ECE 340 Lecture18
Solid State Electronic Devices

Spring 2022 – Section A
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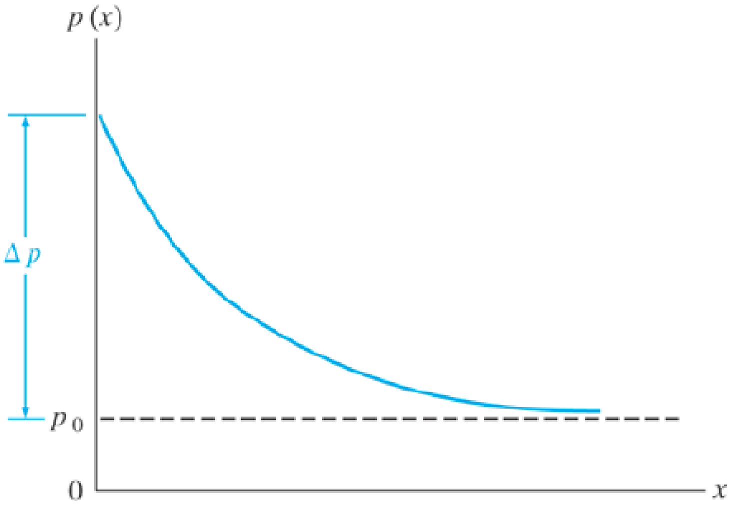
Today’s Discussion

- **Diffusion example**
- **Excess stored charge model**
- **The p-n junction**
- **Contact potential**

Diffusion current density

Excess hole density

$$\delta p(x) = \Delta p \exp\left(-x/L_p\right)$$



Diffusion current density

$$J_p(x) = -qD_p \frac{d\,p(x)}{dx} = -qD_p \frac{d}{dx} \delta p(x) =$$

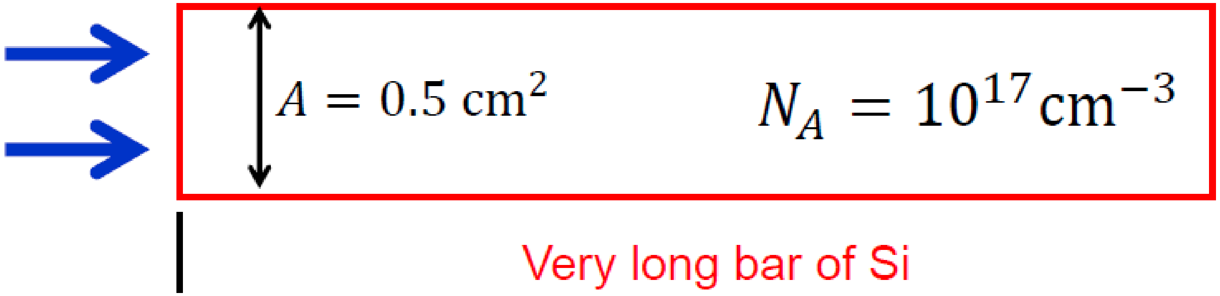
$$= -qD_p \frac{d}{dx}\left(\Delta p \exp\left(-x/L_p\right)\right) =$$

$$= q\frac{D_p}{L_p} \Delta p \exp\left(-x/L_p\right) = q\frac{D_p}{L_p} \delta p(x)$$

Example

$$\delta p(x=0) = \Delta p$$

Steady state excess hole density injection $\Delta p = 5 \times 10^{16} \mathrm{cm}^{-3}$



$x = 0$

Very long bar of Si

$$\mu_p = 500\,\mathrm{cm}^2/\mathrm{V\cdot s}$$

$$\tau_p = 10^{-10}\,\mathrm{s}$$

$$D_p = \frac{k_B T}{q} \mu_p = 0.0259 \times 500 = 12.95\,\mathrm{cm}^2/\mathrm{s}$$

Einstein relation

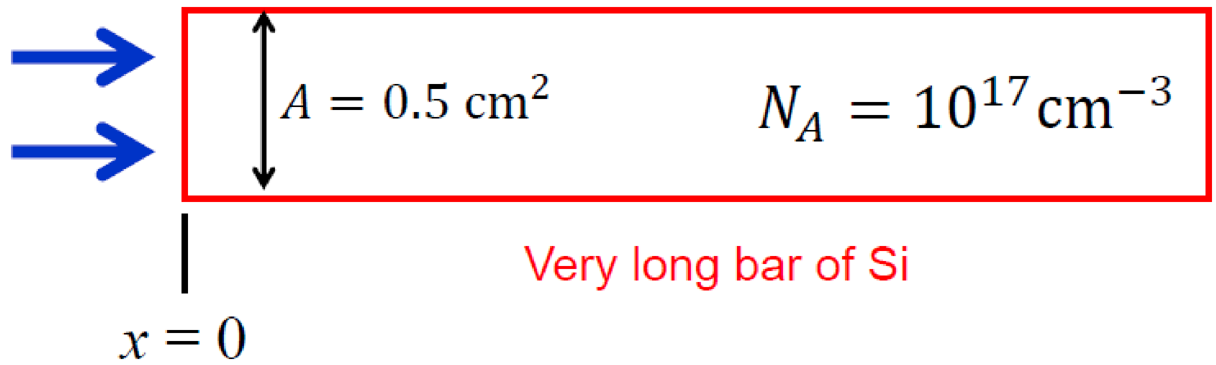
$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.6 \times 10^{-5}\,\mathrm{cm}$$

Question: what is the quasi-Fermi level at $x = 1000\text{Å} = 10^{-5}\mathrm{cm}$

