ECE 340 Lecture 12
Solid State Electronic Devices

Spring 2022
10:00-10:50am
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2062 ECE Building

Today's Discussion

- Mobility dependence on temperature
- Invariance of the Fermi Level at equilibrium
- Junctions between different materials

Mobility dependence on Temperature

There are two competing phenomena which give rise to two mobility component:

Matthiessen's rule

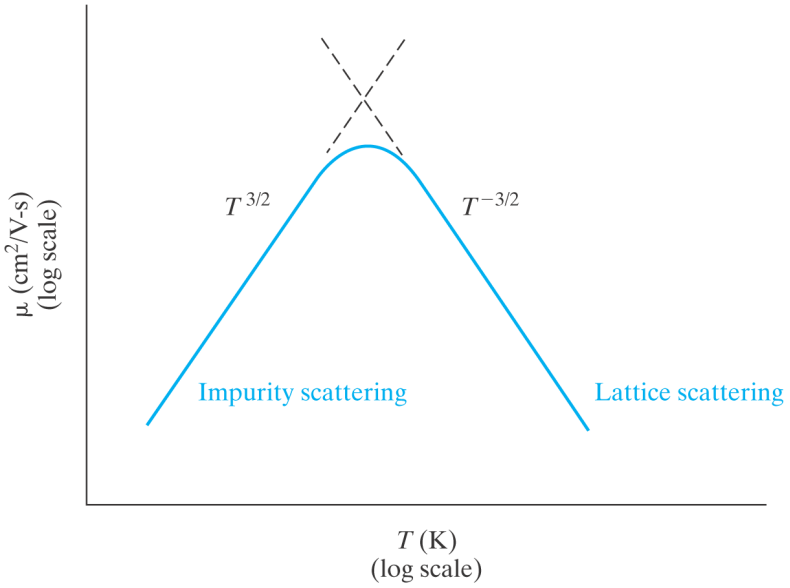
$$\frac{1}{\mu} \approx \frac{1}{\mu_{imp}} + \frac{1}{\mu_{phon}}$$

Impurity scattering

$$\mu_{imp} \propto \frac{T^{\frac{3}{2}}}{N_D + N_A}$$

Phonon scattering

$$\mu_{phon} \propto T^{-\frac{3}{2}}$$



Note:

- Validity of Matthiessen's rule applied to semiconductors is limited to conditions not too far from equilibrium, since the relaxation times (mean free times between collisions) are not simple functions of temperature and energy in realistic conditions. For more advanced applications, it will be better to use:

$$\frac{1}{\tau_c} \approx \frac{1}{\tau_{imp}} + \frac{1}{\tau_{phon}} + \ldots$$

and then calculate the mobility from τ_c

Uniform and non-uniform semiconductors

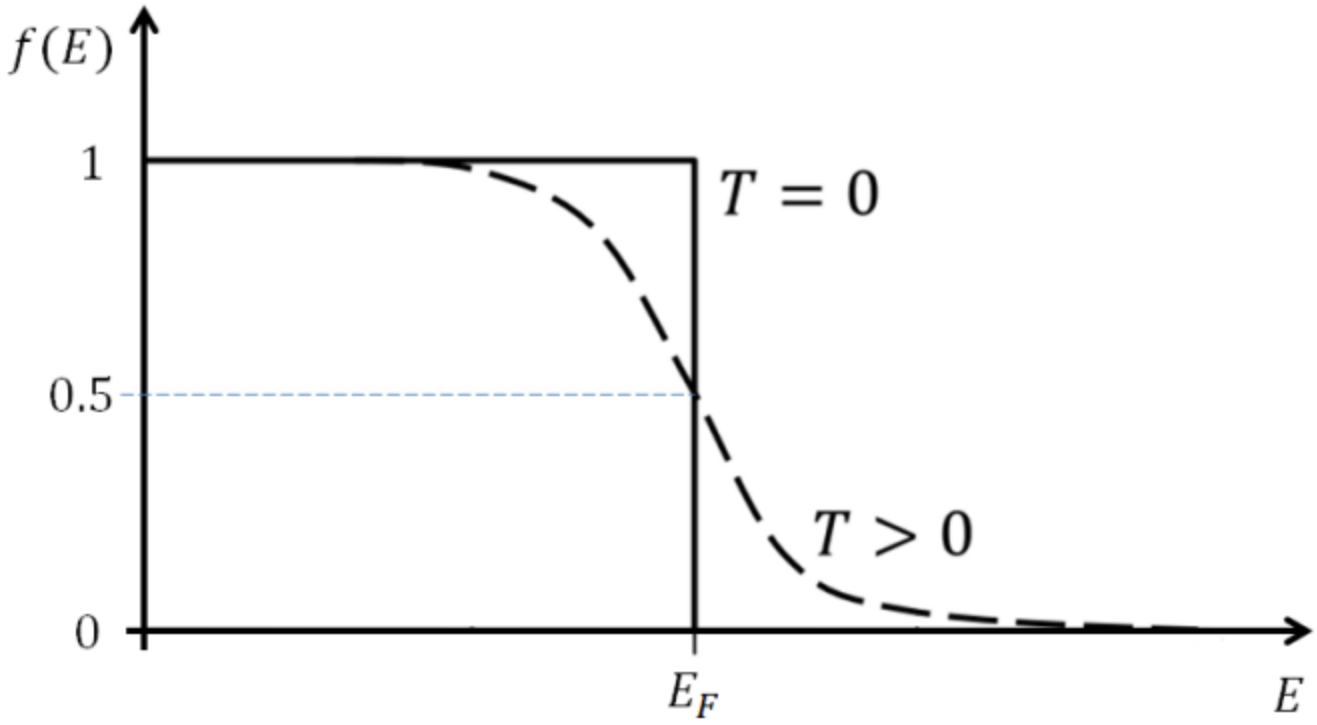
- Until now we have considered **homogeneous** (uniform) materials.
- We have also considered mainly **equilibrium** conditions, except for the case of a semiconductor **"resistor"** which remains uniform even when a field is applied.
- To realize devices we will need **non-uniform** materials systems.

