ECE 340 Lecture 17
Semiconductor Electronics

Spring 2022
10:00-10:50am
Professor Umberto Ravaioli
Department of Electrical and Computer Engineering
2062 ECE Building
Today’s Discussion

• Steady state solution of the diffusion equation
• Examples
Continuity equation

Drift-Diffusion current equations tell us how charge moves “locally” in response to electric field and charge gradient

\[ J_n(x) = q\mu_n n(x) E(x) + qD_n \frac{dn(x)}{dx} \]
\[ J_p(x) = q\mu_p p(x) E(x) - qD_p \frac{dp(x)}{dx} \]

To solve for a “device” structure we need to patch together regions with varying fields and densities. We need a mathematical model which conserves current density.
Divide the domain in patches. Current exiting one side of a patch is the same entering the neighboring patch. Generation/recombination rates may modify the charge density inside the patches.

\[
\mathcal{E}(x, y) = \mathcal{E}(x + \Delta x, y) = \mathcal{E}(x, y - \Delta y) = \mathcal{E}(x - \Delta x, y) - J
\]

We need equations to express current continuity.
From Lecture 16  Simplify to 1-D

$$q \frac{\partial p(x,t)}{\partial t} = \lim_{\Delta x \to 0} \frac{J_p(x - \Delta x / 2) - J_p(x + \Delta x / 2)}{\Delta x} - q \frac{\delta p}{\tau_p}$$

Change (Balance) of hole charge density around point $x$
From Lecture 16  

1-D Continuity Equations

Holes

\[
\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q} \frac{\partial}{\partial x} J_p(x) - \frac{\delta p}{\tau_p}
\]

Electrons

\[
\frac{\partial n(x,t)}{\partial t} = \frac{1}{q} \frac{\partial}{\partial x} J_n(x) - \frac{\delta n}{\tau_n}
\]

In terms of excess carriers

\[
\frac{\partial}{\partial t} \delta p(x,t) = -\frac{1}{q} \frac{\partial}{\partial x} J_p(x) - \frac{\delta p}{\tau_p}
\]

\[
\frac{\partial}{\partial t} \delta n(x,t) = \frac{1}{q} \frac{\partial}{\partial x} J_n(x) - \frac{\delta n}{\tau_n}
\]
From Lecture 16  Diffusive regime (no drift)

**substitute**

\[ J_n(\text{diff.}) = qD_n \frac{\partial}{\partial x} \delta n(x,t) \]

Diffusion Equations

**Electrons**

\[ \frac{\partial}{\partial t} \delta n(x,t) = D_n \frac{\partial^2}{\partial x^2} \delta n(x,t) - \frac{\delta n}{\tau_n} \]

**Holes**

\[ \frac{\partial}{\partial t} \delta p(x,t) = D_p \frac{\partial^2}{\partial x^2} \delta p(x,t) - \frac{\delta p}{\tau_p} \]
Steady State Excess Carrier Distribution

Steady State

\[
\frac{\partial}{\partial t} \rightarrow 0 \quad \text{[we can use} \quad \frac{d^2}{dx^2} \quad \text{instead of} \quad \frac{\partial^2}{\partial x^2}] \]

Electrons

\[
0 = D_n \frac{d^2}{dx^2} \delta n(x) - \frac{\delta n}{\tau_n} \quad \Rightarrow \quad \frac{d^2}{dx^2} \delta n(x) = \frac{\delta n}{D_n \tau_n}
\]

Holes

\[
0 = D_p \frac{d^2}{dx^2} \delta p(x) - \frac{\delta p}{\tau_p} \quad \Rightarrow \quad \frac{d^2}{dx^2} \delta p(x) = \frac{\delta p}{D_p \tau_p}
\]
Steady State Excess Carrier Distribution

Steady State
\[ \frac{d^2}{dx^2} \delta n(x) = \frac{\delta n}{D_n \tau_n} \]
\[ \frac{d^2}{dx^2} \delta p(x) = \frac{\delta n}{D_p \tau_p} \]

Electrons
\[ \sqrt{D_n \tau_n} = L_n = \text{electron diffusion length} \]
\[ \frac{d^2}{dx^2} \delta n(x) = \frac{\delta n}{L_n^2} \]

Holes
\[ \sqrt{D_p \tau_p} = L_p = \text{hole diffusion length} \]
\[ \frac{d^2}{dx^2} \delta p(x) = \frac{\delta p}{L_p^2} \]
Example: steady state hole injection

\[ \delta p(x = 0) = \Delta p \]

\[ x = 0 \]

Semiconductor bar

Holes diffuse and recombine
Example: steady state hole injection

Diffusion equation

\[
\frac{d^2 \delta p(x)}{dx^2} = \frac{\delta p}{L^2_p}
\]

General solution

\[
\delta p(x) = C_1 \exp\left(\frac{x}{L_p}\right) + C_2 \exp\left(-\frac{x}{L_p}\right)
\]

\[
C_1 = 0 \quad \quad C_2 = \Delta p
\]

Diffusion Length
Average distance a hole diffuses before recombining
Proof: Diffusion Length (1)

We have the solution

\[ \delta p(x) = \Delta p \exp\left(-\frac{x}{L_p}\right) \]

**Probability to reach \( x \) without recombining**

\[ \frac{\delta p(x)}{\Delta p} = \exp\left(-\frac{x}{L_p}\right) \quad [A] \]

