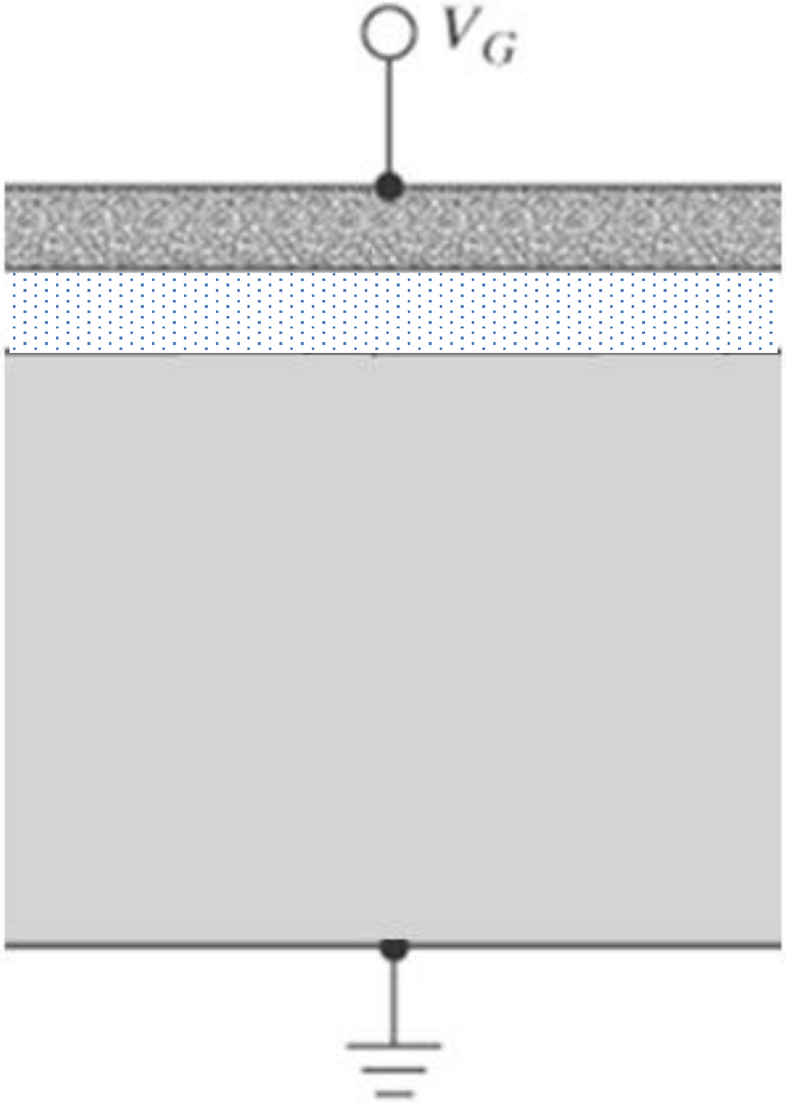
ECE 340 Lecture 32
Semiconductor Electronics

Spring 2022
10:00-10:50am
Professor Umberto Ravaioli
Department of Electrical and Computer Engineering
2062 ECE Building

Today's Discussion

- **MOS Capacitor (ideal model)**
  - **Flat band Voltage**
  - **Threshold Voltage**
  - **Capacitance**

Ideal MOSFET Capacitor



gate contact

oxide

Silicon bulk

substrate
contact

Ideal MOSFET Capacitor (Equilibrium)

flat band condition

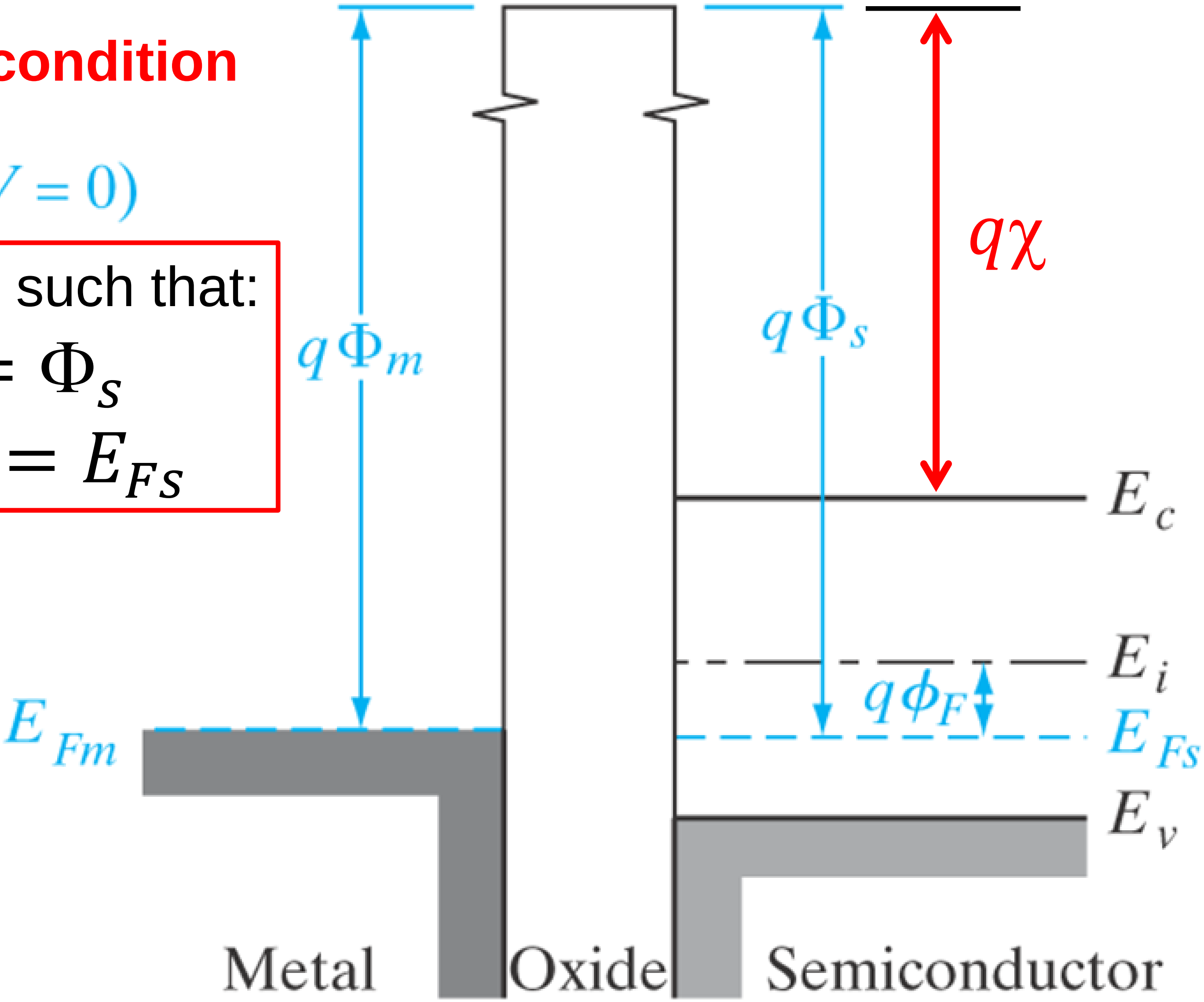
$(V = 0)$

choice of N_A such that:

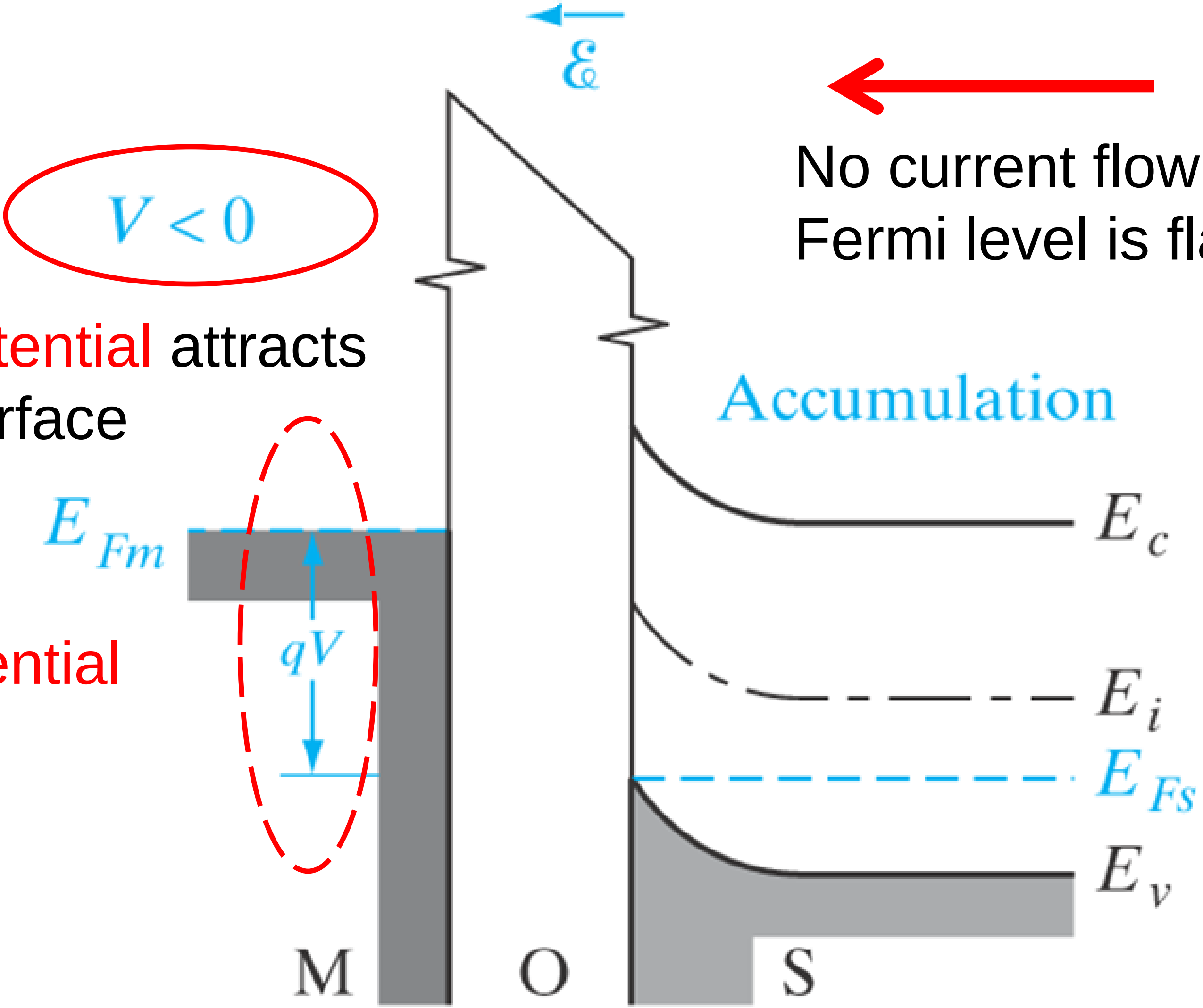
$$\Phi_m = \Phi_s$$

$$\rightarrow E_{Fm} = E_{Fs}$$

(a)



Ideal MOSFET Capacitor (Accumulation)

