ECE 340 Lectures 9-10 Semiconductor Devices

Spring 2022
10:00-10:50am
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Today's Discussion

- Temperature Dependence of carrier concentration

- Compensation and Space Charge Neutrality

- Semiconductor out of equilibrium

- Conductivity

- Mobility

Intrinsic concentration

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2} \qquad N_V = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}$$

$$n_i = \sqrt{N_C \; N_V} \; e^{-E_g/2k_B T} =$$

$$= 2\left(\frac{2\pi m_0 k_B}{h^2}\right)^{3/2} \left(\frac{m_n^* \; m_p^*}{m_0}\right)^{3/4} T^{3/2} \; e^{-E_g/2k_B T}$$

$$= 4.83 \times 10^{15} \left(\frac{m_n^* \; m_p^*}{m_0}\right)^{3/4} T^{3/2} \; e^{-E_g/2k_B T} \quad cm^{-3}$$

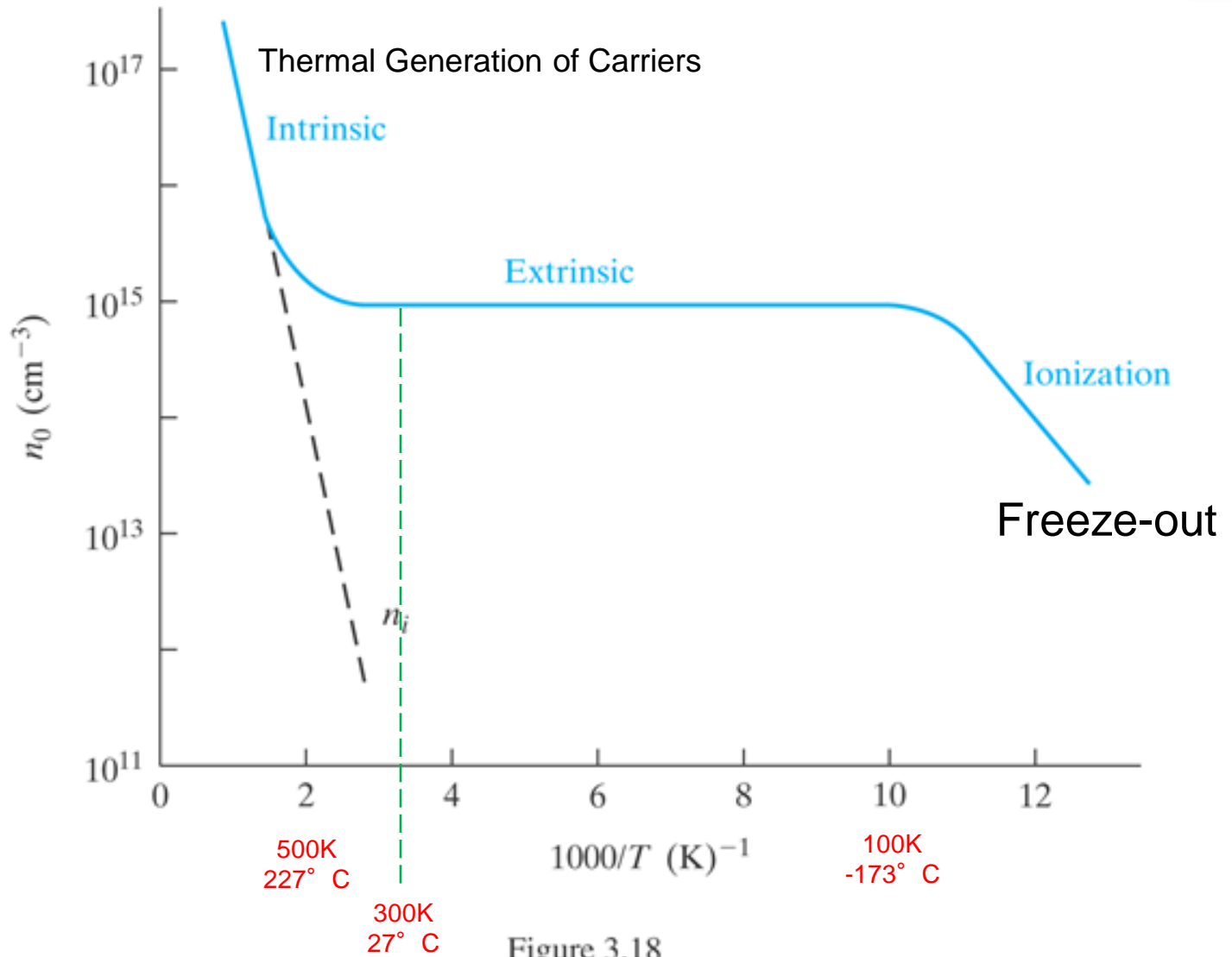# Temperature dependence of n_i
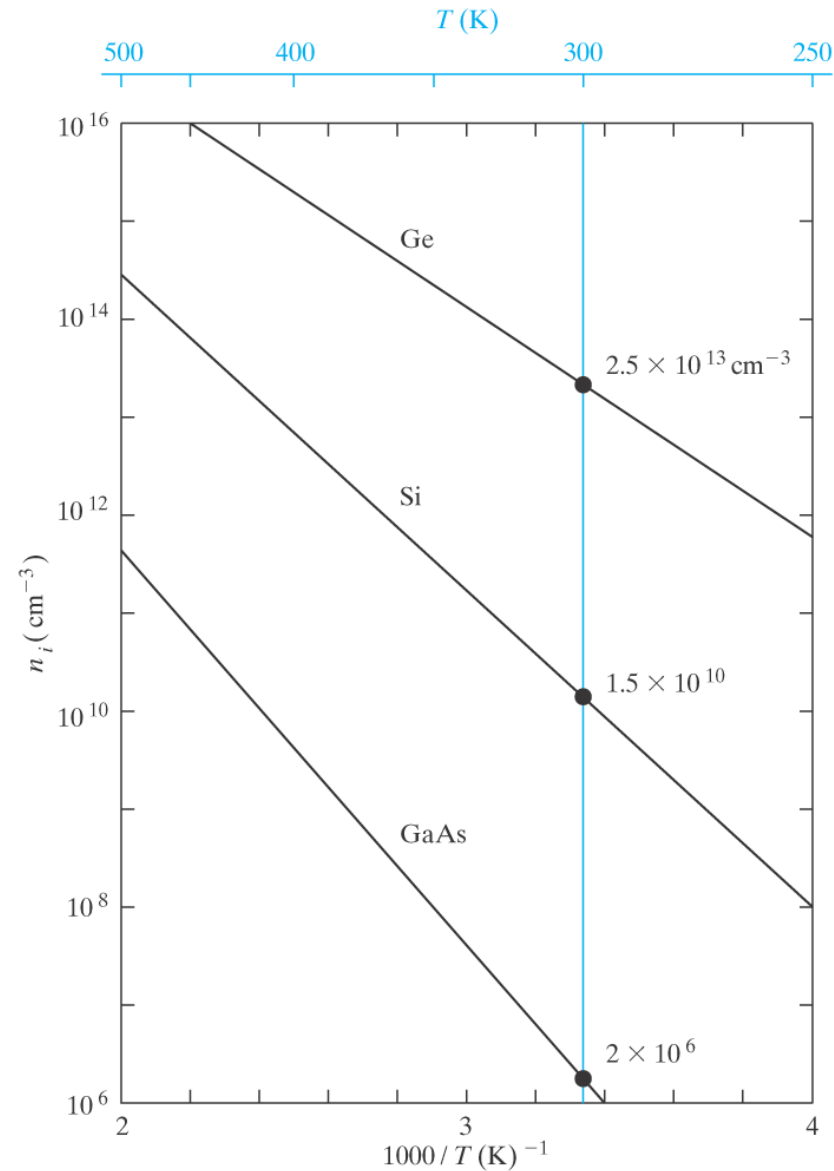
Extrinsic semiconductor case

