ECE 340 Lectures 34
Semiconductor Electronics

Spring 2022
10:00-10:50am
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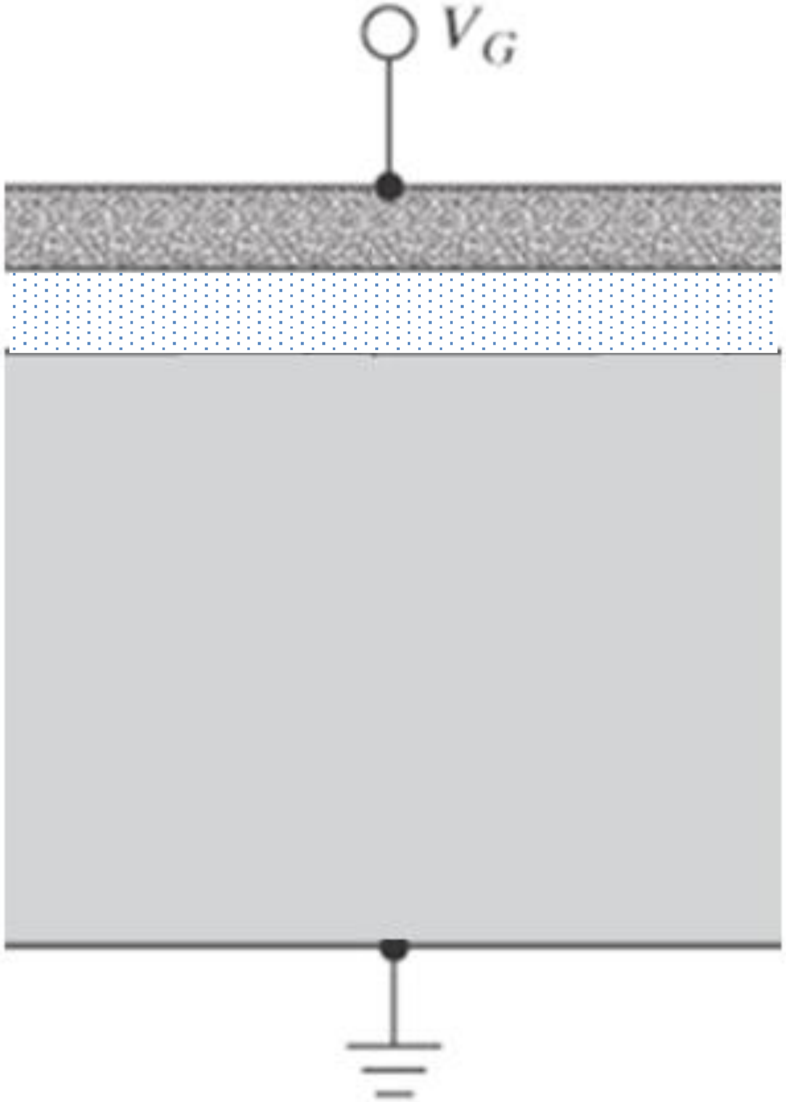
2062 ECE Building

Today's Discussion

- **MOS Capacitor (conclusion)**
  - **Capacitance-Voltage Analysis**
  - **Realistic effects**

We have studied the Ideal MOSFET Capacitor



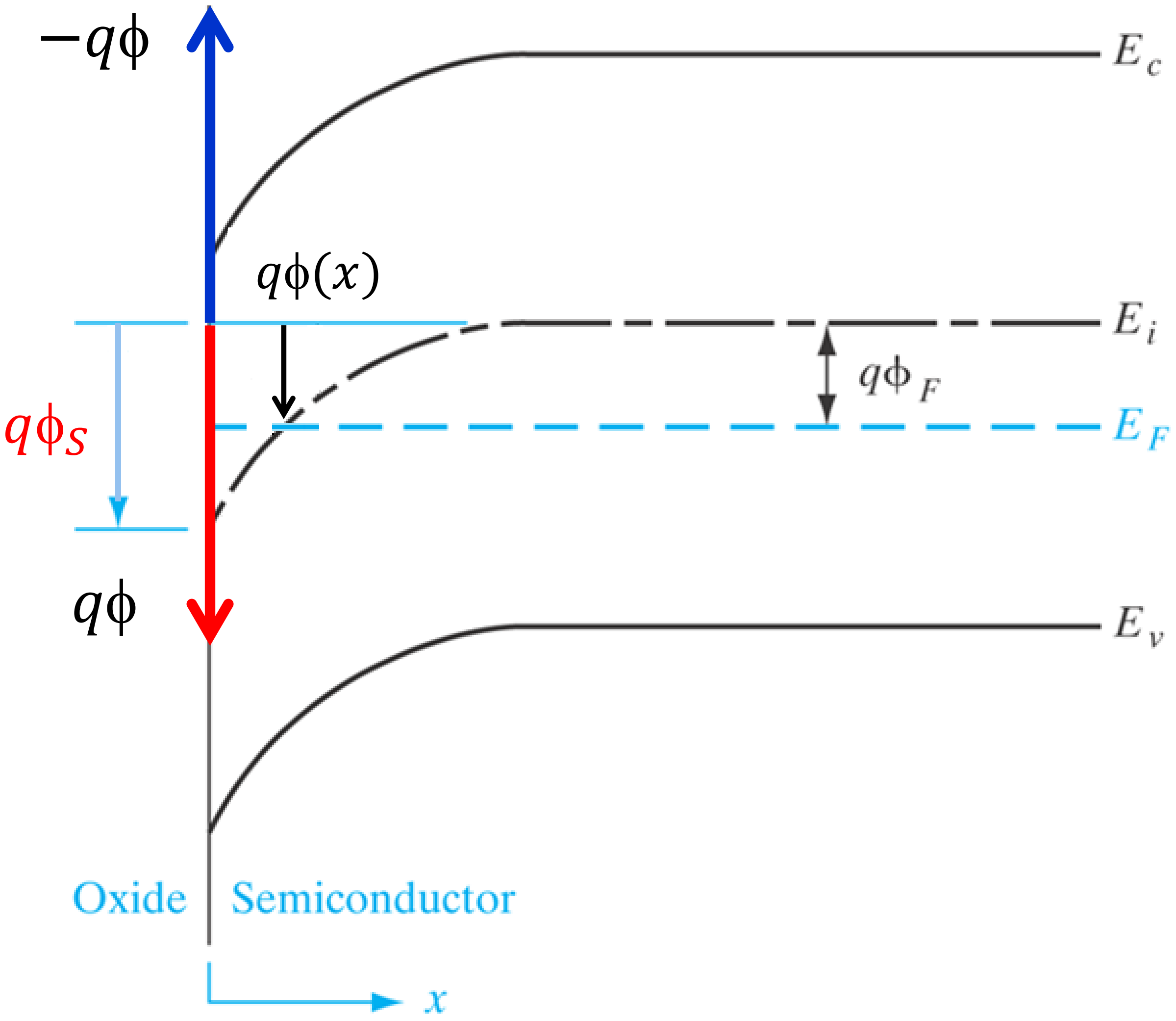
gate contact

oxide

Silicon bulk

substrate
contact

Summary of conditions – surface potential



accumulation
$\phi_S < 0$

flat-band
$\phi_S = 0$

depletion
$0 < \phi_S < \phi_F$

intrinsic
$\phi_S = \phi_F$

weak inversion
$2\phi_F > \phi_S > \phi_F$

strong inversion
$\phi_S \geq 2\phi_F$

Analytical model for $\mathcal{E}(x)$

We do not know the exact behavior of $\phi(x)$ but we know the relationship between charge and potential

Poisson equation

$$\boxed{\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\epsilon_s}}$$

with charge density

$$\boxed{\rho(x) = q[N_D^+ - N_A^- + p(x) - n(x)]}$$

Electric Field

$$\boxed{\mathcal{E} = -\frac{d\phi}{dx}}$$

From Last Lecture: Analytical model for $n(x)$

away from interface

$$n_o = n_i \exp\left(\frac{E_F - E_i}{k_B T}\right) = n_i \exp\left(-\frac{q\phi_F}{k_B T}\right)$$

at any x location

$$n(x) = n_i \exp\left(\frac{E_F - E_i(x)}{k_B T}\right) = n_i \exp\left(-q\frac{\phi_F - \phi(x)}{k_B T}\right) =$$

$$= \underbrace{n_i \exp\left(-q\frac{\phi_F}{k_B T}\right)}_{\boldsymbol{n_o}} \exp\left(q\frac{\phi(x)}{k_B T}\right) = n_0 \exp\left(q\frac{\phi(x)}{k_B T}\right)$$

