# ECE 340 Lecture 20
# Semiconductor Electronics

Spring 2022
10:00-10:50am
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2062 ECE Building

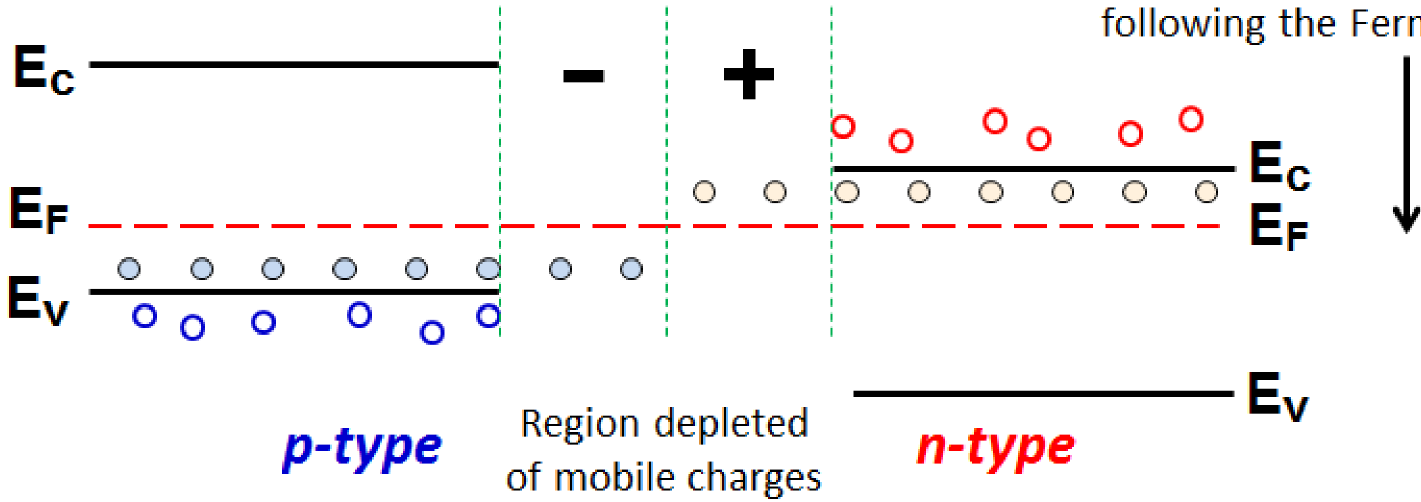
# Today's Discussion

- **The p-n junction**
- **Equilibrium condition**
- **Contact potential**

# Fermi levels equalize

***Electrons** flow to the p-region leaving behind unscreened positive donor charges. **Holes** flow to the n-region leaving behind unscreened negative acceptor charges. Space charge builds up at the junction and a potential barrier opposes diffusive charge flow.*

Potential Energy is lowered following the Fermi level

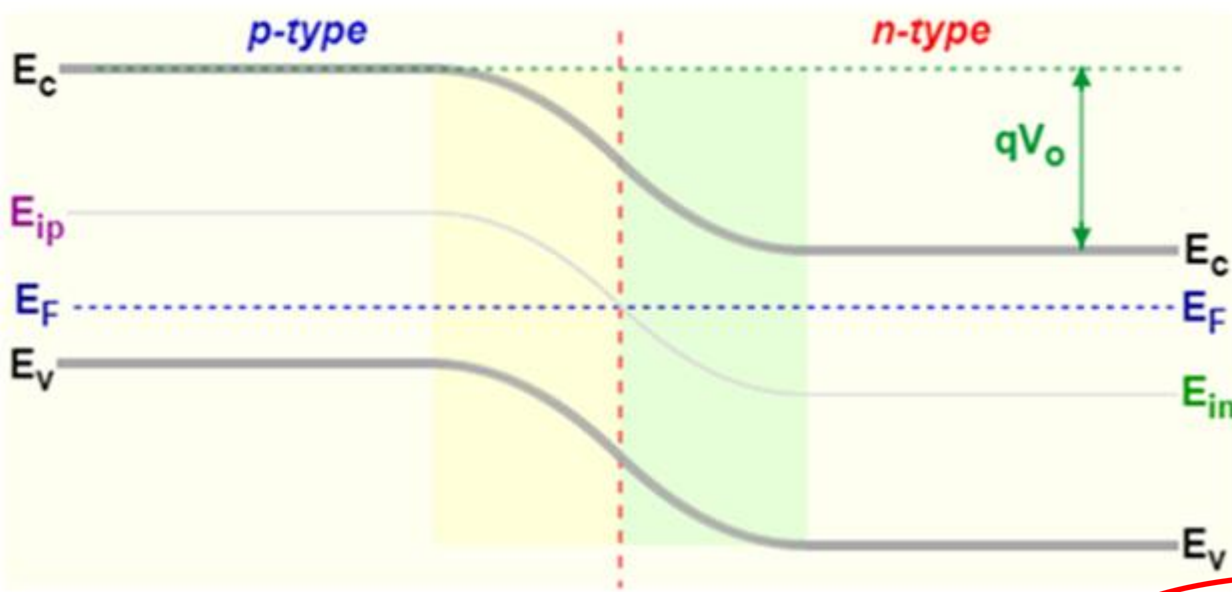



Equilibrium is reached and carrier flow stops

# At equilibrium Fermi level is flat

- The energy levels for $E_C$ , $E_V$ , and $E_i$ exhibit ***band bending***

**potential energy is lowered**




**depletion region**

**by how much?**

# Contact potential $\mathcal{V}_0$

- Let's consider the full drift-diffusion equation (we have both field and density gradient)

$$J_p(x) = q\,p(x)\mu_p \mathcal{E}(x) - qD_p \frac{d\,p(x)}{dx} = 0$$

equilibrium

$$\rightarrow\ p(x)\mu_p \mathcal{E}(x) = qD_p \frac{d\,p(x)}{dx}$$

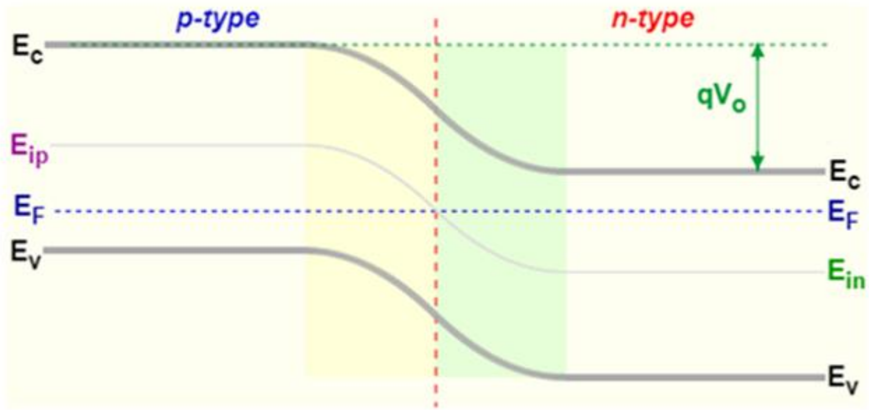
$$\boxed{\frac{\mu_p}{D_p}\mathcal{E}(x) = \frac{1}{p(x)}\frac{d\,p(x)}{dx}}$$

# Contact potential $\mathcal{V}_0$

- Remember: $D_p = \frac{k_B T}{q}\mu_p$ — Einstein relation; $\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx}$ — Definition of field

$$-\frac{q}{k_B T}\frac{d\mathcal{V}(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx}$$

$\mathcal{V}_p$, $p_p$ → $\mathcal{V}_n$, $p_n$



# Contact potential $\mathcal{V}_0$

$$-\frac{q}{k_B T}\int_{\mathcal{V}_p}^{\mathcal{V}_n} d\mathcal{V} = \int_{p_p}^{p_n} \frac{1}{p}\,dp$$

$$-\frac{q}{k_B T}\left(\mathcal{V}_n - \mathcal{V}_p\right) = \ln\left(p_n\right) - \ln\left(p_p\right) = -\ln\frac{p_p}{p_n}$$

$$\left(\mathcal{V}_n - \mathcal{V}_p\right) = \mathcal{V}_0 = \frac{k_B T}{q}\ln\frac{p_p}{p_n}$$

Contact potential

# Contact potential $\mathcal{V}_0$

Step junction: $N_A$ on p-side & $N_D$ on n-side

$$\mathcal{V}_0 = \frac{k_B T}{q} \ln \frac{p_p}{p_n} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$p_p \approx N_A$$

$$p_n \approx \frac{n_i^2}{N_D}$$

Also:

$$\frac{p_p}{p_n} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right)$$

$$\frac{n_n}{n_p} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right)$$

In the regions far away from the junction

$$n_p = \frac{n_i^2}{p_p}$$

$$n_n = \frac{n_i^2}{p_n}$$

# Contact potential $\mathcal{V}_0$

Many equivalent ways to express it

$$\frac{p_p}{p_n} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right) = \frac{N_V \exp\left(-\frac{E_{Fp} - E_{Vp}}{k_B T}\right)}{N_V \exp\left(-\frac{E_{Fn} - E_{Vn}}{k_B T}\right)} = \exp\left(\frac{(E_{Vp} - E_{Vn}) - \overbrace{(E_{Fp} - E_{Fn})}^{=0}}{k_B T}\right)$$

$$\frac{n_n}{n_p} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right) = \frac{N_C \exp\left(-\frac{E_{Cn} - E_{Fn}}{k_B T}\right)}{N_C \exp\left(-\frac{E_{Cp} - E_{Fp}}{k_B T}\right)} = \exp\left(\frac{(E_{Cp} - E_{Cn}) - \overbrace{(E_{Fp} - E_{Fn})}^{=0}}{k_B T}\right)$$

$$\frac{p_p}{p_n} = \exp\left(\frac{q\mathcal{V}_0}{k_B T}\right) = \frac{n_i \exp\left(\frac{E_{ip} - E_{Fp}}{k_B T}\right)}{n_i \exp\left(\frac{E_{in} - E_{Fn}}{k_B T}\right)} = \exp\left(\frac{(E_{ip} - E_{in}) - \overbrace{(E_{Fp} - E_{Fn})}^{=0}}{k_B T}\right)$$

