# ECE 340 Lecture 31
# Semiconductor Electronics

Spring 2022
10:00-10:50am
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Department of Electrical and Computer Engineering
2062 ECE Building

# Today’s Discussion

- **Metal-Insulator-Semiconductor FET**
  - **MISFET or MOSFET**
- **MOS Capacitor**
  - **Flat band Voltage**
  - **Threshold Voltage**

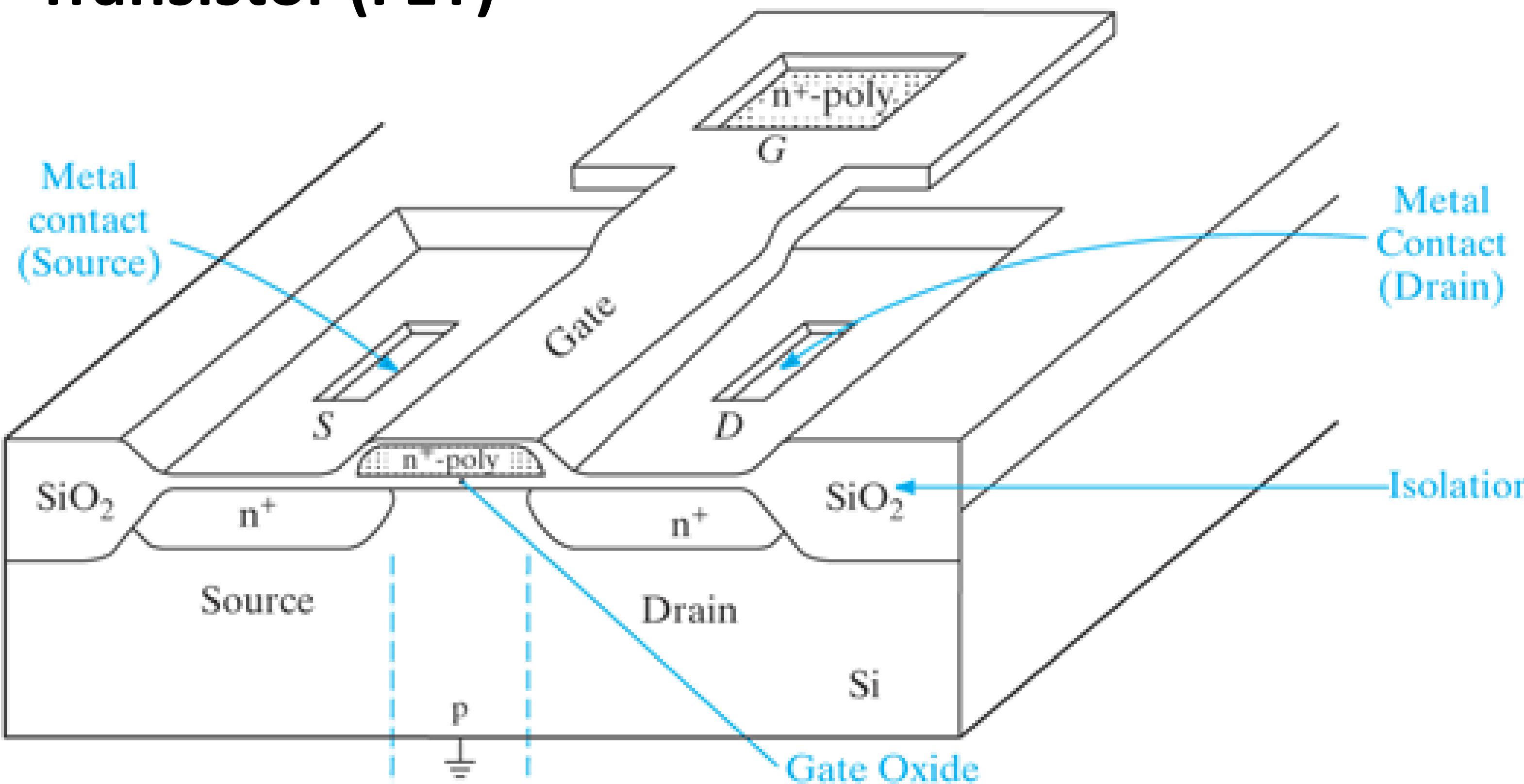
**NO LECTURE ON FRIDAY, APRIL 8**

**(ENGINEERING OPEN HOUSE)**

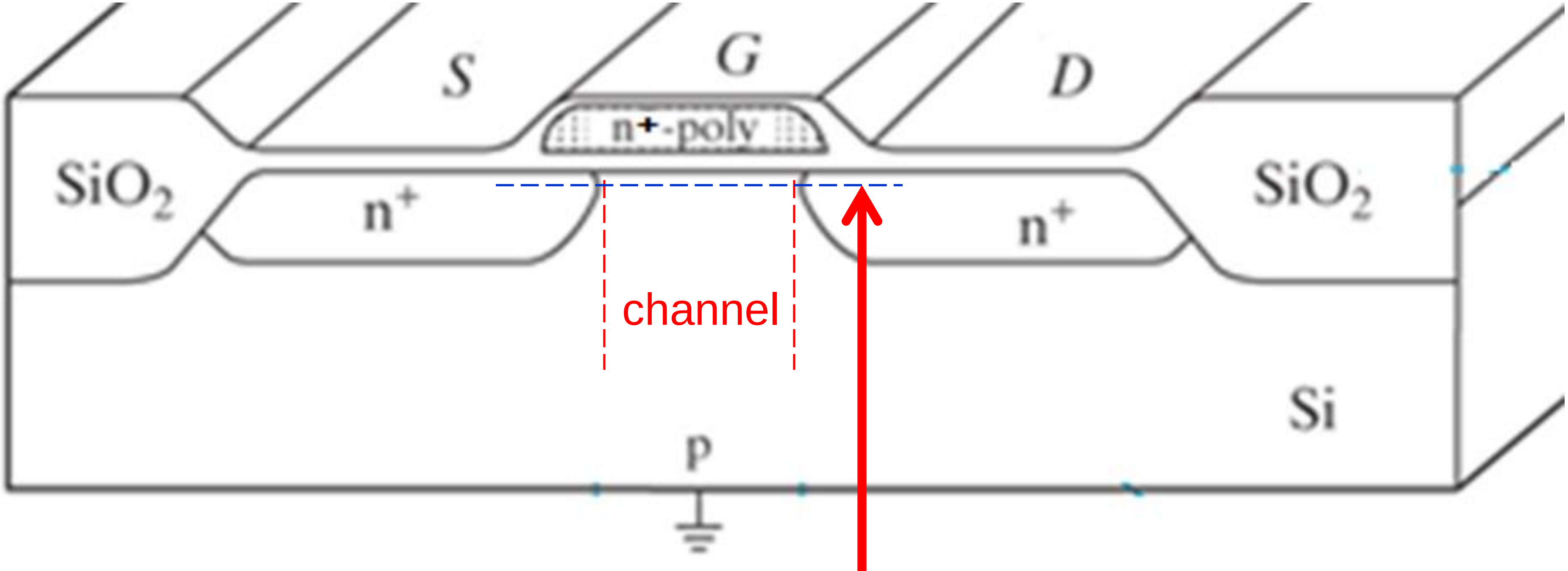
# MISFET or MOSFET

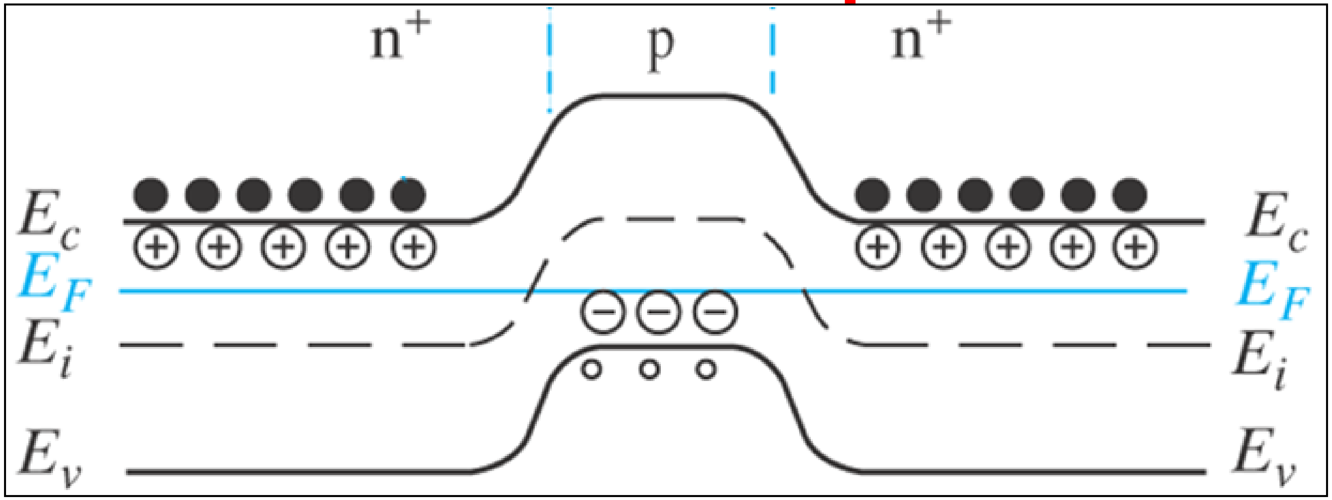
- **Metal-Insulator-Semiconductor (MIS) Field Effect Transistor (FET)**