Carrier concentration vs. inverse temperature for Si doped with 10^{15} donors/cm^3.

Extrinsic semiconductor case

$$N_D^+ = \frac{N_D}{1 + 2\exp\left(\dfrac{E_F - E_D}{k_B T}\right)}$$

$$N_A^- = \frac{N_A}{1 + 4\exp\left(\dfrac{E_A - E_F}{k_B T}\right)}$$

Charge neutrality

$$n_o + N_A^- = p_o + N_D^+$$

Complete ionization

$$n_o + N_A = p_o + N_D$$

In the course we will normally consider complete ionization

Compensation

Donor and Acceptor dopants balance each other out

$$N_D > N_A$$

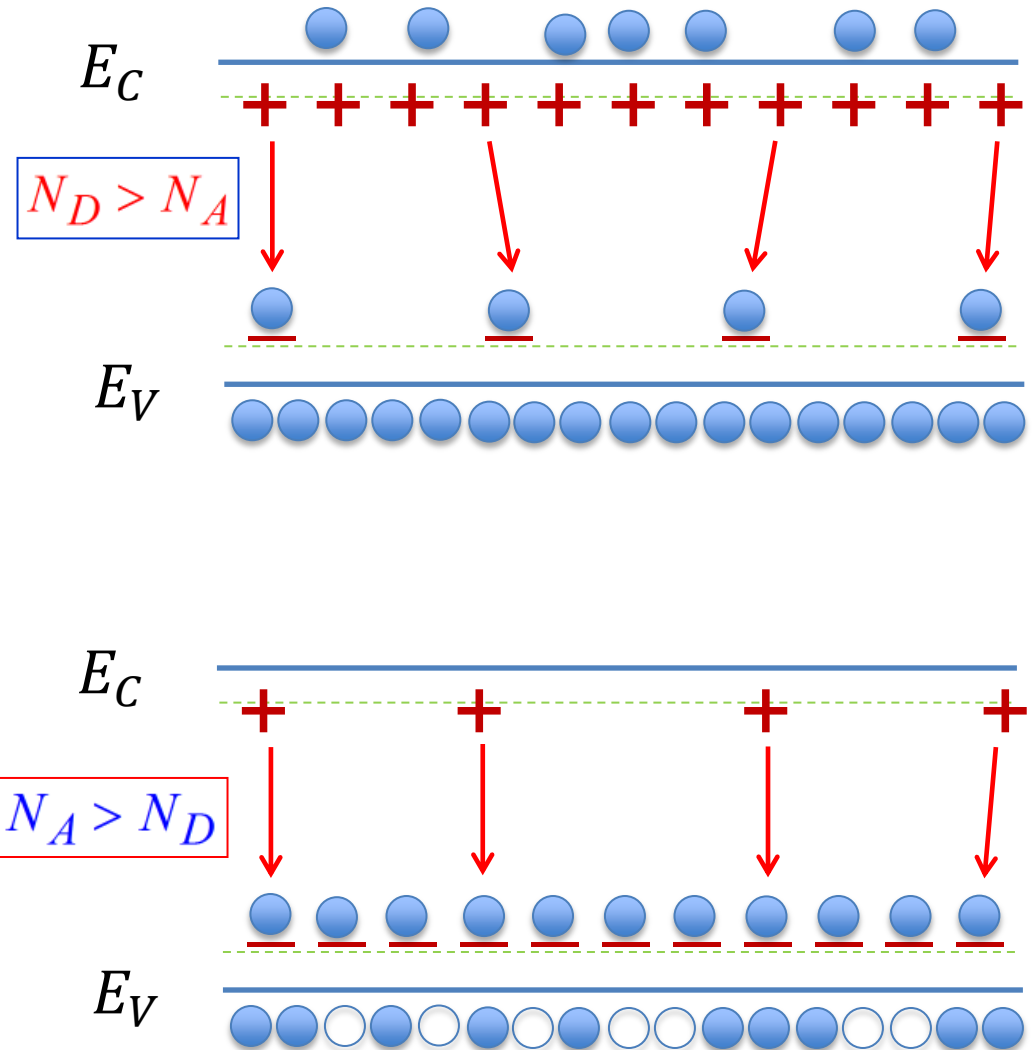$$N_{D,eff} = N_D - N_A$$

$$N_A > N_D$$

$$N_{A,eff} = N_A - N_D$$

$$N_D = N_A$$

$$N_{D,eff} = N_{A,eff} = 0$$

$$n = p = n_i \quad \text{intrinsic!}$$

Compensation

GaAs

T=300K $\qquad n_i \approx 2.0 \times 10^6 \, \text{cm}^{-3}$

$N_D = 10^{17} \, \text{cm}^{-3}$

$N_A = 7 \times 10^{16} \, \text{cm}^{-3}$

$N_D > N_A \quad \& \quad N_D - N_A \gg n_i$

$n_o \simeq N_D^+ - N_A^- = 10^{17} - 7.0 \times 10^{16} \, \text{cm}^{-3}$

$n_o \simeq 3.0 \times 10^{16} \, \text{cm}^{-3}$

$p_o = \dfrac{n_i^2}{n_o} \simeq \dfrac{\left(2.0 \times 10^6\right)^2}{3.0 \times 10^{16}} = 1.\overline{3} \times 10^{-4} \, \text{cm}^{-3}$

Compensation

Si

$T = 300K \qquad n_i \approx 1.5 \times 10^{10} \, \text{cm}^{-3}$

$N_A = 10^{16} \, \text{cm}^{-3}$

$N_D = 5.0 \times 10^{15} \, \text{cm}^{-3}$

$N_A > N_D \quad \& \quad N_A - N_D \gg n_i$

$p_o \simeq N_A^- - N_D^+ = 10^{16} - 5.0 \times 10^{15} \, \text{cm}^{-3}$

$p_o \simeq 5.0 \times 10^{15} \, \text{cm}^{-3}$