Quasi- at $x = 10^{-5}$ cm

$$\delta p(x=0) = \Delta p$$

Steady state excess hole density injection $\Delta p = 5 \times 10^{16} \mathrm{cm}^{-3}$



$$p(x) = p_0 + \Delta p \exp\left(-x/L_p\right) =$$

$$= 10^{17} + 5\times10^{16} \exp\left[\frac{-10^{-5}}{3.6\times10^{-5}}\right] = 1.379\times10^{17}\,\mathrm{cm}^{-3}$$

$$p(x) = n_i \exp\left[\left(E_i - F_p\right)/k_B T\right] = 1.5\times10^{10} \times \exp\left[\left(E_i - F_p\right)/k_B T\right]\mathrm{cm}^{-3}$$

$$E_i - F_p = \left(\ln\frac{1.379\times10^{17}}{1.5\times10^{10}}\right)\times 0.0259 = 0.415\,\mathrm{eV}$$

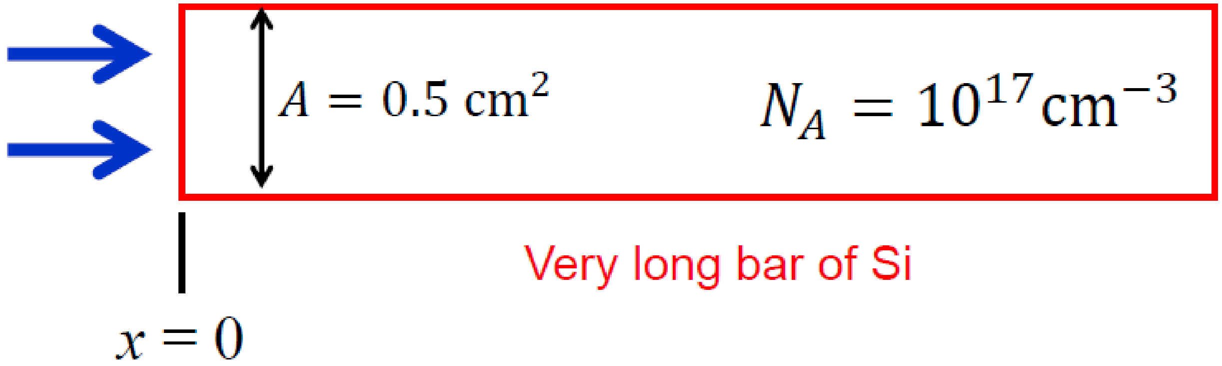
$$E_C - F_p = \frac{E_g}{2} + \left(E_i - F_p\right) = \frac{1.12}{2} + 0.415 = 0.965\,\mathrm{eV}$$

Hole Current at $x = 10^{-5}$ cm

$$\delta p(x=0) = \Delta p$$

Steady state excess hole density injection

$$\Delta p = 5 \times 10^{16} \text{cm}^{-3}$$



$$I_p = -qAD_p \frac{d\,p(x)}{dx} = qA\frac{D_p}{L_p}\Delta p \exp\left(-x/L_p\right)$$

$$I_p = 1.6\times10^{-19}\times0.5\times\frac{12.95}{3.6\times10^{-5}}\times5\times10^{16}\exp\left[\frac{10^{-5}}{3.6\times10^{-5}}\right] = 1.09\times10^{3}\,\text{A}$$

Excess stored hole charge

$$Q_p = q\,A\,\Delta p\,L_p = 1.6\times10^{-19}\times0.5\times5\times10^{16}\times3.6\times10^{-5} = 1.44\times10^{-7}\,\text{C}$$

Excess stored charge - Proof

$$\delta p(x) = \Delta p \exp\left(-x/L_p\right)$$

Total charge along positive *x*

$$A\int_0^\infty q\delta p(x)\,dx = qA\int_0^\infty \Delta p \exp\left(-x/L_p\right)dx$$