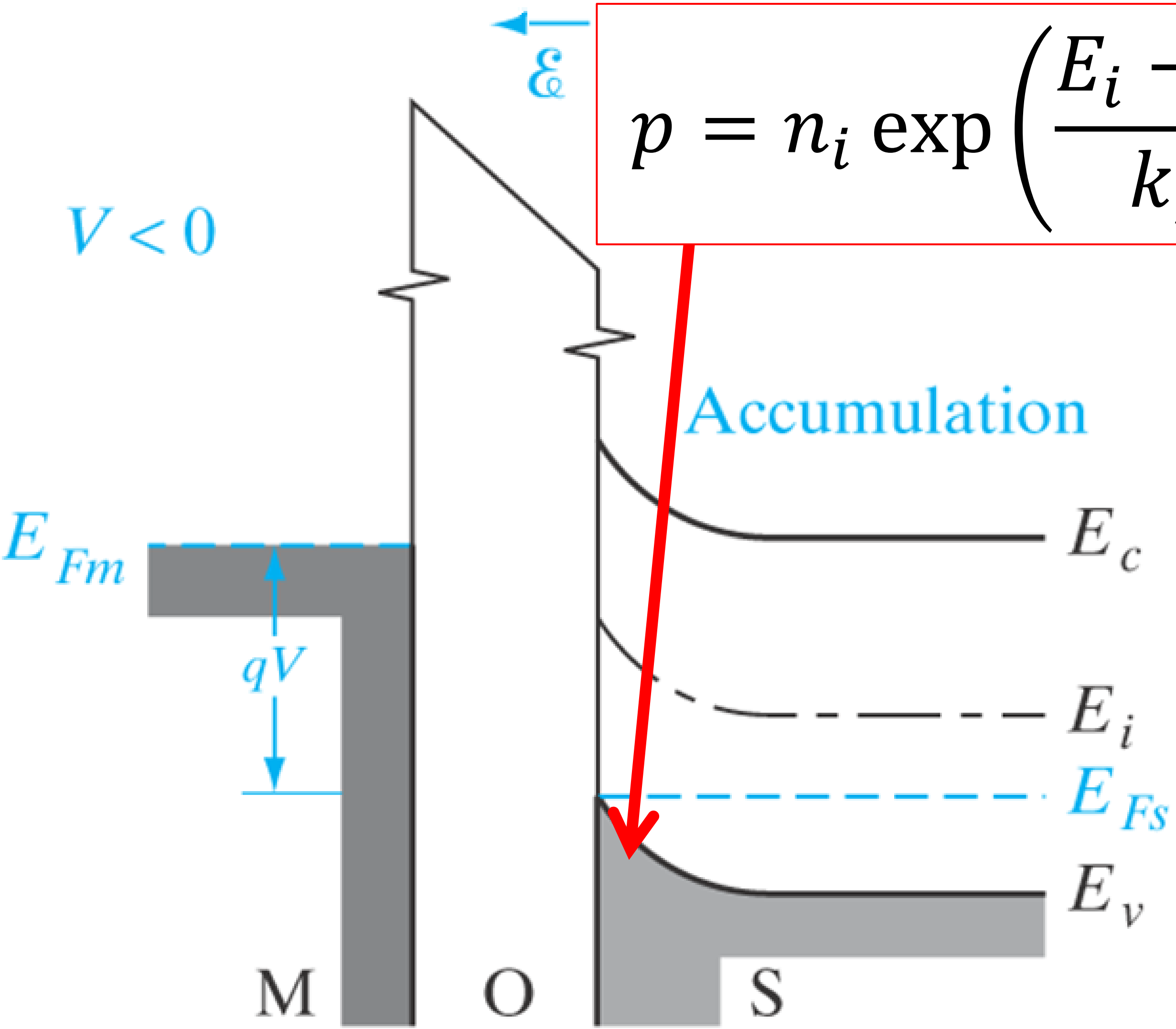


No current flow
Fermi level is flat

Negative potential attracts holes to interface

positive potential energy

Ideal MOSFET Capacitor (Accumulation)



$$p = n_i \exp\left(\frac{E_i - E_{Fs}}{k_B T}\right)$$

Ideal MOSFET Capacitor (Accumulation)

In accumulation:

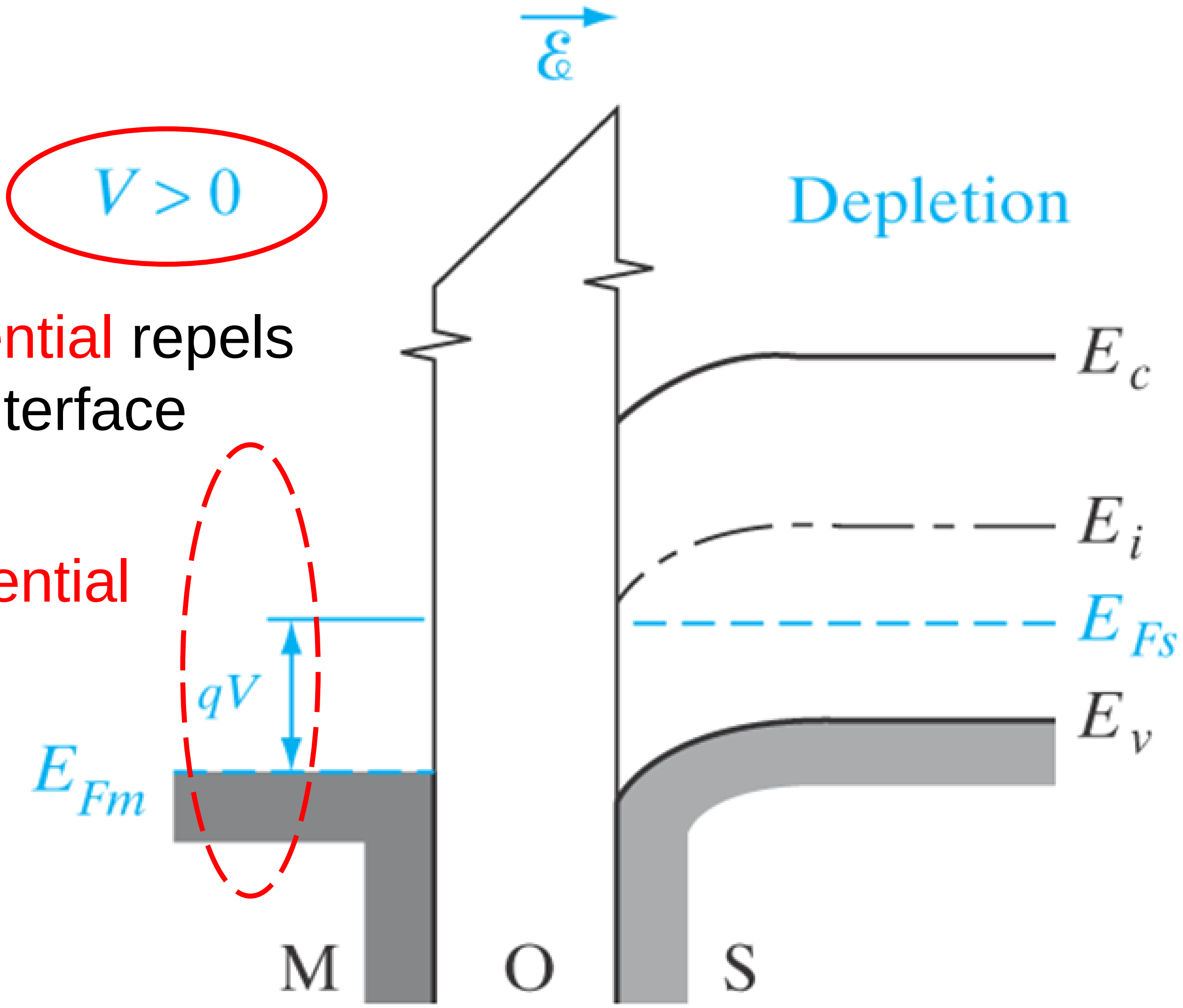
- The MOS capacitor is charged with electrons on the metal side and holes at the interface between *p*-type semiconductor and oxide.
- The capacitance is related to the oxide layer

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_{ox}}{d_{ox}}$$

d_{ox} = thickness of oxide layer

Ideal MOSFET Capacitor (Depletion)

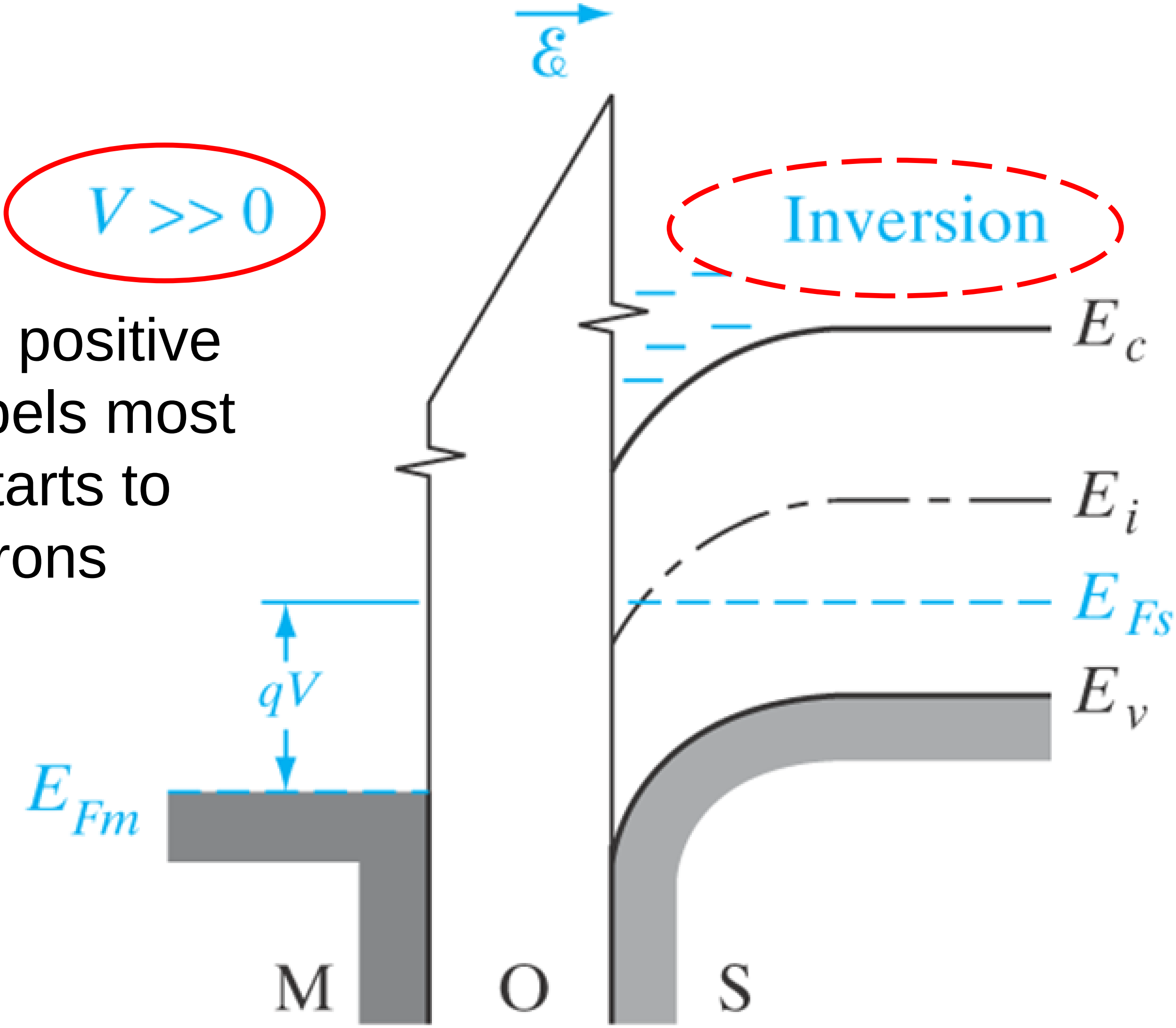


Positive potential repels holes from interface

negative potential energy

Ideal MOSFET Capacitor (Inversion)