  **or Metal-Oxide-Semiconductor (MOS) Field Effect Transistor (FET)**

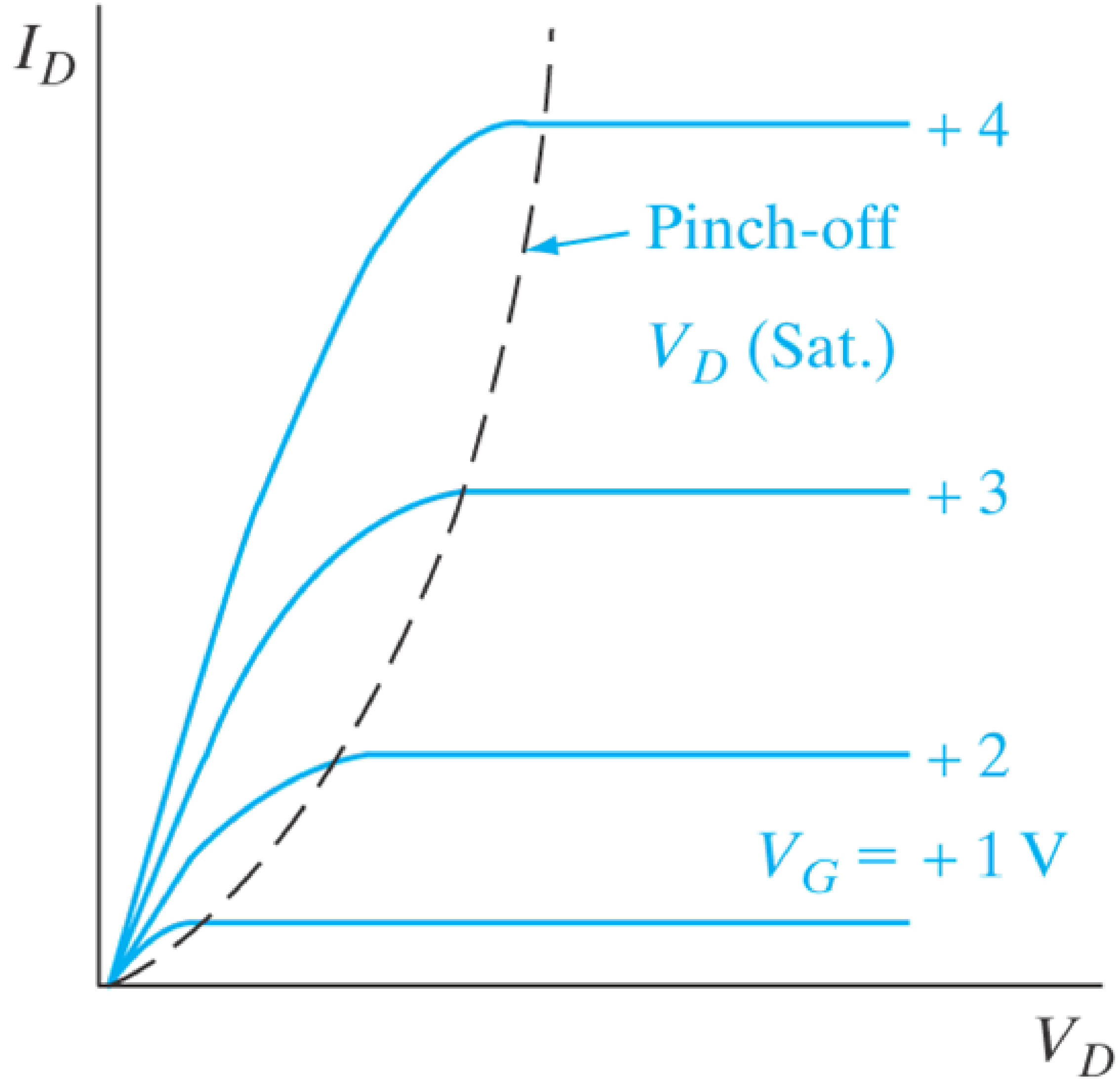


# MOSFET 2D cross-section

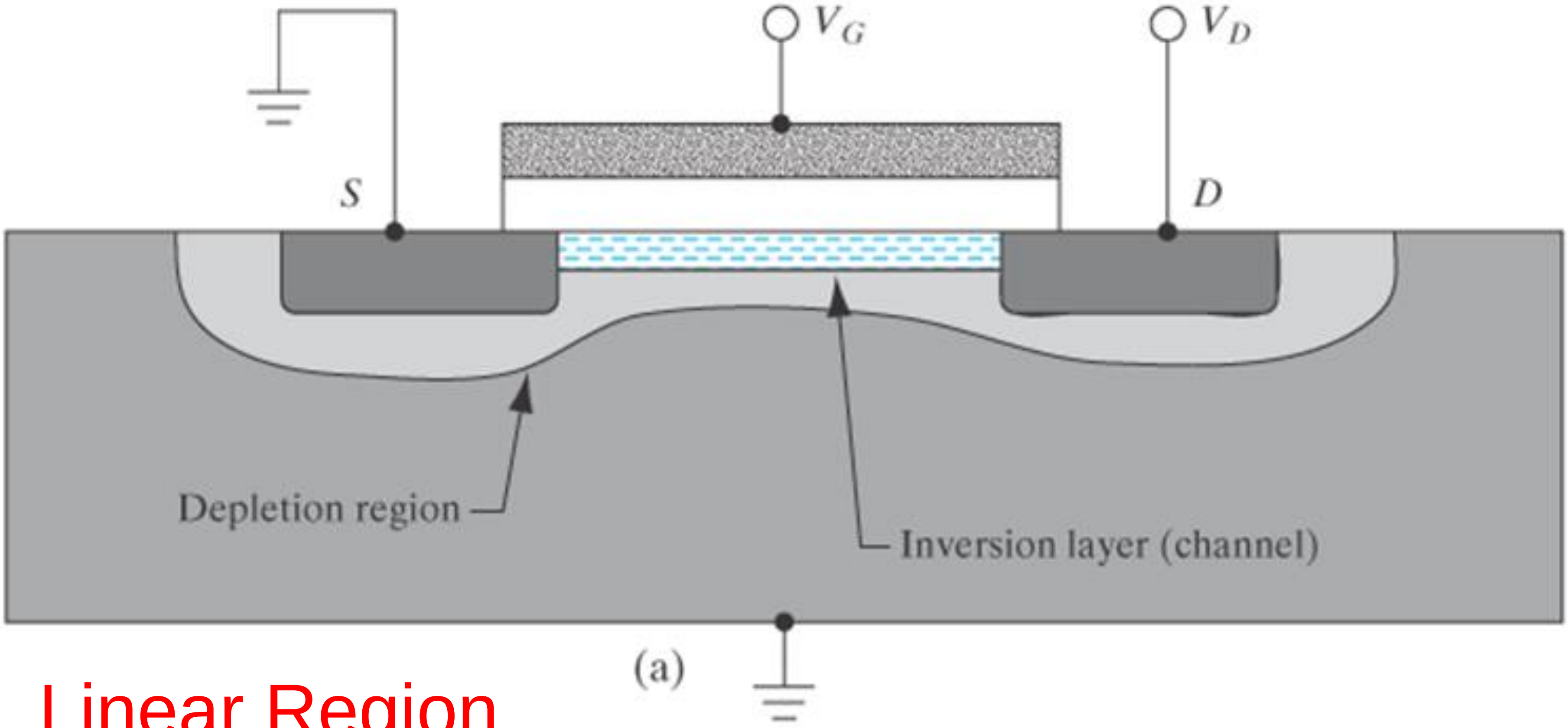


# MOSFET I-V curves



# MOSFET in linear region

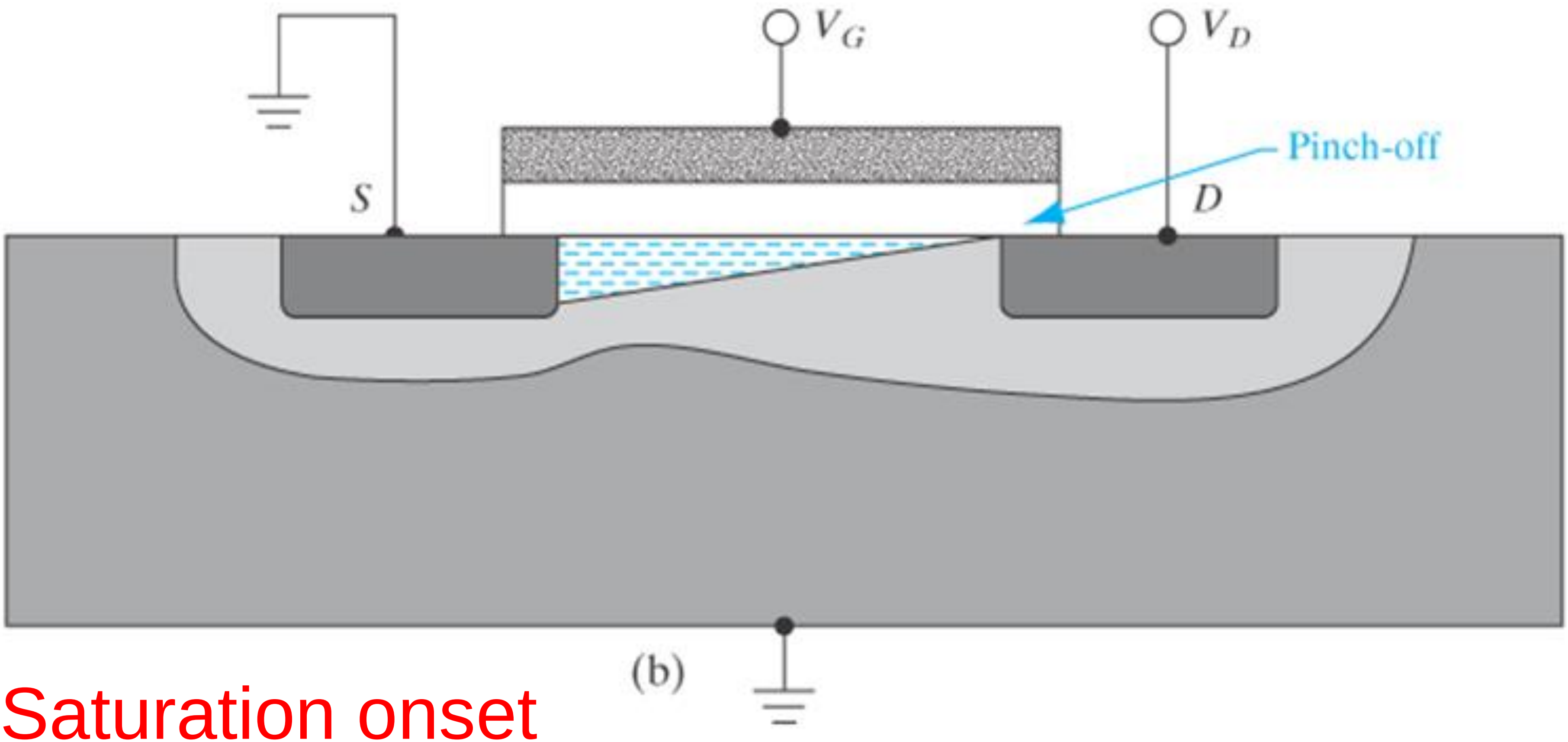



Linear Region

$$V_G > V_T$$

$$V_D < (V_G - V_T)$$

# MOSFET at pinch-off saturation onset

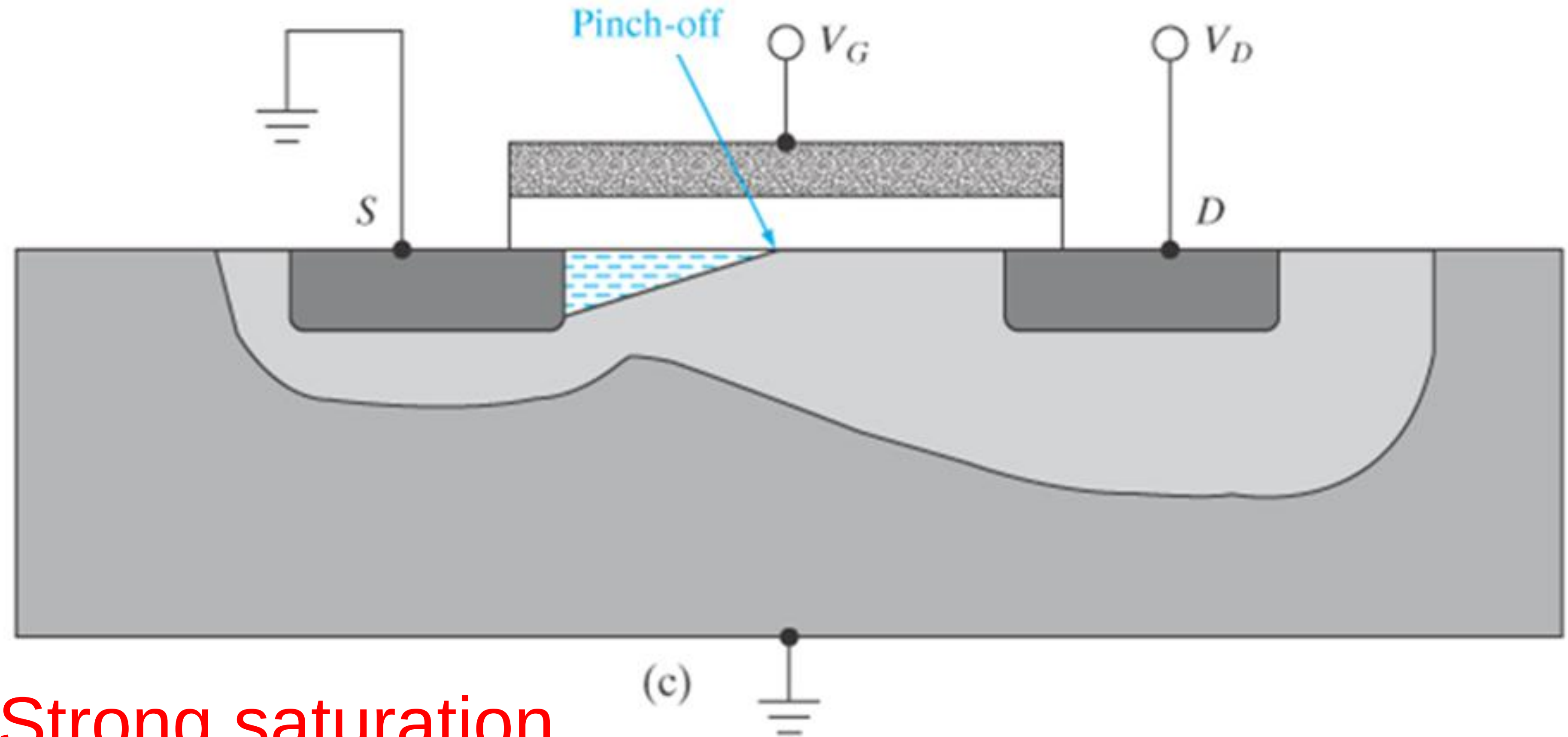



Saturation onset

$$V_G > V_T$$

$$V_D = (V_G - V_T)$$

# MOSFET in strong saturation



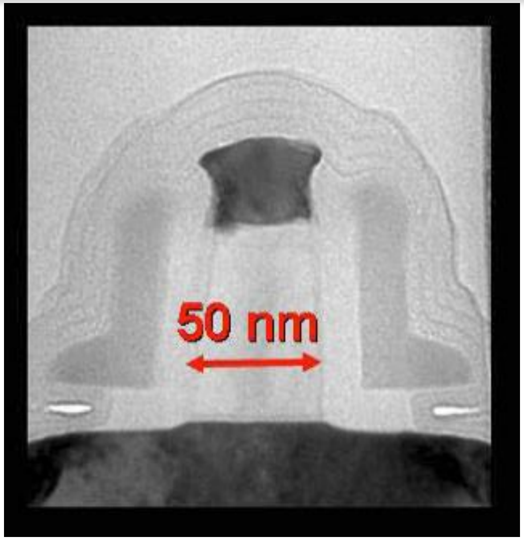

(c)

Strong saturation

$$V_G > V_T$$

$$V_D > (V_G - V_T)$$

# Modern scaled MOSFETs for digital applications

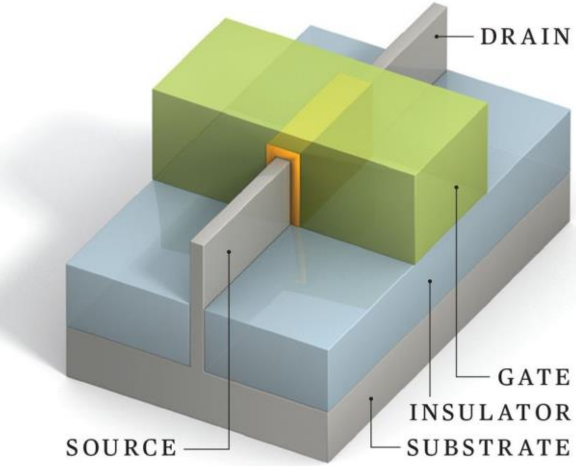



Intel (2005)

Micrograph of MOSFET cross-section for one of the last generations of Intel devices based on standard planar technology.

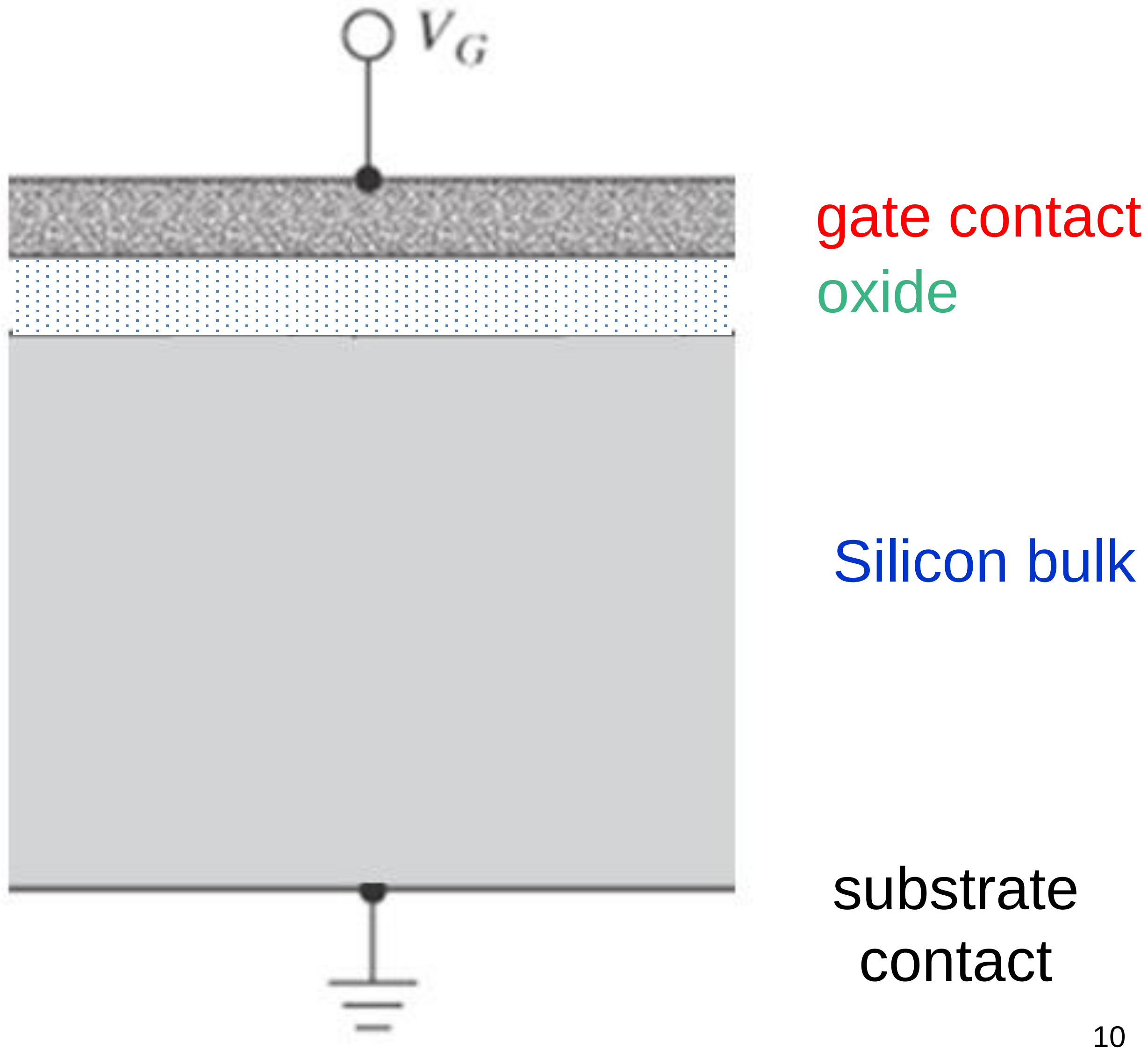
**FinFET** – the latest 3D embodiment of Intel MOS transistors which realizes a “double gate structure”, now down to 7nm channel length.

Intel (2012)

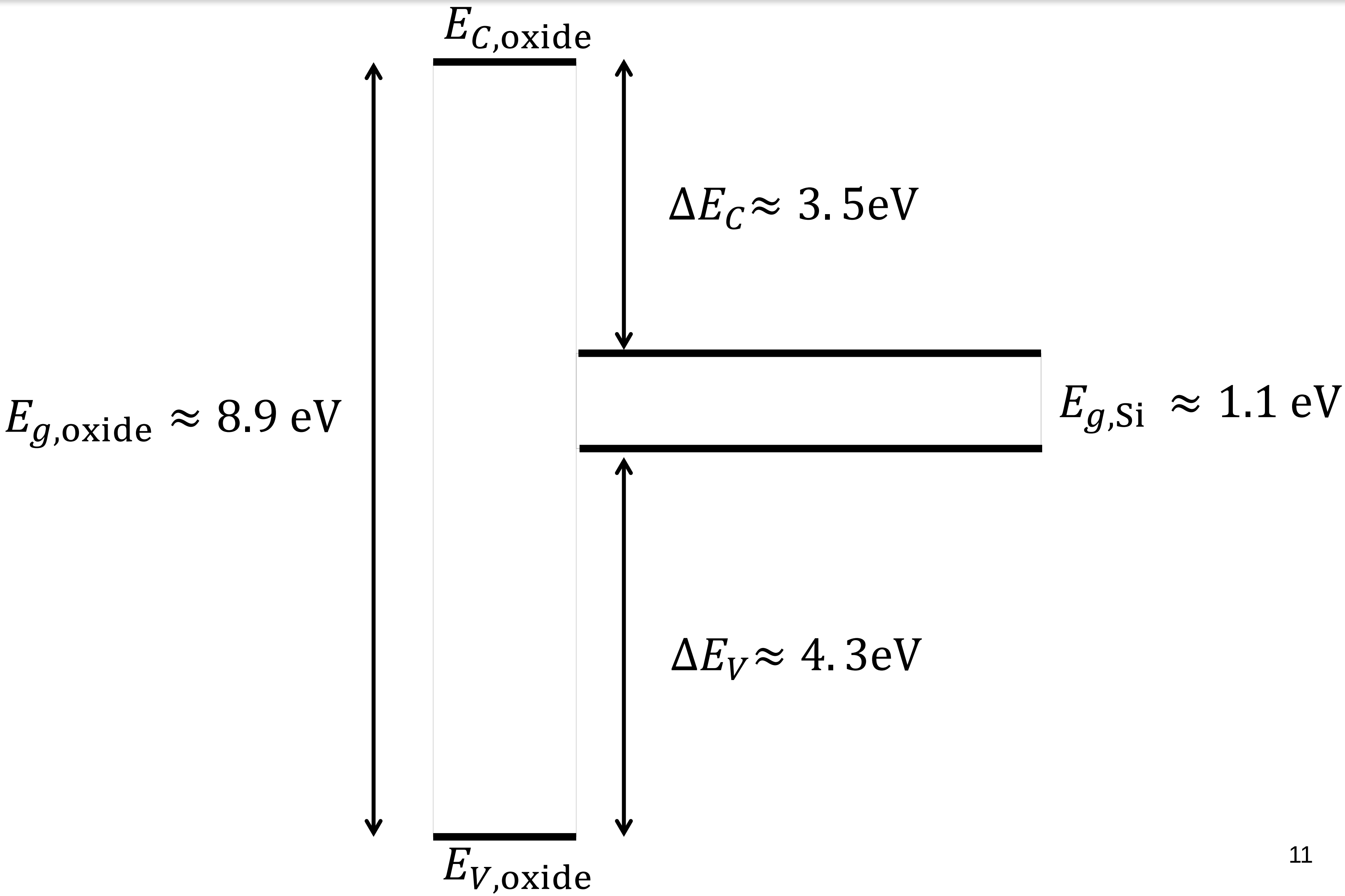



(IEEE Spectrum, April 2020)

