ECE 340 Lecture 16
Semiconductor Electronics

Spring 2022
10:00-10:50am
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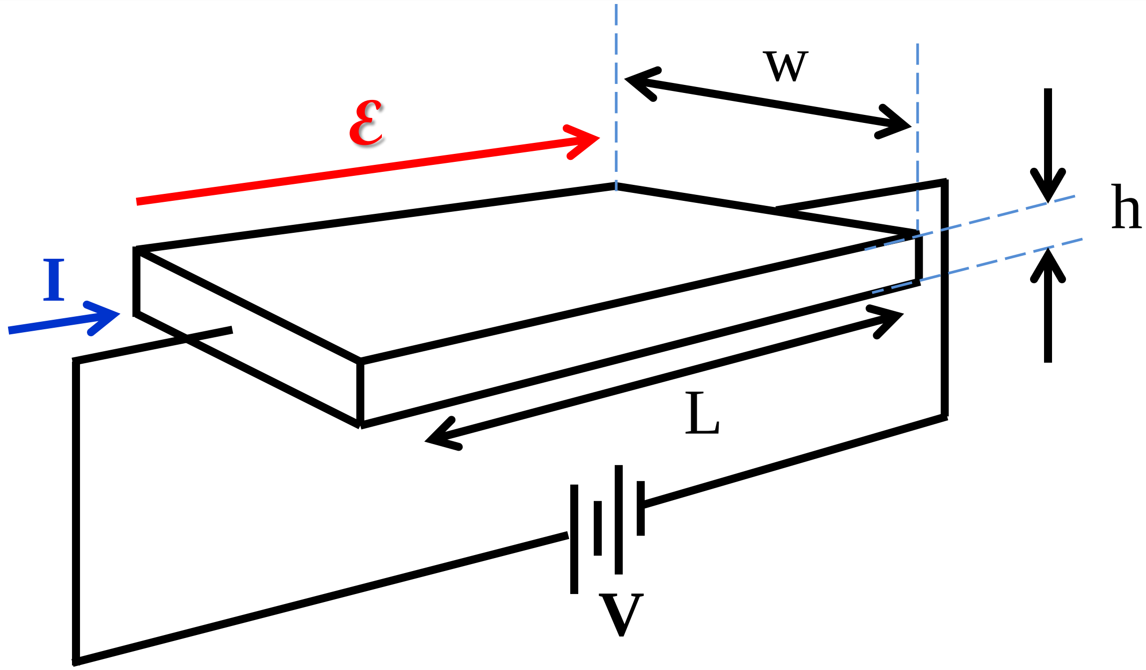
Today’s Discussion

- Diffusion transport
- Diffusion coefficient
- Einstein’s relation

Carrier Transport Mechanisms

- **Mobile carriers (electrons in conduction band and holes in valence band) move in space according to two basic physical processes:**
  - **DRIFT**
  - **DIFFUSION**
- In general these two phenomena coexist and interplay with each other

