ECE 340 Lectures 37
Semiconductor Electronics

Spring 2022
10:00-10:50am
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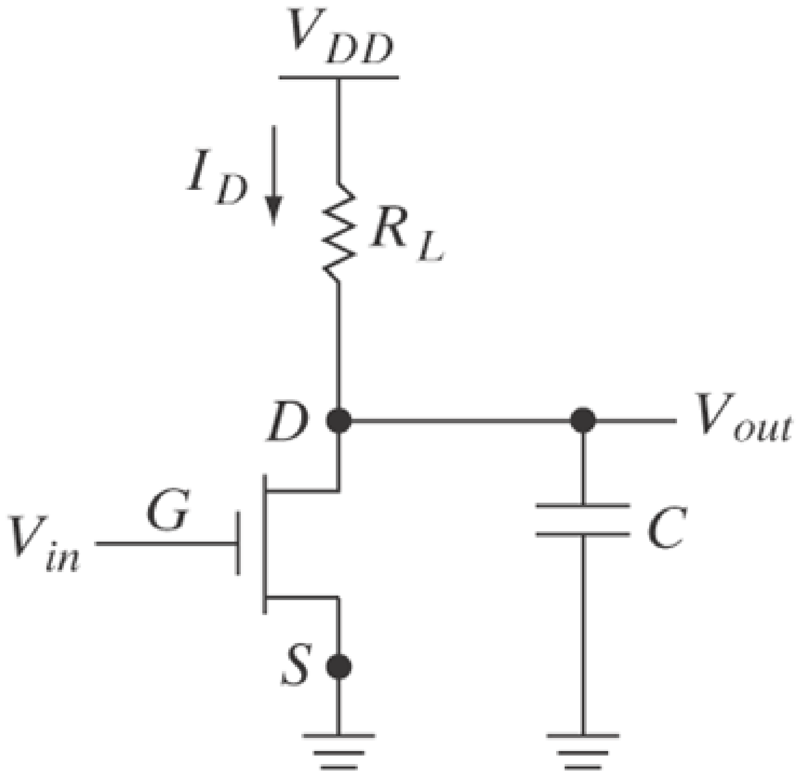
2062 ECE Building

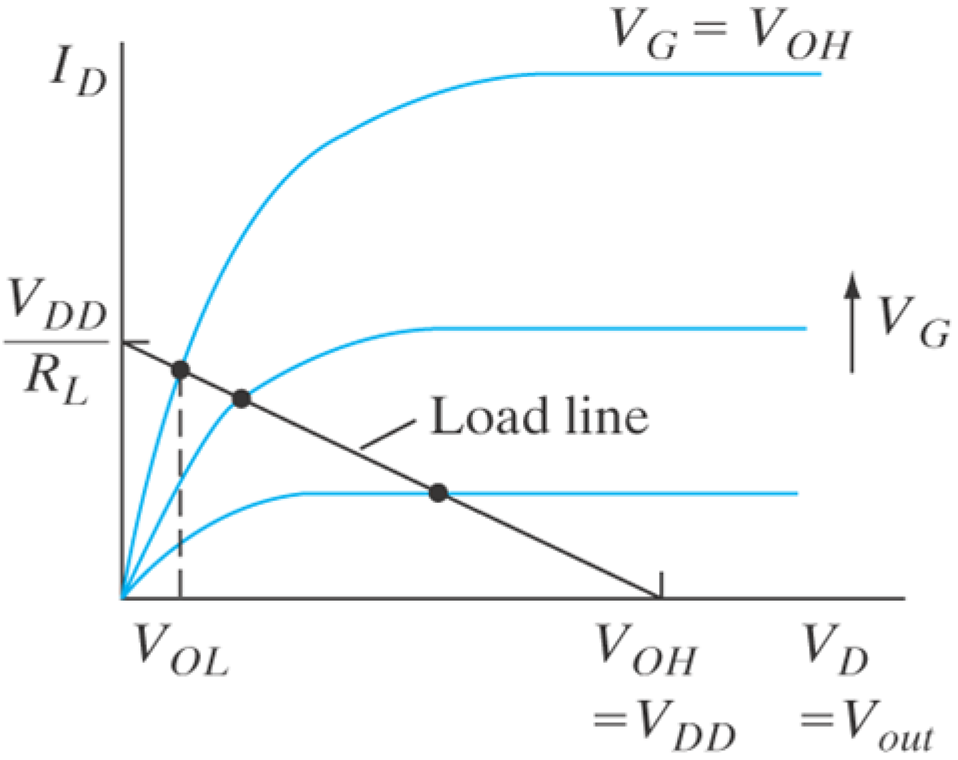
Today’s Discussion

- **Finish MOSFET**
- **Narrow Base Diode**
  - **Motivation: this structure can be considered a precursor to understanding the Bipolar Junction transistor**

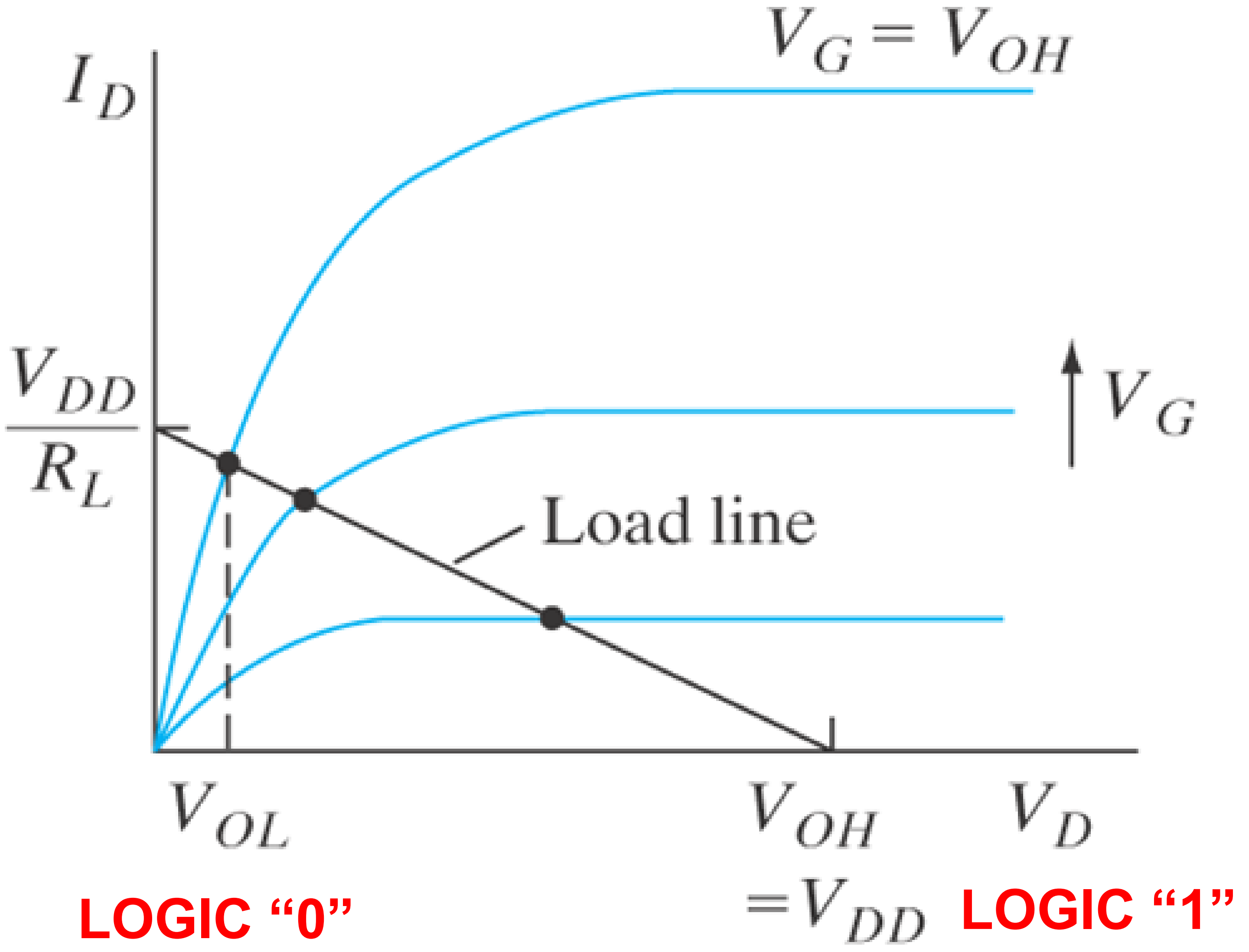
- **Introduction to the Bipolar Junction Transistor (BJT)**

MOS inverter





MOS inverter – Voltage transfer characteristics

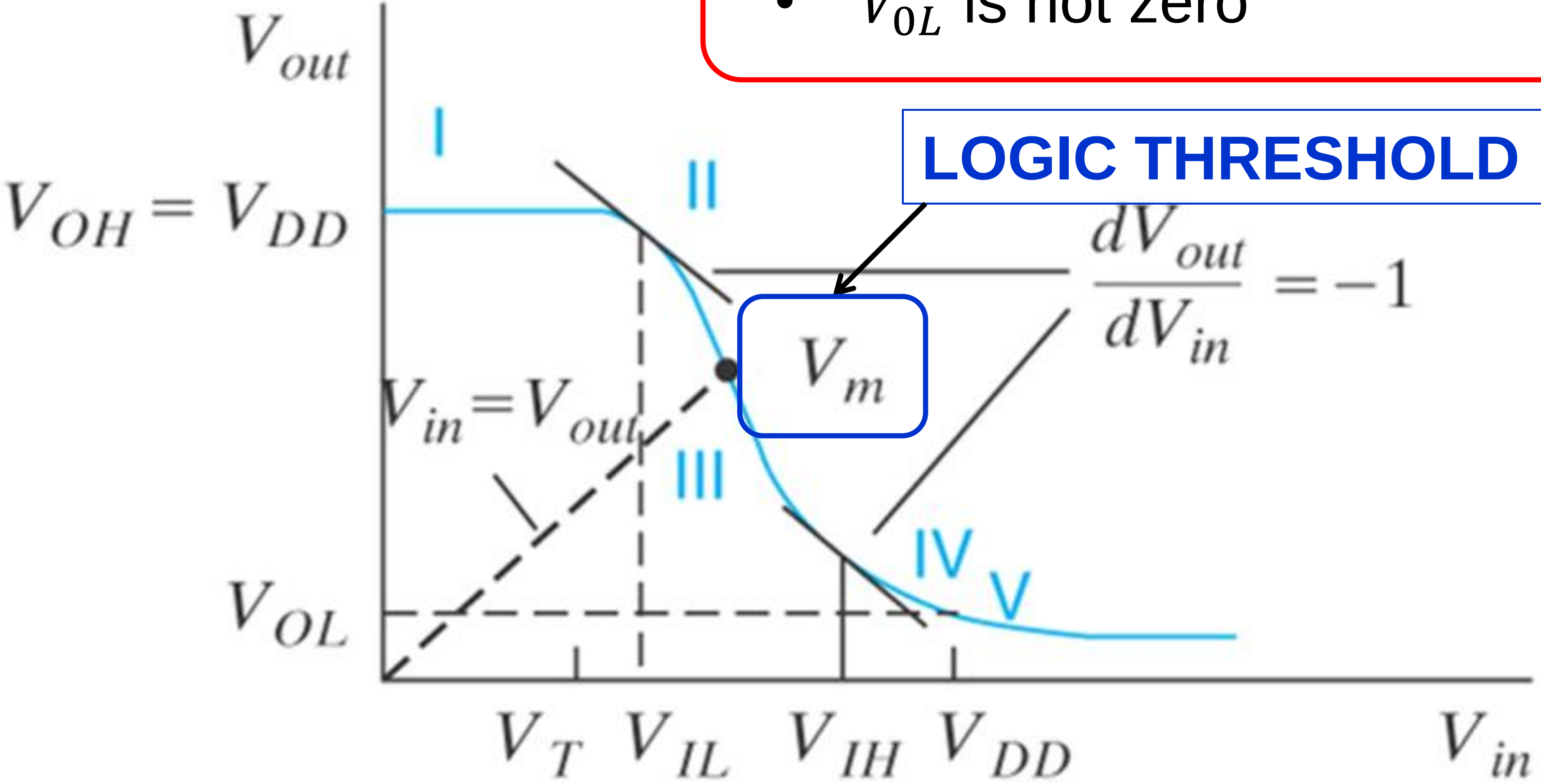


LOGIC "0"