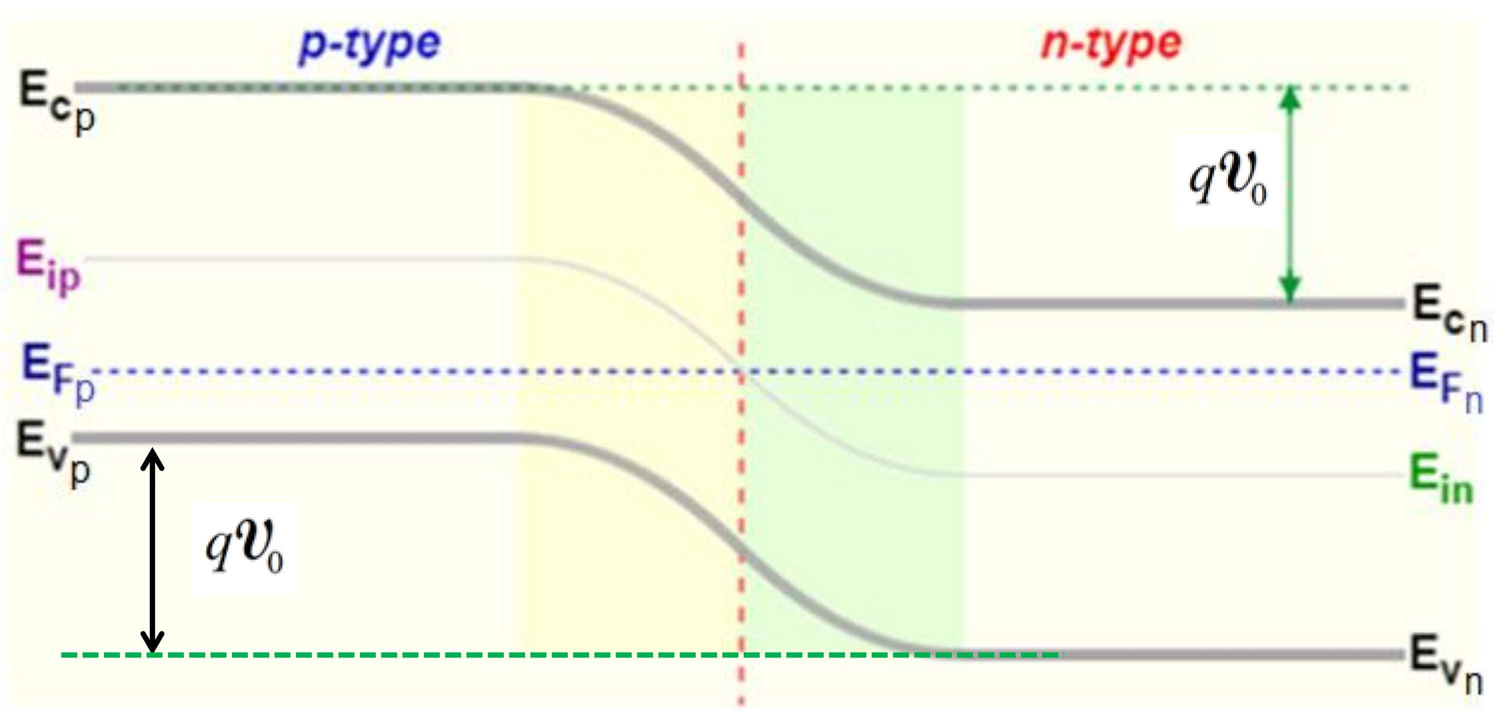
# Contact potential $\mathcal{V}_0$

$$q\mathcal{V}_0 = E_{Vp} - E_{Vn}$$

$$q\mathcal{V}_0 = E_{Cp} - E_{Cn}$$

$$q\mathcal{V}_0 = E_{ip} - E_{in}$$

$$E_{Fp} = E_{Fn}$$



# Example 1 – Si abrupt junction

$N_A = 10^{18}\text{cm}^{-3}$
$N_D = 5 \times 10^{15}\text{cm}^{-3}$

$$E_{ip} - E_F = k_B T \ln \frac{p_p}{n_i} = 0.0259 \ln \frac{10^{18}}{1.5 \times 10^{10}} = 0.467\,\text{eV}$$

$$E_F - E_{in} = k_B T \ln \frac{n_n}{n_i} = 0.0259 \ln \frac{5 \times 10^{15}}{1.5 \times 10^{10}} = 0.329\,\text{eV}$$

$$q\mathcal{V}_0 = E_{ip} - E_{in} = 0.467\,\text{eV} + 0.329\,\text{eV} = 0.796\,\text{eV}$$

$$q\mathcal{V}_0 = k_B T \ln \frac{N_A N_D}{n_i^2} = 0.0259 \ln \frac{5 \times 10^{33}}{2.25 \times 10^{20}} = 0.796\,\text{eV}$$

# My computer solution with more accurate parameters

**Built-in Potential** $qV_o$ = 0.7945 eV

**Intrinsic Fermi Level** $E_c - E_i$ = 0.573183 eV

$E_i - E_v$ = 0.546817 eV

**Fermi Level**

$E_{ip} - E_F$ = 0.465729 eV

$E_F - E_v$ = 0.081088 eV

$E_F - E_{in}$ = 0.328757 eV

$E_c - E_F$ = 0.244426 eV

**Depletion Width**

W = 0.4564 μm

$x_{po}$ = 0.0023 μm (p-side)

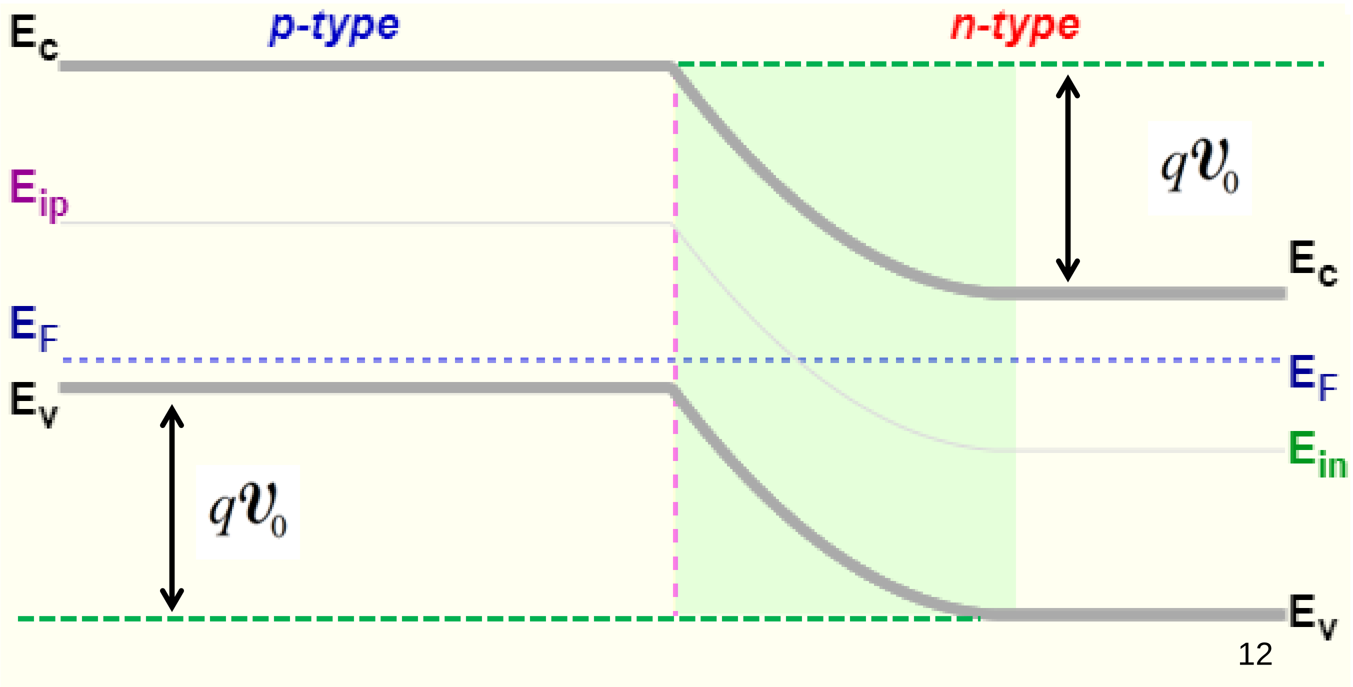
$x_{no}$ = 0.4541 μm (n-side)

**Space Charge**

$Q_- = - q N_A x_{po} = - 3.638 \times 10^{-8}$ C cm$^{-2}$ (p-side)

$Q_+ = q N_D x_{no} = 3.638 \times 10^{-8}$ C cm$^{-2}$ (n-side)

$$N_A = 10^{18}\text{cm}^{-3}$$
$$N_D = 5 \times 10^{15}\text{cm}^{-3}$$



# Significance of Contact Potential

*The **built-in potential** $qV_o$ quantifies the potential energy barrier of the p-n junction. It is determined by the **Fermi level** position, relative to the **intrinsic Fermi level**, in the neutral regions.*

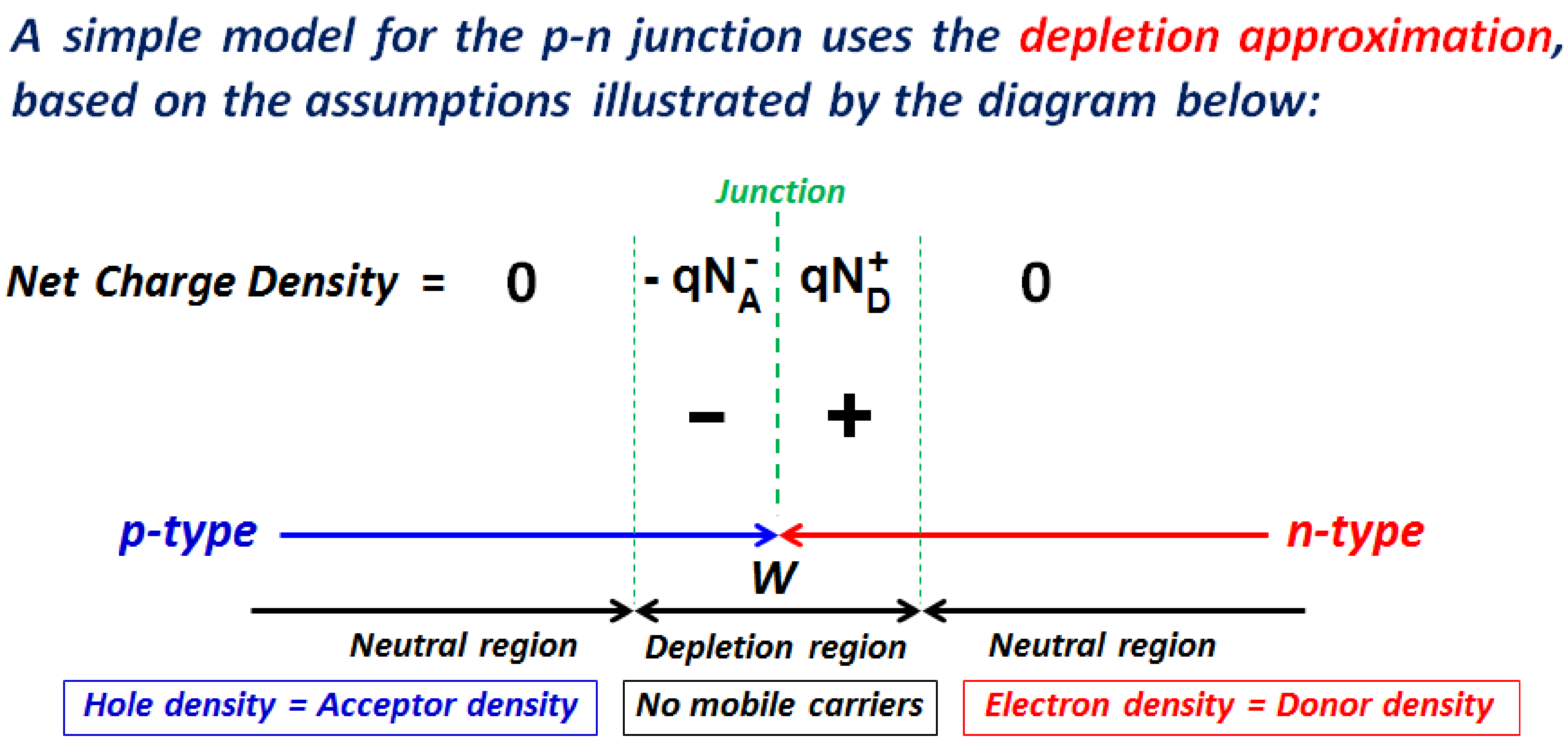
$$E_{ip} - E_F = k_B T \ln \frac{p_p}{n_i} \approx k_B T \ln \frac{N_A}{n_i}$$

$$E_F - E_{in} = k_B T \ln \frac{n_n}{n_i} \approx k_B T \ln \frac{N_D}{n_i}$$

$$qV_o = E_{ip} - E_{in} \approx k_B T \ln \frac{N_A N_D}{n_i^2}$$

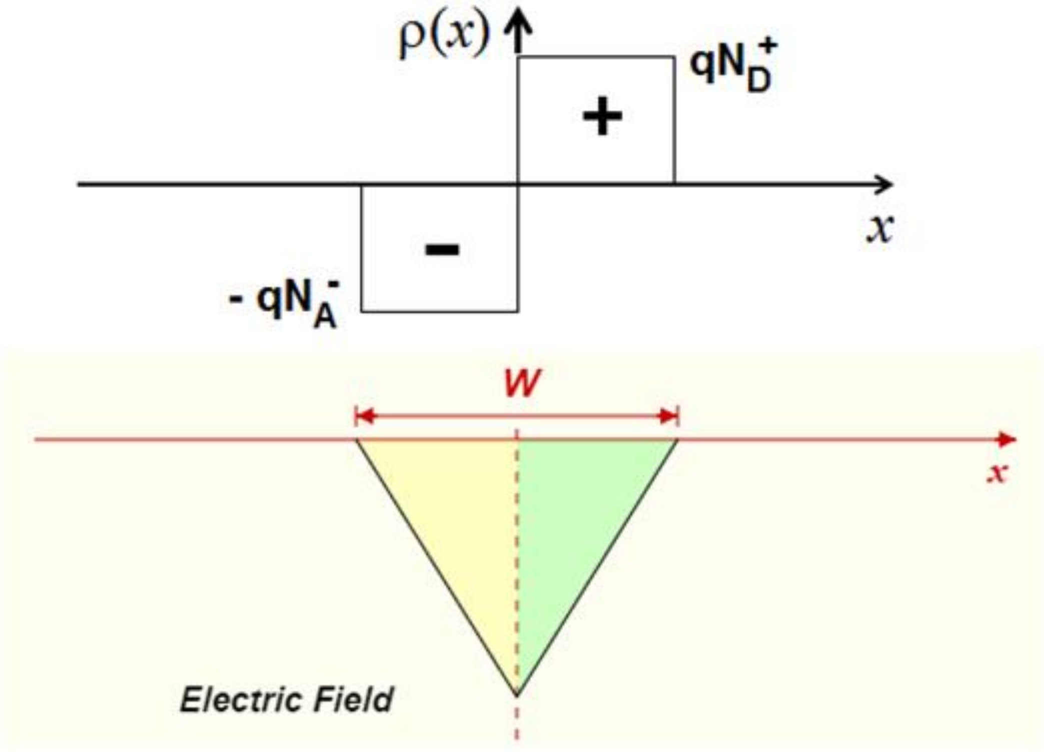
# Depletion near the junction

A space-charge region has been created at the junction



# Space charge originate a field distribution

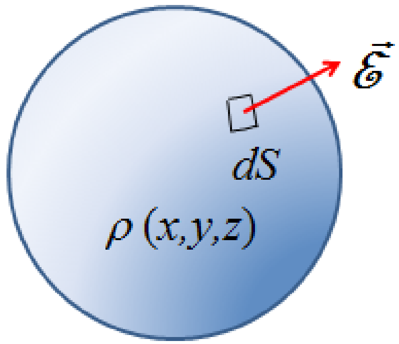
***Simple application of Gauss' law gives a triangular field distribution***