From Last Lecture: Analytical model for $p(x)$

away from interface

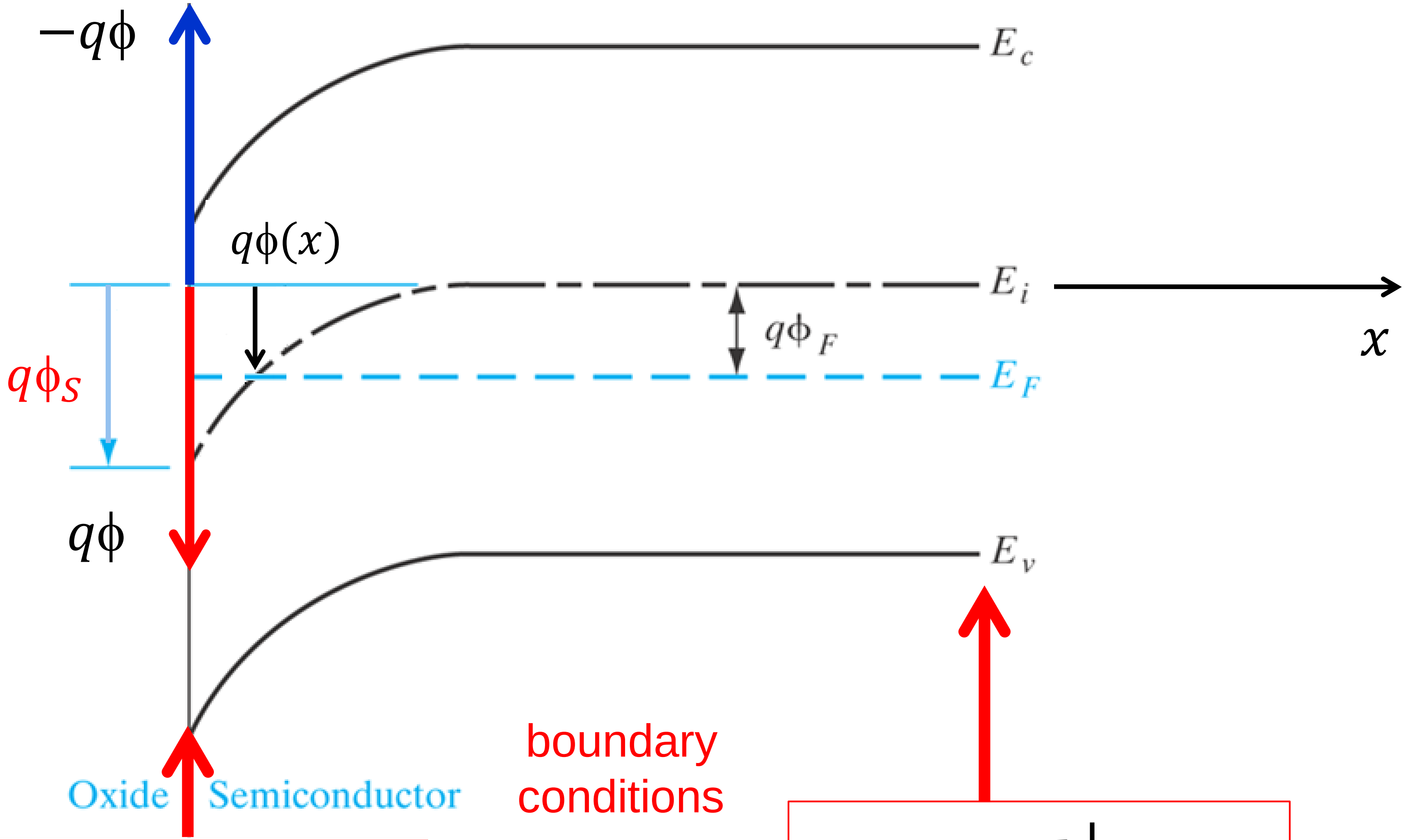
$$p_o = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right) = n_i \exp\left(\frac{q\phi_F}{k_B T}\right)$$

at any x location

$$p(x) = n_i \exp\left(\frac{E_i(x) - E_F}{k_B T}\right) = n_i \exp\left(-q\frac{\phi(x) - \phi_F}{k_B T}\right) =$$

$$= \underbrace{n_i \exp\left(q\frac{\phi_F}{k_B T}\right)}_{\boldsymbol{p_o}} \exp\left(-q\frac{\phi(x)}{k_B T}\right) = p_0 \exp\left(-q\frac{\phi(x)}{k_B T}\right)$$

Analytical model for $\mathcal{E}(x)$



boundary conditions

$$\phi(0) = \phi_s$$

$$\mathcal{E} = -\frac{d\phi}{dx} = 0$$

Analytical model for $\mathcal{E}(x)$

$$\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi}{dx}\right) = -\frac{\rho(x)}{\epsilon_s}$$

$$\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi}{dx}\right) = -\frac{q}{\epsilon_s}[\underbrace{N_D^+ - N_A^-} + p(x) - n(x)]$$

$$\boxed{N_D^+ - N_A^- = n_o - p_o}$$

$$\boxed{p(x) = p_0 \exp\left(-q\frac{\phi(x)}{k_B T}\right)}$$

$$\boxed{n(x) = n_0 \exp\left(q\frac{\phi(x)}{k_B T}\right)}$$

Analytical model for $\mathcal{E}(x)$

$$\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi}{dx}\right) =$$

$$= \overbrace{-\frac{q}{\epsilon_s}\left\{p_o\left[\exp\left(-\frac{q\phi}{k_BT}\right)\underbrace{-1}_{-p_o}\right] - n_o\left[\exp\left(\frac{q\phi}{k_BT}\right)\underbrace{-1}_{n_o}\right]\right\}}^{\color{red}{RHS}\ \text{(Right Hand Side)}}$$

$$\boxed{N_D^+ - N_A^- = n_o - p_o}$$

Analytical model for $\mathcal{E}(x)$

$$\int_{0}^{\frac{d\phi}{dx}} \frac{d}{dx}\left(\frac{d\phi}{dx}\right) = \int_{0}^{\phi(x)} (RHS)\, d\phi$$

$$\boxed{\mathcal{E} = -\frac{d\phi}{dx} = 0} \longrightarrow \boxed{\phi(0) = \phi_s}$$

$x \to \infty$

Integrate from the bottom to any coordinate x, toward the oxide interface, where we have

$x = 0$

$$\boxed{\mathcal{E}\,(x)} \qquad \boxed{\phi(x)}$$

Analytical model for $\mathcal{E}(x)$

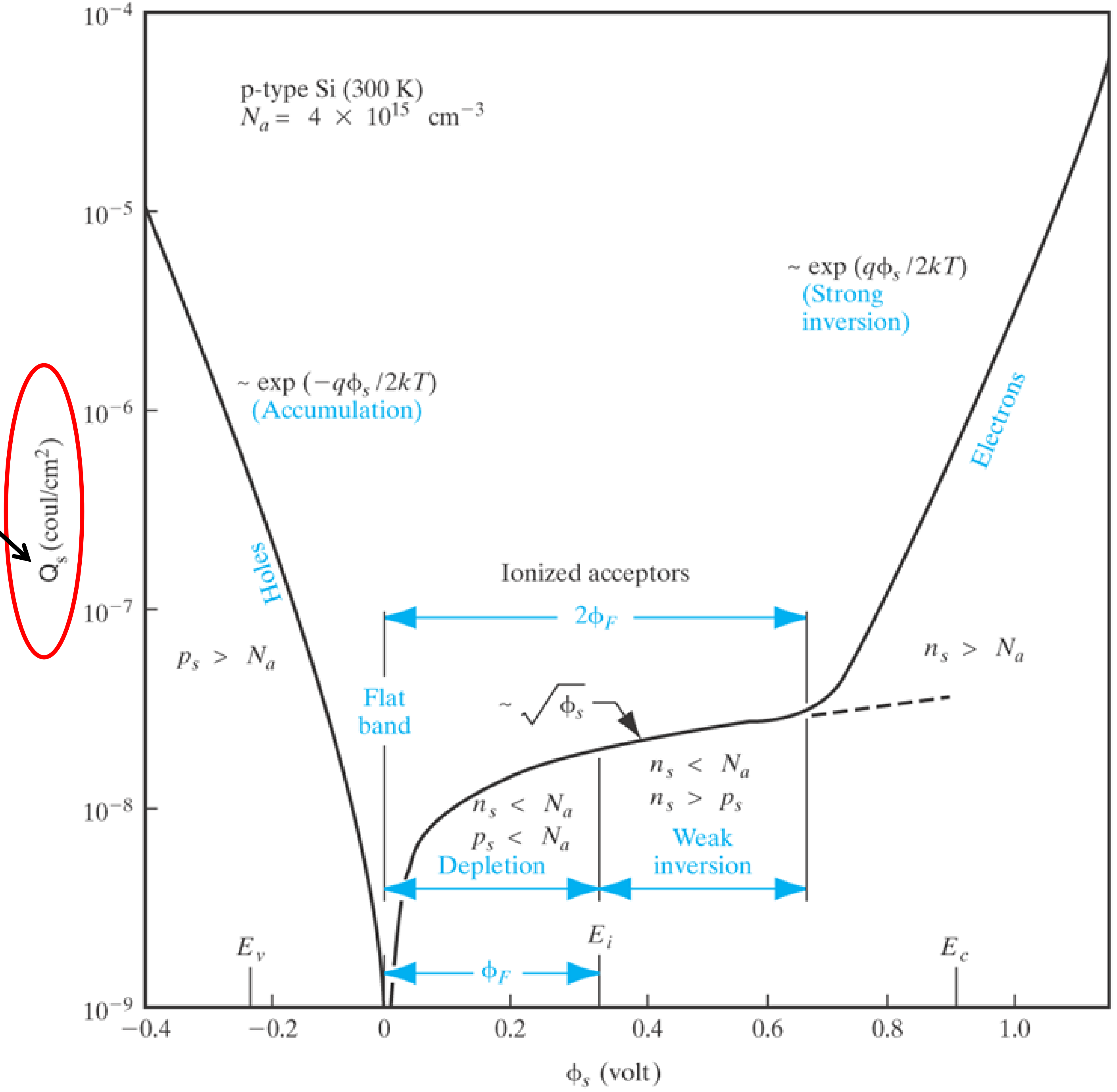
Solution result:

$$\mathcal{E}^2 = \frac{2k_BT}{\varepsilon_s} p_0 \left[\left(\exp\left(-\frac{q\phi}{k_BT} \right) + \frac{q\phi}{k_BT} - 1 \right) + \frac{n_0}{p_0} \left(\exp\left(\frac{q\phi}{k_BT} \right) - \frac{q\phi}{k_BT} - 1 \right) \right]$$

At the surface where $x = 0$, ϕ_S , $\mathcal{E}_S$

$$\mathcal{E}_S = \frac{\sqrt{2}k_BT}{qL_D} \sqrt{ \left(\exp\left(-\frac{q\phi_S}{k_BT} \right) + \frac{q\phi_S}{k_BT} - 1 \right) + \frac{n_0}{p_0} \left(\exp\left(\frac{q\phi_S}{k_BT} \right) - \frac{q\phi_S}{k_BT} - 1 \right) }$$

where $L_D = \sqrt{\frac{\varepsilon_S k_BT}{q^2 p_0}}$ is the Debye Length



Absolute value

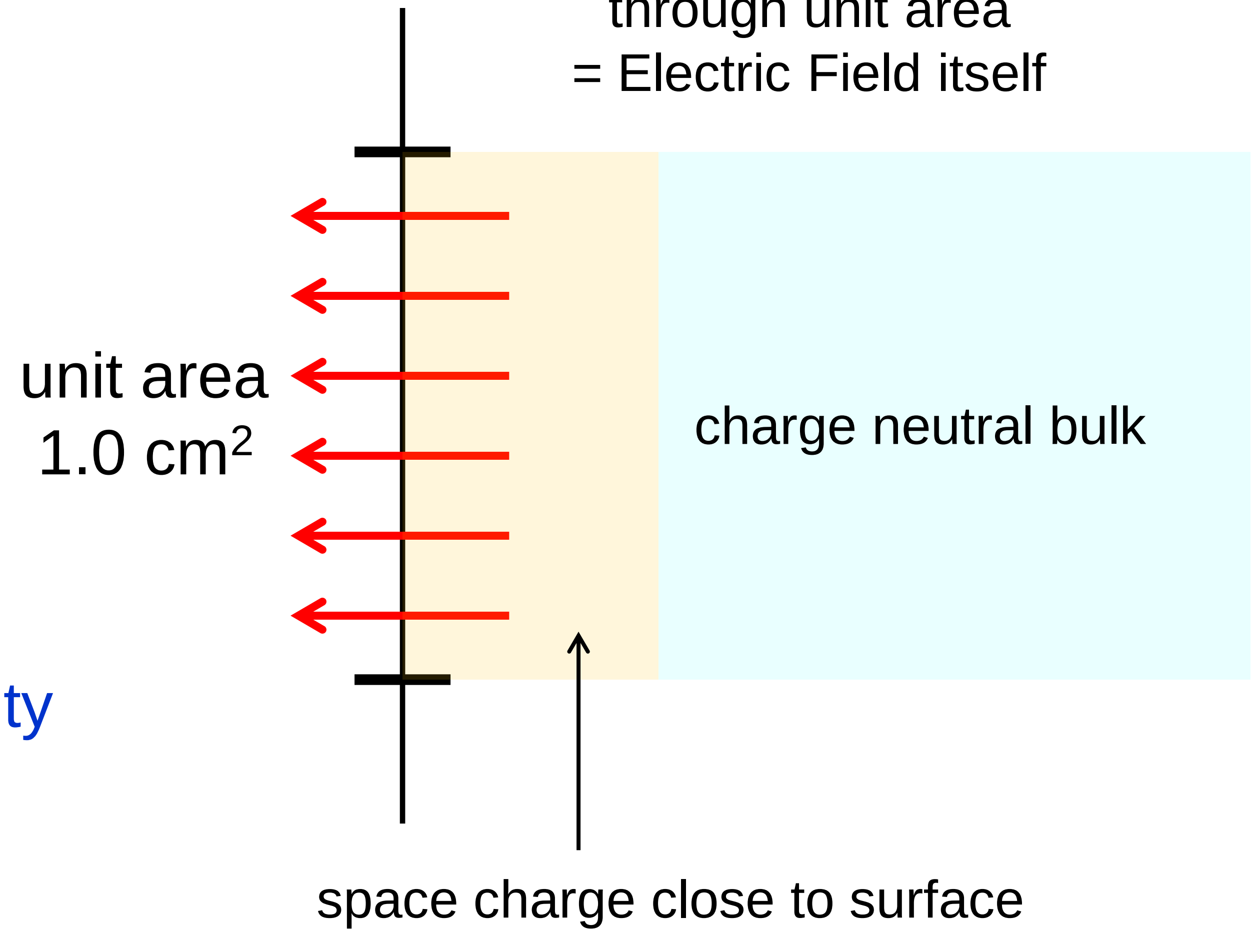
Analytical model for $\mathcal{E}(x)$

Gauss Law at the Surface

$$\boxed{\mathcal{E}_S = -\frac{Q_S}{\varepsilon_S}}$$

Space charge density per unit area

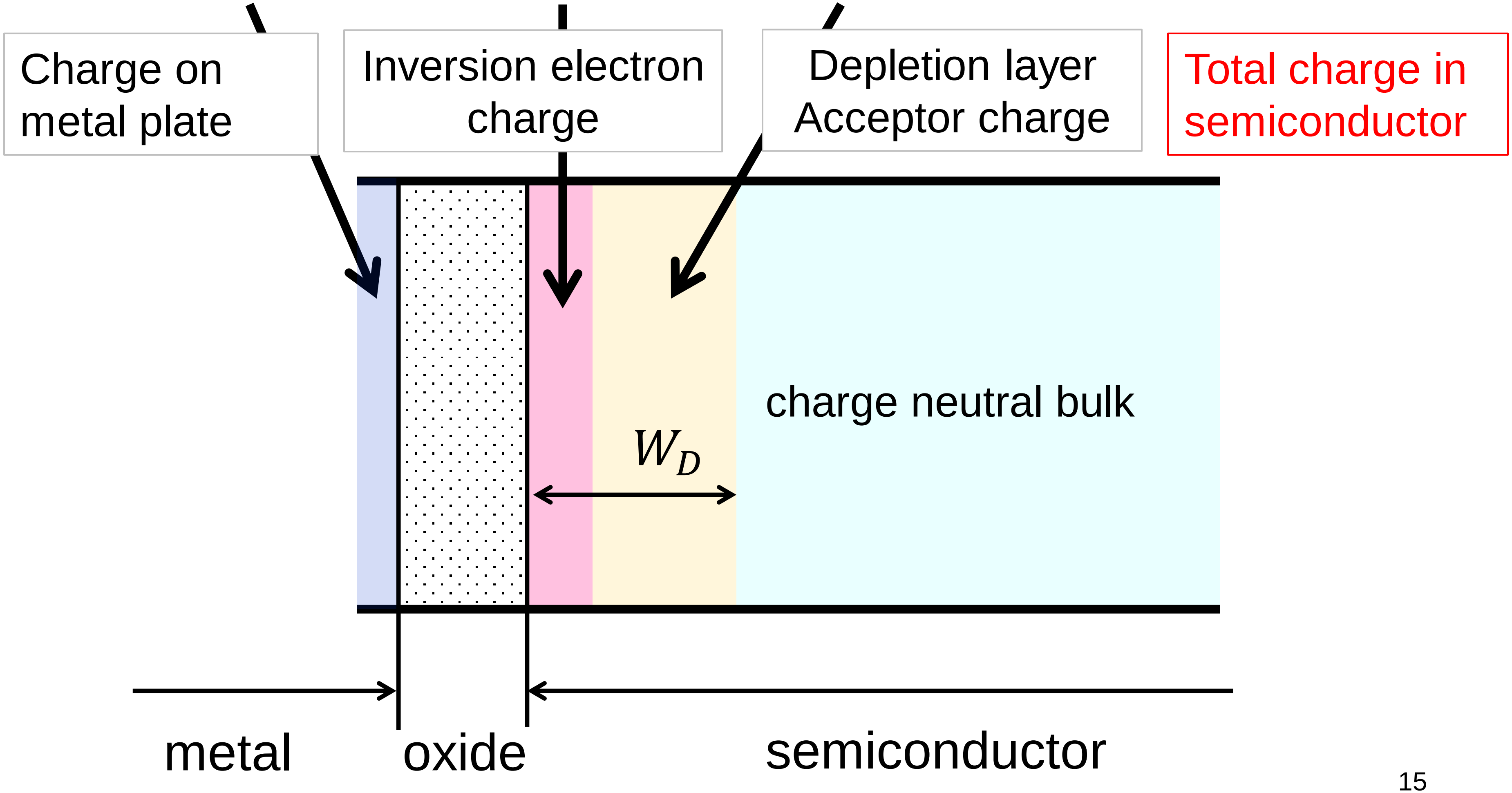
$$Q_S = -\varepsilon_S \mathcal{E}_S$$



flux of the Electric Field through unit area = Electric Field itself

Charge density distribution

$$Q_M = -Q_n + qN_A W_D = -Q_S$$



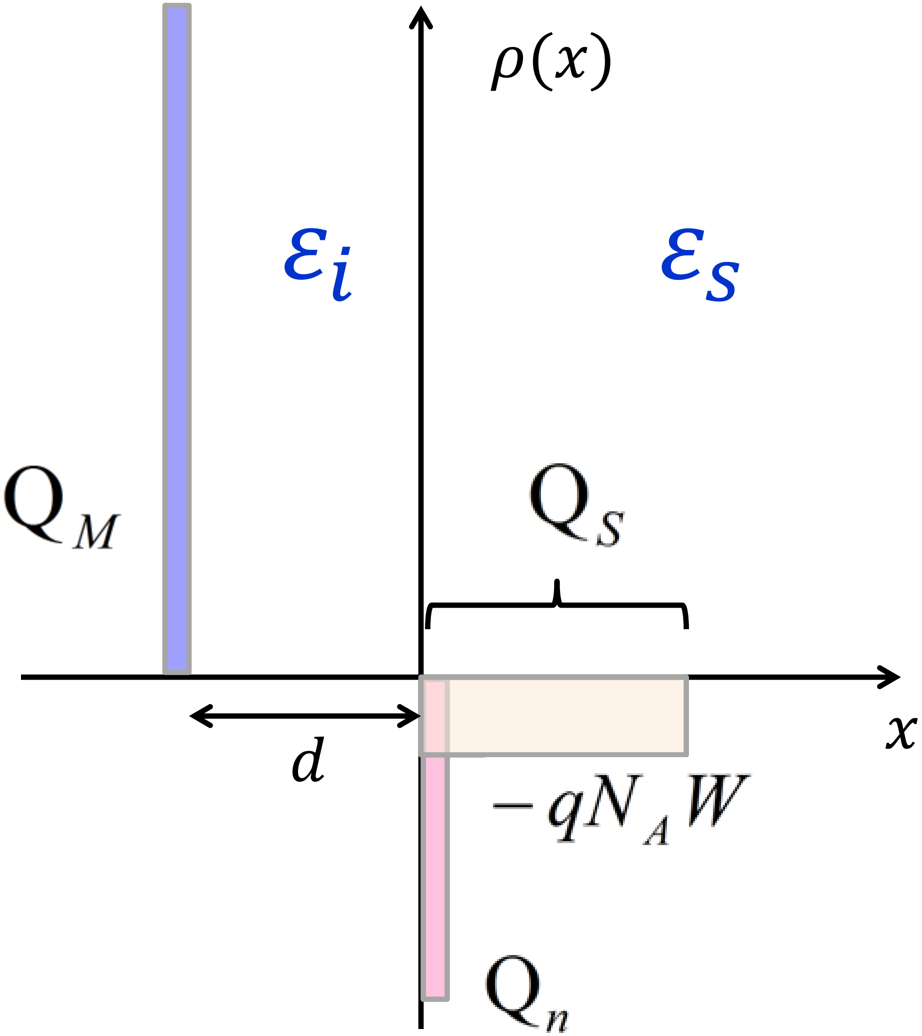
Charge on metal plate

Inversion electron charge

Depletion layer Acceptor charge

Total charge in semiconductor

Charge density distribution



d = thickness of oxide

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_i}{d}$$

Voltage across oxide

$$V_i = \frac{-Q_s}{C_i} = \frac{-Q_s d}{\varepsilon_i}$$

Applied voltage

$$V = V_i + \phi_s$$

