ECE 340 Lecture 17
Semiconductor Electronics

Spring 2022
10:00-10:50am
Professor Umberto Ravaioli
Department of Electrical and Computer Engineering
2062 ECE Building

Today's Discussion

- Steady state solution of the diffusion equation
- Examples

From Lecture 16 Continuity equation

Drift-Diffusion current equations tell us how charge moves "locally" in response to electric field and charge gradient

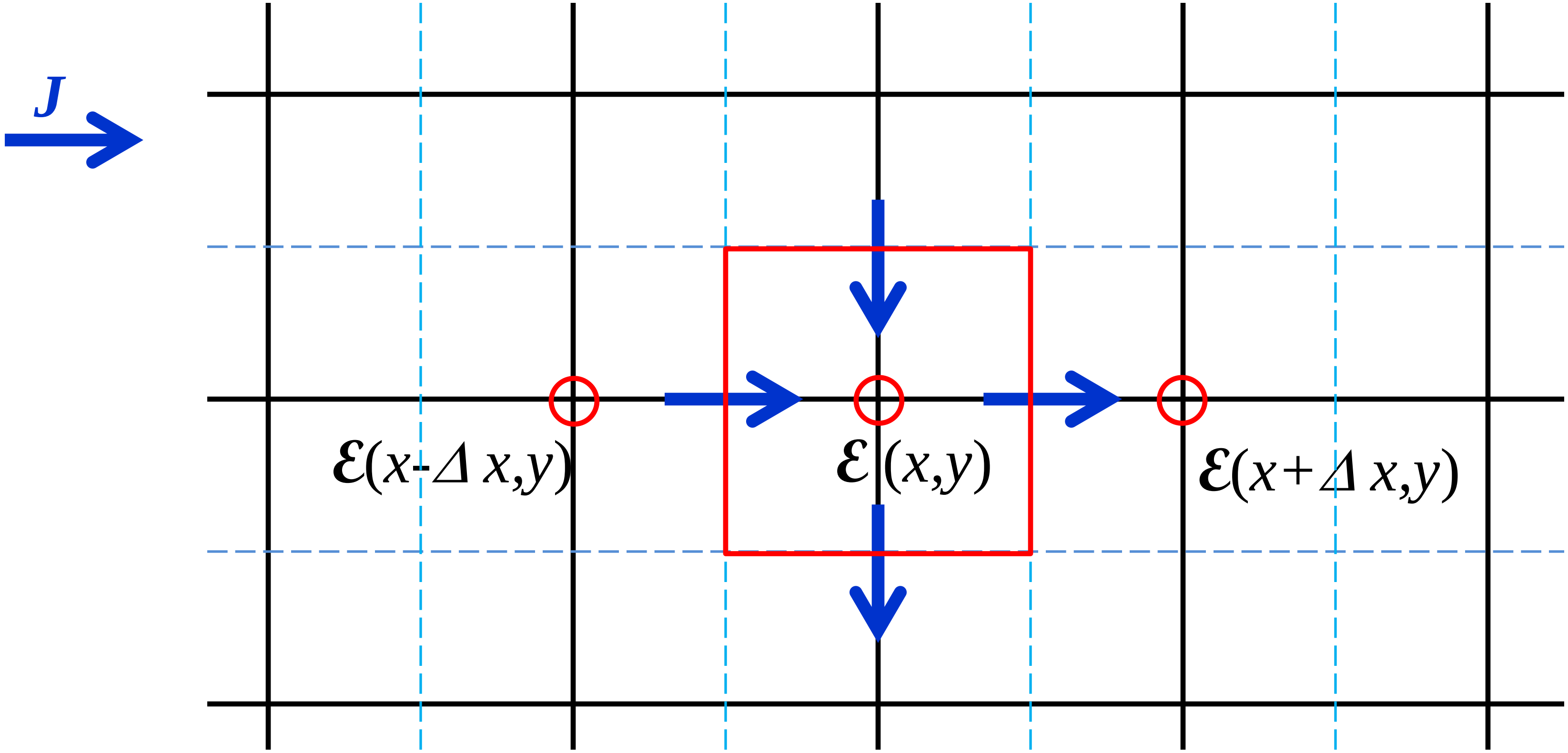
$$J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx}$$

To solve for a "device" structure we need to patch together regions with varying fields and densities. We need a mathematical model which conserves current density.

Charge conservation

Divide the domain in patches. Current exiting one side of a patch is the same entering the neighboring patch. Generation/recombination rates may modify the charge density inside the patches.