$$= qA\,\Delta p\left[-L_p \exp\left(-x/L_p\right)\right]_0^\infty =$$

$$= qA\,\Delta p\left[0-\left(-L_p \exp\left(-0/L_p\right)\right)\right]$$

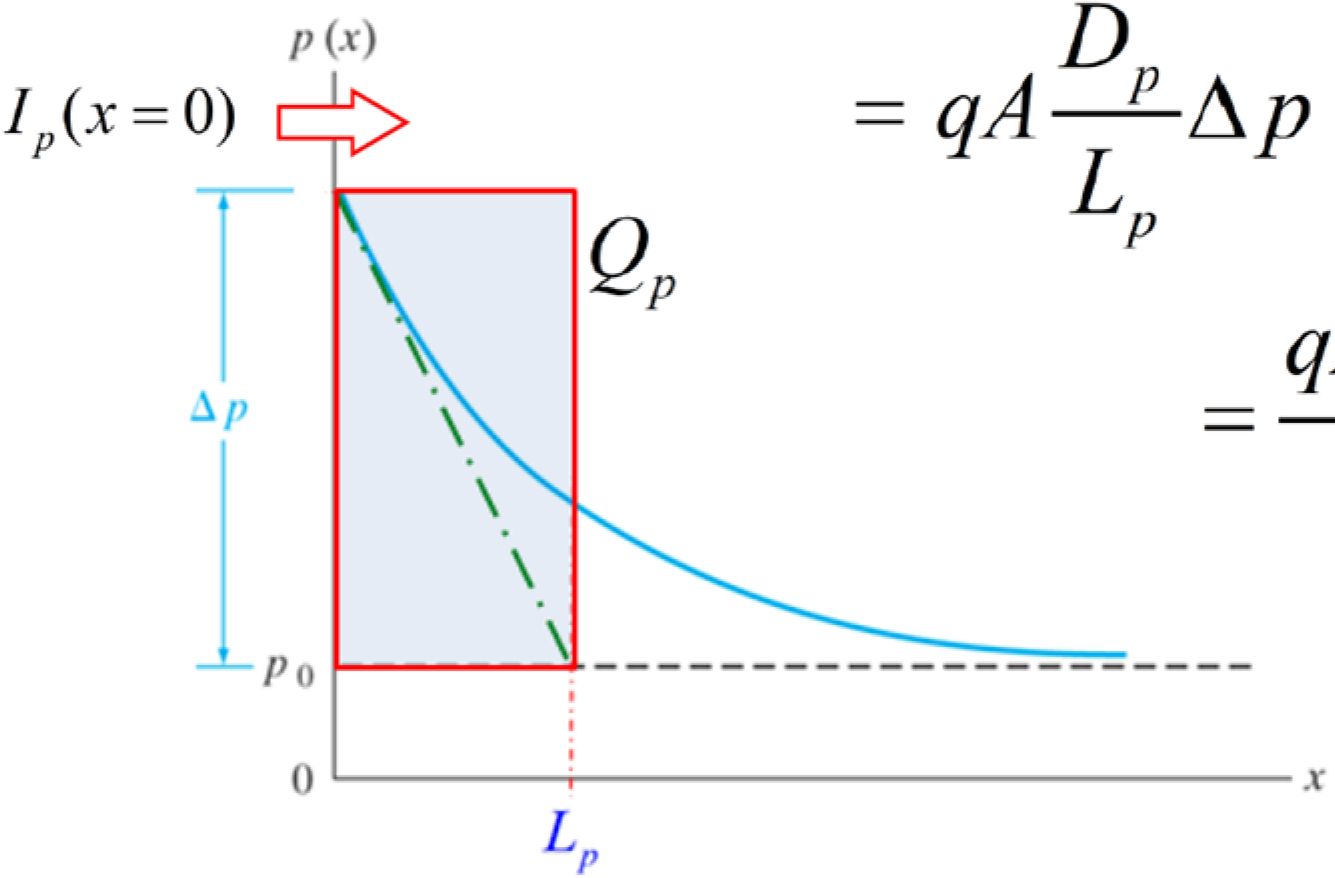
$$= qA\,\Delta p\,L_p$$

Relationship between current and Q_p

$$I_p(x=0) = qA\frac{D_p}{L_p}\Delta p \exp\left(-0/L_p\right)$$

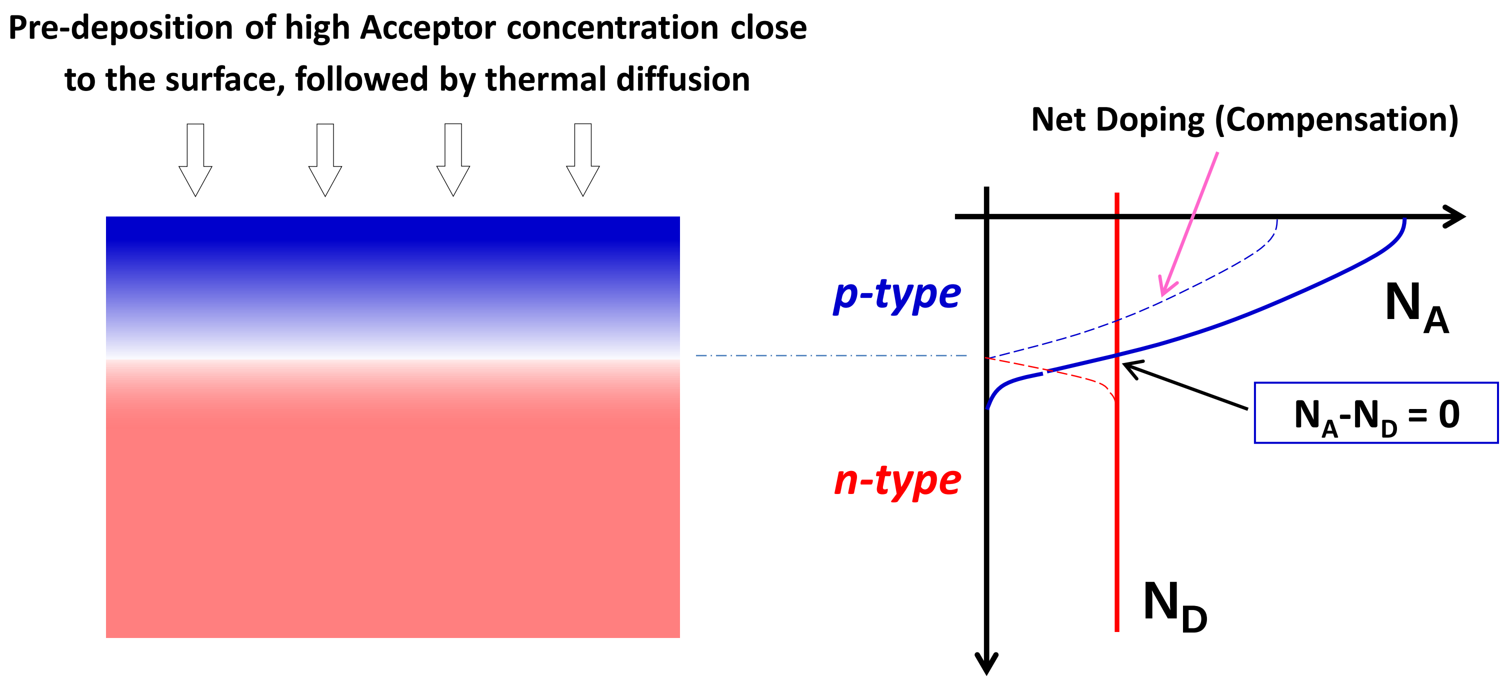
$$= qA\frac{D_p}{L_p}\Delta p = qA\frac{L_p^2}{L_p\tau_p}\Delta p$$