LOGIC "1"

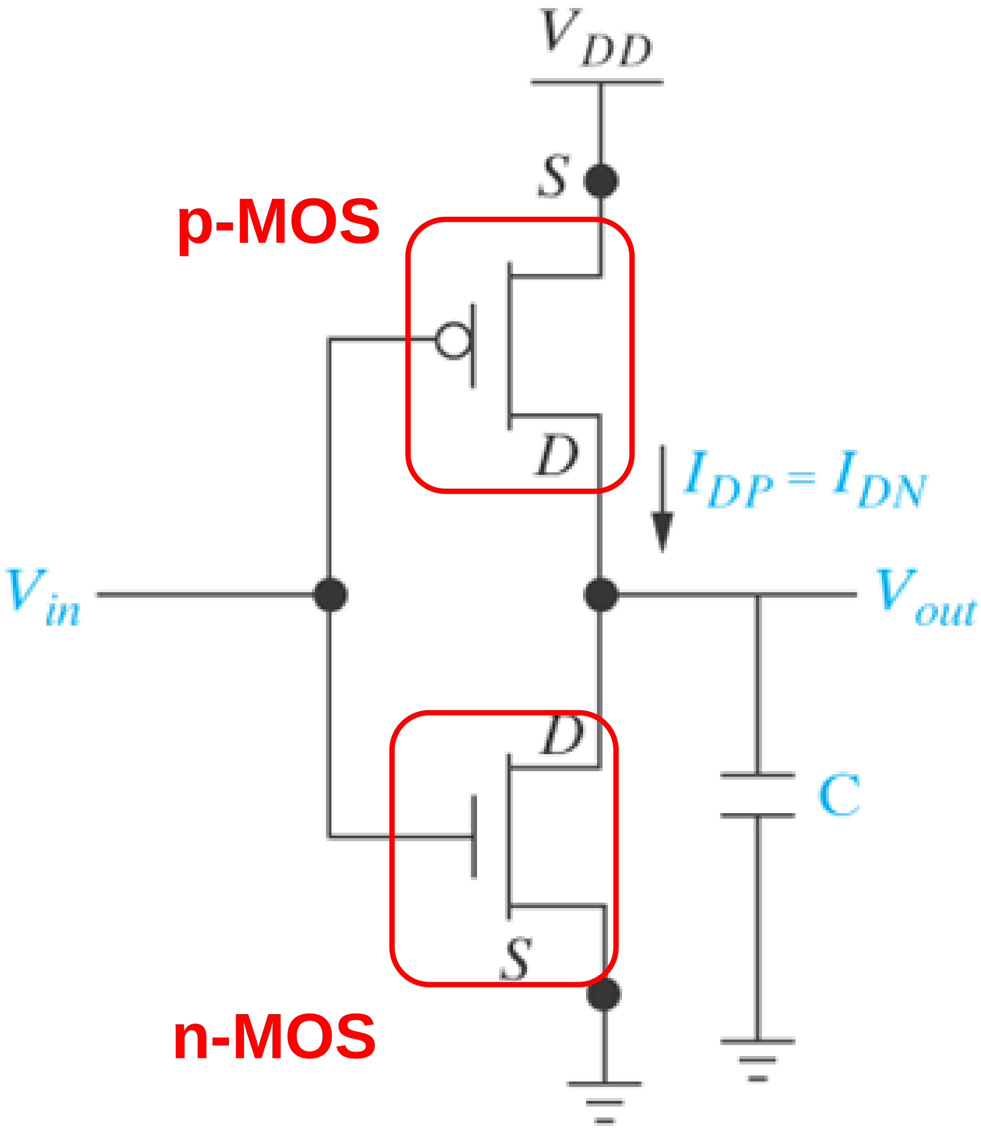
MOS inverter – Voltage transfer characteristics

- Power dissipation in resistor
- V_{0L} is not zero

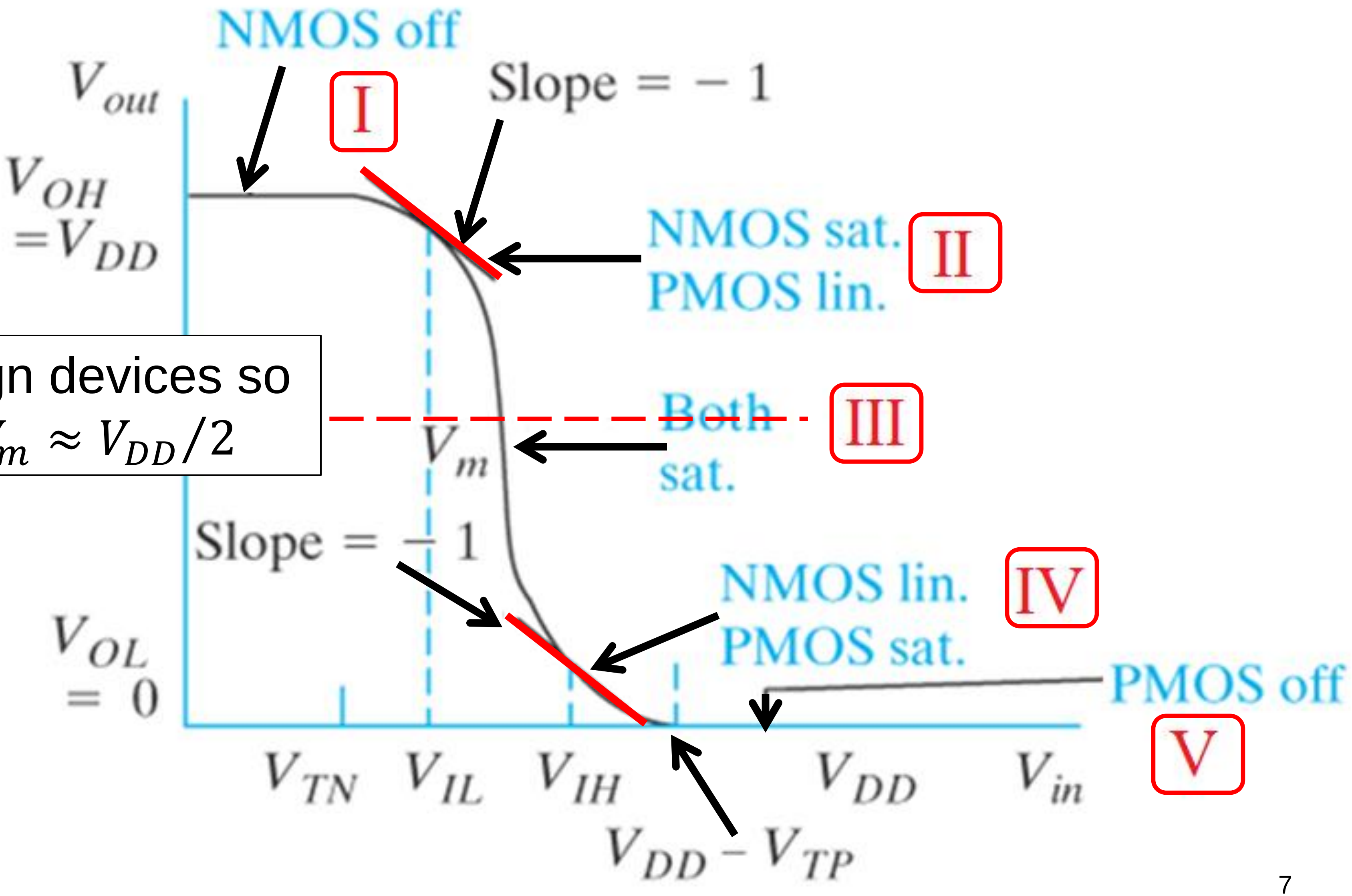


LOGIC THRESHOLD

CMOS inverter



CMOS inverter – Voltage transfer characteristics



Design devices so that $V_m \approx V_{DD}/2$

CMOS inverter – Voltage transfer characteristics

Design devices so that $\boldsymbol{V_m \approx V_{DD}/2}$ REGION III

Setting $I_D(\text{NMOSFET}) = I_D(\text{PMOSFET})$

$$\chi = \left(\frac{k_N}{k_P}\right)^{1/2} = \frac{\left[\overline{\mu_n} C_i \left(\frac{Z}{L}\right)_n\right]^{1/2}}{\left[\overline{\mu_p} C_i \left(\frac{Z}{L}\right)_p\right]^{1/2}} = 1$$

CMOS inverter – Voltage transfer characteristics

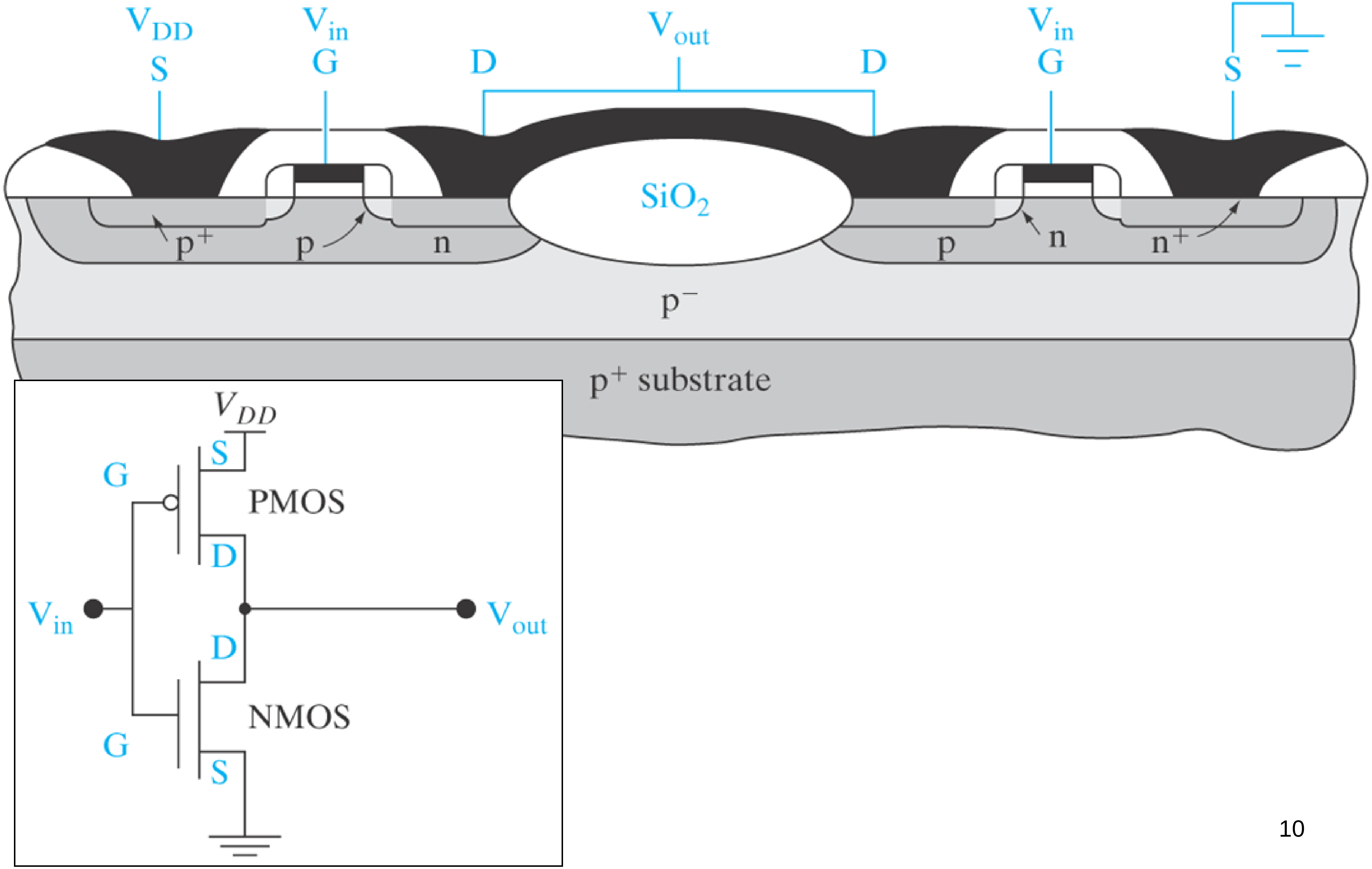
Design devices so that $\boldsymbol{V_m \approx V_{DD}/2}$ REGION III