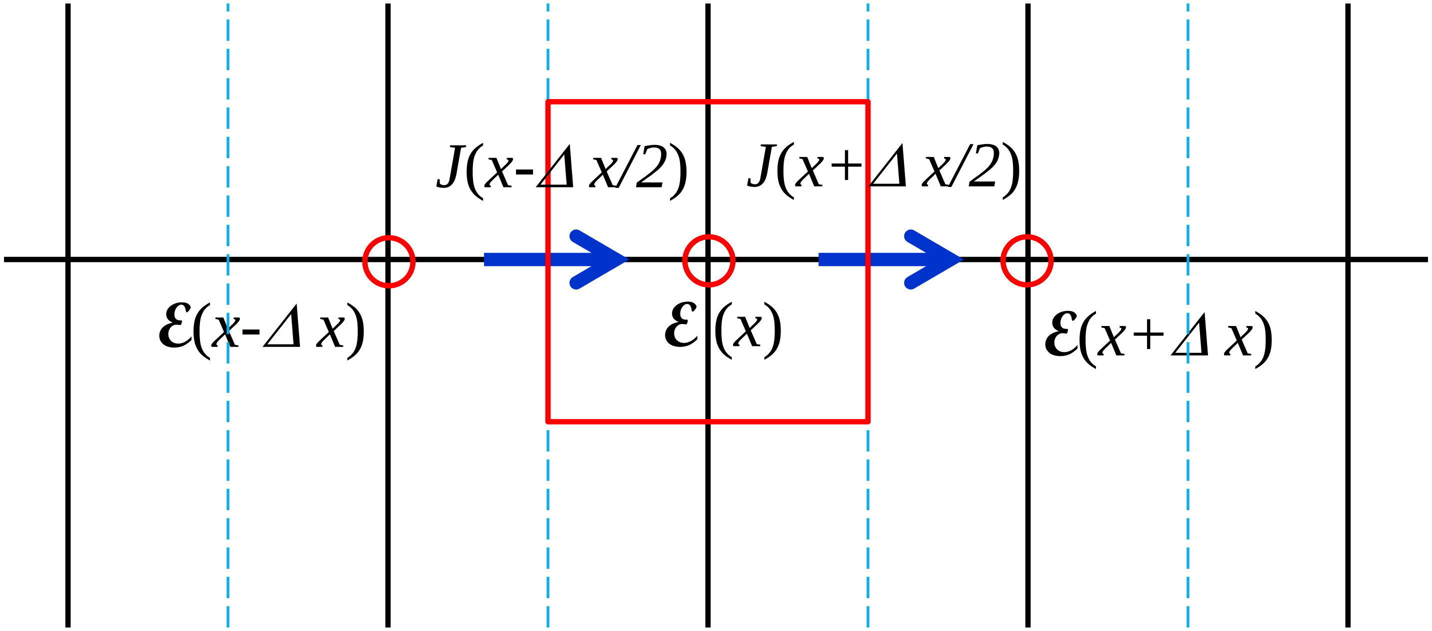


We need equations to express current continuity.

Simplify to 1-D

$$q\frac{\partial p(x,t)}{\partial t} = \lim_{\Delta x \to 0} \frac{J_p(x-\Delta x/2) - J_p(x+\Delta x/2)}{\Delta x} \underbrace{-q\frac{\delta p}{\tau_p}}_{\text{simple recombination rate}}$$

Change (Balance) of hole charge density around point x



From Lecture 16 1-D Continuity Equations

Holes

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x}J_p(x) - \frac{\delta p}{\tau_p}$$

Electrons

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}J_n(x) - \frac{\delta n}{\tau_n}$$

In terms of excess carriers

$$\frac{\partial}{\partial t}\delta p(x,t) = -\frac{1}{q}\frac{\partial}{\partial x}J_p(x) - \frac{\delta p}{\tau_p}$$

$$\frac{\partial}{\partial t}\delta n(x,t) = \frac{1}{q}\frac{\partial}{\partial x}J_n(x) - \frac{\delta n}{\tau_n}$$

From Lecture 16 Diffusive regime (no drift)

substitute $$J_n(diff.) = qD_n \frac{\partial}{\partial x}\delta n(x,t)$$

Diffusion Equations

Electrons $$\frac{\partial}{\partial t}\delta n(x,t) = D_n \frac{\partial^2}{\partial x^2}\delta n(x,t) - \frac{\delta n}{\tau_n}$$

