Today’s Discussion

• Temperature Dependence of carrier concentration
• Compensation and Space Charge Neutrality
• Semiconductor out of equilibrium
• Conductivity
• Mobility
Intrinsic concentration

\[ N_C = 2 \left( \frac{2\pi m_n^* k_B T}{h^2} \right)^{3/2} \]

\[ N_V = 2 \left( \frac{2\pi m_p^* k_B T}{h^2} \right)^{3/2} \]

\[ n_i = \sqrt{N_C N_V} e^{-E_g / 2k_B T} = \]

\[ = 2 \left( \frac{2\pi m_0 k_B}{h^2} \right)^{3/2} \left( \frac{m_n^* m_p^*}{m_0} \right)^{3/4} T^{3/2} e^{-E_g / 2k_B T} \]

\[ = 4.83 \times 10^{15} \left( \frac{m_n^* m_p^*}{m_0} \right)^{3/4} T^{3/2} e^{-E_g / 2k_B T} \text{ cm}^{-3} \]
Temperature dependence of $n_i$
Extrinsic semiconductor case

Thermal Generation of Carriers

Figure 3.18

Carrier concentration vs. inverse temperature for Si doped with $10^{15}$ donors/cm$^3$. 

- Intrinsic
- Extrinsic
- Ionization
- Freeze-out

Key points:
- 500K - 227°C
- 300K - 27°C
- 100K - 173°C
\[ N_D^+ = \frac{N_D}{1 + 2\exp\left(\frac{E_F - E_D}{k_B T}\right)} \]

\[ N_A^- = \frac{N_A}{1 + 4\exp\left(\frac{E_A - E_F}{k_B T}\right)} \]

Charge neutrality:

\[ n_o + N_A^- = p_o + N_D^+ \]

Complete ionization:

\[ n_o + N_A = p_o + N_D \]

In the course we will normally consider complete ionization.
Compensation

Donor and Acceptor dopants balance each other out

\[ N_D > N_A \]
\[ N_{D,\text{eff}} = N_D - N_A \]

\[ N_A > N_D \]
\[ N_{A,\text{eff}} = N_A - N_D \]

\[ N_D = N_A \]
\[ N_{D,\text{eff}} = N_{A,\text{eff}} = 0 \]
\[ n = p = n_i \quad \text{intrinsic!} \]
GaAs

\[ T = 300\text{K} \quad n_i \approx 2.0 \times 10^6 \text{cm}^{-3} \]

\[ N_D = 10^{17} \text{cm}^{-3} \]

\[ N_A = 7 \times 10^{16} \text{cm}^{-3} \]

\[ N_D > N_A \quad \& \quad N_D - N_A \gg n_i \]

\[ n_o \approx N_D^+ - N_A^- = 10^{17} - 7.0 \times 10^{16} \text{cm}^{-3} \]

\[ n_o \approx 3.0 \times 10^{16} \text{cm}^{-3} \]

\[ p_o = \frac{n_i^2}{n_o} \approx \frac{(2.0 \times 10^6)^2}{3.0 \times 10^{16}} = 1.3 \times 10^{-4} \text{cm}^{-3} \]
Compensation

Si

T=300K \quad n_i \approx 1.5 \times 10^{10} \text{ cm}^{-3}

N_A = 10^{16} \text{ cm}^{-3}

N_D = 5.0 \times 10^{15} \text{ cm}^{-3}

N_A > N_D \quad \& \quad N_A - N_D \gg n_i

p_o \approx N_A^- - N_D^+ = 10^{16} - 5.0 \times 10^{15} \text{ cm}^{-3}

p_o \approx 5.0 \times 10^{15} \text{ cm}^{-3}

n_o = \frac{n_i^2}{p_o} \approx \frac{(1.5 \times 10^{10})^2}{5.0 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}
Compensation

Ge

\[ T = 300 \text{K} \quad n_i \approx 2.5 \times 10^{13} \]

\[ N_D > N_A \quad \text{and} \quad N_D - N_A \approx n_i \]

\[ N_D = 2.0 \times 10^{14} \text{cm}^{-3} \]

\[ N_A = 1.5 \times 10^{14} \text{cm}^{-3} \]

\[
\begin{align*}
    n_o - p_o &= N_D^+ - N_A^- = 2 \times 10^{14} - 1.5 \times 10^{14} \text{cm}^{-3} \\
    n_o - p_o &= 5.0 \times 10^{13} \text{cm}^{-3} \\
    p_o &= \frac{n_i^2}{n_o} = \frac{(2.5 \times 10^{13})^2}{n_o} \\
    n_o - \frac{(2.5 \times 10^{13})^2}{n_o} &= 5.0 \times 10^{13} \text{cm}^{-3}
\end{align*}
\]
Compensation

\[
n_o - \frac{(2.5 \times 10^{13})^2}{n_o} = 5.0 \times 10^{13} \text{ cm}^{-3}
\]