# Ideal MOSFET Capacitor

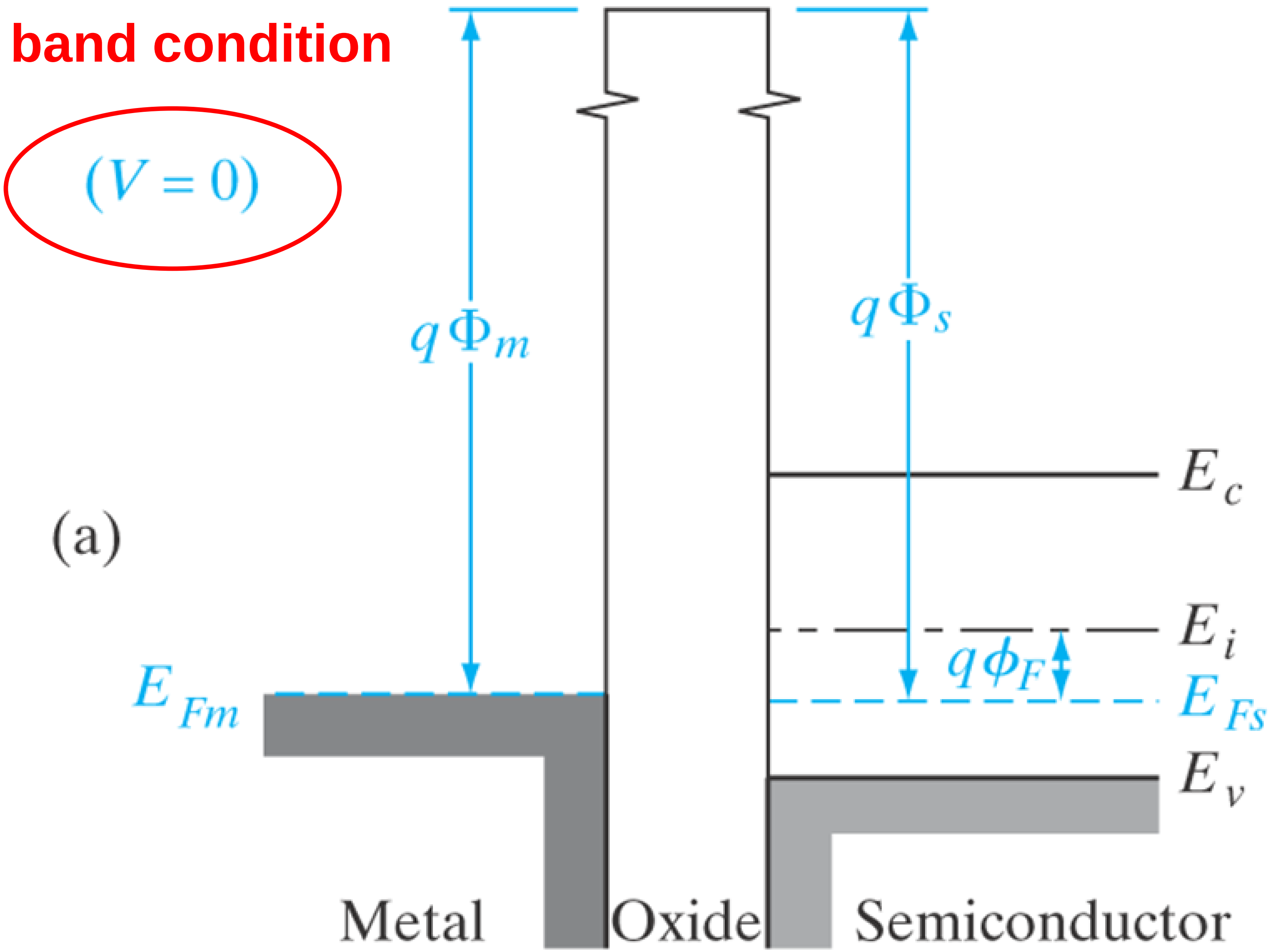


# $SiO_2$ – Si system (band gaps)



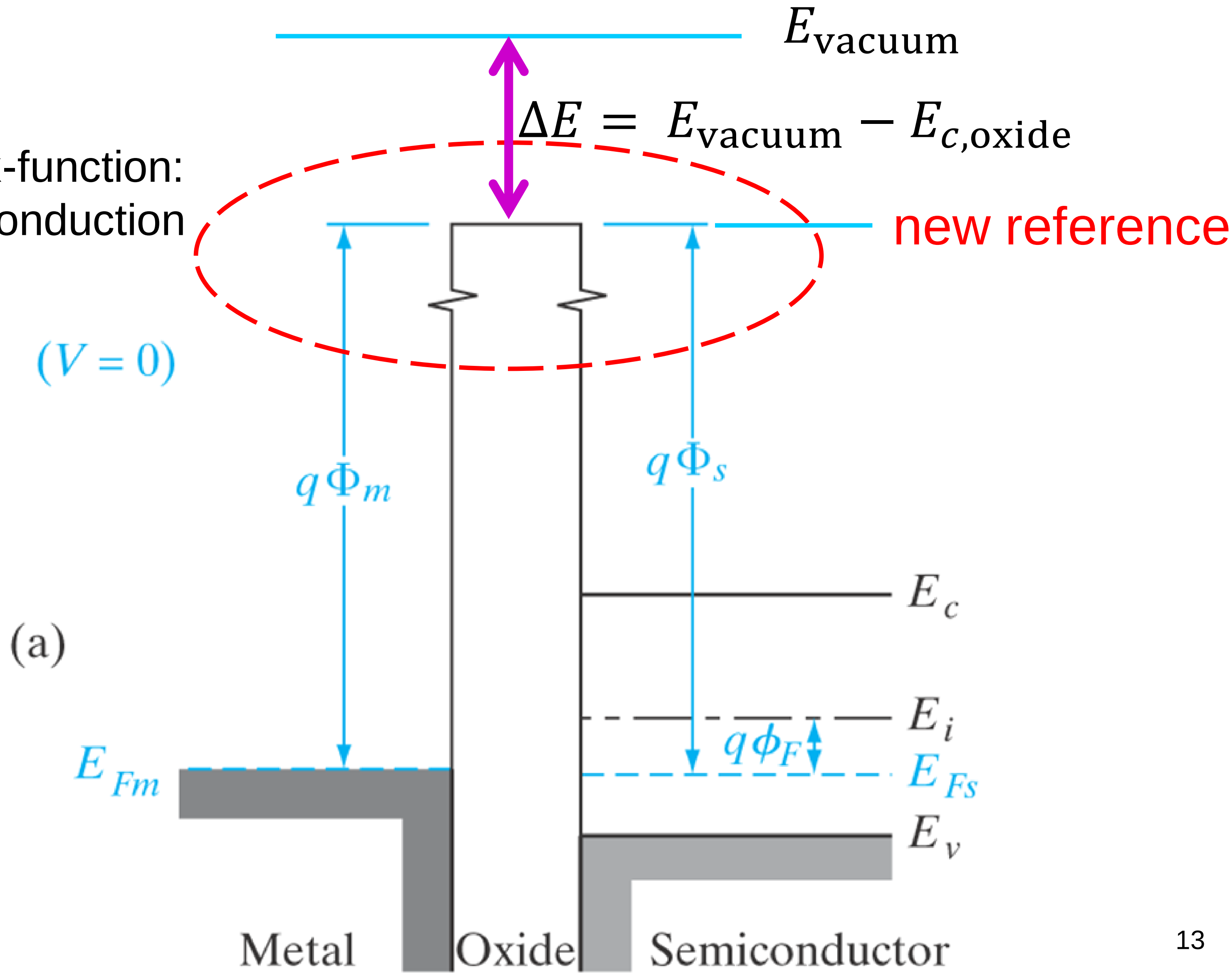
# Ideal MOSFET Capacitor (Equilibrium)

**flat band condition**

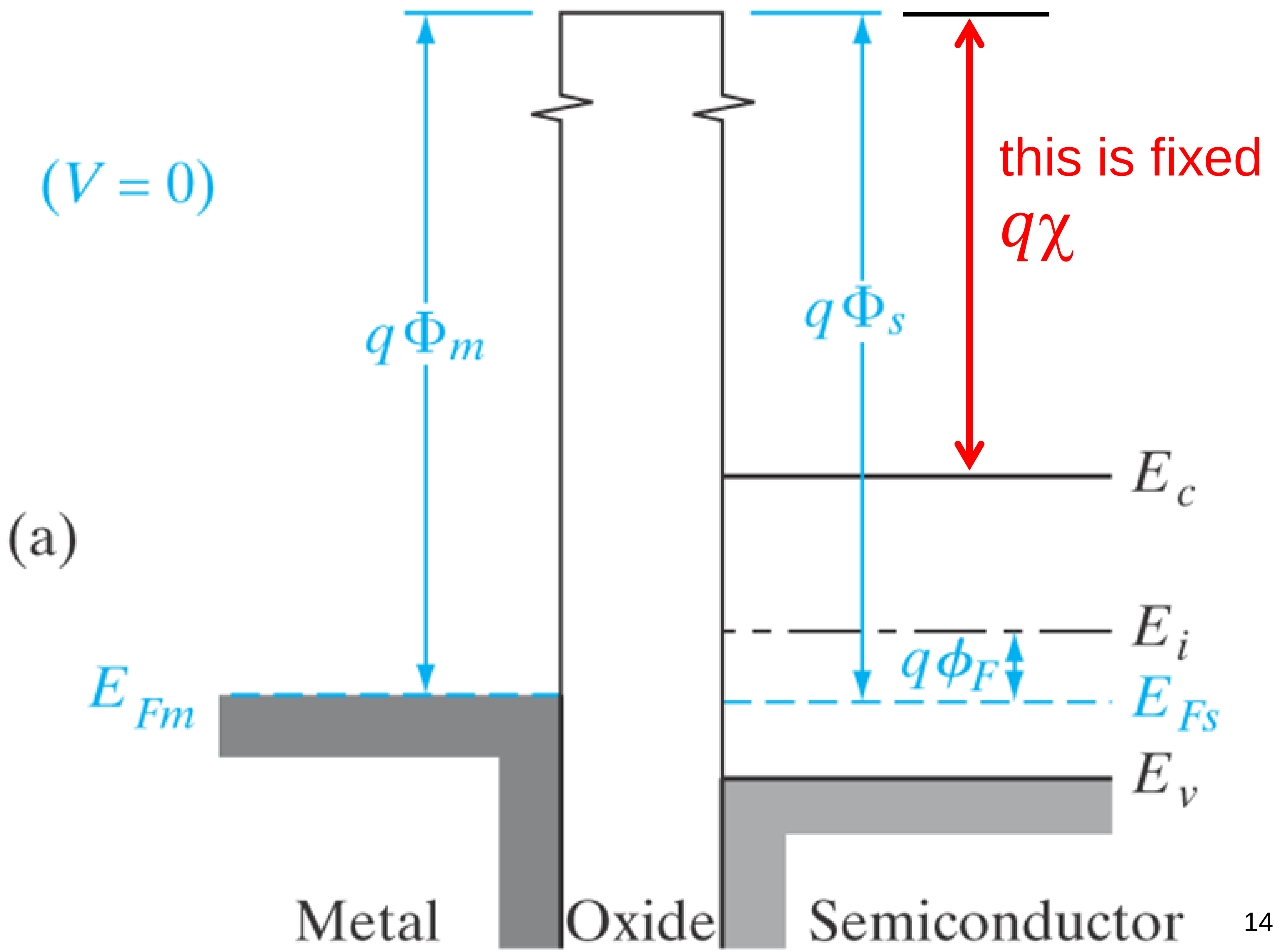


# Ideal MOSFET Capacitor (Equilibrium)

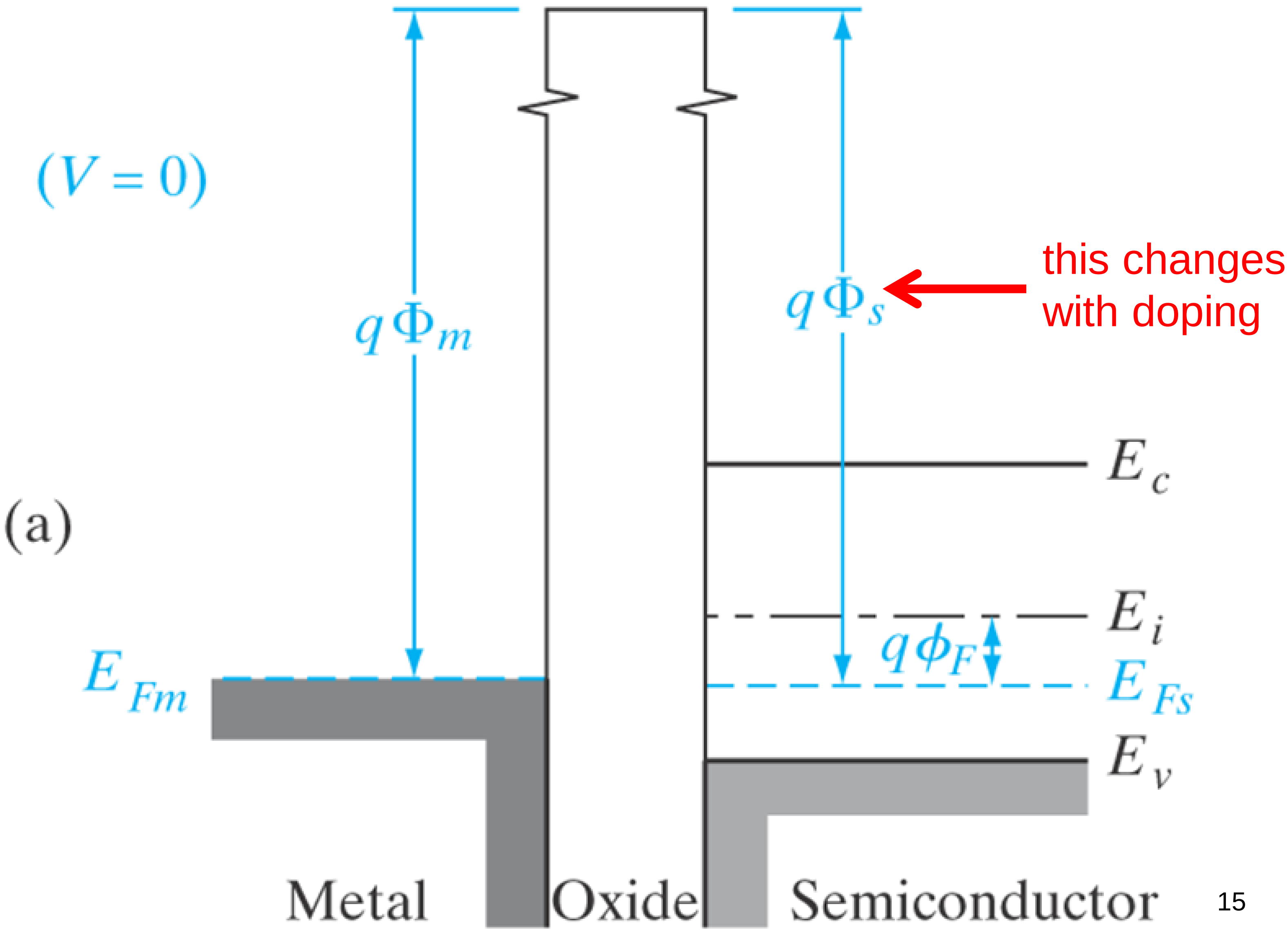
Modified work-function: reference is conduction band of oxide



# Ideal MOSFET Capacitor (Equilibrium)



# Ideal MOSFET Capacitor (Equilibrium)

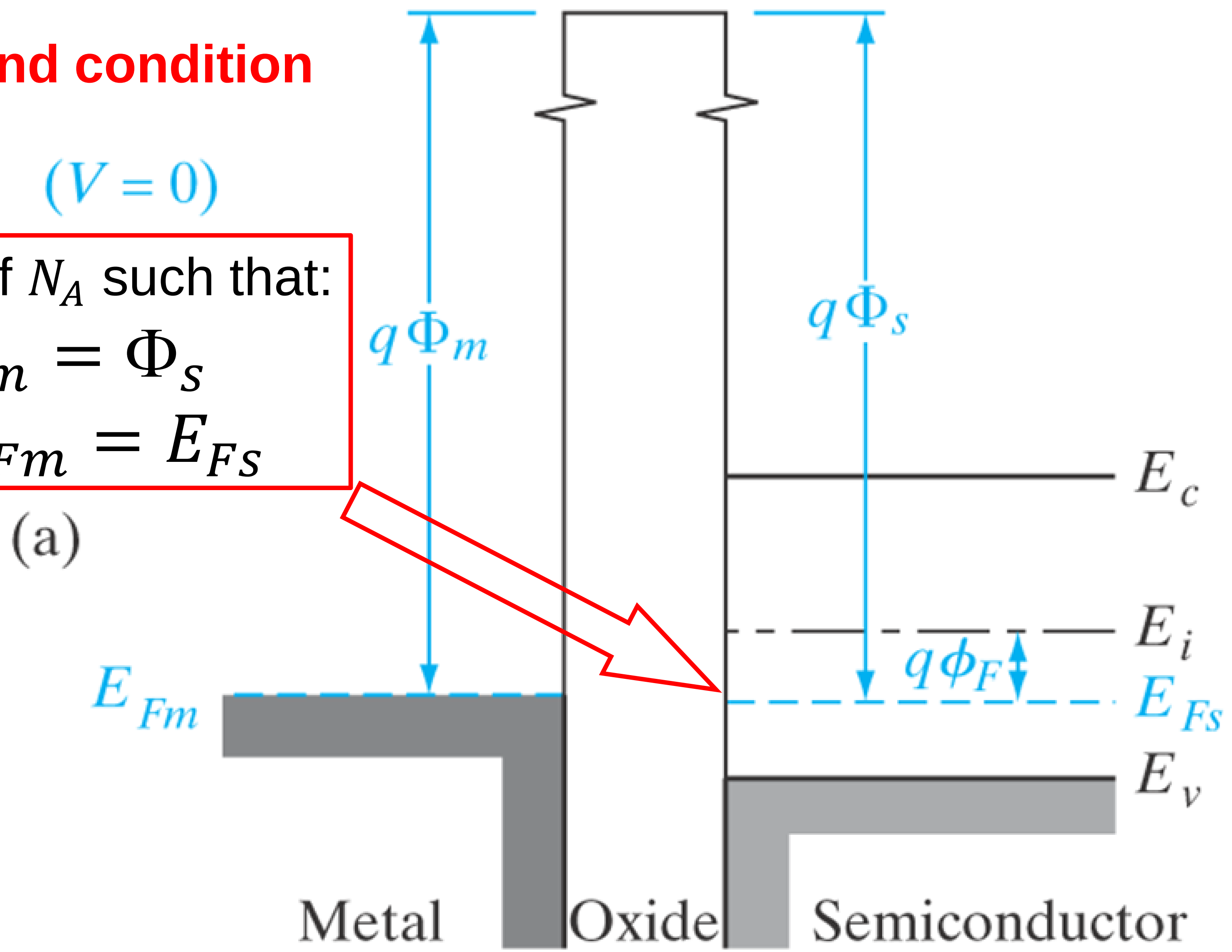


# Ideal MOSFET Capacitor (Equilibrium)

**flat band condition**

choice of $N_A$ such that:

$$\Phi_m = \Phi_s$$
$$\rightarrow E_{Fm} = E_{Fs}$$

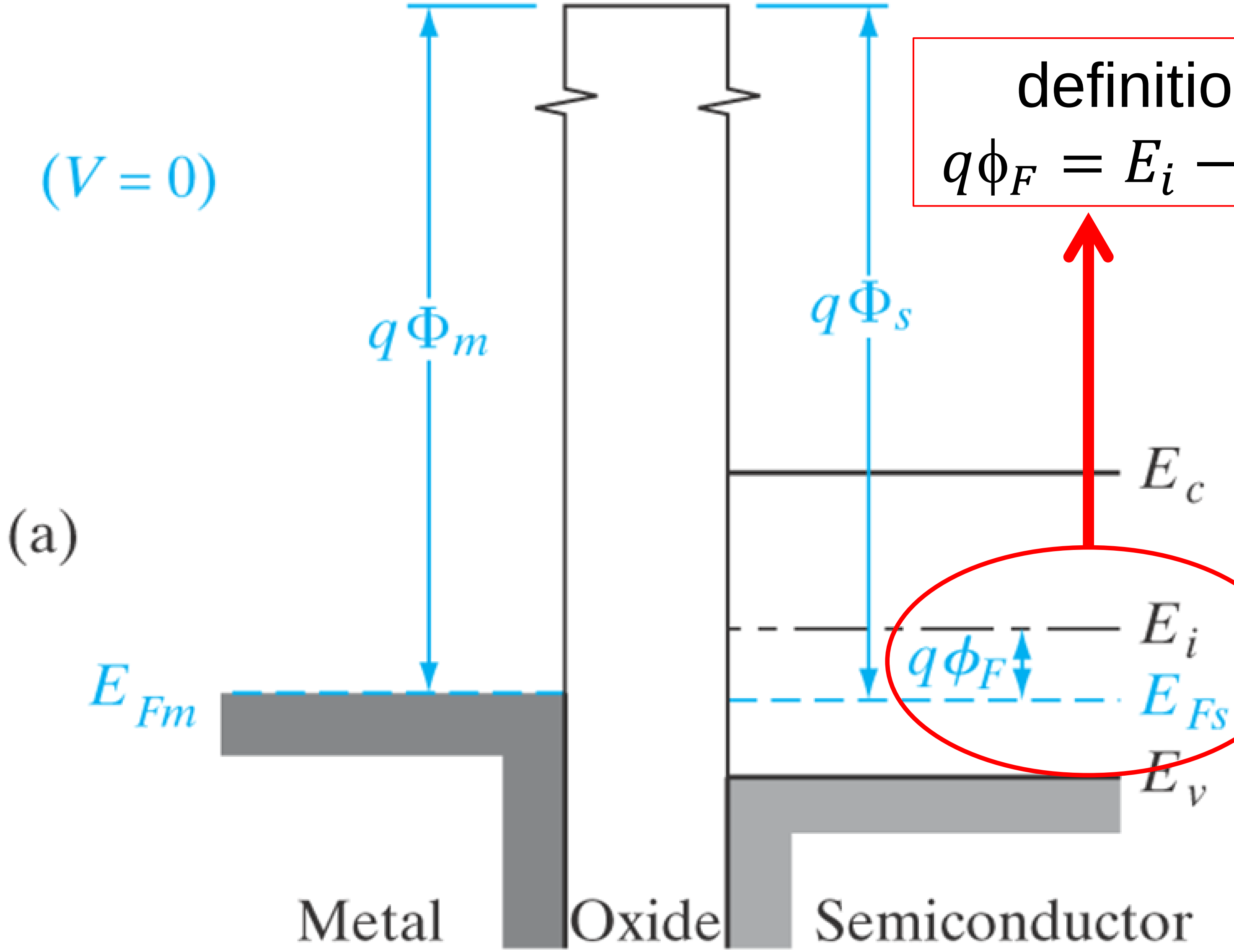


# Ideal MOSFET Capacitor (Equilibrium)

**If the doping choice is a general one:**

- The Fermi level in the semiconductor is not the same as the Fermi level in the metal
- Alignment of the Fermi levels gives rise to a field across the oxide and a charged region at the semiconductor interface in equilibrium
- A **potential** must be applied on the metal gate to achieve flat band condition (called "**flat band potential**")

# Ideal MOSFET Capacitor (Equilibrium)




definition

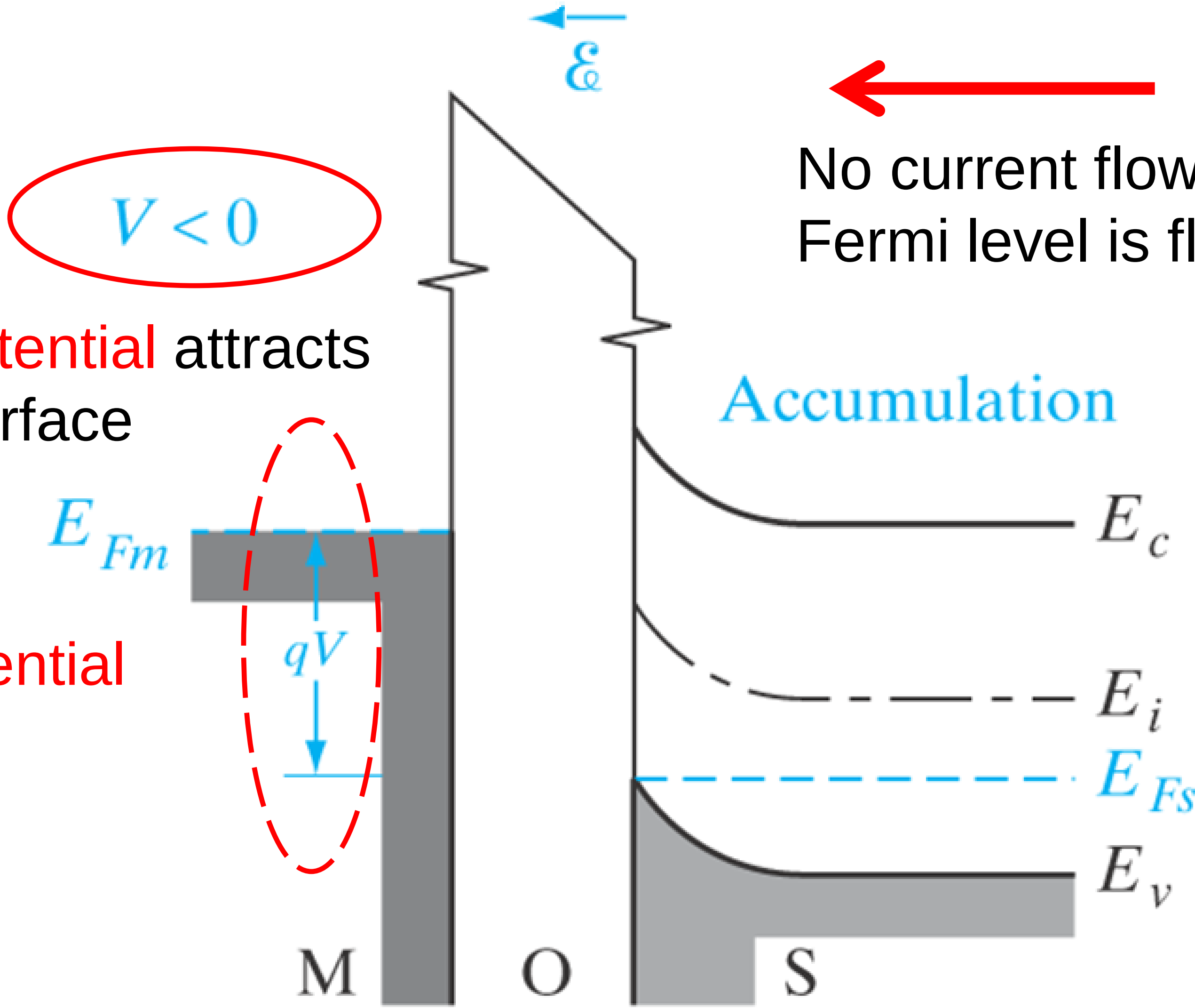
$$q\phi_F = E_i - E_{Fs}$$

# Ideal MOSFET Capacitor (Accumulation)

No current flow
Fermi level is flat

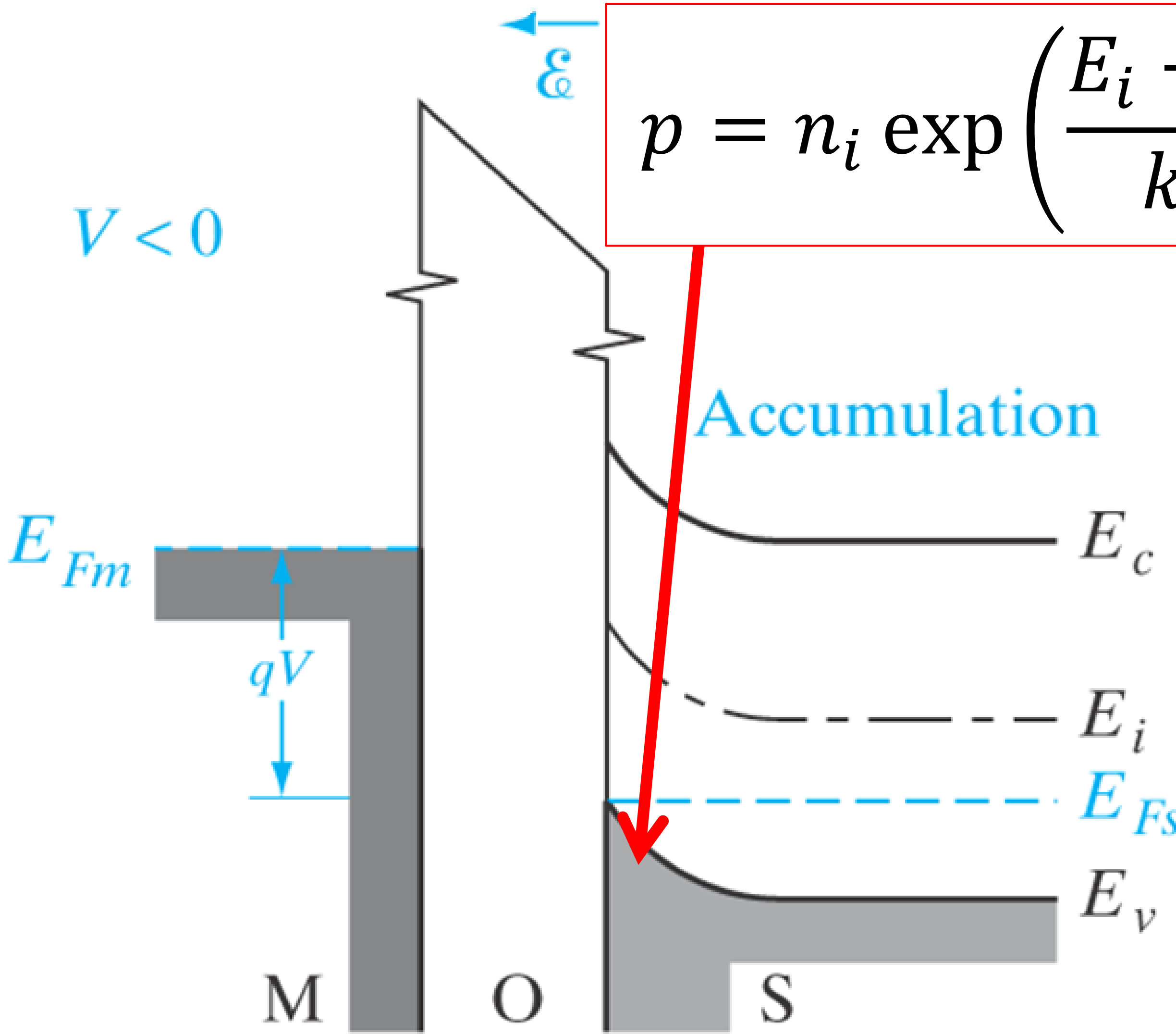
Negative potential attracts holes to interface

positive potential energy



# Ideal MOSFET Capacitor (Accumulation)

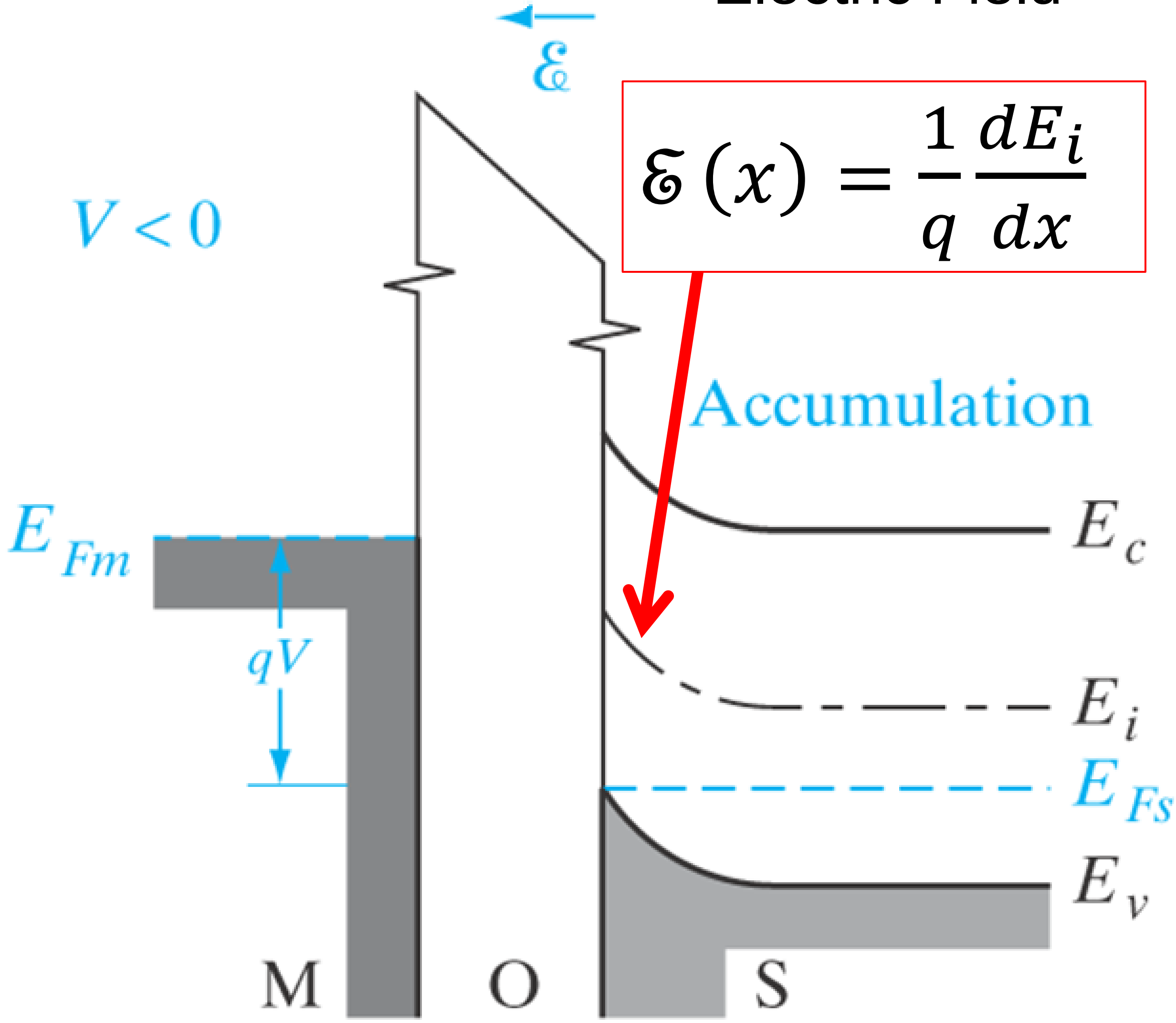
$$p = n_i \exp\left(\frac{E_i - E_{Fs}}{k_B T}\right)$$



# Ideal MOSFET Capacitor (Accumulation)

Electric Field

$$\mathcal{E}(x) = \frac{1}{q}\frac{dE_i}{dx}$$



# Ideal MOSFET Capacitor (Accumulation)

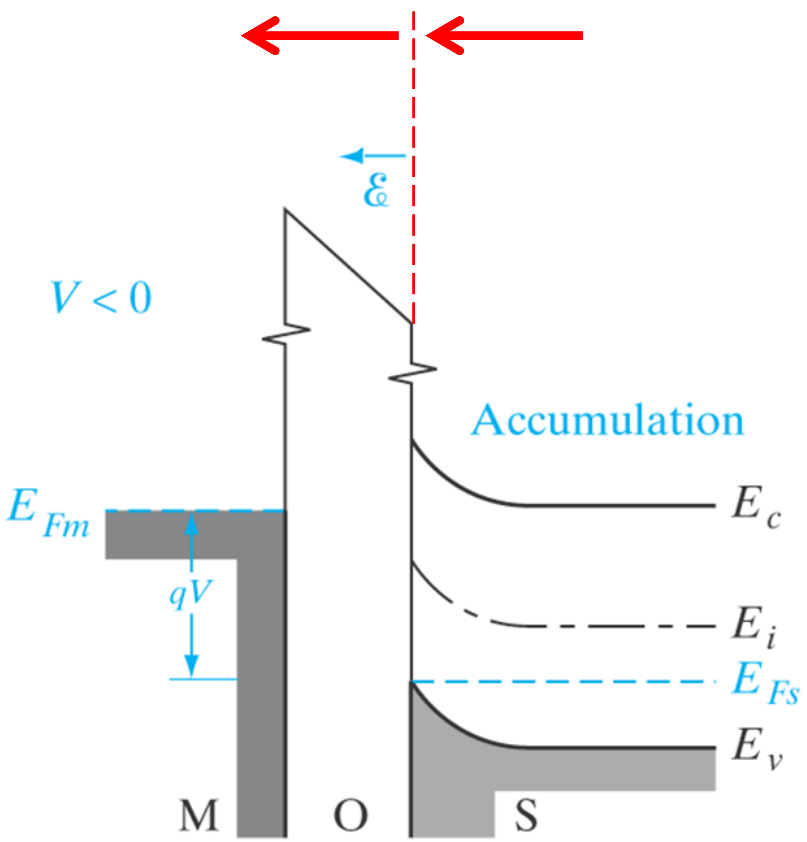
**From ECE 329: Displacement vector is conserved at the $SiO_2$/Si interface**

$$D_{Si} = \varepsilon_{Si}\mathcal{E}_{Si}$$

$$D_{ox} = \varepsilon_{ox}\mathcal{E}_{ox}$$

$$D_{ox} = D_{Si}$$

$$\mathcal{E}_{ox} = \frac{\varepsilon_{Si}}{\varepsilon_{ox}}\mathcal{E}_{Si}$$




$$\varepsilon_{Si} \approx 11.8\ \varepsilon_0$$

$$\varepsilon_{ox} \approx 3.9\ \varepsilon_0$$

# Ideal MOSFET Capacitor (Accumulation)

**In accumulation:**

- The MOS capacitor is charged with electrons on the metal side and holes at the interface between *p*-type semiconductor and oxide.