Depletion Layer Width

Similar to result for n^+- p junction

BEFORE strong inversion

$$W = \sqrt{\frac{2\epsilon_s \phi_s}{q N_A}}$$

$$\phi_S < 2\phi_F$$

At strong inversion, depletion region no longer grows, due to screening of interface electrons

strong inversion

$$\phi_S = 2\phi_F$$

$$W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_F}{q N_A}} = 2\sqrt{\frac{\epsilon_s k_B T \ln \frac{N_A}{n_i}}{q^2 N_A}}$$

Depletion charge at threshold

$$W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}}$$

strong inversion limit

$$\boldsymbol{\phi_S = 2\phi_F}$$

$$Q_D = -qN_A W_{max} = -qN_A\sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}}$$

$$Q_D = -2\sqrt{q\epsilon_s N_A \phi_F}$$

Threshold Voltage (ideal case)

$$\boxed{Q_D = -qN_A W_{max} = -2\sqrt{q\epsilon_s N_A \phi_F}}$$

maximum value

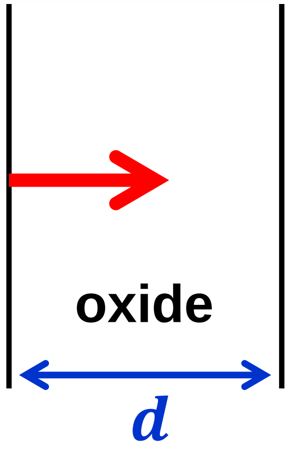
$$V_T = \underbrace{-\frac{\overbrace{Q_D}^{\textbf{Depletion layer charge}}}{C_i}}_{\textbf{Voltage drop across oxide}} + \underbrace{2\phi_F}_{\textbf{Strong inversion condition}}$$

$$\boxed{Q_s \approx Q_D}$$

(Assuming that depletion charge dominates Q_s at threshold)

Above, $-Q_D$ represents Q_m at threshold, to give the potential drop across the oxide.

Potential drop across the oxide



unit area

$\mathcal{E}_{ox} = Q_m/\epsilon_{ox}$

Electric Field in the oxide is constant in absence of charge

Voltage drop across oxide

C_i^{-1} (per unit area)