\[
n_o^2 - 5.0 \times 10^{13} n_o - (2.5 \times 10^{13})^2 = 0
\]

\[
n_o = \frac{5.0 \times 10^{13} \pm \sqrt{(5.0 \times 10^{13})^2 + 4(2.5 \times 10^{13})^2}}{2}
\]

\[
= \frac{5.0 \times 10^{13} \pm 7.071 \times 10^{13}}{2} = 6.036 \times 10^{13} \text{ cm}^{-3}
\]

\[
p_o = \frac{n_i^2}{n_o} = 1.035 \times 10^{13} \text{ cm}^{-3}
\]

\[
n_o p_o = 6.036 \times 10^{13} \times 1.035 \times 10^{13} = 6.25 \times 10^{26} = n_i^2
\]
JAVA App – Doping in Silicon

Conduction Band

Fermi Level

Valence Band

Donor Doping

\[ N_D = 0.0 \text{ [cm}^{-3}\text{]} = 0.0 \text{ [m}^{-3}\text{]} \]

Acceptor Doping

\[ N_A = 0.0 \text{ [cm}^{-3}\text{]} = 0.0 \text{ [m}^{-3}\text{]} \]

Temperature

\[ T = 300.0 \text{ K} = 27.0^\circ \text{C} = 80.6^\circ \text{F} \]

\[ 1000 / T = 3.33 \text{ K}^{-1} \]

Silicon

\[ n_i = 1.5 \times 10^{10} \text{ [cm}^{-3}\text{]} \]

(intrinsic semiconductor)
Experimental measurements show that the bandgaps of most semiconductors decrease with increasing temperature. This can be described with an empirical relation [C.D. Thurmond, 1975]

\[ E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta} \]

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>( E_g(0) ) [eV]</th>
<th>( \alpha )</th>
<th>( \beta )</th>
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<tr>
<td>Silicon</td>
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<td>Gallium Arsenide</td>
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<td>( 5.405 \times 10^{-4} )</td>
<td>204</td>
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Materials out of Equilibrium

Until now we have considered materials in equilibrium.

Let’s apply an electric field. At first we will not worry about the detailed physics but focus on macroscopic properties.

A material can be characterized electrically using its conductivity or its resistivity.
Conductivity and Resistivity

- A current density is induced in the material by applying an Electric Field

\[ \mathbf{E}_x \quad \text{Force on electron} \]

\[ -q \mathbf{E}_x \]

\[ J_x = \sigma \mathbf{E}_x \]

\[ \sigma = \frac{1}{\rho} = \text{conductivity} \quad \left[ \frac{S}{m} \right] \]

\[ \rho = \text{resistivity} \quad [\Omega \text{ m}] \]

S = Siemens = \Omega^{-1}
Conductivity and Resistivity

• We can measure the resistivity, but how do we relate it to the physics of motion of electrons?

• We have said that through the introduction of the band structure we have created a model for a new equivalent “free space” where the “quasi-particles” electrons and holes follow a new energy-momentum law $E(\vec{k})$.

• But inside a solid we also need to consider the collisions (scatterings) experienced by particles.
Scattering

- What kind of collision events scatter the particles?
- The most important ones are:
  - Vibrations of crystal atoms (phonons) scattering increases with temperature
  - Ionized impurity scattering (Coulomb interaction) scattering increases with carrier concentration
  - Electron-electron interaction (Coulomb interaction) scattering increases with doping concentration
Electrons in crystal in equilibrium

- Electrons behave like a “gas” and obey the Boltzmann distribution at a specific temperature (this is our initial condition)

**Average kinetic energy**

\[
E_{\text{kinetic}} = \frac{1}{2} m^*_n \, v_{th}^2
\]

\[
3D) \quad E_{\text{thermal}} = \frac{3}{2} k_B T
\]

\[
\frac{1}{2} m^*_n \, v_{th}^2 = \frac{3}{2} k_B T
\]

\[
v_{th} = \sqrt{\frac{3k_B T}{m^*_n}}
\]

thermal velocity
\( \nu_{th} \) in Si at 300K for electrons

\[
\nu_{th} = \sqrt{\frac{3k_B T}{m^*_n}} = \\
= \sqrt{\frac{3 \times 1.3806 \times 10^{-23} \times 300}{0.26 \times 9.11 \times 10^{-31}}} \text{ conductivity mass} \\
= 2.29 \times 10^5 \text{ m/s} = 2.29 \times 10^7 \text{ cm/s}
\]

- This can be interpreted as the root-mean-square of the total speed in 3D.
Scattering

• For a given field there is going to be a random distribution of “times between collisions” and a corresponding “mean free path”.

