ECE 340 Lecture 23
Semiconductor Electronics

Spring 2022
10:00-10:50am
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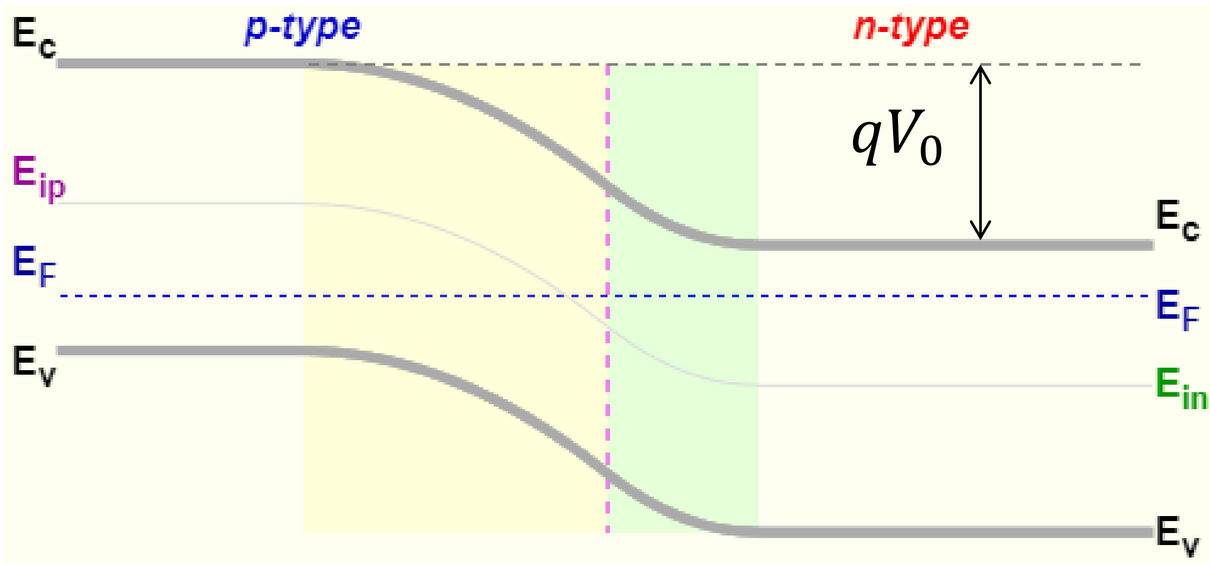
Department of Electrical and Computer Engineering
2062 ECE Building

Today’s Discussion

- **The p-n junction out of equilibrium - 2**

Asymmetric doping: $N_D > N_A$ (equilibrium)