Even higher positive potential repels most holes and starts to attract electrons



Ideal MOSFET Capacitor (Inversion)

Deep in strong inversion:

- The depletion layer no longer responds to changes in the potential because of screening due to the strong electron layer at the interface

- The capacitance is again due only to the oxide

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_{ox}}{d_{ox}}$$

d_{ox} = thickness of oxide layer

More detailed capacitance analysis

The MOS capacitance is the series of a fixed oxide (insulator) parallel plate capacitance, independent of voltage

$$\boxed{C_i = \frac{\varepsilon_{ox}}{d_{ox}}}$$

and of a voltage-dependent semiconductor depletion layer capacitance

$$\boxed{C_d = \frac{dQ}{dV} = \frac{dQ_s}{d\phi_s}}$$

$$\boxed{C_d = \frac{\varepsilon_s}{W}}$$

← check if typo in your edition

series of C_i and C_d

$$\boxed{C = C_i C_d / (C_i + C_d)}$$

More detailed capacitance analysis

Depletion capacitance model is approximate

$$\boxed{C_d = \frac{\varepsilon_S}{W}}$$

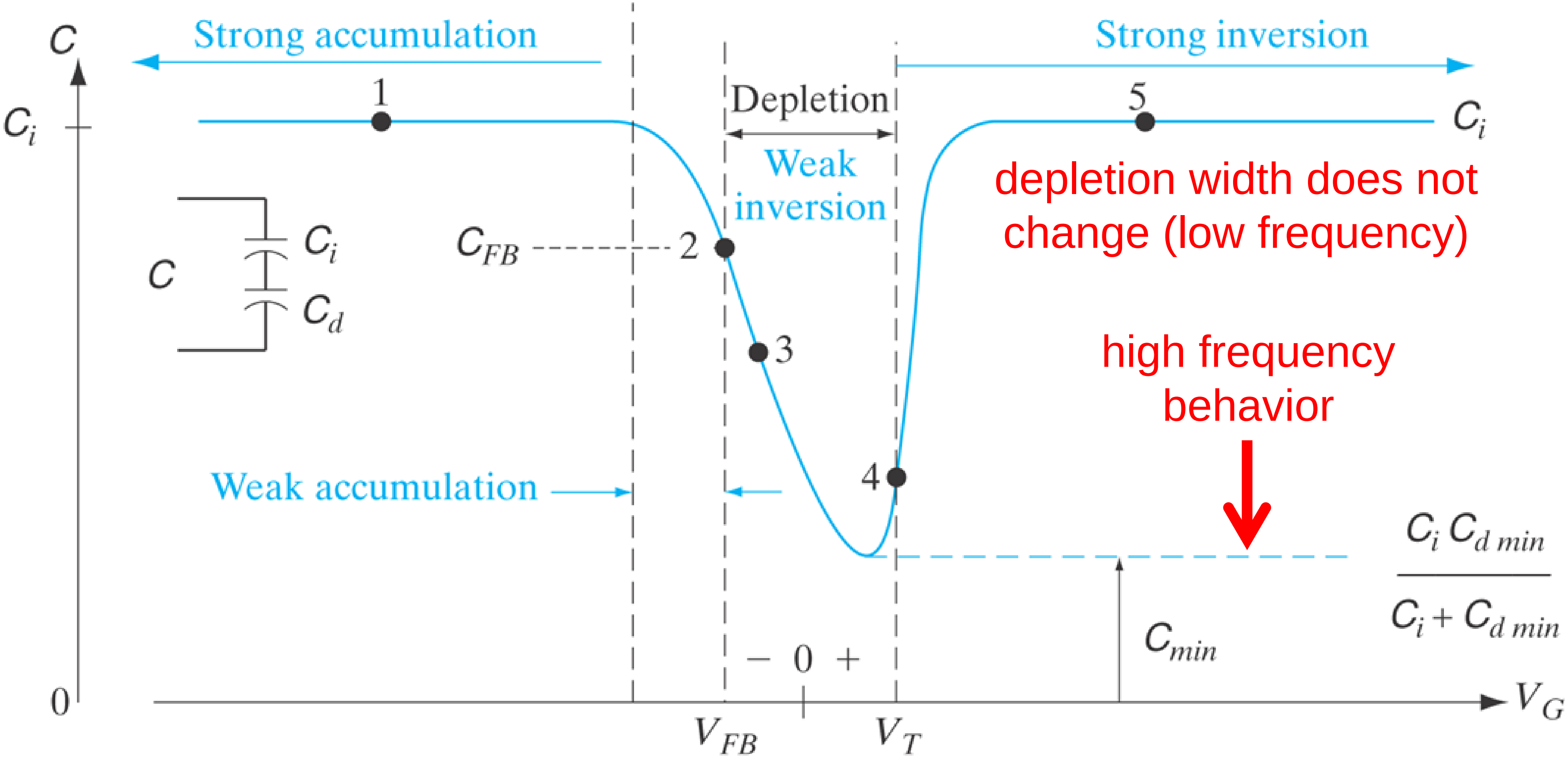
(it has highest error near flat band condition)

A better model for flat band condition indicates:

$$\boxed{C_{d,FB} = \frac{\varepsilon_S}{L_D}}$$

where L_D is the Debye length $L_D = \sqrt{\frac{\varepsilon_S k_B T}{q^2 p_0}}$

MOS Capacitance Measurement



depletion width does not change (low frequency)

high frequency behavior

oxide capacitance

$$C_i = \frac{\varepsilon_{ox}}{d}$$

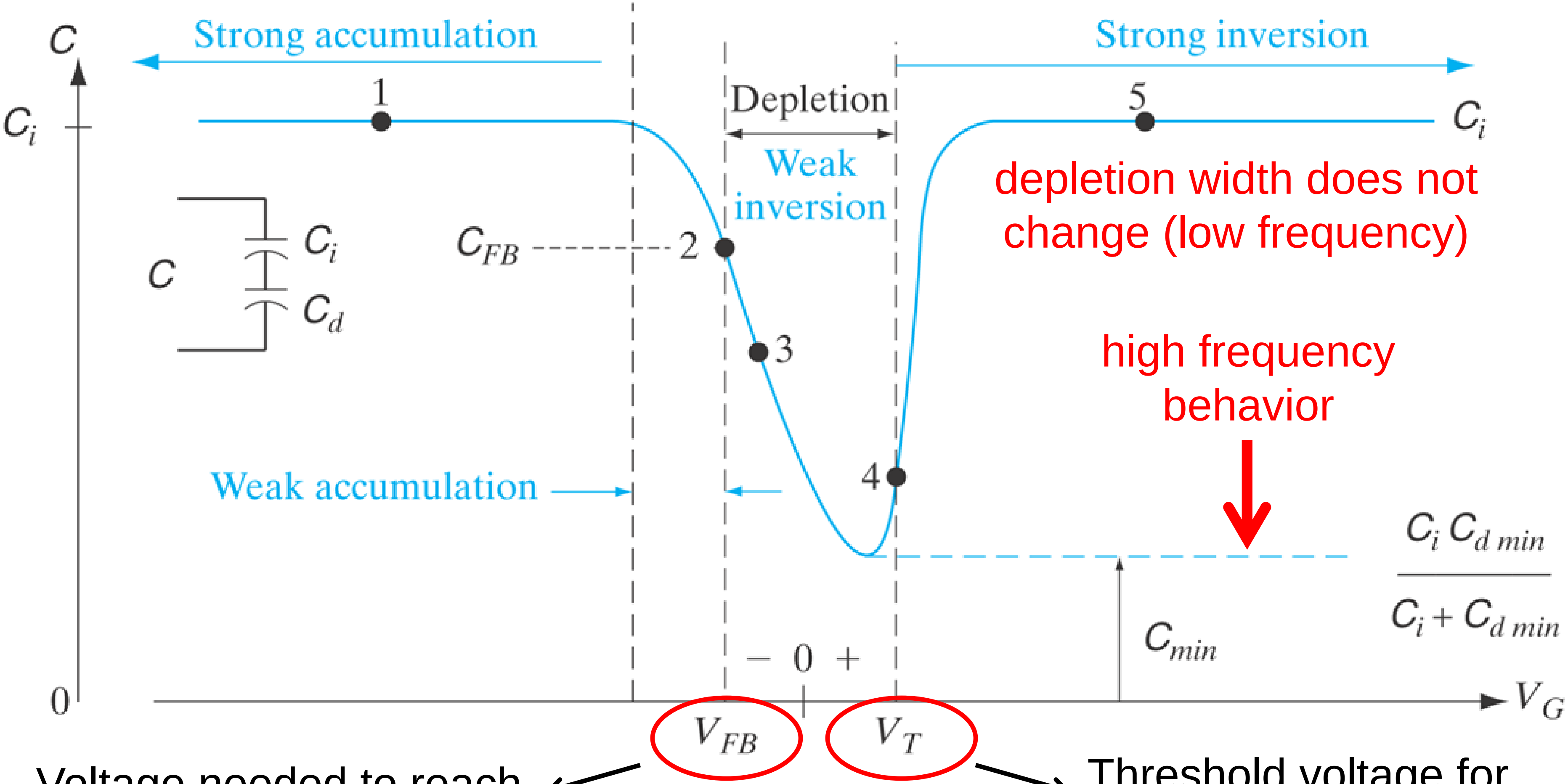
depletion capacitance

$$C_d = \frac{\varepsilon_s}{W}$$

series of C_i and C_d

$$C = C_i C_d/(C_i + C_d)$$

MOS Capacitance Measurement



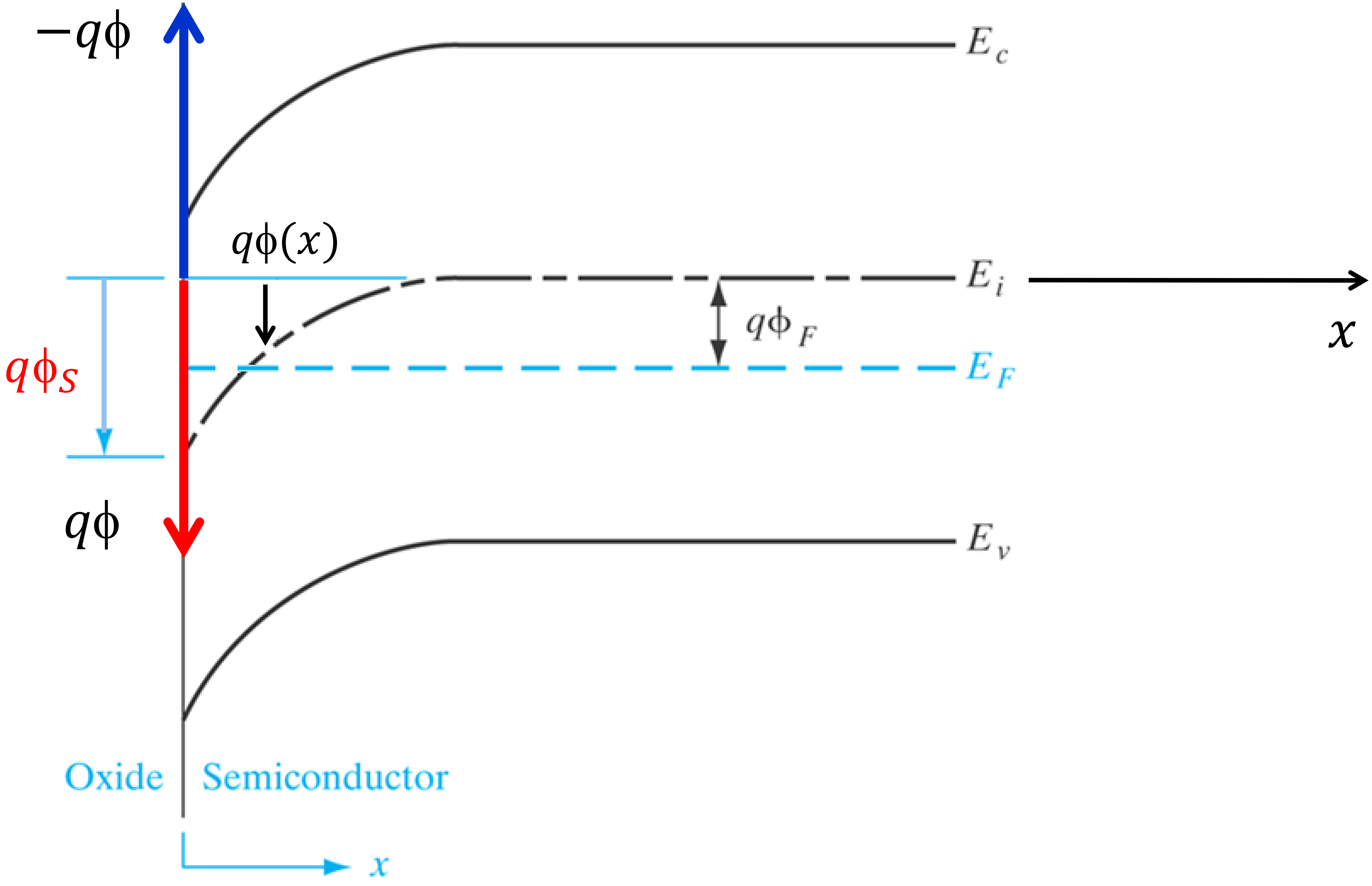
depletion width does not change (low frequency)