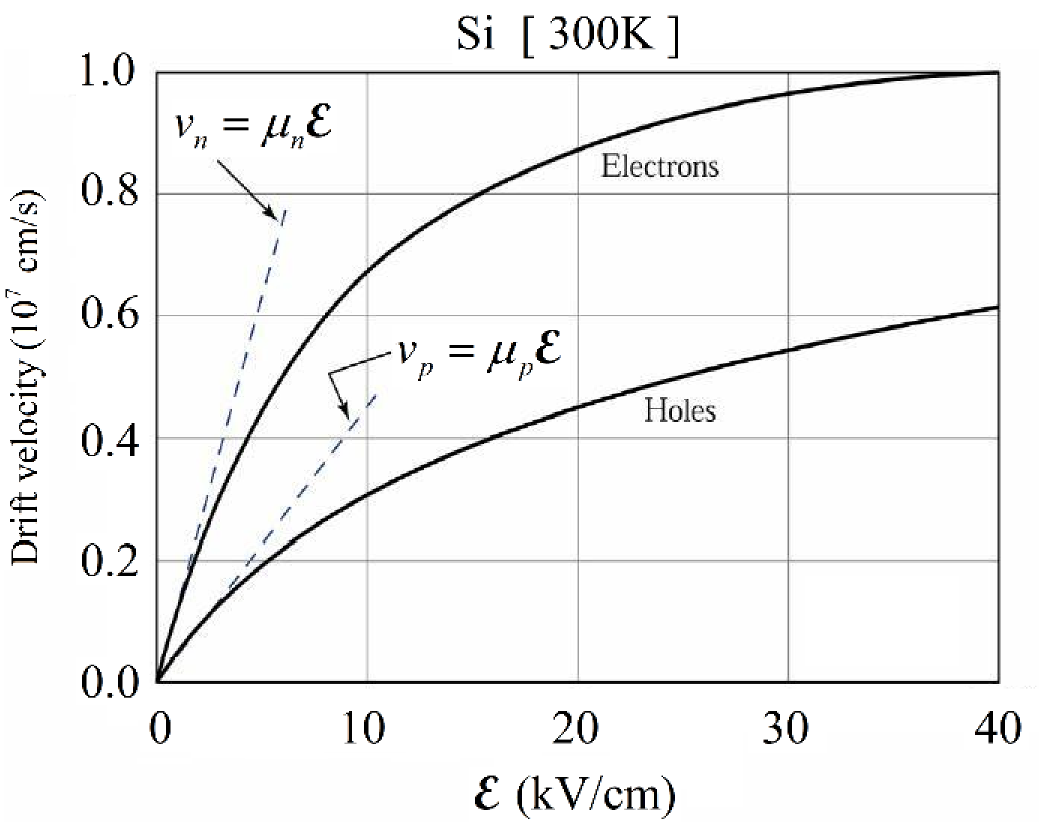
We have seen already the DRIFT process



$$J_n^{drift} = -qn\langle v_{dn}\rangle = qn\mu_n\mathcal{E}_x$$

$$J_p^{drift} = qp\langle v_{dp}\rangle = qp\mu_p\mathcal{E}_x$$

We have seen already the DRIFT process



$$\mu_n = -\frac{\langle v_{dn} \rangle}{\mathcal{E}_x} = \frac{q\,\tau_c}{m_n^*}$$

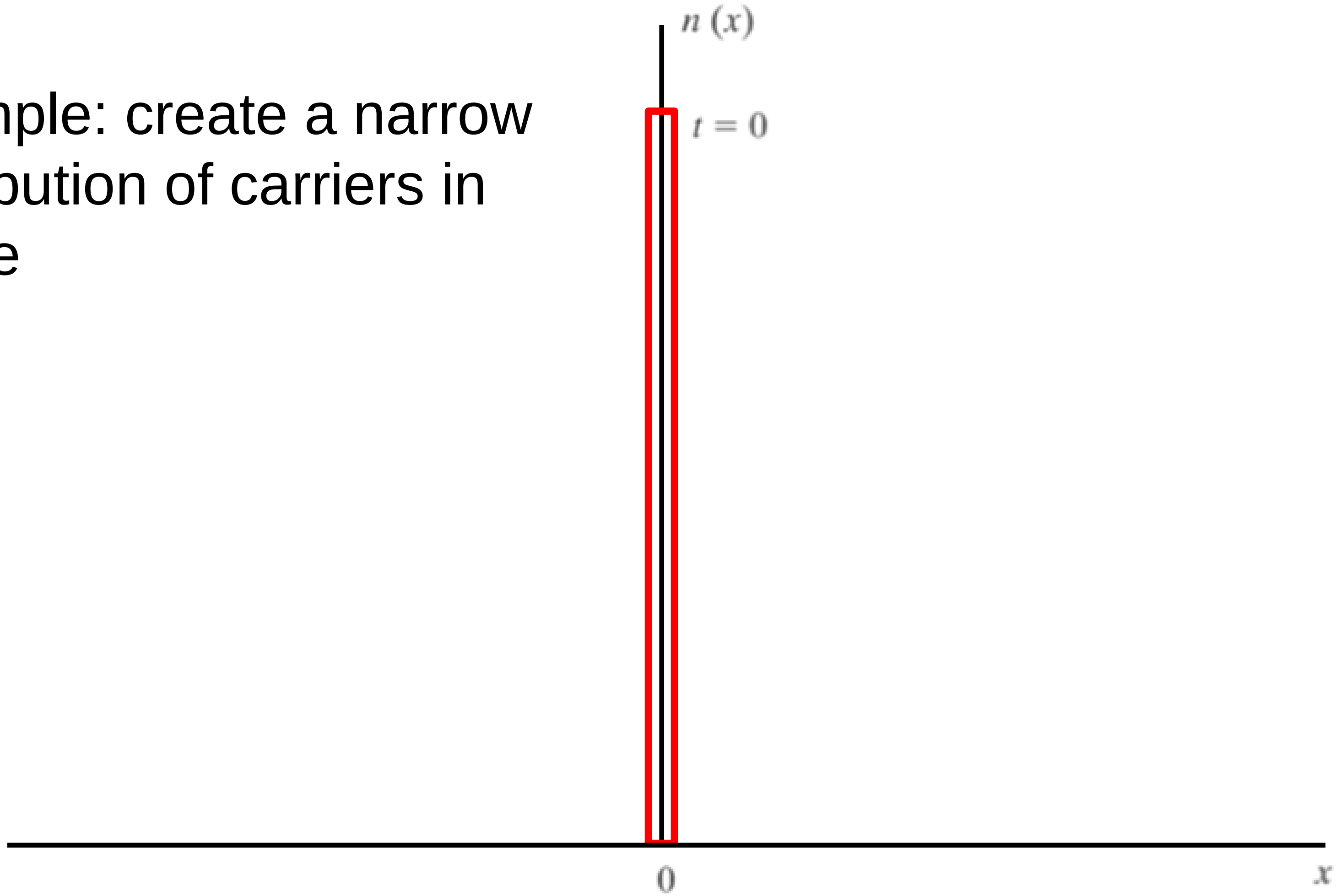
$$\mu_p = \frac{\langle v_{dp} \rangle}{\mathcal{E}_x} = \frac{q\,\tau_c}{m_p^*}$$

