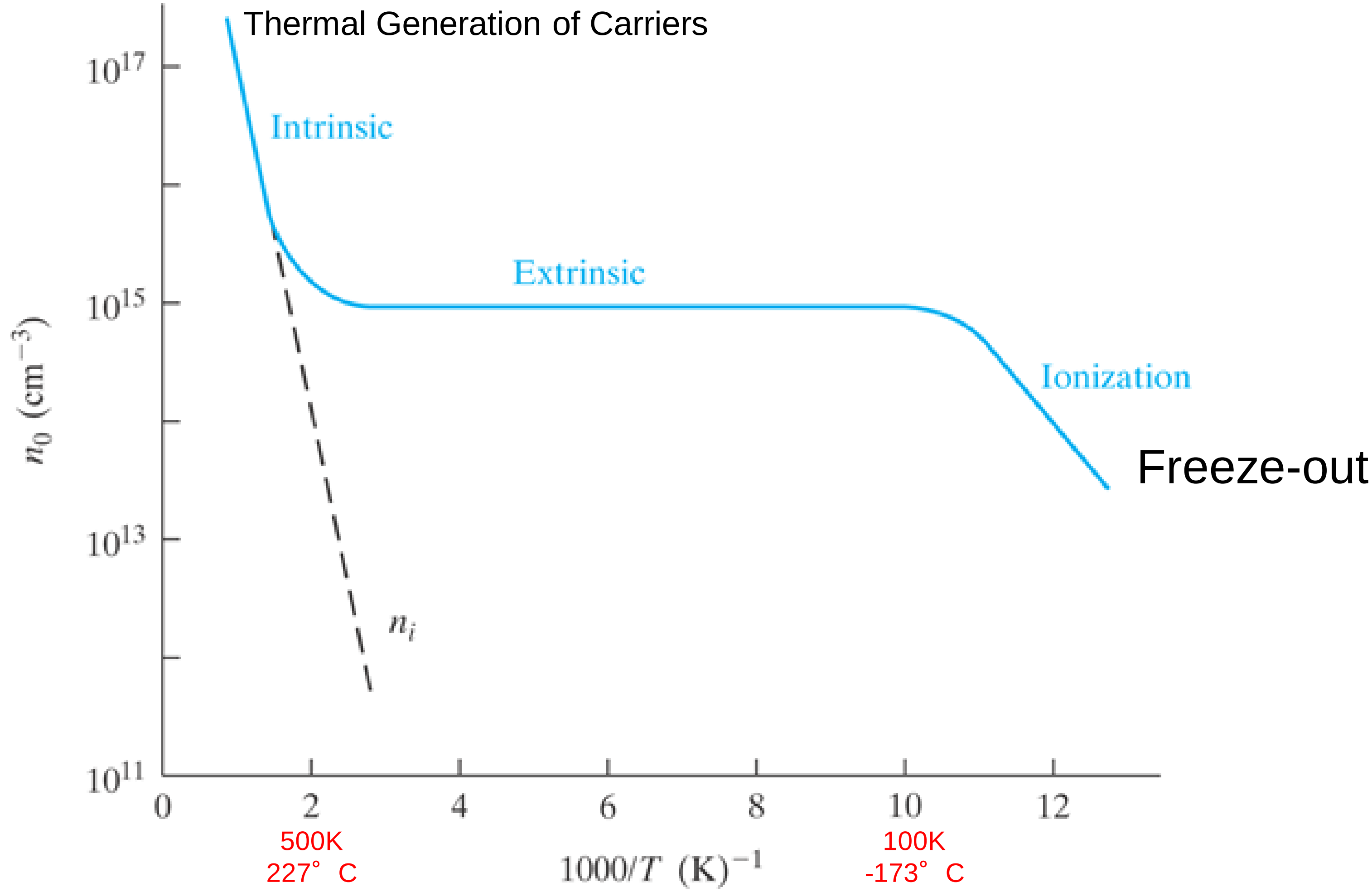


Figure 3.18

Carrier concentration vs. inverse temperature for Si doped with 10^{15} donors/cm^3.