Holes $$\frac{\partial}{\partial t}\delta p(x,t) = D_p \frac{\partial^2}{\partial x^2}\delta p(x,t) - \frac{\delta p}{\tau_p}$$

Steady State Excess Carrier Distribution

Steady State

$$\frac{\partial}{\partial t} \to 0 \qquad [\text{we can use } \frac{d^2}{dx^2} \text{ instead of } \frac{\partial^2}{\partial x^2}]$$

Electrons $$0 = D_n \frac{d^2}{dx^2}\delta n(x) - \frac{\delta n}{\tau_n} \quad \Rightarrow \frac{d^2}{dx^2}\delta n(x) = \frac{\delta n}{D_n \tau_n}$$

Holes $$0 = D_p \frac{d^2}{dx^2}\delta p(x) - \frac{\delta p}{\tau_p} \quad \Rightarrow \frac{d^2}{dx^2}\delta p(x) = \frac{\delta p}{D_p \tau_p}$$

Steady State Excess Carrier Distribution

Steady State
$$\frac{d^2}{dx^2}\delta n(x) = \frac{\delta n}{D_n \tau_n}$$

$$\frac{d^2}{dx^2}\delta p(x) = \frac{\delta n}{D_p \tau_p}$$

units of D are **cm²/s**

Electrons
$$\sqrt{D_n \tau_n} = L_n = \text{ electron diffusion length}$$

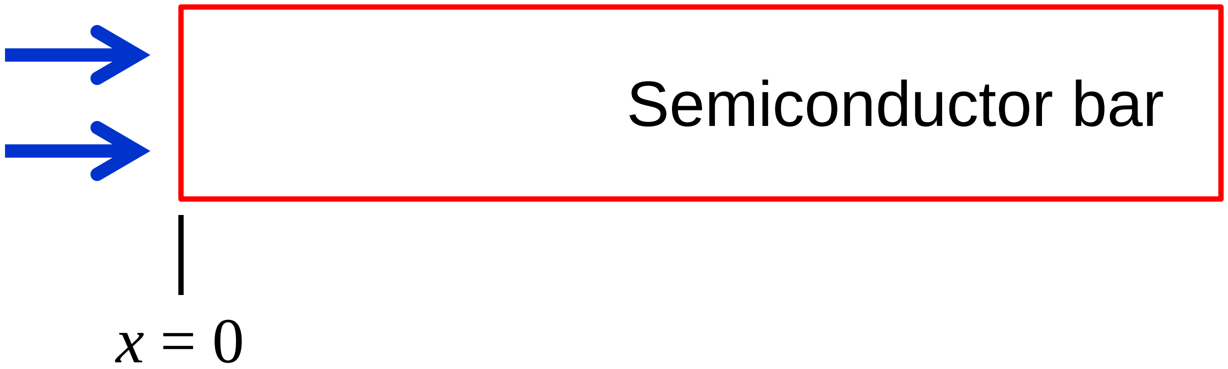
$$\frac{d^2}{dx^2}\delta n(x) = \frac{\delta n}{L_n^2}$$

Holes
$$\sqrt{D_p \tau_p} = L_p = \text{ hole diffusion length}$$

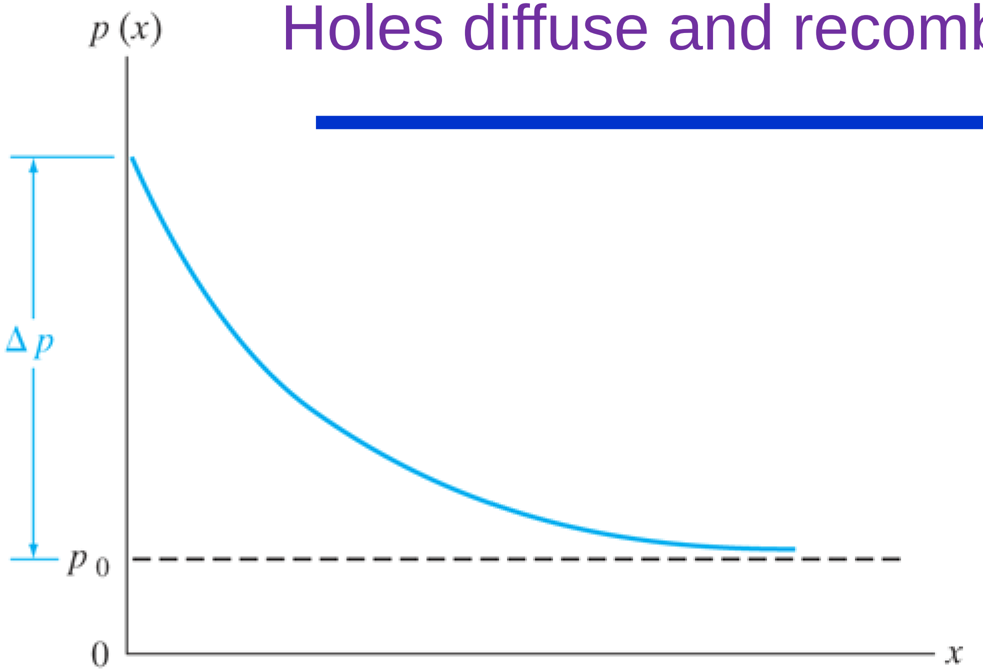
$$\frac{d^2}{dx^2}\delta p(x) = \frac{\delta p}{L_p^2}$$