In a uniform semiconductor system with an applied electric field drift is the only transport phenomenon

Diffusion

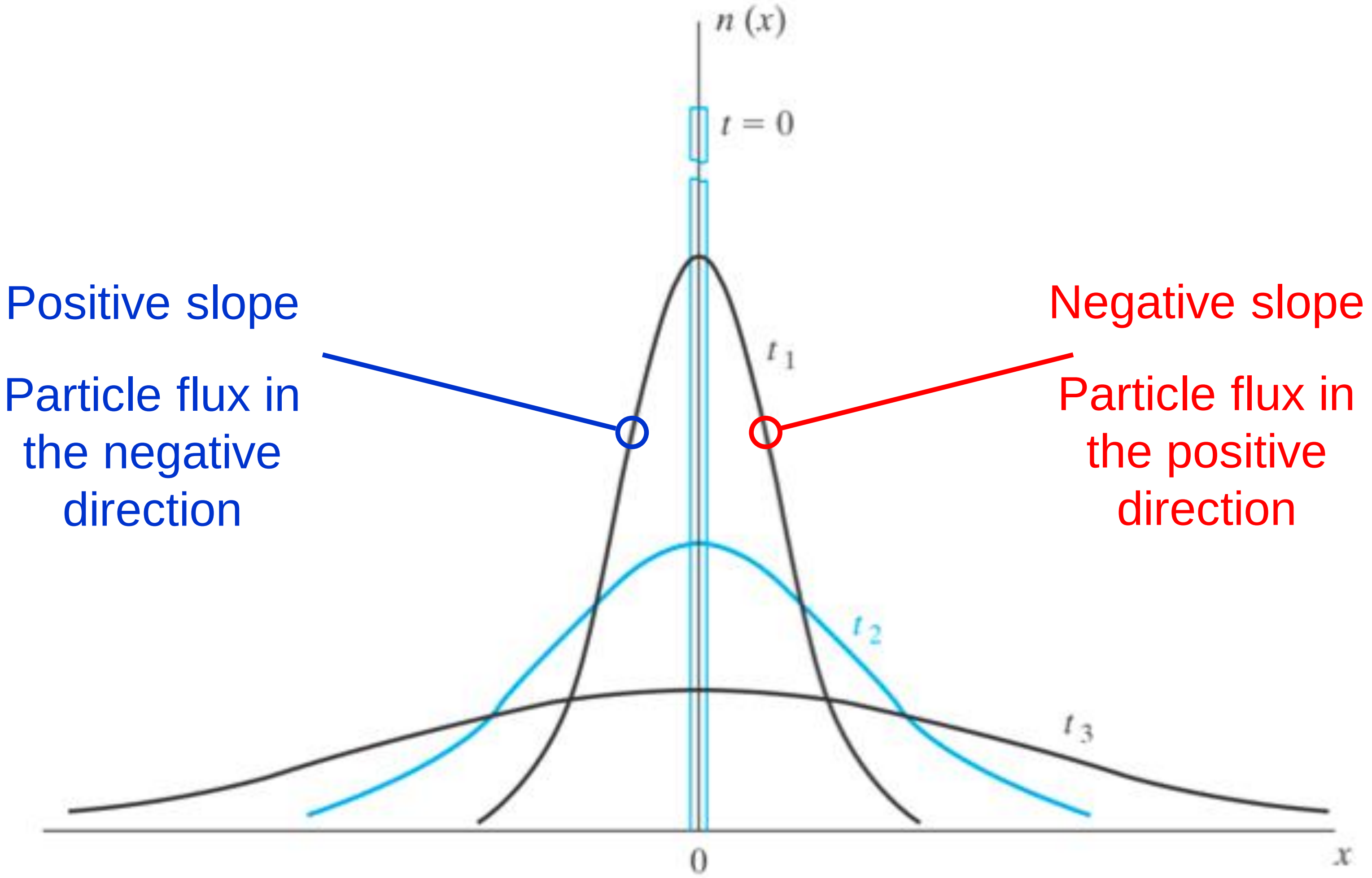
Diffusion occurs when there is non-uniformity in carrier density