high frequency behavior

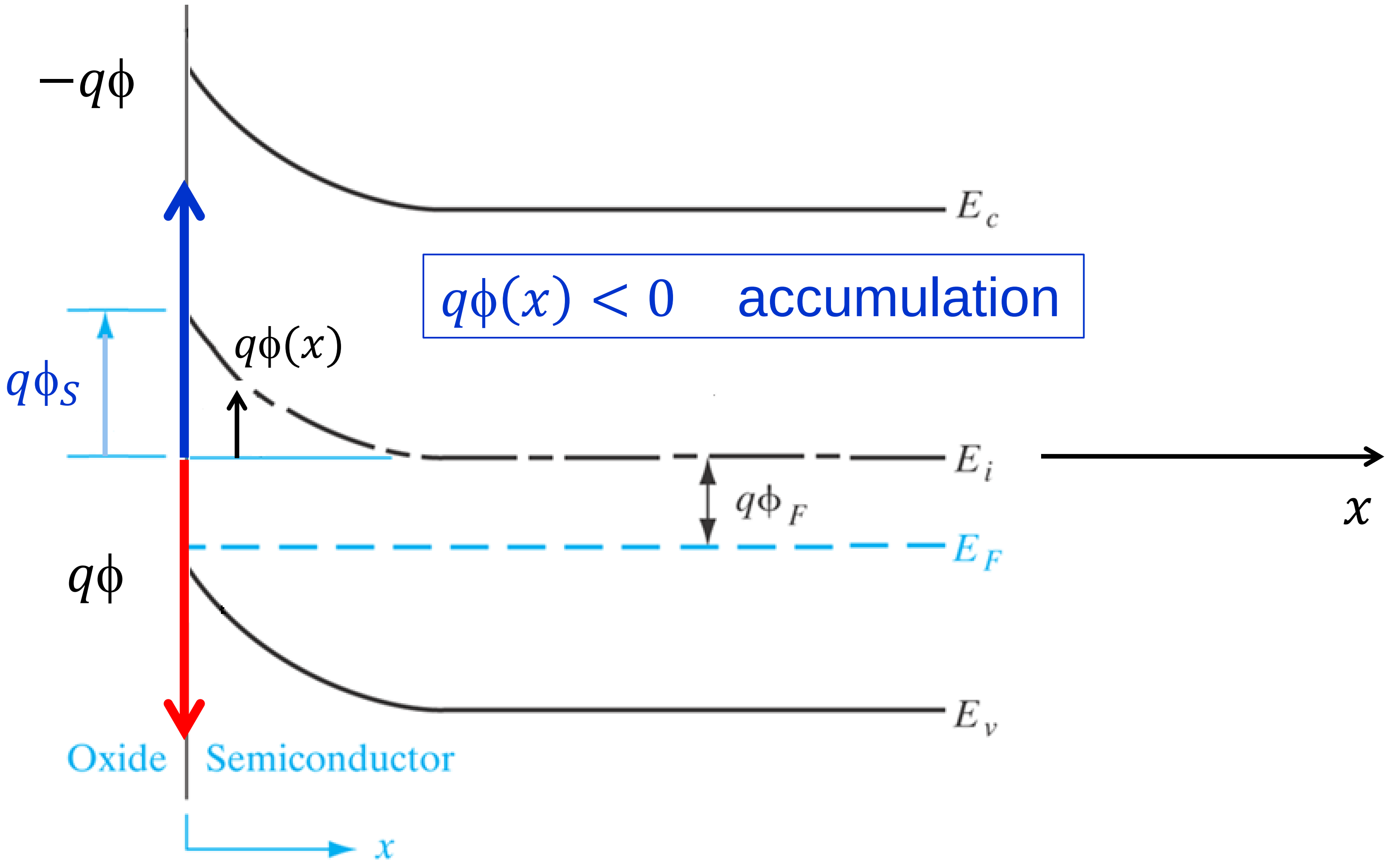
Voltage needed to reach flat band. This is zero if $E_{Fm} = E_{Fs}$