Example: steady state hole injection

$$\delta p(x=0) = \Delta p$$



Holes diffuse and recombine



Example: steady state hole injection

Diffusion equation

$$\frac{d^2}{dx^2}\delta p(x) = \frac{\delta p}{L_p^2}$$

General solution

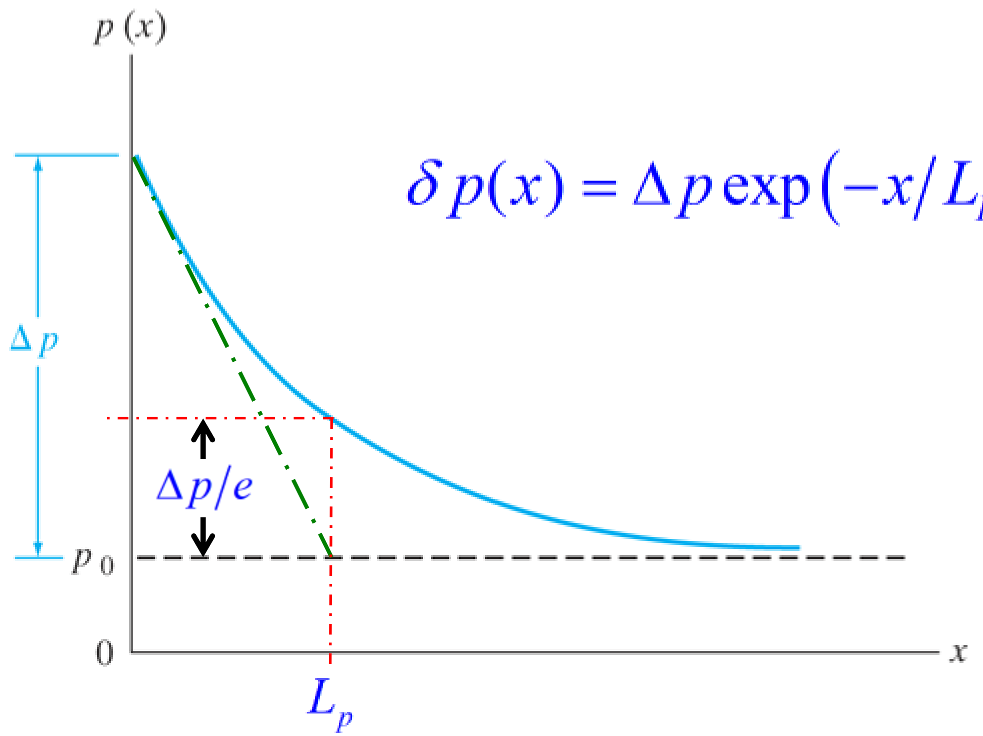
$$\delta p(x) = C_1 \exp\left(x/L_P\right) + C_2 \exp\left(-x/L_P\right)$$

$C_1 = 0$ $C_2 = \Delta p$

L_p

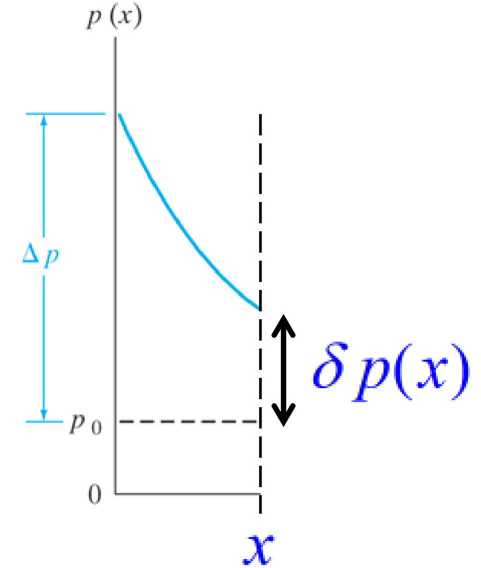
Diffusion Length

Average distance a hole diffuses before recombining



Proof: Diffusion Length (1)