Example: create a narrow distribution of carriers in space



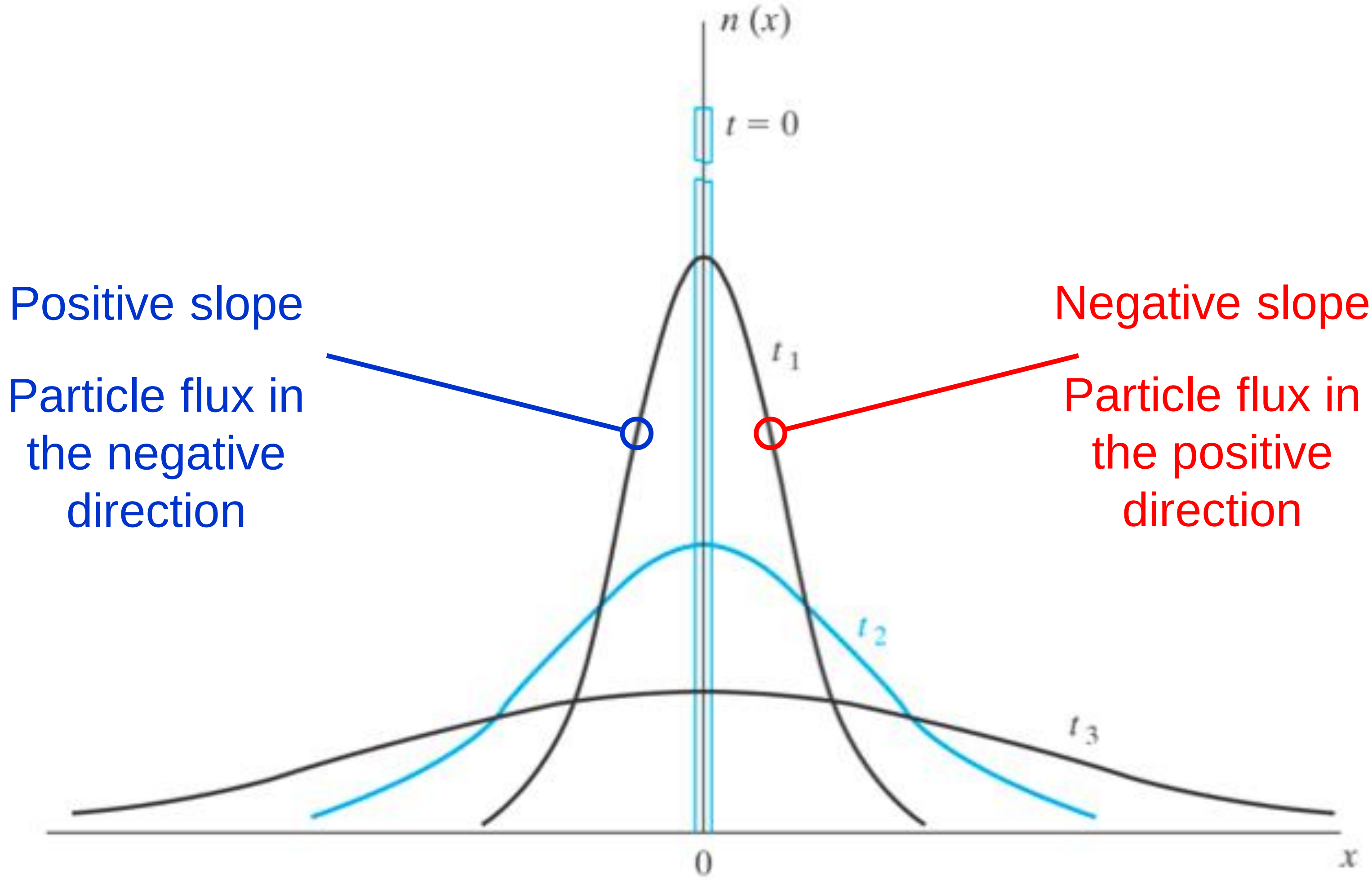
Diffusion

Diffusion occurs when there is non-uniformity in carrier density



Diffusion

Charge is moving $\rightarrow$ is there a net current flowing?

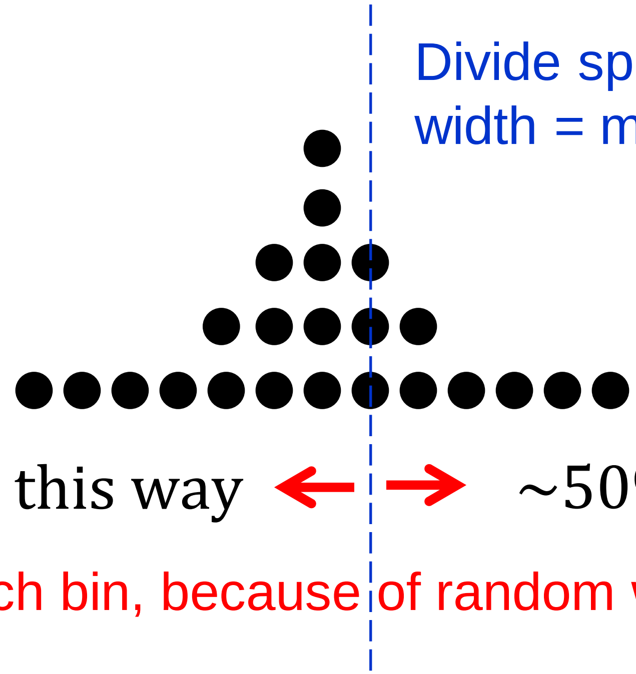


Diffusion – simple microscopic picture

SIMPLIFYING ASSUMPTIONS:

1) One-dimensional model for motion and concentration gradients
2) All carriers have the same average thermal speed $|v_{th}|$
3) Distance travelled between collisions is fixed at mean free path

Divide space in bins of width = mean free path



$\sim$50% go this way ← → $\sim$50% go this way

From each bin, because of random walk of particles

Average time between collisions $= \tau_c$

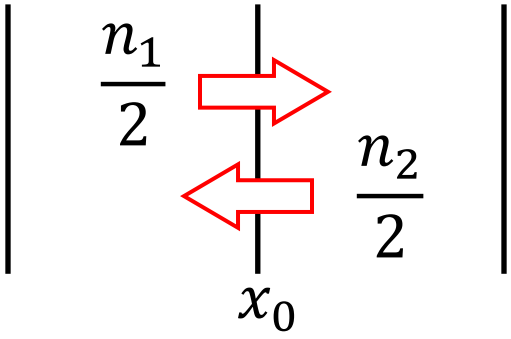
Mean free path $= \lambda = v_{th}\,\tau_c$

$$\lambda = \underbrace{\bar{l}}_{\text{book uses}} \qquad \tau_c = \underbrace{\bar{t}}_{\text{book uses}}$$

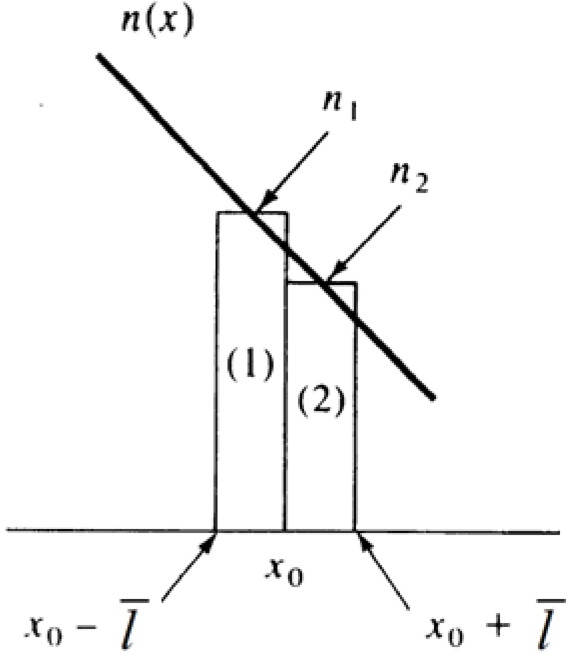
Flux of particles per unit area

$$\text{Flux} = \Phi_n(x_o) = \frac{\bar{l}}{2\bar{t}}\left(n_1 - n_2\right)$$

$$\left(n_1 - n_2\right) = \frac{n_1 - n_2}{\bar{l}}\,\bar{l} = \underbrace{\frac{n(x) - n\left(x + \Delta x\right)}{\Delta x}}_{\approx -\frac{dn(x)}{dx}}\,\bar{l} \qquad (\Delta x \to \bar{l})$$



$$Flux = \Phi_n(x_o) = -\underbrace{\frac{\bar{l}^2}{2\bar{t}}}_{\text{diffusion coefficient}}\frac{dn(x)}{dx}$$



Discretize space in terms of mean-free path, which is a small quantity

Diffusion coefficient

$$Flux = \Phi_n(x_o) = -\underbrace{\frac{\overline{l}^2}{2\overline{t}}}_{\text{diffusion coefficient}} \frac{dn(x)}{dx}$$

This diffusion coefficient is approximate and it is based on a 1D model (in 3D, factor of “3” at denominator for the x-component result) but this gives a qualitative idea of what is involved in diffusion processes. More physics is needed, anyway, for rigor.

We will use

D_n = Diffusion coefficient for electrons

D_p = Diffusion coefficient for holes

A more rigorous diffusion coefficient will be derived soon from the total current in equilibrium conditions

From particle flux to diffusion current density

$$\varphi_n(x) = -D_n \frac{dn(x)}{dx}$$

$$\varphi_p(x) = -D_p \frac{dp(x)}{dx}$$

$$J^n_{diff} = qD_n \frac{dn(x)}{dx}$$

$$J^p_{diff} = -qD_p \frac{dp(x)}{dx}$$

Complete 1-D drift-diffusion equations

DRIFT DIFFUSION

$$J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx}$$

Total current density $J(x) = J_n(x) + J_p(x)$

These currents represent the carrier density as a “compressible fluid of charged particles”. The flow model will be assembled later in the form of “continuity equations” (conservation of charge).

The field depends on charges (fixed and mobile) so the model is to be completed with the addition of Poisson equation.