- The capacitance is related to the oxide layer

Oxide capacitance (unit area)

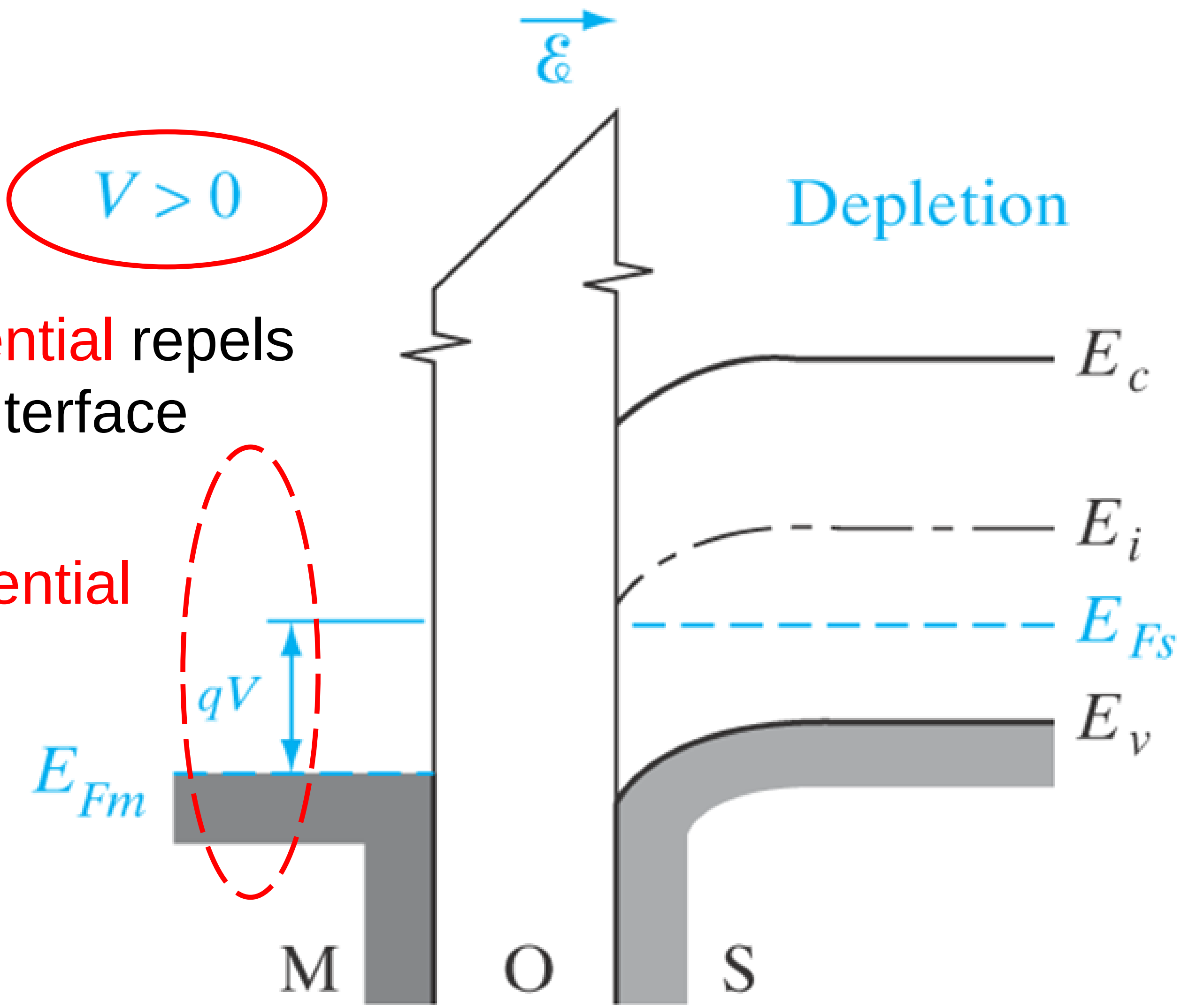
$$C_i = \frac{\varepsilon_{ox}}{d_{ox}}$$

$d_{ox}$ = thickness of oxide layer

# Ideal MOSFET Capacitor (Depletion)

Positive potential repels holes from interface

negative potential energy



# Ideal MOSFET Capacitor (Depletion)

**In depletion:**

- The MOS capacitor is charged with positive ions on the metal side and mainly negative acceptor ions at the interface between *p*-type semiconductor and oxide.
- The depletion width grows with the potential.
- Holes move away from the oxide interface region and minority carrier electrons start increasing there

# Ideal MOSFET Capacitor (Depletion)

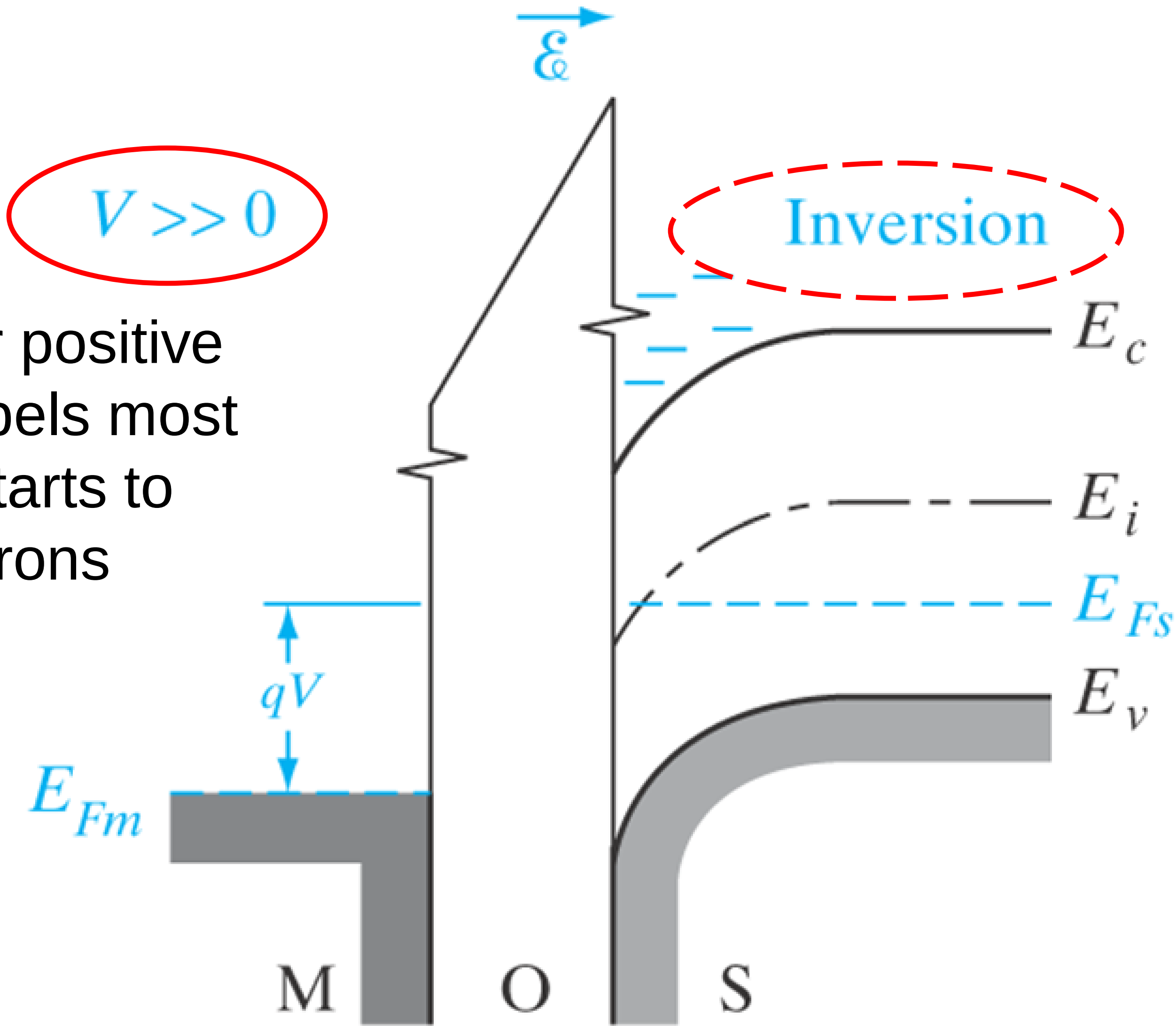
**In depletion:**

- The negative acceptor ions charge is distributed through the depletion layer, not just at the interface between *p*-type semiconductor and oxide.

- The total capacitance becomes approximately the series between the oxide capacitance and the depletion capacitance associated with the depletion layer

# Ideal MOSFET Capacitor (Inversion)

Even higher positive potential repels most holes and starts to attract electrons



# Ideal MOSFET Capacitor (Inversion)

**In weak inversion:**

- The MOS capacitor is charged with positive ions on the metal side and with negative acceptor ions plus an electron layer at the interface between *p*-type semiconductor and oxide. The depletion width keeps growing

- The semiconductor Fermi level crosses over the intrinsic Fermi level, causing electrons to become majority carriers in the vicinity of the oxide interface

# Ideal MOSFET Capacitor (Inversion)

**At the onset of strong inversion (threshold):**

- The semiconductor Fermi level has crossed over to the point that electrons near the interface **equal** the hole density in p-type bulk

- In the vicinity of the threshold the minimum total capacitance is reached. Electrons at the interface start taking over and responding more to the change in gate potential

# Ideal MOSFET Capacitor (Inversion)

**Deep in strong inversion:**

- The semiconductor Fermi level has crossed over so much that electrons near the interface **exceed** the hole density in p-type bulk

- The MOS capacitor is charged with positive ions on the metal side and with an electron layer at the oxide interface. Acceptor ion charge in the depletion layer no longer changes

- The depletion width **STOPS** growing

# Ideal MOSFET Capacitor (Inversion)

**Deep in strong inversion:**

- The depletion layer no longer responds to changes in the potential because of screening due to the strong electron layer at the interface

- The capacitance is again due only to the oxide

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_{ox}}{d_{ox}}$$

$d_{ox}$ = thickness of oxide layer

# More detailed capacitance analysis

The MOS capacitance is the series of a fixed oxide (insulator) parallel plate capacitance, independent of voltage

$$C_i = \frac{\varepsilon_{ox}}{d_{ox}}$$

and of a voltage-dependent semiconductor depletion layer capacitance

$$C_d = \frac{dQ}{dV} = \frac{dQ_s}{d\phi_s}$$

$$C_d = \frac{\varepsilon_s}{W}$$

series of $C_i$ and $C_d$

$$C = C_i C_d / (C_i + C_d)$$

# More detailed capacitance analysis

Depletion capacitance model is approximate

$$C_d = \frac{\varepsilon_S}{W}$$

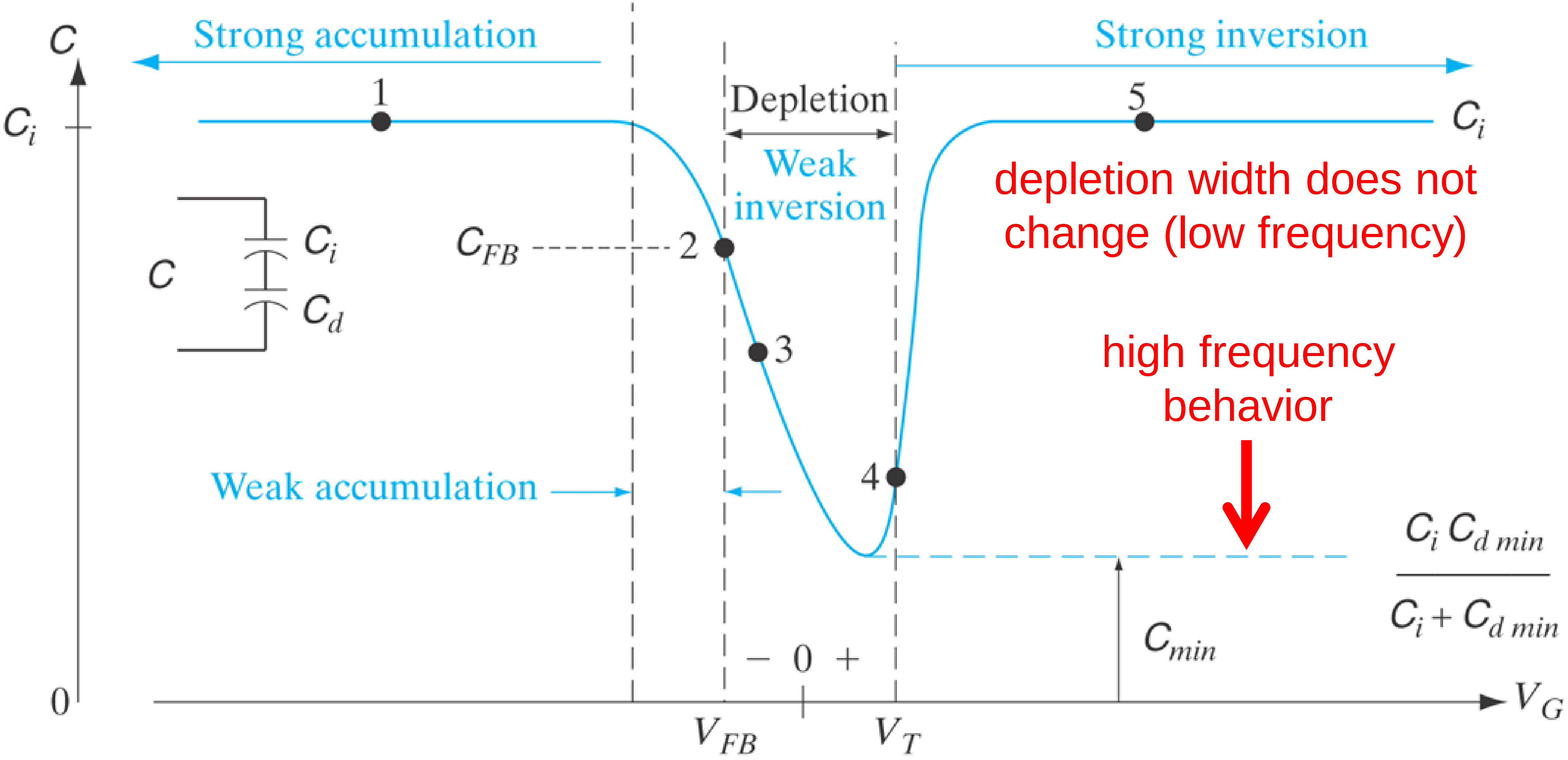
(it has highest error near flat band condition)

A better model for flat band condition indicates:

$$C_{d,FB} = \frac{\varepsilon_S}{L_D}$$

where $L_D$ is the Debye length $L_D = \sqrt{\frac{\varepsilon_S k_B T}{q^2 p_0}}$

