ECE 340 Lecture 21
Semiconductor Electronics

Spring 2022
10:00-10:50am
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2062 ECE Building

Today’s Discussion

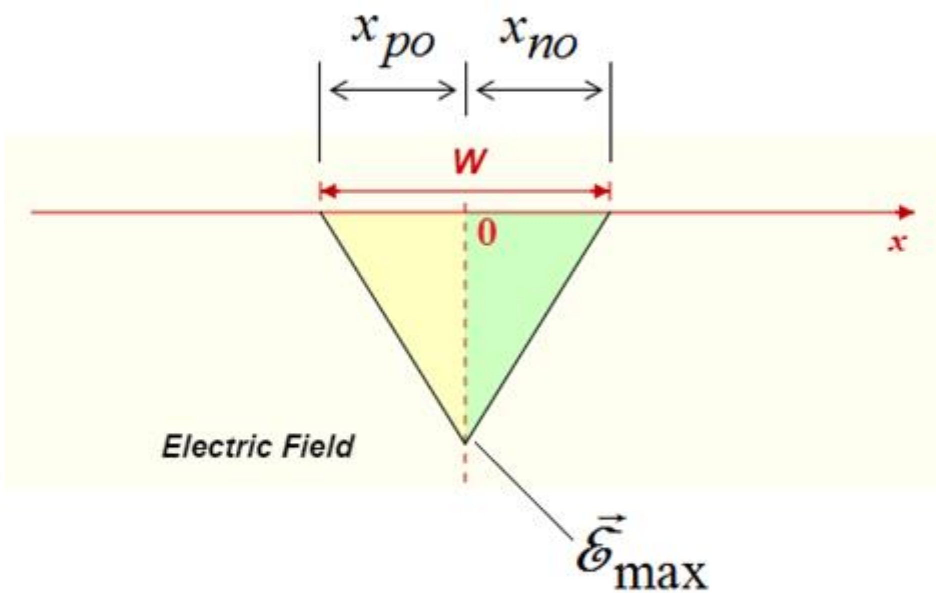
- **The p-n junction in equilibrium (finish)**
- **The p-n junction out of equilibrium (begin - if time allows)**

Application of Gauss law

$$\int_{0}^{\mathcal{E}_0} d\mathcal{E} = -\frac{q}{\varepsilon} N_A \int_{-x_{p0}}^{0} dx \qquad [-x_{p0} < x < 0]$$

$$\int_{\mathcal{E}_0}^{0} d\mathcal{E} = \frac{q}{\varepsilon} N_D \int_{0}^{x_{n0}} dx \qquad [0 < x < x_{n0}]$$

$$\mathcal{E}_0 = \mathcal{E}_{max} = -\frac{q}{\varepsilon} N_D\, x_{n0} = -\frac{q}{\varepsilon} N_A\, x_{p0}$$

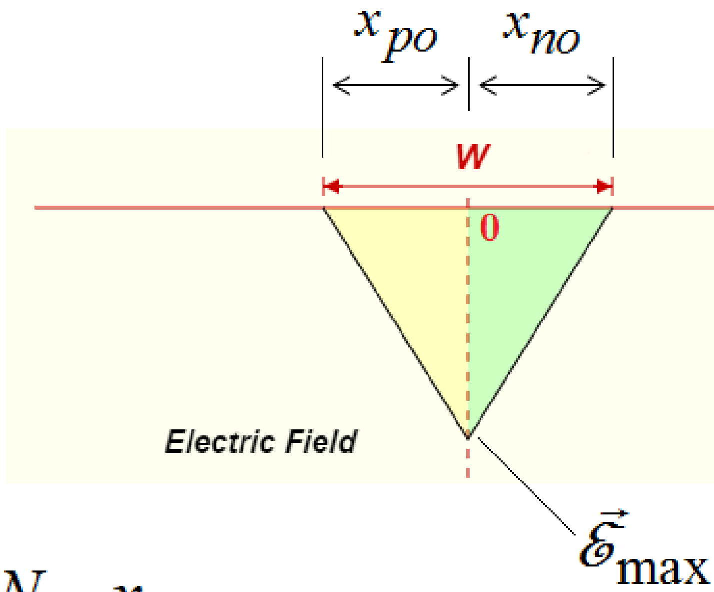


Depletion near the junction

The built-in potential is also related to the width of the depletion region. Since the field distribution has a triangular shape, the integral is simply

$$V_o = -\int_{-x_{po}}^{x_{no}} \vec{\mathscr{E}}(x)\,dx$$

$$= -\frac{1}{2}\vec{\mathscr{E}}_{\max} W$$



From Gauss Law

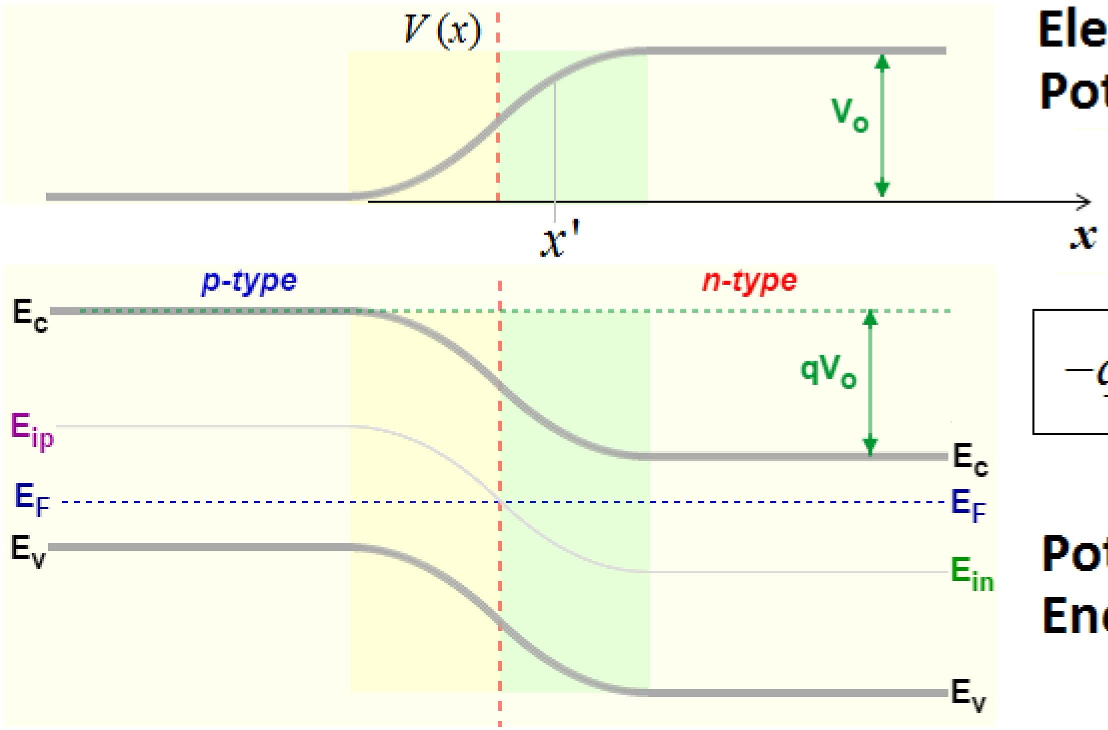
$$\left|\vec{\mathscr{E}}_{\max}\right| = q\frac{N_A\, x_{po}}{\varepsilon} = q\frac{N_D\, x_{no}}{\varepsilon}$$

Details of band bending from potential

$V(x)$ completes the band diagram in the depletion region

$$-\frac{dV}{dx} = \vec{\mathscr{E}}(x)$$

$$-\int_{-\infty}^{x'} \vec{\mathscr{E}}(x)\, dx = V(x')$$



Electrostatic Potential

$$-qV(x) = \mathbf{E}$$

Potential Energy

Depletion width

And with a little algebra

$$\left|\vec{\mathcal{E}}_o\right| = q\frac{N_A\, x_{po}}{\varepsilon} = q\frac{N_D\, x_{no}}{\varepsilon} \;\Rightarrow\; N_A\, x_{po} = N_D\, x_{no}$$

$$\Rightarrow\; x_{po} = x_{no}\, N_D / N_A$$

$$W = x_{no} + x_{po} = x_{no} + x_{no}\, N_D / N_A = x_{no}\left(N_A + N_D\right)/N_A$$

$$\Rightarrow\; x_{no} = \frac{N_A}{\left(N_A + N_D\right)}W$$

$$V_o = \frac{1}{2}\left|\vec{\mathcal{E}}_o\right| W = \frac{1}{2}\, q\frac{N_D\, x_{no}}{\varepsilon}\, W = \frac{q}{2\varepsilon}\frac{N_A N_D}{\left(N_A + N_D\right)}W^2$$

$$\Rightarrow\; W = \sqrt{\frac{2\varepsilon V_o}{q}\frac{N_A + N_D}{N_A N_D}}$$

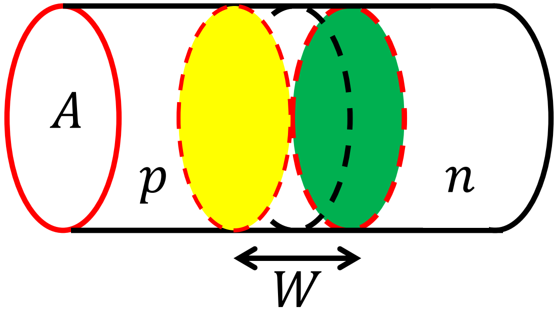
Depletion Width

Back to Example 1 (previous lecture)

$$N_A = 10^{18}\text{cm}^{-3}$$
$$N_D = 5 \times 10^{15}\text{cm}^{-3}$$

$$qV_0 = k_B T \ln \frac{N_A N_D}{n_i^2} = 0.796\,\text{eV}$$

Assume circular cross-section



Area A = circle with diameter 10 μm

$$A = \pi \frac{D^2}{4} = \pi\left(5\times10^{-4}\right)^2 = 7.85\times10^{-7}\,\text{cm}^2$$

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\frac{N_A+N_D}{N_A N_D}} = \sqrt{\frac{2\times\left(11.8\times8.85\times10^{-14}\right)\times0.796}{1.6\times10^{-19}}\cdot\frac{10^{18}+5\times10^{15}}{10^{18}\times5\times10^{15}}}$$

$$= 0.457\,\mu\text{m}$$

Back to Example 1 (previous lecture)

$$N_A = 10^{18}\text{cm}^{-3}$$
$$N_D = 5 \times 10^{15}\text{cm}^{-3}$$

$$qV_0 = k_B T \ln\frac{N_A N_D}{n_i^2} = 0.796\,\text{eV}$$

$$x_{po} = \frac{N_D}{N_A + N_D} W = \frac{5\times10^{15}}{10^{18} + 5\times10^{15}} 0.457 = 2.27\times10^{-3}\,\mu\text{m}$$

$$x_{no} = \frac{N_A}{N_A + N_D} W = \frac{10^{18}}{10^{18} + 5\times10^{15}} 0.457 = 0.455\,\mu\text{m}$$

$$\mathcal{E}_0 = -\frac{q}{\varepsilon} N_A\, x_{p0} = -\frac{1.6\times10^{-19}\times10^{18}}{11.8\times8.85\times10^{-14}}\times2.27\times10^{-3}\times10^{-4}$$