$$\chi = \left(\frac{k_N}{k_P}\right)^{1/2} = \frac{\left[\overline{\mu_n} C_i \left(\frac{Z}{L}\right)_n\right]^{1/2}}{\left[\overline{\mu_p} C_i \left(\frac{Z}{L}\right)_p\right]^{1/2}} = 1$$

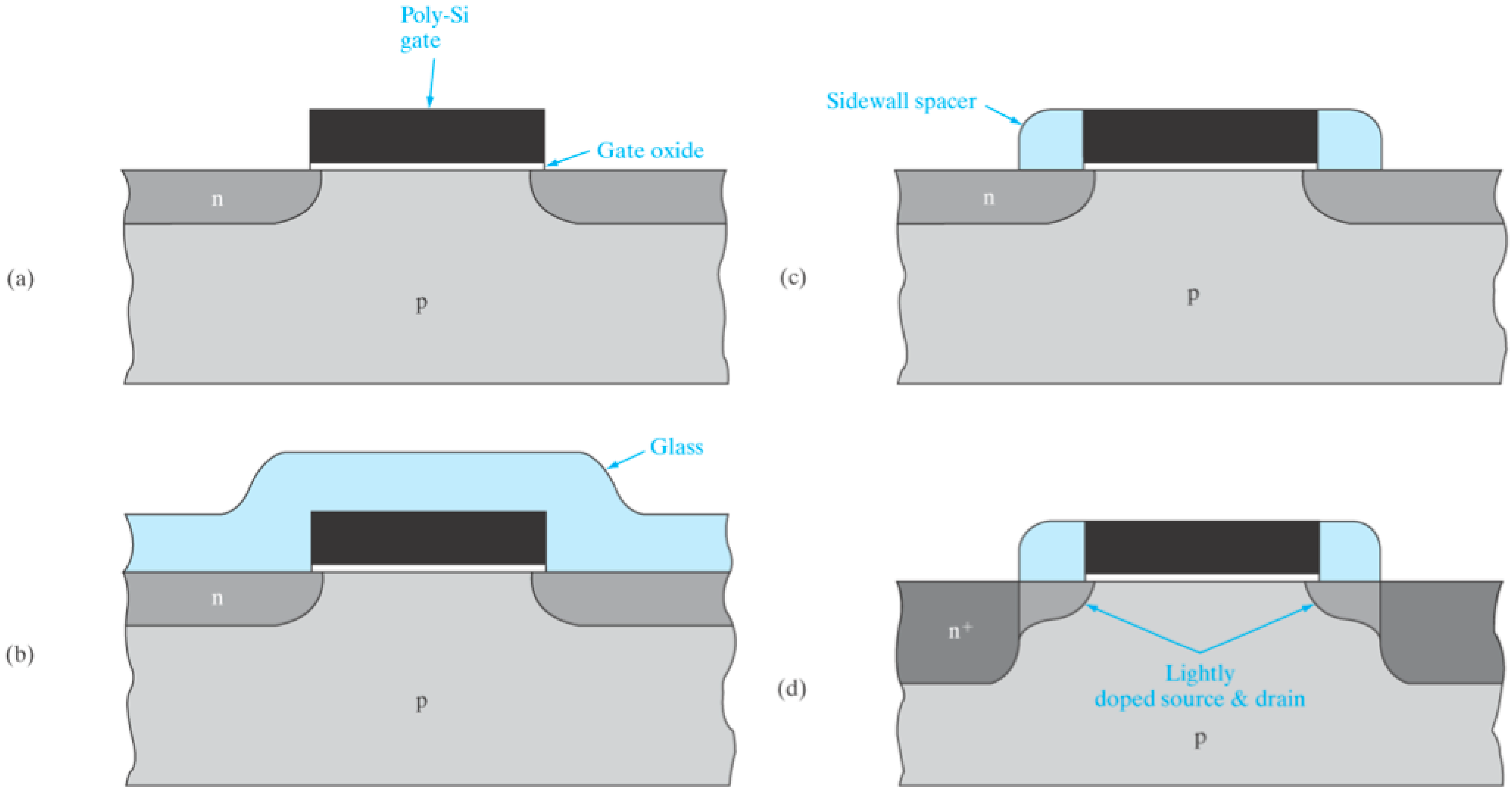
Typically $\overline{\mu_n} \approx 2\overline{\mu_p}$