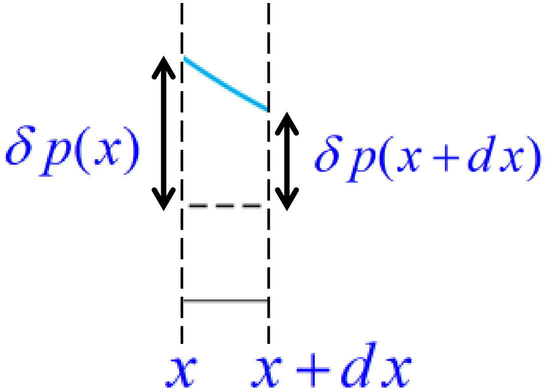
We have the solution $\delta p(x) = \Delta p \exp(-x/L_p)$



Probability to reach x without recombining

$$\frac{\delta p(x)}{\Delta p} = \exp(-x/L_p) \qquad [A]$$

Probability to recombine between x and $x + dx$ after having reached x



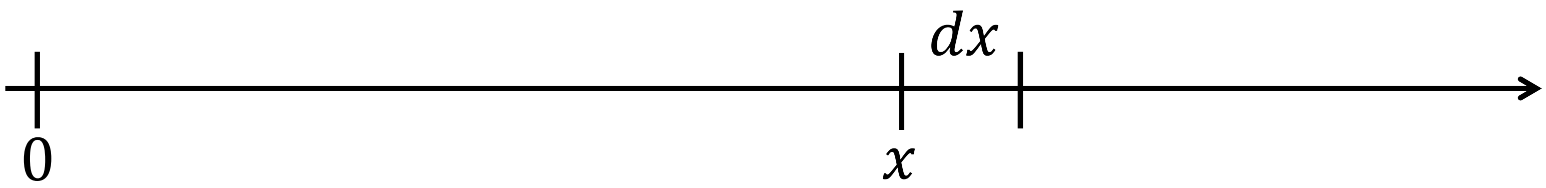
$$1 - \frac{\delta p(x+dx)}{\delta p(x)} = \frac{\delta p(x) - \delta p(x+dx)}{\delta p(x)} =$$

$$= \frac{-\left(\frac{d}{dx}\delta p(x)\right)dx}{\delta p(x)} = \frac{-\Delta p \exp(-x/L_p)(-1/L_p)}{\Delta p \exp(-x/L_p)} dx = \frac{1}{L_p} dx$$

[B]

Proof: Diffusion Length (2)

Probability that a hole injected at $\boldsymbol{x = 0}$ will recombine in a given $\boldsymbol{dx}$



$$[\mathrm{A}]\times[\mathrm{B}] = \exp\left(-x/L_p\right)\times\frac{1}{L_p}dx = \frac{\exp\left(-x/L_p\right)}{L_p}dx$$

Average distance a hole diffuses before recombining

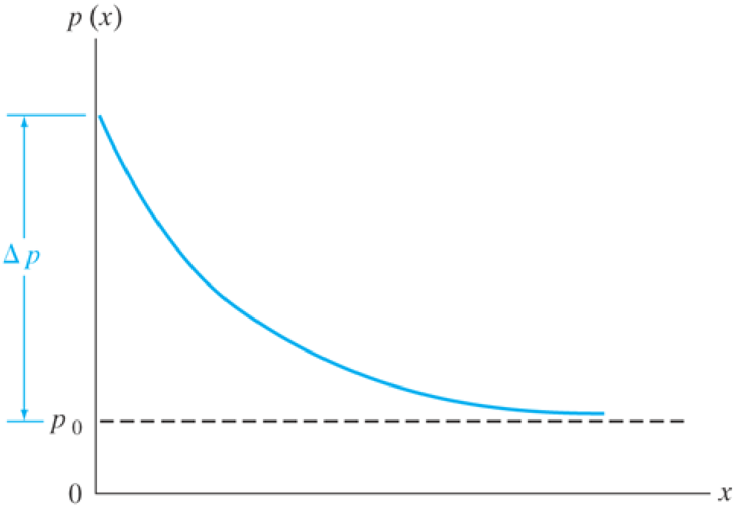
$$\langle x\rangle = \int_0^\infty x\frac{\exp\left(-x/L_p\right)}{L_p}dx = L_p$$