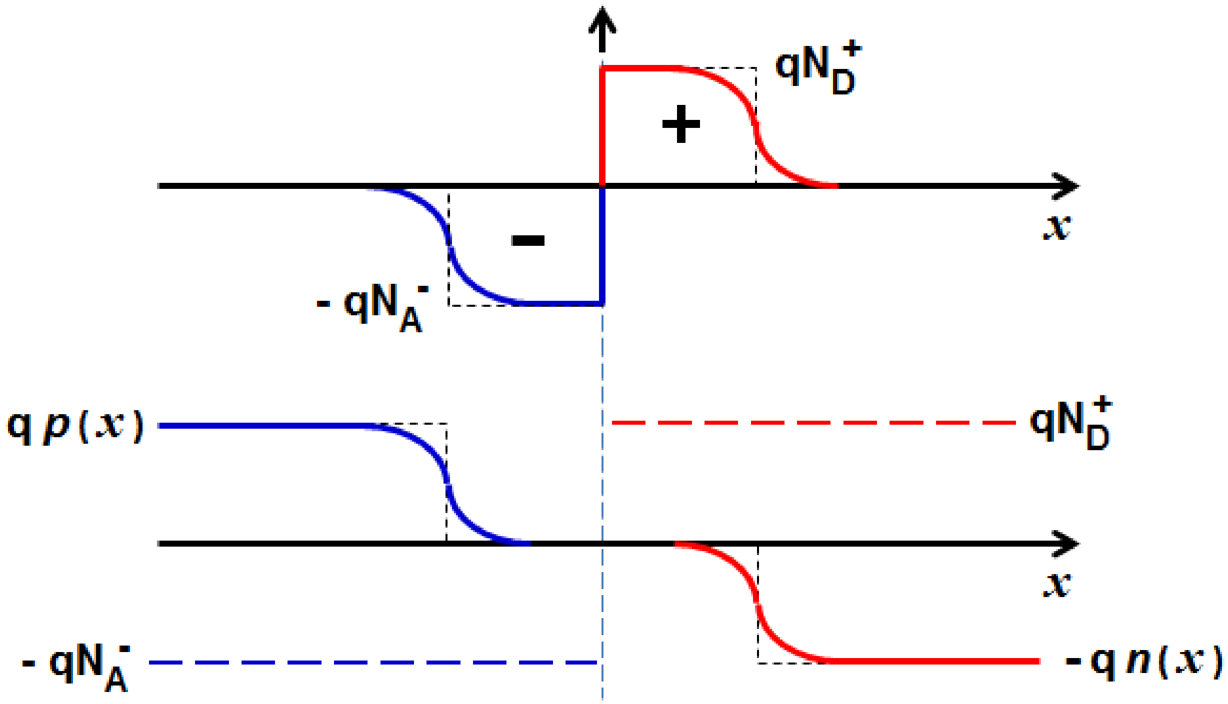
$$= -3.48\times10^{4}\,\text{V/cm}$$

$$Q = -qAN_A\, x_{p0} = -1.6\times10^{-19}\times7.85\times10^{-7}\times10^{18}\times2.27\times10^{-7}$$

$$= -2.85\times10^{-14}\,\text{C}$$

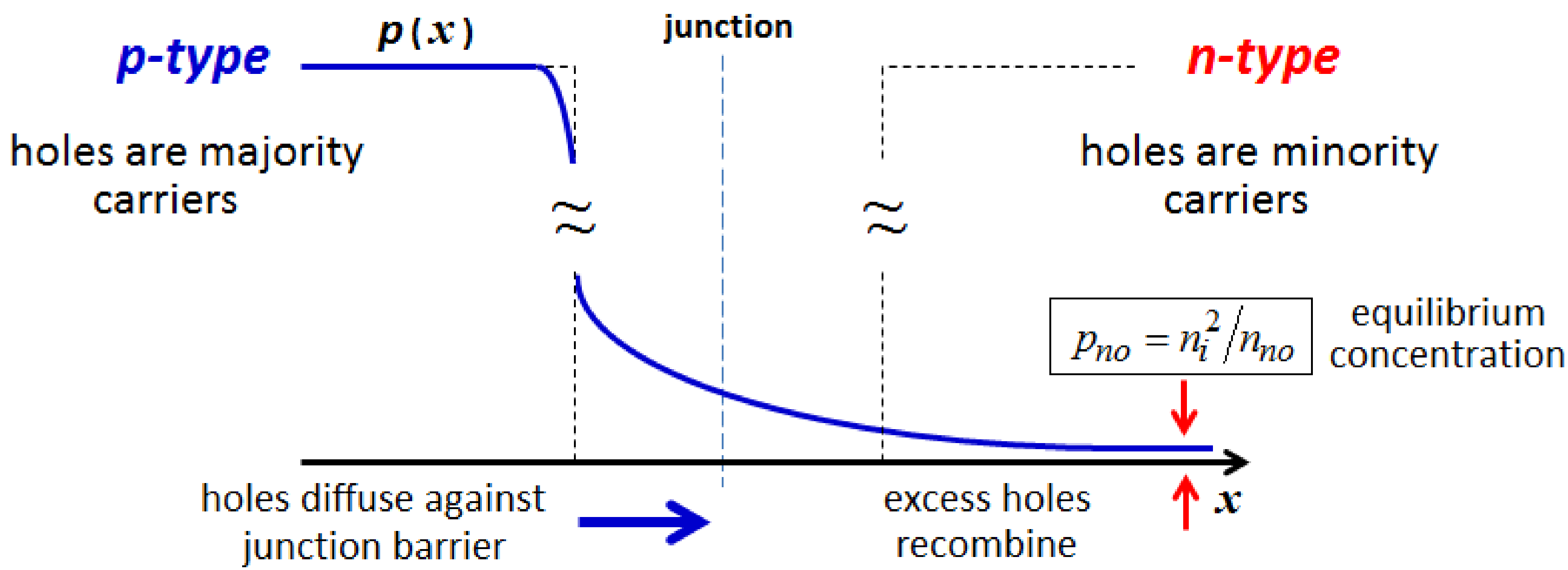
Beyond depletion approximation

In reality, the transitions between space charge region and neutral regions are not as abrupt as stated by the depletion approximation

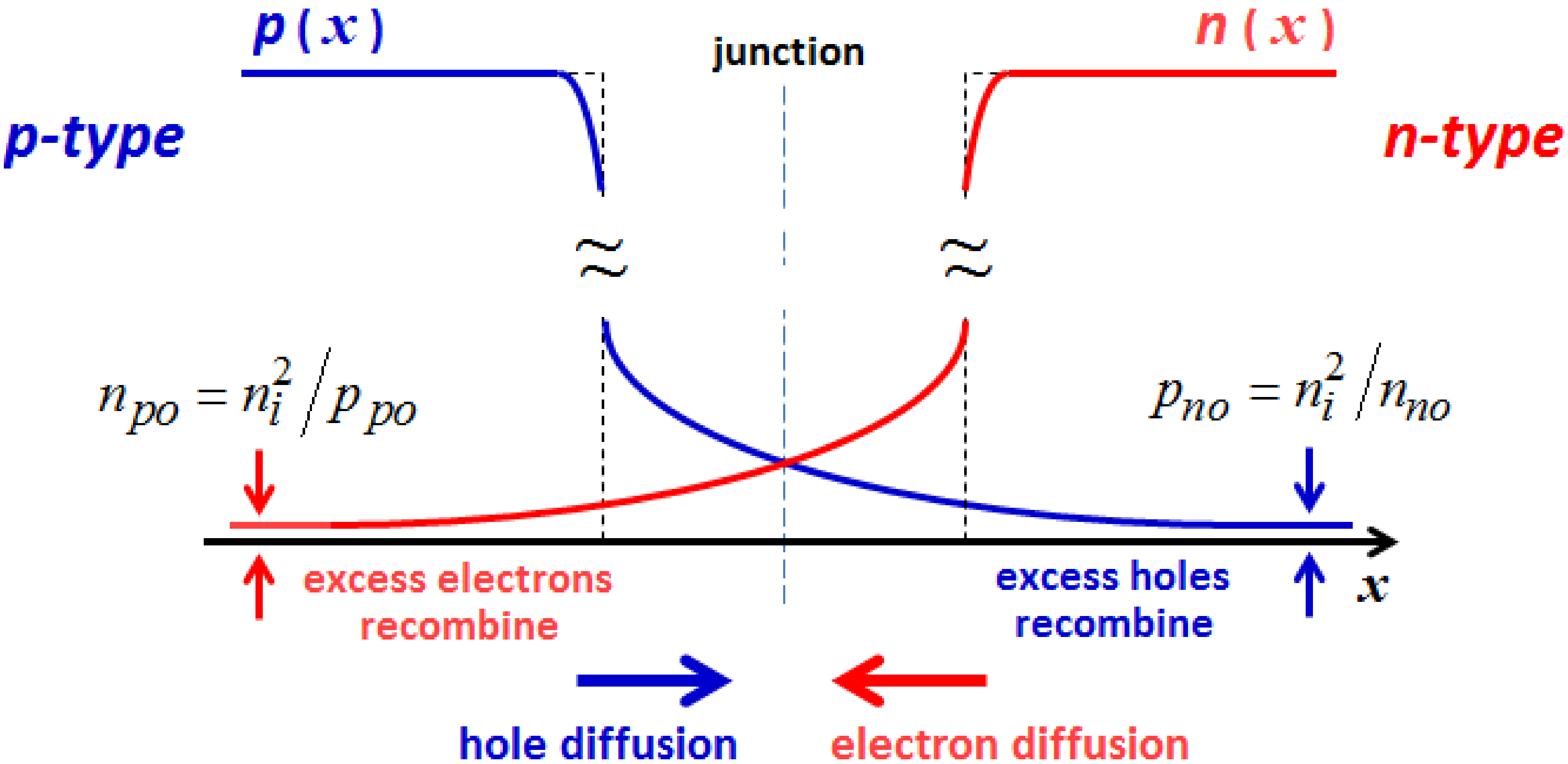


Beyond depletion approximation

The minority concentration is very minute and it has been neglected in the equilibrium analysis, but minority carriers will be important to understand current injection when a bias voltage is applied.