$$\chi = 1 \rightarrow (Z/L)_p = 2(Z/L)_n$$

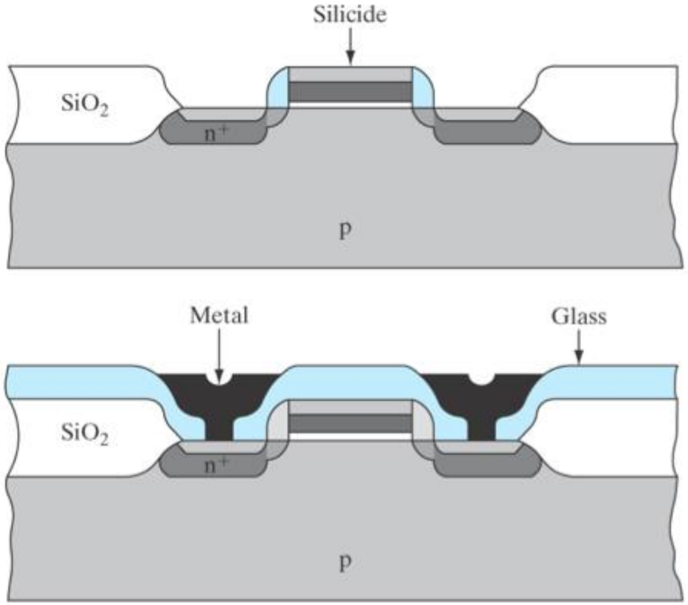
CMOS integration



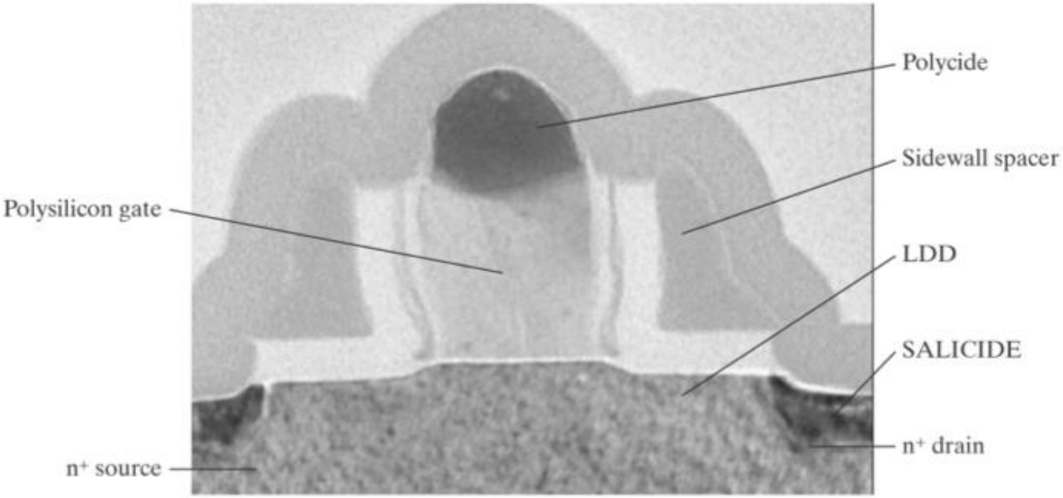
NMOS Fabrication in p-Well



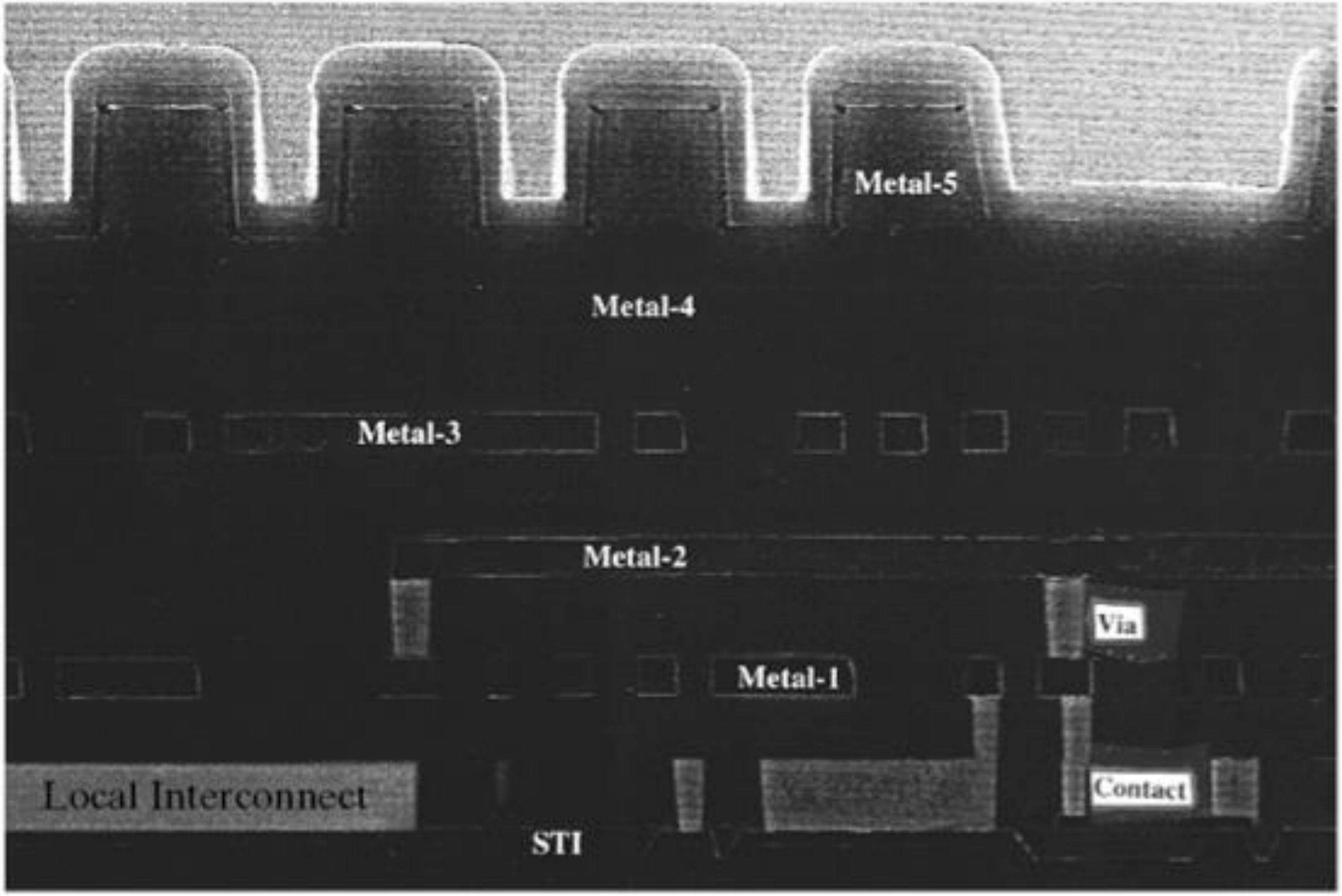
MOSFET – metal contacts



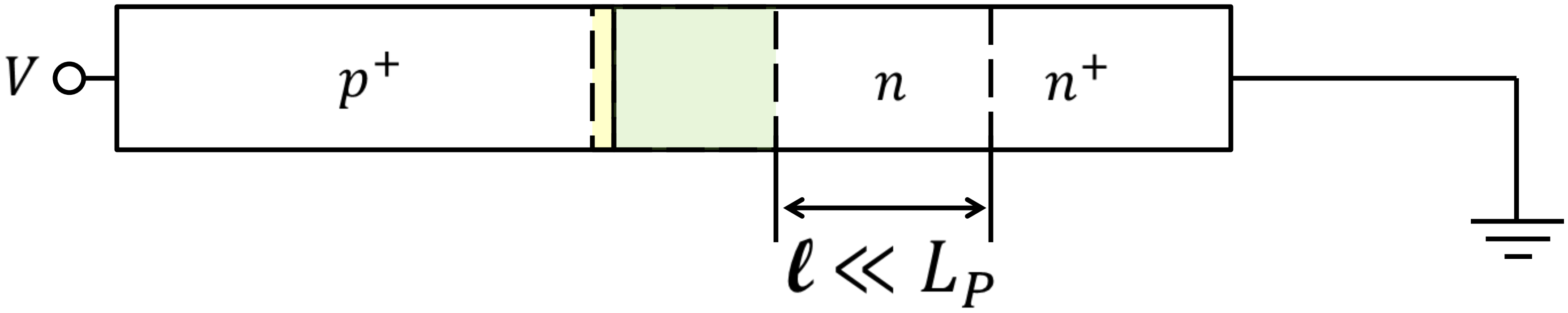
(c)

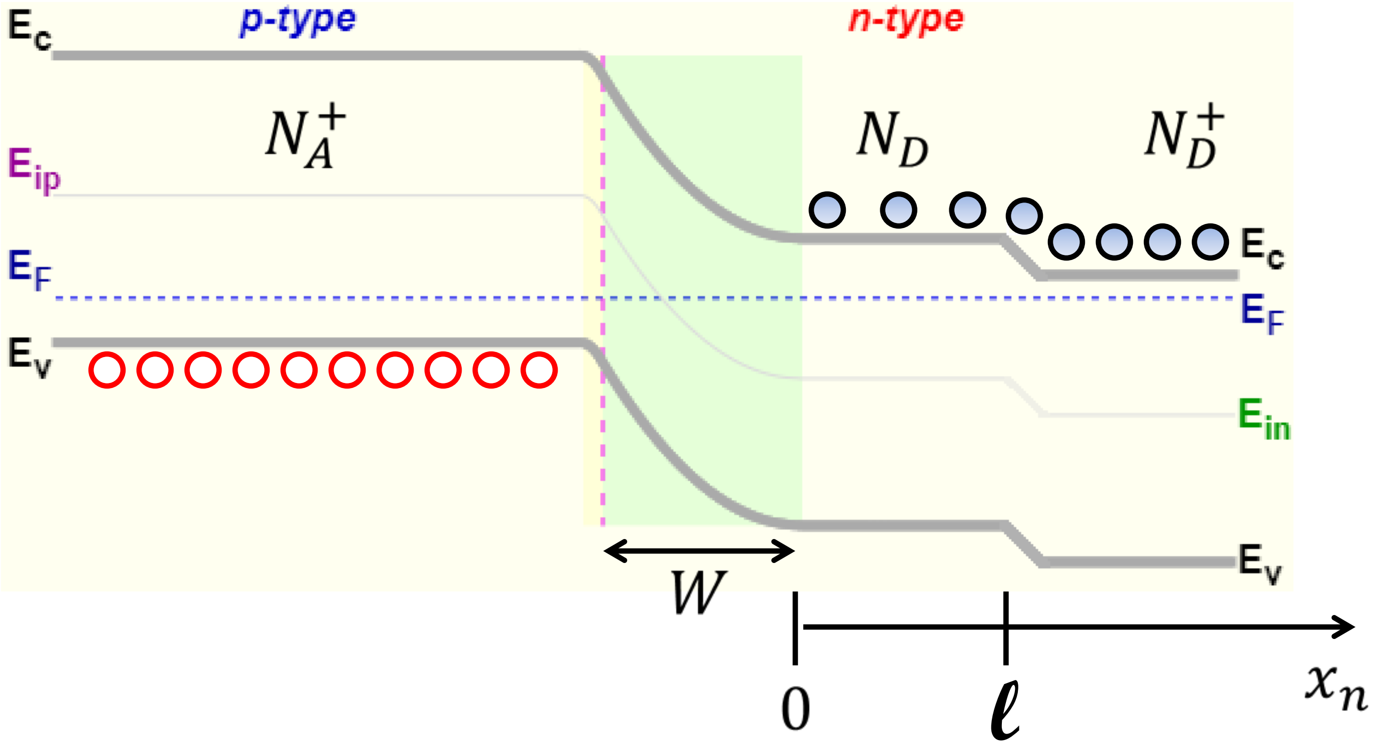


Multilevel Interconnects

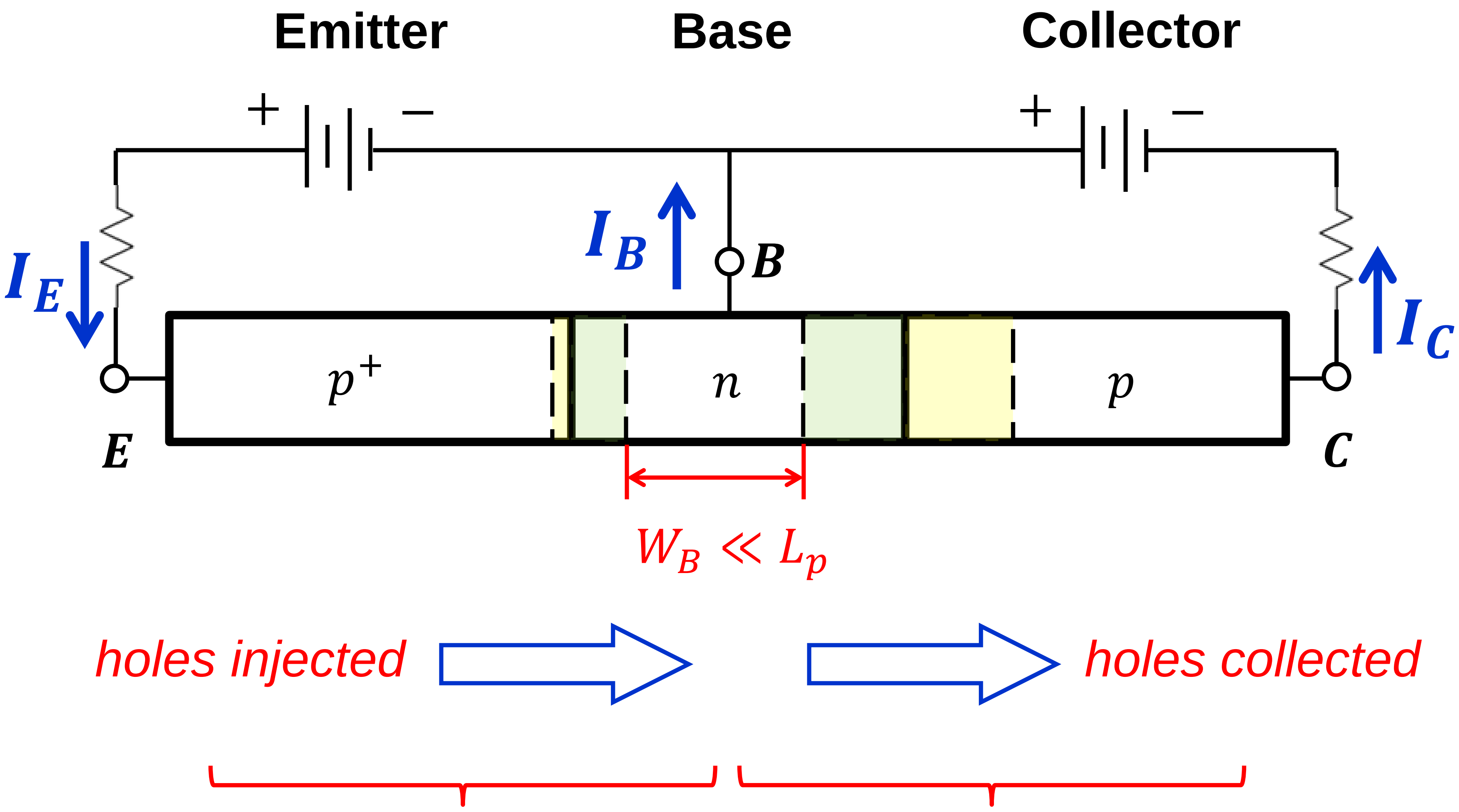


Narrow Base Diode (N-B-D)





p-n-p Bipolar Junction Transistor (B-J-T)