# Gauss Law

**Gauss Law Refresher**

3D



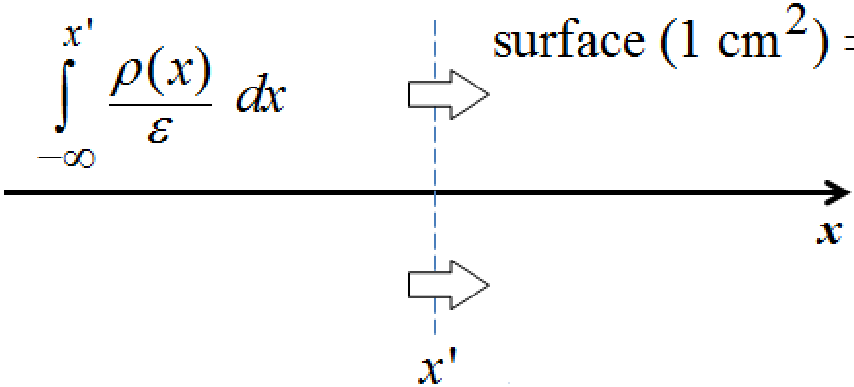

$\rho =$ net charge density enclosed

$\vec{\mathscr{E}} =$ Electric Field

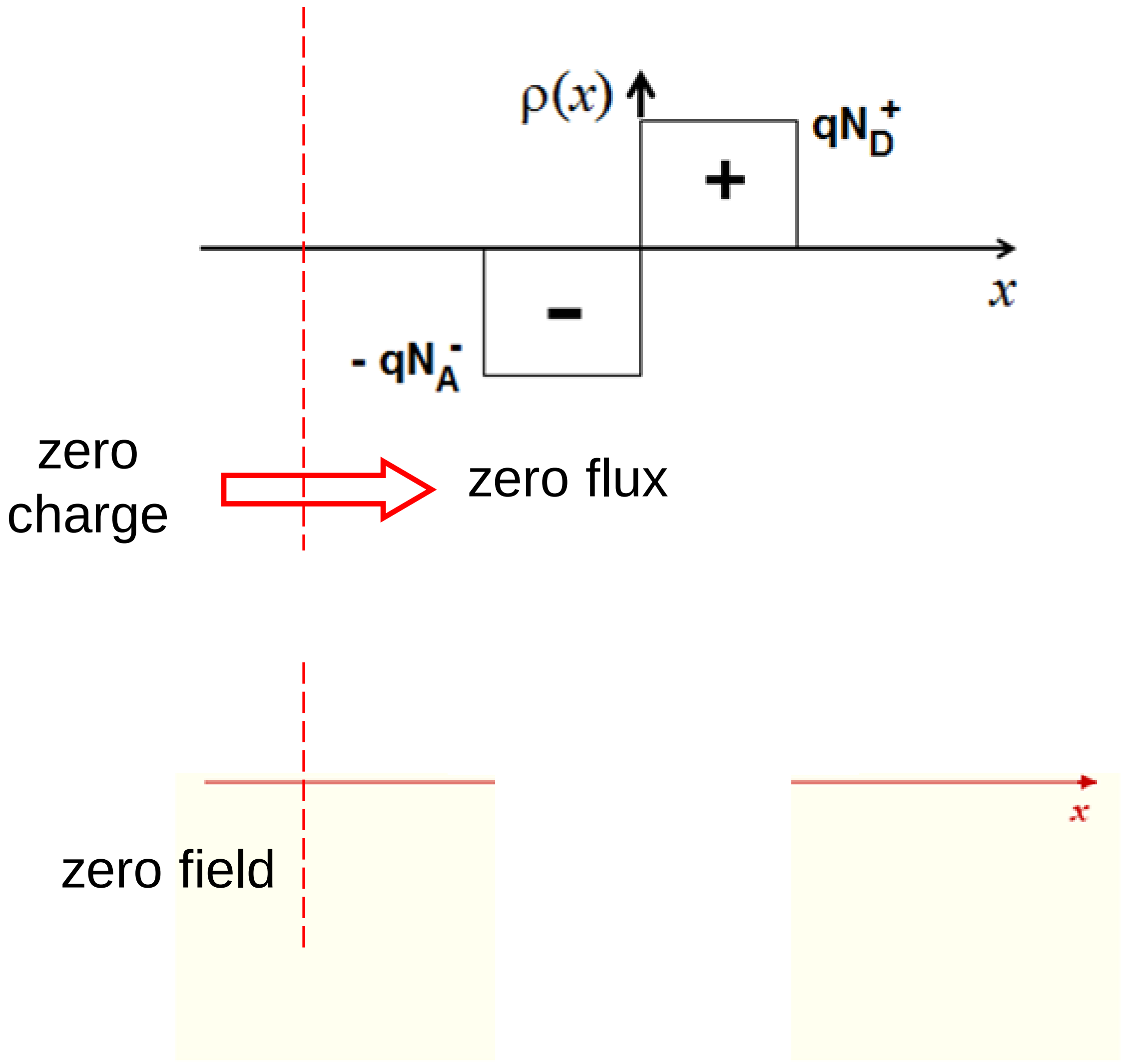
$$\iiint_V \frac{\rho}{\varepsilon}\, dV = \oiint_S \vec{\mathscr{E}} \cdot dS = \text{Flux of } \vec{\mathscr{E}} \text{ through } S$$

1D Enclosed charge density/$\varepsilon$ = Flux of $\vec{\mathscr{E}}$ through unitary surface (1 cm$^2$) = $\vec{\mathscr{E}}$ [V/cm]

$$\int_{-\infty}^{x'} \frac{\rho(x)}{\varepsilon}\, dx$$



# Gauss Law



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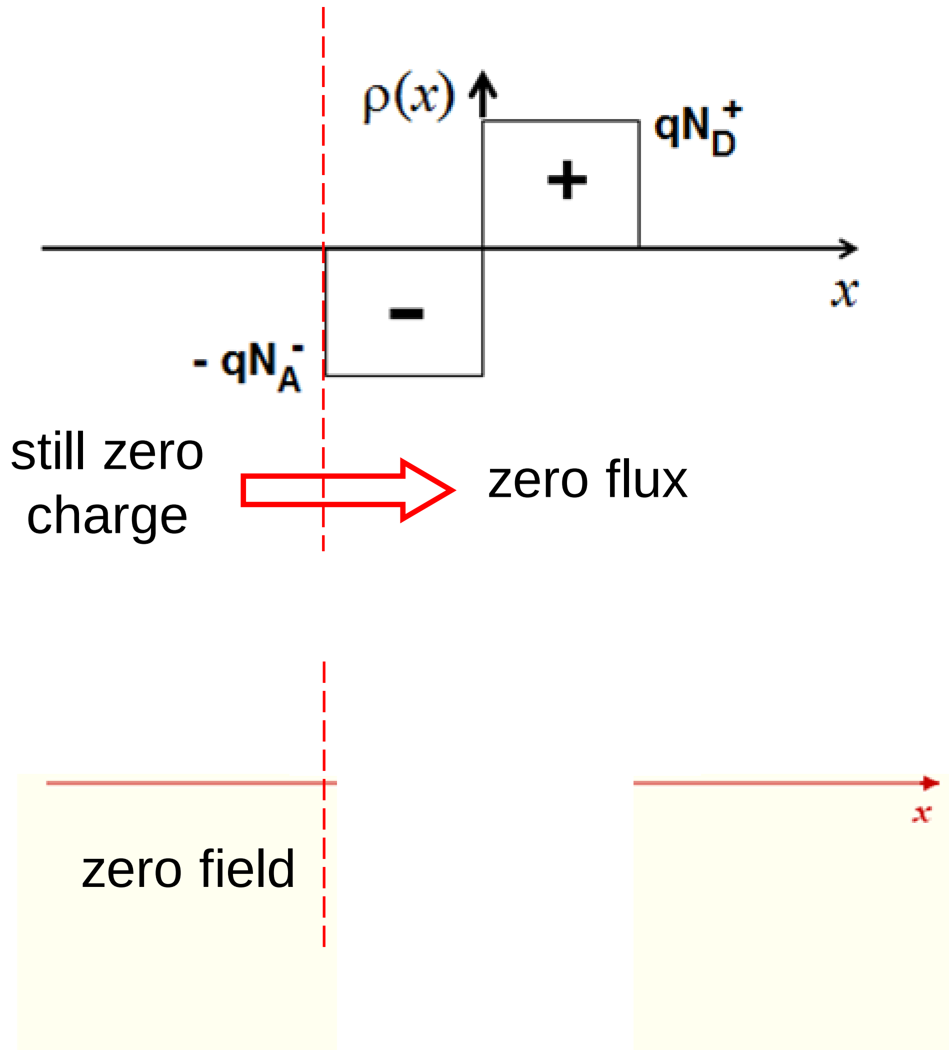
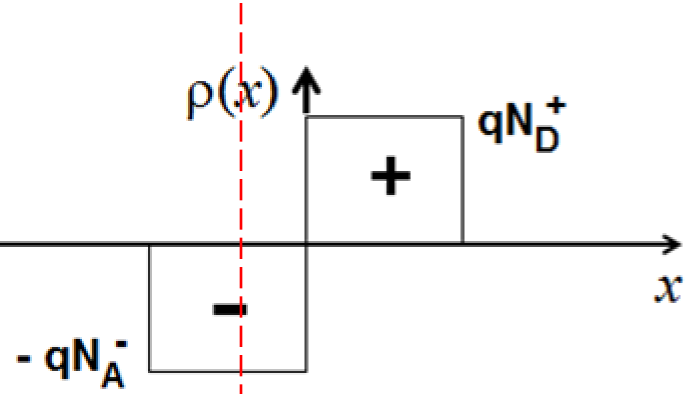
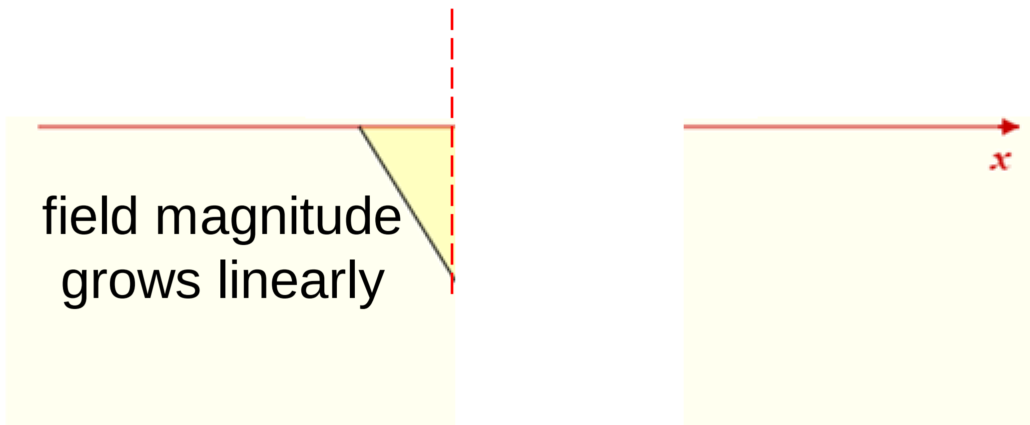