Invariance of the Fermi level at equilibrium

- At equilibrium, the **Fermi Level** E_F is always "invariant", that is, it cannot exhibit any **discontinuity** or **gradient**.
- In simple words, the (equilibrium) Fermi Level is always **flat** (in space) in an energy diagram.
- It should be noted that the Fermi Level is actually a property of systems in equilibrium, although the concept will be extend in some form to non-equilibrium situations.

What is the Fermi level?

- So far, we have defined the Fermi level as the energy at which the Fermi-Dirac probability distribution equal to $1/2$.



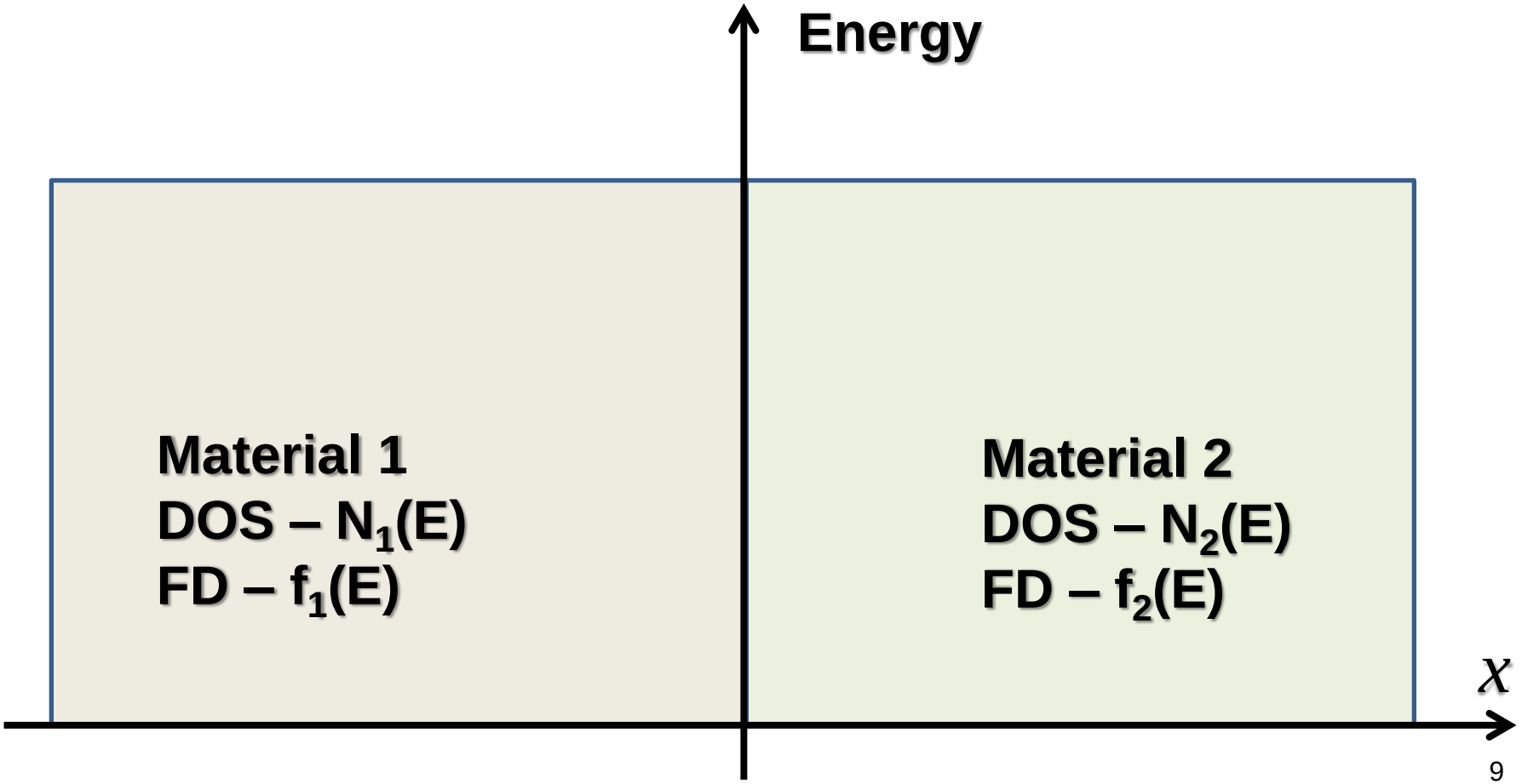
What is the Fermi level?

- The **Fermi level** of a solid-state system is related to the thermodynamic work required to add one electron to the system.

- So, the Fermi level is the same as the electro-chemical potential in equilibrium

Two materials in contact at equilibrium

- The two materials have their own density of states and Fermi-Dirac statistics.