# MOS Capacitance Measurement




oxide capacitance

$$C_i = \frac{\varepsilon_{ox}}{d}$$

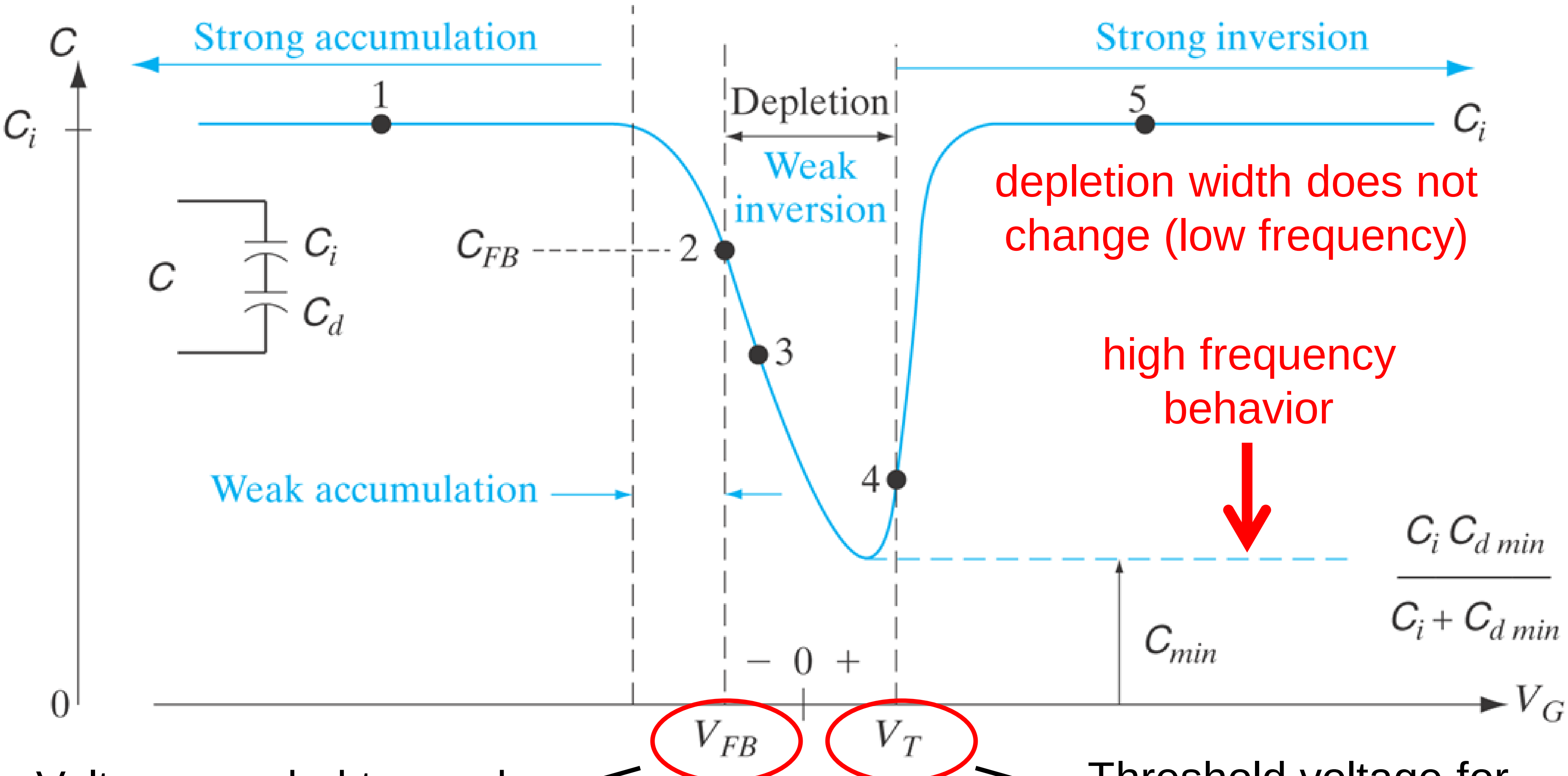
depletion capacitance

$$C_d = \frac{\varepsilon_s}{W}$$

series of $C_i$ and $C_d$

$$C = C_i C_d / (C_i + C_d)$$

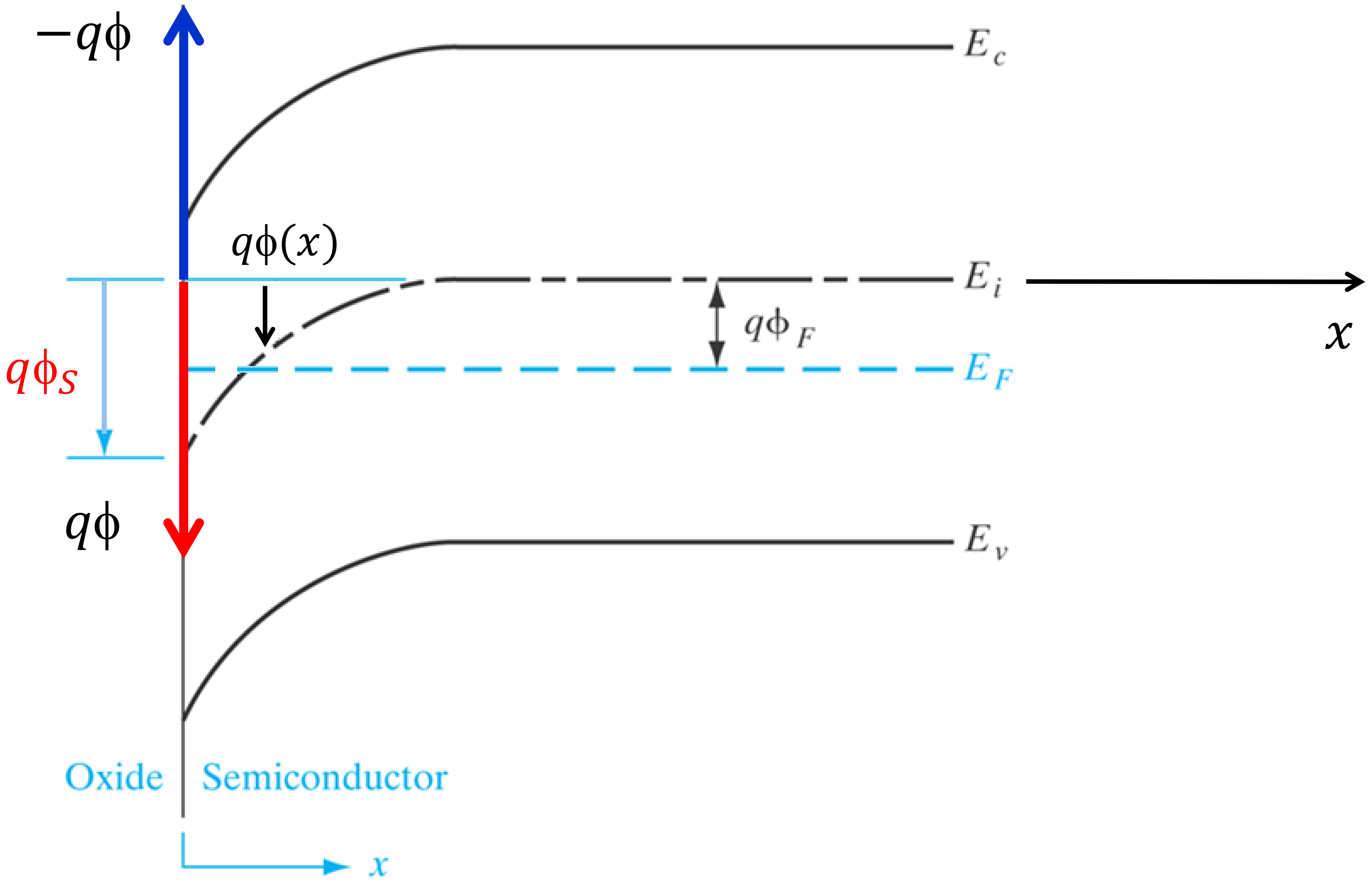
# MOS Capacitance Measurement



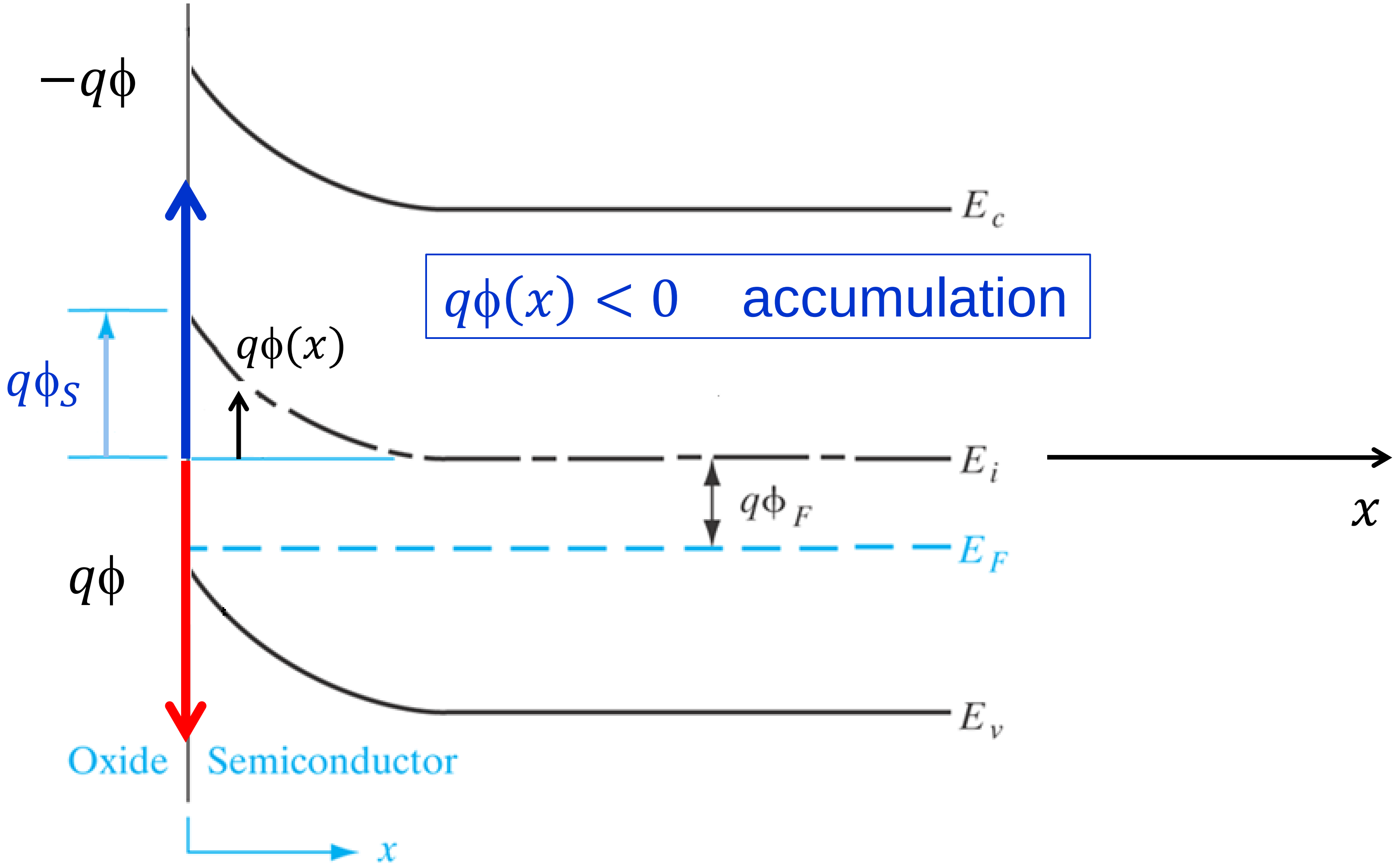

Voltage needed to reach flat band. This is zero if $E_{Fm} = E_{Fs}$

Threshold voltage for strong inversion

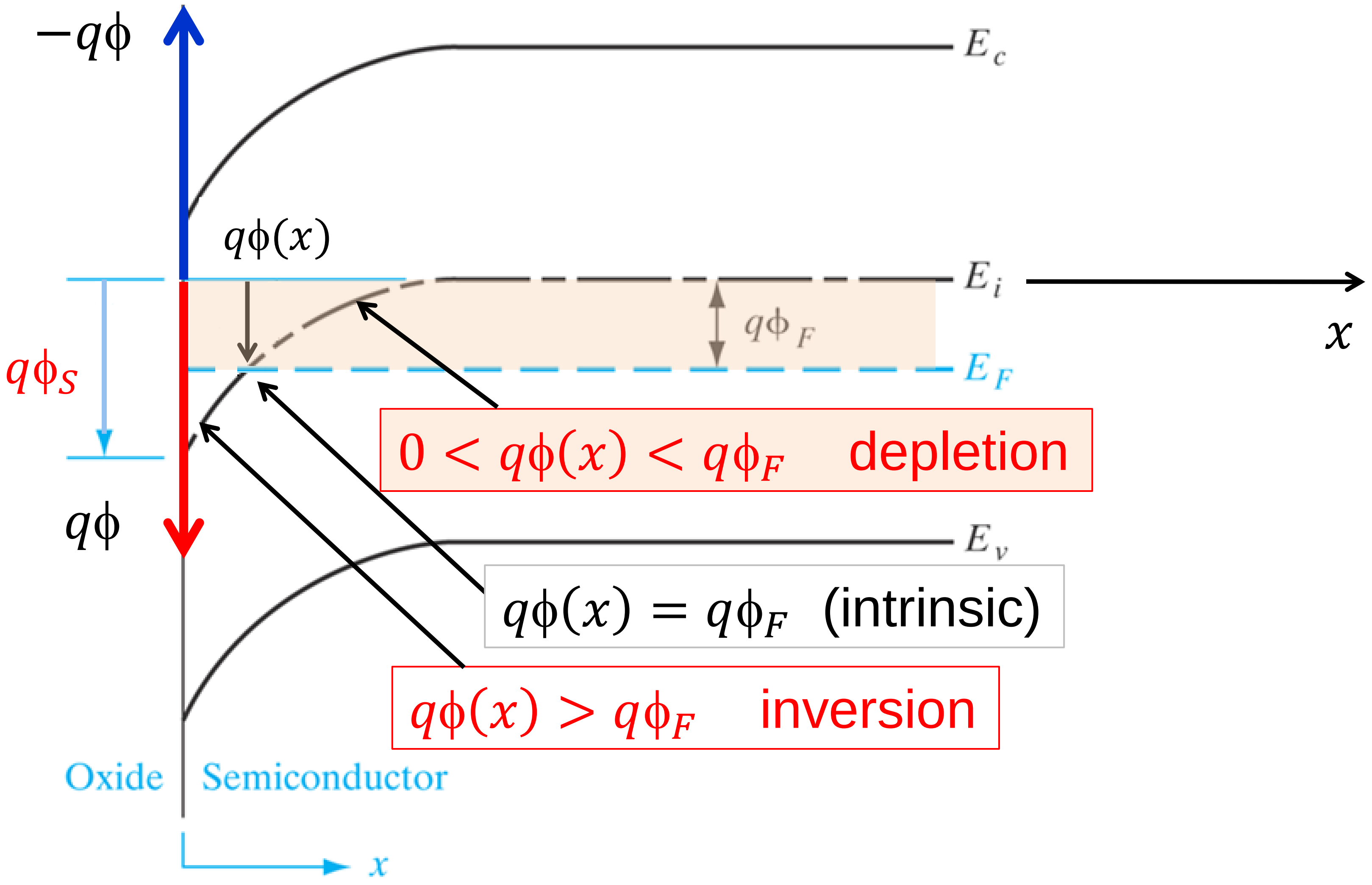
# Potential energy system of reference



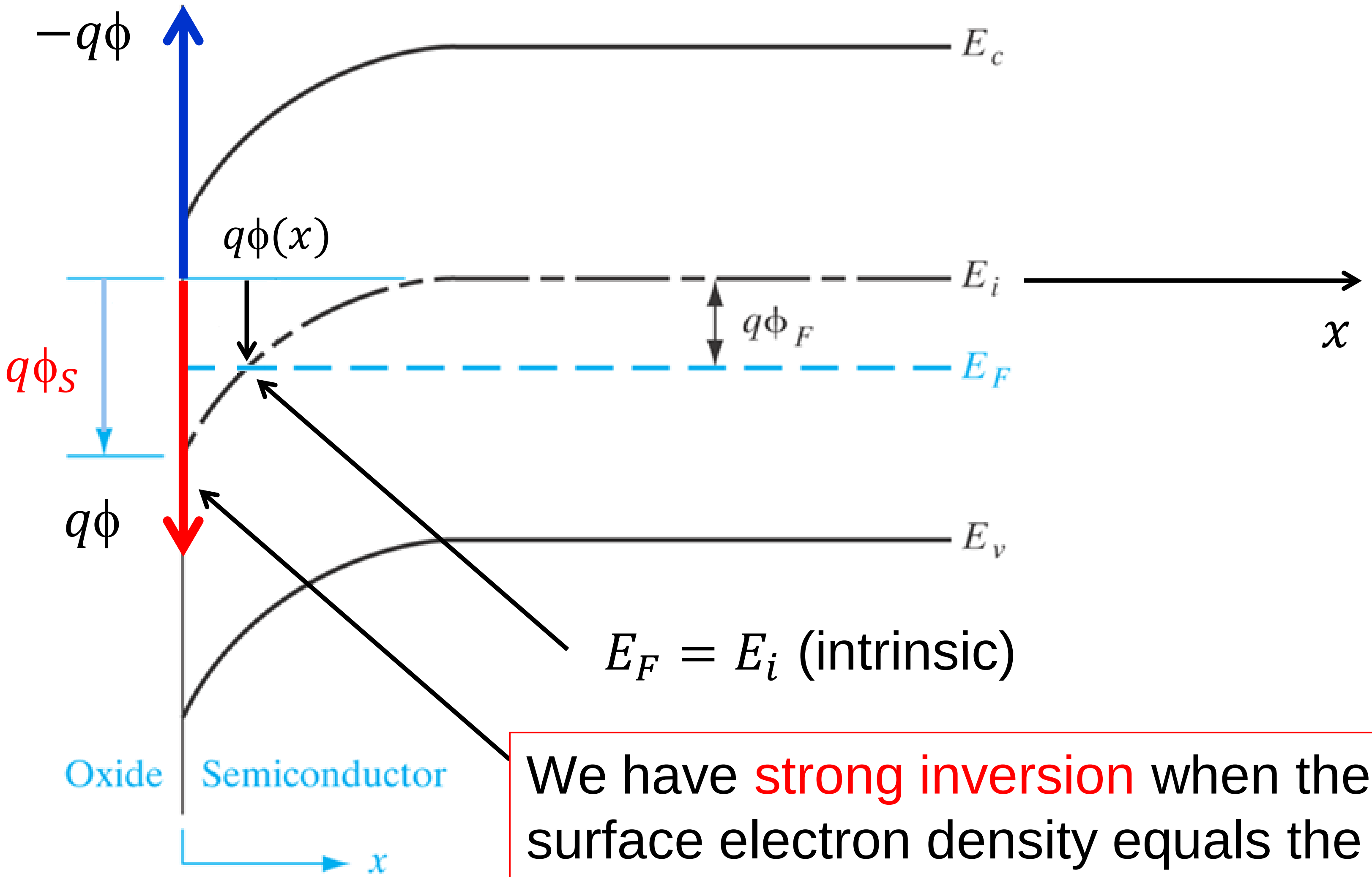
# Potential energy system of reference



# Potential energy system of reference

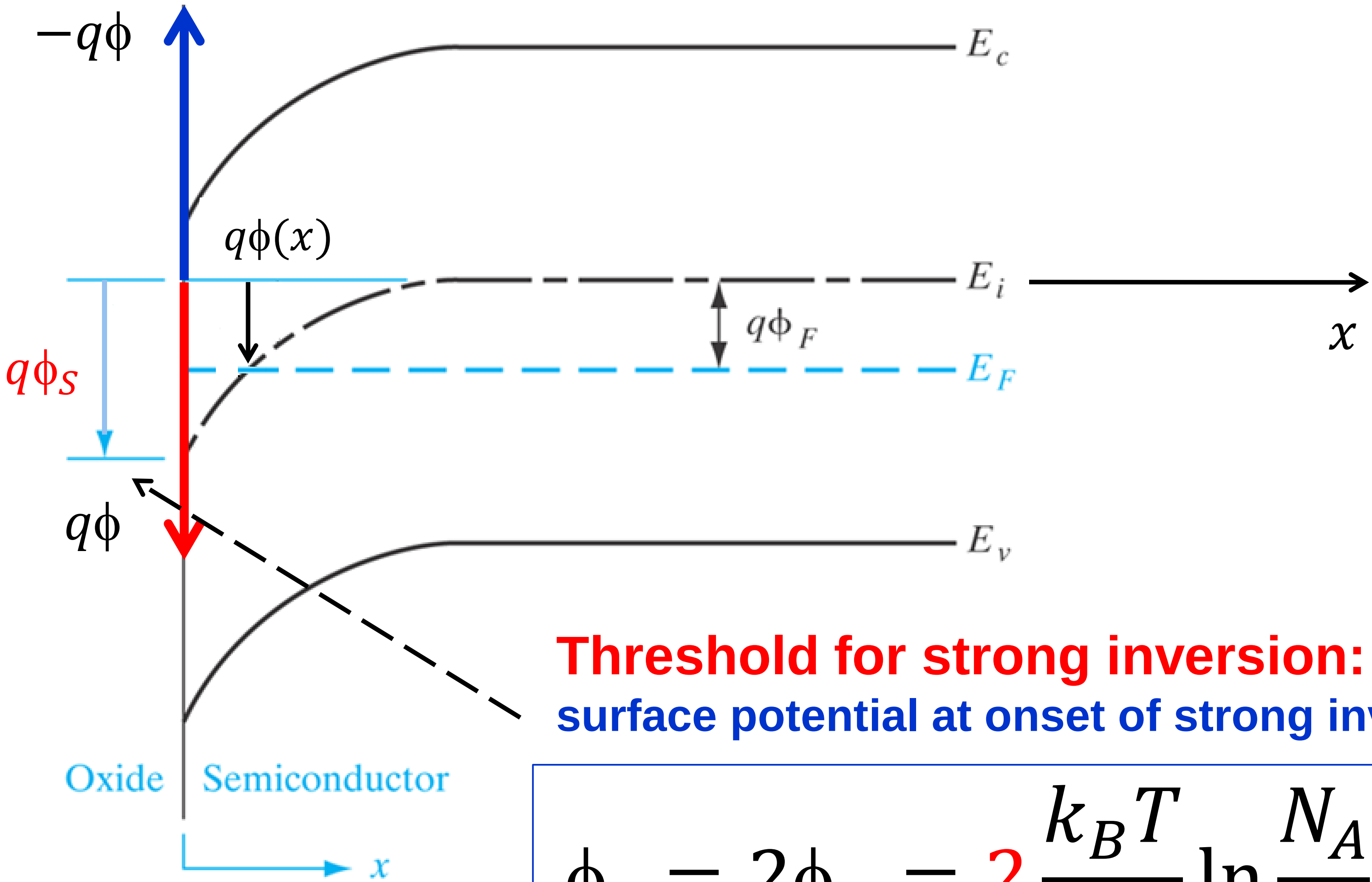


# Strong inversion condition (definition)




We have strong inversion when the surface electron density equals the hole density in the bulk

# Strong inversion condition (definition)

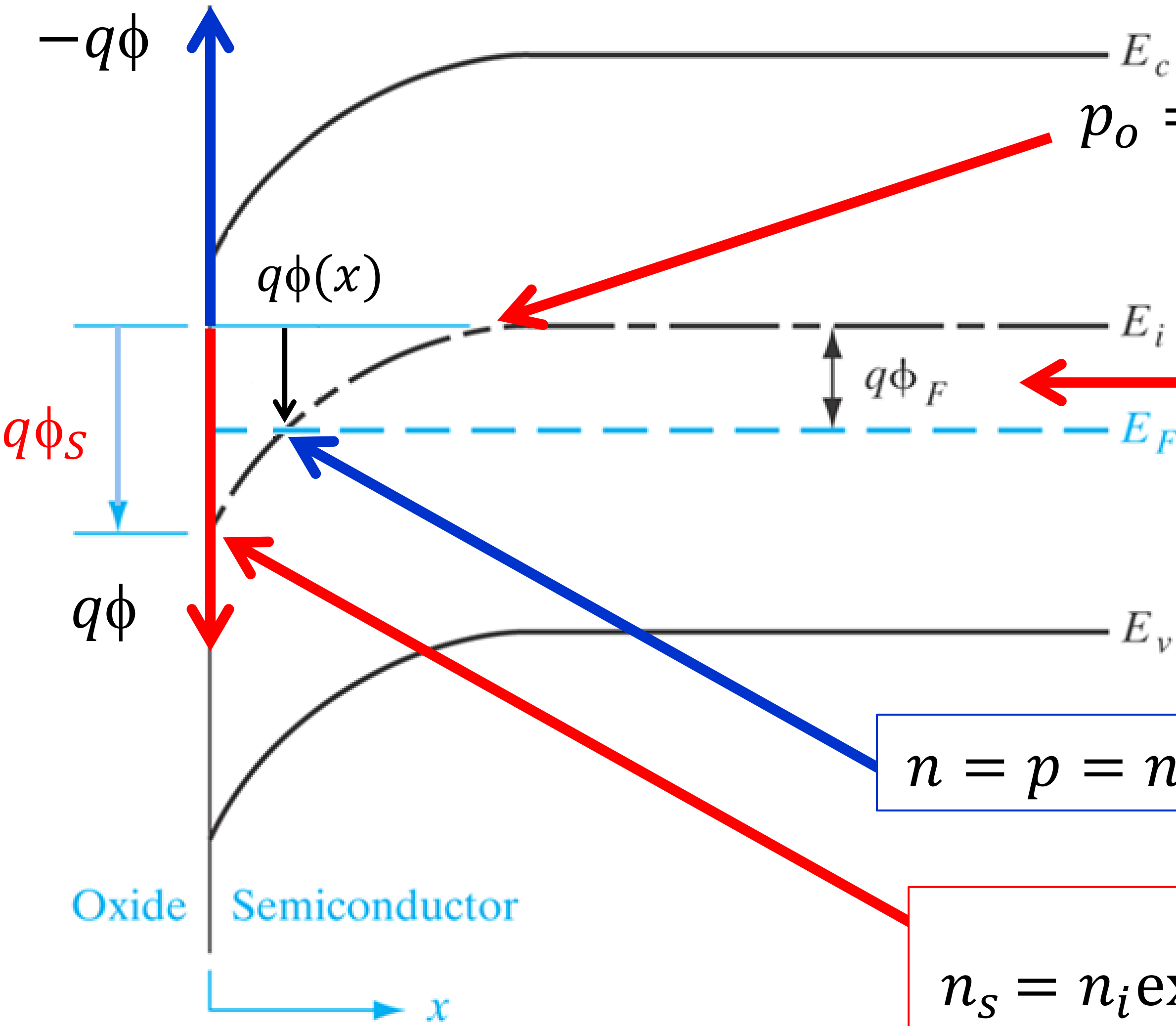



**Threshold for strong inversion:**

**surface potential at onset of strong inversion**

$$\phi_s = 2\phi_F = 2\frac{k_B T}{q}\ln\frac{N_A}{n_i}$$

# Strong inversion condition




$$p_o = N_A = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)$$

$$q\phi_F = (E_i - E_F) = \\ = k_B T \ln \frac{N_A}{n_i}$$

$$n = p = n_i$$

$$n_s = n_i \exp\left(\frac{E_F - E_i(0)}{k_B T}\right) = p_o$$

# Analytical model for $n(x)$

away from interface

$$n_o = n_i \exp\left(\frac{E_F - E_i}{k_B T}\right) = n_i \exp\left(-\frac{q\phi_F}{k_B T}\right)$$

at any $x$ location

$$n(x) = n_i \exp\left(\frac{E_F - E_i(x)}{k_B T}\right) = n_i \exp\left(-q\,\frac{\phi_F - \phi(x)}{k_B T}\right) =$$

$$= \underbrace{n_i \exp\left(-q\,\frac{\phi_F}{k_B T}\right)}_{\boldsymbol{n_o}} \exp\left(q\,\frac{\phi(x)}{k_B T}\right) = n_0 \exp\left(q\,\frac{\phi(x)}{k_B T}\right)$$

# Analytical model for $p(x)$

away from interface

$$p_o = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right) = n_i \exp\left(\frac{q\phi_F}{k_B T}\right)$$