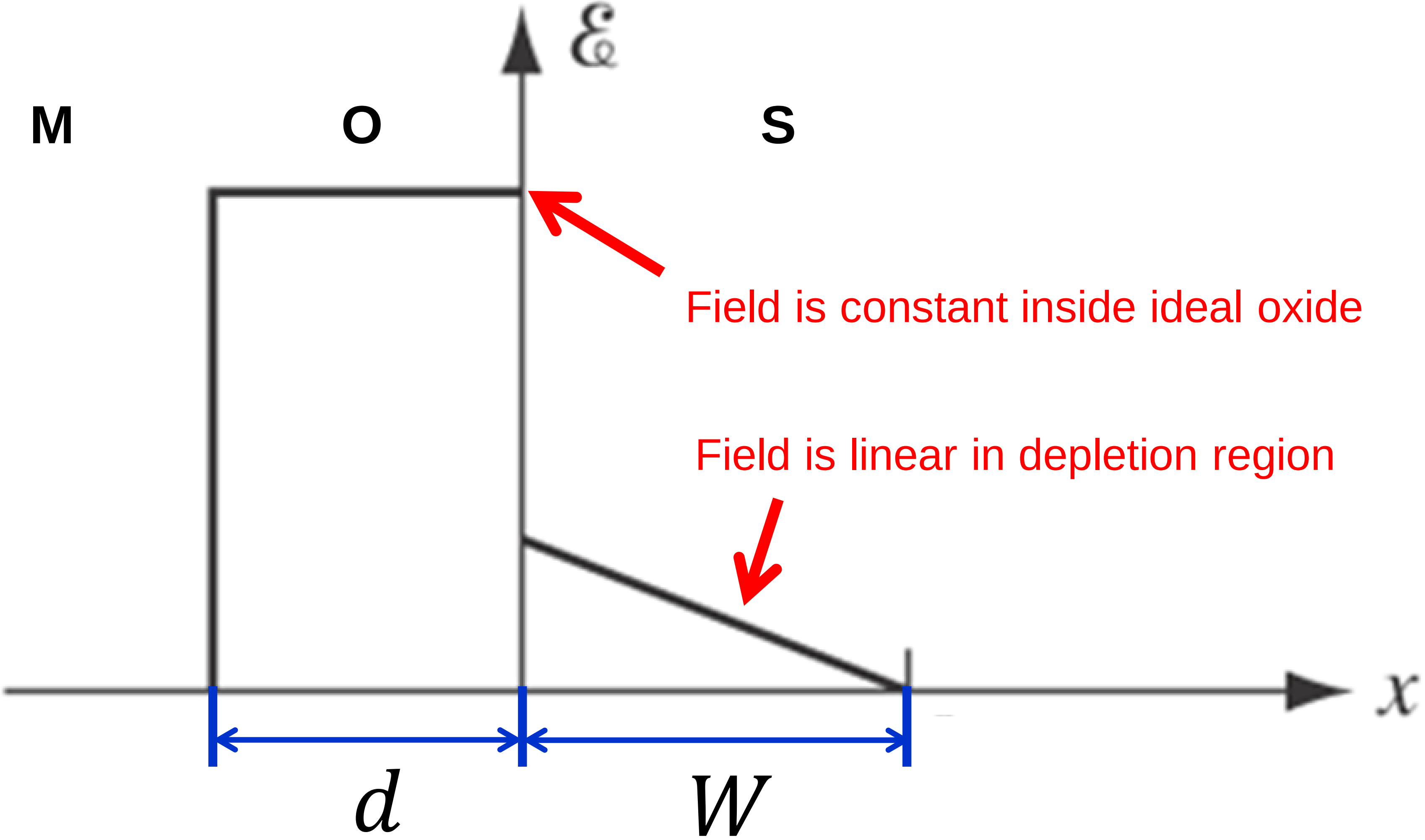
$$V_i = \int_0^d \frac{Q_m}{\epsilon_{ox}}\,dx = Q_m \boxed{\frac{d}{\epsilon_{ox}}} \approx -\frac{Q_D}{C_i}$$