**Probability to recombine between \( x \) and \( x + dx \) after having reached \( x \)**

\[ 1 - \frac{\delta p(x + dx)}{\delta p(x)} = \frac{\delta p(x) - \delta p(x + dx)}{\delta p(x)} = \]

\[ = \frac{-\left(\frac{d}{dx} \delta p(x)\right)}{\delta p(x)} \quad [B] \]

\[ = \frac{-\Delta p \exp\left(-\frac{x}{L_p}\right)\left(-1/L_p\right)}{\Delta p \exp\left(-x/L_p\right)} \quad dx = \frac{1}{L_p} \quad dx \]
Proof: Diffusion Length (2)

Probability that a hole injected at $x = 0$ will recombine in a given $dx$

$$[A] \times [B] = \exp\left(-\frac{x}{L_p}\right) \times \frac{1}{L_p} \, dx = \frac{\exp\left(-\frac{x}{L_p}\right)}{L_p} \, dx$$

Average distance a hole diffuses before recombining

$$\langle x \rangle = \int_0^\infty x \frac{\exp\left(-\frac{x}{L_p}\right)}{L_p} \, dx = L_p$$

use

$$\int x e^{ax} \, dx = \frac{e^{ax}}{a} \left(x - \frac{1}{a}\right)$$
Diffusion current density

Excess hole density

\[ \delta p(x) = \Delta p \exp\left(-x/L_p\right) \]

Diffusion current density

\[ J_p(x) = -qD_p \frac{dp(x)}{dx} = -qD_p \frac{dp}{dx} \delta p(x) = \]

\[ = -qD_p \frac{d}{dx} \left( \Delta p \exp\left(-x/L_p\right) \right) = \]

\[ = q \frac{D_p}{L_p} \Delta p \exp\left(-x/L_p\right) = q \frac{D_p}{L_p} \delta p(x) \]
Example

Steady state excess hole density injection

\[ \Delta p = 5 \times 10^{16} \text{cm}^{-3} \]

Very long bar of Si

\[ A = 0.5 \text{ cm}^2 \]
\[ N_A = 10^{17} \text{ cm}^{-3} \]

Einstein relation

\[ D_p = \frac{k_B T}{q} \mu_p = 0.0259 \times 500 = 12.95 \text{ cm}^2/\text{s} \]

\[ L_p = \sqrt{D_p \tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.6 \times 10^{-5} \text{ cm} \]

Question: what is the quasi-Fermi level at \( x = 1000 \text{Å} = 10^{-5} \text{cm} \)
quasi-Fermi level at $x = 10^{-5}$ cm

\[ \delta p(x = 0) = \Delta p \]

Steady state excess hole density injection \( \Delta p = 5 \times 10^{16} \text{cm}^{-3} \)

\[ p(x) = p_0 + \Delta p \exp\left(-x/L_p\right) = \]

\[ = 10^{17} + 5 \times 10^{16} \exp\left[\frac{-10^{-5}}{3.6 \times 10^{-5}}\right] = 1.379 \times 10^{17} \text{cm}^{-3} \]

\[ p(x) = n_i \exp\left[\left(E_i - F_p\right)/k_B T\right] = 1.5 \times 10^{10} \times \exp\left[\left(E_i - F_p\right)/k_B T\right] \text{cm}^{-3} \]

\[ E_i - F_p = \left(\ln \frac{1.379 \times 10^{17}}{1.5 \times 10^{10}}\right) \times 0.0259 = 0.415 \text{ eV} \]

\[ E_C - F_p = \frac{E_g}{2} + (E_i - F_p) = \frac{1.12}{2} + 0.415 = 0.965 \text{ eV} \]
Hole Current at $x = \SI{10^{-5}}{\text{cm}}$

\[
\delta p(x = 0) = \Delta p
\]

Steady state excess hole density injection
\[
\Delta p = 5 \times 10^{16} \text{ cm}^{-3}
\]

\[
I_p = -qAD_p \frac{d p(x)}{dx} = qA \frac{D_p}{L_p} \Delta p \exp \left(-\frac{x}{L_p}\right)
\]

\[
I_p = 1.6 \times 10^{-19} \times 0.5 \times \frac{12.95}{3.6 \times 10^{-5}} \times 5 \times 10^{16} \exp \left[\frac{10^{-5}}{3.6 \times 10^{-5}}\right] = 1.09 \times 10^3 \text{ A}
\]

Excess stored hole charge
\[
Q_p = q \ A \Delta p \ L_p = 1.6 \times 10^{-19} \times 0.5 \times 5 \times 10^{16} \times 3.6 \times 10^{-5} = 1.44 \times 10^{-7} \text{ C}
\]
Excess stored charge

\[ Q_p = q \ A \Delta p \ L_p \]

\[ \delta p(x) = \Delta p \exp\left(-\frac{x}{L_p}\right) \]
Excess stored charge – Proof

Total charge along positive $x$

\[
A \int_{0}^{\infty} q \delta p(x) \, dx = qA \int_{0}^{\infty} \Delta p \exp\left(-x/L_p\right) \, dx
\]

\[
= qA \Delta p \left[-L_p \exp\left(-x/L_p\right)\right]_{0}^{\infty} = 
\]

\[
= qA \Delta p \left[0 - \left(-L_p \exp\left(-0/L_p\right)\right)\right] = 
\]

\[
= qA \Delta p \ L_p
\]
Relationship between current and $Q_p$

\[ I_p(x = 0) = qA \frac{D_p}{L_p} \Delta p \exp \left( -\frac{0}{L_p} \right) \]

\[ = qA \frac{D_p}{L_p} \Delta p = qA \frac{L^2}{L_p \tau_p} \Delta p \]

\[ = \frac{qA \Delta p L_p}{\tau_p} = \frac{Q_p}{\tau_p} \]