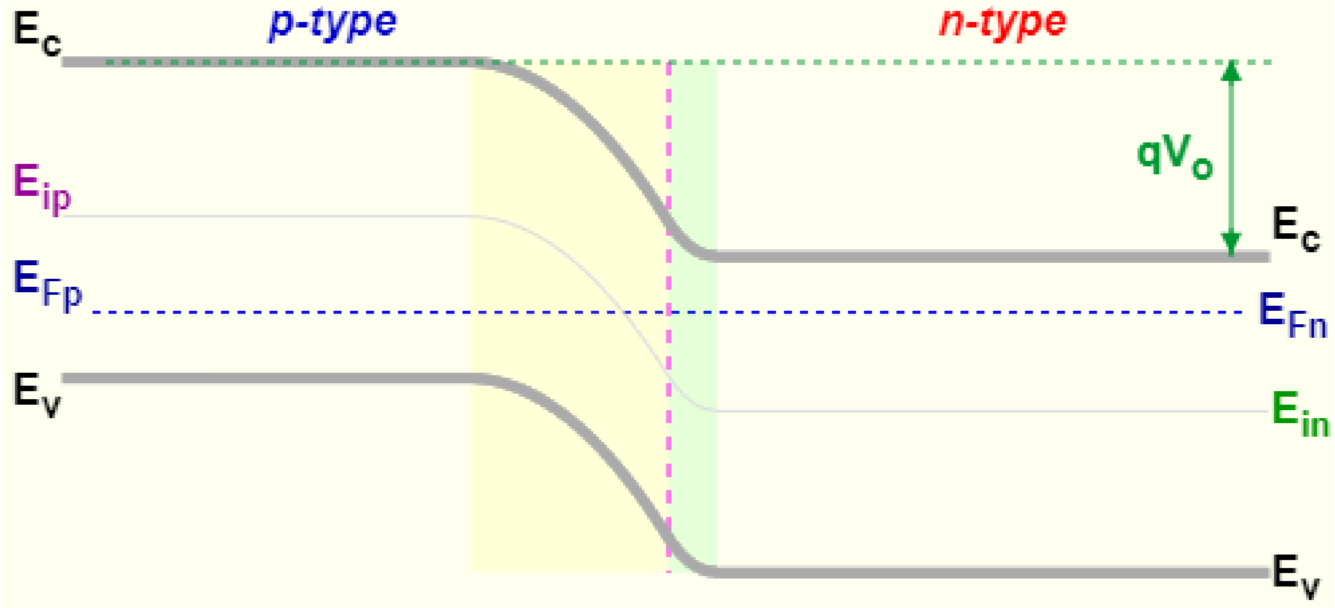
Beyond depletion approximation



Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$



$$p_p = ?$$

$$n_p = ?$$

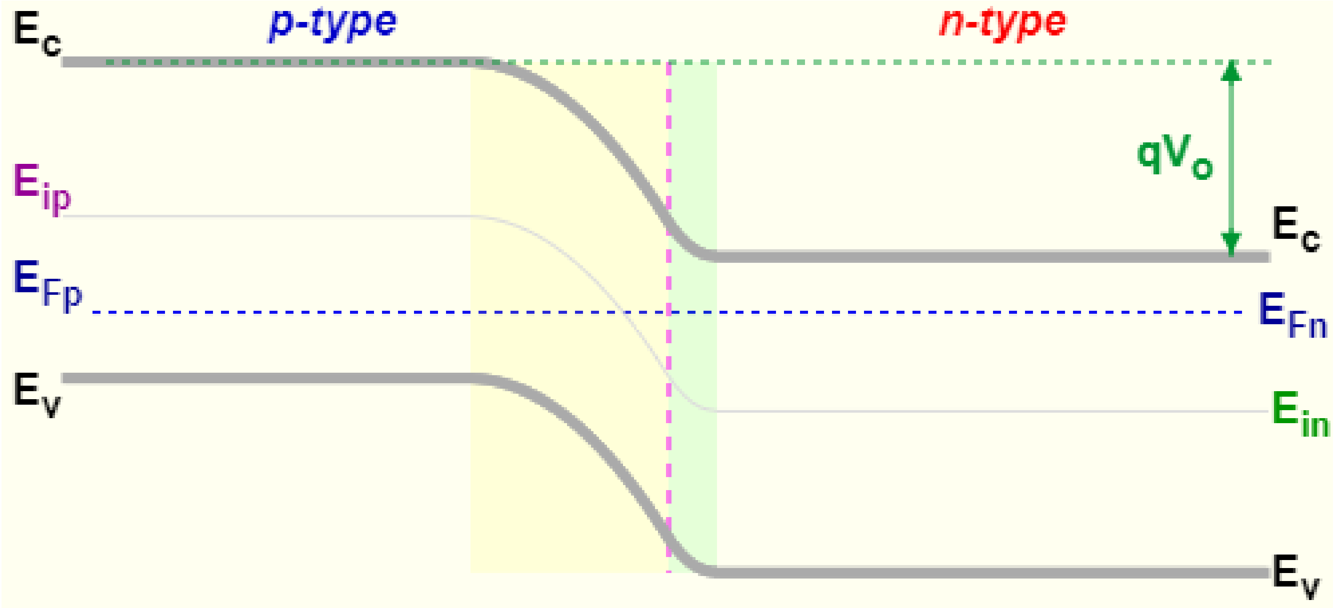
$$n_n = ?$$

$$p_n = ?$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$



$$p_p \approx N_A = 5\times10^{15}\,\text{cm}^{-3}$$

$$n_p = \frac{n_i^2}{N_A} = 4.5\times10^{4}\,\text{cm}^{-3}$$

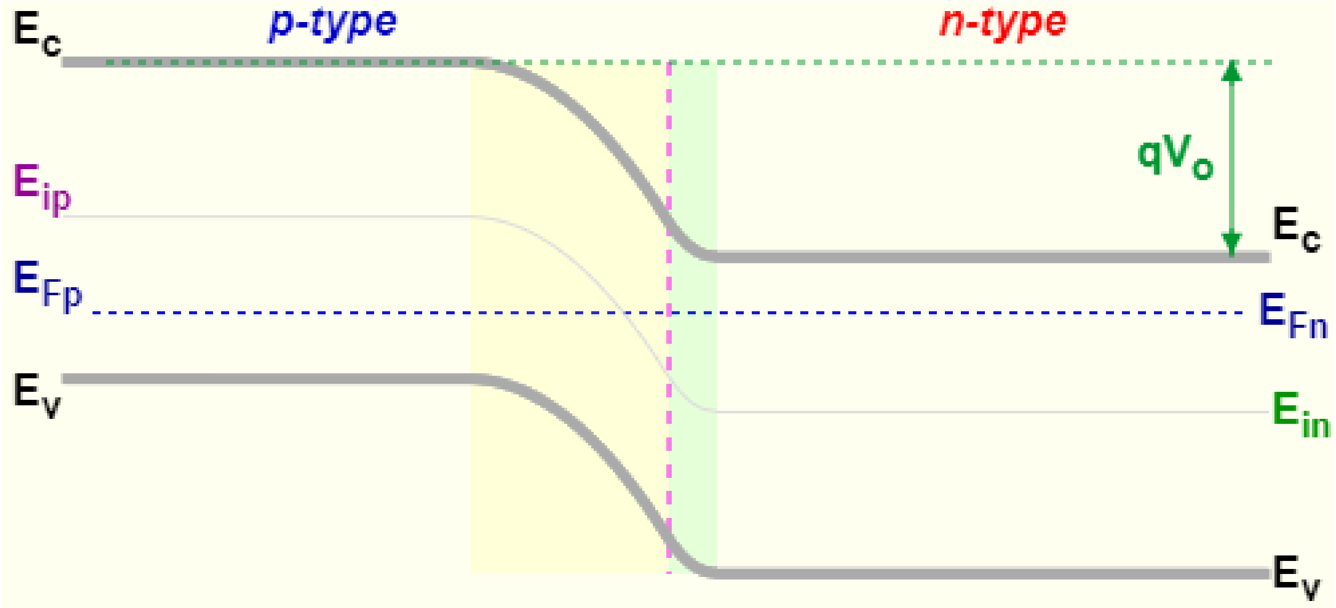
$$n_n \approx N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$p_n = \frac{n_i^2}{N_D} = 1.125\times10^{4}\,\text{cm}^{-3}$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$