$W_B \ll L_p$

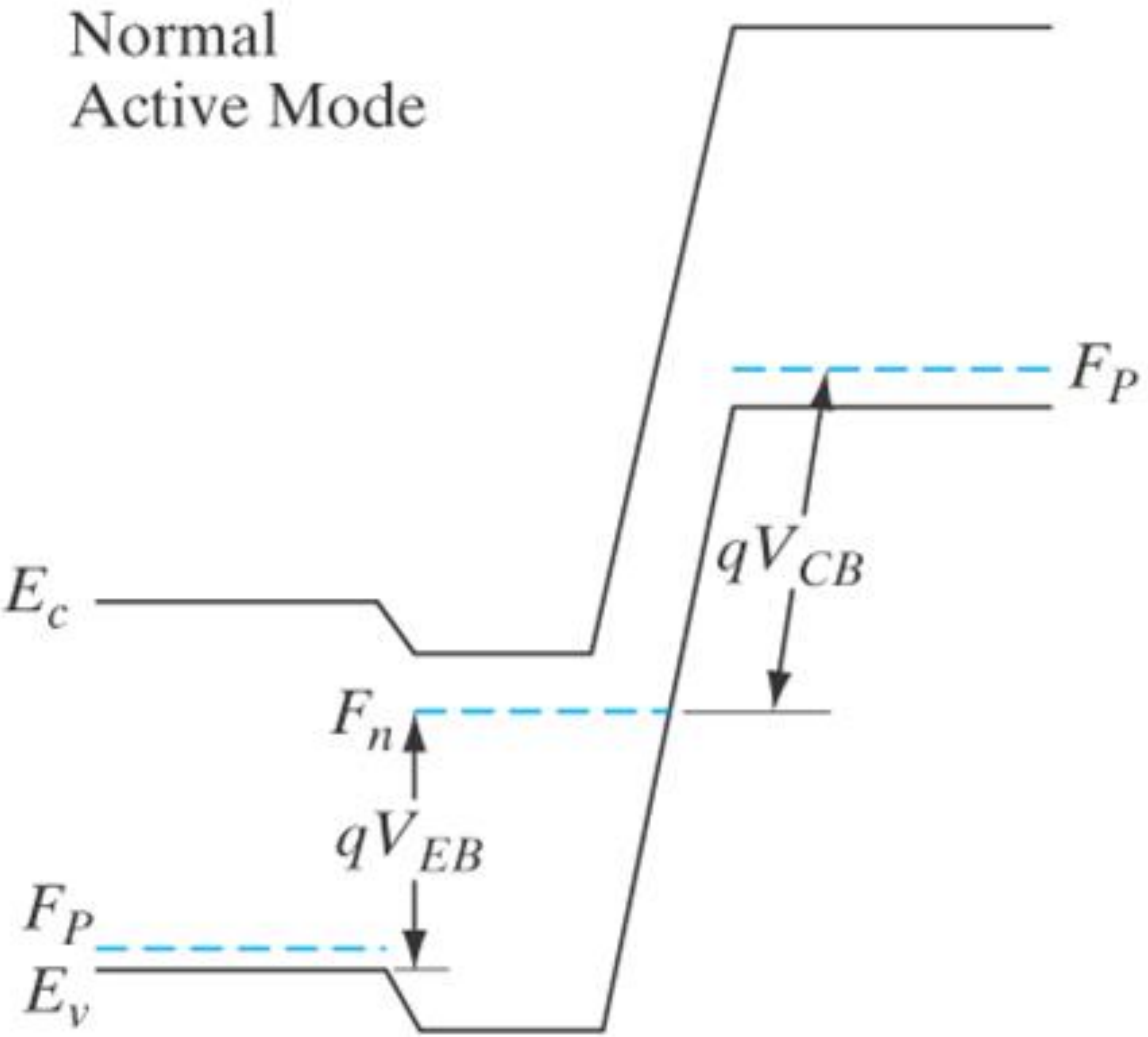
holes injected

holes collected

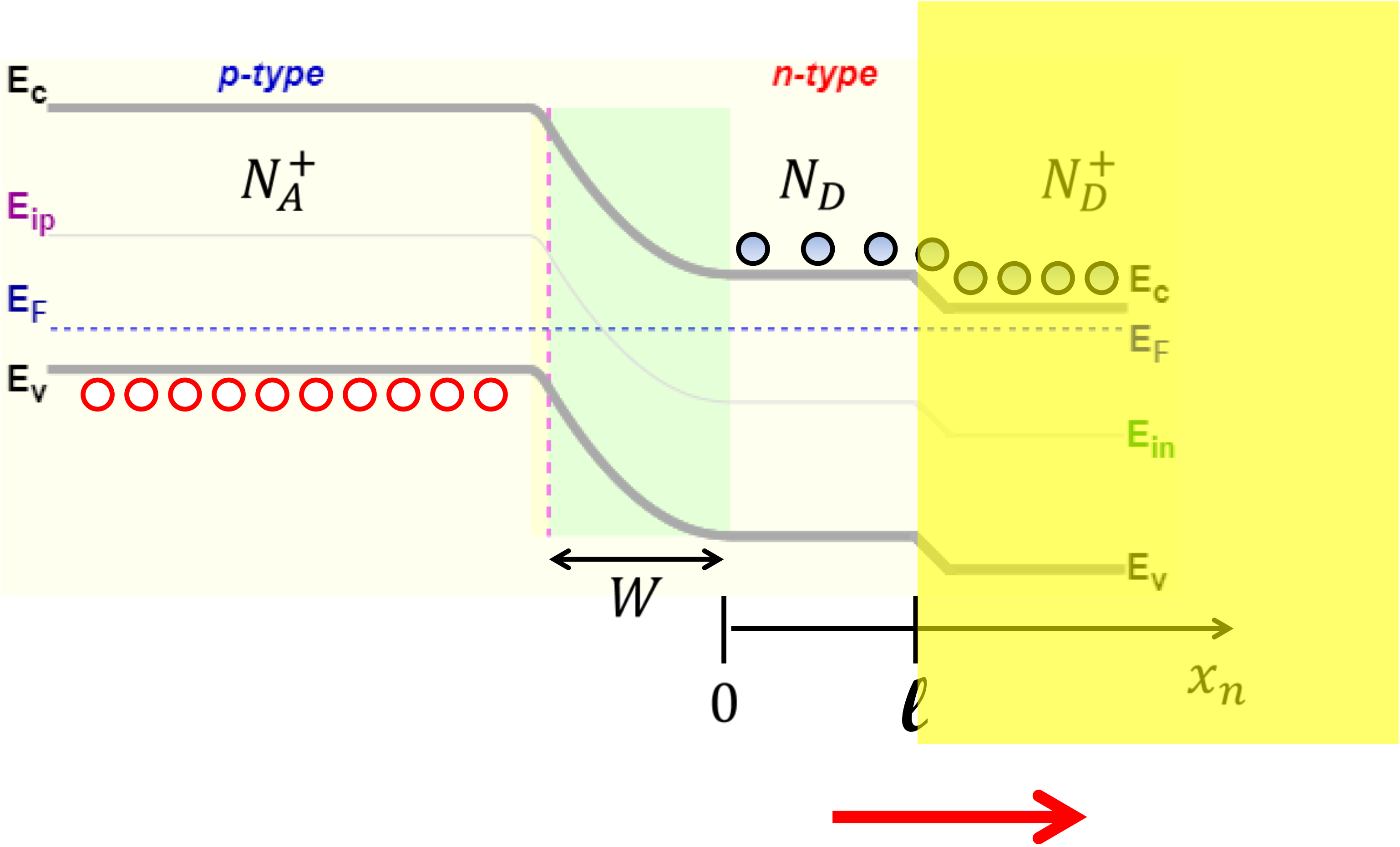
forward biased

reverse biased

p-n-p B-J-T band diagram

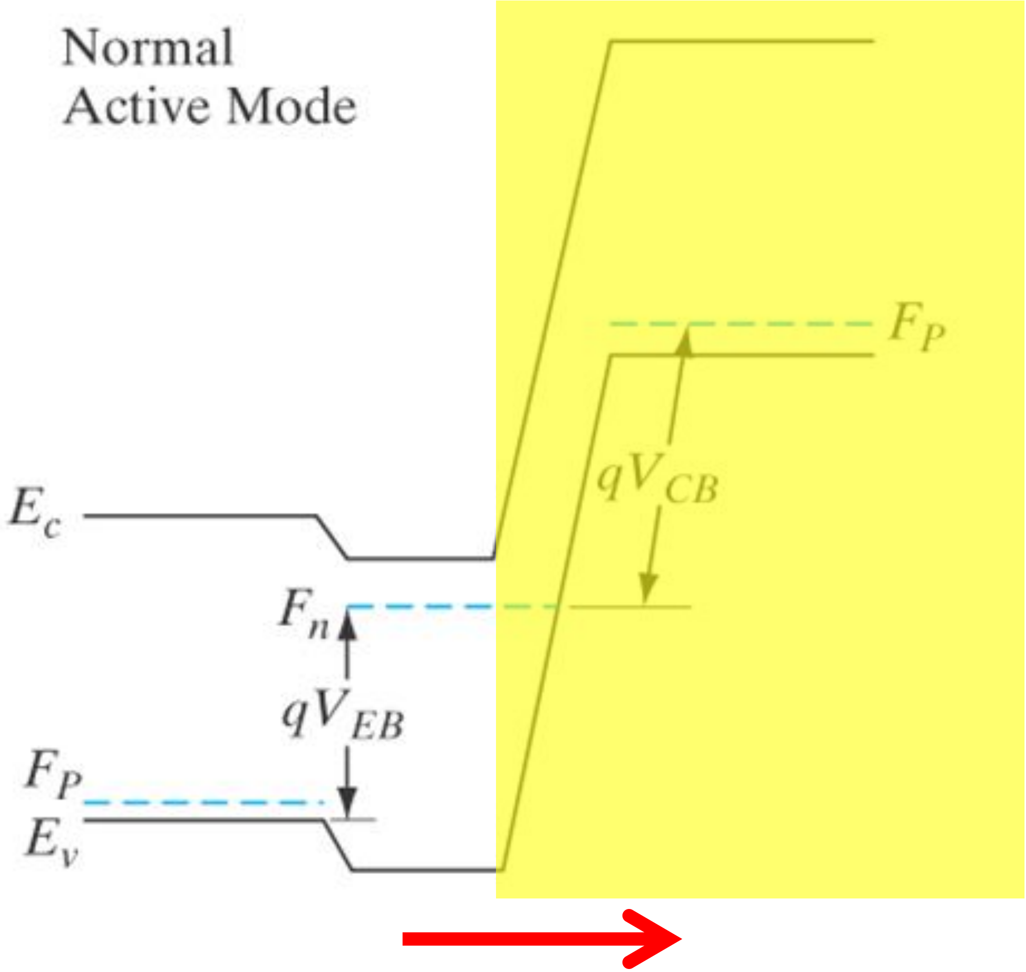


Similarity between N-B-D and B-J-T



Injected holes are quickly removed by recombination with high n^+ concentration

Similarity between N-B-D and B-J-T



Holes are quickly removed by high field of reverse biased p-n junction

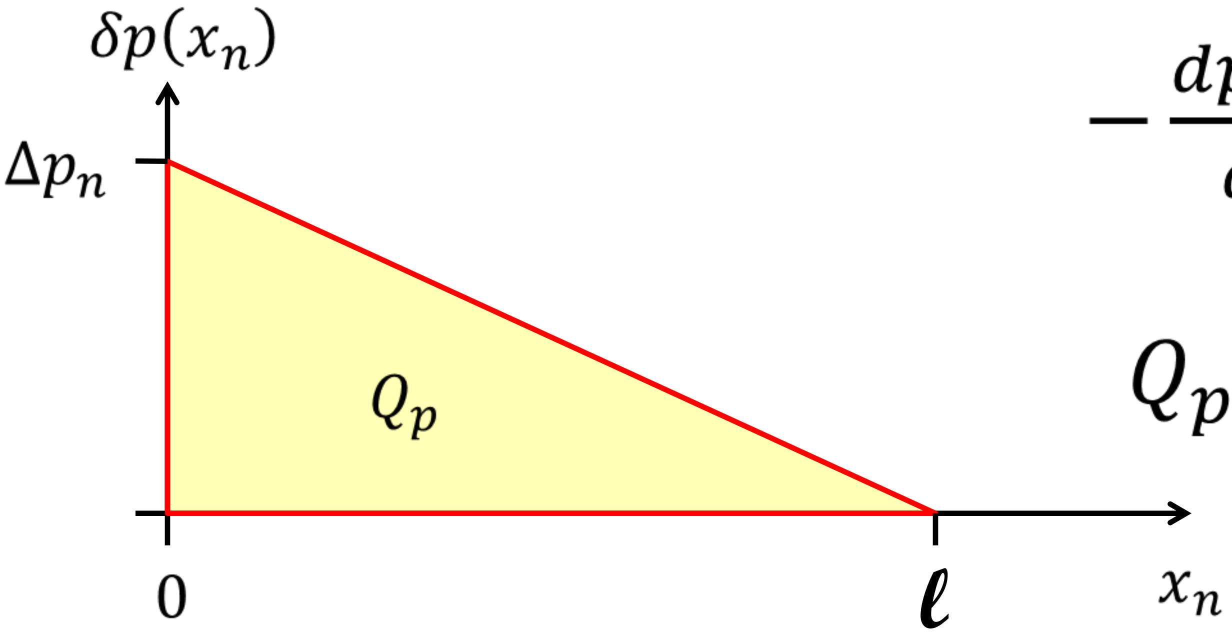
Narrow Base Diode

- $\ell \ll L_P \sim 1 \text{ to } 10\ \mu\text{m}$
- **Boundary conditions for holes is modified by presence of heavily doped n$^+$-region**
- **Assume that minority holes recombine immediately when entering the n$^+$-region**
- **Hole lifetime is smaller in n$^+$-region, approximately according to the ratio** $\frac{N_D^+}{N_D}$