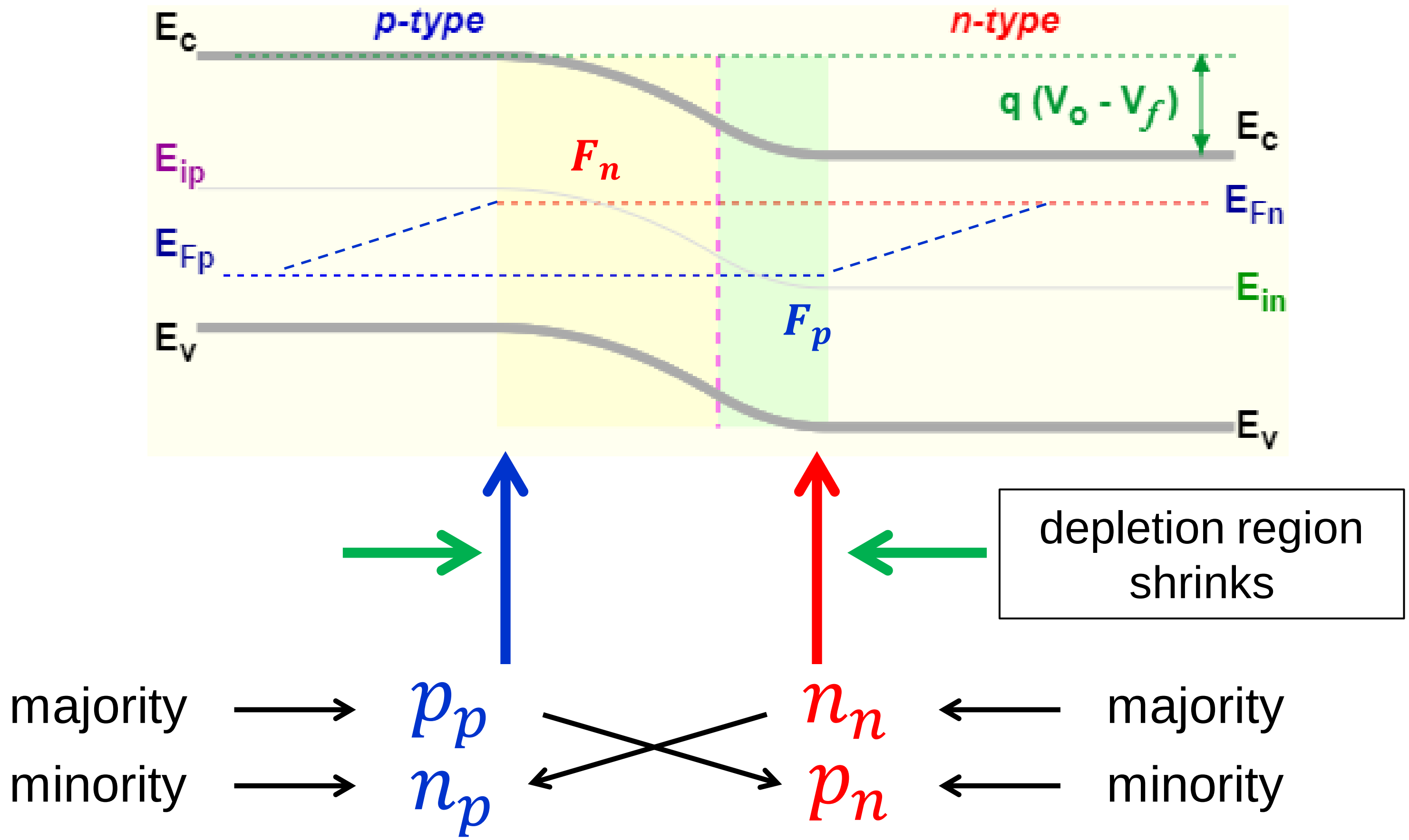


majority → p_p n_n ← majority

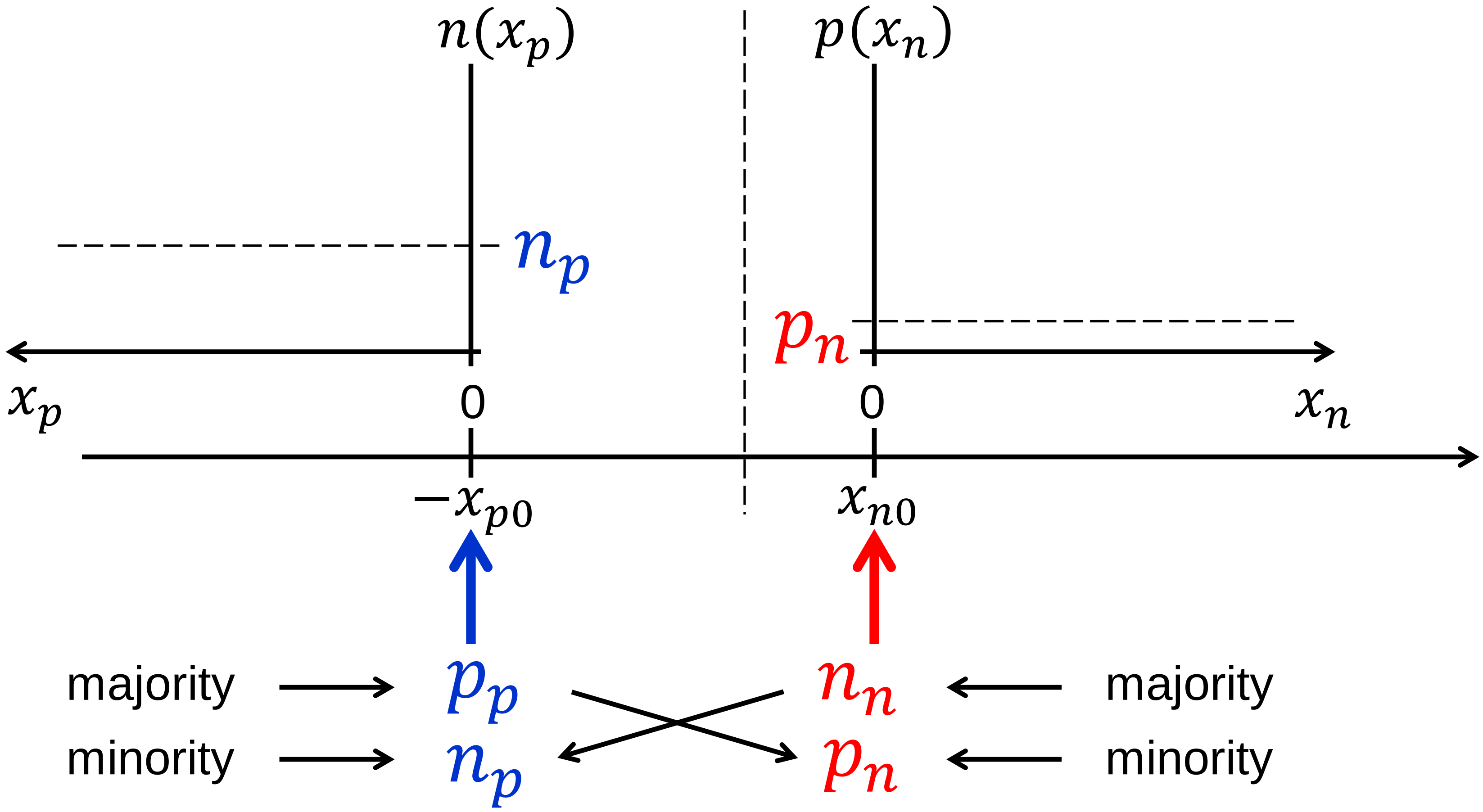
minority → n_p p_n ← minority

$$\frac{p_p}{p_n} = \exp\left(\frac{qV_0}{k_B T}\right) \qquad \frac{n_n}{n_p} = \exp\left(\frac{qV_0}{k_B T}\right)$$

Asymmetric doping: $N_D > N_A$ (forward bias)