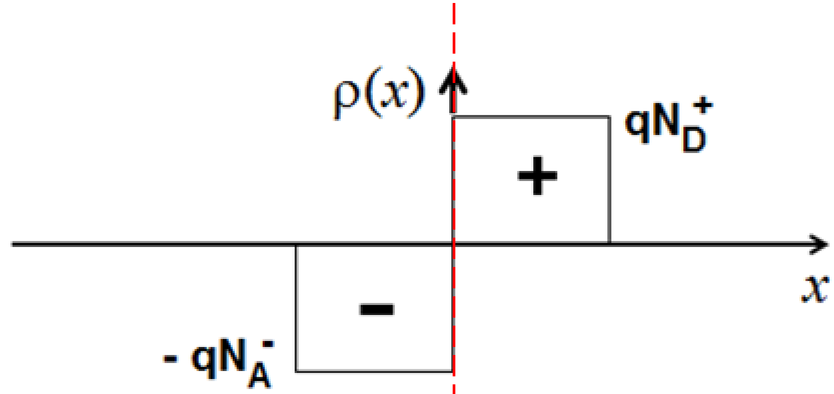
# Gauss Law




negative charge grows linearly ⇒ flux magnitude grows linearly

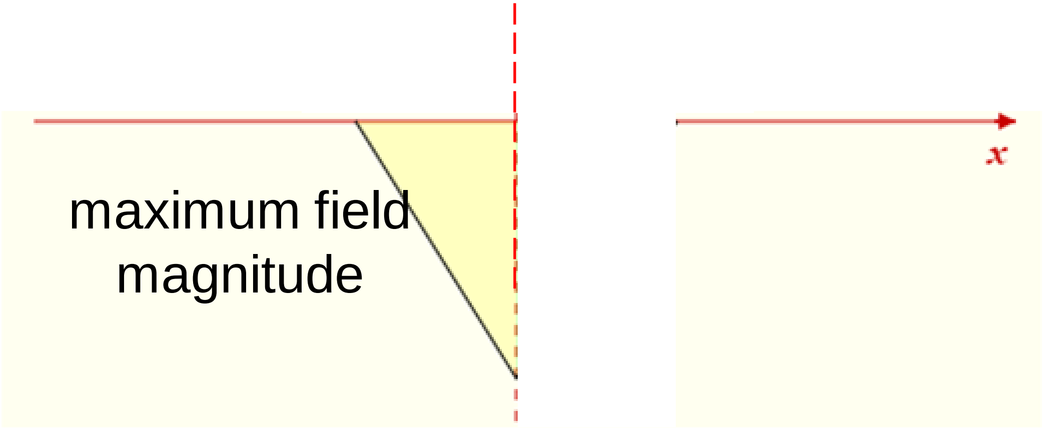


# Gauss Law



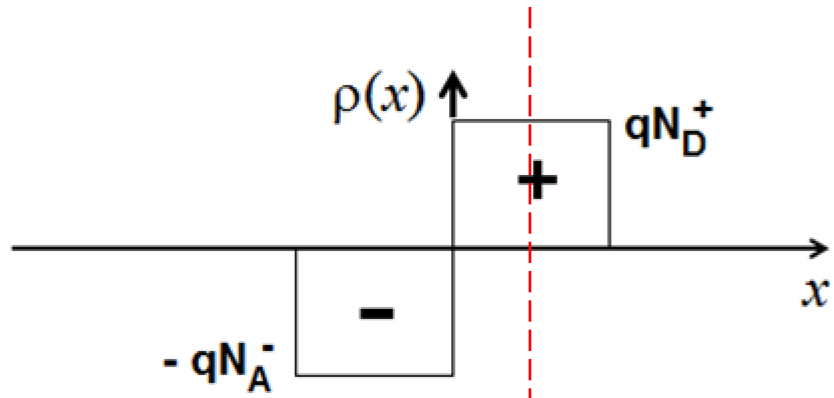

all negative charge on left of surface

flux reaches maximum magnitude

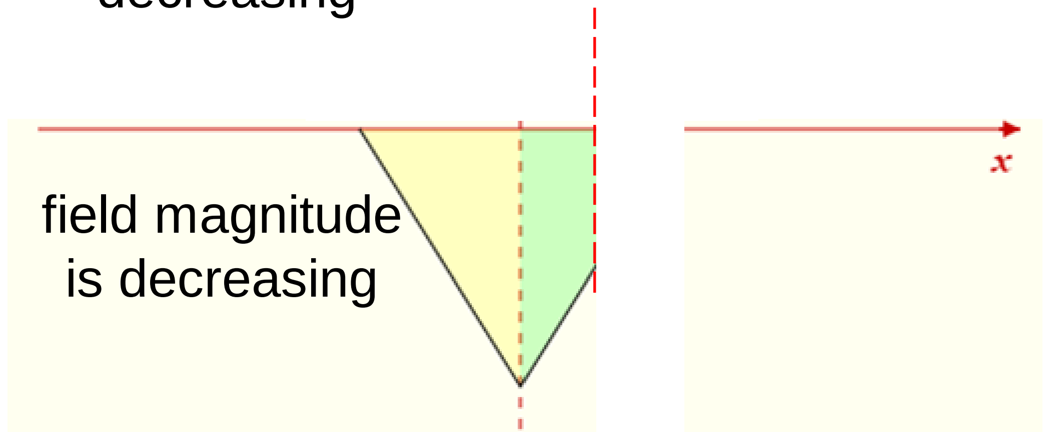



maximum field magnitude

# Gauss Law

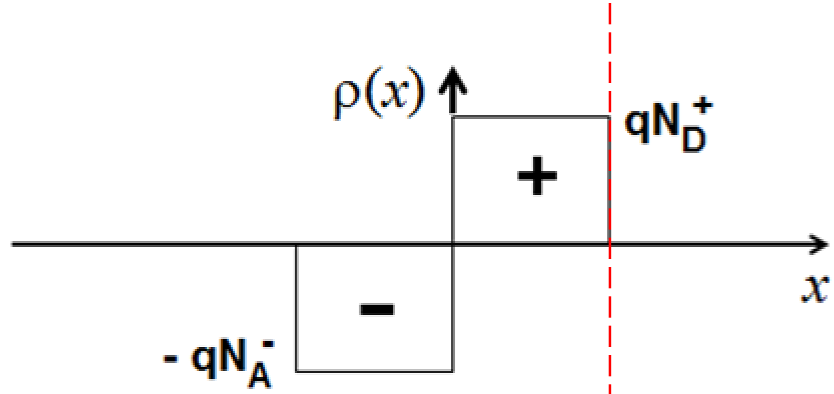



some positive charge added, total charge magnitude is decreasing → flux magnitude is decreasing



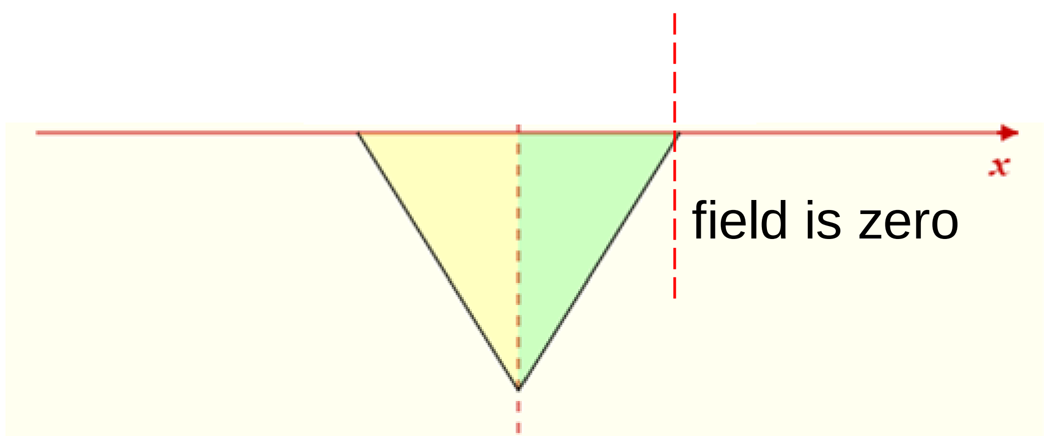

field magnitude is decreasing

# Gauss Law

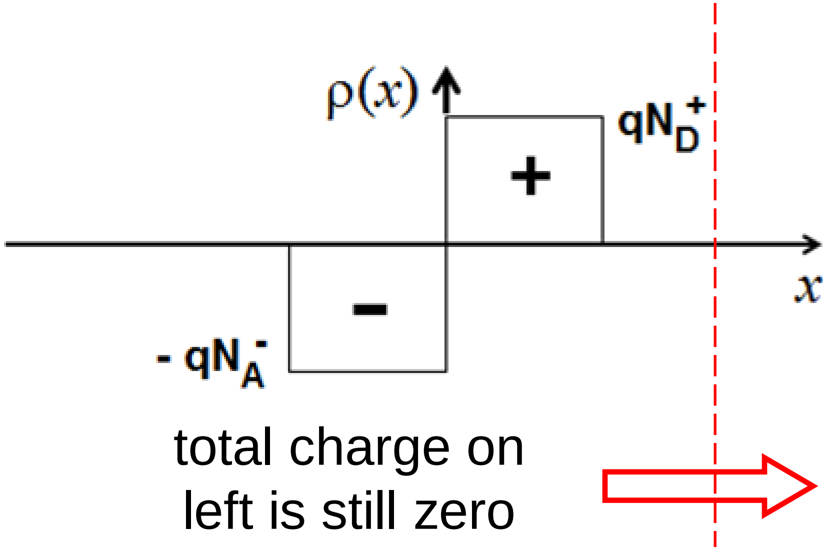



end of depletion region, total charge on left is zero

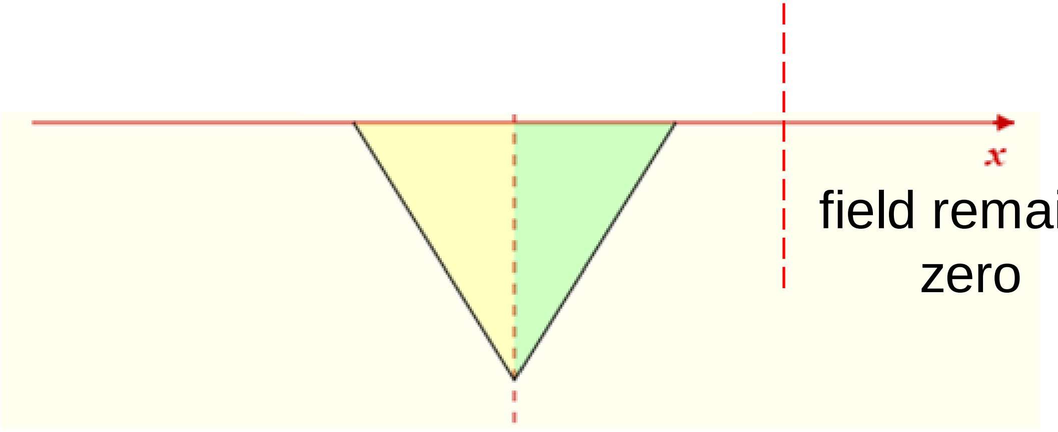
flux is zero again



# Gauss Law




total charge on left is still zero → flux remains zero




field remains zero

# Poisson equation

Differential form of Gauss Law

$$\frac{d\mathcal{E}(x)}{dx} = \frac{q}{\varepsilon}(p - n + N_D^+ - N_A^-)$$

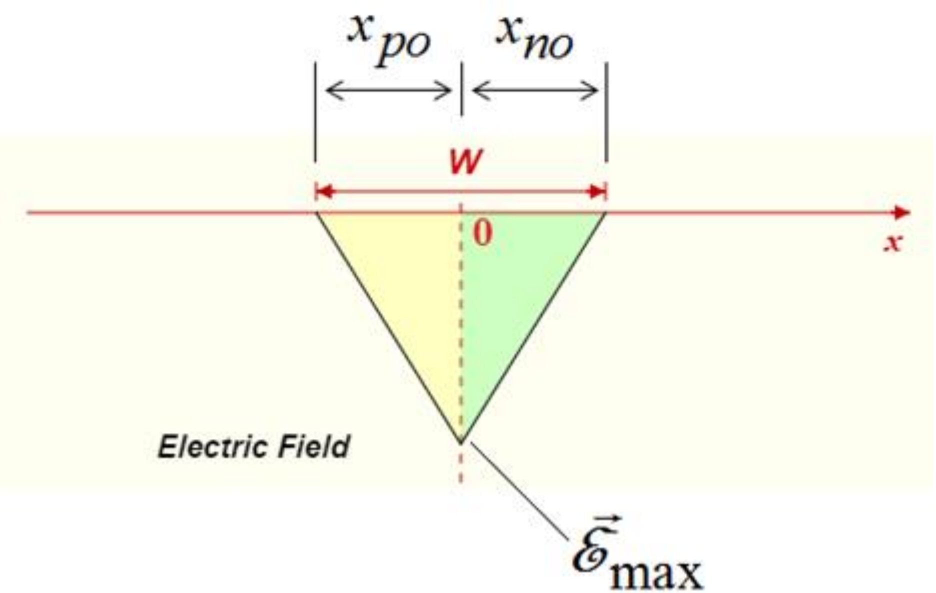
$$\frac{d^2\mathcal{V}(x)}{dx^2} = -\frac{q}{\varepsilon}(p - n + N_D^+ - N_A^-)$$