Hole boundary conditions

$$\delta p(x_n = 0) = \Delta p_n = p_n\left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

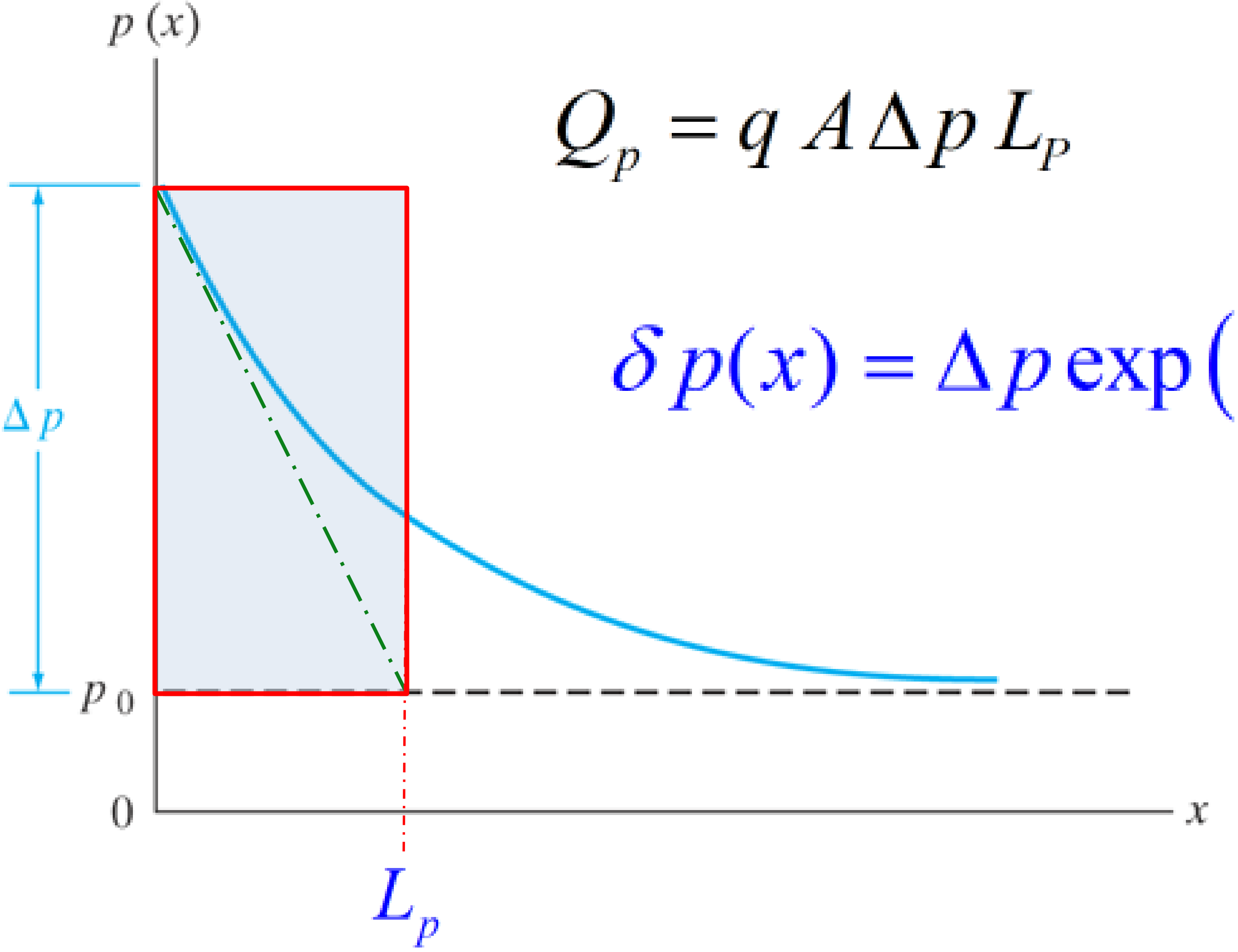
$$\delta p(x_n = \ell\,) \approx 0$$



$$-\frac{dp(x_n)}{dx_n} \approx \Delta p_n / \,\ell$$

$$Q_p \approx \frac{1}{2} qA\,\ell\,\Delta p_n$$

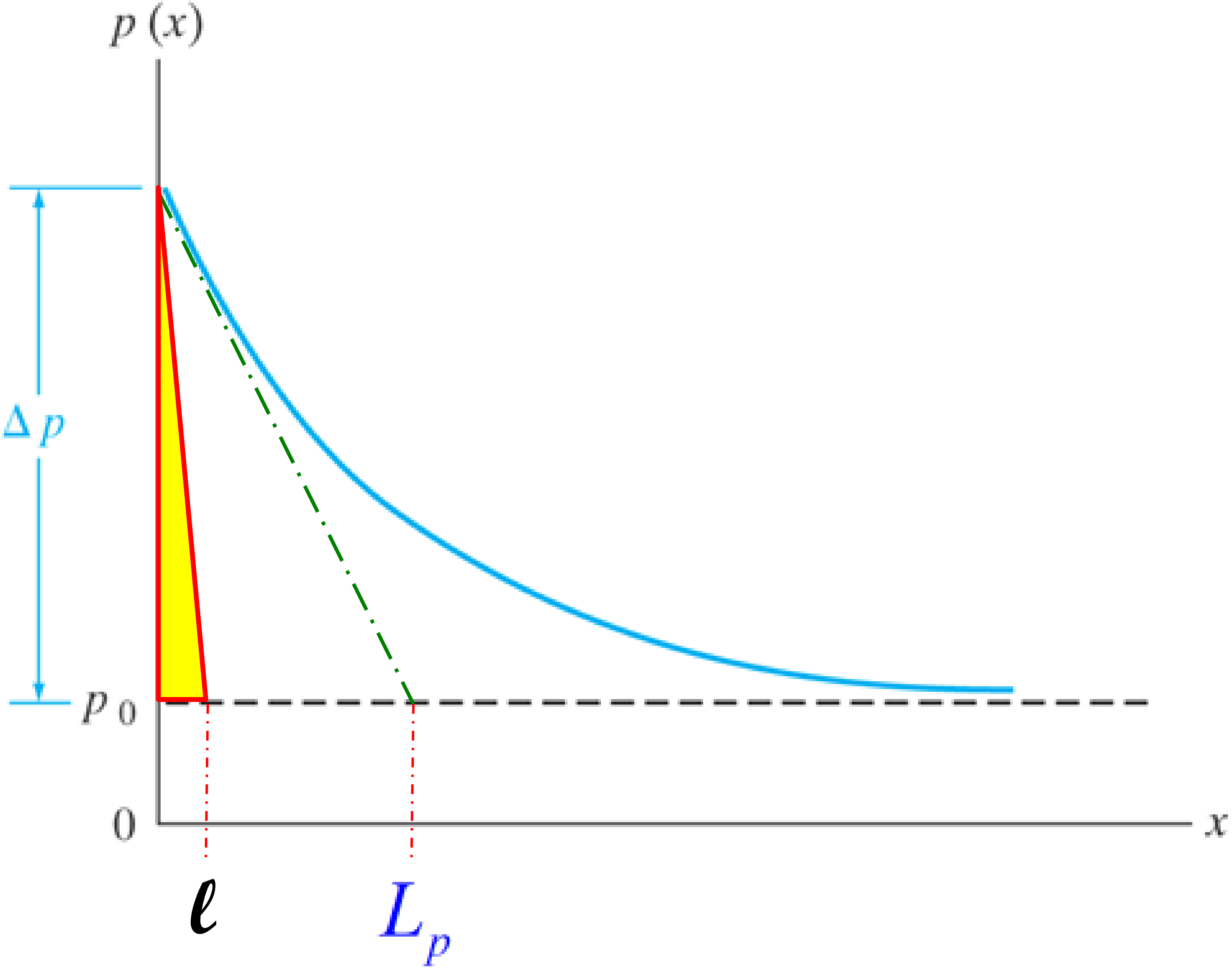
Remember solution for long material



$$Q_p = q\,A\,\Delta p\,L_p$$

$$\delta p(x) = \Delta p \exp\left(-x/L_p\right)$$

Compare with narrow base



Minority hole current – straight line approximation

current density

$$J_p(x_n) \approx J_p(\text{diff}) = -qD_p \frac{d\,p(x_n)}{d\,x_n} = qD_p \frac{\Delta p_n}{\ell}$$

total current

$$I_p(x_n) \approx A\,J_p(\text{diff}) = q\;A\,\frac{D_p}{\ell}\,p_n\left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

Minority hole current

- Diffusion current in this narrow base structure is much larger than in an ordinary p^+-n diode for the same voltage since $\ell \ll L_P$ and ℓ replaces the diffusion length at denominator.
- Essentially, more electrons are dragged in from the n^+-region to satisfy the $\Delta p_n \approx 0$ boundary condition.

Checking straight line approximation

- In straight-line approximation, the hole diffusion current is the same throughout the *n*-region.

- In reality there is a small decrease which one can estimate examining the recombination rate for stored minority hole charge.

Checking straight line approximation

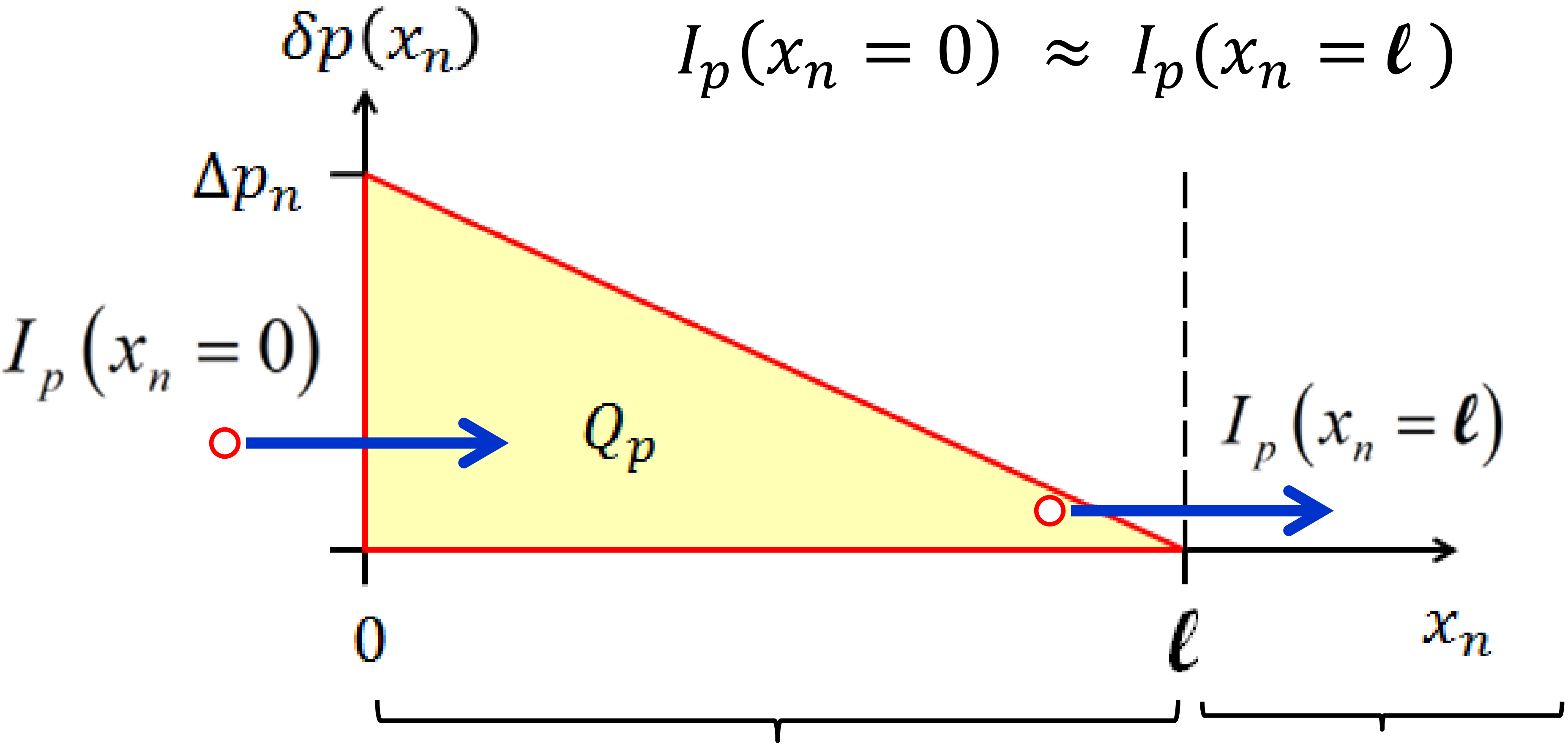
- Each time a hole recombines in the narrow *n*-region another electron flows from the n^+ contact region to maintain space-charge neutrality