$n_o = \dfrac{n_i^2}{p_o} \simeq \dfrac{\left(1.5 \times 10^{10}\right)^2}{5.0 \times 10^{15}} = 4.5 \times 10^4 \, \text{cm}^{-3}$

Compensation

Ge

$T=300K$ $n_i \approx 2.5 \times 10^{13}$

$N_D > N_A$ and $N_D - N_A \simeq n_i$

$N_D = 2.0 \times 10^{14} \, cm^{-3}$

$N_A = 1.5 \times 10^{14} \, cm^{-3}$

$$n_o - p_o = N_D^+ - N_A^- = 2 \times 10^{14} - 1.5 \times 10^{14} \, cm^{-3}$$

$$n_o - p_o = 5.0 \times 10^{13} \, cm^{-3}$$

apply charge neutrality

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(2.5 \times 10^{13}\right)^2}{n_o}$$

apply mass action law

$$n_o - \frac{\left(2.5 \times 10^{13}\right)^2}{n_o} = 5.0 \times 10^{13} \, cm^{-3}$$

Compensation

$$n_o - \frac{\left(2.5\times10^{13}\right)^2}{n_o} = 5.0\times10^{13}\,cm^{-3}$$

$$n_o^2 - 5.0\times10^{13}\,n_o - \left(2.5\times10^{13}\right)^2 = 0$$

$$n_o = \frac{5.0\times10^{13} \pm \sqrt{\left(5.0\times10^{13}\right)^2 + 4\left(2.5\times10^{13}\right)^2}}{2}$$

$$= \frac{5.0\times10^{13} \pm 7.071\times10^{13}}{2} = 6.036\times10^{13}\,cm^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = 1.035\times10^{13}\,cm^{-3}$$

$$n_o p_o = 6.036\times10^{13} \times 1.035\times10^{13} = 6.25\times10^{26} = n_i^2$$

JAVA App – Doping in Silicon
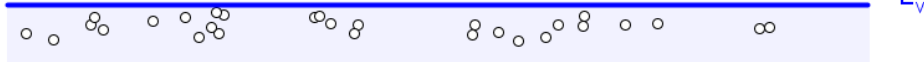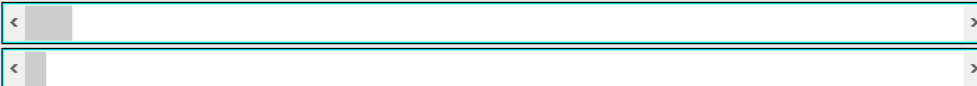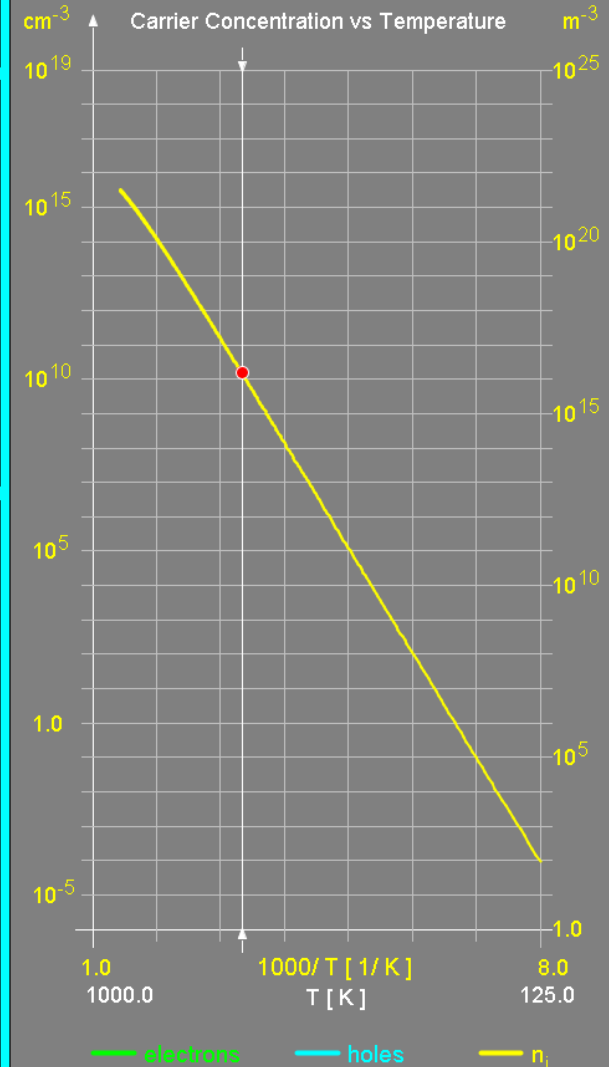
Experimental measurements show that the bandgaps of most semiconductors decrease with increasing temperature. This can be described with an empirical relation [C.D. Thurmond, 1975]