Threshold voltage for strong inversion

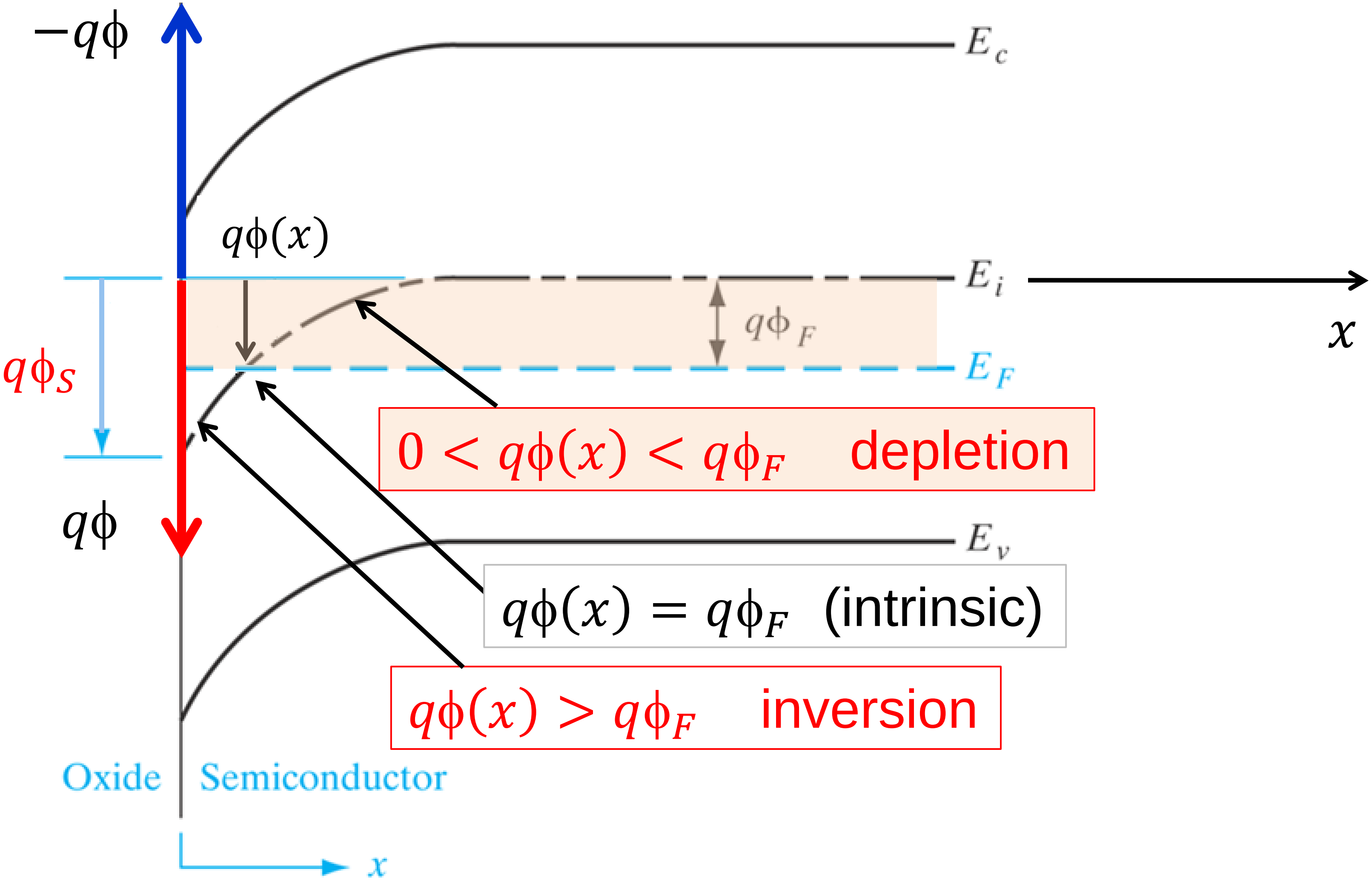
Potential energy system of reference



Potential energy system of reference



Potential energy system of reference

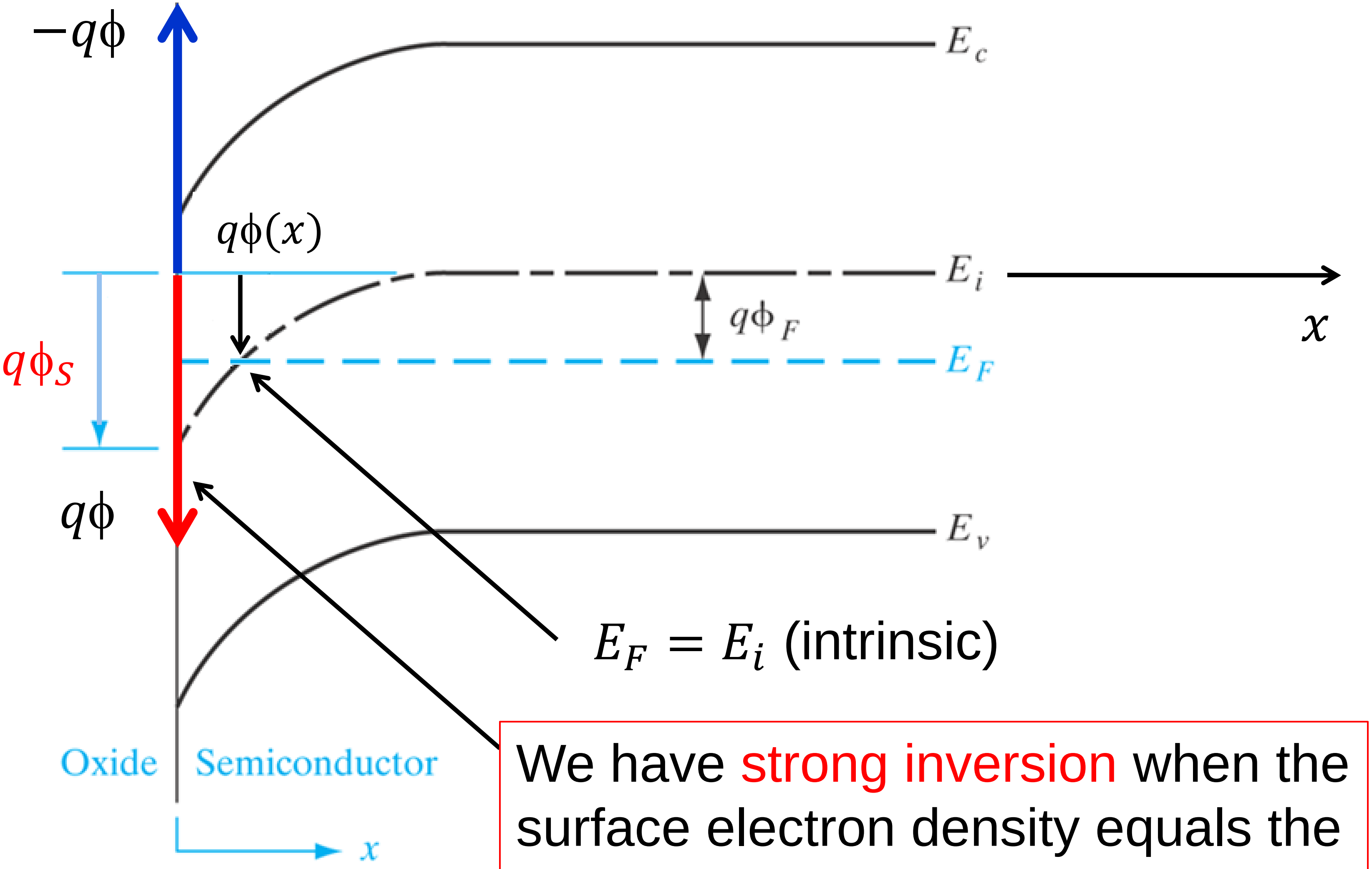


$0 < q\phi(x) < q\phi_F$ depletion

$q\phi(x) = q\phi_F$ (intrinsic)

$q\phi(x) > q\phi_F$ inversion

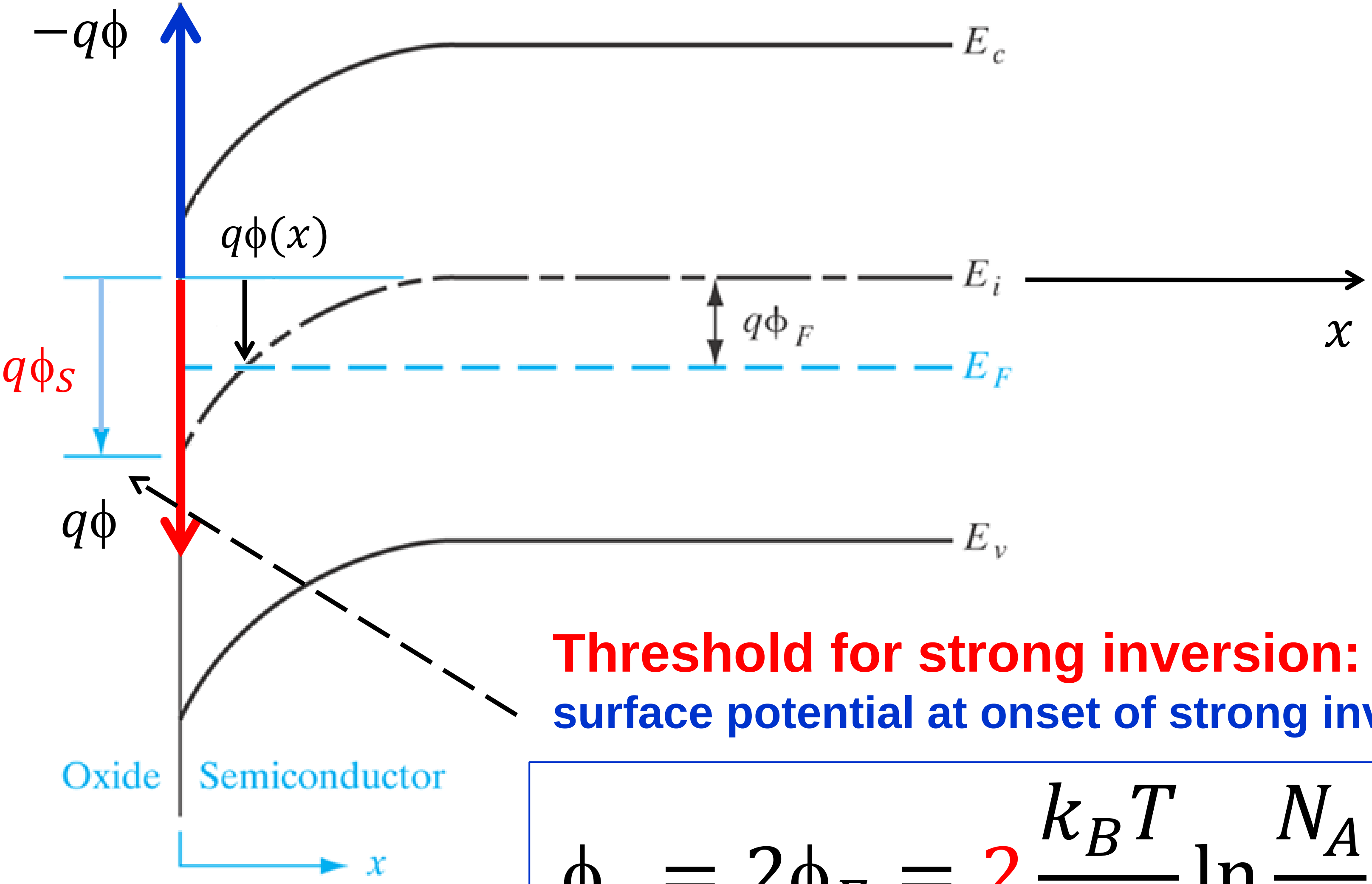
Strong inversion condition (definition)



$E_F = E_i$ (intrinsic)

We have strong inversion when the surface electron density equals the hole density in the bulk

Strong inversion condition (definition)

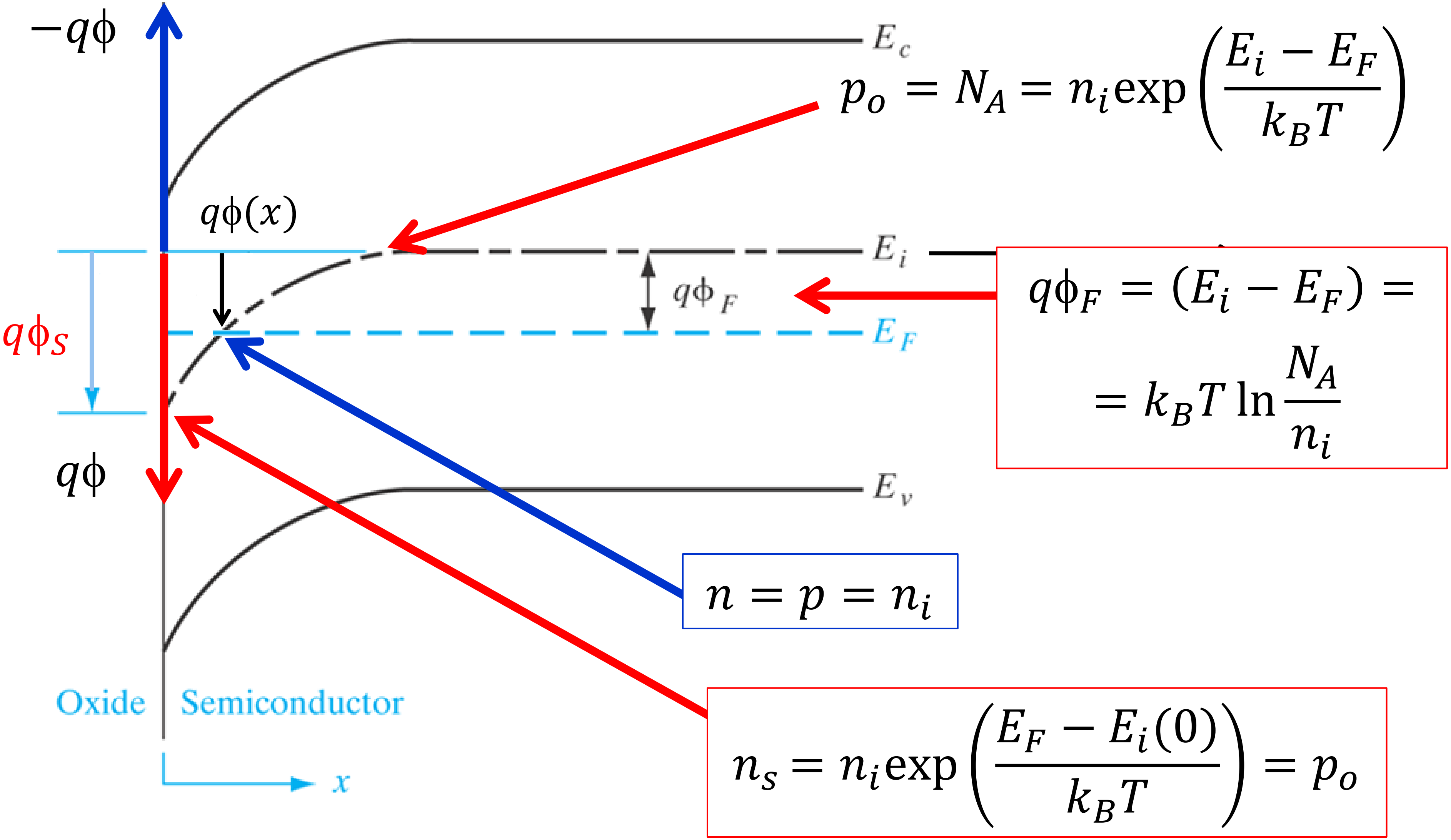


Threshold for strong inversion:

surface potential at onset of strong inversion

$$\boxed{\phi_s = 2\phi_F = 2\frac{k_B T}{q}\ln\frac{N_A}{n_i}}$$

Strong inversion condition



$$p_o = N_A = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)$$

$$q\phi_F = (E_i - E_F) = $$
$$= k_B T \ln \frac{N_A}{n_i}$$

$$n = p = n_i$$

$$n_s = n_i \exp\left(\frac{E_F - E_i(0)}{k_B T}\right) = p_o$$

Analytical model for $n(x)$

away from interface

$$n_o = n_i \exp\left(\frac{E_F - E_i}{k_B T}\right) = n_i \exp\left(-\frac{q\phi_F}{k_B T}\right)$$

at any x location

$$n(x) = n_i \exp\left(\frac{E_F - E_i(x)}{k_B T}\right) = n_i \exp\left(-q\frac{\phi_F - \phi(x)}{k_B T}\right) =$$

$$= \underbrace{n_i \exp\left(-q\frac{\phi_F}{k_B T}\right)}_{\boldsymbol{n_o}} \exp\left(q\frac{\phi(x)}{k_B T}\right) = n_0 \exp\left(q\frac{\phi(x)}{k_B T}\right)$$

Analytical model for $p(x)$

away from interface

$$p_o = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right) = n_i \exp\left(\frac{q\phi_F}{k_B T}\right)$$

at any x location

$$p(x) = n_i \exp\left(\frac{E_i(x) - E_F}{k_B T}\right) = n_i \exp\left(-q\frac{\phi(x) - \phi_F}{k_B T}\right) =$$

$$= \underbrace{n_i \exp\left(q\frac{\phi_F}{k_B T}\right)}_{\boldsymbol{p_o}} \exp\left(-q\frac{\phi(x)}{k_B T}\right) = p_0 \exp\left(-q\frac{\phi(x)}{k_B T}\right)$$