Diffusivity – Consider Equilibrium ($J = 0$)

Current density for holes

$$0 = qp(x)\mu_p \mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p} \frac{1}{p(x)} \frac{dp(x)}{dx}$$

Use:

$$p = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)$$

$$\frac{dp(x)}{dx} = \underbrace{n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)}_{p(x)} \frac{1}{k_B T}\left[\frac{dE_i}{dx} - \frac{dE_F}{dx}\right]$$

Diffusivity – Consider Equilibrium ($J = 0$)

Current density for holes

$$0 = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p} \frac{1}{p(x)} \frac{dp(x)}{dx} \qquad p = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)$$

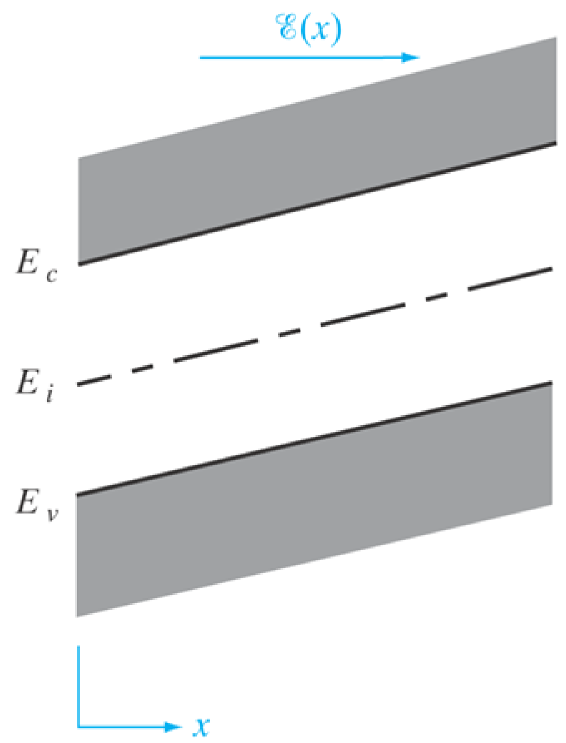
$$\mathcal{E}(x) = \frac{D_p}{\mu_p} \frac{1}{k_B T}\left[\ \underbrace{\frac{dE_i}{dx}}_{=q\mathcal{E} \text{ at equilibrium}} - \underbrace{\cancel{\frac{dE_F}{dx}}}_{=0 \text{ at equilibrium}}\ \right]$$

$$D_p = \frac{k_B T}{q}\mu_p$$

Einstein relation

Diffusivity – Summarizing

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}p(x)\frac{1}{k_B T}\left[\underbrace{\frac{dE_i}{dx}}_{=q\mathcal{E}} - \underbrace{\cancel{\frac{dE_F}{dx}}}_{=0}\right]$$



$$D_p = \frac{k_B T}{q}\mu_p$$

Einstein relation

Analogous result for electrons

$$\mathcal{E} = -\frac{dV}{dx} = -\frac{d}{dx}\left[\frac{E_i}{-q}\right] = \frac{1}{q}\frac{dE_i}{dx}$$