Rate of electron transfer for any energy

rate from 1 to 2 is proportional to

$$\underbrace{N_1(E)\,f_1(E)}_{\text{occupied states in Material 1}} \cdot \underbrace{N_2(E)\left[1-f_2(E)\right]}_{\text{non-occupied states in Material 2}}$$

rate from 2 to 1 is proportional to

$$\underbrace{N_2(E)\,f_2(E)}_{\text{occupied states in Material 2}} \cdot \underbrace{N_1(E)\left[1-f_1(E)\right]}_{\text{non-occupied states in Material 1}}$$

At equilibrium there is no current

at equilibrium the two rates must be equal

$$N_1(E)\,f_1(E)\cdot N_2(E)\left[1-f_2(E)\right]=$$

$$N_2(E)\,f_2(E)\cdot N_1(E)\left[1-f_1(E)\right]$$

Fermi level has to be flat

$$N_1(E)f_1(E)N_2(E) - N_1(E)f_1(E)N_2(E)f_2(E) =$$

$$N_2(E)f_2(E)N_1(E) - N_2(E)f_2(E)N_1(E)f_1(E)$$

$$\Rightarrow \quad N_1(E)N_2(E)f_1(E) = N_1(E)N_2(E)f_2(E)$$

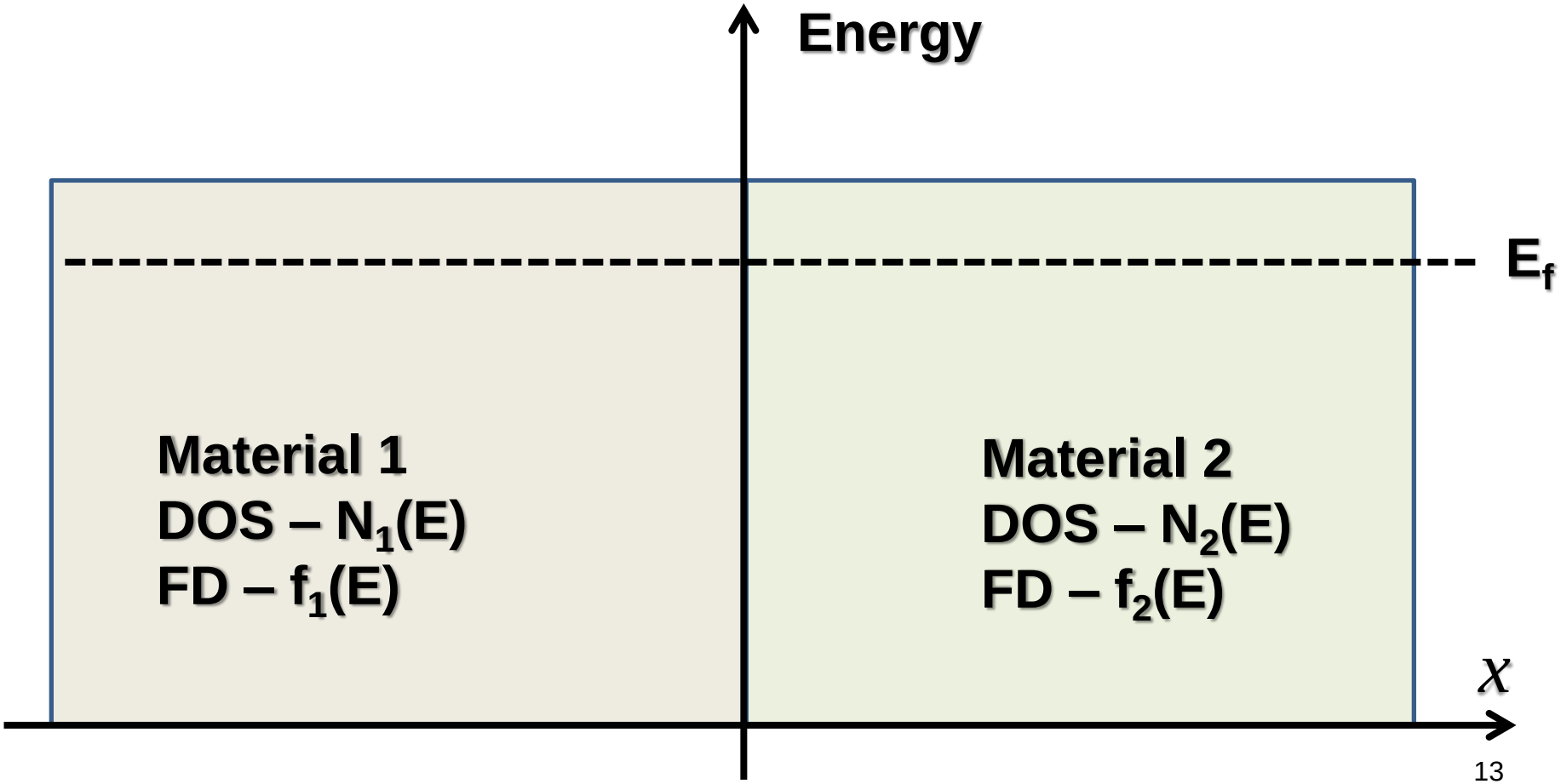
$$\Rightarrow \quad \left[1+e^{\left(E-E_{F1}\right)/k_BT}\right] = \left[1+e^{\left(E-E_{F2}\right)/k_BT}\right]$$

$$\Rightarrow \quad \boxed{E_{F1} = E_{F2}}$$

$$\Rightarrow \quad \boxed{\frac{dE_F}{dx} = 0}$$

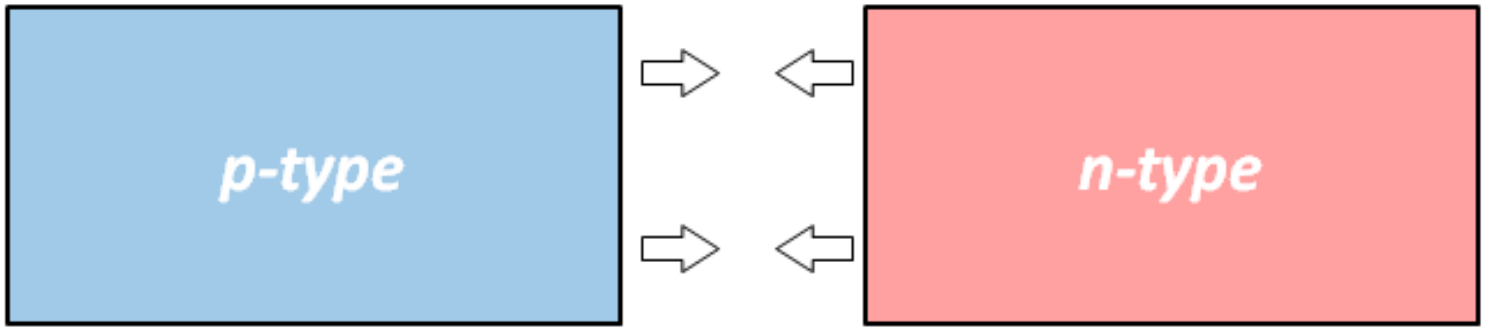
But what happens at the interface?