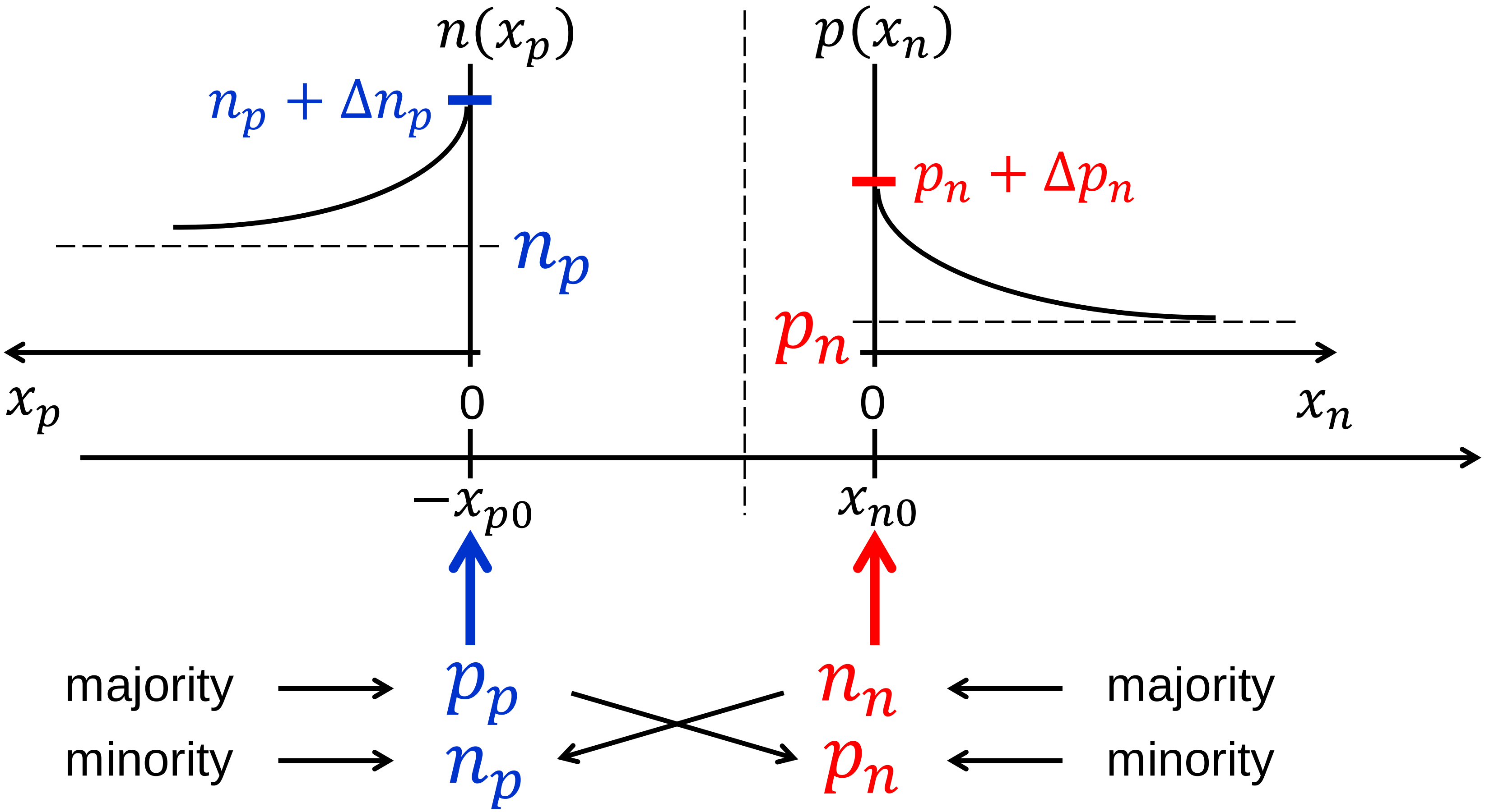


Asymmetric doping: $N_D > N_A$ (forward bias)



Reference minority carrier densities up to the edges of the depletion layer

Asymmetric doping: $N_D > N_A$ (forward bias)



Excess carriers as boundary conditions for the two sides

→ We do not model in detail the space-charge region

Asymmetric doping: $N_D > N_A$ (forward bias)

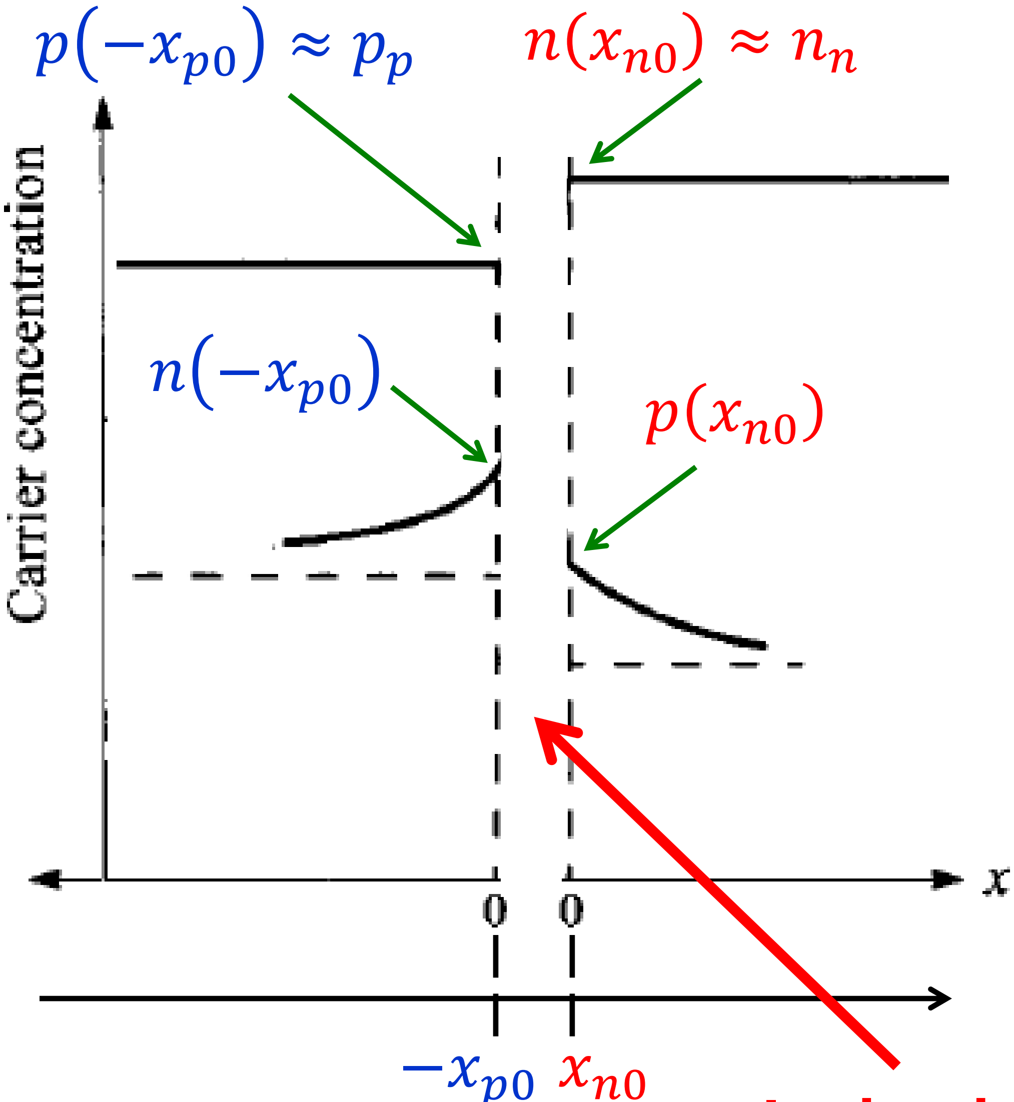
If we can determine the boundary conditions for minority carriers at the edges of the quasi-neutral regions, then we can circumvent the need to model the space-charge (depletion) region in detail.

Let's zoom out to see the complete carrier picture and set up the boundary conditions.