Analytical model for $\mathcal{E}(x)$

We do not know the exact behavior of $\phi(x)$ but we know the relationship between charge and potential

Poisson equation

$$\boxed{\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\epsilon_s}}$$

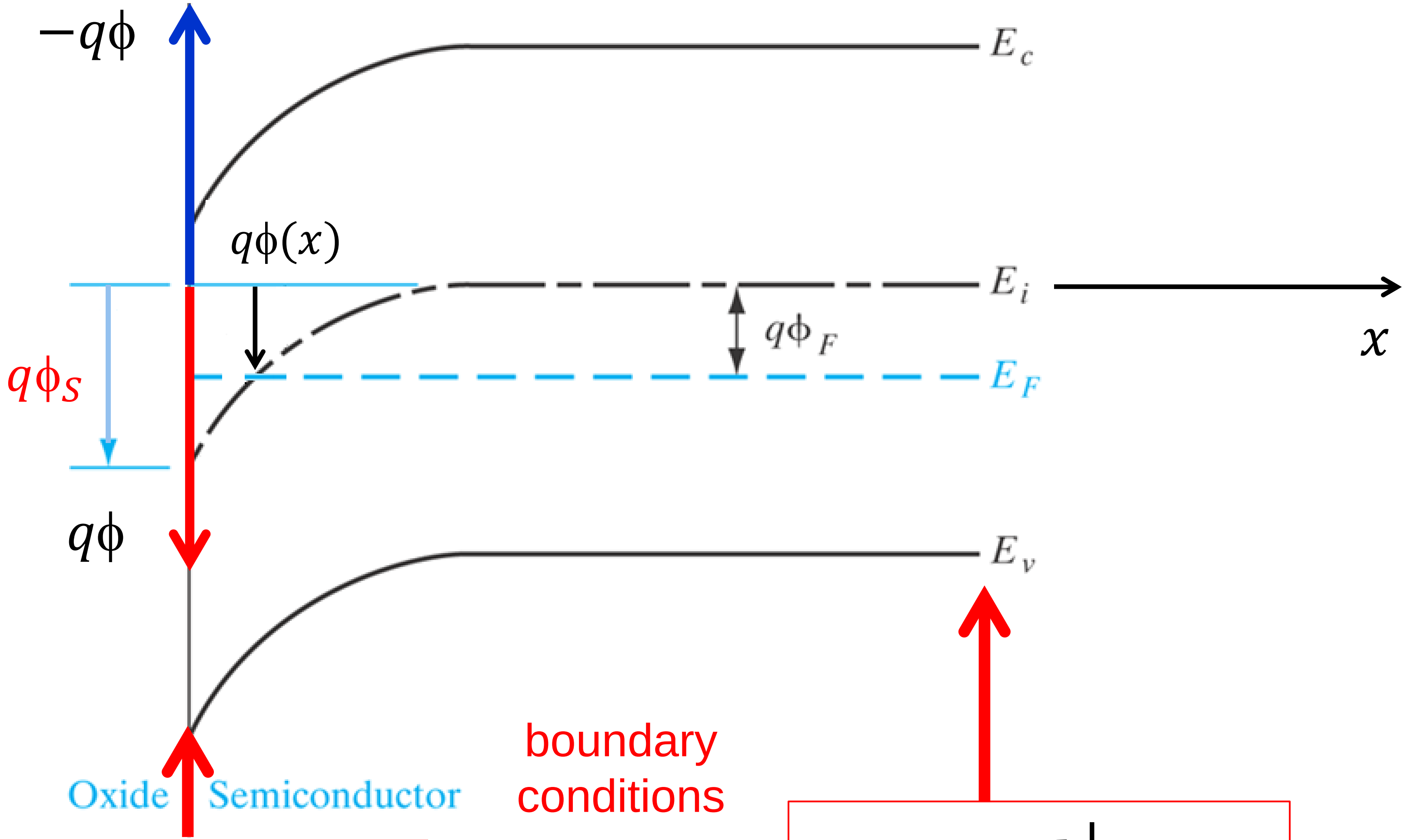
with charge density

$$\boxed{\rho(x) = q[N_D^+ - N_A^- + p(x) - n(x)]}$$

Electric Field

$$\boxed{\mathcal{E} = -\frac{d\phi}{dx}}$$

Analytical model for $\mathcal{E}(x)$



boundary conditions

$$\phi(0) = \phi_s$$

$$\mathcal{E} = -\frac{d\phi}{dx} = 0$$

Analytical model for $\mathcal{E}(x)$

$$\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi}{dx}\right) =$$

$$= -\frac{q}{\epsilon_s}\left\{p_o\left[\exp\left(-\frac{q\phi}{k_B T}\right) - 1\right] - n_o\left[\exp\left(\frac{q\phi}{k_B T}\right) - 1\right]\right\}$$

$$\boxed{N_D^+ - N_A^- = n_o - p_o}$$

$$\int_0^{\frac{d\phi}{dx}} \frac{d}{dx}\left(\frac{d\phi}{dx}\right) = \int_0^{\phi} RHS$$

$x = 0$

$$\boxed{\phi(0) = \phi_s}$$

← integrate

$x \to \infty$

$$\boxed{\mathcal{E} = -\frac{d\phi}{dx} = 0}$$

Analytical model for $\mathcal{E}(x)$

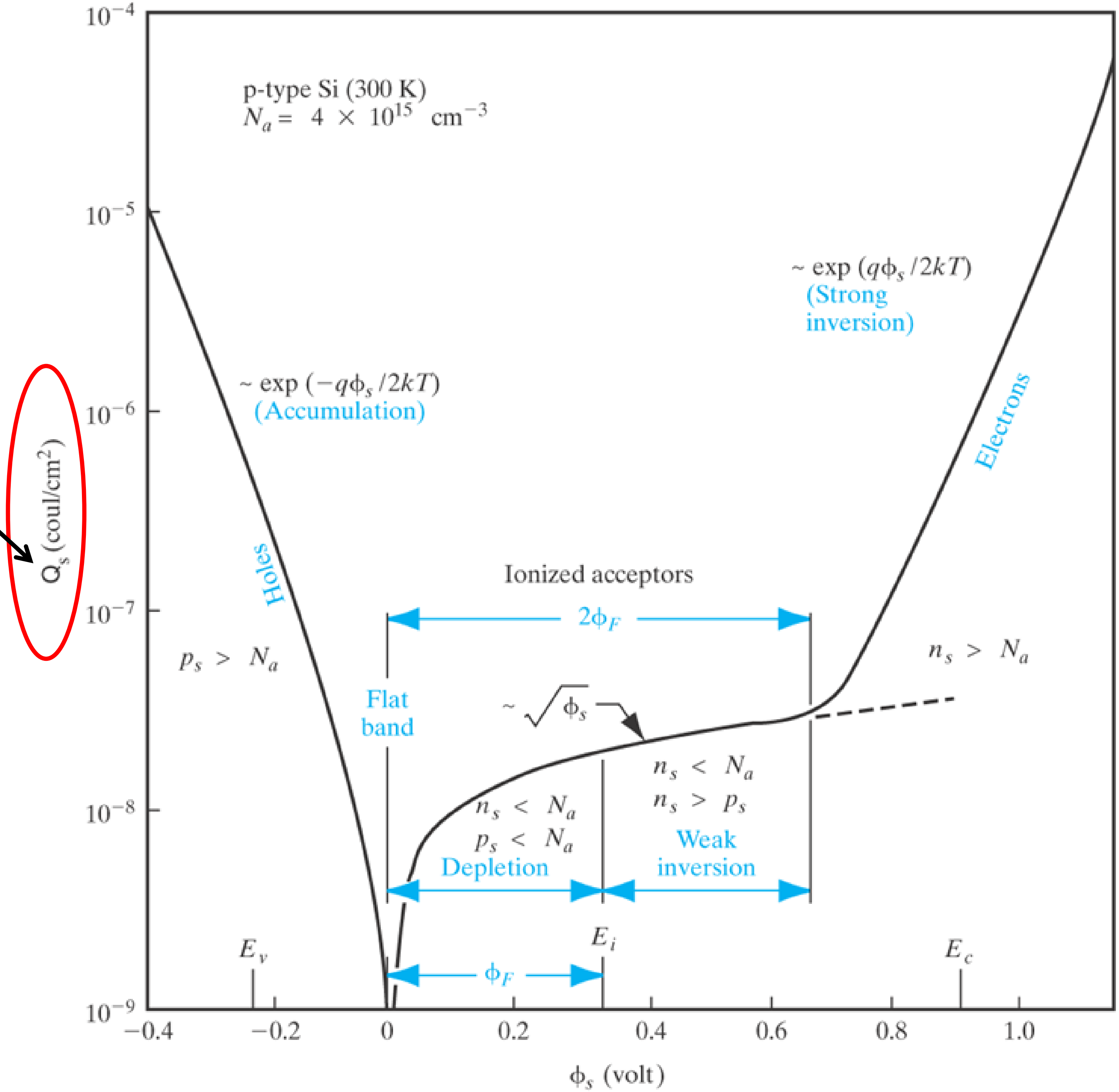
Solution result:

$$\mathcal{E}^2 = \frac{2k_BT}{\varepsilon_s} p_0 \left[\left(\exp\left(-\frac{q\phi}{k_BT} \right) + \frac{q\phi}{k_BT} - 1 \right) + \frac{n_0}{p_0} \left(\exp\left(\frac{q\phi}{k_BT} \right) - \frac{q\phi}{k_BT} - 1 \right) \right]$$

At the surface where $x = 0$, ϕ_S , $\mathcal{E}_S$

$$\mathcal{E}_S = \frac{\sqrt{2}k_BT}{qL_D} \sqrt{\left(\exp\left(-\frac{q\phi_S}{k_BT} \right) + \frac{q\phi_S}{k_BT} - 1 \right) + \frac{n_0}{p_0} \left(\exp\left(\frac{q\phi_S}{k_BT} \right) - \frac{q\phi_S}{k_BT} - 1 \right)}$$