$$= \frac{qA\Delta p L_p}{\tau_p} = \frac{Q_p}{\tau_p}$$



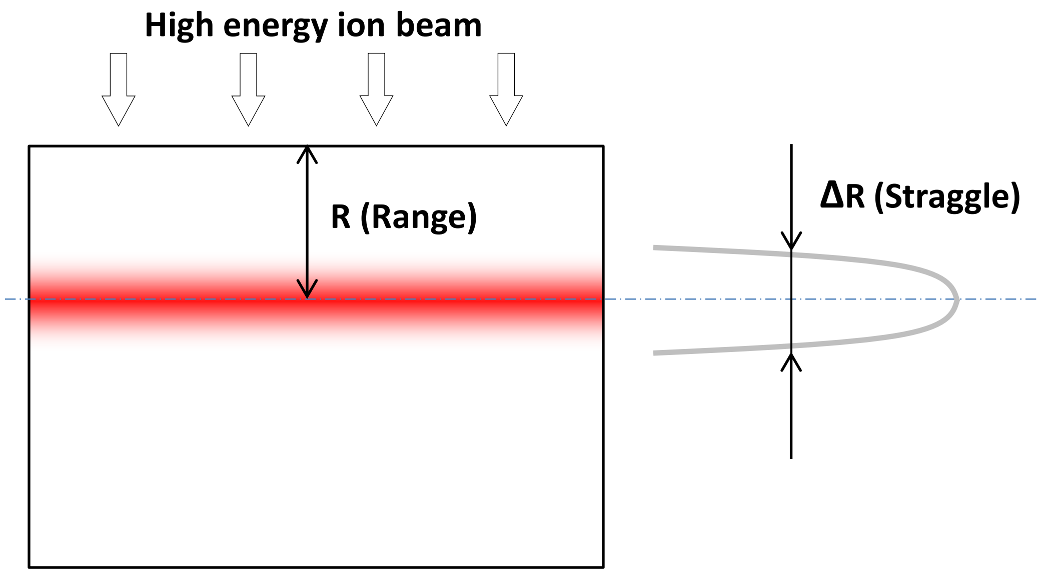
p-n junction

- ***Starting from an n-type substrate with uniform donor doping, a p-n junction may be obtained, for instance, by diffusion of acceptor doping***



p-n junction

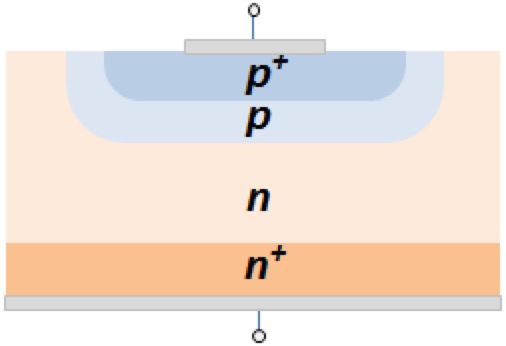
- ***Ion implantation* is commonly used today to obtain a more controlled spatial distribution of dopants.**



A beam of ions at a specific energy creates a nearly Gaussian dopant distribution centered at a depth R (Range). Specific doping profiles can be achieved by repeating the process at different beam energies.

p-n junction

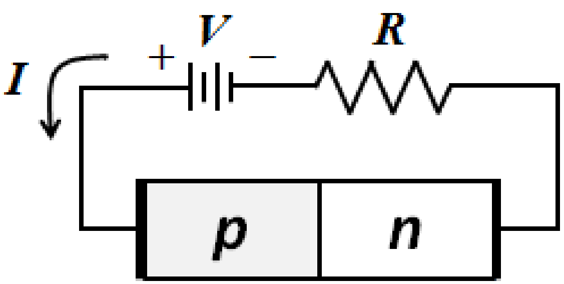
- *An ideal rectifying diode is a perfect voltage-controlled "valve" behaving as a short circuit when a positive voltage (forward bias) is applied and as an open circuit when a negative voltage (reverse bias) is applied. A p-n junction diode approximates this behavior.*