at threshold

$$Q_m \approx -Q_D$$

$$\epsilon_{ox} = \epsilon_{rox}\epsilon_0$$

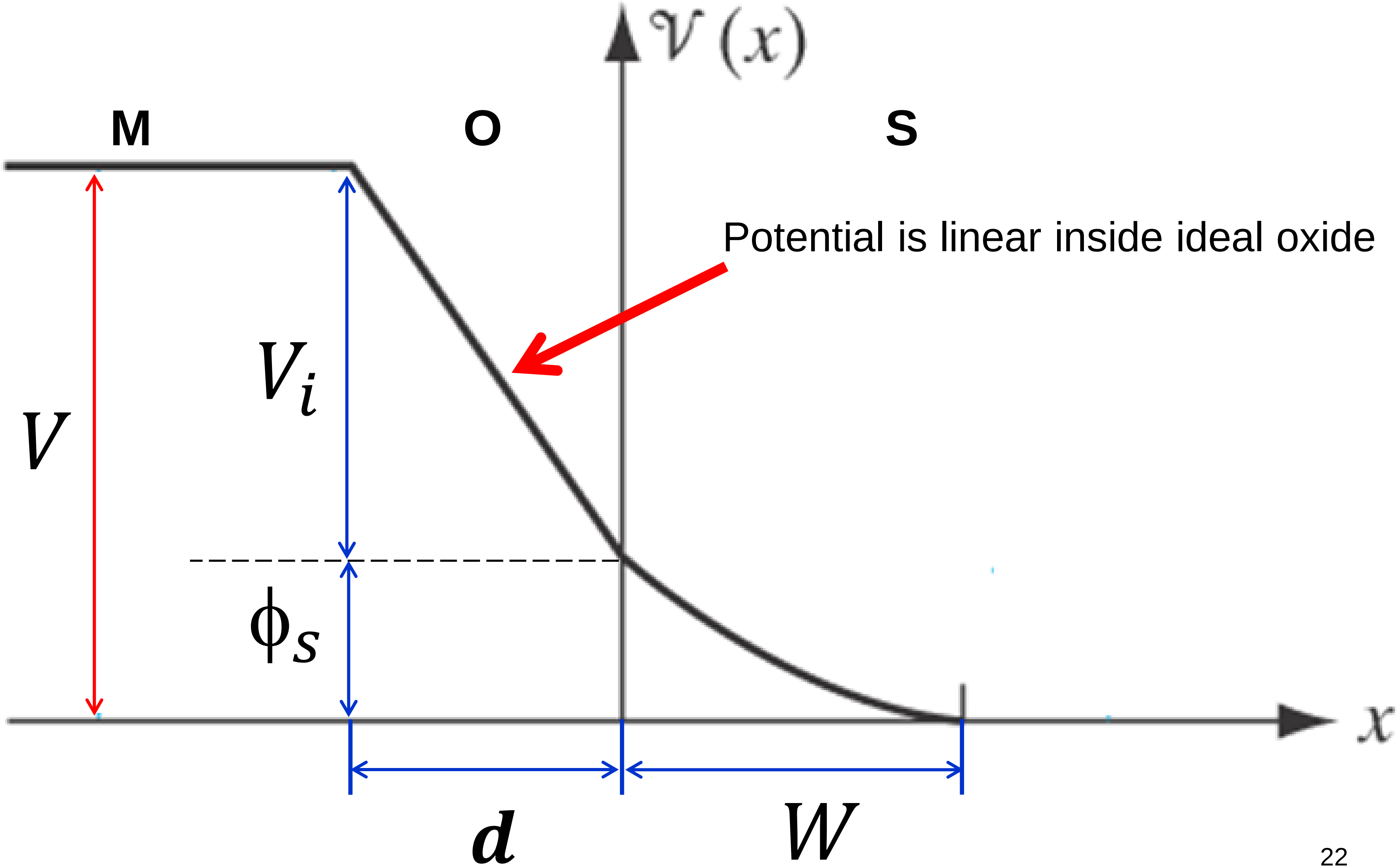
Electric Field distribution



Field is constant inside ideal oxide

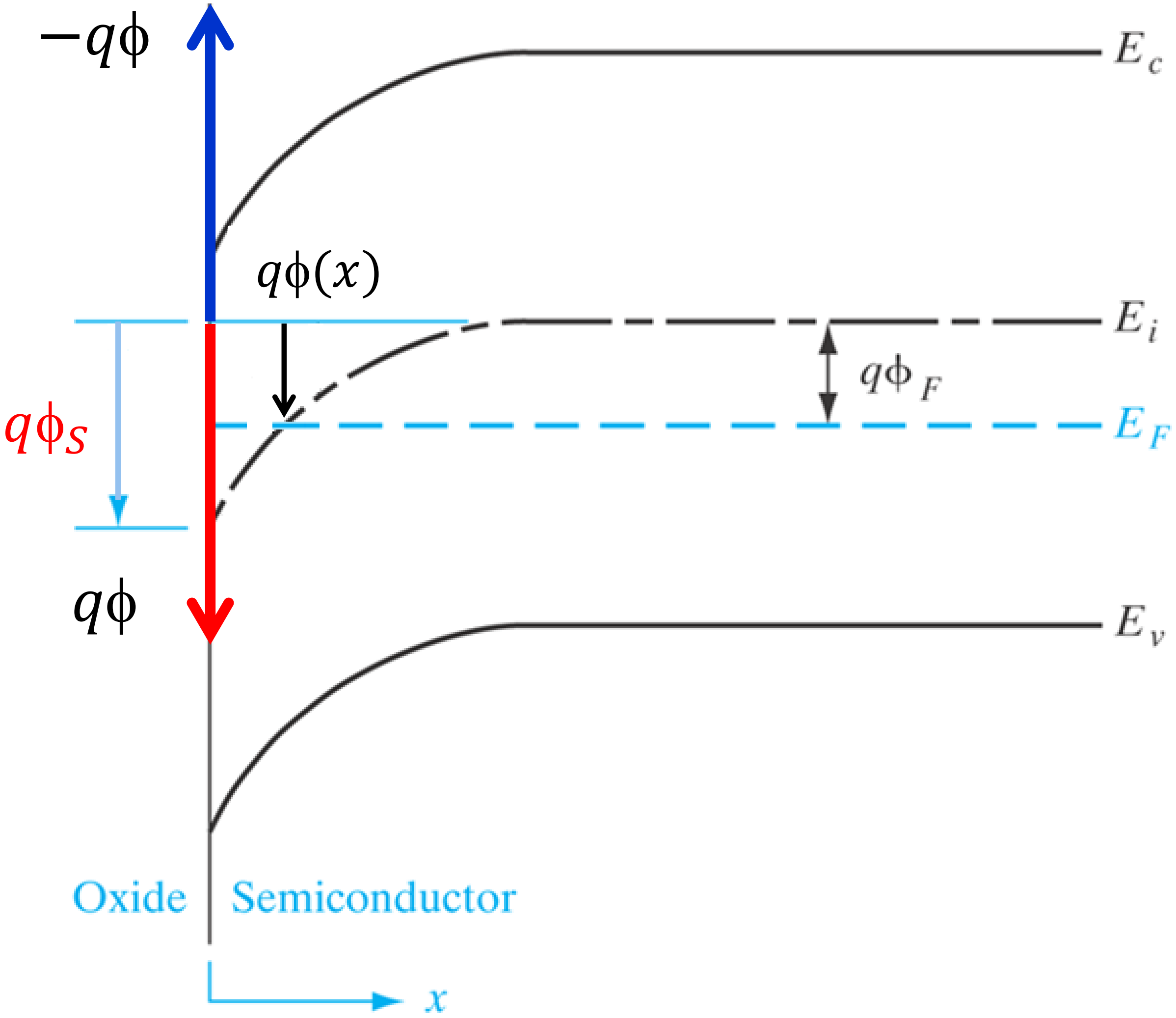
Field is linear in depletion region

Electric Potential distribution



Potential is linear inside ideal oxide

Summary of conditions – surface potential



accumulation
$\phi_S < 0$

flat-band
$\phi_S = 0$

depletion
$0 < \phi_S < \phi_F$

intrinsic
$\phi_S = \phi_F$

weak inversion
$2\phi_F > \phi_S > \phi_F$

strong inversion
$\phi_S \geq 2\phi_F$

Real Surface effects – Work function difference

- **We assumed in the previous analysis for simplicity**

$$\boxed{\Phi_m = \Phi_s}$$

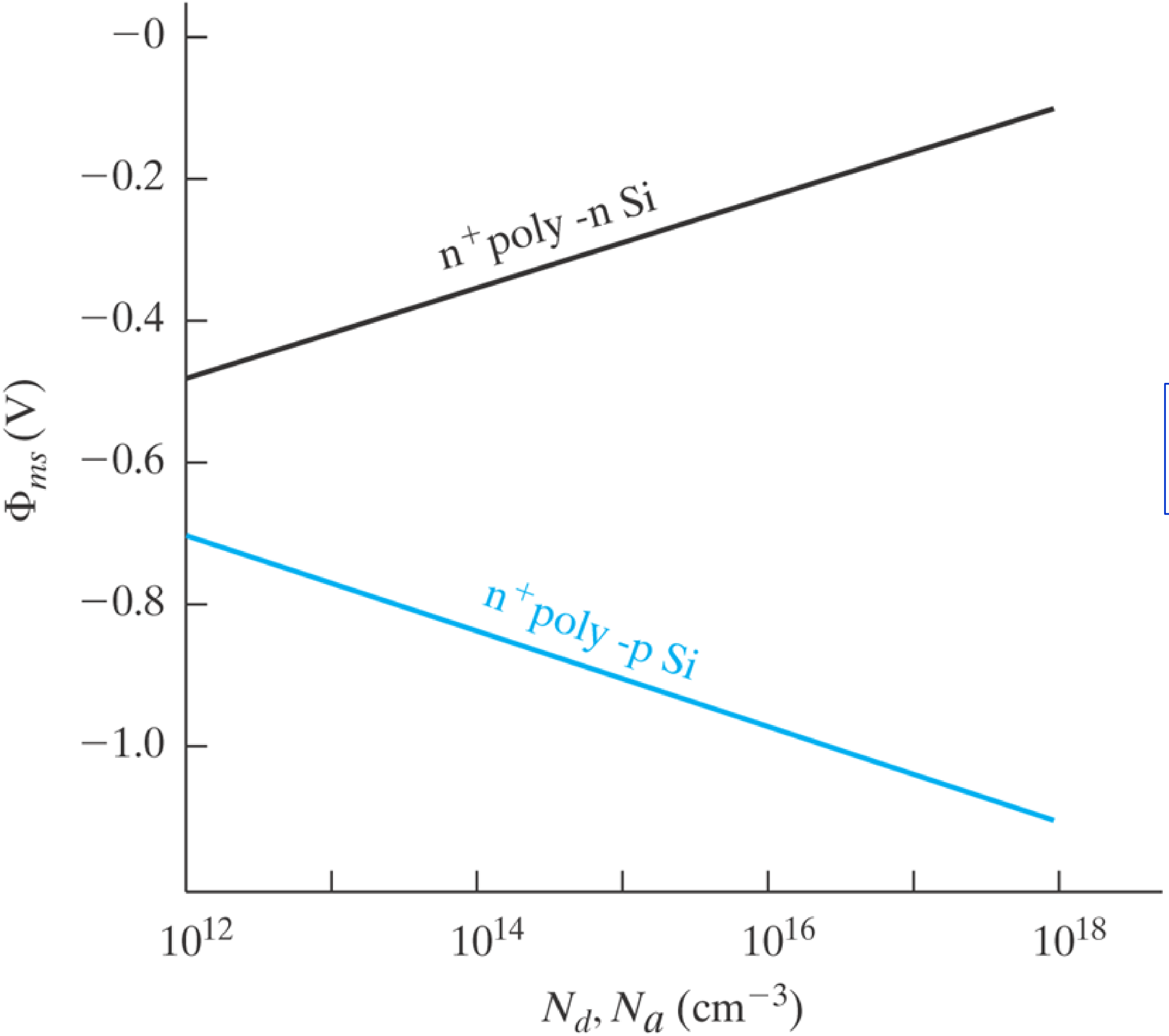
- **In general, we are limited in the choice of metal by technological constraints and**