at any $x$ location

$$p(x) = n_i \exp\left(\frac{E_i(x) - E_F}{k_B T}\right) = n_i \exp\left(-q\frac{\phi(x) - \phi_F}{k_B T}\right) =$$

$$= \underbrace{n_i \exp\left(q\frac{\phi_F}{k_B T}\right)}_{p_o} \exp\left(-q\frac{\phi(x)}{k_B T}\right) = p_0 \exp\left(-q\frac{\phi(x)}{k_B T}\right)$$

# Analytical model for $\mathcal{E}(x)$

We do not know the exact behavior of $\phi(x)$ but we know the relationship between charge and potential

Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\epsilon_s}$$

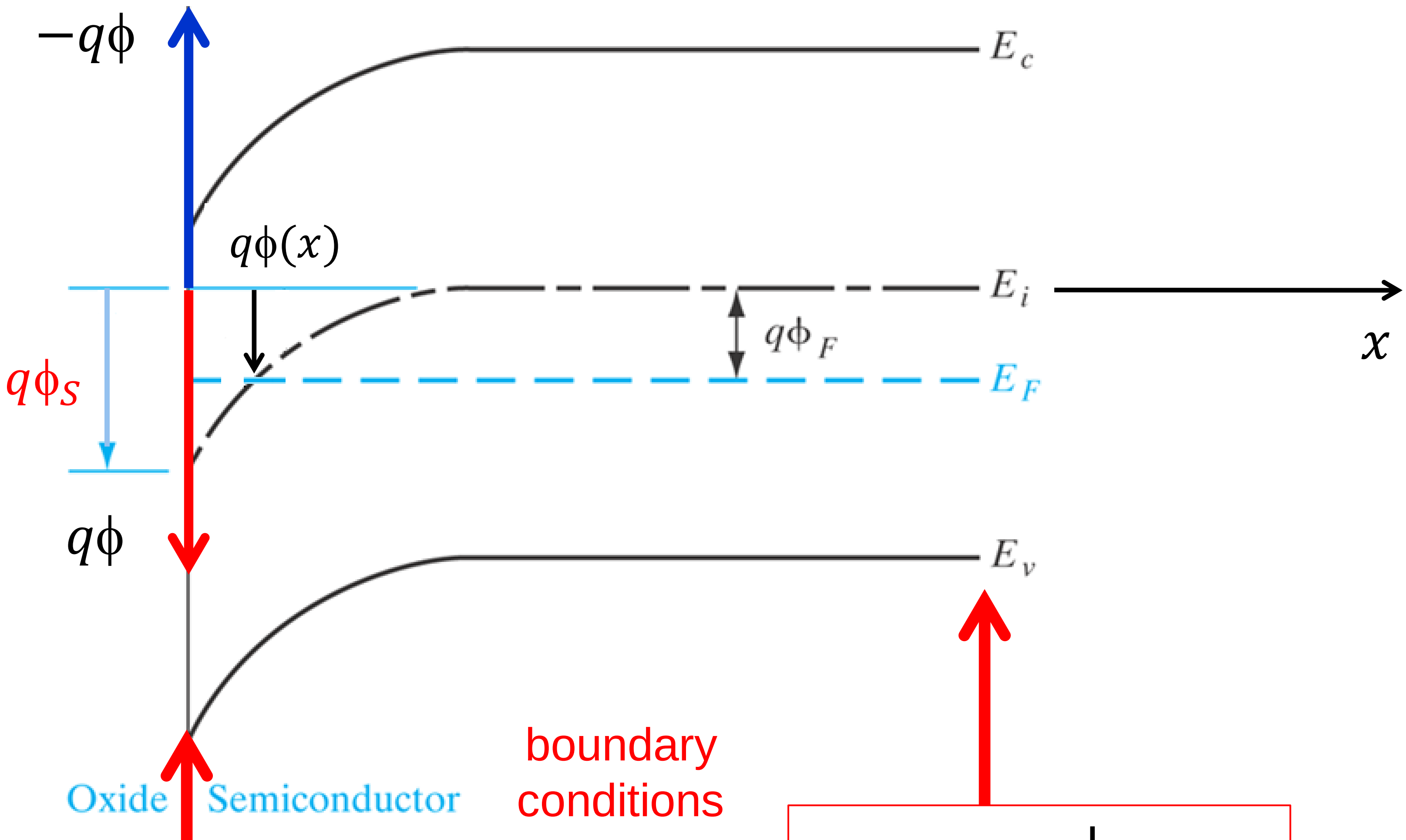
with charge density

$$\rho(x) = q[N_D^+ - N_A^- + p(x) - n(x)]$$

Electric Field

$$\mathcal{E} = -\frac{d\phi}{dx}$$

# Analytical model for $\mathcal{E}(x)$




boundary conditions

$$\phi(0) = \phi_s$$

$$\mathcal{E} = -\frac{d\phi}{dx} = 0$$

# Analytical model for $\mathcal{E}(x)$

$$\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi}{dx}\right) =$$

$$= -\frac{q}{\epsilon_s}\left\{p_o\left[\exp\left(-\frac{q\phi}{k_B T}\right) - 1\right] - n_o\left[\exp\left(\frac{q\phi}{k_B T}\right) - 1\right]\right\}$$

$$N_D^+ - N_A^- = n_o - p_o$$

$$\int_0^{\frac{d\phi}{dx}} \frac{d}{dx}\left(\frac{d\phi}{dx}\right) = \int_0^{\phi} RHS$$

$x = 0$

$$\phi(0) = \phi_s$$

integrate

$x \to \infty$

$$\mathcal{E} = -\frac{d\phi}{dx} = 0$$

# Analytical model for $\mathcal{E}(x)$

Solution

$$\mathcal{E}^2 = \frac{2k_BT}{\varepsilon_s} p_0 \left[ \left( \exp\left( -\frac{q\phi}{k_BT} \right) + \frac{q\phi}{k_BT} - 1 \right) + \frac{n_0}{p_0} \left( \exp\left( \frac{q\phi}{k_BT} \right) - \frac{q\phi}{k_BT} - 1 \right) \right]$$

At the surface where $x = 0$, $\phi_s$ , $\mathcal{E}_s$

$$\mathcal{E}_S = \frac{\sqrt{2}k_BT}{qL_D} \sqrt{\left( \exp\left( -\frac{q\phi_S}{k_BT} \right) + \frac{q\phi_S}{k_BT} - 1 \right) + \frac{n_0}{p_0} \left( \exp\left( \frac{q\phi_S}{k_BT} \right) - \frac{q\phi_S}{k_BT} - 1 \right)}$$