\[
\text{Average time between collisions} = \tau_c
\]
\[
\text{Mean free path} \quad = \lambda = v_{th} \tau_c
\]
\[
\tau_c \approx \left[10^{-14}, 10^{-13}\right] \text{ seconds}
\]
\[
v_{th} \approx 10^5 \text{ m/s}
\]
\[
\lambda \approx 1 \text{ to } 10 \text{ nm}
\]
Velocity distribution in equilibrium

Symmetric distribution, no net current (no field applied)
Electron drift under a field

- The individual path of the electron is apparently chaotic, but it will tend to “drift” on the average in the direction opposite to the field.
Energy balance

- The applied field tries to direct and accelerate the individual velocities along a given direction, thus increasing the energy of the system.
  - Scattering randomizes the direction of individual electron velocities.
  - Phonon scattering exchanges energy between gas of electrons and vibrating lattice atoms.
  - Scattering with ionized impurities only changes directions, does not exchange energy.
  - Electron-electron interaction redistributes energy among electrons (important in contact regions).
Simple model for momentum scattering loss

**Assumptions:** $\tau_c$ is constant for all energies; momentum increase due to field is all lost in a collision (Drude model) so that particle relaxes to thermal equilibrium conditions.

\[ p_x(t) = \begin{cases} n \frac{\Delta t}{\tau_c} \text{ electrons scatter} & \text{lost} \\ n \left(1 - \frac{\Delta t}{\tau_c}\right) \text{ electrons do not scatter} \end{cases} \]

\[ p_x \left(1 - \frac{\Delta t}{\tau_c}\right) \text{ momentum} \]
Simple model for momentum scattering loss

**Assumptions:** \( \tau_c \) is constant for all energies; momentum increase due to field is all lost in a collision (Drude model) so that particle relaxes to thermal equilibrium conditions.

\[
\begin{align*}
\Delta n &= \frac{\Delta t}{\tau_c} n \\
# of particles that scatter and thermalize
\end{align*}
\]

\[
\begin{align*}
t + \Delta t &\) n - \Delta n \\
# of particles that have acquired momentum
\end{align*}
\]

\[
\begin{align*}
p_x(t) + \Delta p_x &= p_x(t) - \frac{p_x(t)}{\tau_c} \Delta t \\
\Delta p_x &= -\frac{p_x(t)}{\tau_c} \Delta t
\end{align*}
\]
Steady-state condition

- In steady state a specific current density is established for a given electric field.
- Thus, the net acceleration of electrons due to the field produces an increase of the average momentum which has to be balanced out by the deceleration due to the collisions.

The electron velocity distribution is slightly shifted following the field direction.
Momentum change

Consider a group of $n$ electrons

- Rate of change of momentum due to Field

\[
\Delta p_x = -nq\mathcal{E}_x \Delta t \quad \Rightarrow \quad \frac{dp_x}{dt}\bigg|_{\text{Field}} = -nq\mathcal{E}_x
\]

- Rate of change of momentum due to scattering

\[
\Delta p_x = \frac{-p_x}{\tau_c} \Delta t \quad \Rightarrow \quad \frac{dp_x}{dt}\bigg|_{\text{Collisions}} = -\frac{p_x}{\tau_c}
\]
Steady-state = constant current

\[ \frac{dp_x}{dt}_{\text{Field}} + \frac{dp_x}{dt}_{\text{Collisions}} = 0 \]

\[-nqE_x - \frac{p_x}{\tau_c} = 0\]

\[\langle p_x \rangle = \frac{p_x}{n}\]

Average momentum per electron

**Drift velocity**

\[\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = -\frac{q\tau_c E_x}{m_n^*}\]

Similarly for holes

\[\langle v_x \rangle = \frac{\langle p_x \rangle}{m_p^*} = \frac{q\tau_c E_x}{m_p^*}\]
Define Drift Current Density

\[ J = \text{Charge per unit time per unit area} \]

\[ J_{\text{drift}} = -qn \langle v_{dn} \rangle = qn \mu_n E_x \]

\[ J_{\text{drift}} = qp \langle v_{dp} \rangle = qp \mu_p E_x \]
Mobility

\[ \mu_n = - \frac{\langle v_{dn} \rangle}{\mathcal{E}_x} = \frac{q \tau_c}{m_n^*} \]

\[ \mu_p = \frac{\langle v_{dp} \rangle}{\mathcal{E}_x} = \frac{q \tau_c}{m_p^*} \]

units

\[ \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \]
\( \mu \) is the slope of the curve at \( \mathcal{E}=0 \) and is usually called low-field mobility.
<table>
<thead>
<tr>
<th>Material</th>
<th>$E_g$ [eV]</th>
<th>$\mu_n$ [cm²/V·s]</th>
<th>$\mu_p$ [cm²/V·s]</th>
<th>$m_n^*/m_o$</th>
<th>$m_p^*/m_o$</th>
<th>$\alpha$ [Å]</th>
<th>$\varepsilon_r$</th>
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All values at 300 K.

* Vaporizes