$$\boxed{\Phi_m \neq \Phi_s}$$

- **It is convenient to define the quantity**

$$\boxed{\Phi_{ms} = \Phi_m - \Phi_s}$$

n+ plus polysilicon for gate electrode

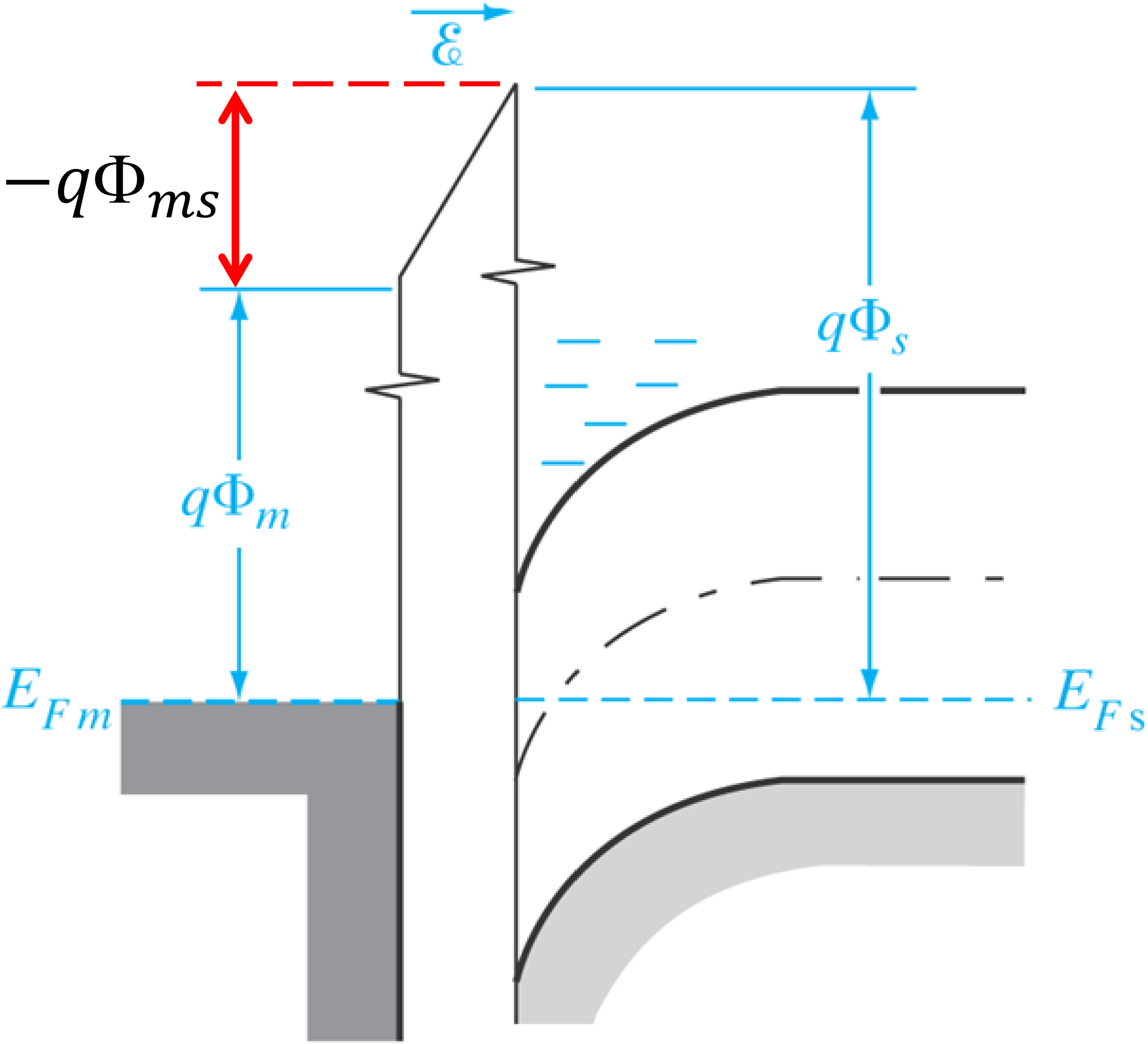


common choice

instead of metal

$$\boxed{\Phi_{ms} < 0}$$

Effect of negative workfunction difference

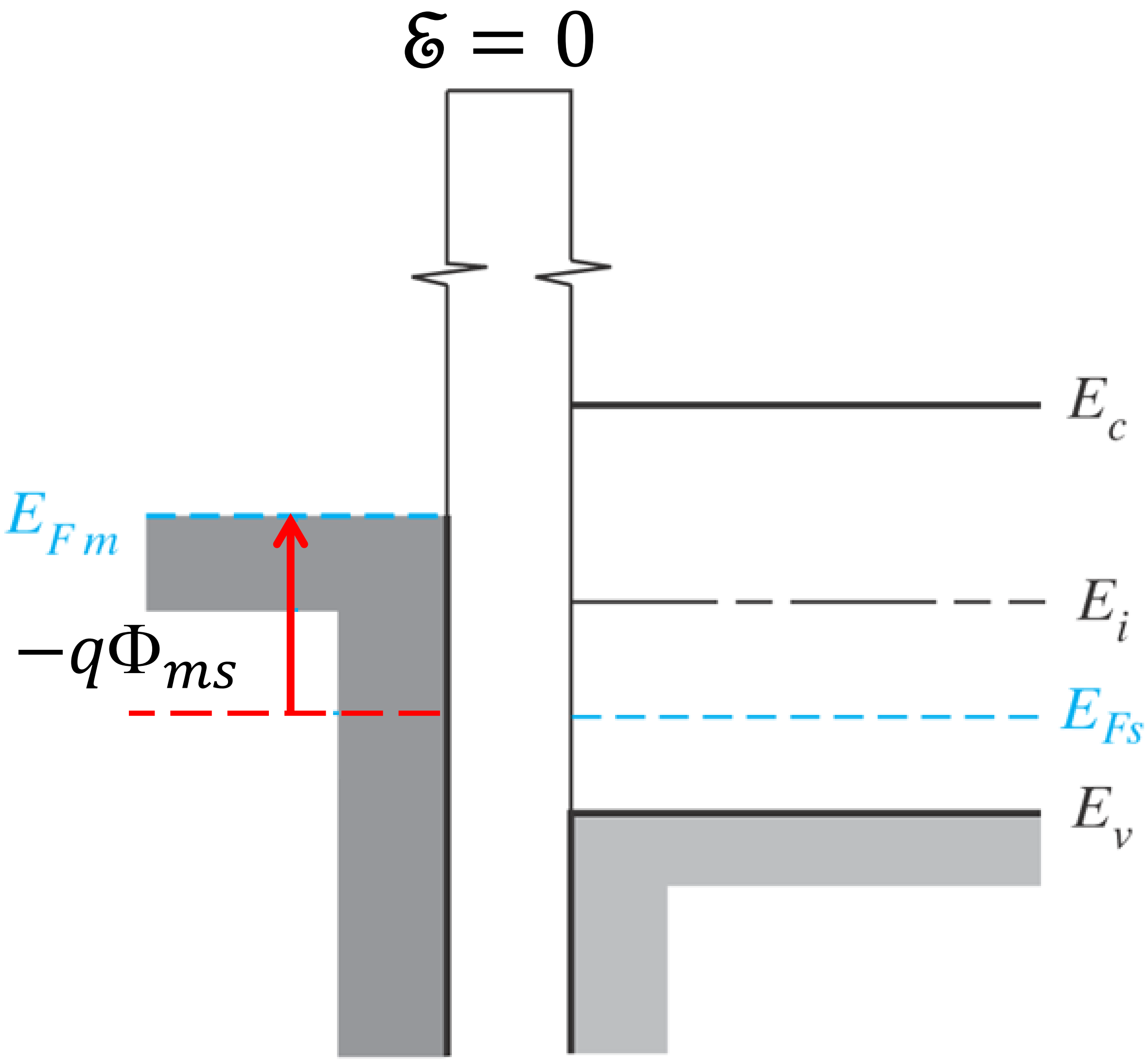


EQUILIBRIUM

$\boldsymbol{V = 0}$

$$\boxed{\Phi_{ms} < 0}$$

Apply $V_{FB} = \Phi_{ms}$ to obtain flat band



FLAT BAND

$V = V_{FB} = \Phi_{ms}$

$$\boxed{\Phi_{ms} < 0}$$

Real Surface effects – Interface charge (1)

- **Alkali metal ions (e.g. Na^+) inside the oxide cause a mobile charge $\boldsymbol{Q_m}$ inducing negative charge in Si (reduced by careful processing)**

- **Imperfections in the $\mathrm{SiO_2}$ material cause positive trapped charges $\boldsymbol{Q_{ot}}$ ($\approx 10^{10}\,\mathrm{cm}^{-3}$).**

Real Surface effects – Interface charge (2)

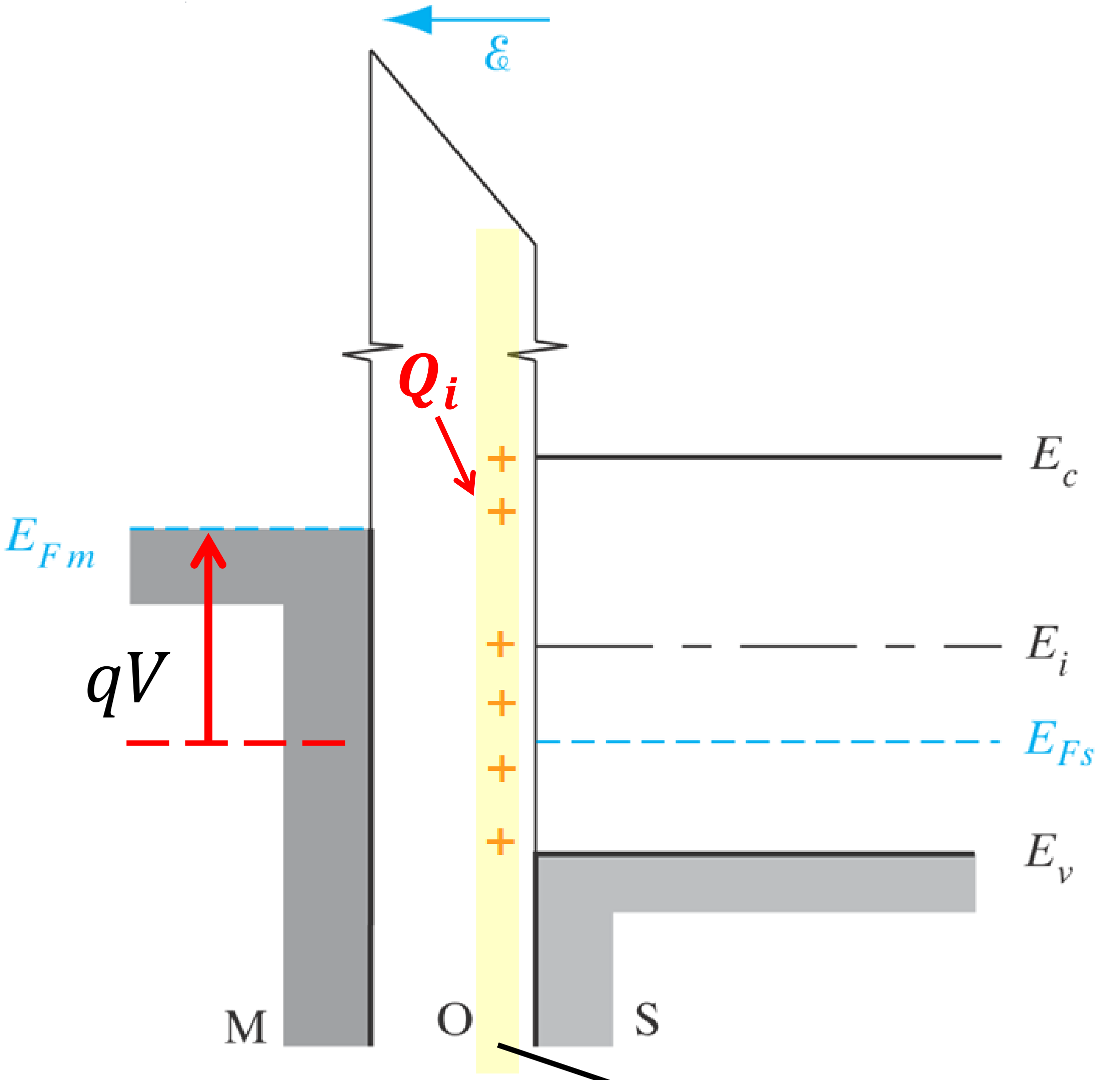
- **Positive fixed charges $\boldsymbol{Q_f}$ in a transition layer at the interface.**