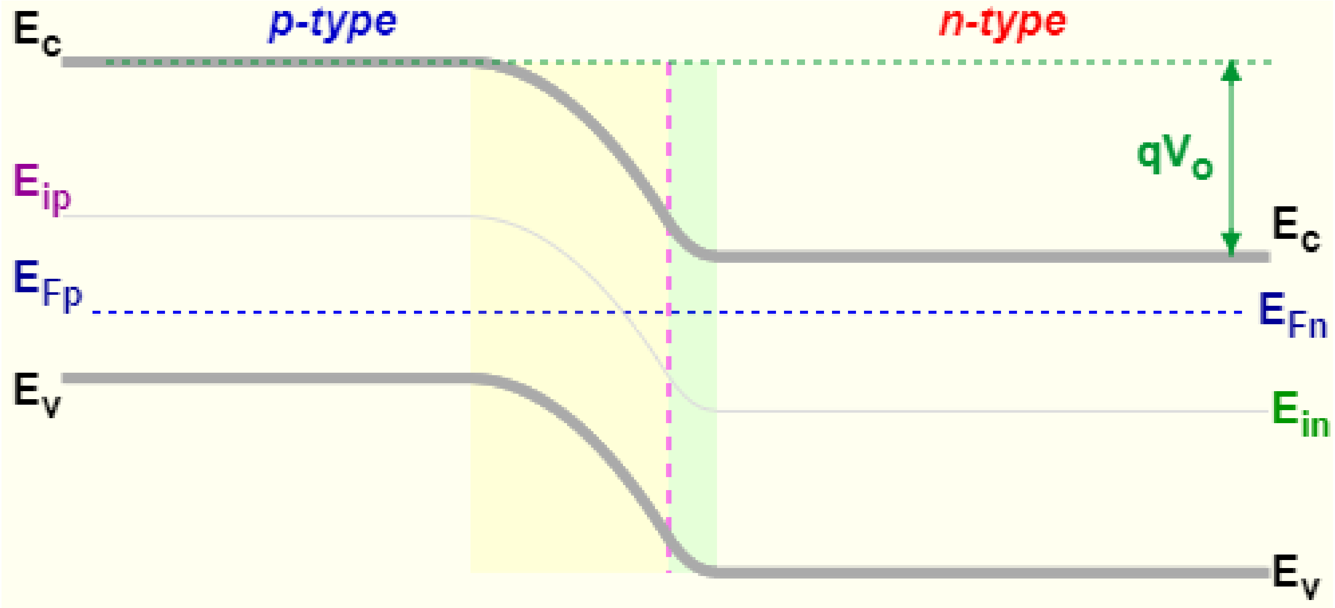


$$qV_0 = ?$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$



$$E_{ip} - E_F = k_B T \ln\frac{p_p}{n_i} \approx k_B T \ln\frac{N_A}{n_i}$$

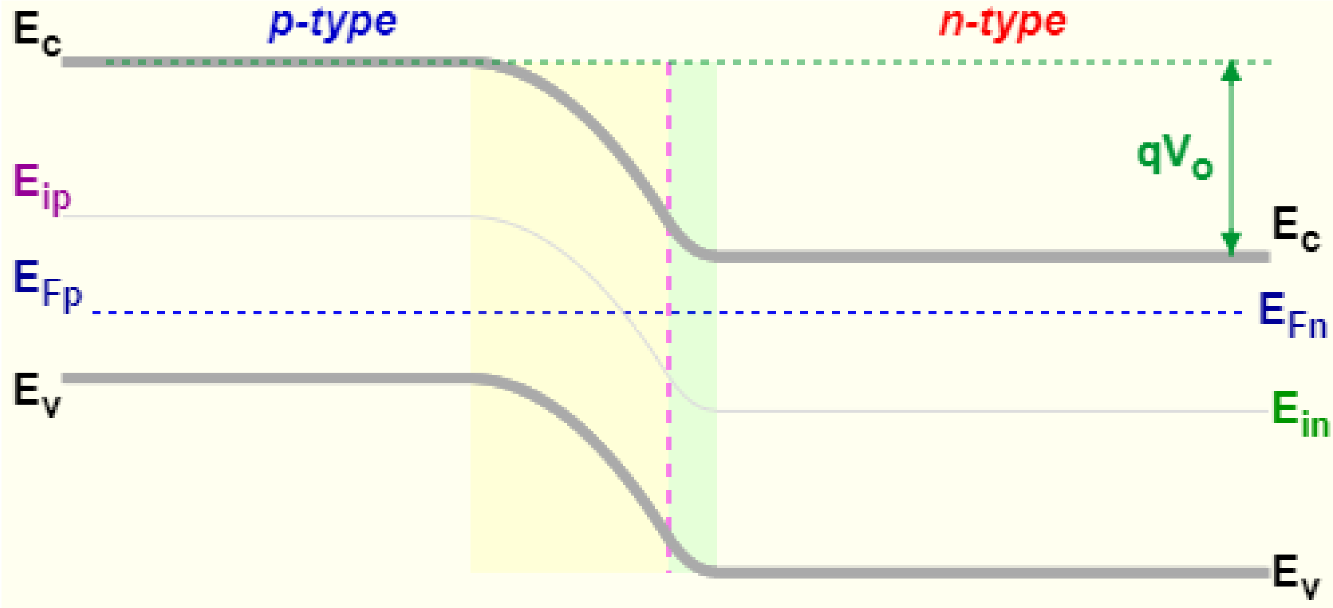
$$E_F - E_{in} = k_B T \ln\frac{n_n}{n_i} \approx k_B T \ln\frac{N_D}{n_i}$$

$$qV_o = E_{ip} - E_{in} \approx k_B T \ln\frac{N_A N_D}{n_i^2}$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$



$$E_{ip} - E_F = k_B T \ln\frac{p_p}{n_i} \approx k_B T \ln\frac{N_A}{n_i}$$

$$E_F - E_{in} = k_B T \ln\frac{n_n}{n_i} \approx k_B T \ln\frac{N_D}{n_i}$$

$$qV_o = E_{ip} - E_{in} \approx k_B T \ln\frac{N_A N_D}{n_i^2}$$

$$k_B T \ln\frac{N_D}{n_i} = 0.3653\ \text{eV}$$

$$k_B T \ln\frac{N_A}{n_i} = 0.3294\ \text{eV}$$

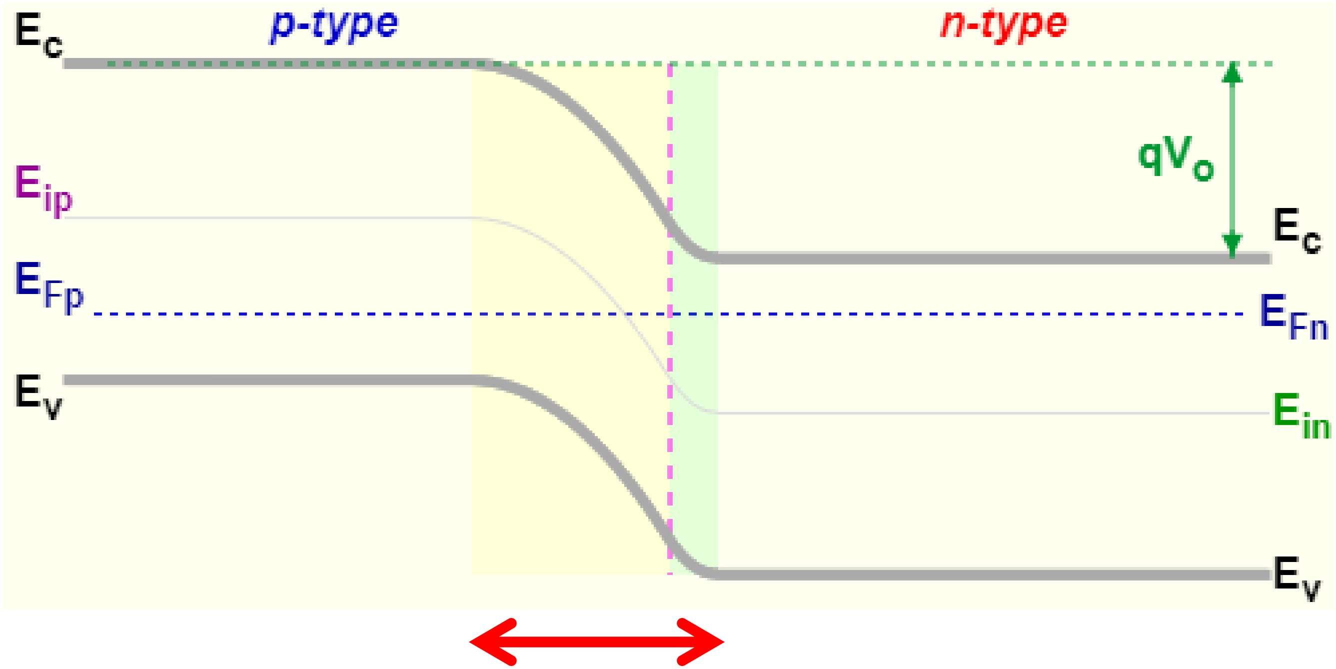
$$qV_o = 0.0259\times\ln\left(4.\overline{4}\times10^{11}\right) =$$

$$= 0.0259\times26.82 = 0.695\ \text{eV}$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$

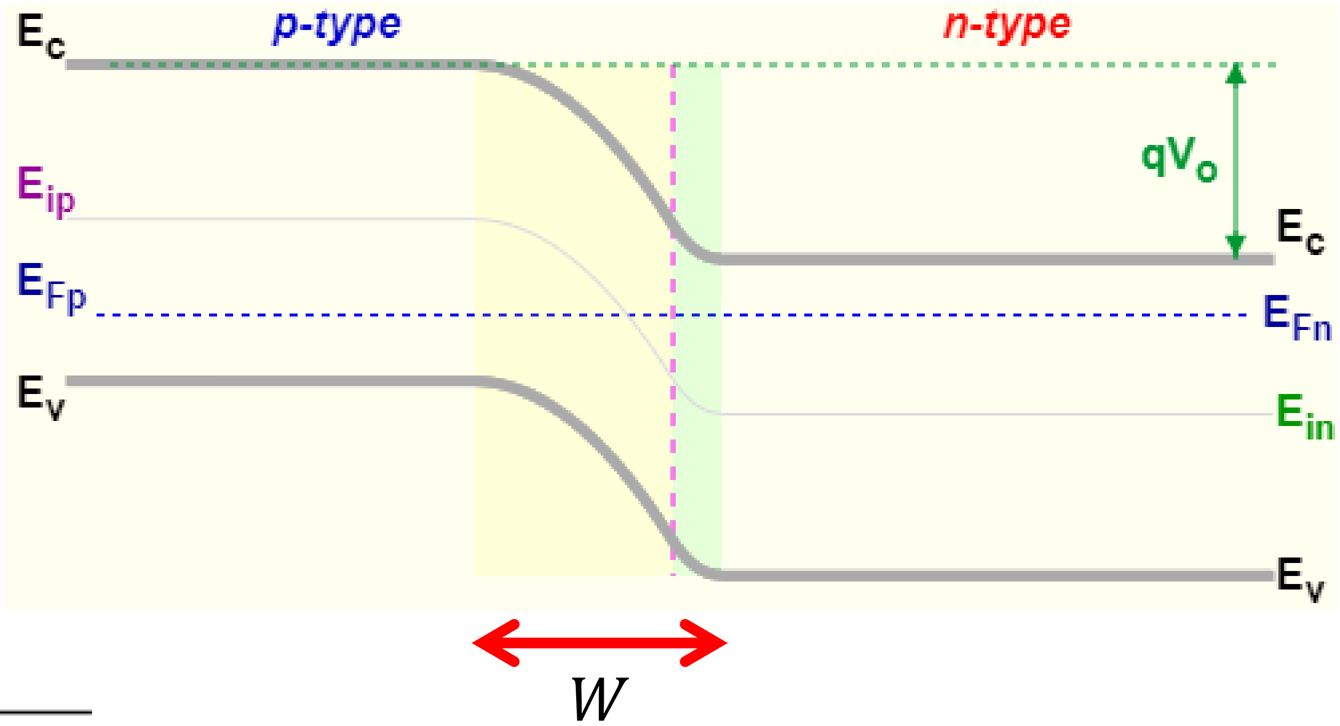


$$W = ?$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$



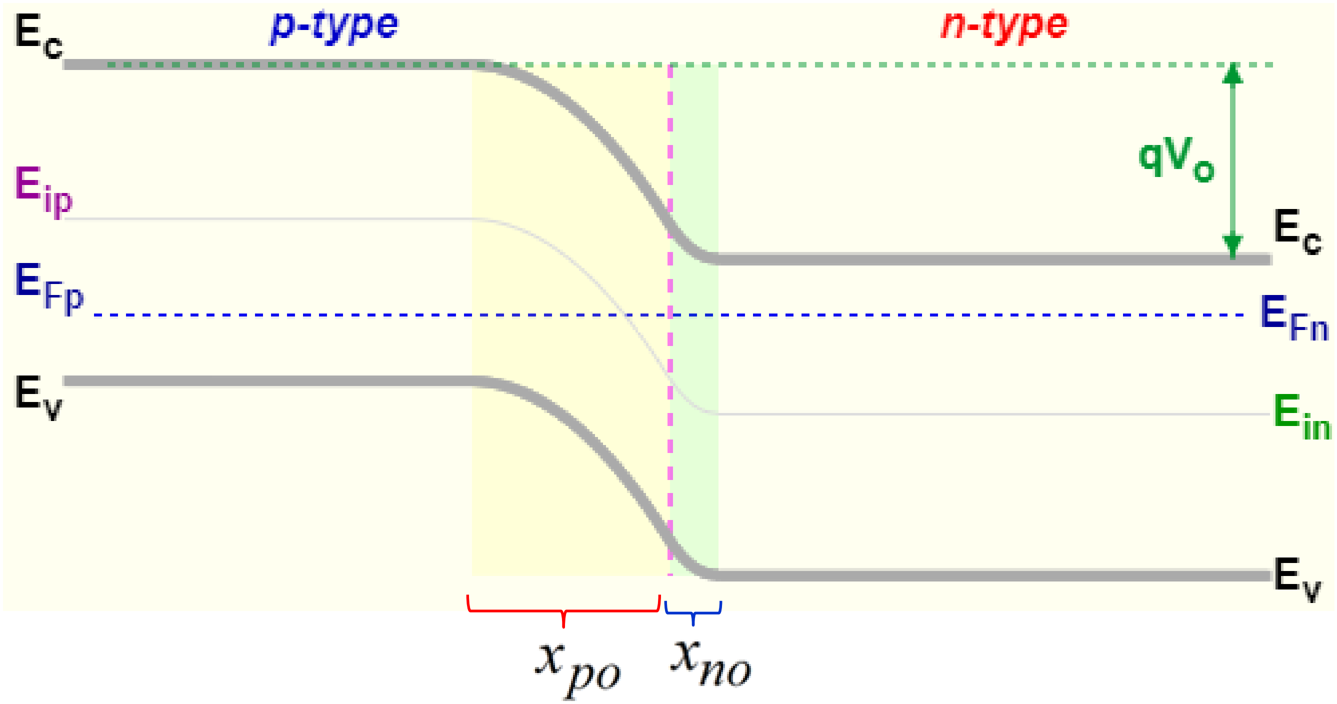
$$W = \sqrt{\frac{2\varepsilon V_0}{q}\frac{N_A+N_D}{N_A N_D}} =$$