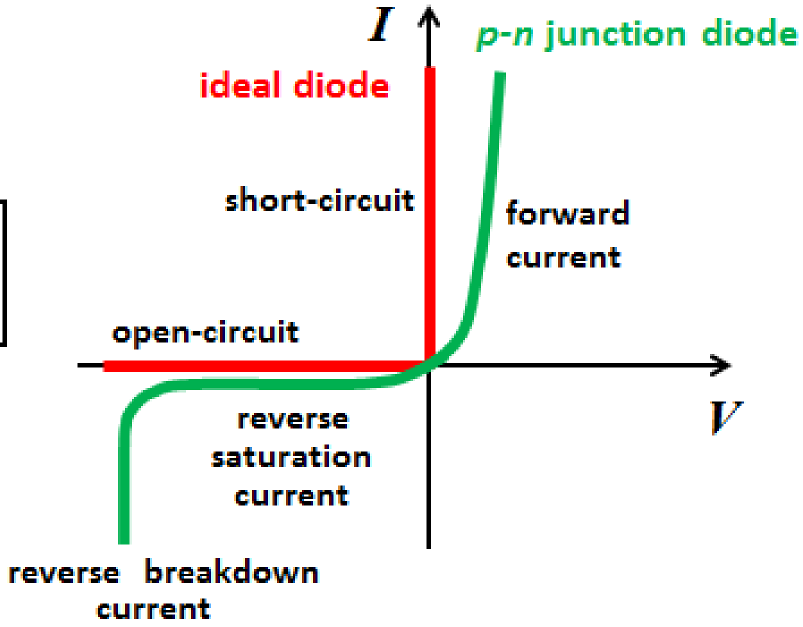


Simple physical realization

n^+ and p^+ facilitate ohmic contact behavior

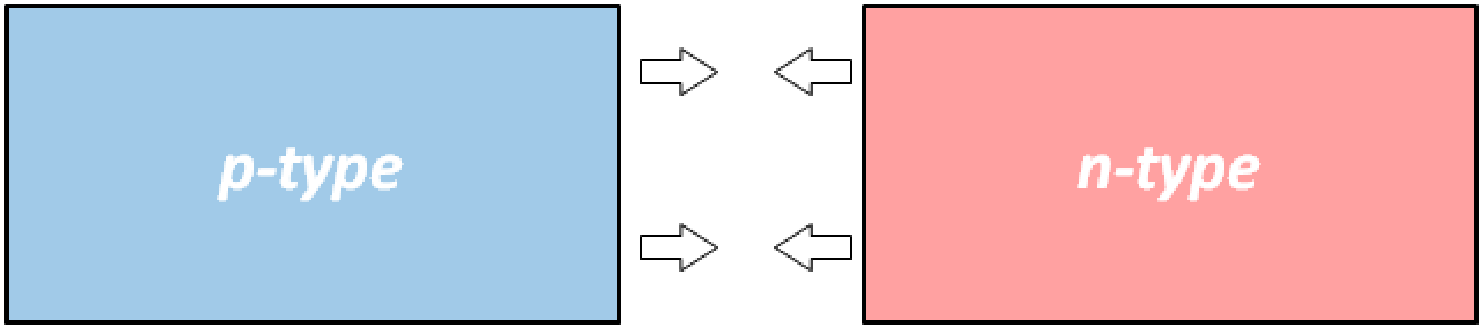
Forward bias circuit





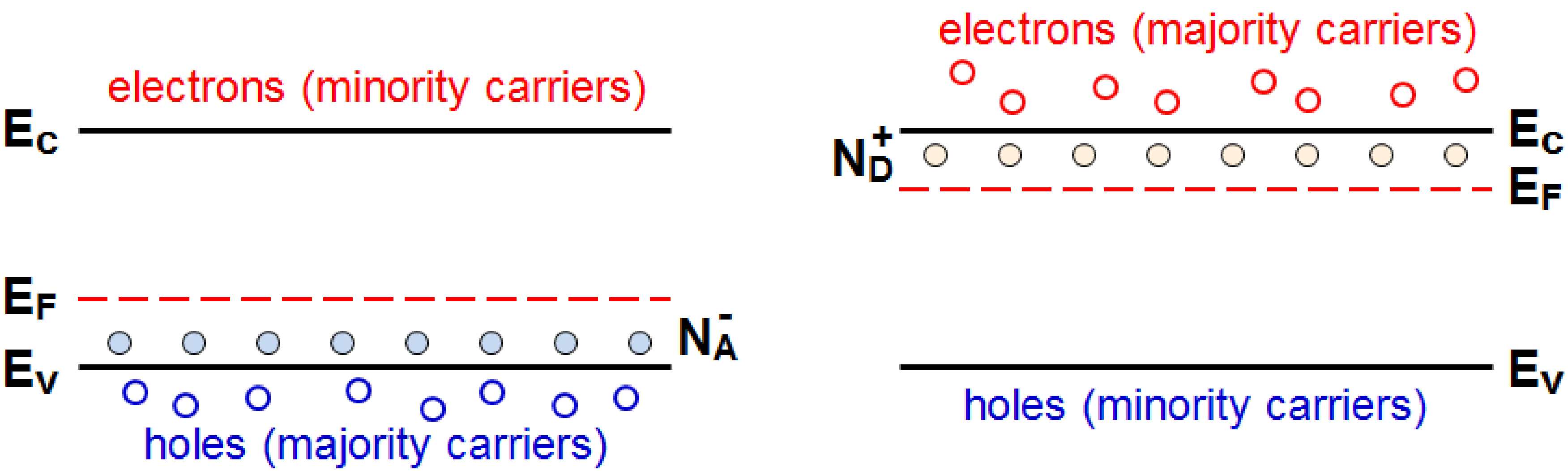
We saw this earlier in Lecture 12

- Imagine to bring into contact two separate sample materials of the same semiconductor in equilibrium but with different doping.



Two systems in equilibrium

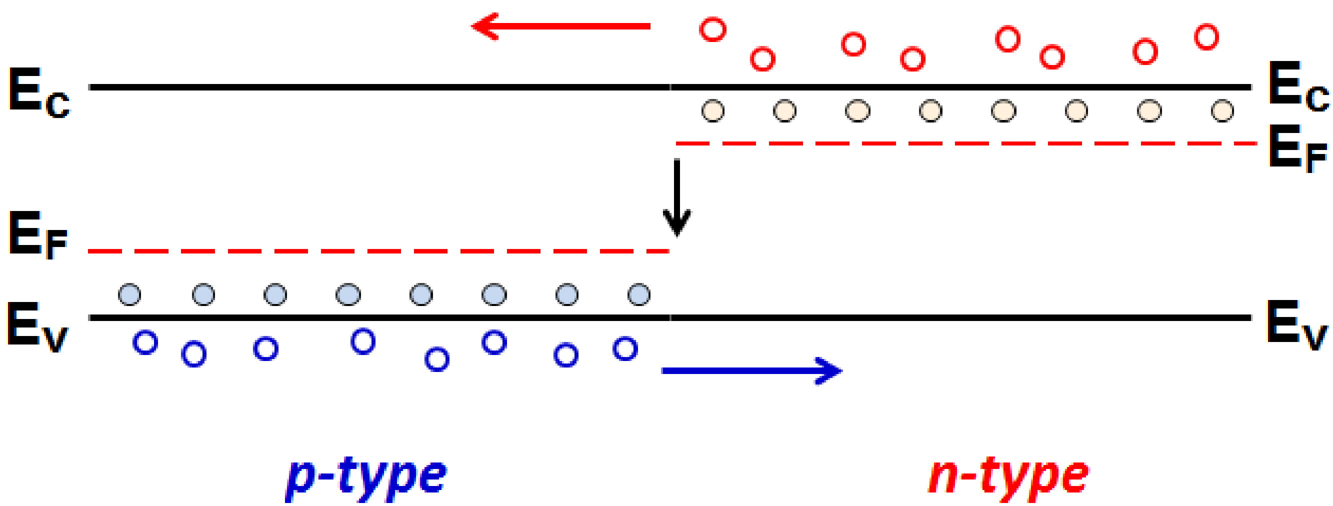
In p-type semiconductor materials, the Fermi level is closer to the valence band. In n-type semiconductor materials, the Fermi level is closer to the conduction band.