- So far we only know that the Fermi level is constant...



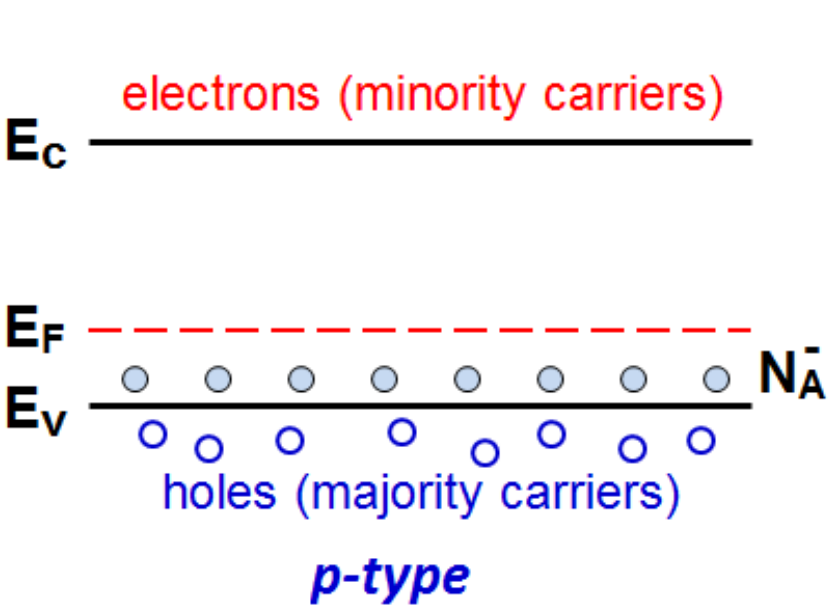
Gedanken Experiment

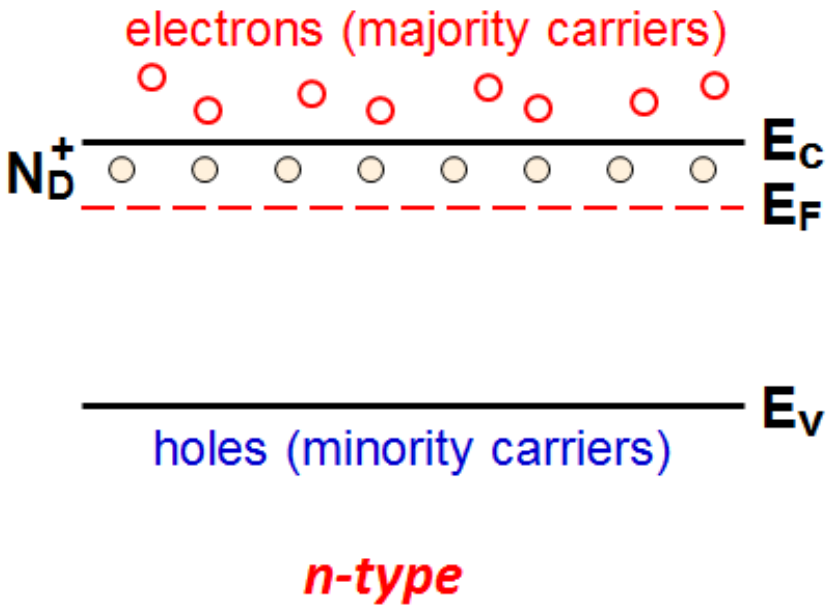
- Imagine to bring into contact two separate sample materials of the same semiconductor in equilibrium but with different doping.



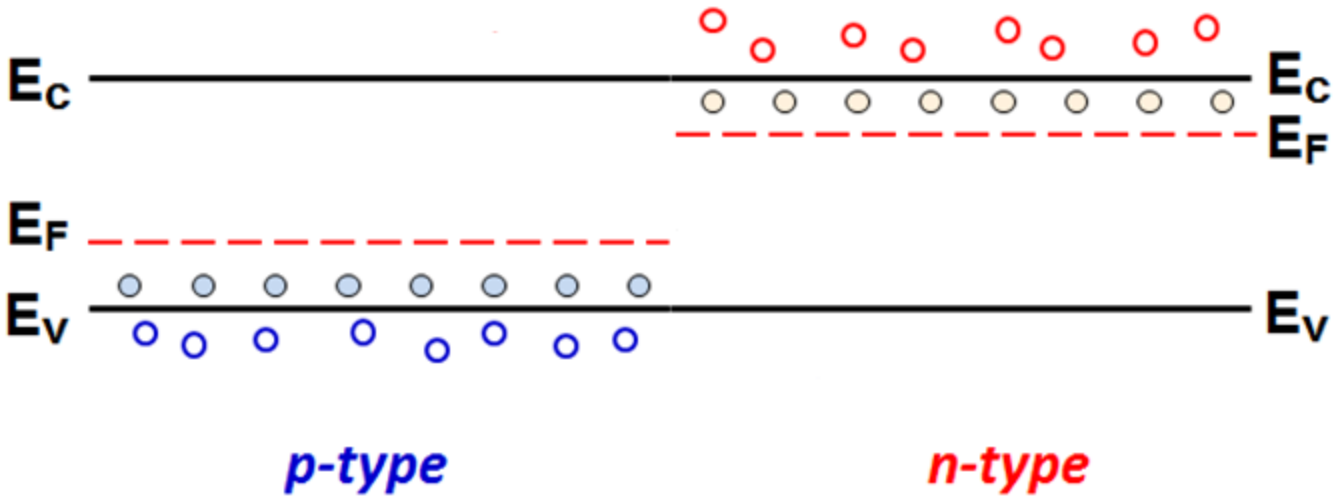
Two systems in equilibrium

In p-type semiconductor materials, the Fermi level is closer to the valence band. In n-type semiconductor materials, the Fermi level is closer to the conduction band.