$$D_n = \frac{k_B T}{q}\mu_n$$

Einstein relation

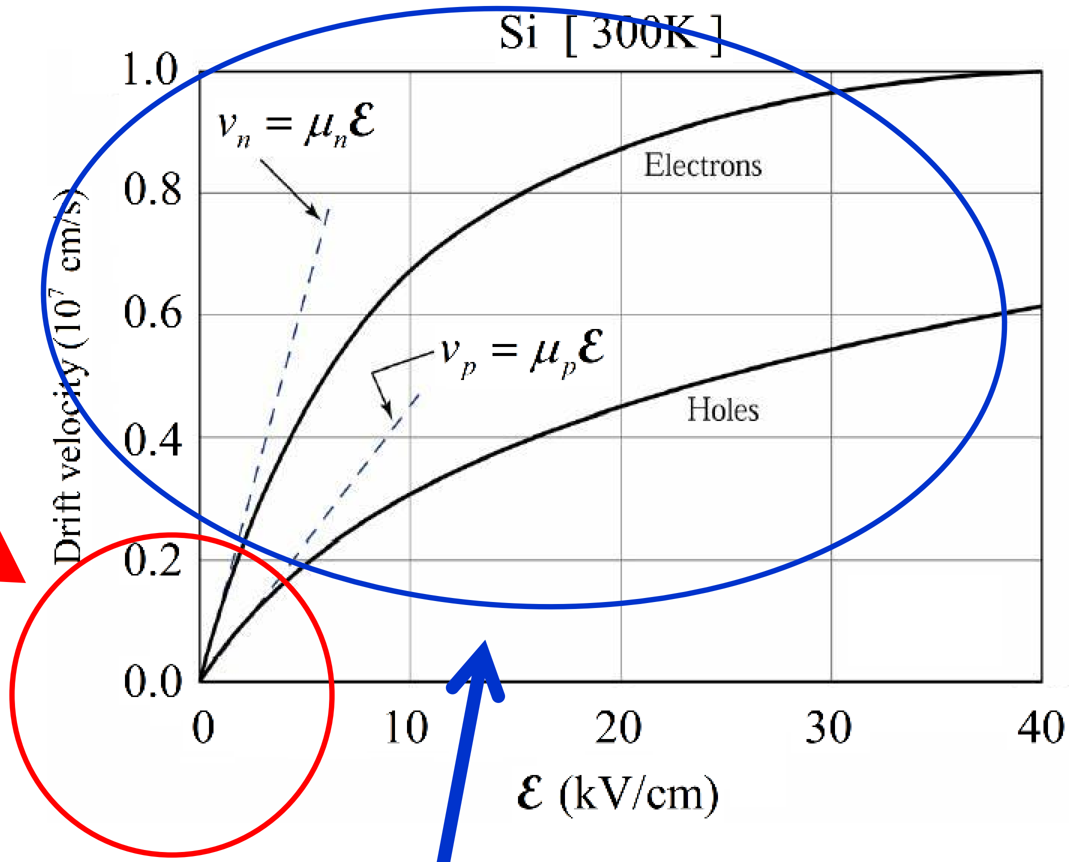
Diffusivity – THIS IS EXTRA FYI

Einstein relations

$$D_p = \frac{k_B T}{q} \mu_p$$

$$D_n = \frac{k_B T}{q} \mu_n$$

Strictly valid in this linear region close to equilibrium



$$\mu(\mathcal{E}) = \frac{\langle v(\mathcal{E}) \rangle}{\mathcal{E}}$$

$$D(\mathcal{E}) = \frac{k_B T}{q} \mu(\mathcal{E})$$

Empirical extension to nonlinear region

Continuity equation

Drift-Diffusion current equations tell us how charge moves "locally" in response to electric field and charge gradient

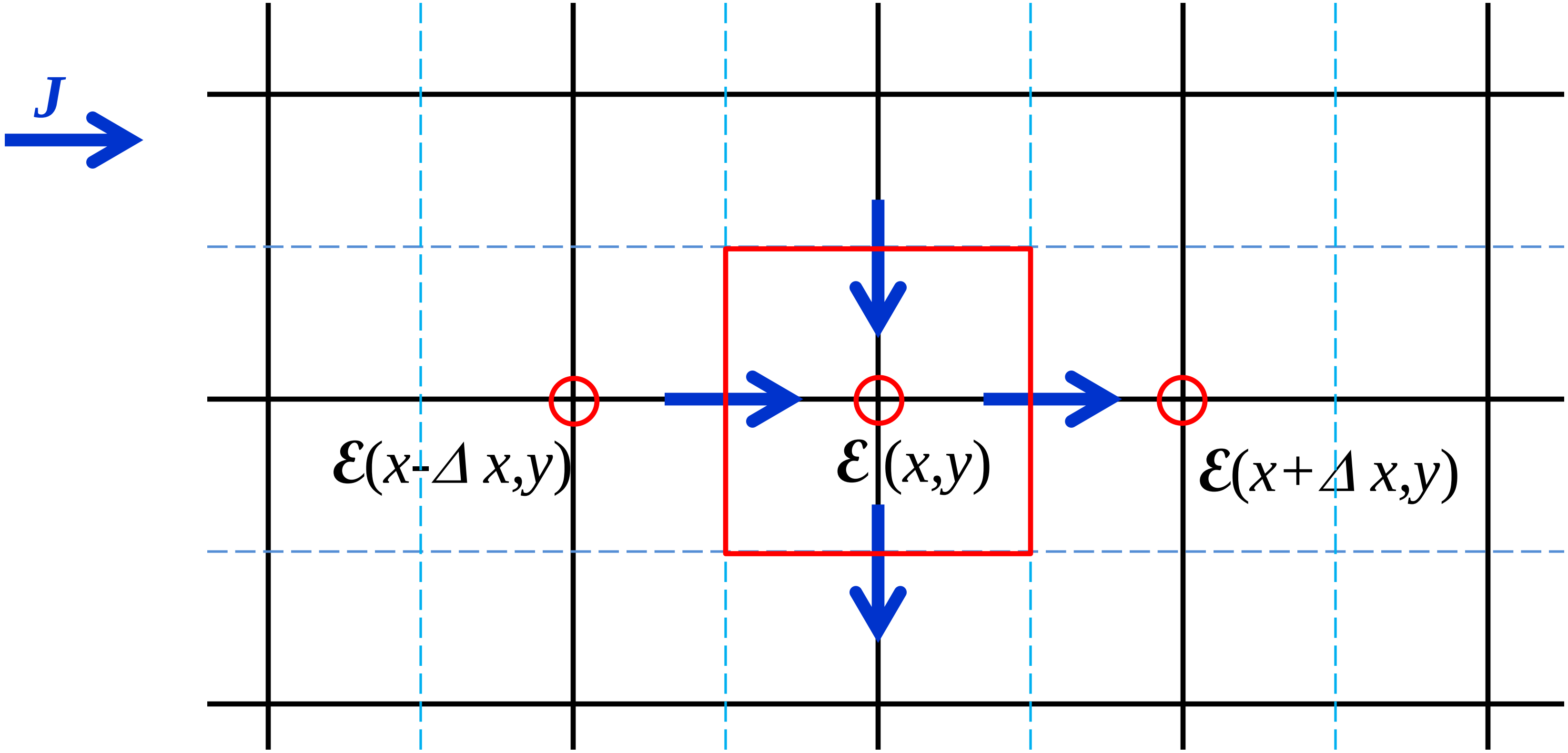
$$J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx}$$

To solve for a "device" structure we need to patch together regions with varying fields and densities. We need a mathematical model which conserves current density.