$$E_g(T) = E_g(0) - \frac{\alpha\,T^2}{T + \beta}$$

MATERIAL	$E_g(0)$ [eV]	α	β
Silicon	1.170	4.73×10^{-4}	636
Germanium	0.7437	4.774×10^{-4}	235
Gallium Arsenide	1.519	5.405×10^{-4}	204

Materials out of Equilibrium

Until now we have considered materials in equilibrium.

Let's apply an electric field. At first we will not worry about the detailed physics but focus on macroscopic properties.

A material can be characterized electrically using its conductivity or its resistivity.

Conductivity and Resistivity

- A current density is induced in the material by applying an Electric Field

$$\mathcal{E}_x$$

Force on electron

$$-q\mathcal{E}_x$$

$$J_x = \sigma \mathcal{E}_x$$

$$\sigma = \frac{1}{\rho} = \text{conductivity} \left[\frac{\text{S}}{\text{m}}\right] \qquad \rho = \text{resistivity} \, [\Omega \, \text{m}]$$

$$\text{S} = \text{Siemens} = \Omega^{-1}$$

Conductivity and Resistivity

- We can measure the resistivity, but how do we relate it to the physics of motion of electrons?

- We have said that through the introduction of the band structure we have created a model for a new equivalent "free space" where the "quasi-particles" electrons and holes follow a new energy-momentum law $E(\vec{k})$.

- But inside a solid we also need to consider the collisions (scatterings) experienced by particles.

Scattering

- **What kind of collision events scatter the particles?**

- The most important ones are:

  - **Vibrations of crystal atoms (phonons)**

    scattering increases with temperature

  - **Ionized impurity scattering (Coulomb interaction)**

    scattering increases with carrier concentration

  - **Electron-electron interaction (Coulomb interaction)**

    scattering increases with doping concentration

Electrons in crystal in equilibrium

- Electrons behave like a "gas" and obey the Boltzmann distribution at a specific temperature (this is our initial condition)

Average kinetic energy

$$E_{kinetic} = \frac{1}{2} m_n^* v_{th}^2$$

$$3D) \quad E_{thermal} = \frac{3}{2} k_B T$$

thermal velocity

$$\frac{1}{2} m_n^* v_{th}^2 = \frac{3}{2} k_B T \implies v_{th} = \sqrt{\frac{3 k_B T}{m_n^*}}$$

v_{th} in Si at 300K for electrons

$$v_{th} = \sqrt{\frac{3k_B T}{m_n^*}} =$$

$$= \sqrt{\frac{3 \times 1.3806 \times 10^{-23} \times 300}{\boxed{0.26 \times 9.11 \times 10^{-31}}}} \quad \text{conductivity mass}$$

$$= 2.29 \times 10^5 \text{ m/s} = 2.29 \times 10^7 \text{ cm/s}$$

- This can be interpreted as the root-mean-square of the total speed in 3D.

Scattering

- For a given field there is going to be a random distribution of "times between collisions" and a corresponding "mean free path".

Average time between collisions $= \tau_c$

Mean free path $= \lambda = v_{th} \tau_c$

$\tau_c \approx \left[10^{-14}, 10^{-13} \right]$ seconds

$v_{th} \approx 10^5$ m/s

$\lambda \approx 1$ to 10 nm

50 nm
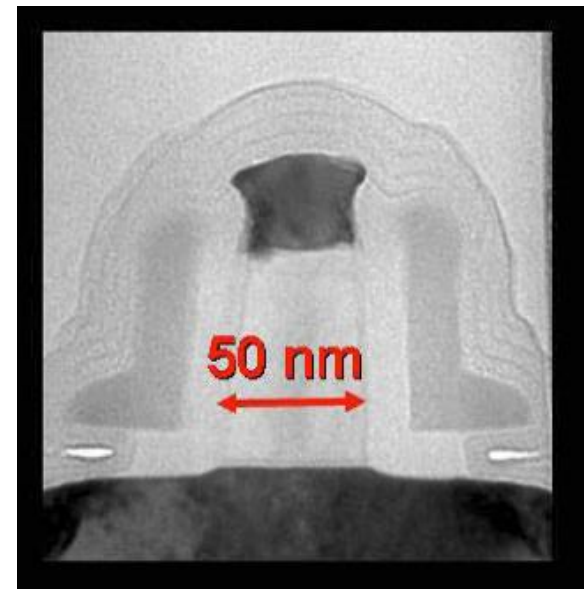
Intel (2005)

Velocity distribution in equilibrium

Symmetric distribution, no net current (no field applied)

Electron drift under a field

- The individual path of the electron is apparently chaotic, but it will tend to "drift" on the average in the direction opposite to the field.