# Application of Gauss law

$$\int_0^{\mathcal{E}_0} d\mathcal{E} = -\frac{q}{\varepsilon} N_A \int_{-x_{p0}}^{0} dx \qquad [-x_{p0} < x < 0]$$

$$\int_{\mathcal{E}_0}^{0} d\mathcal{E} = \frac{q}{\varepsilon} N_D \int_0^{x_{n0}} dx \qquad [0 < x < x_{n0}]$$

$$\mathcal{E}_0 = \mathcal{E}_{\max} = -\frac{q}{\varepsilon} N_D\, x_{n0} = -\frac{q}{\varepsilon} N_A\, x_{p0}$$

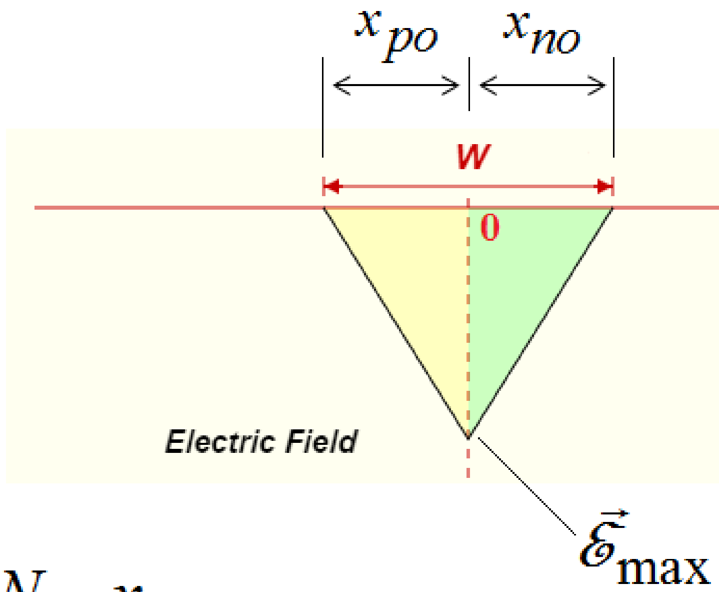


# Depletion near the junction

*The **built-in potential** is also related to the width of the **depletion region**. Since the field distribution has a triangular shape, the integral is simply*

$$V_o = -\int_{-x_{po}}^{x_{no}} \vec{\mathscr{E}}(x)\, dx$$

$$= -\frac{1}{2}\vec{\mathscr{E}}_{\max} W$$




***From Gauss Law***

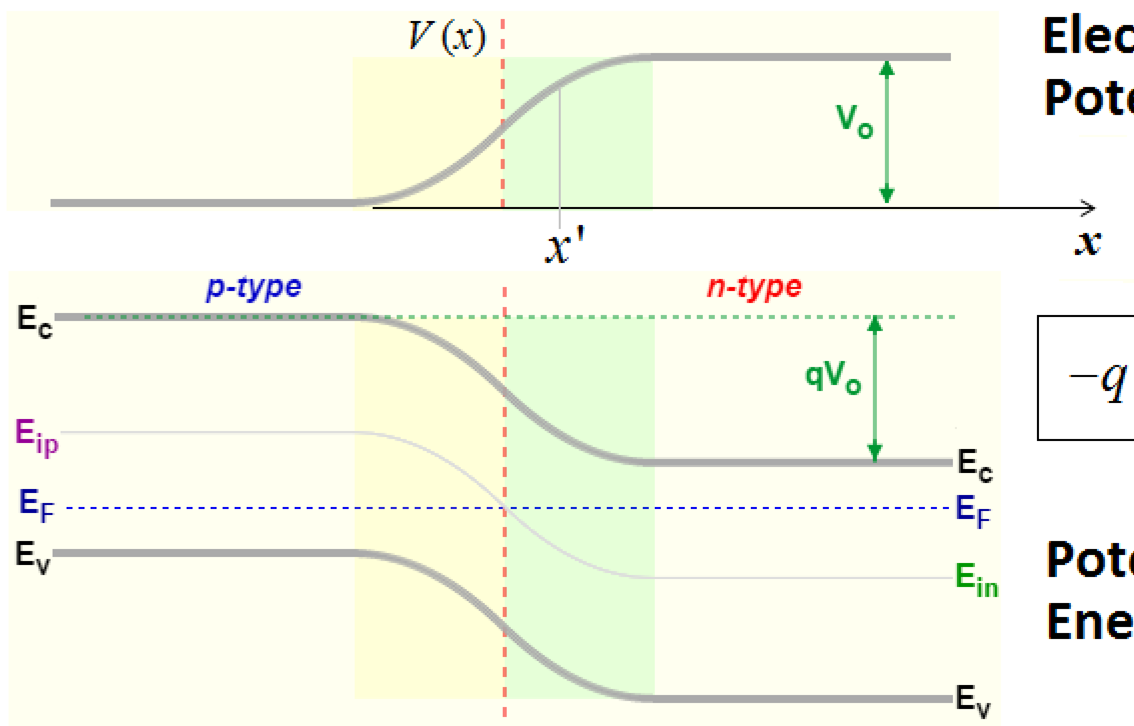
$$\left|\vec{\mathscr{E}}_{\max}\right| = q\frac{N_A\, x_{po}}{\varepsilon} = q\frac{N_D\, x_{no}}{\varepsilon}$$

# Details of band bending from potential

**$V(x)$ completes the band diagram in the depletion region**

$$-\frac{dV}{dx} = \vec{\mathscr{E}}(x)$$

$$-\int_{-\infty}^{x'} \vec{\mathscr{E}}(x)\, dx = V(x')$$




**Electrostatic Potential**

$$-qV(x) = \mathbf{E}$$

**Potential Energy**

# Depletion width

And with a little algebra

$$\left|\vec{\mathscr{E}}_o\right| = q\frac{N_A x_{po}}{\varepsilon} = q\frac{N_D x_{no}}{\varepsilon} \Rightarrow N_A x_{po} = N_D x_{no}$$

$$\Rightarrow x_{po} = x_{no} N_D / N_A$$

$$W = x_{no} + x_{po} = x_{no} + x_{no} N_D / N_A = x_{no}(N_A + N_D)/N_A$$

$$\Rightarrow x_{no} = \frac{N_A}{(N_A + N_D)} W$$

$$V_o = \frac{1}{2}\left|\vec{\mathscr{E}}_o\right| W = \frac{1}{2} q \frac{N_D x_{no}}{\varepsilon} W = \frac{q}{2\varepsilon}\frac{N_A N_D}{(N_A + N_D)} W^2$$

$$\Rightarrow \boxed{W = \sqrt{\frac{2\varepsilon V_o}{q}\frac{N_A + N_D}{N_A N_D}}}$$

Depletion Width

# Back to Example 1

$$N_A = 10^{18}\text{cm}^{-3}$$
$$N_D = 5 \times 10^{15}\text{cm}^{-3}$$

$$qV_0 = k_B T \ln \frac{N_A N_D}{n_i^2} = 0.796\,\text{eV}$$

Area $A$ = circle with diameter 10 μm

$$A = \pi \frac{D^2}{4} = \pi \left(5\times10^{-4}\right)^2 = 7.85\times10^{-7}\,\text{cm}^2$$

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\frac{N_A + N_D}{N_A N_D}} = \sqrt{\frac{2\times\left(11.8\times8.85\times10^{-14}\right)\times0.796}{1.6\times10^{-19}}\cdot\frac{10^{18}+5\times10^{15}}{10^{18}\times5\times10^{15}}}$$
$$= 0.457\,\mu\text{m}$$

# Back to Example 1

$$N_A = 10^{18}\,\text{cm}^{-3}$$
$$N_D = 5\times10^{15}\,\text{cm}^{-3}$$

$$qV_0 = k_B T \ln\frac{N_A N_D}{n_i^2} = 0.796\,\text{eV}$$

$$x_{po} = \frac{N_D}{N_A+N_D}W = \frac{5\times10^{15}}{10^{18}+5\times10^{15}}0.457 = 2.27\times10^{-3}\,\mu\text{m}$$

$$x_{no} = \frac{N_A}{N_A+N_D}W = \frac{10^{18}}{10^{18}+5\times10^{15}}0.457 = 0.455\,\mu\text{m}$$

$$\mathcal{E}_0 = -\frac{q}{\varepsilon}N_A\,x_{p0} = -\frac{1.6\times10^{-19}\times10^{18}}{11.8\times8.85\times10^{-14}}\times2.27\times10^{-3}\times10^{-4}$$

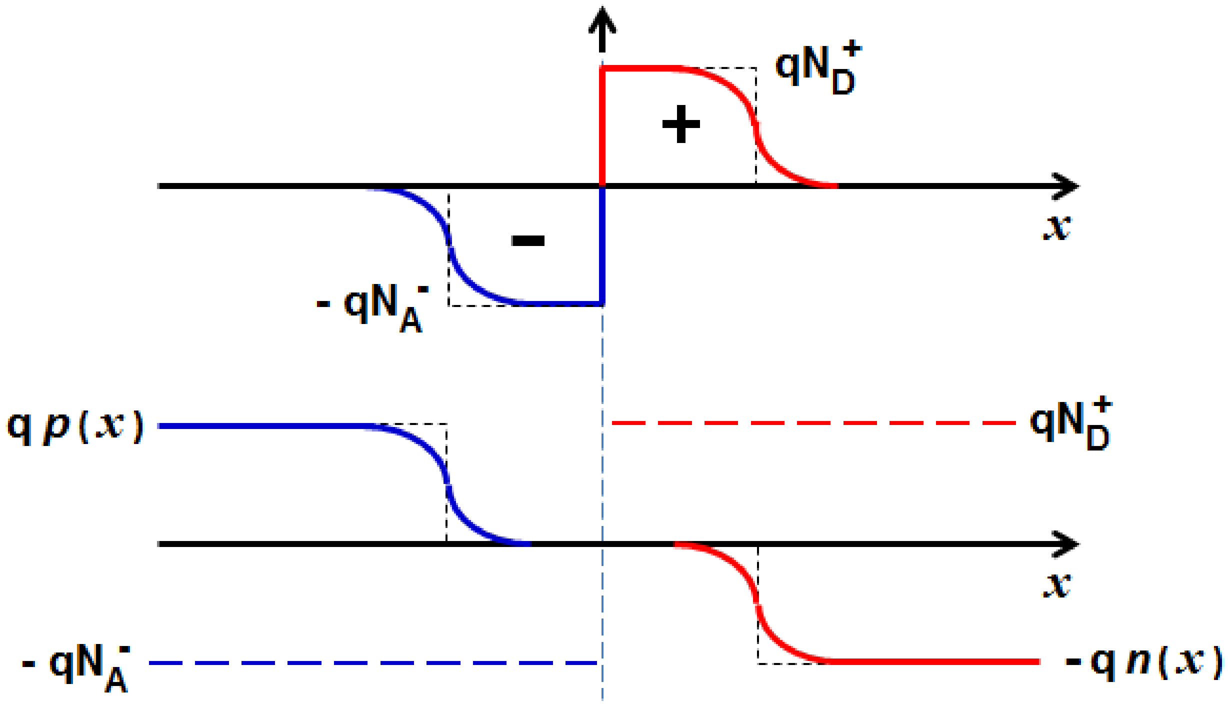
$$= -3.48\times10^{4}\,\text{V/cm}$$

$$Q = -qAN_A\,x_{p0} = -1.6\times10^{-19}\times7.85\times10^{-7}\times10^{18}\times2.27\times10^{-7}$$