Remember, in equilibrium we had:

$$\frac{p_p}{p_n} = \exp\left(\frac{qV_0}{k_B T}\right) \qquad \frac{n_n}{n_p} = \exp\left(\frac{qV_0}{k_B T}\right)$$

Approximate majority carriers at depletion layer boundary



bias

$$\frac{p(-x_{p0})}{p(x_{n0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

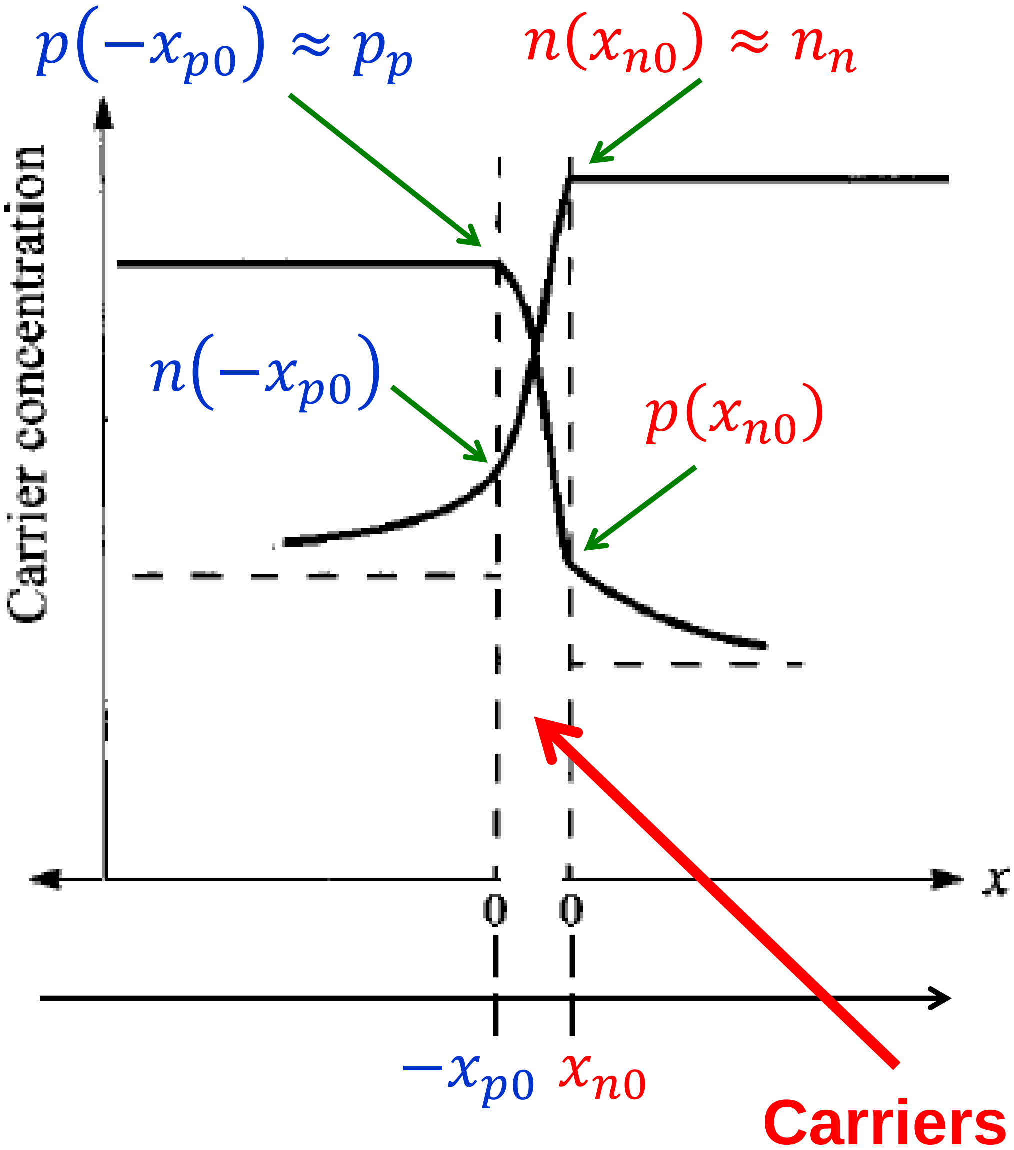
$$\frac{p_p}{p(x_{n0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

$$\frac{n(x_{n0})}{n(-x_{p0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

$$\frac{n_n}{n(-x_{p0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

In the depletion region?

Approximate majority carriers at depletion layer boundary



bias

$$\frac{p(-x_{p0})}{p(x_{n0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

$$\frac{p_p}{p(x_{n0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

$$\frac{n(x_{n0})}{n(-x_{p0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

$$\frac{n_n}{n(-x_{p0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$

Find excess hole density at depletion layer boundary - 1

We have the following expressions

$$\frac{p_p}{p_n} = \exp\left(\frac{qV_0}{k_B T}\right)$$ Equilibrium **A**

$$\frac{p_p}{p(x_{n0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$ Forward Bias **B**

$$\frac{\text{A}}{\text{B}} = \frac{p_p}{p_n}\frac{p(x_{n0})}{p_p} = \exp\left(\frac{qV_0}{k_B T}\right)\exp\left(\frac{q(V - V_0)}{k_B T}\right)$$

$$= \frac{p(x_{n0})}{p_n} = \exp\left(\frac{qV}{k_B T}\right)$$

Find excess hole density at depletion layer boundary - 2

$$\frac{p(x_{n0})}{p_n} = \exp\left(\frac{qV}{k_B T}\right) \quad \rightarrow \quad p(x_{n0}) = p_n \exp\left(\frac{qV}{k_B T}\right)$$