p-type ***n-type***

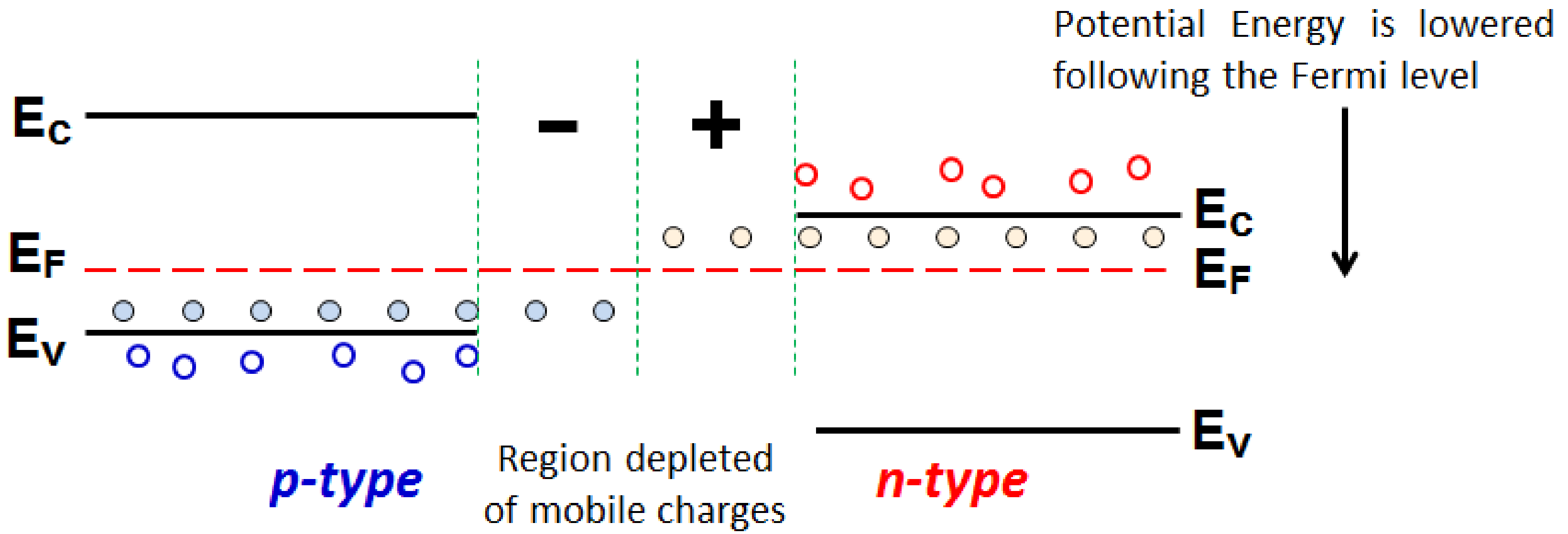
Conduction bands are the same initially

If the two materials are joined together, in order to reach equilibrium there has to be an equalization of the Fermi level (chemical potential) with mobile charges diffusing to regions of lower density.



Fermi levels equalize

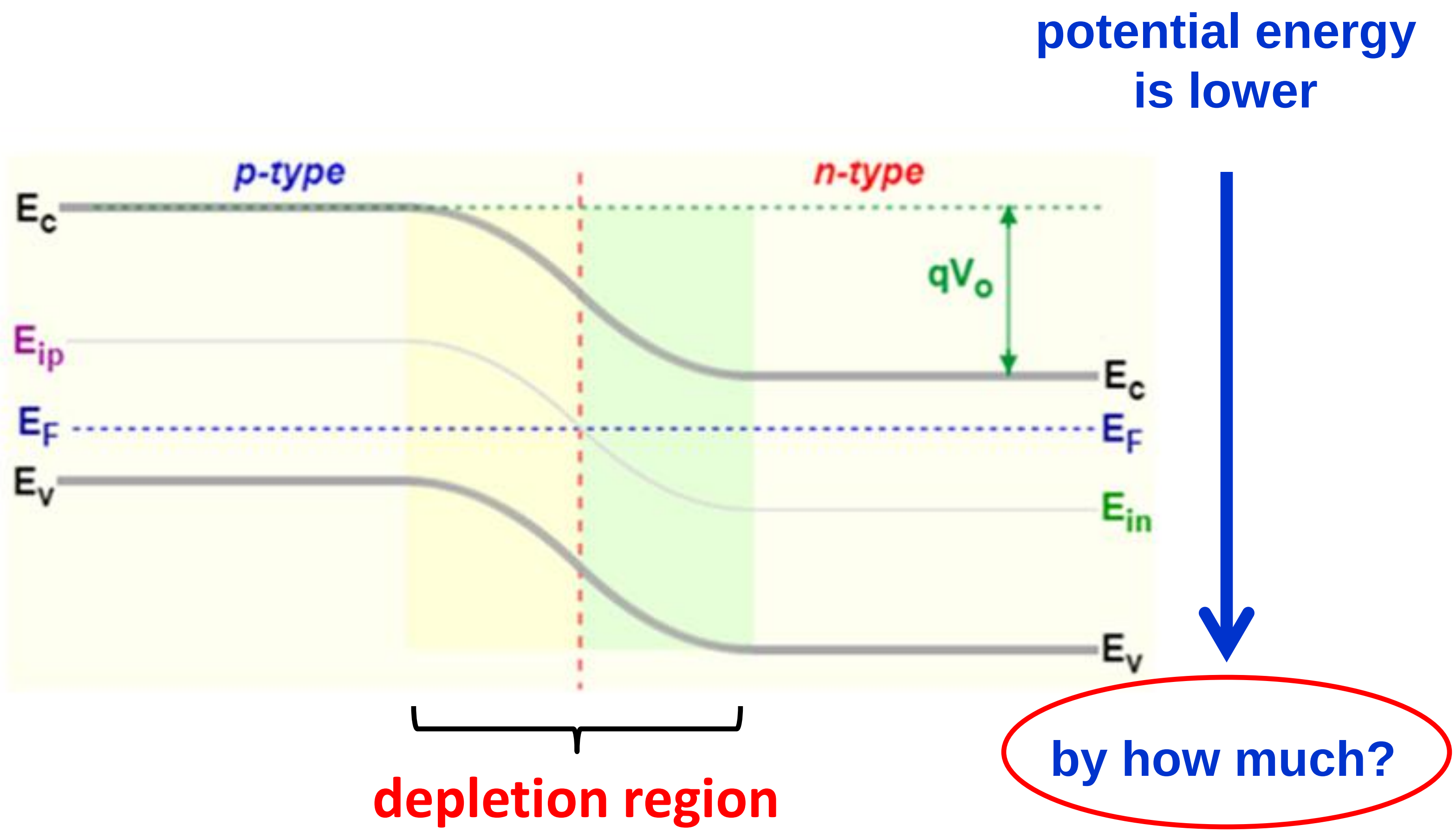
Electrons ***flow to the p-region leaving behind unscreened positive donor charges.*** ***Holes*** ***flow to the n-region leaving behind unscreened negative acceptor charges. Space charge builds up at the junction and a potential barrier opposes diffusive charge flow.***