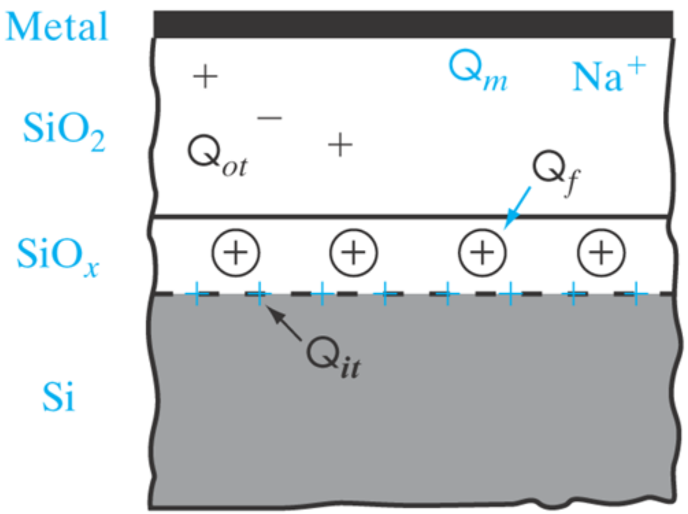
- **Positive charges $\boldsymbol{Q_{it}}$ at the Si- SiO_2 interface (interface states) due to mismatch causing "ionic" Si atoms with incomplete bonds.**

$$\boldsymbol{Q_{it} + Q_f \approx 10^{10}\mathrm{cm}^{-3}}$$ [100] *preferred for devices*

$$\boldsymbol{Q_{it} + Q_f \approx 10^{11}\mathrm{cm}^{-3}}$$ [111]

Effect of interface charge

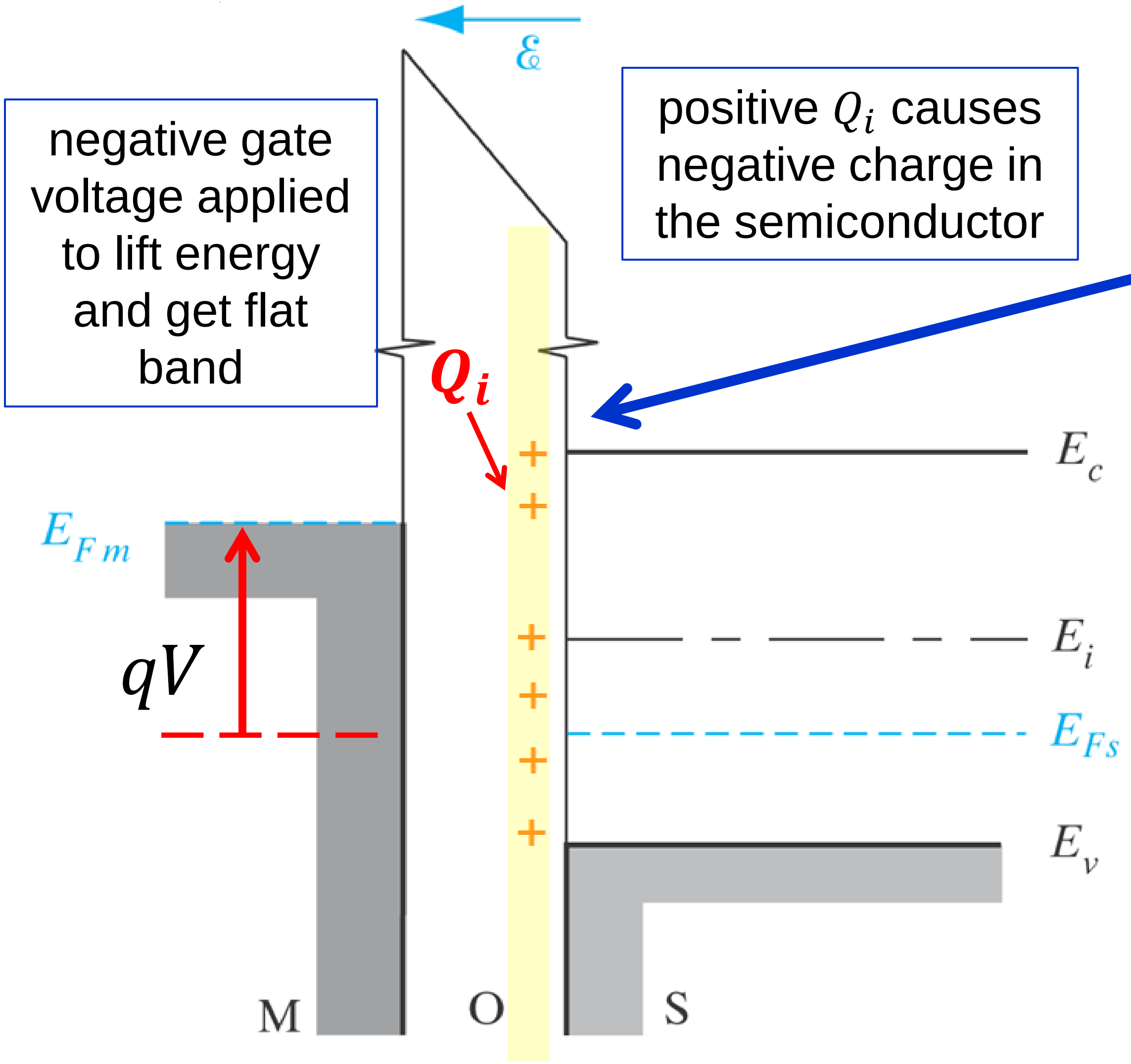




$$V = V_{FB} = -\frac{Q_i}{C_i}$$

Equivalent sheet of charge Q_i at the interface **accounting for all**

Effect of interface charge



negative gate voltage applied to lift energy and get flat band

positive Q_i causes negative charge in the semiconductor

Electric displacement $\boldsymbol{\varepsilon\mathcal{E}}$ discontinuity because of interface charge

$$\varepsilon_{ox}\mathcal{E}_{ox}-\varepsilon_s\mathcal{E}_s=\rho_i$$

$$V = V_{FB} = -\frac{Q_i}{C_i}$$

Threshold Voltage

$$\underbrace{V_T}_{\text{Threshold Voltage}} = \underbrace{\Phi_{ms}}_{\text{Work function difference}} \underbrace{- \frac{Q_i}{C_i}}_{\text{Interface charge}} \overbrace{\underbrace{- \frac{Q_d}{C_i}}_{\text{Depletion layer charge}} \underbrace{+ 2\phi_F}_{\text{Strong inversion condition}}}^{\text{Ideal case}}$$

Enhancement and Depletion MOSFET

- **Enhancement-mode MOS usually employed for switching elements. These devices are off at zero gate–source voltage.**

- **Depletion-mode MOS, usually employed to realize “resistors” in logic circuits. These devices are normally on on at zero gate–source voltage.**