$$= -2.85\times10^{-14}\,\text{C}$$

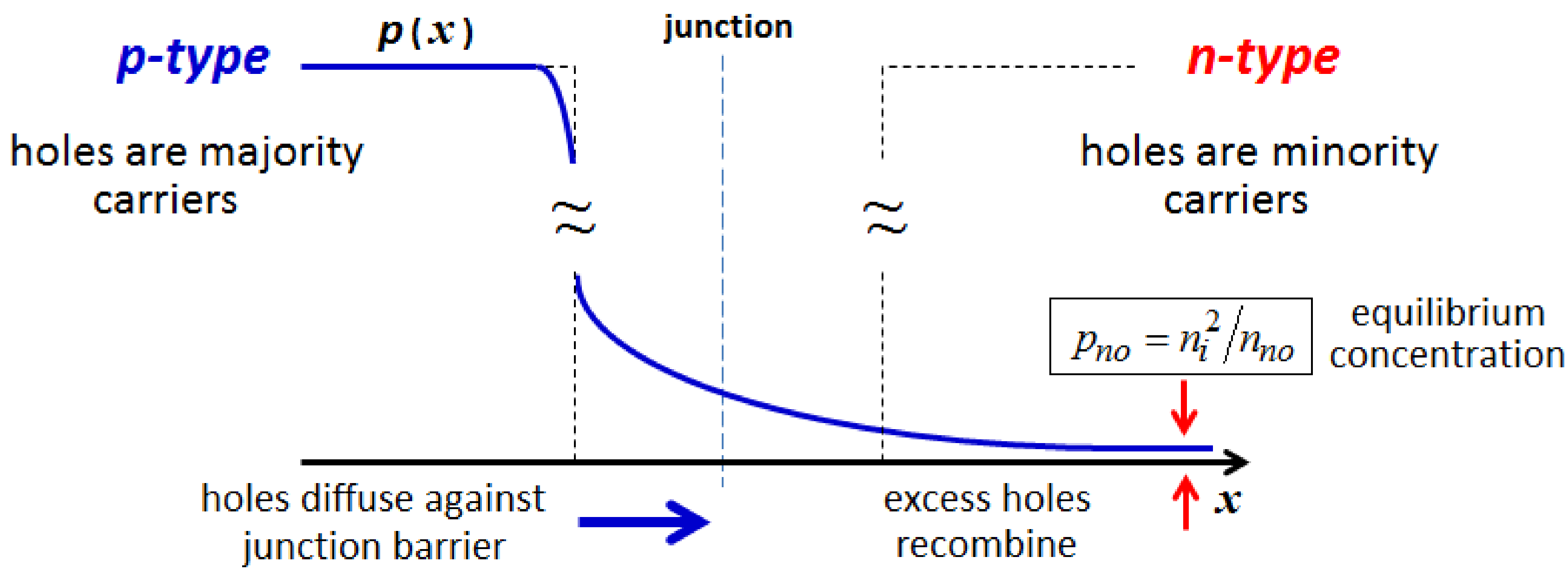
# Beyond depletion approximation

*In reality, the transitions between space charge region and neutral regions are not as abrupt as stated by the depletion approximation*

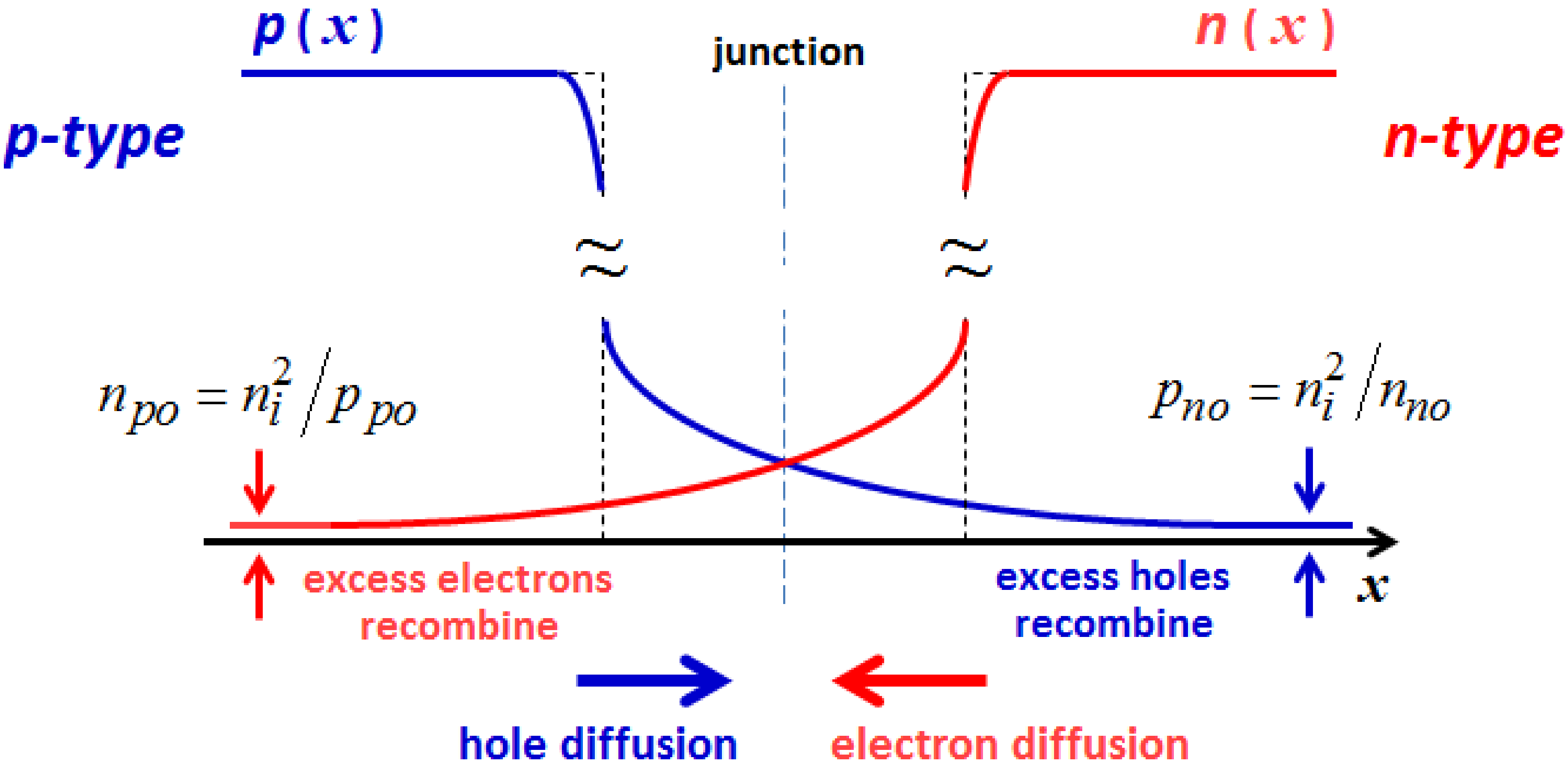


# Beyond depletion approximation

*The minority concentration is very minute and it has been neglected in the equilibrium analysis, but minority carriers will be important to understand current injection when a bias voltage is applied.*