$$\sqrt{\frac{2\times\left(11.8\times8.85\times10^{-14}\right)\times0.695}{1.6\times10^{-19}}\cdot\frac{5\times10^{15}+2\times10^{16}}{5\times10^{15}\times2\times10^{16}}} = 0.476\ \mu\text{m}$$

Example 2

$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$



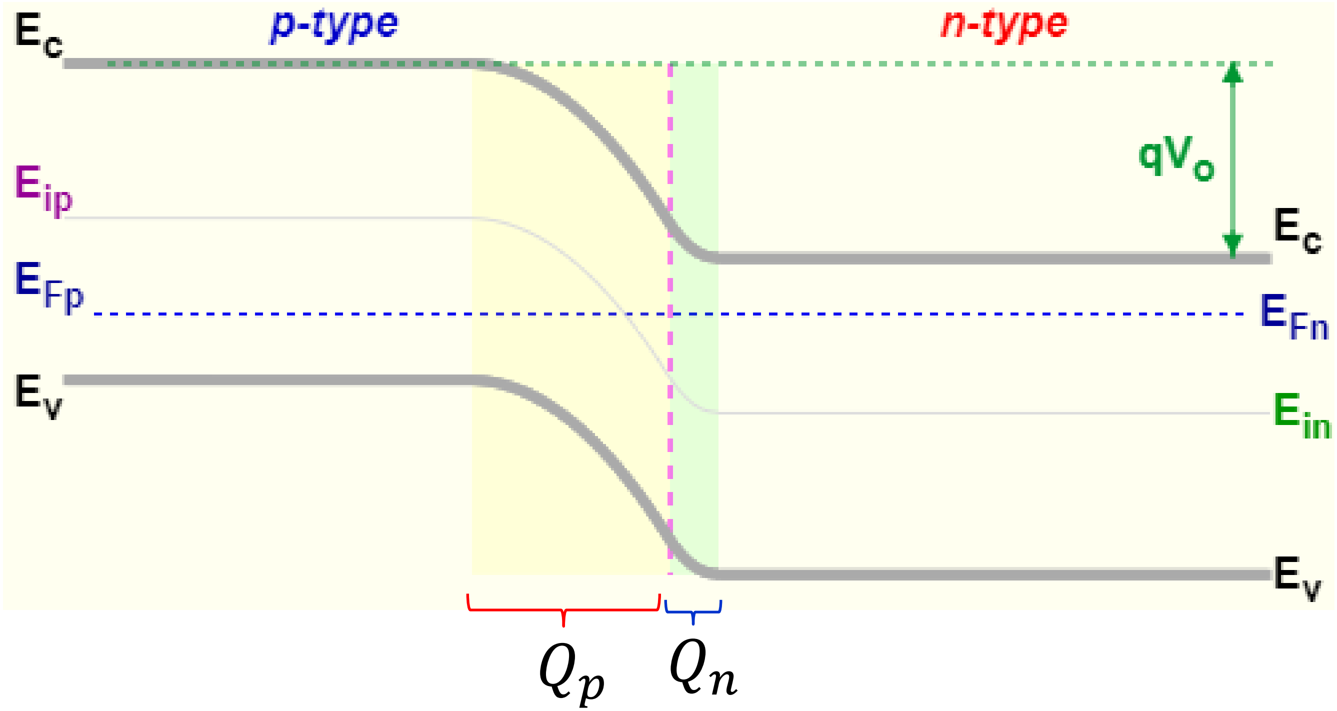
$$x_{no} = \frac{N_A}{N_A + N_D} W = \frac{5\times10^{15}}{5\times10^{15} + 2\times10^{16}} 0.476 = 0.095\ \mu\text{m}$$

$$x_{po} = \frac{N_D}{N_A + N_D} W = \frac{2\times10^{16}}{5\times10^{15} + 2\times10^{16}} 0.476 = 0.381\ \mu\text{m}$$

Example 2

$N_D = 2\times10^{16}\,\text{cm}^{-3}$

$N_A = 5\times10^{15}\,\text{cm}^{-3}$



Charge density per unit area

$$Q_p = -qN_A\, x_{p0} = -1.6\times10^{-19}\times5\times10^{15}\times0.381\times10^{-4} = -3.04\times10^{-8}\ \text{C cm}^{-2}$$

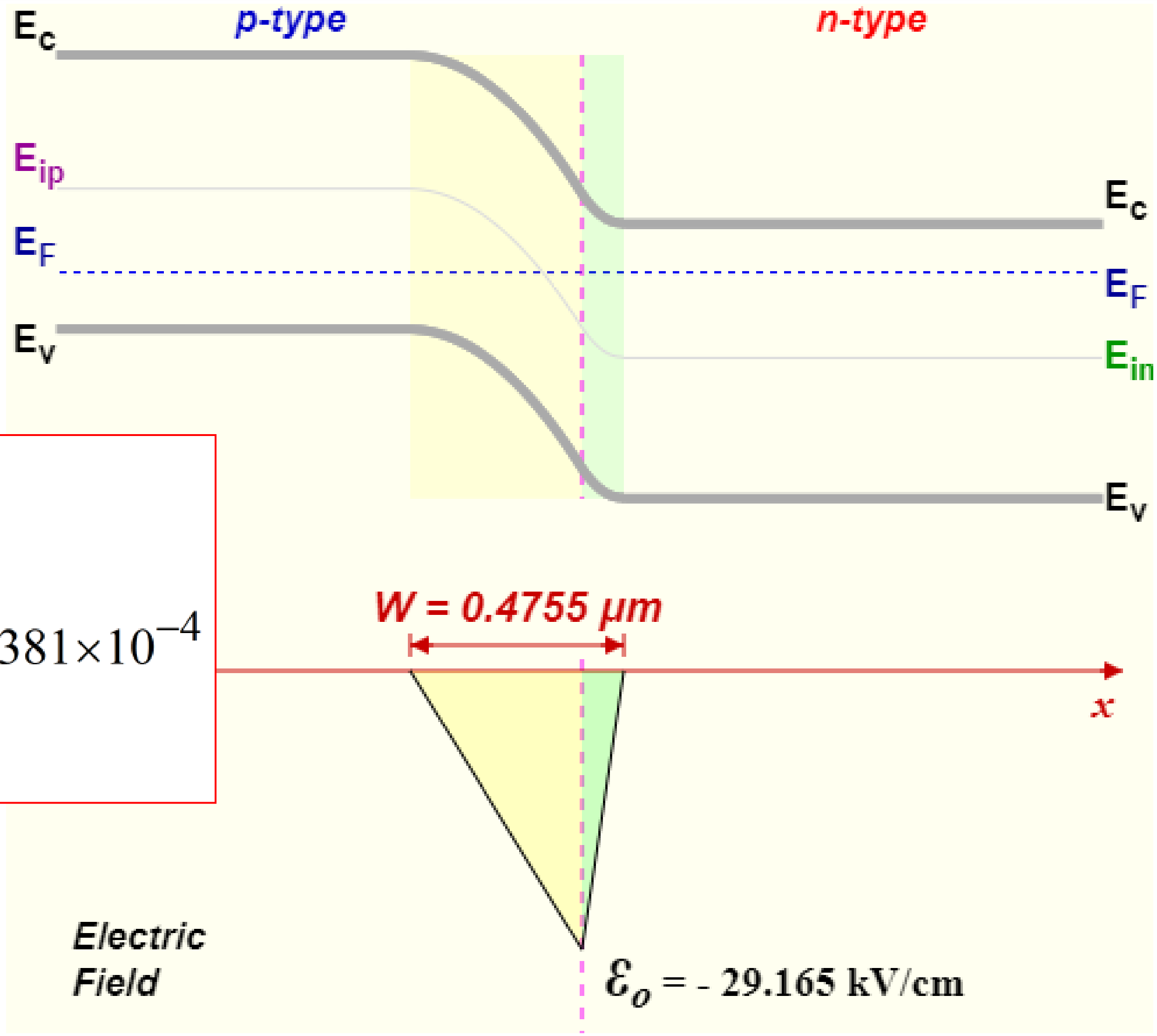
$$Q_n = qN_D\, x_{n0} = 1.6\times10^{-19}\times2\times10^{16}\times0.095\times10^{-4} = 3.04\times10^{-8}\ \text{C cm}^{-2}$$

Example 2

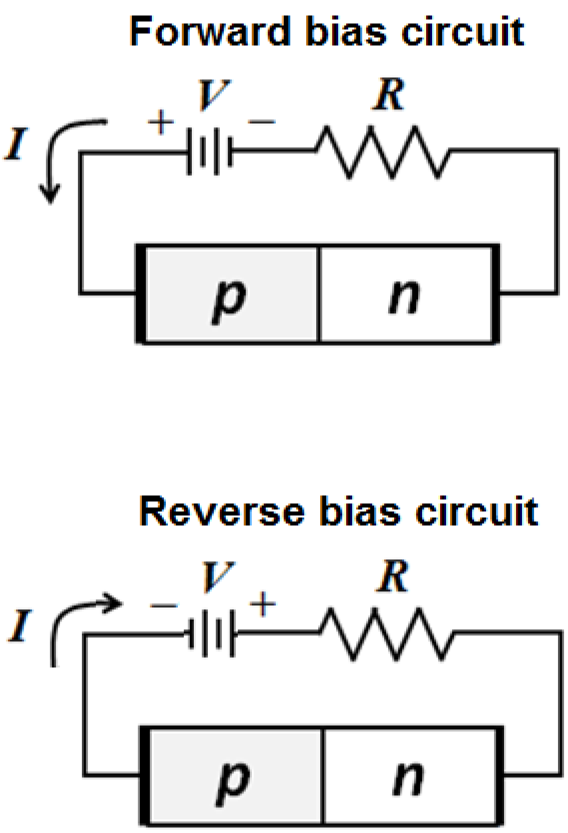
$$N_D = 2\times10^{16}\,\text{cm}^{-3}$$