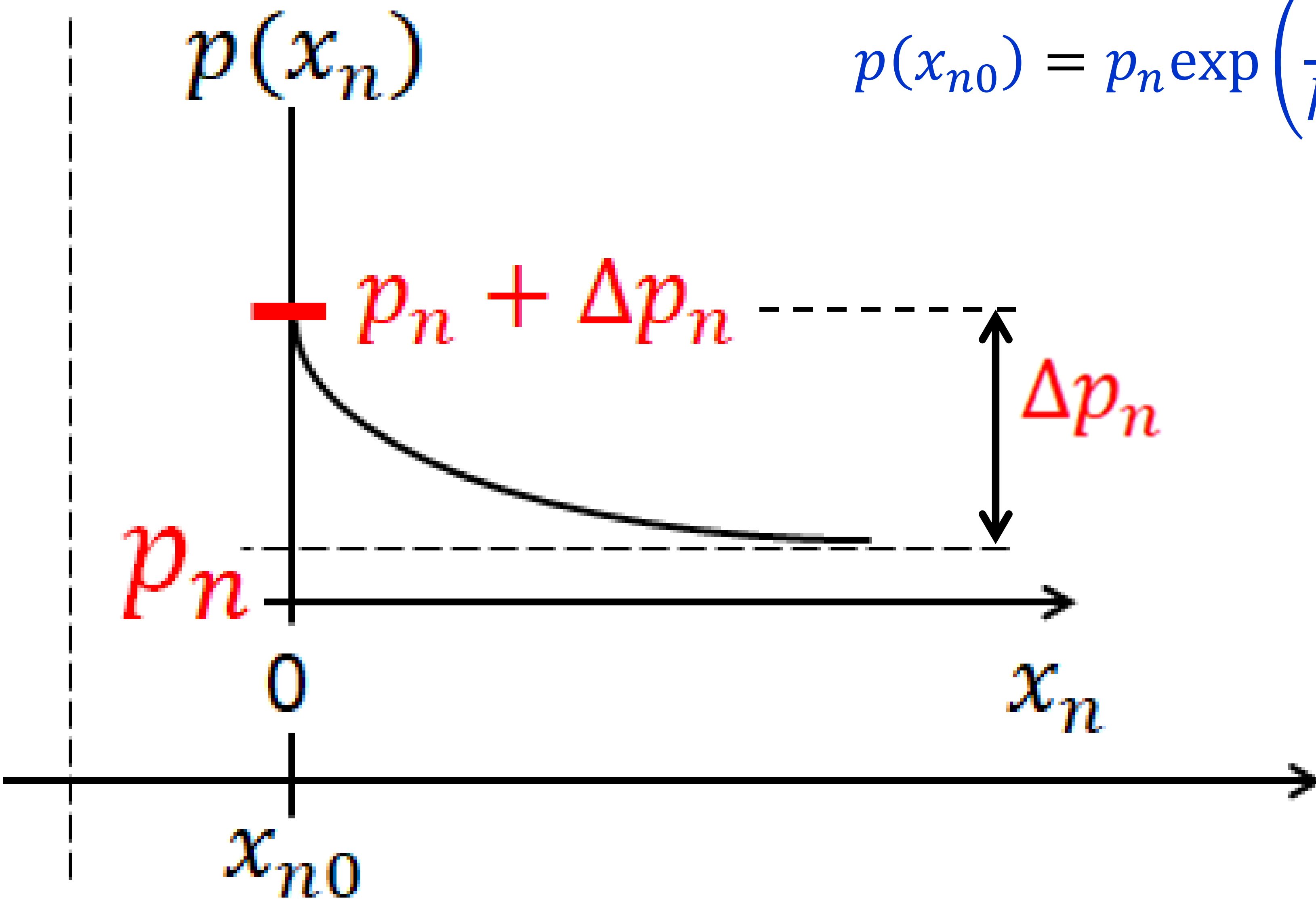
$$p(x_{n0}) = p_n + \Delta p_n = p_n \exp\left(\frac{qV}{k_B T}\right)$$

$$\Delta p_n = p_n \exp\left(\frac{qV}{k_B T}\right) - p_n = p_n \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

$$\Delta p_n = p_n \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

Find excess hole density at depletion layer boundary - 3

$$p(x_{n0}) = p_n \exp\left(\frac{qV}{k_B T}\right)$$



Find excess electron density at depletion layer boundary - 1

We have the following expressions

$$\frac{n_n}{n_p} = \exp\left(\frac{qV_0}{k_B T}\right)$$ Equilibrium (A)

$$\frac{n_n}{n(-x_{p0})} = \exp\left(\frac{q(V_0 - V)}{k_B T}\right)$$ Forward Bias (B)

$$\frac{(A)}{(B)} = \frac{n_n}{n_p}\frac{n(-x_{p0})}{n_n} = \exp\left(\frac{qV_0}{k_B T}\right)\exp\left(\frac{q(V - V_0)}{k_B T}\right)$$

$$= \frac{n(-x_{p0})}{n_p} = \exp\left(\frac{qV}{k_B T}\right)$$

Find excess electron density at depletion layer boundary - 2

$$\frac{n(-x_{p0})}{n_p} = \exp\left(\frac{qV}{k_B T}\right) \quad \rightarrow \quad n(-x_{p0}) = n_p \exp\left(\frac{qV}{k_B T}\right)$$

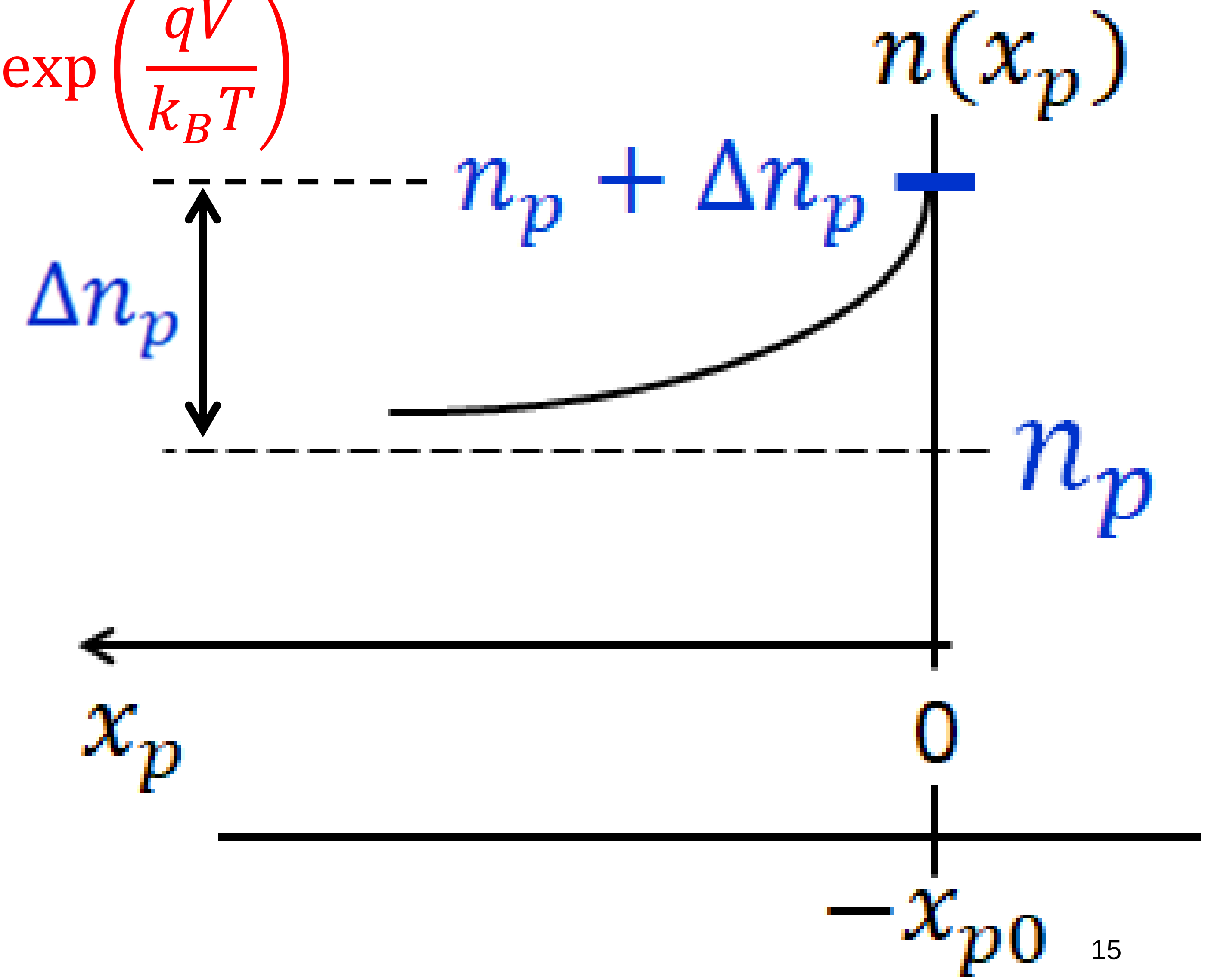
$$n(-x_{p0}) = n_p + \Delta n_p = n_p \exp\left(\frac{qV}{k_B T}\right)$$