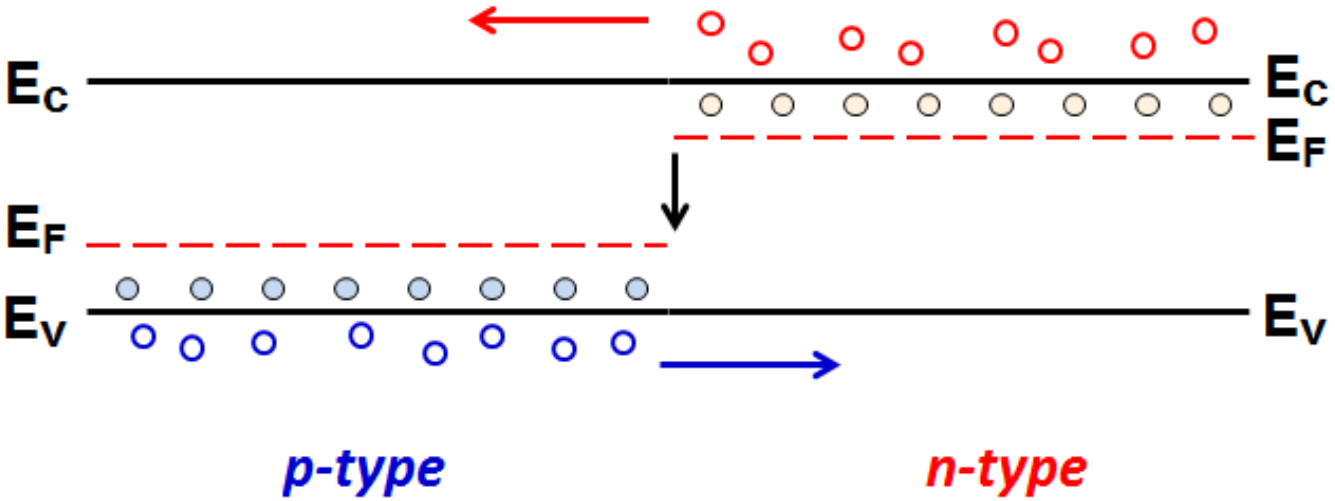


Is the combined system in equilibrium?



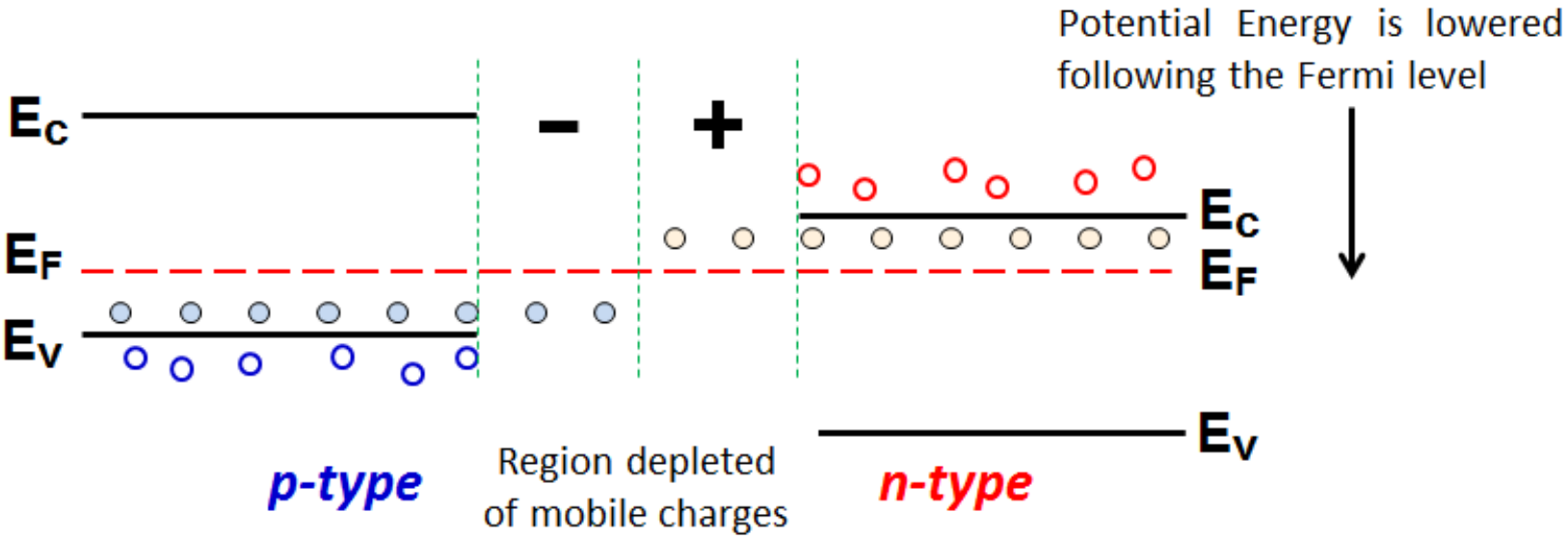
Conduction bands are the same initially

If the two materials are joined together, in order to reach equilibrium there has to be an equalization of the Fermi level (chemical potential) with mobile charges diffusing to regions of lower density.