Extrinsic semiconductor case

Dopant ionization – more complete model including effect of temperature

$$N_D^+ = \frac{N_D}{1+2\exp\left(\frac{E_F - E_D}{k_B T}\right)}$$

$$N_A^- = \frac{N_A}{1+4\exp\left(\frac{E_A - E_F}{k_B T}\right)}$$

Charge neutrality condition $n_o + N_A^- = p_o + N_D^+$

Assuming complete ionization $n_o + N_A = p_o + N_D$

In the course we will normally consider complete ionization

Compensation

Donor and Acceptor dopants balance each other out

$$N_D > N_A$$

$$N_{D,eff} = N_D - N_A$$

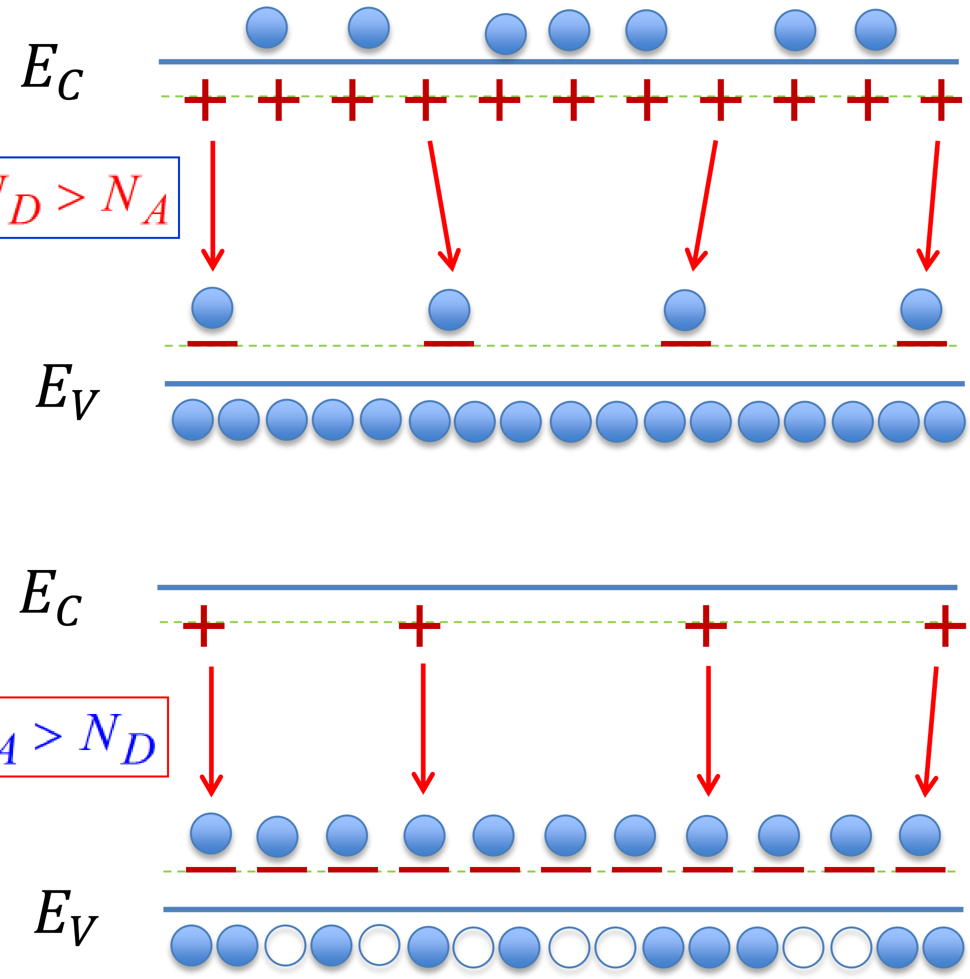
$$N_A > N_D$$

$$N_{A,eff} = N_A - N_D$$

$$N_D = N_A$$

$$N_{D,eff} = N_{A,eff} = 0$$

$n = p = n_i$ intrinsic!



Compensation

GaAs

T=300K $\quad n_i \approx 2.0\times10^{6}\,\text{cm}^{-3}$

$N_D = 10^{17}\,\text{cm}^{-3}$

$N_A = 7\times10^{16}\,\text{cm}^{-3}$

$N_D > N_A \quad \& \quad N_D - N_A \gg n_i$

$$n_o \simeq N_D^+ - N_A^- = 10^{17} - 7.0\times10^{16}\,\text{cm}^{-3}$$

$$n_o \simeq 3.0\times10^{16}\,\text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} \simeq \frac{\left(2.0\times10^{6}\right)^2}{3.0\times10^{16}} = 1.\overline{3}\times10^{-4}\,\text{cm}^{-3}$$

Compensation

Si

T=300K $\quad n_i \approx 1.5\times10^{10}\,\text{cm}^{-3}$

$N_A = 10^{16}\,\text{cm}^{-3}$

$N_D = 5.0\times10^{15}\,\text{cm}^{-3}$

$N_A > N_D \quad \& \quad N_A - N_D \gg n_i$

$$p_o \simeq N_A^- - N_D^+ = 10^{16} - 5.0\times10^{15}\,\text{cm}^{-3}$$

$$p_o \simeq 5.0\times10^{15}\,\text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} \simeq \frac{\left(1.5\times10^{10}\right)^2}{5.0\times10^{15}} = 4.5\times10^{4}\ \text{cm}^{-3}$$

Compensation

Ge

T=300K $n_i \approx 2.5\times10^{13}$

$N_D > N_A$ and $N_D - N_A \simeq n_i$

$N_D = 2.0\times10^{14}\,\mathrm{cm}^{-3}$

$N_A = 1.5\times10^{14}\,cm^{-3}$

$$n_o - p_o = N_D^+ - N_A^- = 2\times10^{14} - 1.5\times10^{14}\,\mathrm{cm}^{-3}$$

$$n_o - p_o = 5.0\times10^{13}\,\mathrm{cm}^{-3}$$

apply charge neutrality

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(2.5\times10^{13}\right)^2}{n_o}$$

apply mass action law

$$n_o - \frac{\left(2.5\times10^{13}\right)^2}{n_o} = 5.0\times10^{13}\,\mathrm{cm}^{-3}$$

insert in charge neutrality eq

Compensation

$$n_o - \frac{\left(2.5\times10^{13}\right)^2}{n_o} = 5.0\times10^{13}\,cm^{-3}$$

$$n_o^2 - 5.0\times10^{13}\;n_o\; - \left(2.5\times10^{13}\right)^2 = 0$$

$$n_o = \frac{5.0\times10^{13} \pm \sqrt{\left(5.0\times10^{13}\right)^2 + 4\left(2.5\times10^{13}\right)^2}}{2}$$

$$= \frac{5.0\times10^{13} \pm 7.071\times10^{13}}{2} = 6.036\times10^{13}\,cm^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = 1.035\times10^{13}\,cm^{-3}$$

$$n_o p_o = 6.036\times10^{13} \times 1.035\times10^{13} = 6.25\times10^{26} = n_i^2$$