Charge conservation

Divide the domain in patches. Current exiting one side of a patch is the same entering the neighboring patch. Generation/recombination rates may modify the charge density inside the patches.

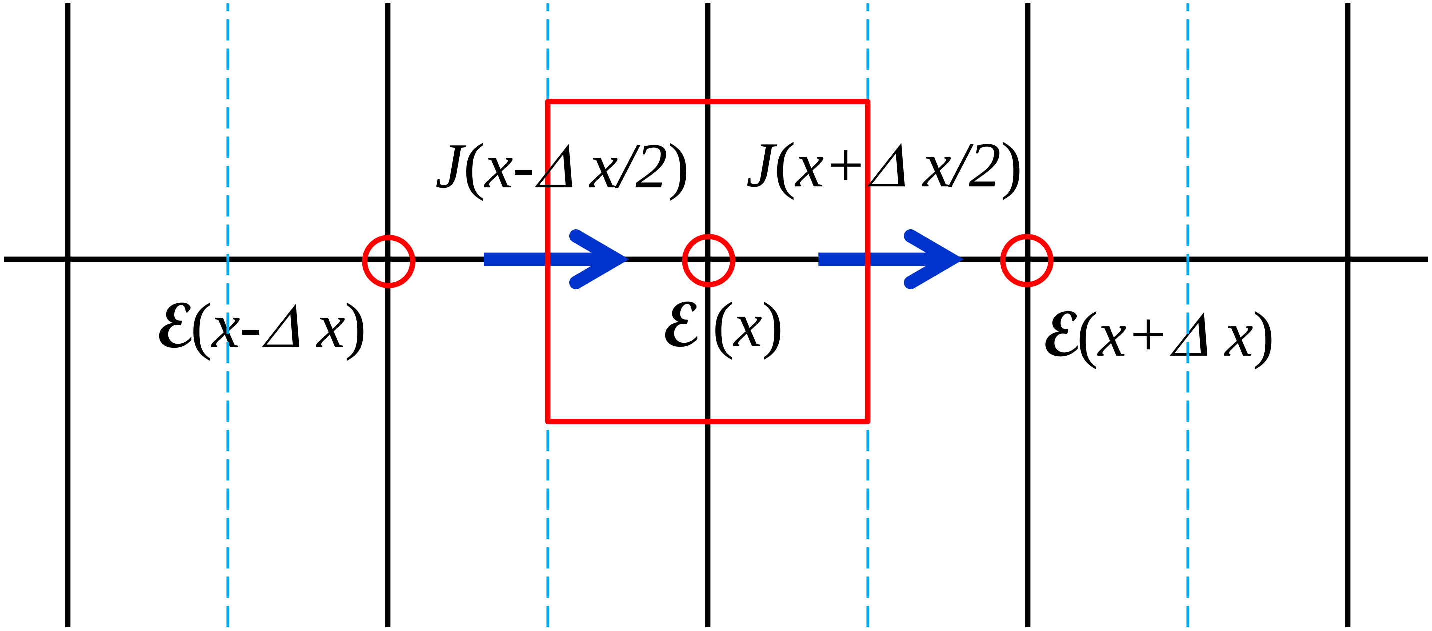


We need equations to express current continuity.

Simplify to 1-D

$$q\frac{\partial p(x,t)}{\partial t} = \lim_{\Delta x \to 0} \frac{J_p(x-\Delta x/2) - J_p(x+\Delta x/2)}{\Delta x} \underbrace{- q\frac{\delta p}{\tau_p}}_{\text{simple recombination rate}}$$

Change (Balance) of hole charge density around point x



1-D Continuity Equations

Holes
$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x}J_p(x) - \frac{\delta p}{\tau_p}$$

Electrons
$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}J_n(x) - \frac{\delta n}{\tau_n}$$

In terms of excess carriers

$$\frac{\partial}{\partial t}\delta p(x,t) = -\frac{1}{q}\frac{\partial}{\partial x}J_p(x) - \frac{\delta p}{\tau_p}$$

$$\frac{\partial}{\partial t}\delta n(x,t) = \frac{1}{q}\frac{\partial}{\partial x}J_n(x) - \frac{\delta n}{\tau_n}$$

Diffusive regime (no drift)

substitute

$$J_n(diff.) = qD_n \frac{\partial}{\partial x} \delta n(x,t)$$

Diffusion Equations

Electrons

$$\frac{\partial}{\partial t} \delta n(x,t) = D_n \frac{\partial^2}{\partial x^2} \delta n(x,t) - \frac{\delta n}{\tau_n}$$

Holes

$$\frac{\partial}{\partial t} \delta p(x,t) = D_p \frac{\partial^2}{\partial x^2} \delta p(x,t) - \frac{\delta p}{\tau_p}$$