scattering event

$$\mathcal{E}_x$$
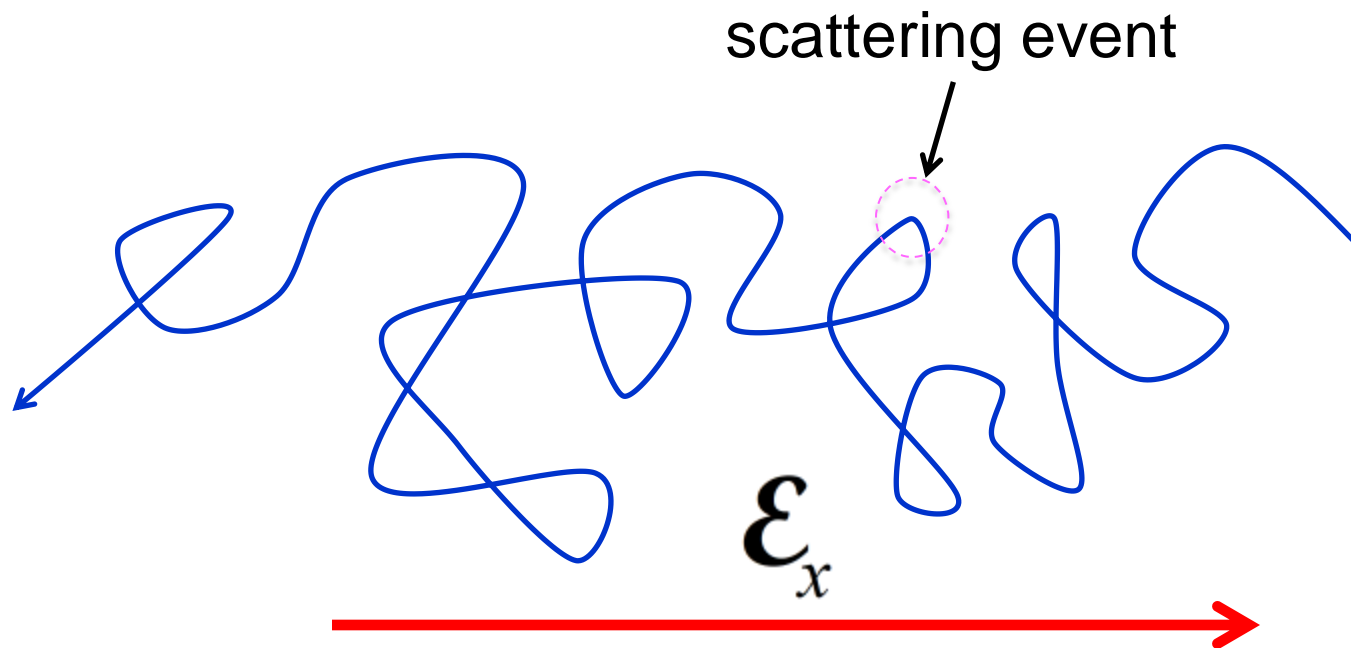
Energy balance

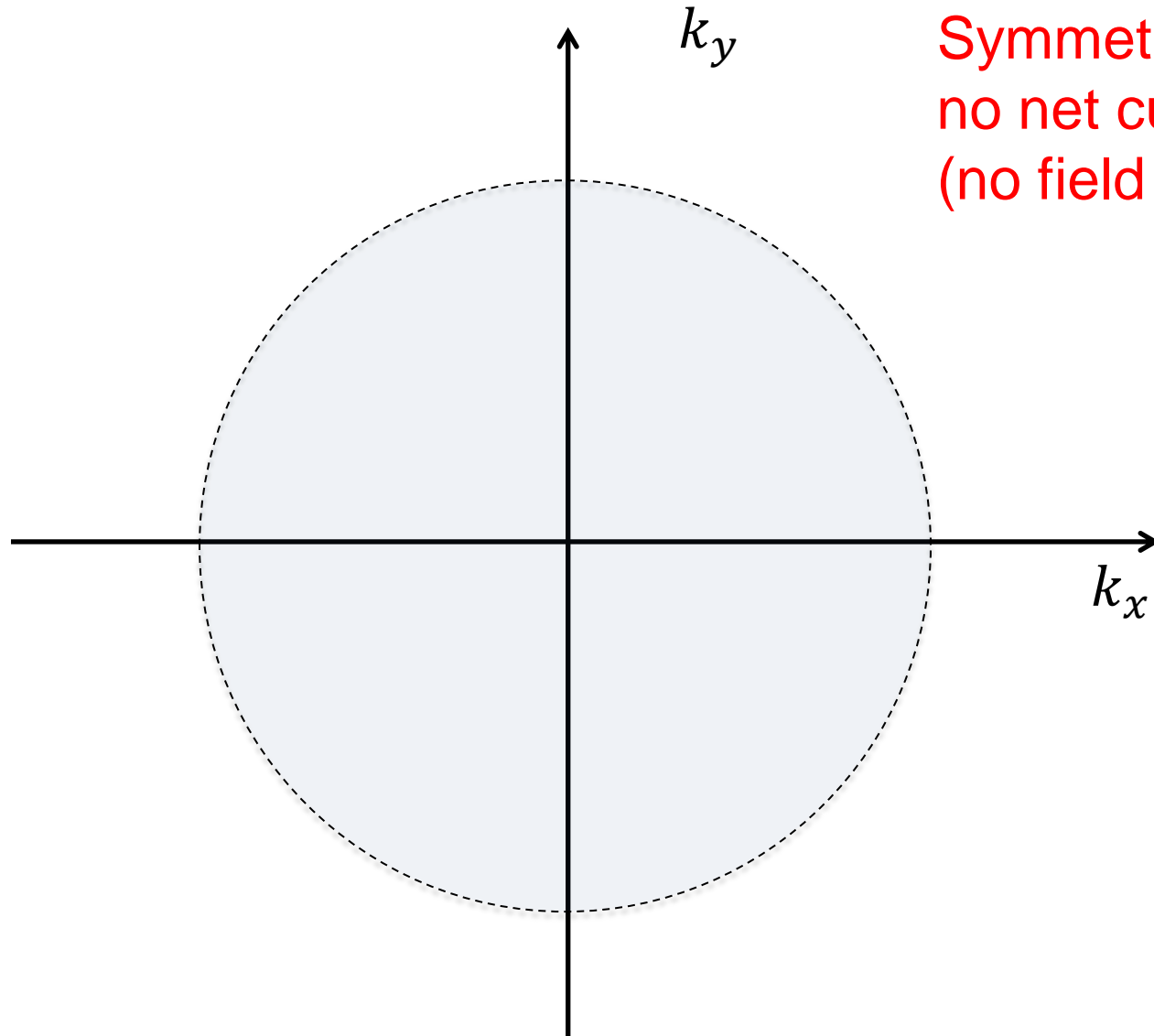- The applied field tries to direct and accelerate the individual velocities along a given direction, thus increasing the energy of the system.
  - Scattering randomizes the direction of individual electron velocities.
  - Phonon scattering exchanges energy between gas of electrons and vibrating lattice atoms.
  - Scattering with ionized impurities only changes directions, does not exchange energy.
  - Electron-electron interaction redistributes energy among electrons (important in contact regions).