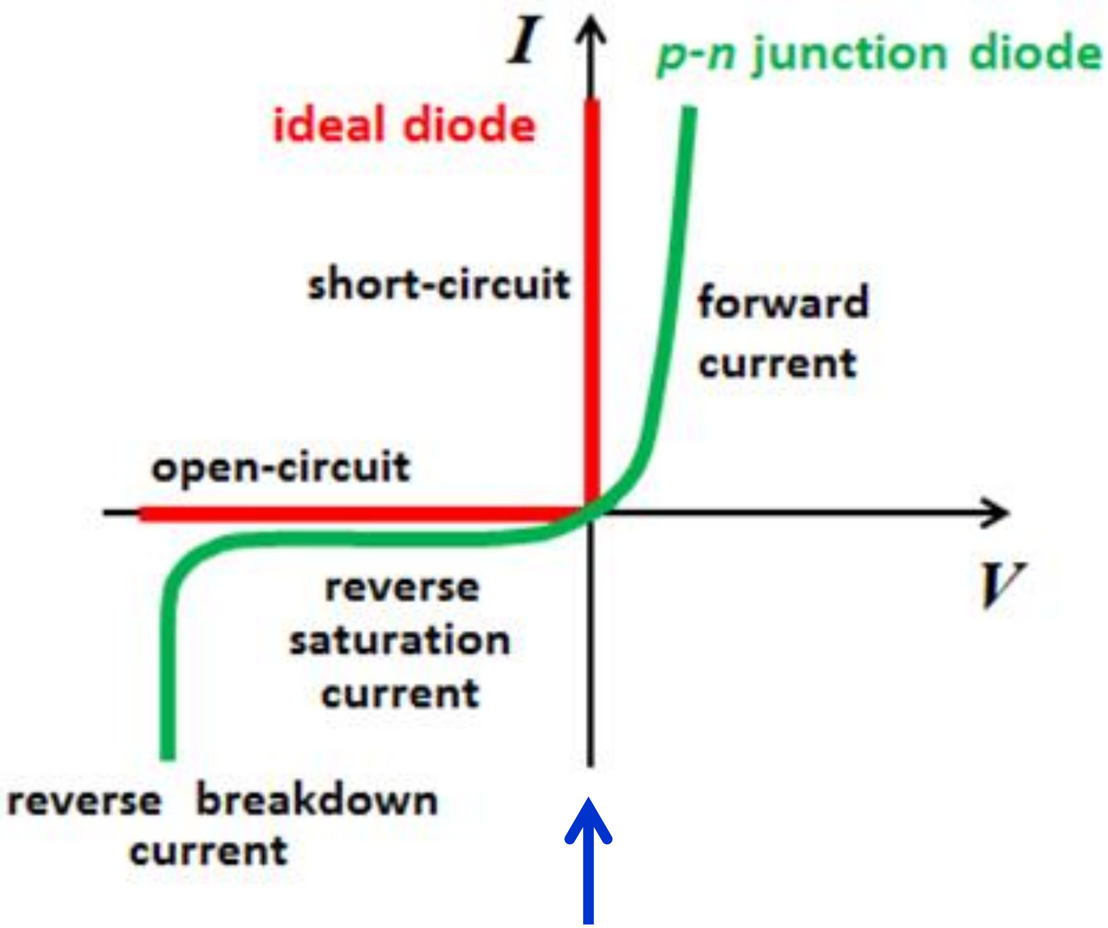
$$N_A = 5\times10^{15}\,\text{cm}^{-3}$$

$$\mathcal{E}_0 = -\frac{q}{\varepsilon}N_A\,x_{p0} = $$
$$= -\frac{1.6\times10^{-19}\times5\times10^{15}}{11.8\times8.85\times10^{-14}}\times0.381\times10^{-4}$$
$$= -29.2\ \text{kV/cm}$$



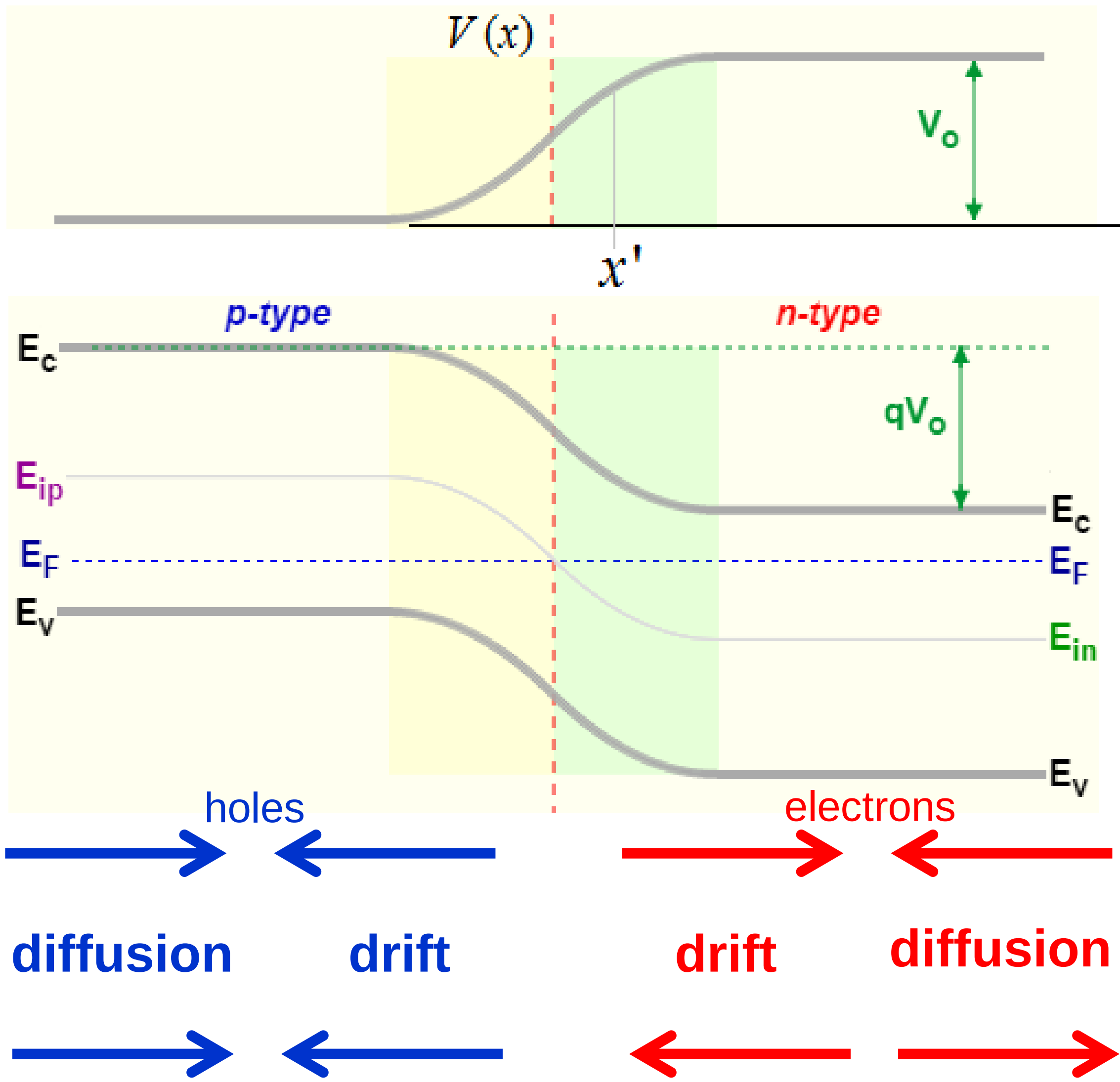
p-n junction *I-V* curve





equilibrium

p-n junction in *equilibrium*



$$-qV(x) = \mathbf{E}$$

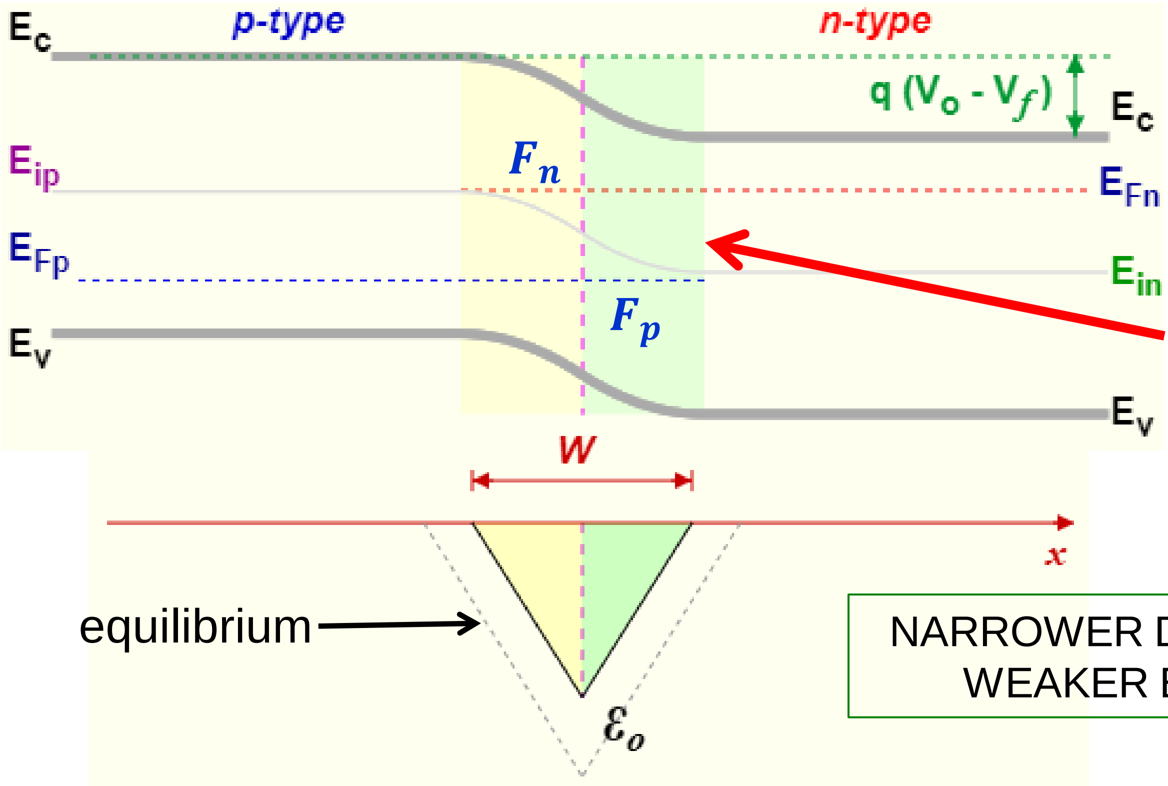
PARTICLE FLOW

net current = 0

CURRENT

p-n junction in *forward bias*

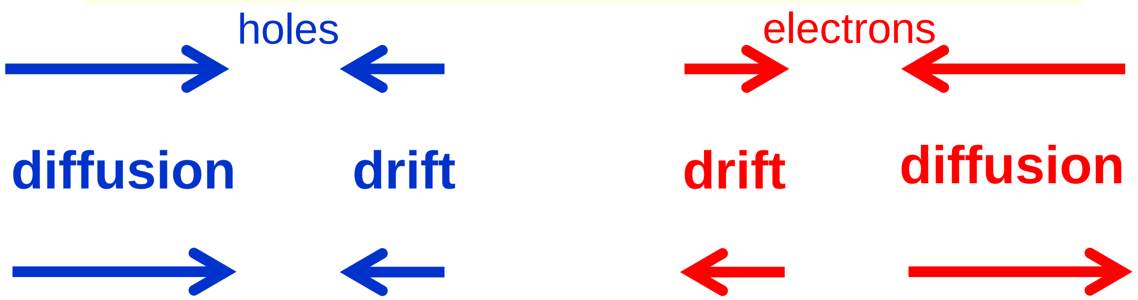
LOWER BARRIER



BIAS REDUCES CONTACT POTENTIAL

What about the details of quasi-Fermi levels?