where $L_D = \sqrt{\frac{\varepsilon_S k_B T}{q^2 p_0}}$ is the Debye Length

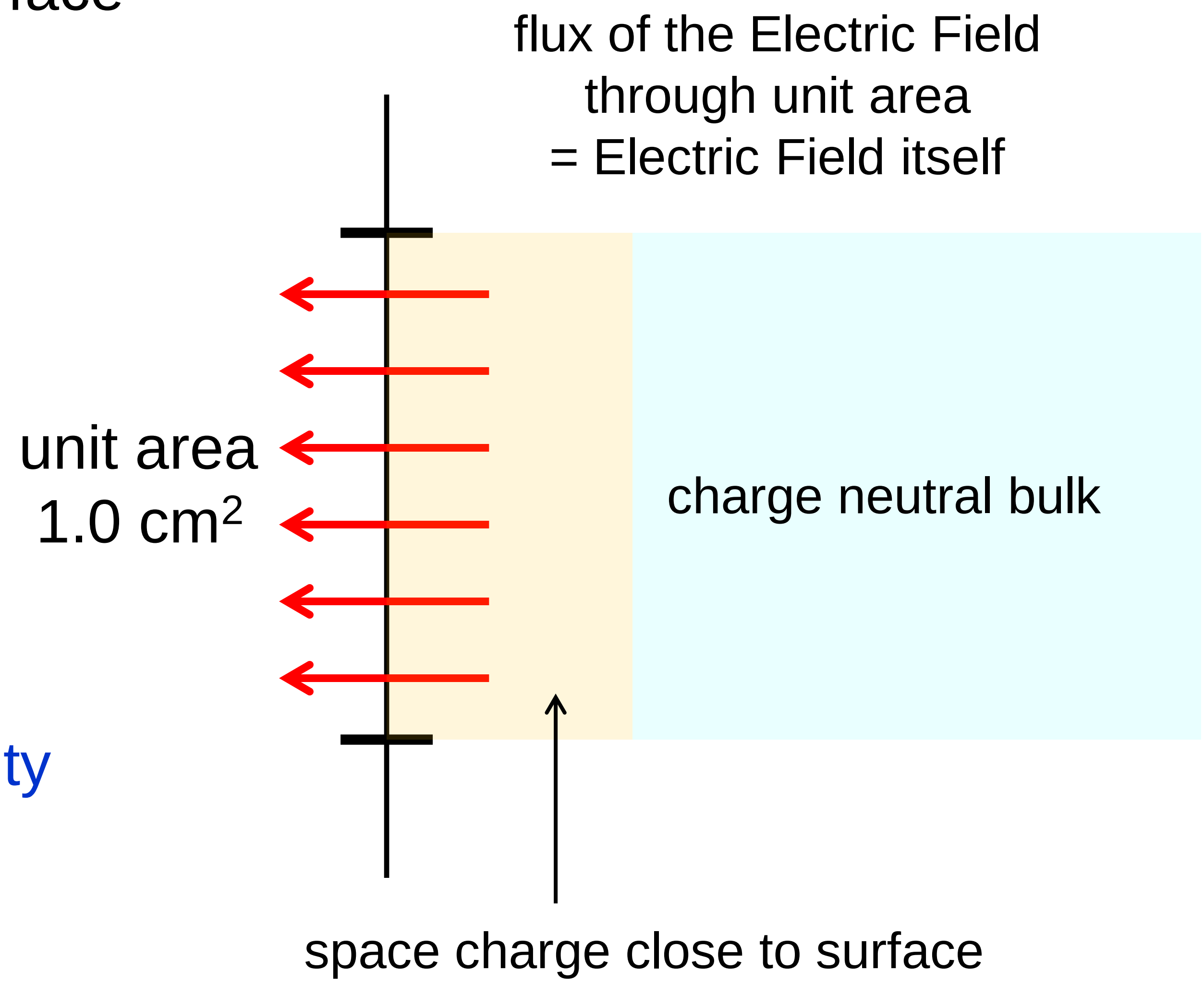
# Analytical model for $\mathcal{E}(x)$

Gauss Law at the Surface

$$\mathcal{E}_S = -\frac{Q_S}{\varepsilon_S}$$

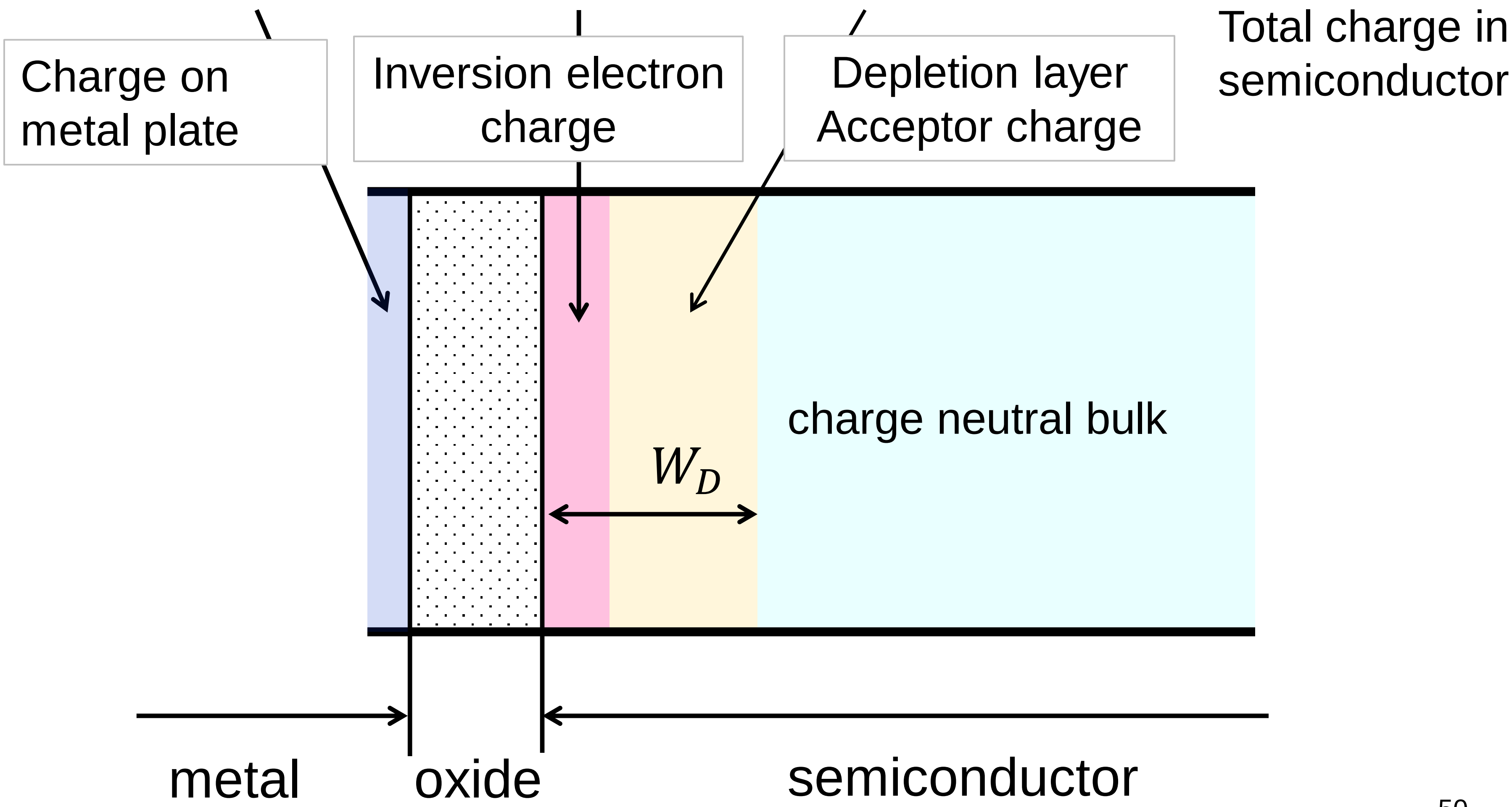
Space charge density per unit area

$$Q_S = -\varepsilon_S \mathcal{E}_S$$

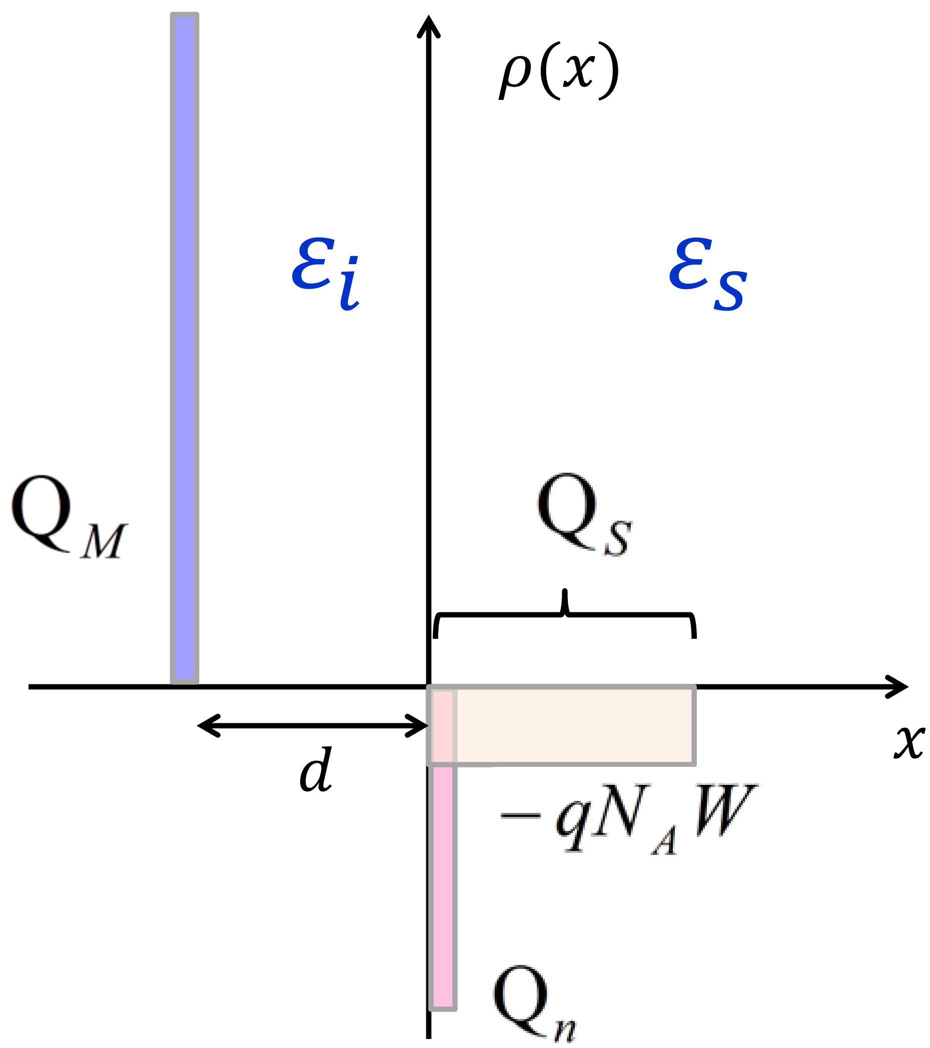


# Charge density distribution

$$Q_M = -Q_n + qN_A W_D = -Q_S$$



# Charge density distribution




$d$ = thickness of oxide

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_i}{d}$$

Voltage across oxide

$$V_i = \frac{-Q_s}{C_i} = \frac{-Q_s d}{\varepsilon_i}$$

Applied voltage

$$V = V_i + \phi_s$$

# Depletion Layer Width

Similar to result for $n^+$- $p$ junction

$$W = \sqrt{\frac{2\epsilon_s \phi_s}{qN_A}}$$

$$\phi_S < 2\phi_F$$

At strong inversion, depletion region no longer grows, due to screening of interface electrons

$$W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}} = 2\sqrt{\frac{\epsilon_s k_B T \ln \frac{N_A}{n_i}}{q^2 N_A}}$$

strong inversion
$\phi_S = 2\phi_F$

# Threshold Voltage (ideal case)

$$Q_D = -qN_A W = 2\sqrt{q\epsilon_s N_A \phi_F}$$

maximum value

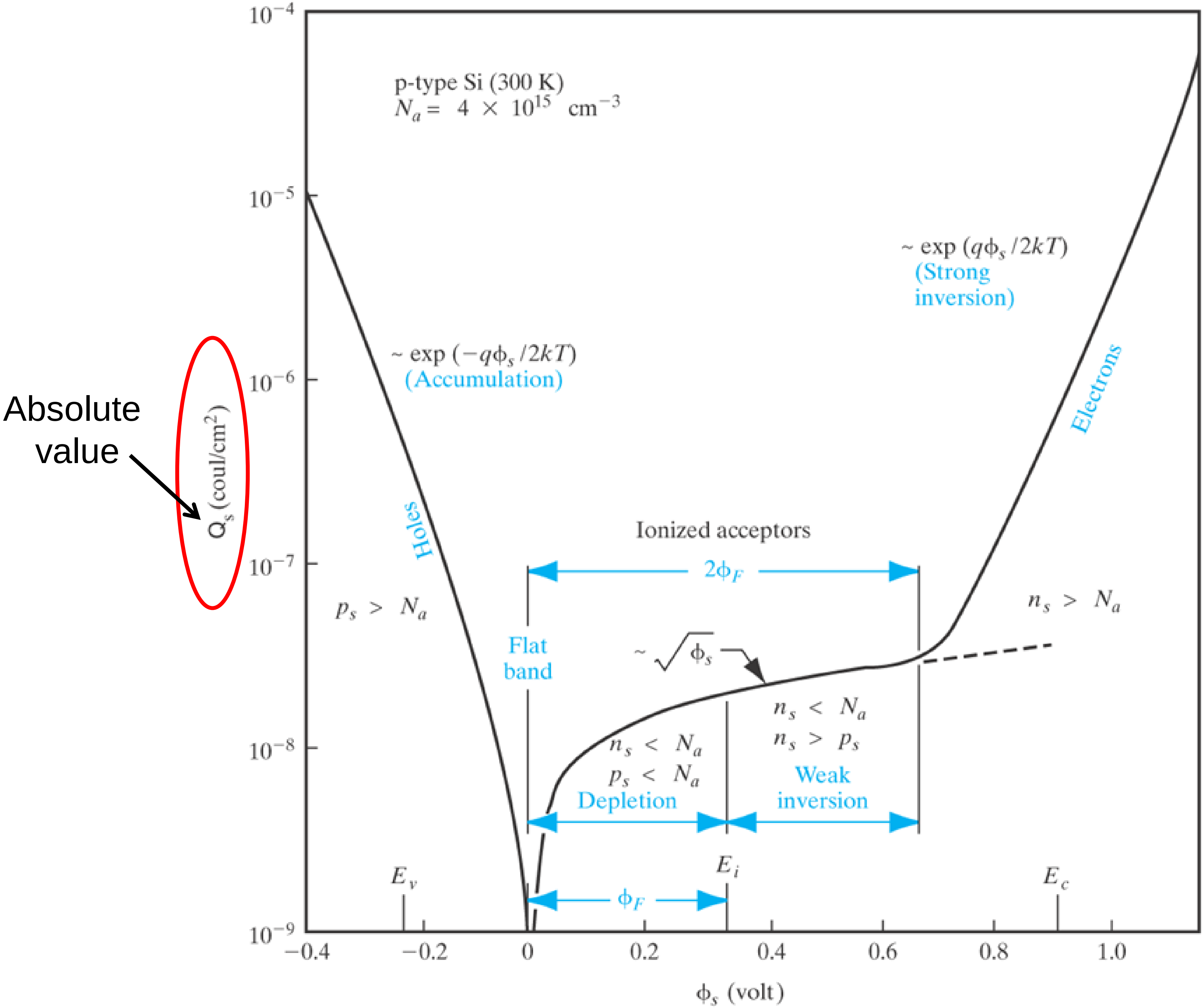
Depletion layer charge

$$V_T = -\frac{Q_d}{C_i} + 2\phi_F$$

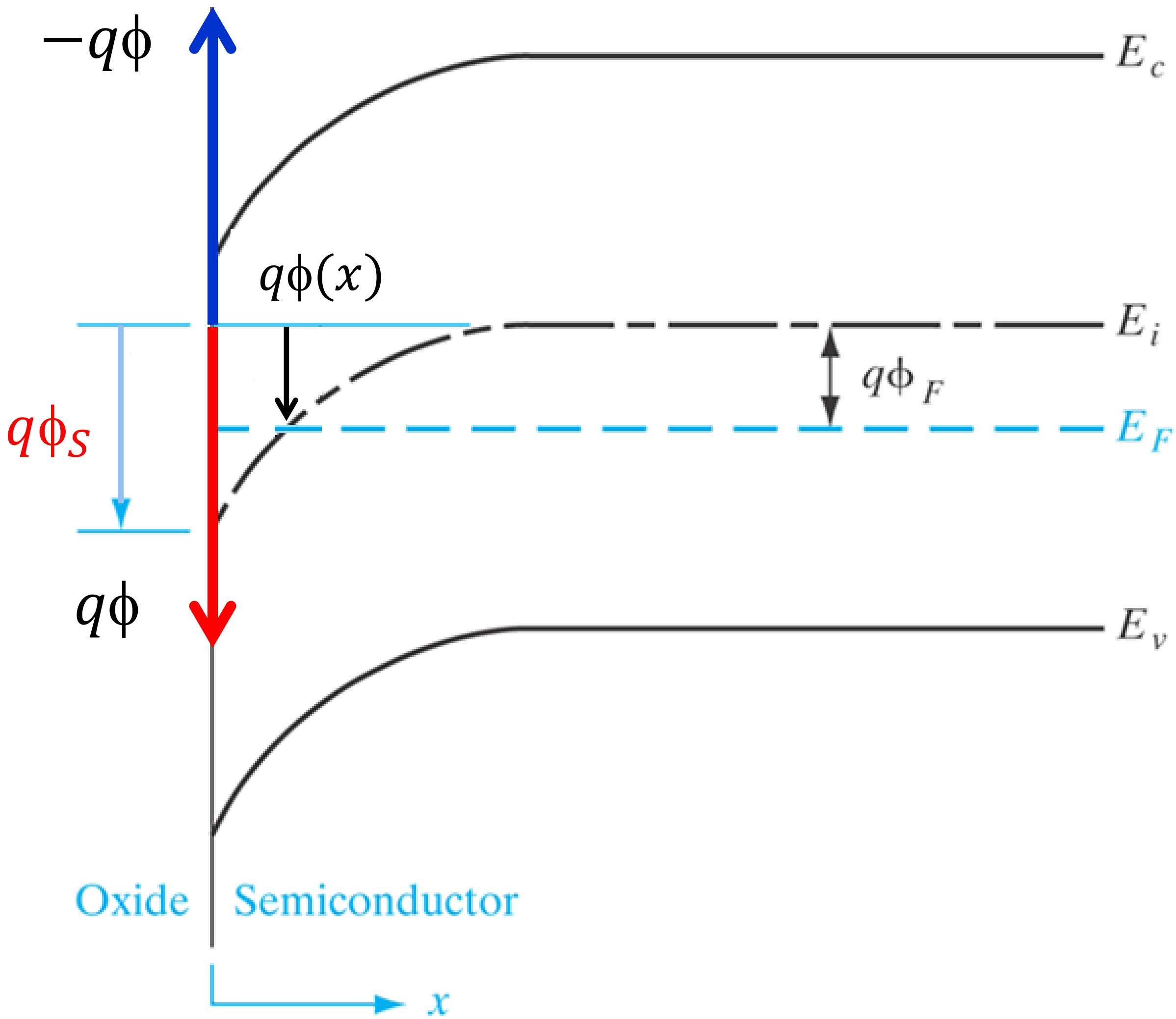
Voltage drop across oxide

Strong inversion condition

(Assuming that depletion charge dominates $Q_s$ at threshold)


Absolute value
p-type Si (300 K)
$N_a = 4 \times 10^{15}$ cm$^{-3}$
~ exp ($-q\phi_s/2kT$)
(Accumulation)
~ exp ($q\phi_s/2kT$)
(Strong inversion)
Holes
Electrons
Ionized acceptors
$2\phi_F$
$p_s > N_a$
$n_s > N_a$
Flat band
~ $\sqrt{\phi_s}$
$n_s < N_a$
$p_s < N_a$
Depletion
$n_s < N_a$
$n_s > p_s$
Weak inversion
$E_v$
$E_i$
$E_c$
$\phi_F$
$Q_s$ (coul/cm$^2$)
$\phi_s$ (volt)
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$
$10^{-9}$
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1.0

# Summary of conditions – surface potential




accumulation
$\phi_S < 0$

flat-band
$\phi_S = 0$

depletion
$0 < \phi_S < \phi_F$

intrinsic
$\phi_S = \phi_F$

weak inversion
$2\phi_F > \phi_S > \phi_F$

strong inversion
$\phi_S \geq 2\phi_F$