use $$\int x e^{ax}dx = \frac{e^{ax}}{a}\left(x-\frac{1}{a}\right)$$

Diffusion current density

Excess hole density

$$\delta p(x) = \Delta p \exp\left(-x/L_p\right)$$



Diffusion current density

$$J_p(x) = -qD_p \frac{d\,p(x)}{dx} = -qD_p \frac{d}{dx}\delta p(x) =$$

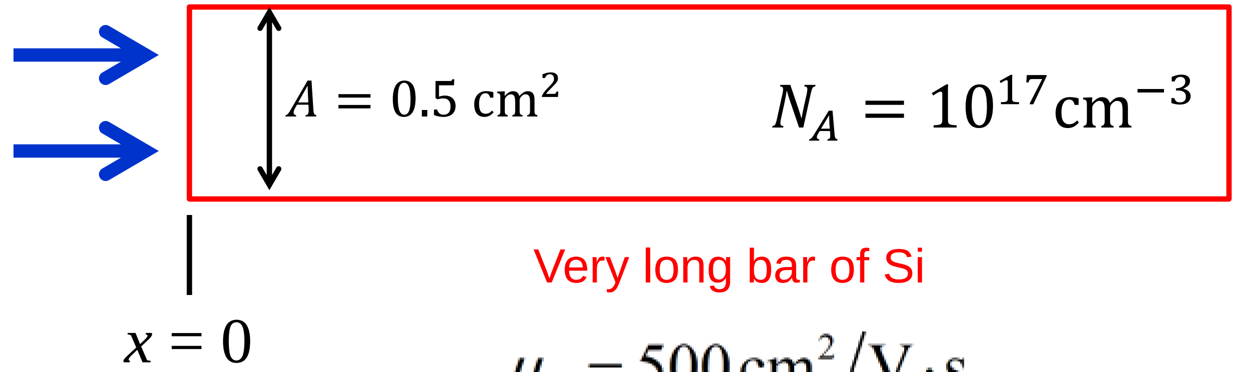
$$= -qD_p \frac{d}{dx}\left(\Delta p \exp\left(-x/L_p\right)\right) =$$

$$= q\frac{D_p}{L_p}\Delta p \exp\left(-x/L_p\right) = q\frac{D_p}{L_p}\delta p(x)$$

Example

$$\delta p(x=0) = \Delta p$$

Steady state excess
hole density injection
$\Delta p = 5 \times 10^{16} \text{cm}^{-3}$



Very long bar of Si

$$\mu_p = 500 \, \text{cm}^2/\text{V}\cdot\text{s}$$

$$\tau_p = 10^{-10} \text{s}$$

$$D_p = \frac{k_B T}{q}\mu_p = 0.0259 \times 500 = 12.95 \, \text{cm}^2/\text{s}$$

Einstein relation

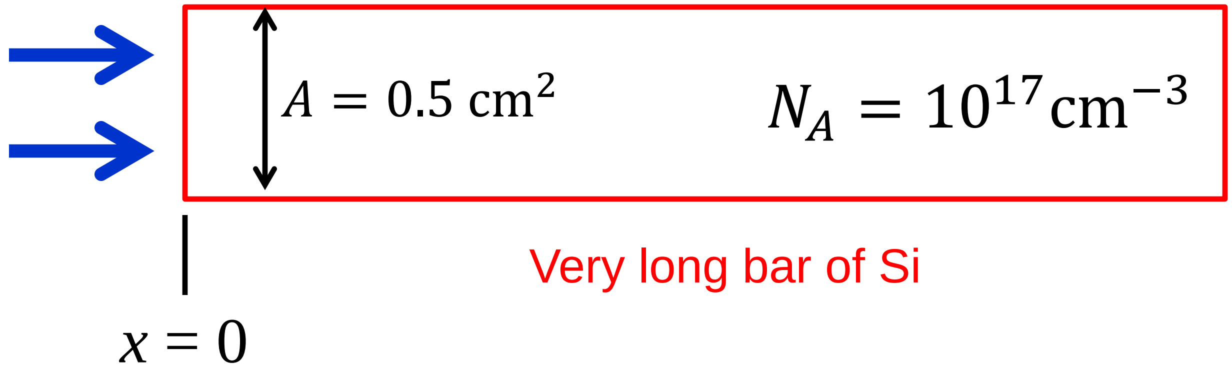
$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.6 \times 10^{-5} \text{cm}$$