$$\Delta n_p = n_p \exp\left(\frac{qV}{k_B T}\right) - n_p = n_p \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

$$\boxed{\Delta n_p = n_p \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]}$$

Find excess electron density at depletion layer boundary - 3

$$n(-x_{p0}) = n_p \exp\left(\frac{qV}{k_B T}\right)$$

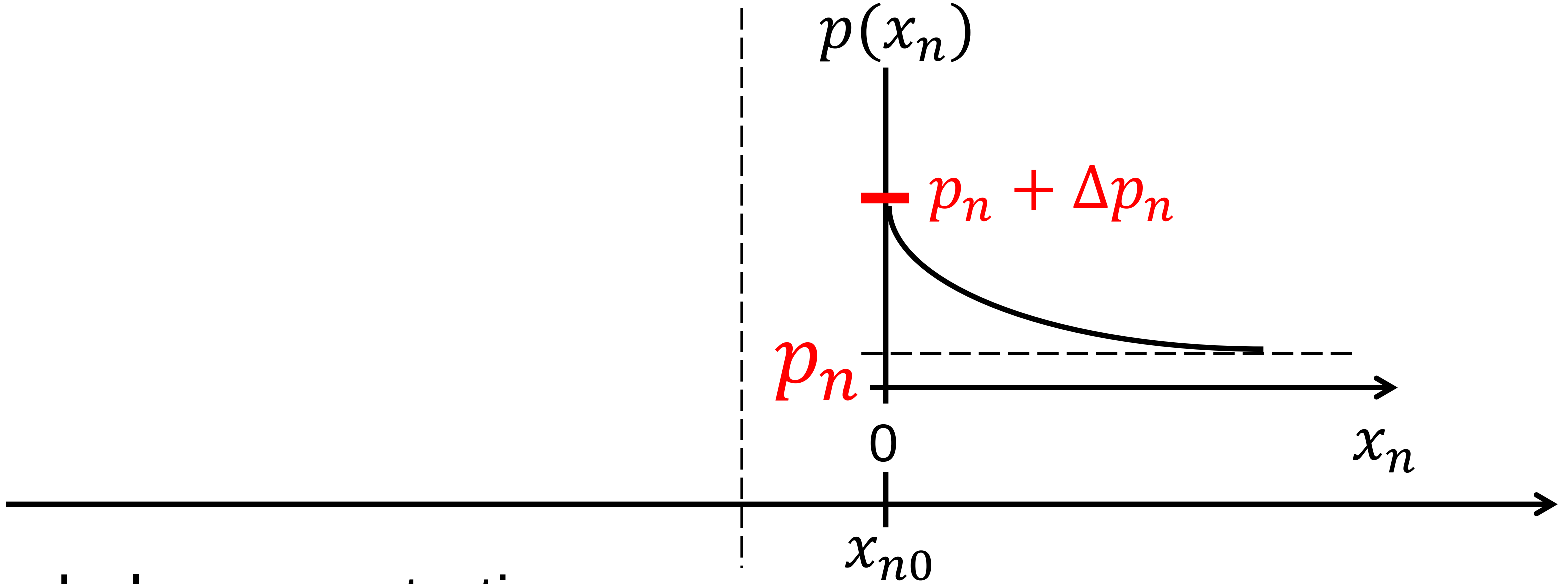


Minority carrier injection boundary conditions - Summary

$$\Delta p_n = p_n \left[\exp\left(\frac{qV}{k_B T} \right) - 1 \right]$$

$$\Delta n_p = n_p \left[\exp\left(\frac{qV}{k_B T} \right) - 1 \right]$$

Diffusion of minority hole in the n-region

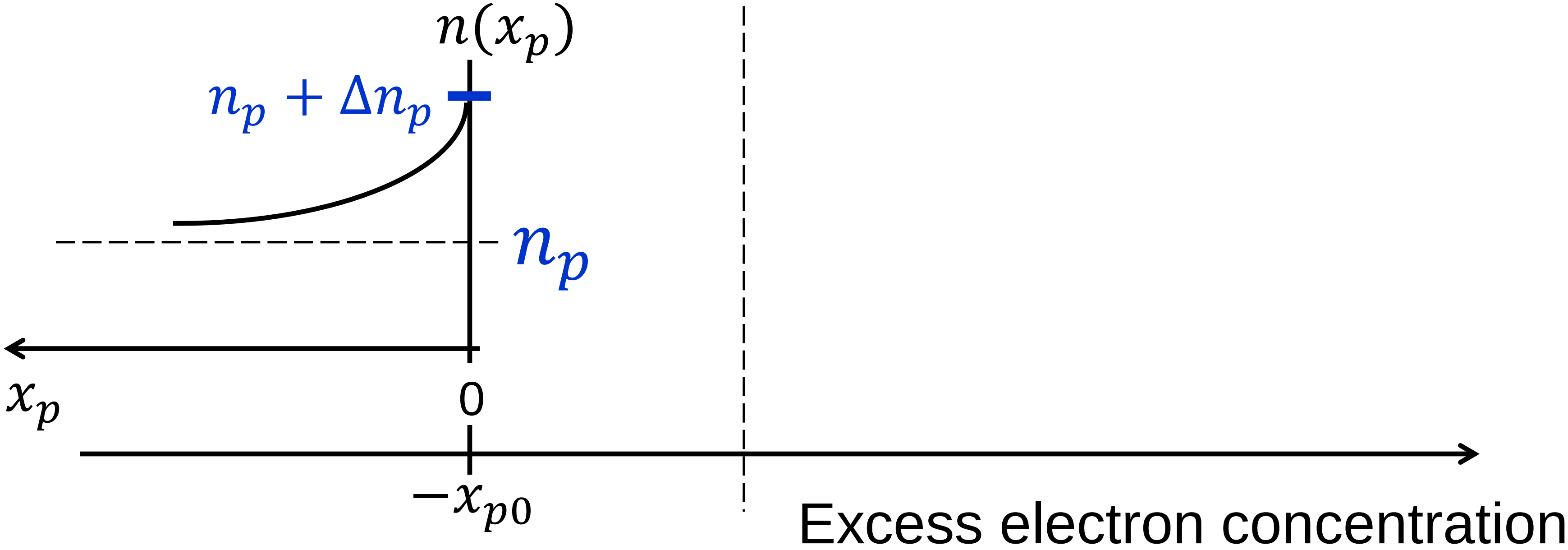


Excess hole concentration

$$\delta p(x_n) = \Delta p_n \exp\left(-\frac{x_n}{L_p}\right) =$$

$$= p_n \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right] \exp\left(-\frac{x_n}{L_p}\right)$$

Diffusion of minority electrons in the *p*-region



$$\delta n(x_p) = \Delta n_p \exp\left(-\frac{x_p}{L_n}\right) =$$