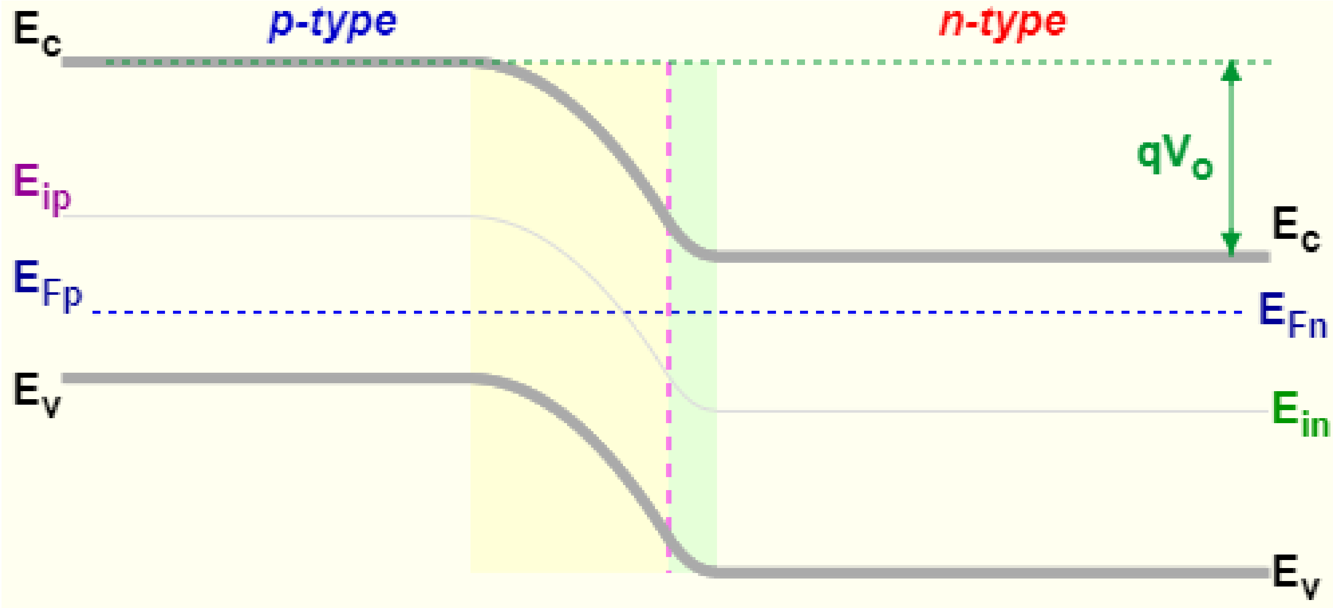
# Beyond depletion approximation



# Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$




$$p_p = ?$$

$$n_p = ?$$

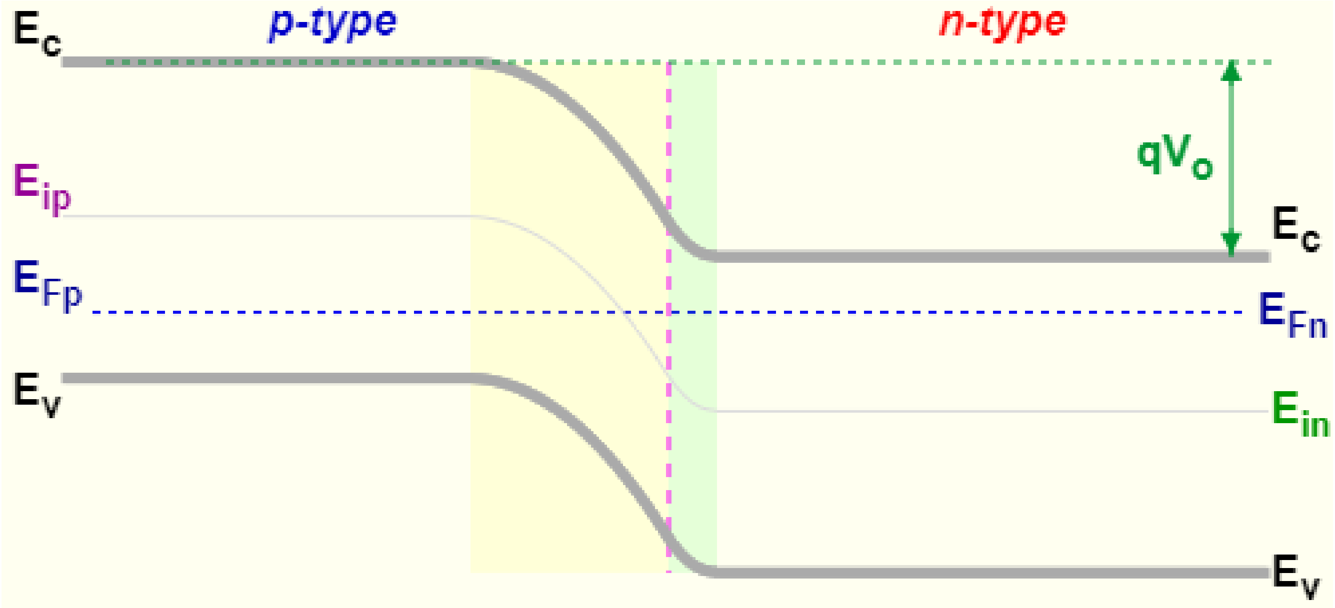
$$n_n = ?$$

$$p_n = ?$$

# Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$




$$p_p \approx N_A = 5\times10^{15}\,\text{cm}^{-3}$$

$$n_n \approx N_D = 2\times10^{16}\,\text{cm}^{-3}$$

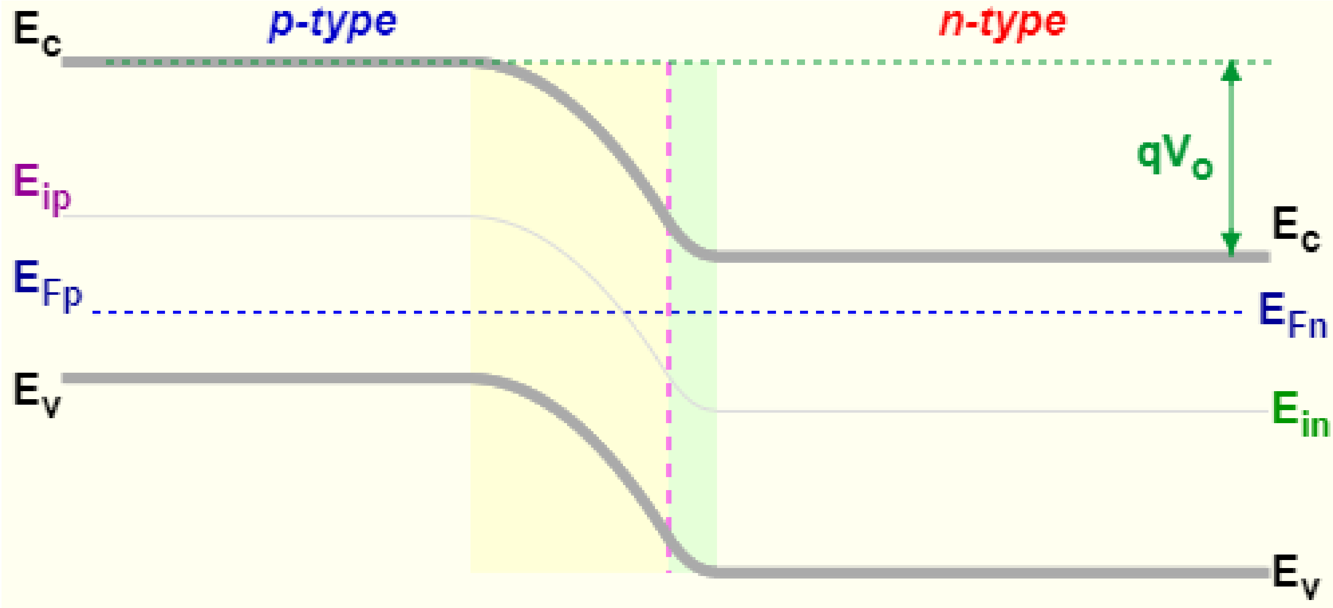
$$n_p = \frac{n_i^2}{N_A} = 4.5\times10^{4}\,\text{cm}^{-3}$$

$$p_n = \frac{n_i^2}{N_D} = 1.125\times10^{4}\,\text{cm}^{-3}$$

# Example 2

$$N_D = 2\times10^{16}\,\mathrm{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\mathrm{cm}^{-3}$$

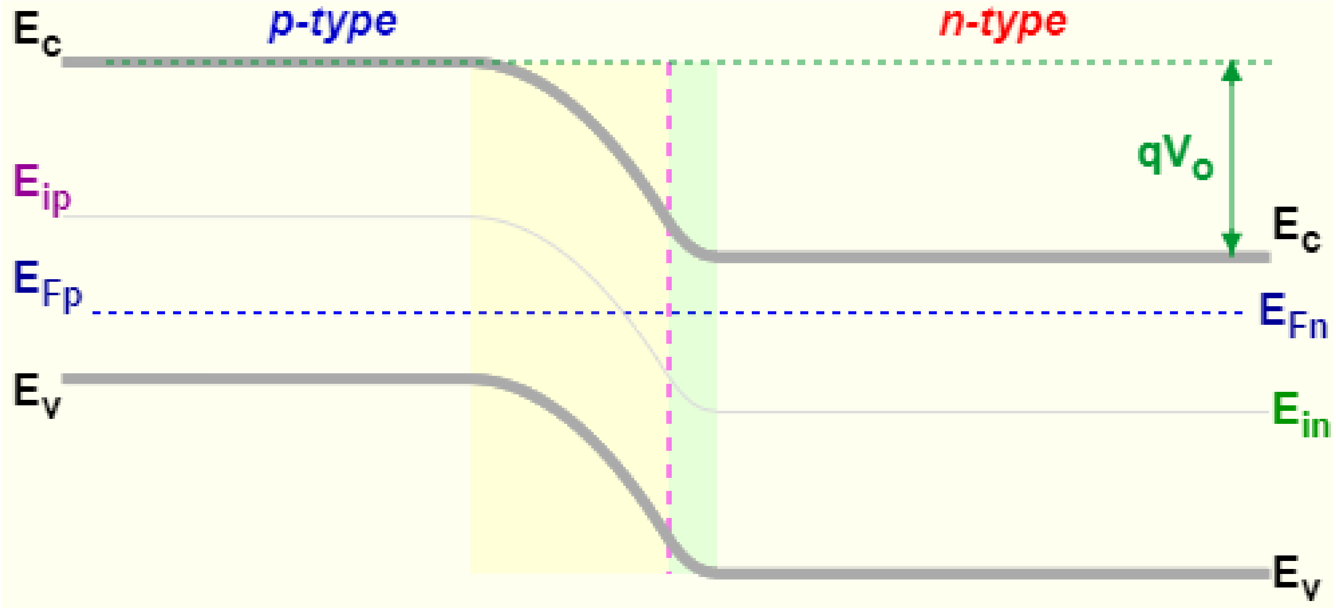



$$qV_0 = ?$$

# Example 2

$$N_D = 2 \times 10^{16} \text{cm}^{-3}$$

$$N_A = 5 \times 10^{15} \text{cm}^{-3}$$




$$E_{ip} - E_F = k_B T \ln \frac{p_p}{n_i} \approx k_B T \ln \frac{N_A}{n_i}$$

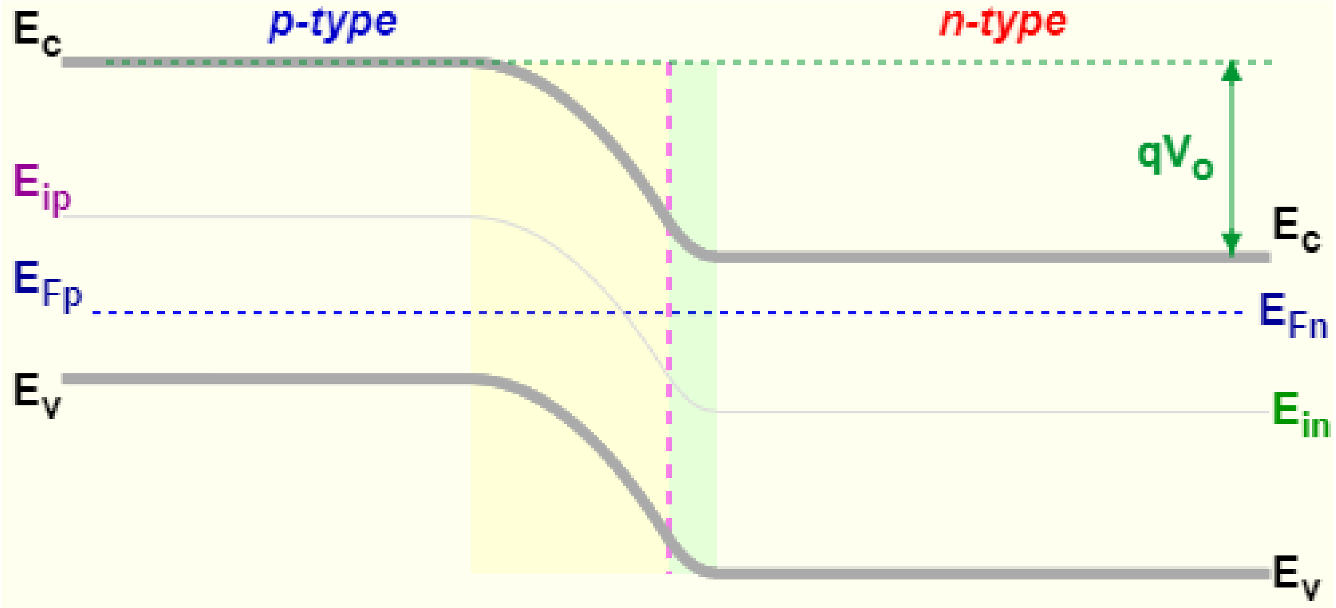
$$E_F - E_{in} = k_B T \ln \frac{n_n}{n_i} \approx k_B T \ln \frac{N_D}{n_i}$$

$$qV_o = E_{ip} - E_{in} \approx k_B T \ln \frac{N_A N_D}{n_i^2}$$

# Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$




$$E_{ip} - E_F = k_B T \ln\frac{p_p}{n_i} \approx k_B T \ln\frac{N_A}{n_i}$$

$$E_F - E_{in} = k_B T \ln\frac{n_n}{n_i} \approx k_B T \ln\frac{N_D}{n_i}$$

$$qV_o = E_{ip} - E_{in} \approx k_B T \ln\frac{N_A N_D}{n_i^2}$$

$$k_B T \ln\frac{N_D}{n_i} = 0.3653\text{ eV}$$

$$k_B T \ln\frac{N_A}{n_i} = 0.3294\text{ eV}$$

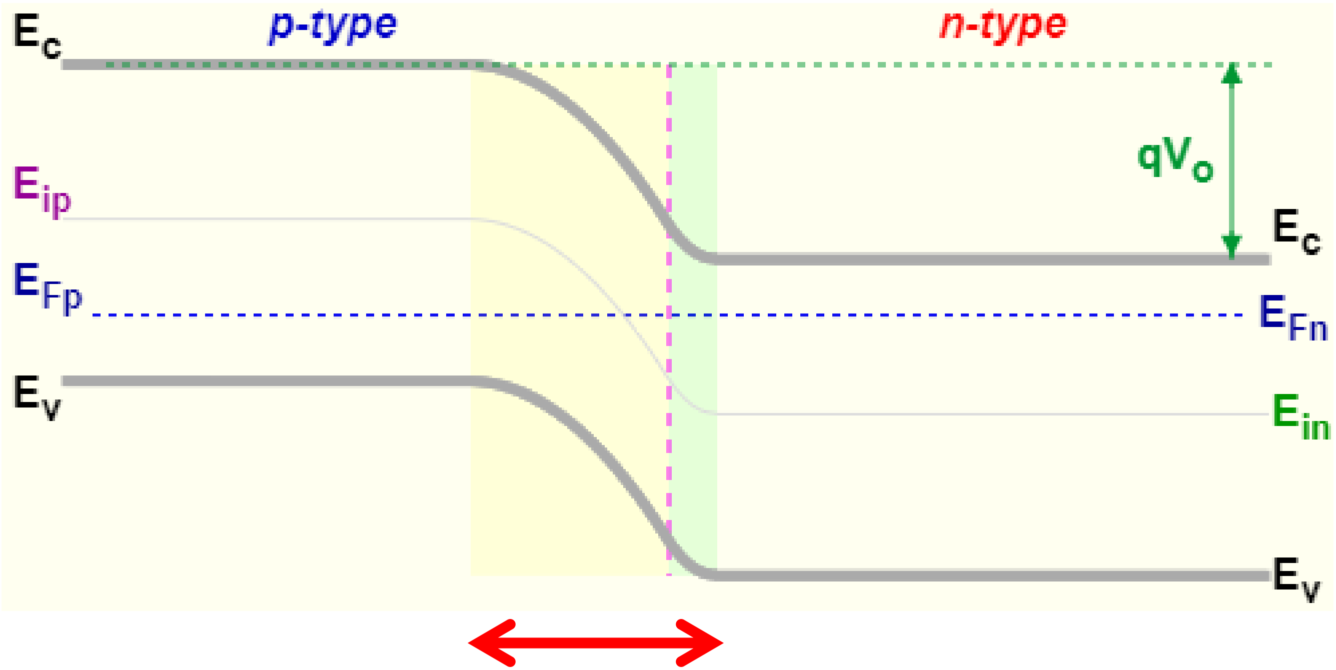
$$qV_o = 0.0259\times\ln\left(4.\bar{4}\times10^{11}\right) =$$