Fermi levels equalize

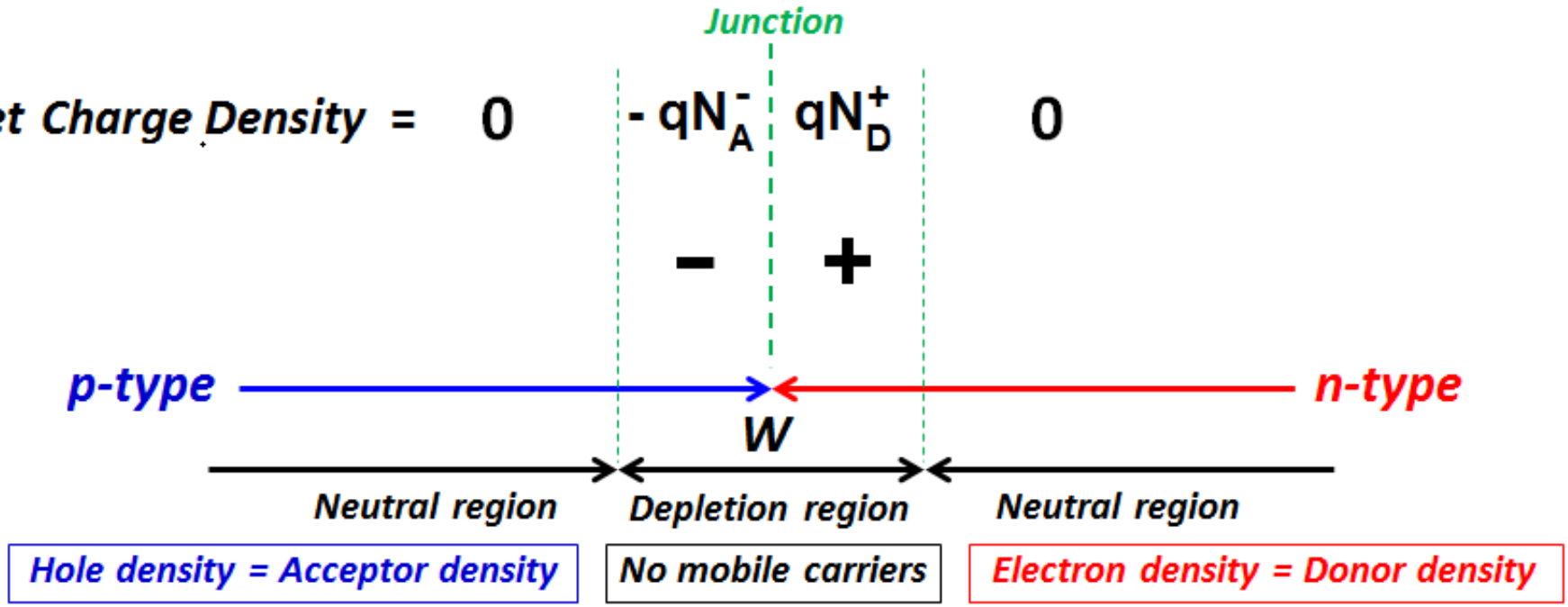
***Electrons** flow to the p-region leaving behind unscreened positive donor charges. **Holes** flow to the n-region leaving behind unscreened negative acceptor charges. Space charge builds up at the junction and a potential barrier opposes diffusive charge flow.*



Equilibrium is reached and carrier flow stops

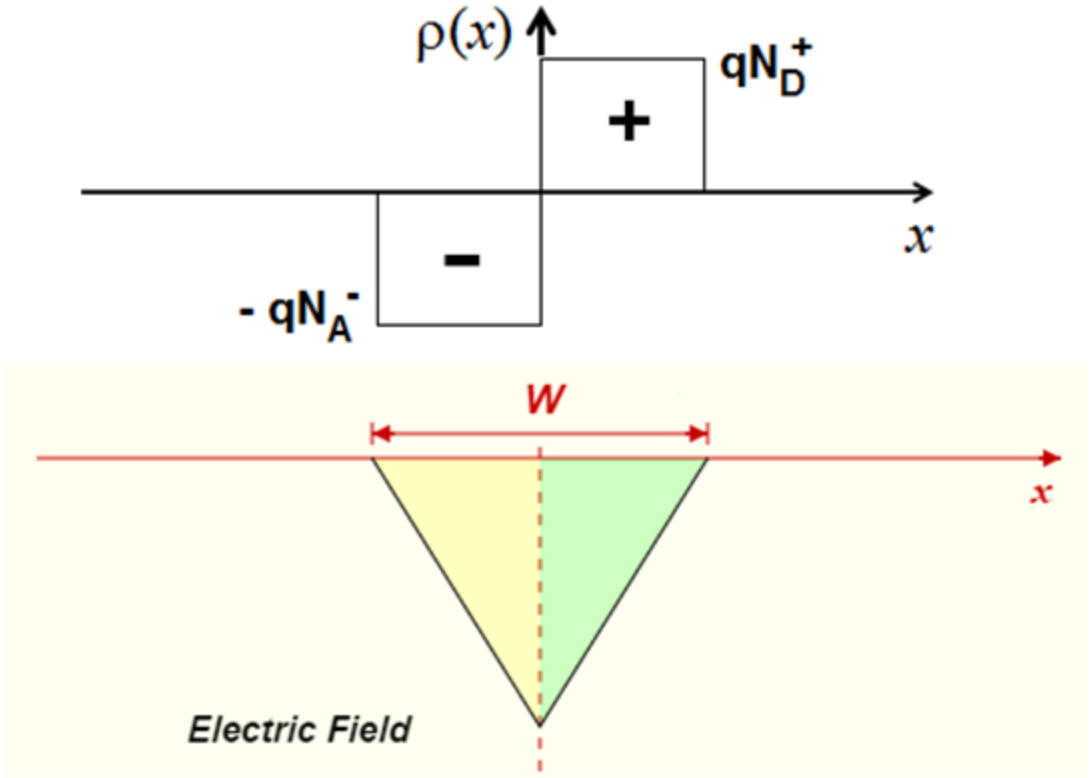
The process stop after equalization

A simple model for the p-n junction uses the depletion approximation, based on the assumptions illustrated by the diagram below:



A region with electric field is established

Simple application of Gauss' law gives a triangular field distribution

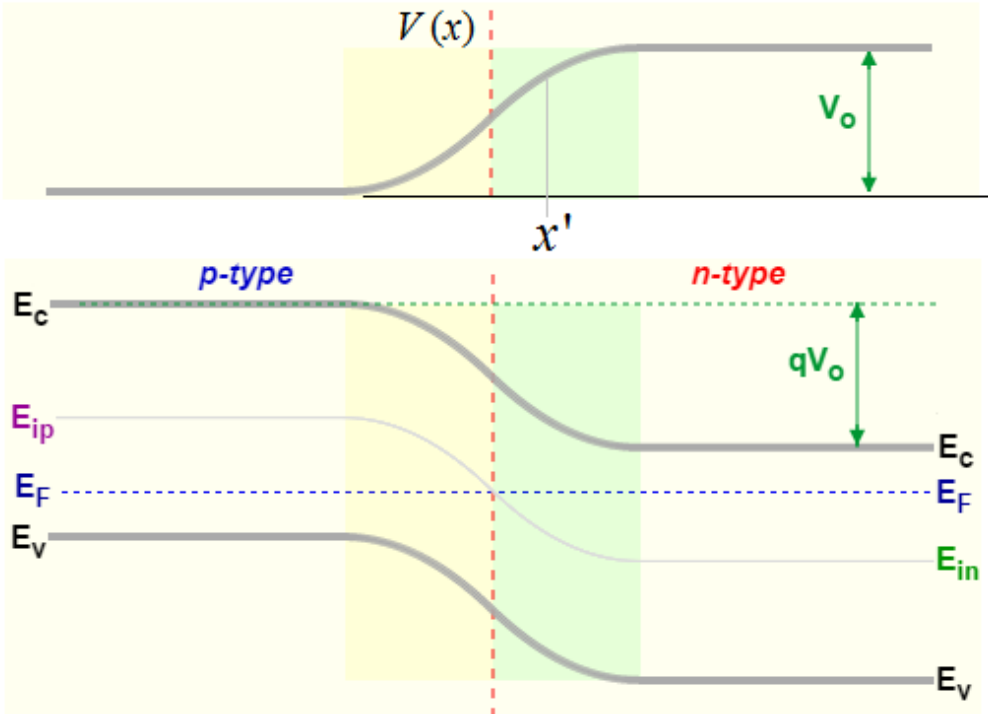


The result is “band bending”