$$= n_p \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right] \exp\left(-\frac{x_p}{L_p}\right)$$

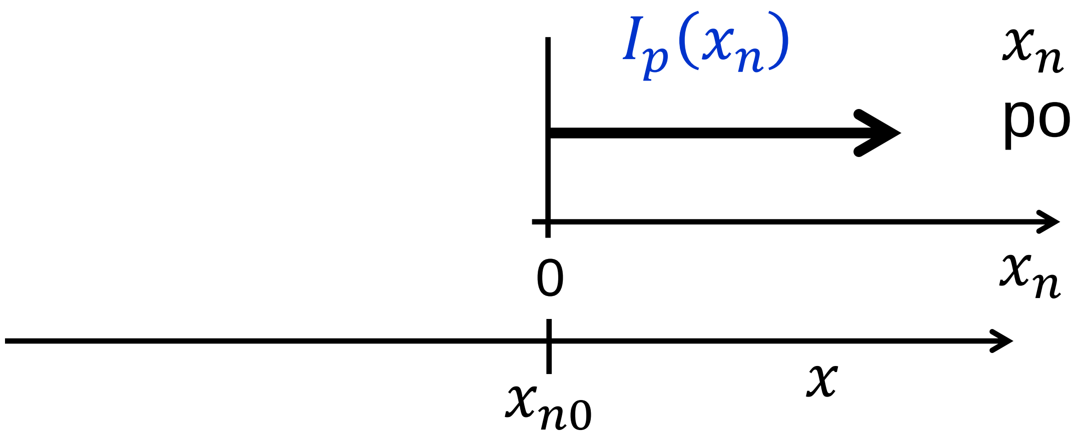
Diffusion Current of holes at any point in *n*-region

$$\delta p(x_n) = \Delta p_n \exp\left(-\frac{x_n}{L_p}\right)$$

Solution of diffusion equation

$$I_p(x_n) = -qAD_p \frac{d\delta p(x_n)}{dx_n} = qA\frac{D_p}{L_p}\delta p(x_n)$$

$$= qA\frac{D_p}{L_p}\Delta p_n \exp\left(-\frac{x_n}{L_p}\right) = qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]\exp\left(-\frac{x_n}{L_p}\right)$$



x_n coordinate along positive x.

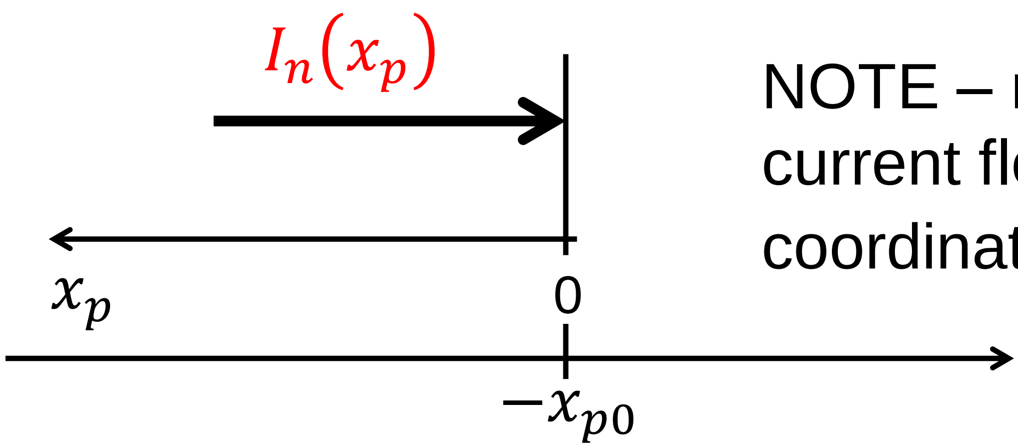
Diffusion Current of electrons at any point in *p*-region

$$\delta n(x_p) = \Delta n_p \exp\left(-\frac{x_p}{L_n}\right)$$

Solution of diffusion equation

$$I_n(x_p) = qAD_n \frac{d\delta n(x_p)}{dx_p} = -qA\frac{D_n}{L_n}\delta n(x_p)$$

$$= -qA\frac{D_n}{L_n}\Delta n_p \exp\left(-\frac{x_p}{L_n}\right) = -qA\frac{D_n}{L_n} n_p \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right] \exp\left(-\frac{x_p}{L_n}\right)$$