$$= 0.0259\times26.82 = 0.695\text{ eV}$$

# Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$

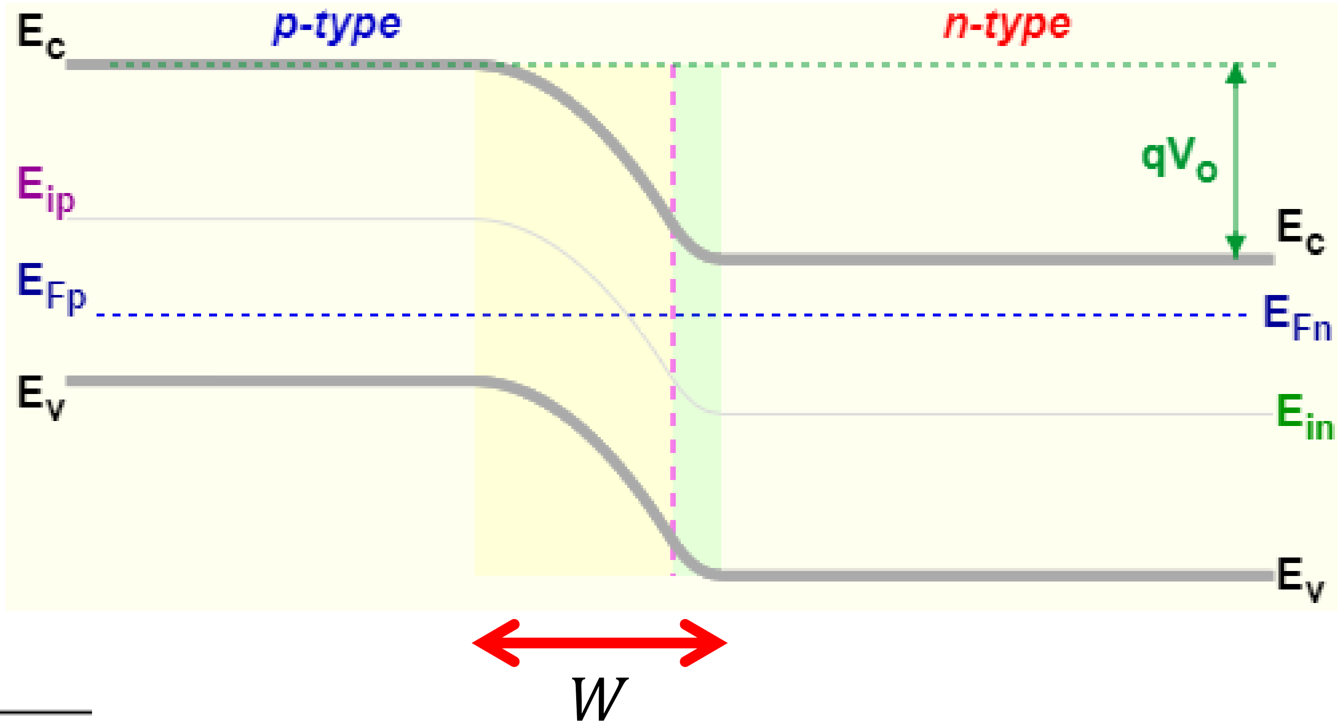



$$W = ?$$

# Example 2

$$N_D = 2\times10^{16}\,\mathrm{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\mathrm{cm}^{-3}$$



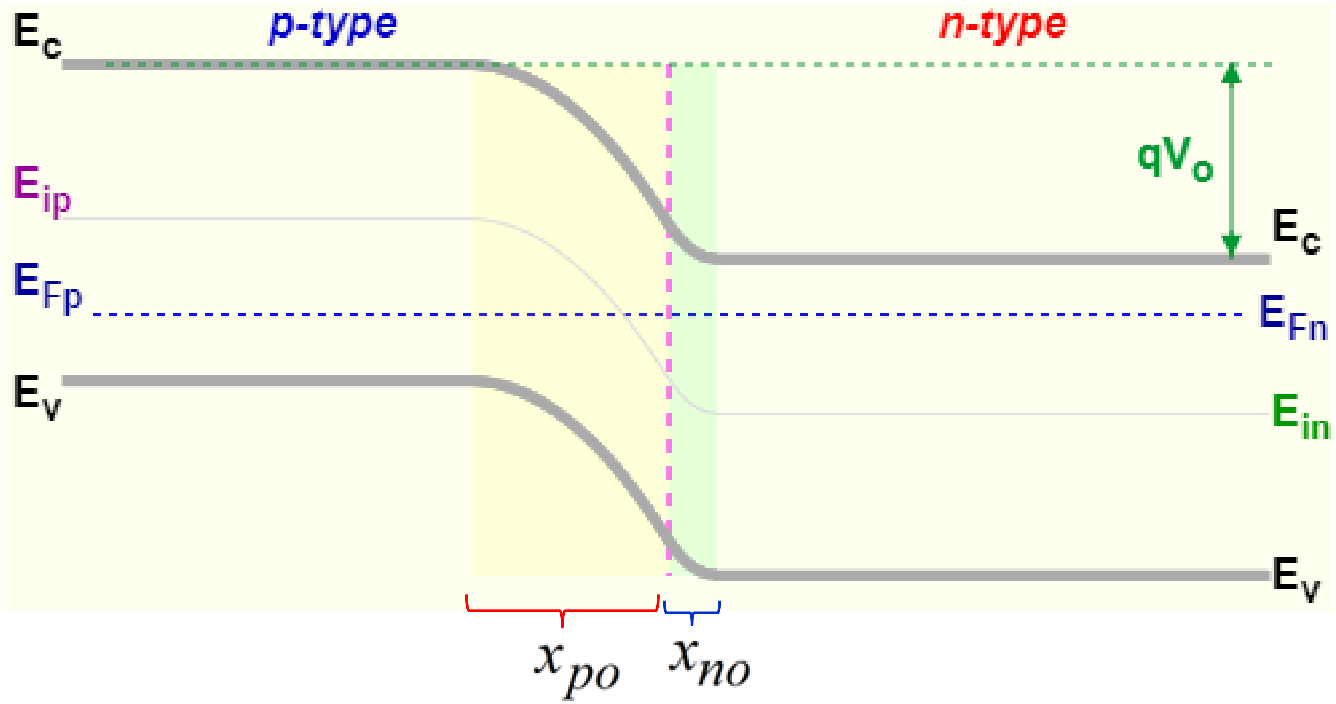

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\frac{N_A + N_D}{N_A N_D}} =$$

$$\sqrt{\frac{2\times\left(11.8\times8.85\times10^{-14}\right)\times0.695}{1.6\times10^{-19}}\cdot\frac{5\times10^{15}+2\times10^{16}}{5\times10^{15}\times2\times10^{16}}} = 0.476\ \mu\mathrm{m}$$

# Example 2

$N_D = 2\times10^{16}\,\text{cm}^{-3}$

$N_A = 5\times10^{15}\,\text{cm}^{-3}$

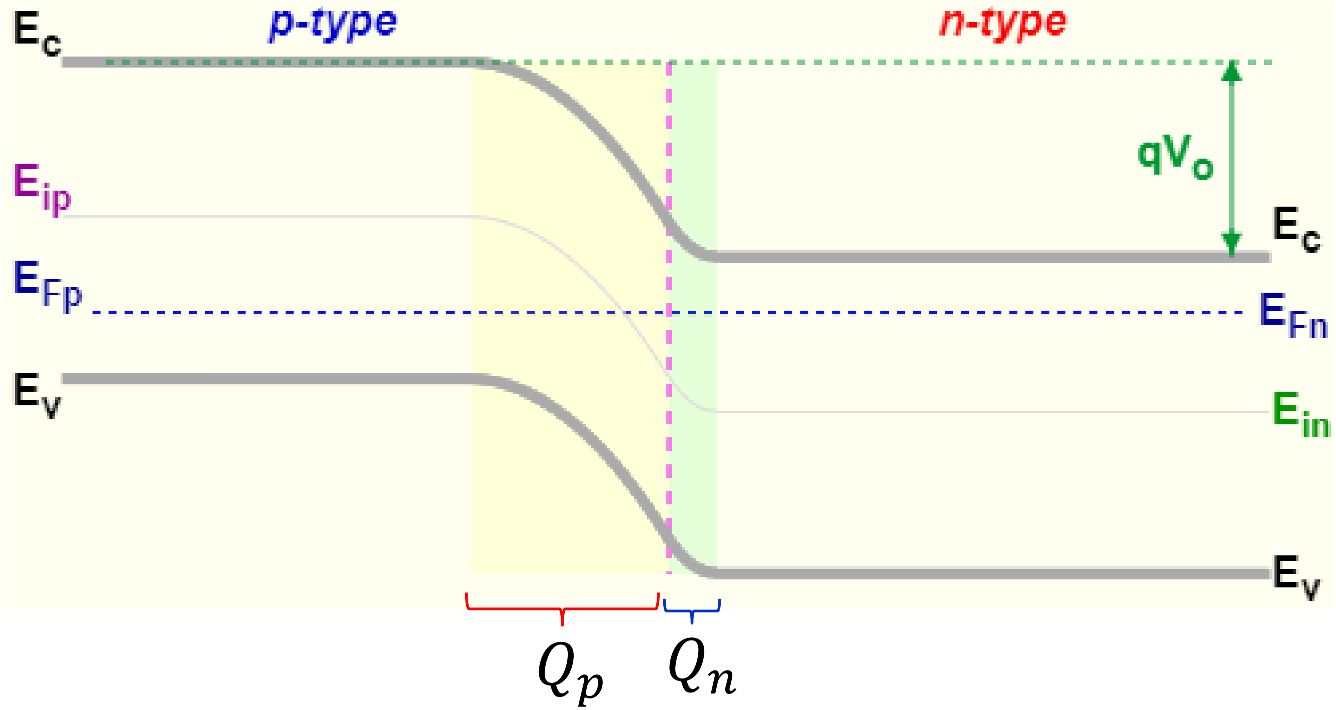



$$x_{no} = \frac{N_A}{N_A + N_D} W = \frac{5\times10^{15}}{5\times10^{15} + 2\times10^{16}} 0.476 = 0.095\ \mu\text{m}$$

$$x_{po} = \frac{N_D}{N_A + N_D} W = \frac{2\times10^{16}}{5\times10^{15} + 2\times10^{16}} 0.476 = 0.381\ \mu\text{m}$$

# Example 2

$$N_D = 2\times10^{16}\,\mathrm{cm}^{-3}$$
$$N_A = 5\times10^{15}\,\mathrm{cm}^{-3}$$




## Charge density per unit area

$$Q_p = -qN_A\, x_{p0} = -1.6\times10^{-19}\times5\times10^{15}\times0.381\times10^{-4} = -3.04\times10^{-8}\ \mathrm{C\ cm^{-2}}$$

$$Q_n = qN_D\, x_{n0} = 1.6\times10^{-19}\times2\times10^{16}\times0.095\times10^{-4} = 3.04\times10^{-8}\ \mathrm{C\ cm^{-2}}$$

# Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$

$$\mathcal{E}_0 = -\frac{q}{\varepsilon} N_A\, x_{p0} =$$

$$= -\frac{1.6\times10^{-19}\times5\times10^{15}}{11.8\times8.85\times10^{-14}}\times0.381\times10^{-4}$$

$$= -29.2\ \text{kV/cm}$$