$$I_n(\text{recomb}) \approx \frac{Q_P}{\tau_p} = \frac{\frac{1}{2} q A \ell \Delta p_n}{\tau_p}$$

$$= \frac{qA\ell}{2\tau_p} p_n \left[\exp\left(\frac{qV}{k_B T} \right) - 1 \right]$$

Hole boundary conditions

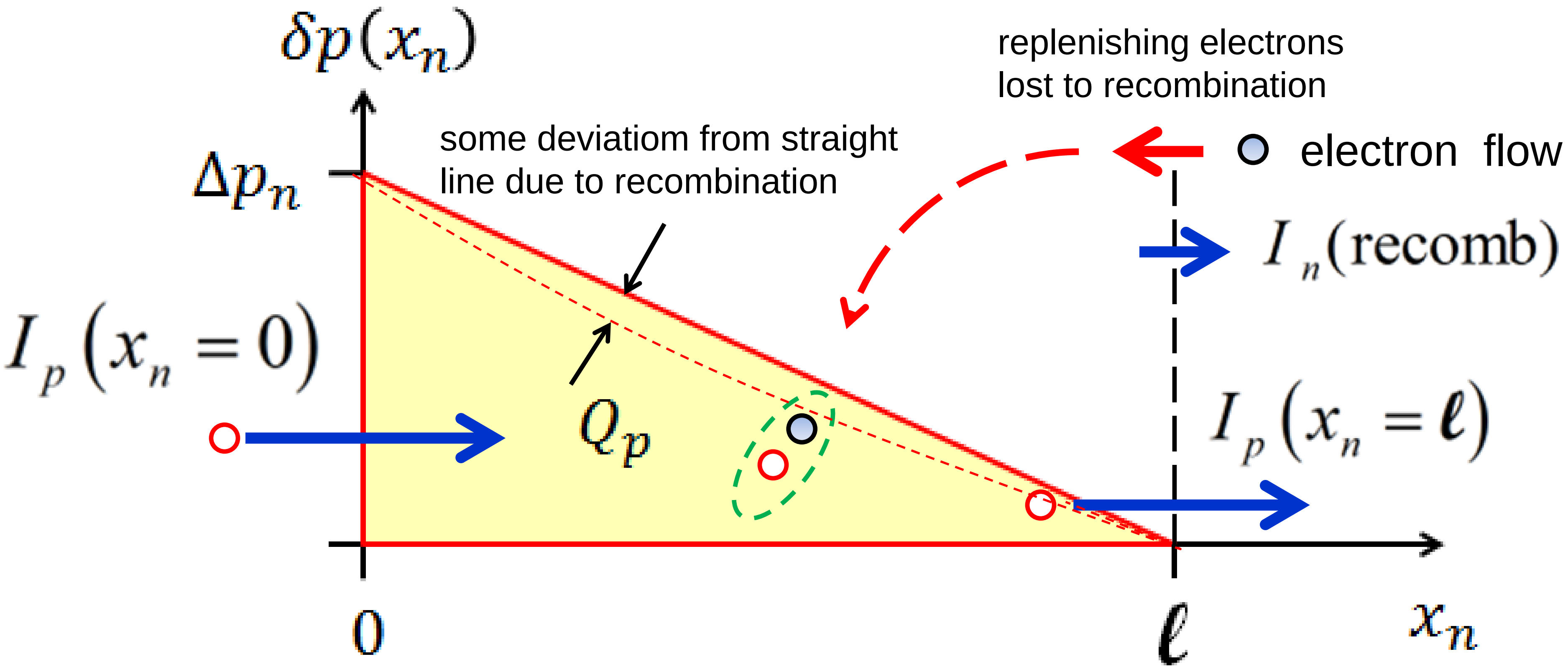
$$I_p(x_n = 0) \approx I_p(x_n = \ell)$$



$$-\frac{dp(x_n)}{dx_n} \approx \Delta p_n / \ell$$

$$\frac{dp(x_n)}{dx_n} \approx 0$$

Beyond straight line approximation



some deviatiom from straight line due to recombination

replenishing electrons lost to recombination

$$\boxed{I_p(x_n = 0) = I_p(x_n = \ell\,) + I_n(\text{recomb})}$$

Majority electron current

- The majority electron current flowing into the *n*-region at $x_n = \ell$ compensates for the decrease in hole diffusion due to recombination in the base region

$$I_n(\text{recomb}) = I_p(x_n = 0) - I_p(x_n = \ell)$$

$$\approx I_p(x_n = 0)\left[\frac{\ell^2}{2L_p^2}\right]$$

result of rigorous analysis

Majority electron current

$$I_n(\text{recomb}) \approx I_p(x_n = 0)\underbrace{\left[\frac{\ell^2}{2L_p^2}\right]}_{\text{veryy small since } \ell \ll L_p}$$

straight-line approximation holds well

$$-\frac{dp(x_n)}{dx_n} \approx \Delta p_n / \ell = \text{constant}$$

- holds fairly well throughout *n*-region

Conclusions

Although an exact solution of the diffusion equation is possible, the straight line approximation offers intuitive understanding of the diode behavior.

For lenghth of the base less than $\sim 0.5\, L_p$ the straight line approximation yields very good results. The approximations for derivative of hole concentration and stored charge can be used safely

$$-\frac{dp(x_n)}{dx_n} \approx \Delta p_n / \ell$$

$$Q_p \approx \frac{1}{2} qA\, \ell\, \Delta p_n$$

Conclusions

There is also a component of electron current injected into the p^+ region which is negligible but which can be calculated with the standard diode equation

$$I_n(\mathrm{inj}) = -q\,A\,\frac{D_n}{L_n} n_p \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

We will see that in the bipolar transistor with a p^+-*n*-*p* structure, a third terminal can inject instead a current into the base to control the much larger hole current, leading to amplification.

Handout

- Download from the website the handout prepared by the ECE 340 staff, with material on the narrow base diode and on the bipolar junction transistor.

Exact Solution (1D diffusion equation) – 1

Assume constant cross sectional area. An exact soution of the diffusion equation is obtained from linear combination of exponentials

$$\delta p(x_n) = \Delta p_n \frac{\exp\left(\dfrac{\ell - x_n}{L_p}\right) - \exp\left(\dfrac{x_n - \ell}{L_p}\right)}{\exp\left(\dfrac{\ell}{L_p}\right) - \exp\left(-\dfrac{\ell}{L_p}\right)}$$

with boundary conditions

$$\delta p(x_n = 0) = \Delta p_n = p_n \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

$$\delta p(x_n = \ell) \approx 0$$

Exact Solution (1D diffusion equation) – 2

At any point of the *n*-region

$$I_p(x_n) = -qAD_p \frac{d}{dx} \delta p(x_n)$$

$$= qA \frac{D_P}{L_p} \Delta p_n \frac{\exp\left(\dfrac{\ell - x_n}{L_P}\right) - \exp\left(\dfrac{x_n - \ell}{L_P}\right)}{\exp\left(\dfrac{\ell}{L_p}\right) - \exp\left(-\dfrac{\ell}{L_p}\right)}$$

Exact Solution (1D diffusion equation) – 3

At $x_n = 0$

$$I_p(x_n = 0) = qA\frac{D_P}{L_p}\Delta p_n \,\mathrm{ctnh}\left(\frac{\ell}{L_p}\right)$$

$$= qA\frac{D_P}{\ell}\Delta p_n\left[1+\frac{\ell^2}{3L_p^2}\right] \qquad \ell \ll L_p$$

using the expansion $\mathrm{ctnh}(y) \sim y^{-1}\left[1+y^2/3+\cdots\right]$

For $\ell \gg L_p$ we have $\mathrm{ctnh}(y) \to 1$

we recover the standard diode equation (long base)

Exact Solution (1D diffusion equation) – 4

At $x_n = \ell$

$$I_p(x_n = \ell) = qA\frac{D_p}{L_p}\Delta p_n \,\mathrm{csch}\left(\frac{\ell}{L_p}\right)$$

$$= qA\frac{D_p}{\ell}\Delta p_n\left[1 - \frac{\ell^2}{6L_p^2}\right] \qquad \ell \ll L_p$$

using the expansion $\mathrm{csch}(y) \sim y^{-1}\left[1 - y^2/6 + \cdots\right]$

slightly less than $I_p(x_n = 0)$

Exact Solution (1D diffusion equation) – 5

majority electron current flowing into the base to offset recombination of holes

$$I_n(\text{recomb}) = I_p(x_n = 0) - I_p(x_n = \ell)$$

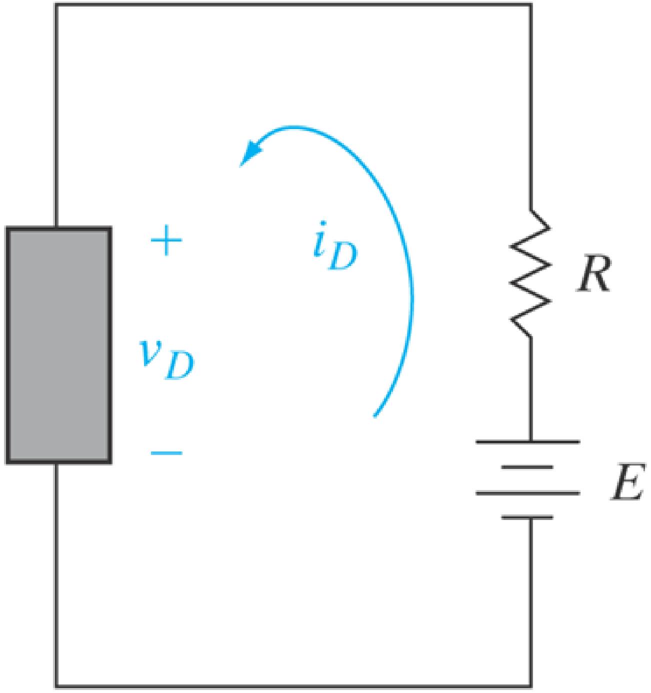
$$= qA\frac{D_p}{L_p}\Delta p_n \tanh\left(\frac{\ell}{2L_p}\right)$$

$$= qA\frac{D_p}{\ell}\Delta p_n \left[\frac{\ell^2}{2L_p^2}\right] \qquad \ell \ll L_p$$