Question: what is the quasi-Fermi level at $x = 1000\text{Å} = 10^{-5} \text{cm}$

quasi-Fermi level at $x = 10^{-5}\text{cm}$

$$\delta p(x=0) = \Delta p$$

Steady state excess
hole density injection
$\Delta p = 5 \times 10^{16}\text{cm}^{-3}$



$$p(x) = p_0 + \Delta p \exp\left(-x/L_p\right) =$$

$$= 10^{17} + 5\times10^{16} \exp\left[\frac{-10^{-5}}{3.6\times10^{-5}}\right] = 1.379\times10^{17}\,\text{cm}^{-3}$$

$$p(x) = n_i \exp\left[\left(E_i - F_p\right)/k_B T\right] = 1.5\times10^{10} \times \exp\left[\left(E_i - F_p\right)/k_B T\right]\text{cm}^{-3}$$

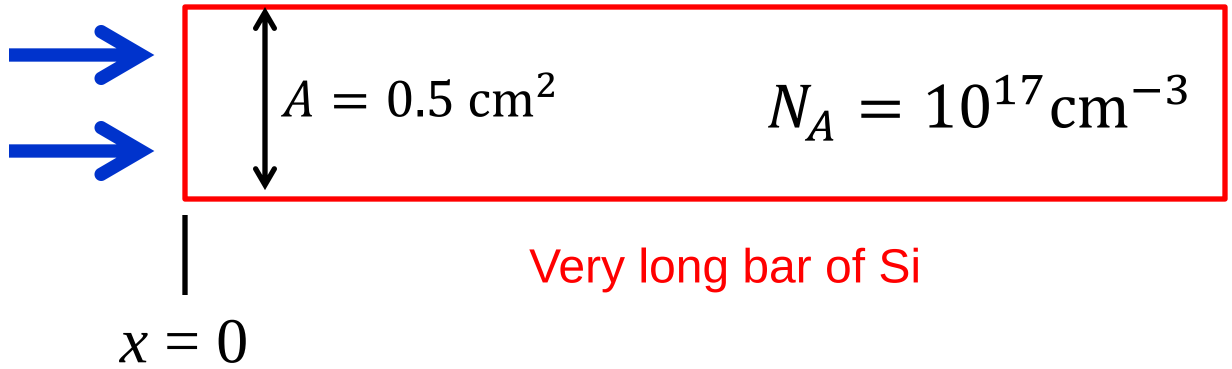
$$E_i - F_p = \left(\ln\frac{1.379\times10^{17}}{1.5\times10^{10}}\right)\times 0.0259 = 0.415\,\text{eV}$$

$$E_C - F_p = \frac{E_g}{2} + \left(E_i - F_p\right) = \frac{1.12}{2} + 0.415 = 0.965\,\text{eV}$$

Hole Current at $x = 10^{-5}\text{cm}$

$\delta p(x=0) = \Delta p$

Steady state excess hole density injection
$\Delta p = 5 \times 10^{16}\text{cm}^{-3}$