$V(x)$ completes the band diagram in the depletion region

$$-\frac{dV}{dx} = \vec{\mathcal{E}}(x)$$

$$-\int_{-\infty}^{x'} \vec{\mathcal{E}}(x)\,dx = V(x')$$



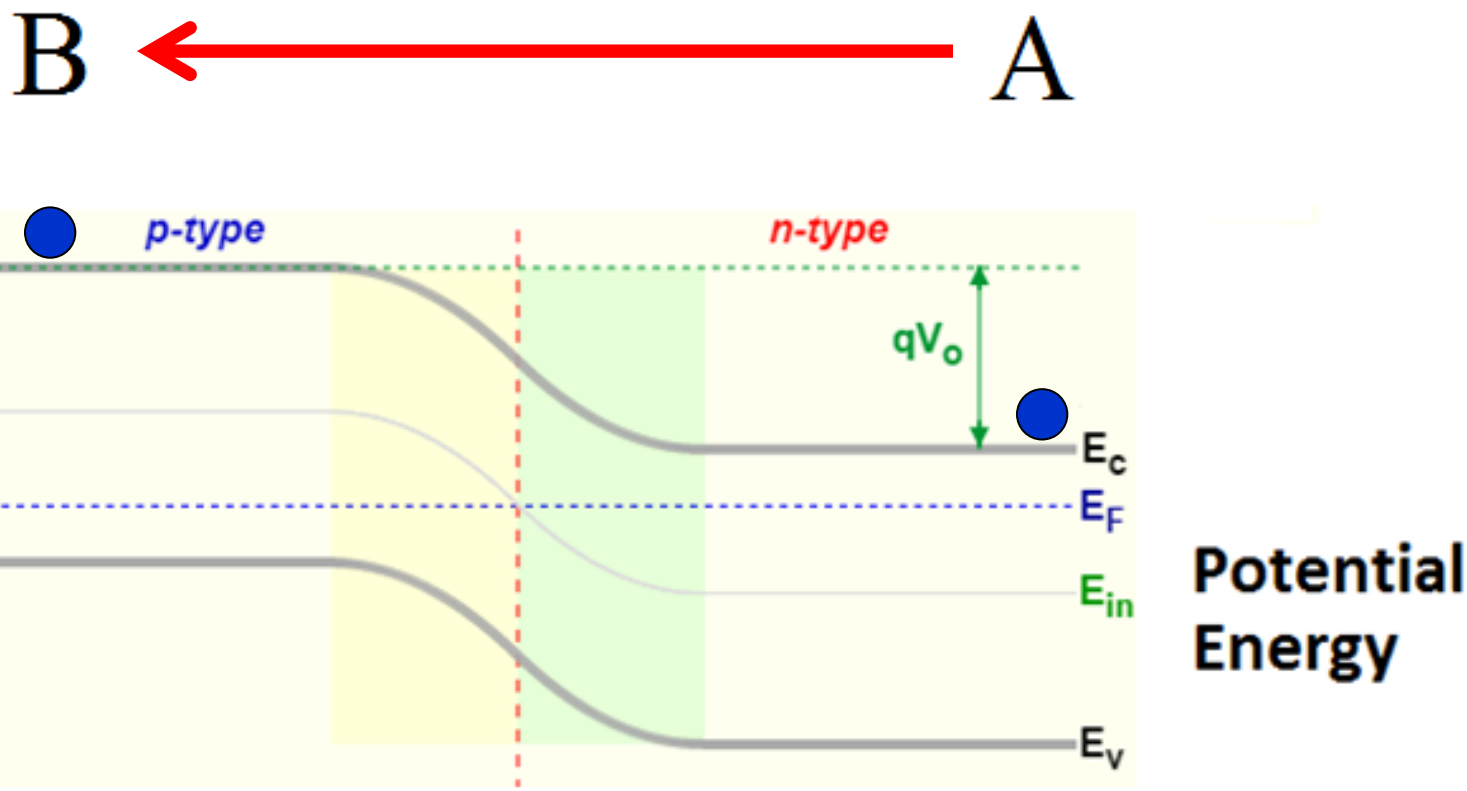
Electrostatic Potential

$$-qV(x) = \mathbf{E}$$

Potential Energy

Fermi level is the electrochemical potential

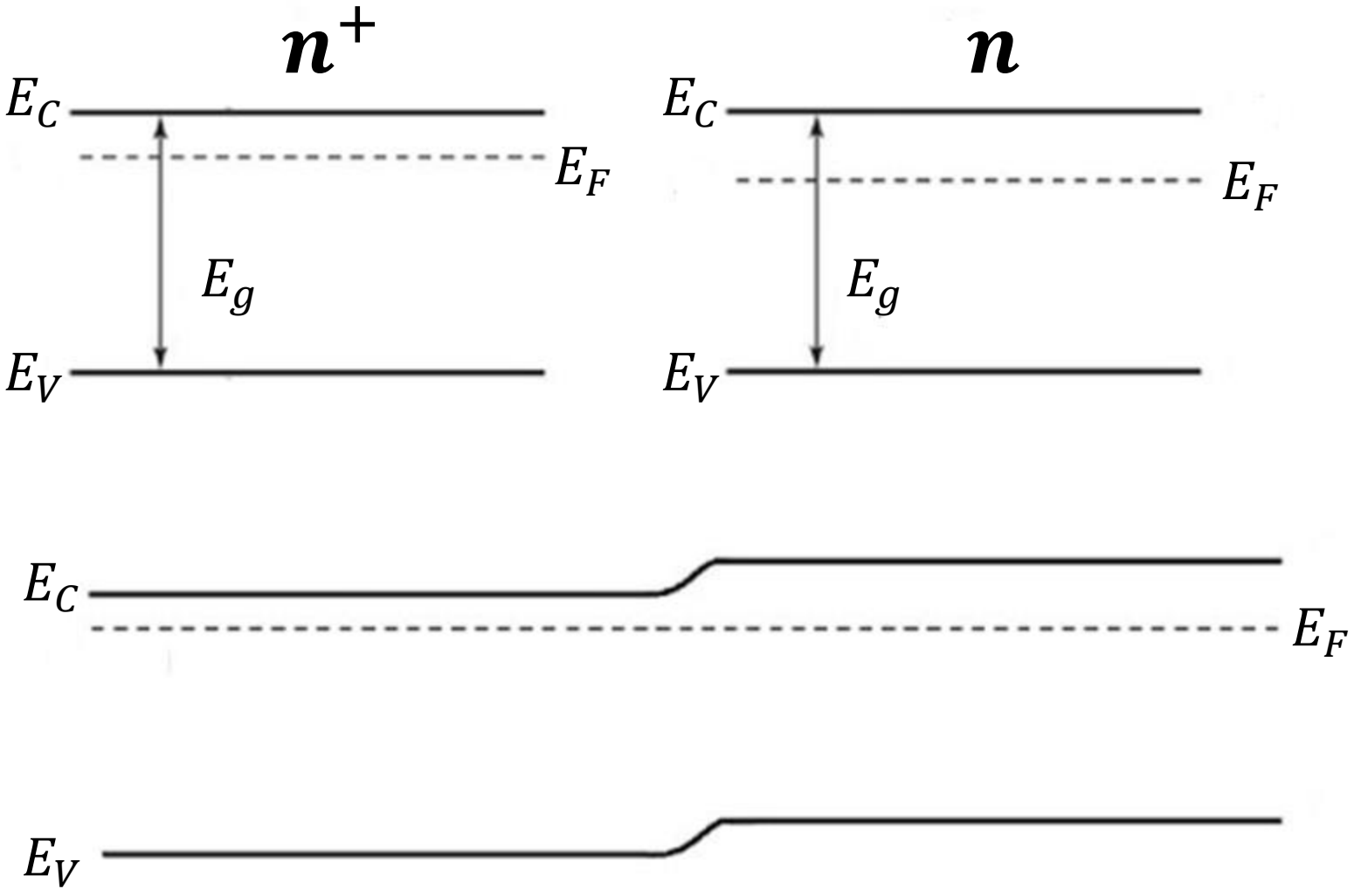
The Fermi level reference gives us a framework to determine how much energy is necessary to move an electron from A to B



NOTE: The intrinsic Fermi level reference is not flat.

Question: Where is the semiconductor intrinsic?

We can make also n^+-n junctions…



… and p^+-p junctions