Equilibrium is reached and carrier flow stops

At equilibrium Fermi level is flat

- The energy levels for E_C , E_V , and E_i exhibit ***band bending***



Contact potential $\mathcal{V}_0$

- Let's consider the full drift-diffusion equation (we have both field and density gradient)

$$J_p(x) = q\,p(x)\mu_p \mathcal{E}(x) - qD_p \frac{d\,p(x)}{dx} = 0$$

equilibrium

$$\rightarrow p(x)\mu_p \mathcal{E}(x) = qD_p \frac{d\,p(x)}{dx}$$

$$\boxed{\frac{\mu_p}{D_p}\mathcal{E}(x) = \frac{1}{p(x)}\frac{d\,p(x)}{dx}}$$

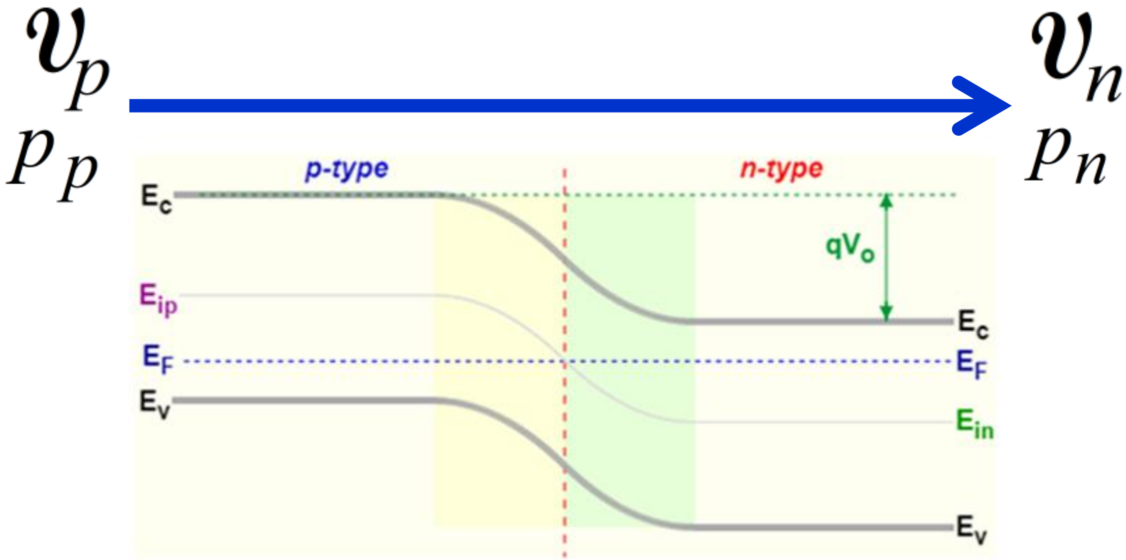
Contact potential $\mathcal{V}_0$

- Remember: $$D_p = \frac{k_B T}{q} \mu_p$$ Einstein relation $$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx}$$ Definition of field

$$-\frac{q}{k_B T}\frac{d\mathcal{V}(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx}$$

$\mathcal{V}_p$ $\mathcal{V}_n$

p_p p_n



Contact potential $\mathcal{V}_0$

$$-\frac{q}{k_B T}\int_{\mathcal{V}_p}^{\mathcal{V}_n} d\mathcal{V} = \int_{p_p}^{p_n} \frac{1}{p}\,dp$$

$$-\frac{q}{k_B T}\left(\mathcal{V}_n - \mathcal{V}_p\right) = \ln\left(p_n\right) - \ln\left(p_p\right) = -\ln\frac{p_p}{p_n}$$

$$\left(\mathcal{V}_n - \mathcal{V}_p\right) = \mathcal{V}_0 = \frac{k_B T}{q}\ln\frac{p_p}{p_n}$$

Contact potential

Contact potential $\mathcal{V}_0$

Step junction: N_A on p-side & N_D on n-side

$$\mathcal{V}_0 = \frac{k_B T}{q} \ln \frac{p_p}{p_n} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$p_p \approx N_A \qquad p_n \approx \frac{n_i^2}{N_D}$$