Simple model for momentum scattering loss

Assumptions: τ_c is constant for all energies; momentum increase due to field is all lost in a collision (Drude model) so that particle relaxes to thermal equilibrium conditions

$$n\frac{\Delta t}{\tau_C} \text{ electrons scatter}$$

$$p_x \frac{\Delta t}{\tau_C} \text{ lost}$$

$$p_x(t)$$

n electrons

$$p_x\left(1-\frac{\Delta t}{\tau_C}\right) \text{ momentum}$$

$$n\left(1-\frac{\Delta t}{\tau_C}\right) \text{ electrons do not scatter}$$

$$t \qquad \Delta t \qquad t+\Delta t$$
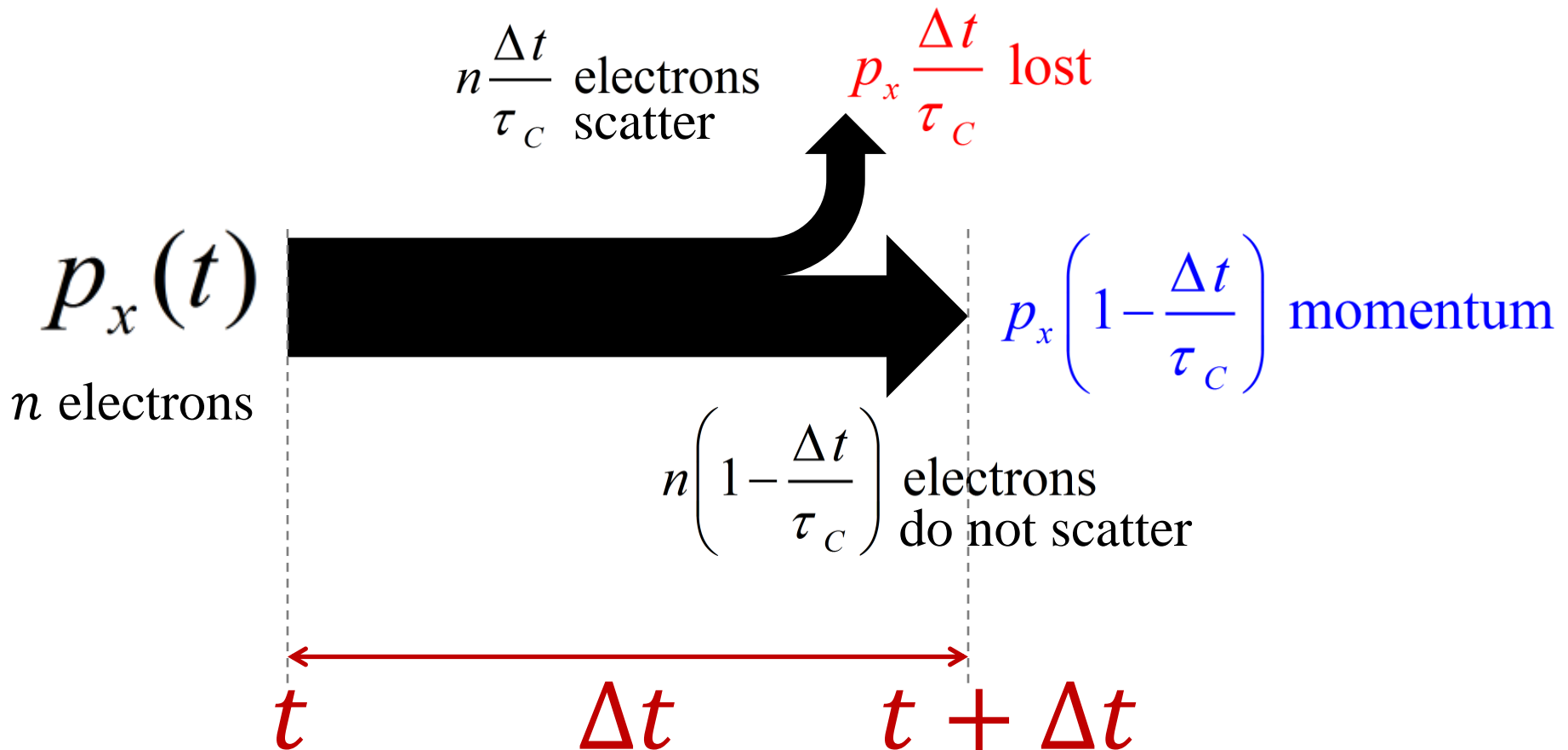
Simple model for momentum scattering loss