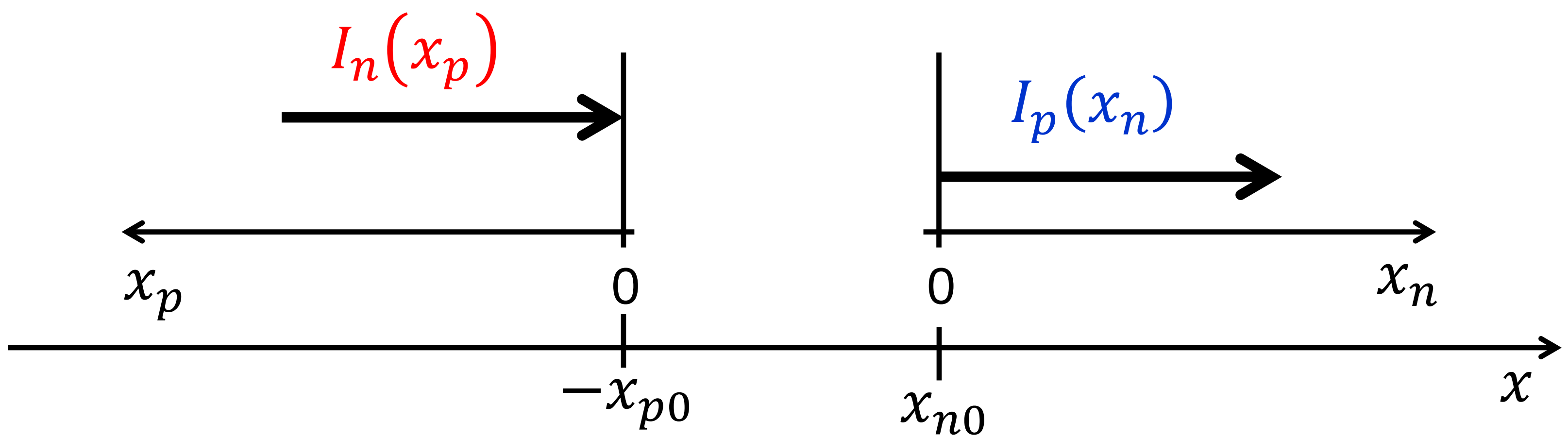
NOTE – negative sign means that current flow goes opposite to x_p coordinate along positive x.

Current components for electrons and holes

At the edges of the depletion region

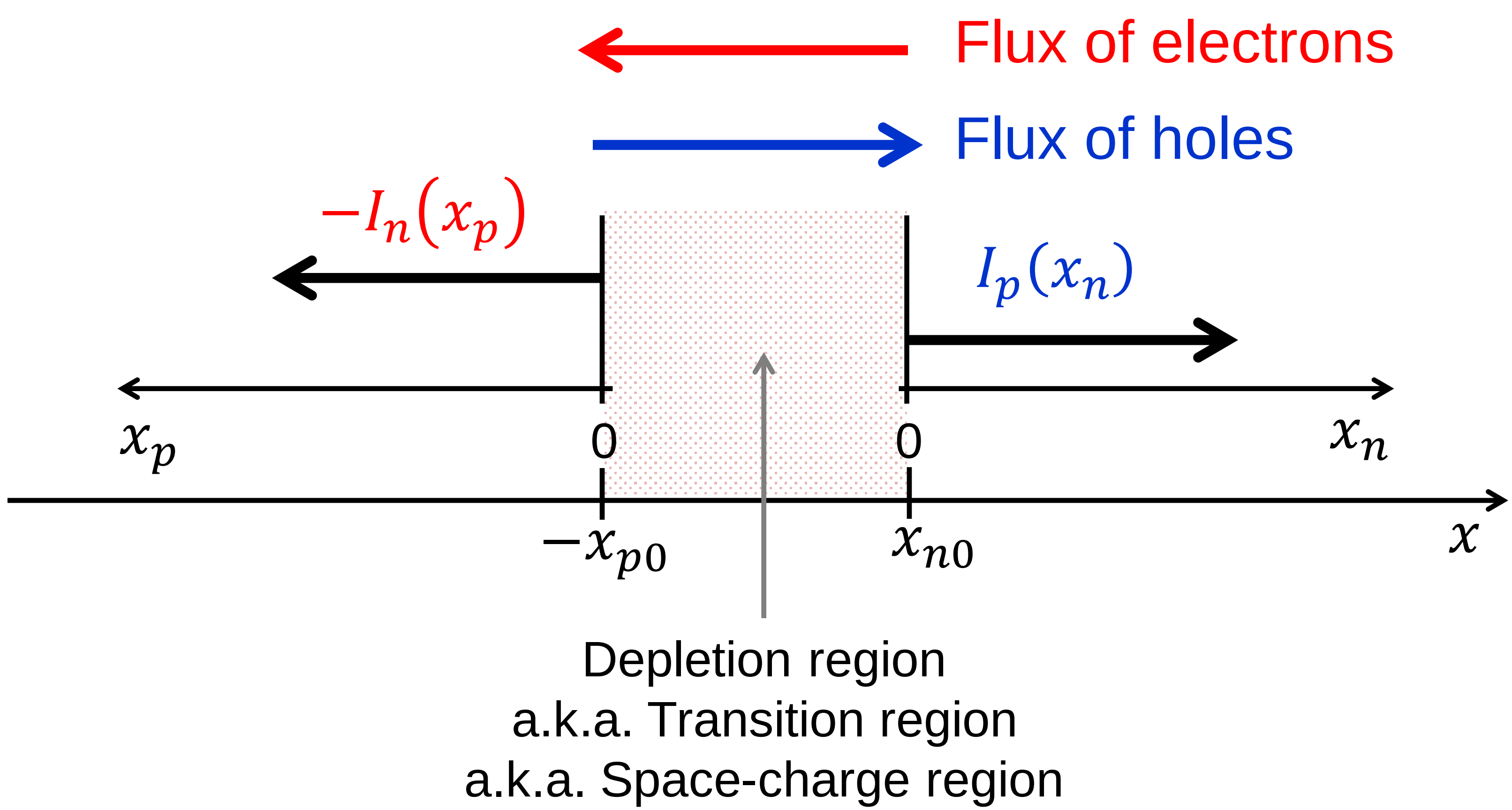
$$I_p(x_n = 0) = qA\frac{D_p}{L_p}\Delta p_n = qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$I_n(x_p = 0) = -qA\frac{D_n}{L_n}\Delta n_p = -qA\frac{D_n}{L_n}n_p\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$



Shockley's ideal diode approximation

- Neglect recombination in the depletion region
  - Holes that reach $-x_{p0}$ continue to x_{n0}.
  - Electrons that reach x_{n0} continue to $-x_{p0}$.



Current components of electrons and holes

$$I_p(x_n = 0) = qA\frac{D_p}{L_p}\Delta p_n = qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$I_n(x_p = 0) = -qA\frac{D_n}{L_n}\Delta n_p = -qA\frac{D_n}{L_n}n_p\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

Neglecting recombination in the depletion region (Shockley's approximation)

$$I = I_p(x_n = 0) - I_n(x_p = 0)$$

$$= qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] + qA\frac{D_n}{L_n}n_p\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

Ideal diode current equation

$$I = I_p(x_n = 0) - I_n(x_p = 0)$$

$$= qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] + qA\frac{D_n}{L_n}n_p\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$= \underbrace{qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)}_{I_0}\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$I = I_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$