NARROWER DEPLETION REGION
WEAKER ELECTRIC FIELD



PARTICLE FLOW

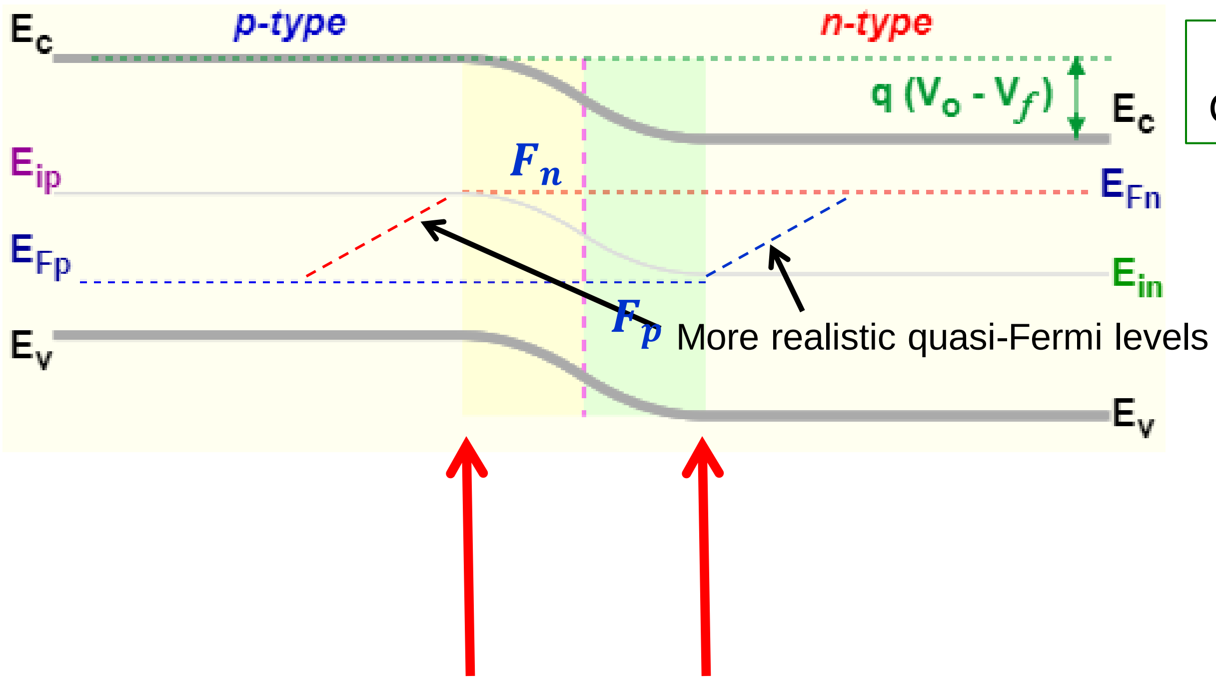
CURRENT

Limits of depletion approximation

- **The depletion approximation is sufficient to evaluate the electrostatic behavior of the junction reasonably well.**
- **However, it is insufficient to determine current flow.**
- ***No mobile charge* inside the depletion region would imply that there is never current flow.**

p-n junction in *forward bias*

LOWER BARRIER

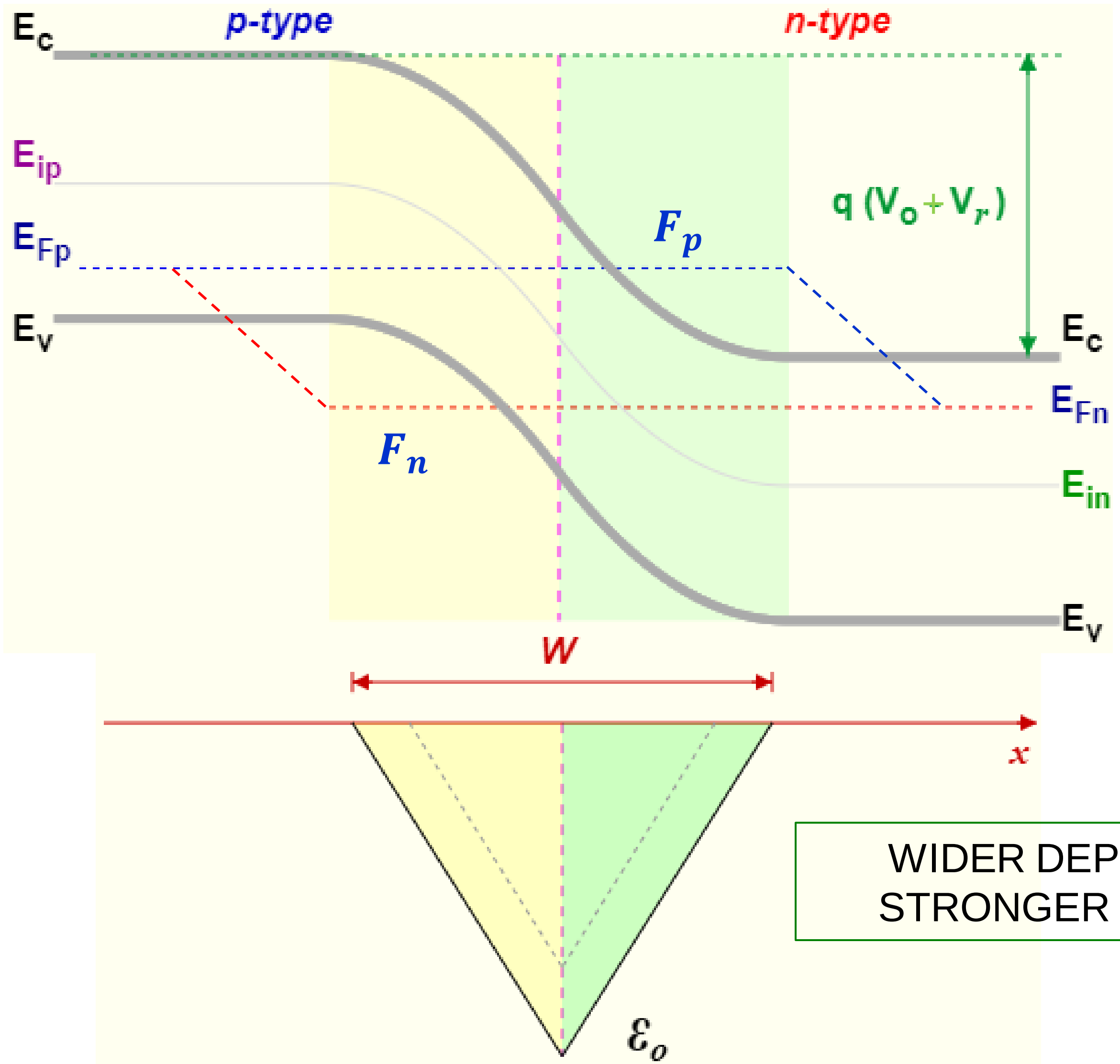


BIAS REDUCES CONTACT POTENTIAL

The behavior of the quasi-Fermi levels indicates that there is excess minority carrier concentration at the “boundaries” of the depletion region.