Assumptions: τ_c is constant for all energies; momentum increase due to field is all lost in a collision (Drude model) so that particle relaxes to thermal equilibrium conditions

$$\underbrace{p_x\left(t + \Delta t\right)}_{\substack{\text{cumulative} \\ \text{momentum} \\ \text{after } \Delta t}} = \underbrace{\left(1 - \frac{\Delta t}{\tau_C}\right)}_{\substack{\text{fraction not} \\ \text{scattered} \\ \text{after } \Delta t}} \underbrace{p_x(t)}_{\substack{\text{cumulative} \\ \text{momentum} \\ \text{for } n \text{ particles}}}$$

$$p_x(t) + \Delta p_x = p_x(t) - \frac{p_x(t)}{\tau_c}\Delta t$$

$$\Delta p_x = -\frac{p_x(t)}{\tau_c}\Delta t$$

$t)\ \ n\ particles$

$$\Delta n = \frac{\Delta t}{\tau_c} n$$

# of particles that scatter and thermalize

$t + \Delta t)\ \ n - \Delta n$

# of particles that have acquired momentum

Steady-state condition

- In steady state a specific current density is established for a given electric field.

- Thus, the net acceleration of electrons due to the field produces an increase of the average momentum which has to be balanced out by the deceleration due to the collisions.
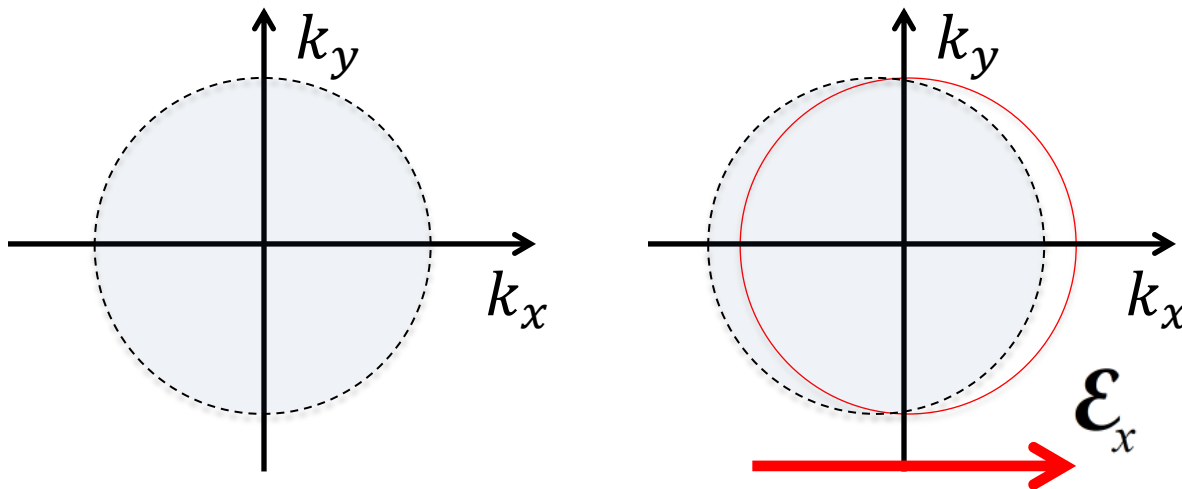
The electron velocity distribution is slightly shifted following the field direction.

Momentum change

Consider a group of n electrons

- Rate of change of momentum due to Field

$$\Delta p_x = -nq\mathcal{E}_x \Delta t \quad \longrightarrow \quad \frac{dp_x}{dt}\bigg|_{Field} = -nq\mathcal{E}_x$$

- Rate of change of momentum due to scattering

$$\Delta p_x = -\frac{p_x}{\tau_c}\Delta t \quad \longrightarrow \quad \frac{dp_x}{dt}\bigg|_{Collisions} = -\frac{p_x}{\tau_c}$$

Steady-state = constant current

$$\frac{dp_x}{dt}\bigg|_{Field} + \frac{dp_x}{dt}\bigg|_{Collisions} = 0$$

$$-nq\mathcal{E}_x - \frac{p_x}{\tau_c} = 0 \qquad \longrightarrow \qquad \frac{p_x}{n} = -q\,\tau_c\,\mathcal{E}_x$$

Average momentum
per electron

$$\langle p_x \rangle = \frac{p_x}{n}$$

Drift velocity

$$\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = -\frac{q\,\tau_c\,\mathcal{E}_x}{m_n^*}$$

Similarly for holes

$$\langle v_x \rangle = \frac{\langle p_x \rangle}{m_p^*} = \frac{q\,\tau_c\,\mathcal{E}_x}{m_p^*}$$

Define Drift Current Density
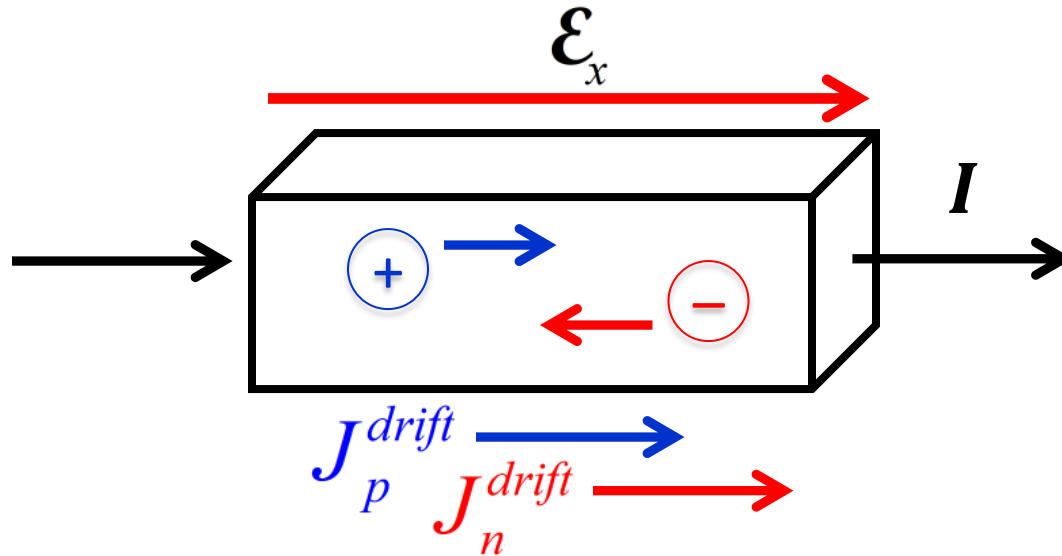
J = Charge per unit time per unit area