where $L_D = \sqrt{\dfrac{\varepsilon_S k_BT}{q^2 p_0}}$ is the Debye Length



Absolute value

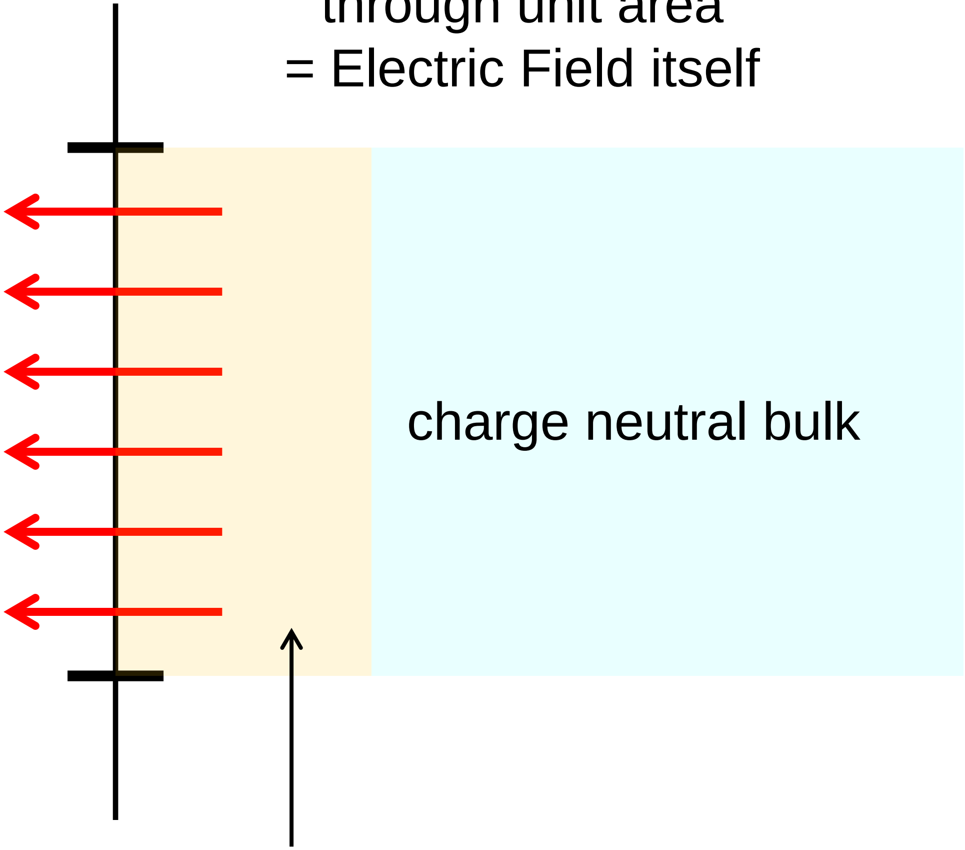
Analytical model for $\mathscr{E}(x)$

Gauss Law at the Surface

$$\boxed{\mathscr{E}_S = -\frac{Q_S}{\varepsilon_S}}$$

Space charge density per unit area

$$Q_S = -\varepsilon_S \mathscr{E}_S$$



flux of the Electric Field
through unit area
= Electric Field itself

unit area
1.0 cm^2

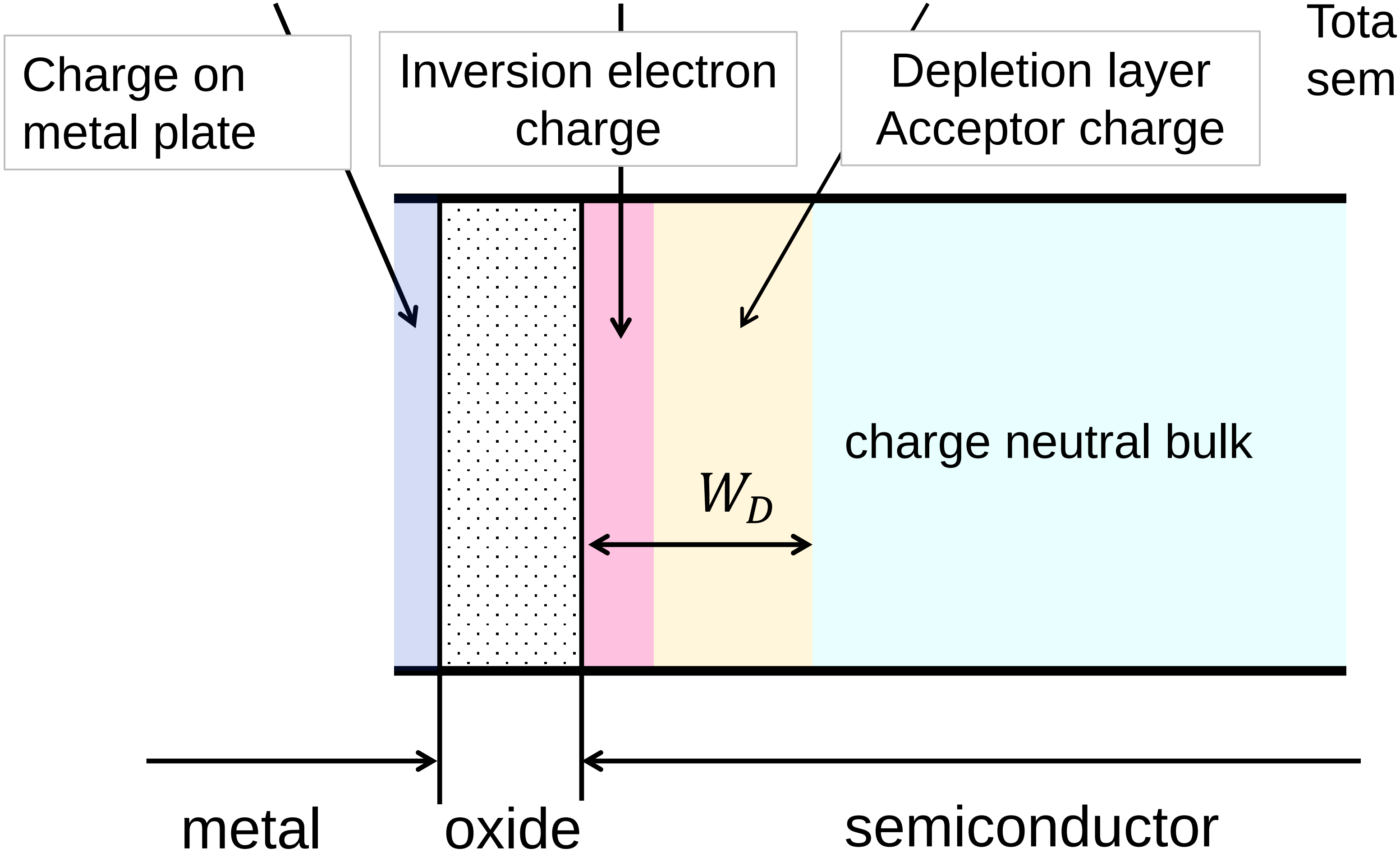
charge neutral bulk

space charge close to surface

Charge density distribution

$$Q_M = -Q_n + qN_A W_D = -Q_S$$

Total charge in semiconductor



Charge on metal plate

Inversion electron charge

Depletion layer Acceptor charge

charge neutral bulk

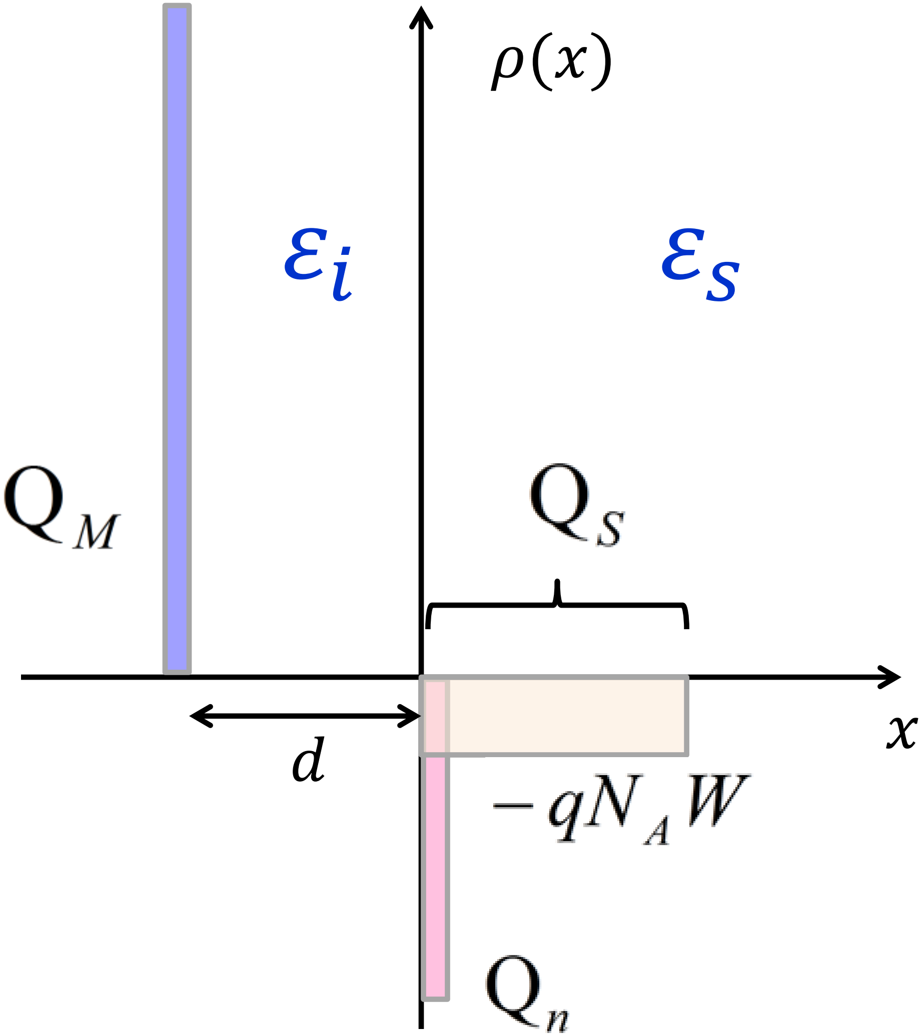
W_D

metal

oxide

semiconductor

Charge density distribution



d = thickness of oxide

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_i}{d}$$

Voltage across oxide

$$V_i = \frac{-Q_s}{C_i} = \frac{-Q_s d}{\varepsilon_i}$$

Applied voltage

$$V = V_i + \phi_s$$

Depletion Layer Width

Similar to result for n^+- p junction

$$W = \sqrt{\frac{2\epsilon_s \phi_s}{qN_A}}$$

$$\boldsymbol{\phi_S < 2\phi_F}$$

At strong inversion, depletion region no longer grows, due to screening of interface electrons

strong inversion
$\phi_S = 2\phi_F$

$$W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}} = 2\sqrt{\frac{\epsilon_s k_B T \ln \frac{N_A}{n_i}}{q^2 N_A}}$$

Depletion charge at threshold

$$W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}}$$

strong inversion limit
$\phi_S = 2\phi_F$

$$Q_D = -qN_A W_{max} = -qN_A \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}}$$

$$Q_D = -2\sqrt{q\epsilon_s N_A \phi_F}$$

Threshold Voltage (ideal case)

$$\boxed{Q_D = -qN_AW = -2\sqrt{q\epsilon_s N_A \phi_F}}$$

maximum value

$$V_T = \underbrace{-\frac{\overbrace{Q_D}^{\text{Depletion layer charge}}}{C_i}}_{\text{Voltage drop across oxide}} + \underbrace{2\phi_F}_{\text{Strong inversion condition}}$$

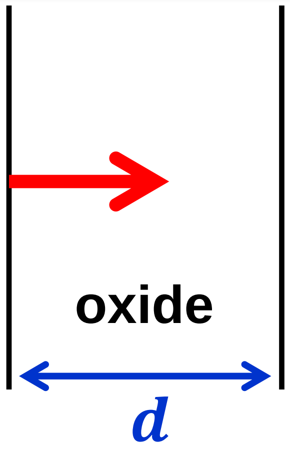
(Assuming that depletion charge dominates Q_s at threshold)

Above, $-Q_D$ represents Q_m to give the potential drop across the oxide.