p-n junction in *reverse bias*

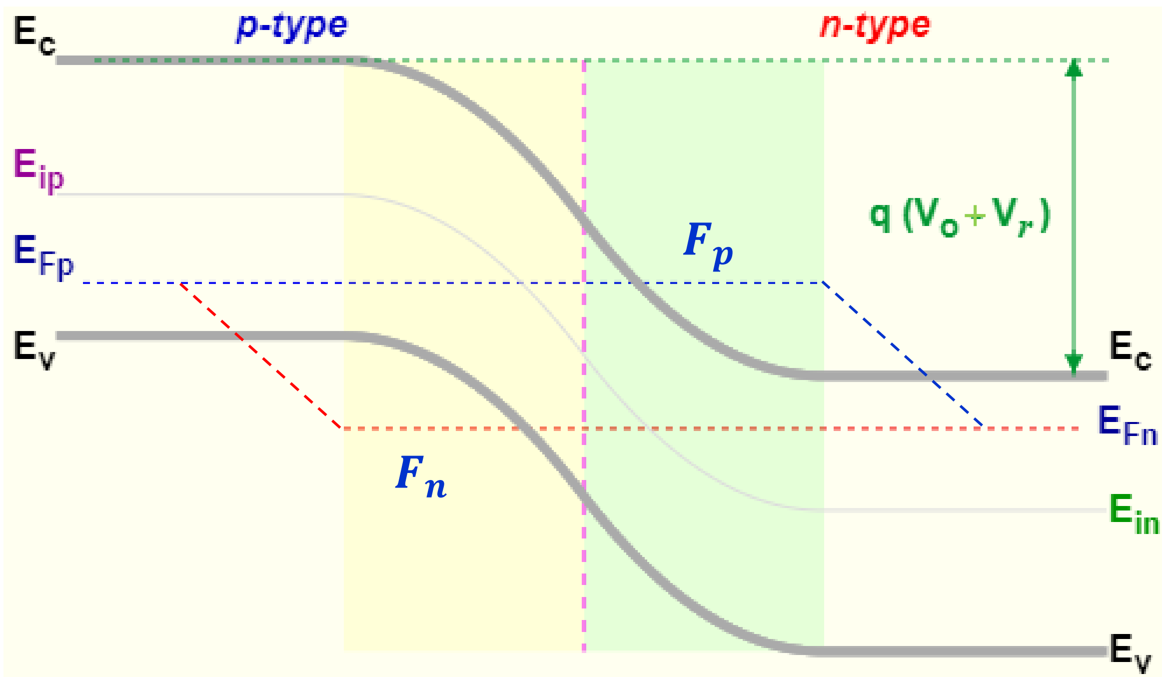
HIGHER BARRIER



BIAS INCREASES CONTACT POTENTIAL

WIDER DEPLETION REGION
STRONGER ELECTRIC FIELD

p-n junction in *reverse bias*



holes → ← | electrons → ←

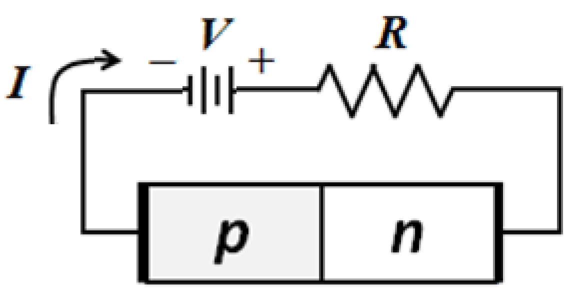
PARTICLE FLOW

diffusion **drift** **drift** **diffusion**

→ ← ← →

CURRENT

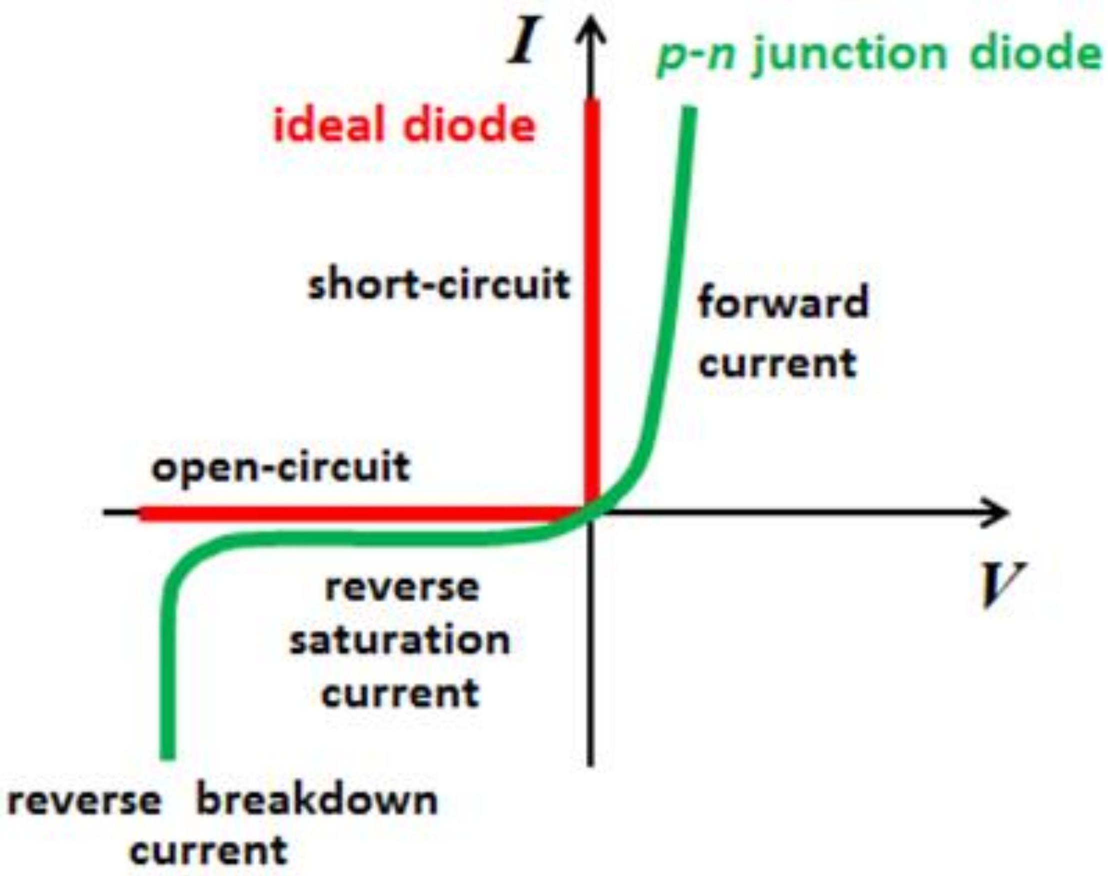
p-n junction in *reverse bias*



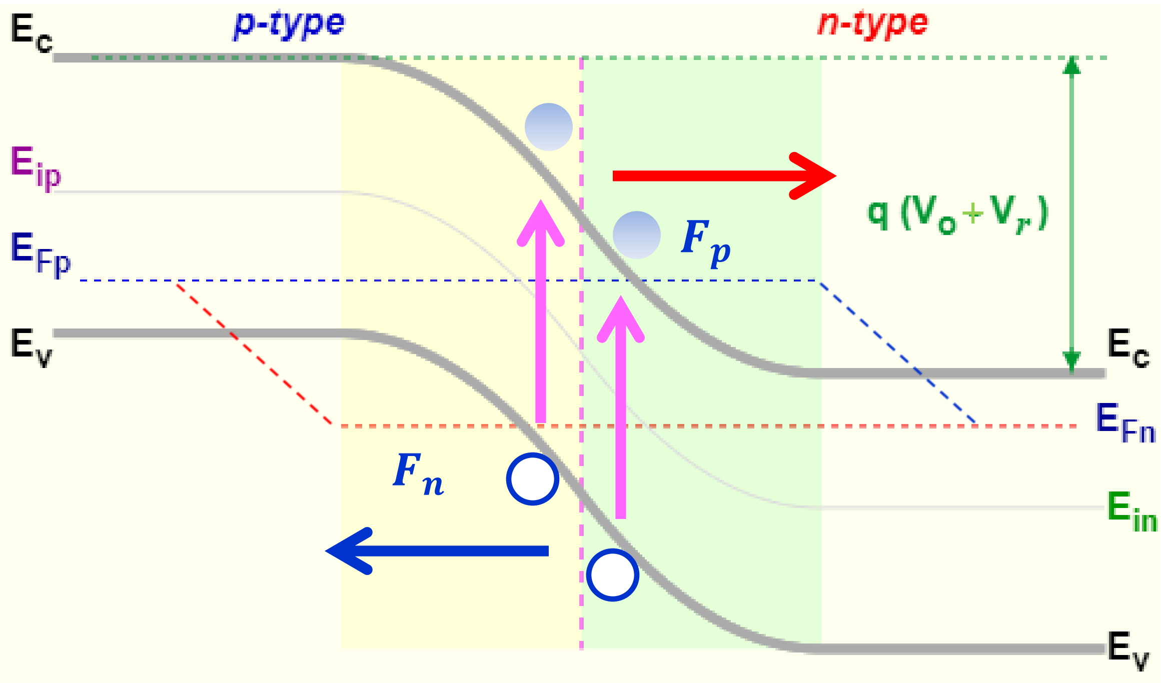
Reverse saturation current

$$I = I_0$$

Where does this current come from?
Why is it constant?



p-n junction in *reverse bias*



The field sweeps away the carriers generated in the depletion region

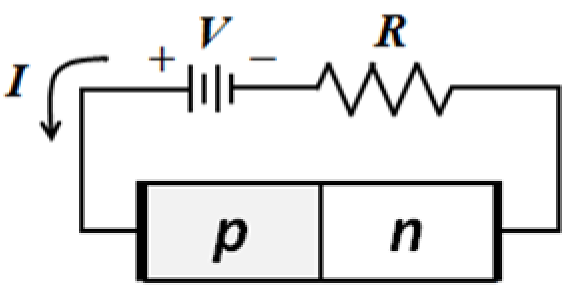
Thermal generation

At fixed temperature, thermal generation of EHP's is steady.
If the reverse bias is increased, carriers in the space charge region may be faster, but the number of available carriers per unit time stays the same.

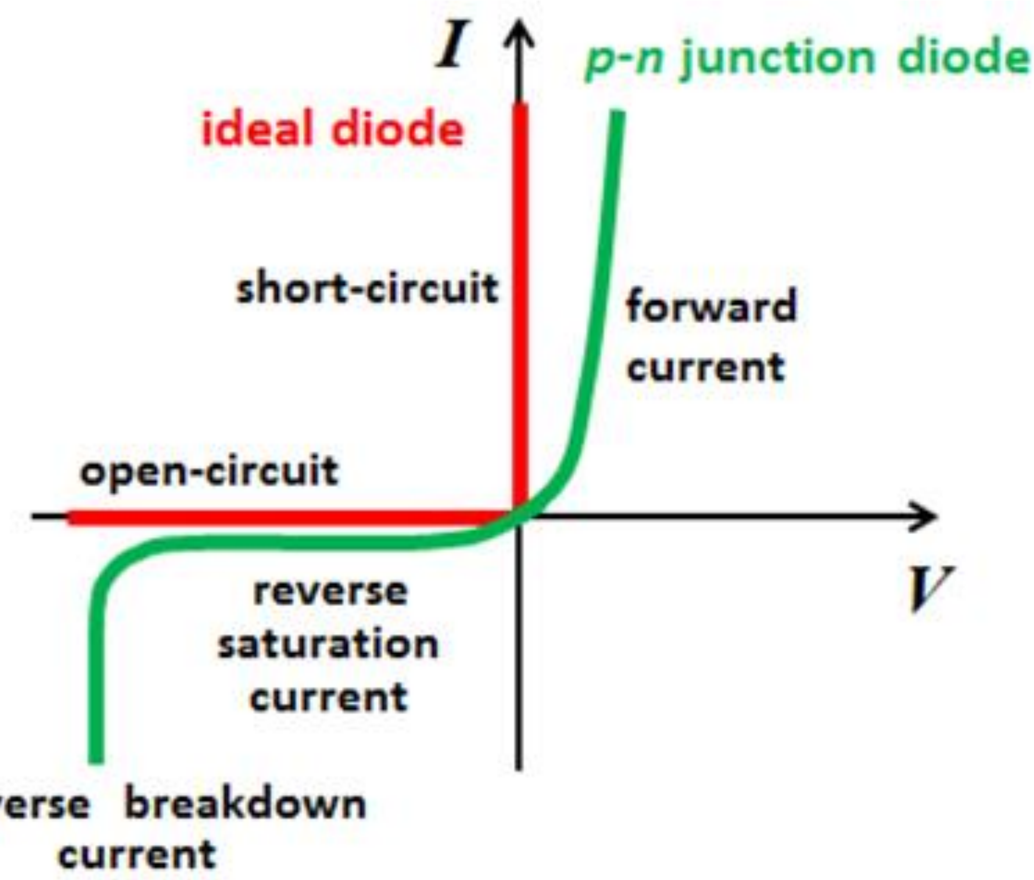
→ **The reverse saturation current remains constant.**

p-n junction in *forward bias*

Forward bias circuit



$$I = I_0\left(e^{qV/k_BT} - 1\right)$$
$$= I_0\, e^{qV/k_BT} - I_0$$



Thermal generation current is always there but it becomes negligible for $V \gg k_BT/q$