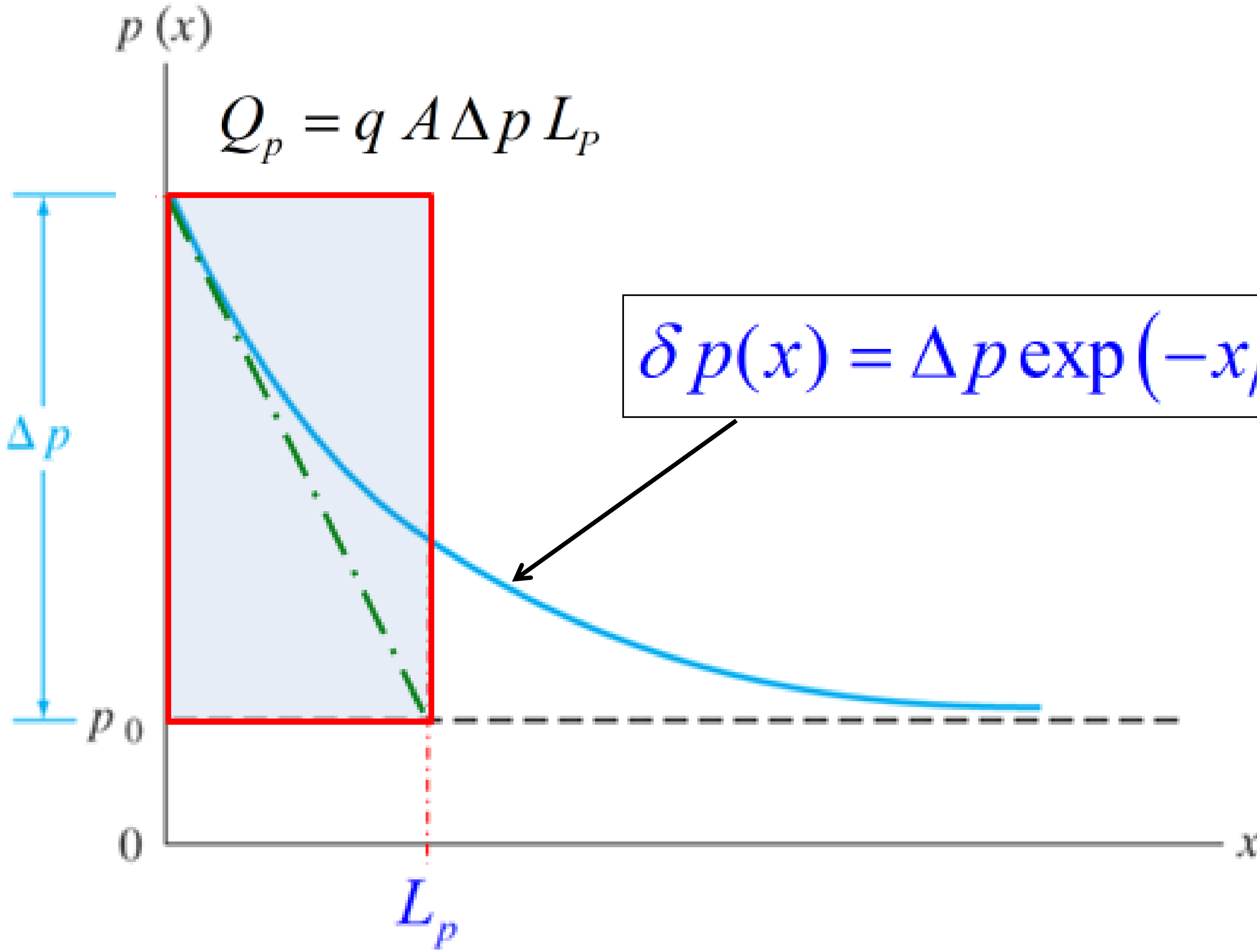
$$I_p = -qAD_p \frac{d\,p(x)}{dx} = qA\frac{D_p}{L_p}\Delta p \exp\left(-x/L_p\right)$$

$$I_p = 1.6\times 10^{-19}\times 0.5\times \frac{12.95}{3.6\times 10^{-5}}\times 5\times 10^{16}\exp\left[\frac{10^{-5}}{3.6\times 10^{-5}}\right] = 1.09\times 10^{3}\,\text{A}$$

Excess stored hole charge

$$Q_p = q\,A\,\Delta p\,L_p = 1.6\times 10^{-19}\times 0.5\times 5\times 10^{16}\times 3.6\times 10^{-5} = 1.44\times 10^{-7}\,\text{C}$$

Excess stored charge



Excess stored charge – Proof

$$\delta p(x) = \Delta p \exp\left(-x/L_p\right)$$

Total charge along positive *x*

$$A\int_0^\infty q\delta p(x)\,dx = qA\int_0^\infty \Delta p \exp\left(-x/L_p\right) dx$$

$$= qA\Delta p\left[-L_p \exp\left(-x/L_p\right)\right]_0^\infty =$$

$$= qA\Delta p\left[0-\left(-L_p \exp\left(-0/L_p\right)\right)\right]$$

$$= qA\Delta p\, L_P$$

Relationship between current and Q_p

$$I_p(x=0) = qA\frac{D_p}{L_p}\Delta p \exp\left(-0/L_p\right)$$

$$= qA\frac{D_p}{L_p}\Delta p = qA\frac{L_p^2}{L_p\tau_p}\Delta p$$

$$= \frac{qA\Delta p L_p}{\tau_p} = \frac{Q_p}{\tau_p}$$