Potential drop across the oxide

unit area

$\mathcal{E}_{ox} = Q_m / \epsilon_{ox}$



semiconductor

Electric Field in the oxide is constant in absence of charge

Voltage drop across oxide

C_i^{-1} (per unit area)

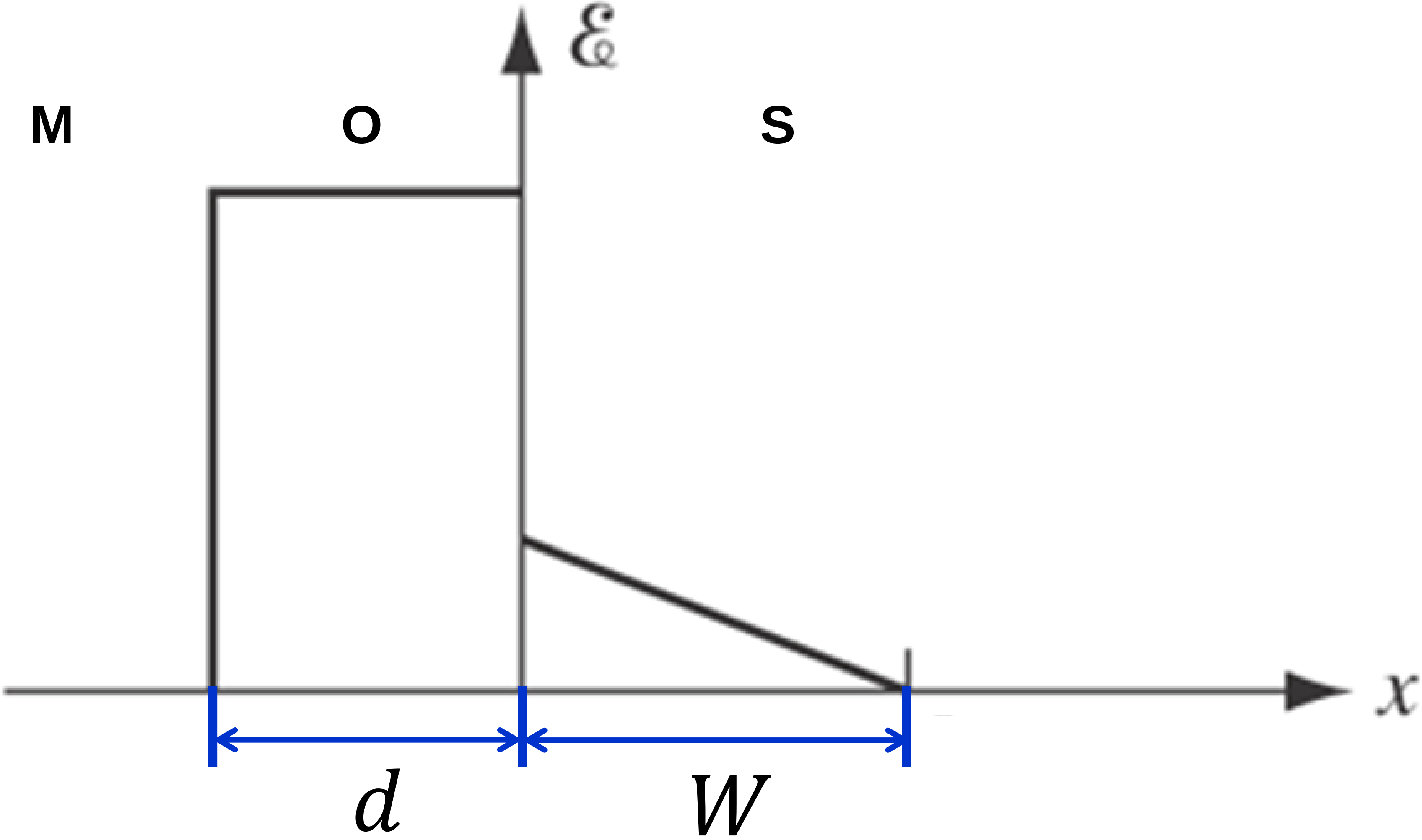
$$V_i = \int_0^d \frac{Q_m}{\epsilon_{ox}}\, dx = Q_m \boxed{\frac{d}{\epsilon_{ox}}} \approx -\frac{Q_D}{C_i}$$

at threshold

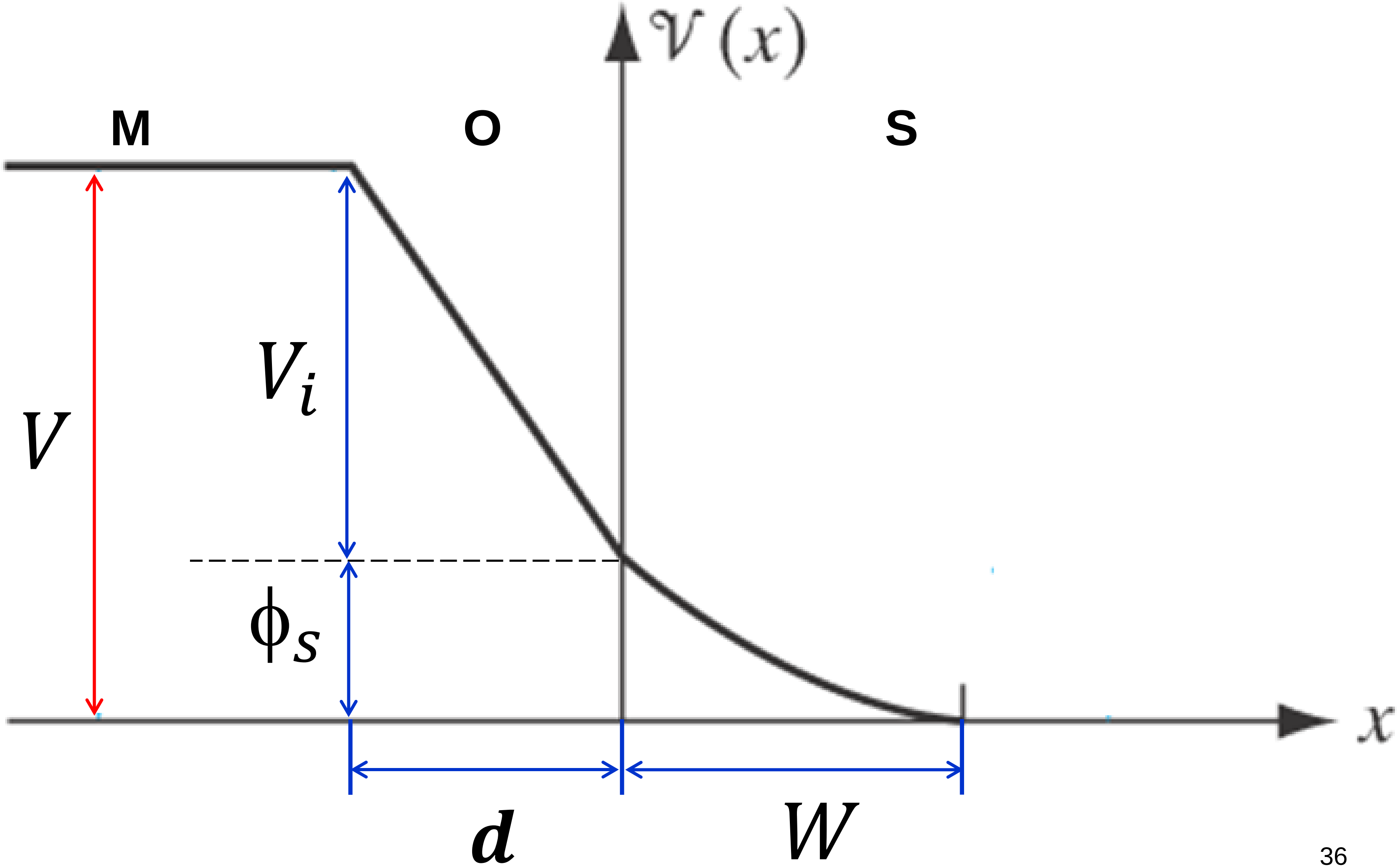
$Q_m \approx -Q_D$

$\epsilon_{ox} = \epsilon_{rox} \epsilon_0$

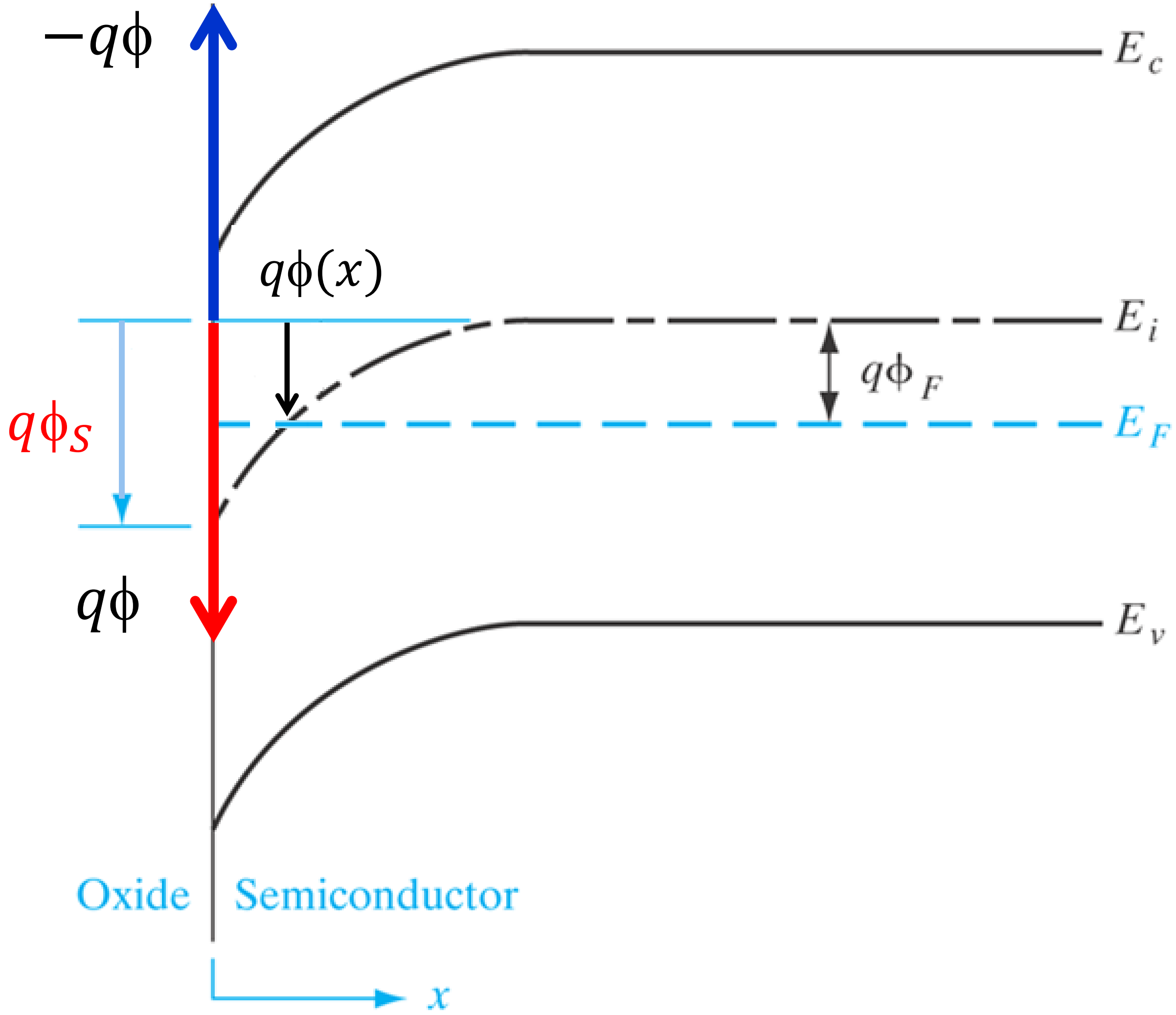
Electric Field distribution



Electric Potential distribution



Summary of conditions – surface potential



accumulation
$\phi_S < 0$

flat-band
$\phi_S = 0$

depletion
$0 < \phi_S < \phi_F$

intrinsic
$\phi_S = \phi_F$

weak inversion
$2\phi_F > \phi_S > \phi_F$

strong inversion
$\phi_S \geq 2\phi_F$