Also:

$$\frac{p_p}{p_n} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right)$$

$$\frac{n_n}{n_p} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right)$$

In the regions far away from the junction

$$n_p = \frac{n_i^2}{p_p} \qquad n_n = \frac{n_i^2}{p_n}$$

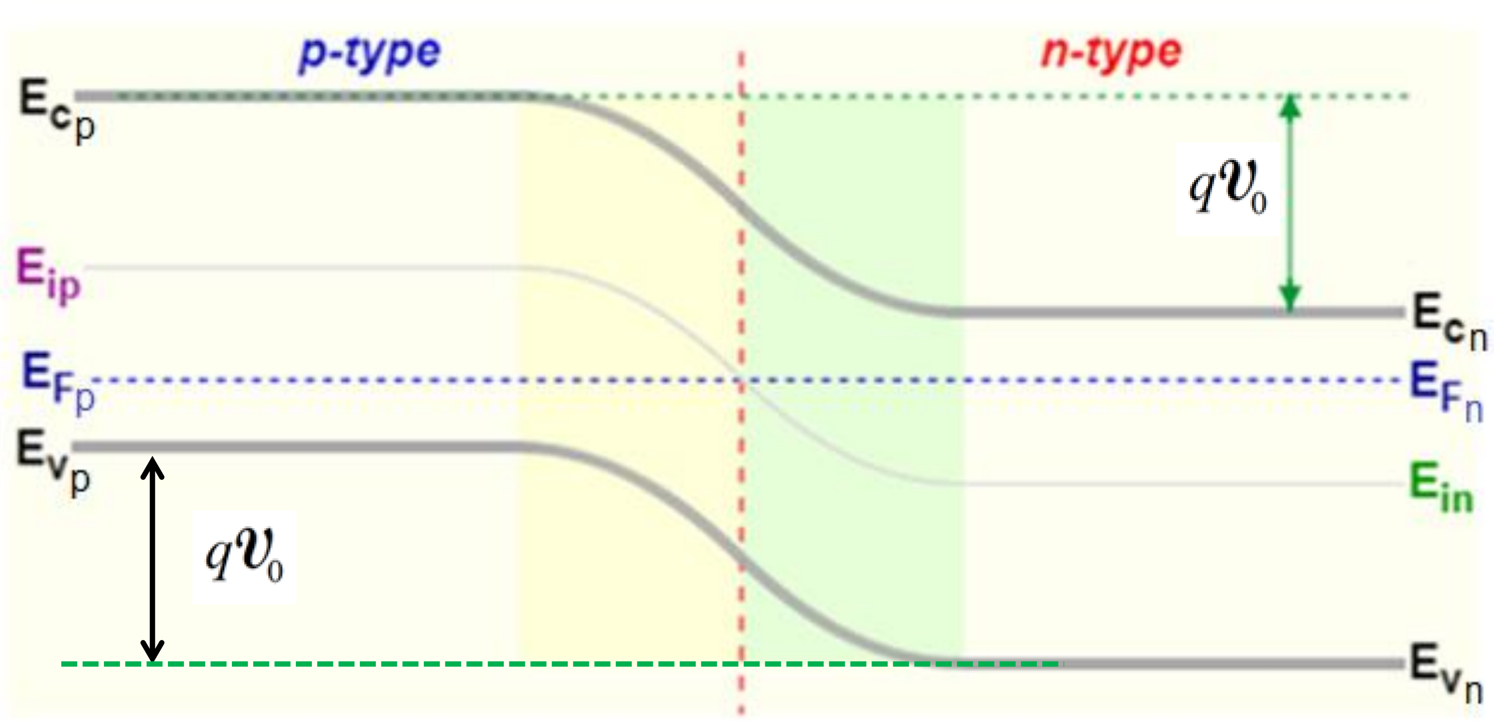
Contact potential $\mathcal{V}_0$

$$q\mathcal{V}_0 = E_{Vp} - E_{Vn}$$

$$q\mathcal{V}_0 = E_{Cp} - E_{Cn}$$

$$q\mathcal{V}_0 = E_{ip} - E_{in}$$

$$E_{Fp} = E_{Fn}$$



Example – Si abrupt junction

$N_A = 10^{18}\text{cm}^{-3}$
$N_D = 5 \times 10^{15}\text{cm}^{-3}$

$$E_{ip} - E_F = k_B T \ln \frac{p_p}{n_i} = 0.0259 \ln \frac{10^{18}}{1.5 \times 10^{10}} = 0.467\,\text{eV}$$

$$E_F - E_{in} = k_B T \ln \frac{n_n}{n_i} = 0.0259 \ln \frac{5 \times 10^{15}}{1.5 \times 10^{10}} = 0.329\,\text{eV}$$

$$q\mathcal{V}_0 = E_{ip} - E_{in} = 0.467eV + 0.329eV = 0.796\,\text{eV}$$

$$q\mathcal{V}_0 = k_B T \ln \frac{N_A N_D}{n_i^2} = 0.0259 \ln \frac{5 \times 10^{33}}{2.25 \times 10^{20}} = 0.796\,\text{eV}$$

My computer solution with more accurate parameters (we will discuss additional theory next week)

Built-in Potential	$qV_o = 0.7945$ eV
Intrinsic Fermi Level	$E_c - E_i = 0.573183$ eV
	$E_i - E_v = 0.546817$ eV

Fermi Level

$E_{ip} - E_F = 0.465729$ eV	$E_F - E_{in} = 0.328757$ eV
$E_F - E_v = 0.081088$ eV	$E_c - E_F = 0.244426$ eV

Depletion Width

$W = 0.4564\ \mu m$

$x_{po} = 0.0023\ \mu m$ (p-side)

$x_{no} = 0.4541\ \mu m$ (n-side)

Space Charge

$Q_- = -q N_A x_{po} = -3.638 \times 10^{-8}$ C cm^{-2} (p-side)

$Q_+ = q N_D x_{no} = 3.638 \times 10^{-8}$ C cm^{-2} (n-side)

$$N_A = 10^{18}\text{cm}^{-3}$$
$$N_D = 5 \times 10^{15}\text{cm}^{-3}$$