$$J_n^{drift} = -qn\langle v_{dn} \rangle = qn\mu_n \mathcal{E}_x$$

$$J_p^{drift} = qp\langle v_{dp} \rangle = qp\mu_p \mathcal{E}_x$$

Mobility

$$\mu_n = -\frac{\langle v_{dn} \rangle}{\mathcal{E}_x} = \frac{q\,\tau_c}{m_n^*}$$

$$\mu_p = \frac{\langle v_{dp} \rangle}{\mathcal{E}_x} = \frac{q\,\tau_c}{m_p^*}$$

units

$$\left[\frac{\text{cm}^2}{\text{V}\cdot\text{s}} \right]$$

Low Field Mobility

Si [300K]

$v_n = \mu_n \mathcal{E}$

$v_p = \mu_p \mathcal{E}$

Electrons

Holes

Drift velocity (10^7 cm/s)

$\mathcal{E}$ (kV/cm)
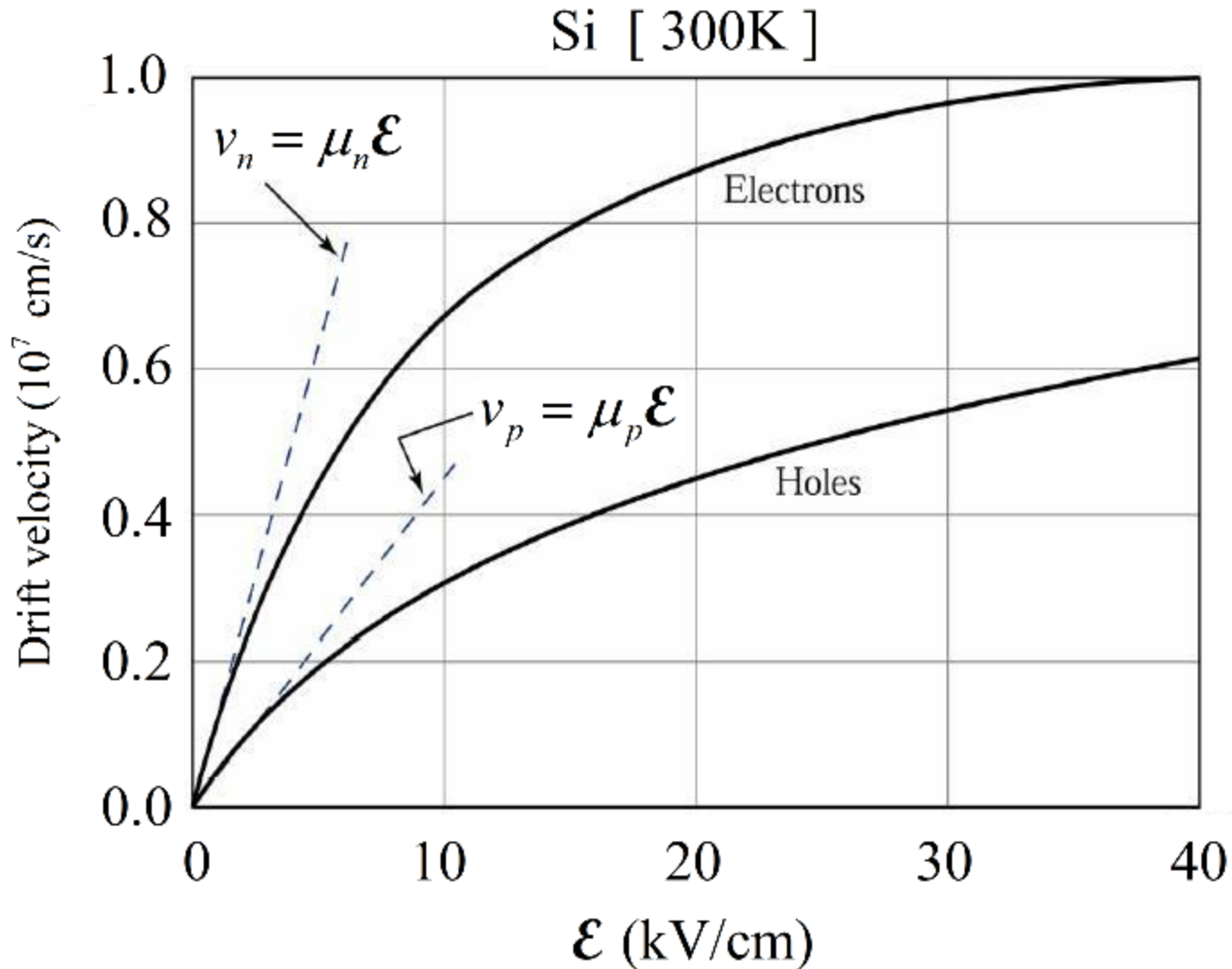
μ is the slope of the curve at $\mathcal{E}=0$ and is usually called low-field mobility

Semiconductor Material Properties

		E_g (eV)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)	m^*_n/m_o (m_l,m_t)	m^*_p/m_o (m_{lh},m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

All values at 300 K.

*Vaporizes