- **Transistor Operation**
- **Introduction to the Bipolar Junction Transistor (BJT)**

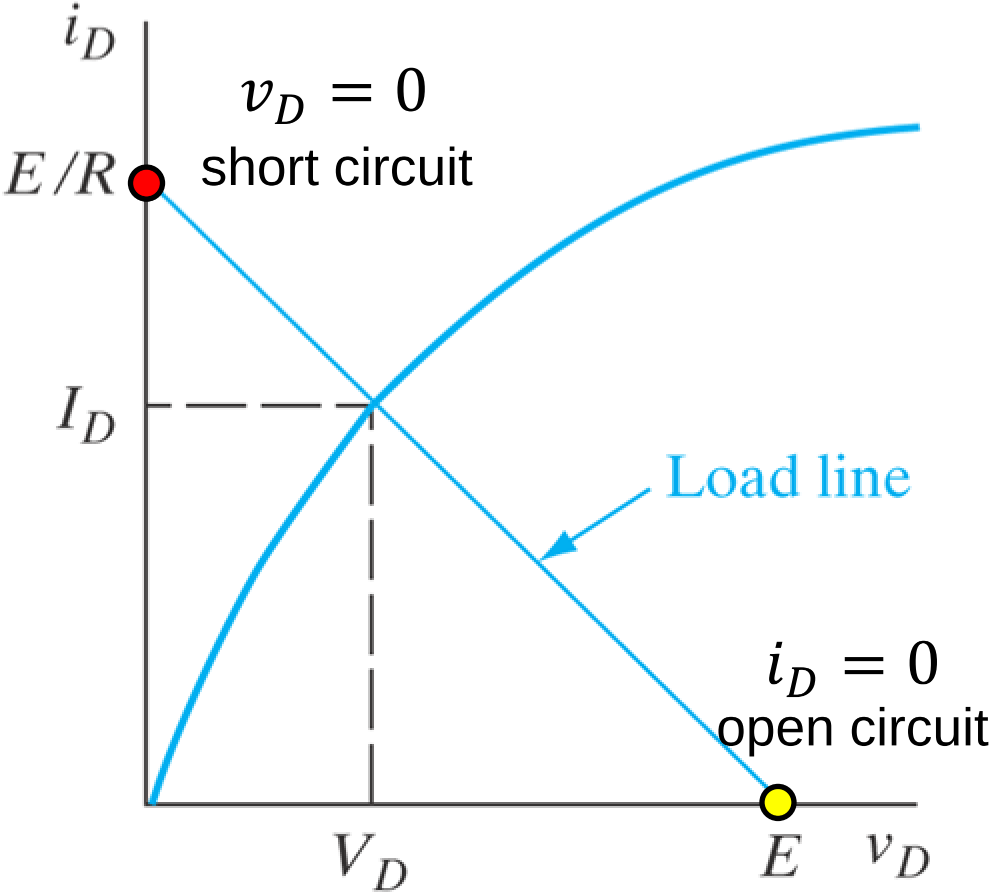
Graphical solution of circuit

- **The load line in a two-terminal device**



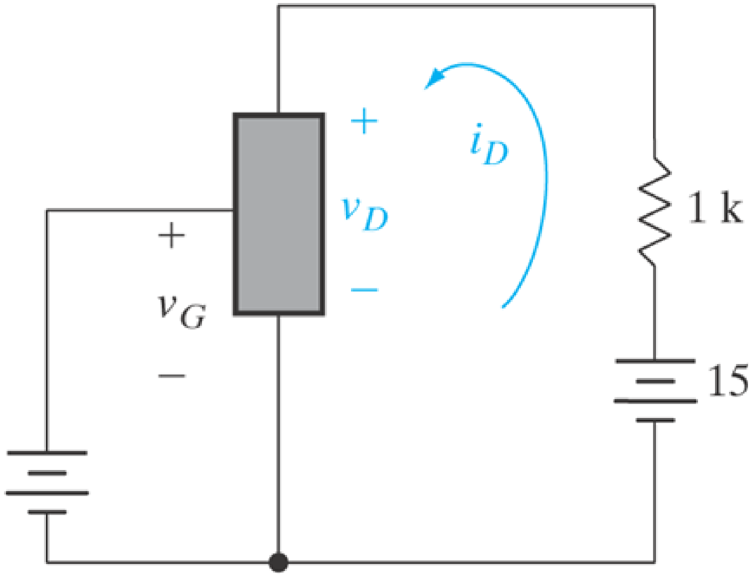
$$E = i_D R + v_D$$

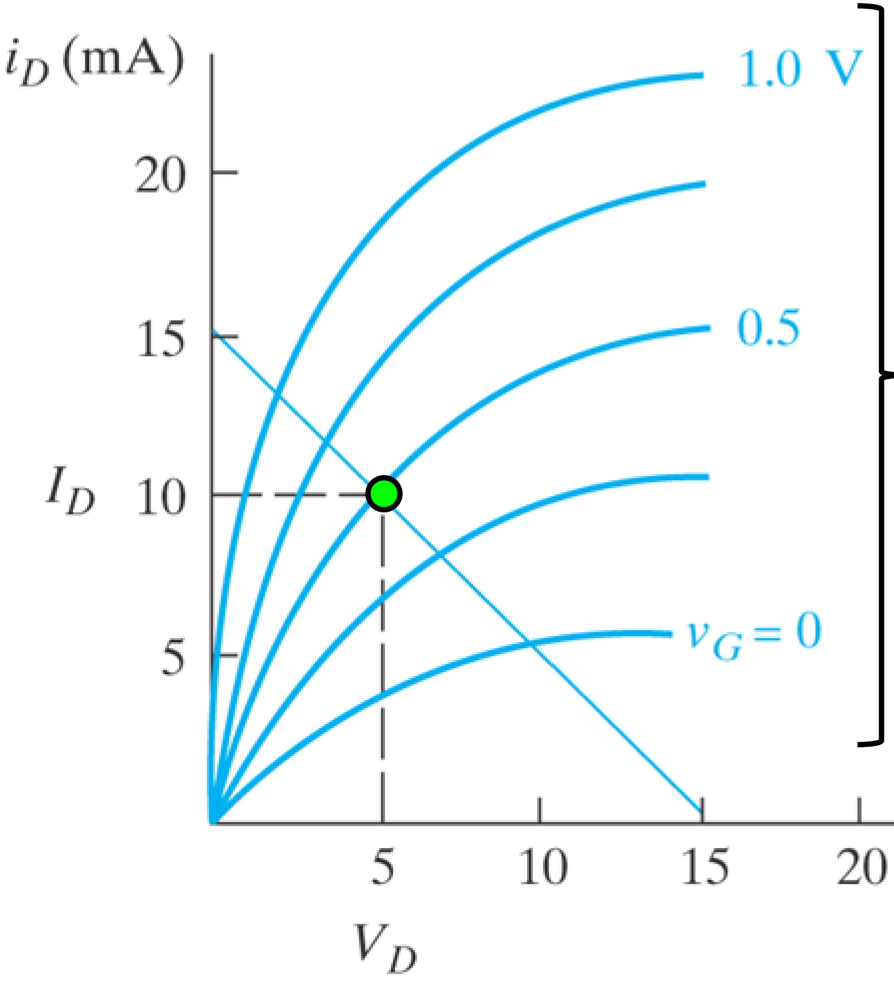
load line equation



Graphical solution of circuit

- **The load line in a three-terminal device**





control voltage

$$\boxed{V_G = 0.5\ \mathrm{V}}$$

$$V_D = 5\ \mathrm{V}$$

$$I_D = 10\ \mathrm{mA}$$

Amplification

a.c. component added to the control input generates large variation of the output

From the graph

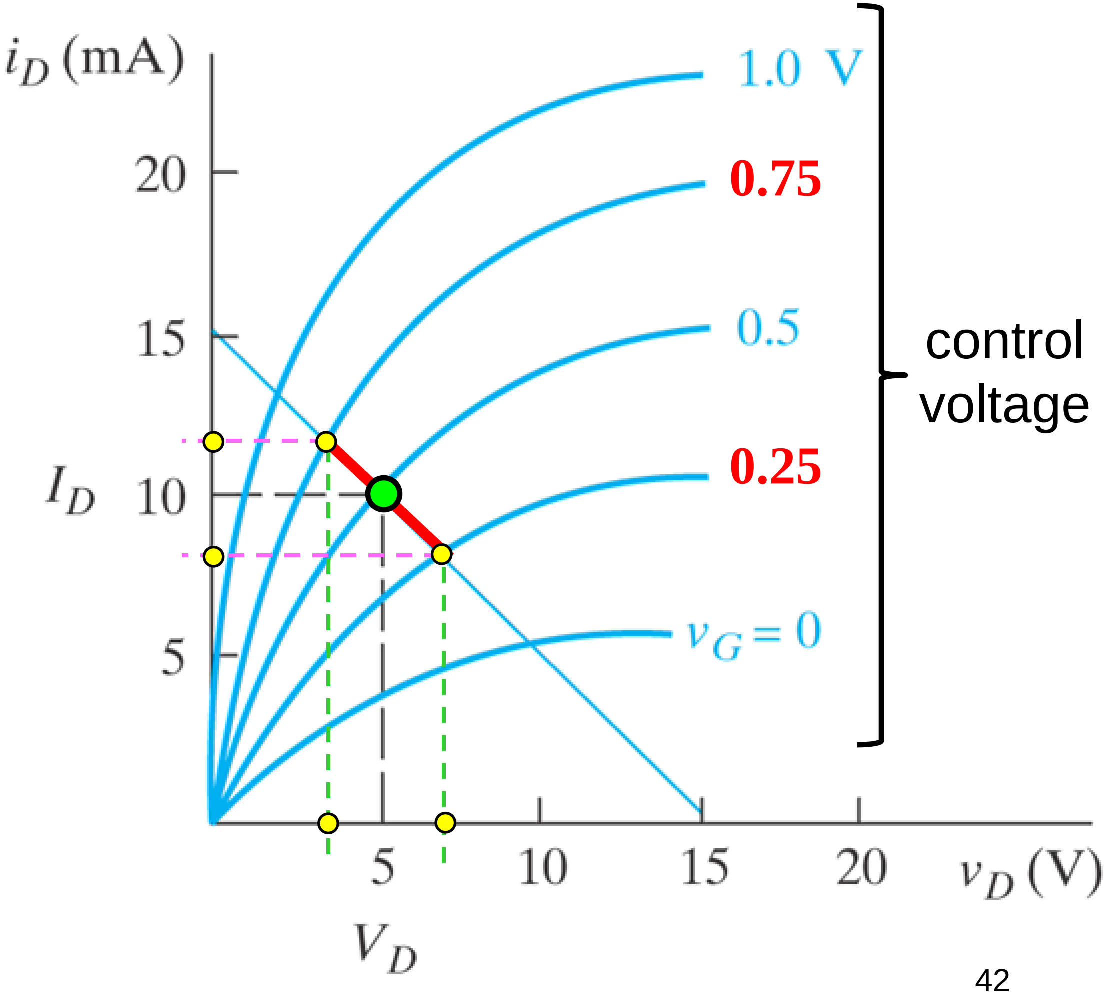
$$\Delta V_G = 0.75 - 0.25 = 0.5\ \mathrm{V}$$

$$\Delta V_D \approx 7.0 - 3.0 = 4\ \mathrm{V}$$

$$\Delta I_D \approx 12.0 - 8.0 = 4\ \mathrm{mA}$$

Gain

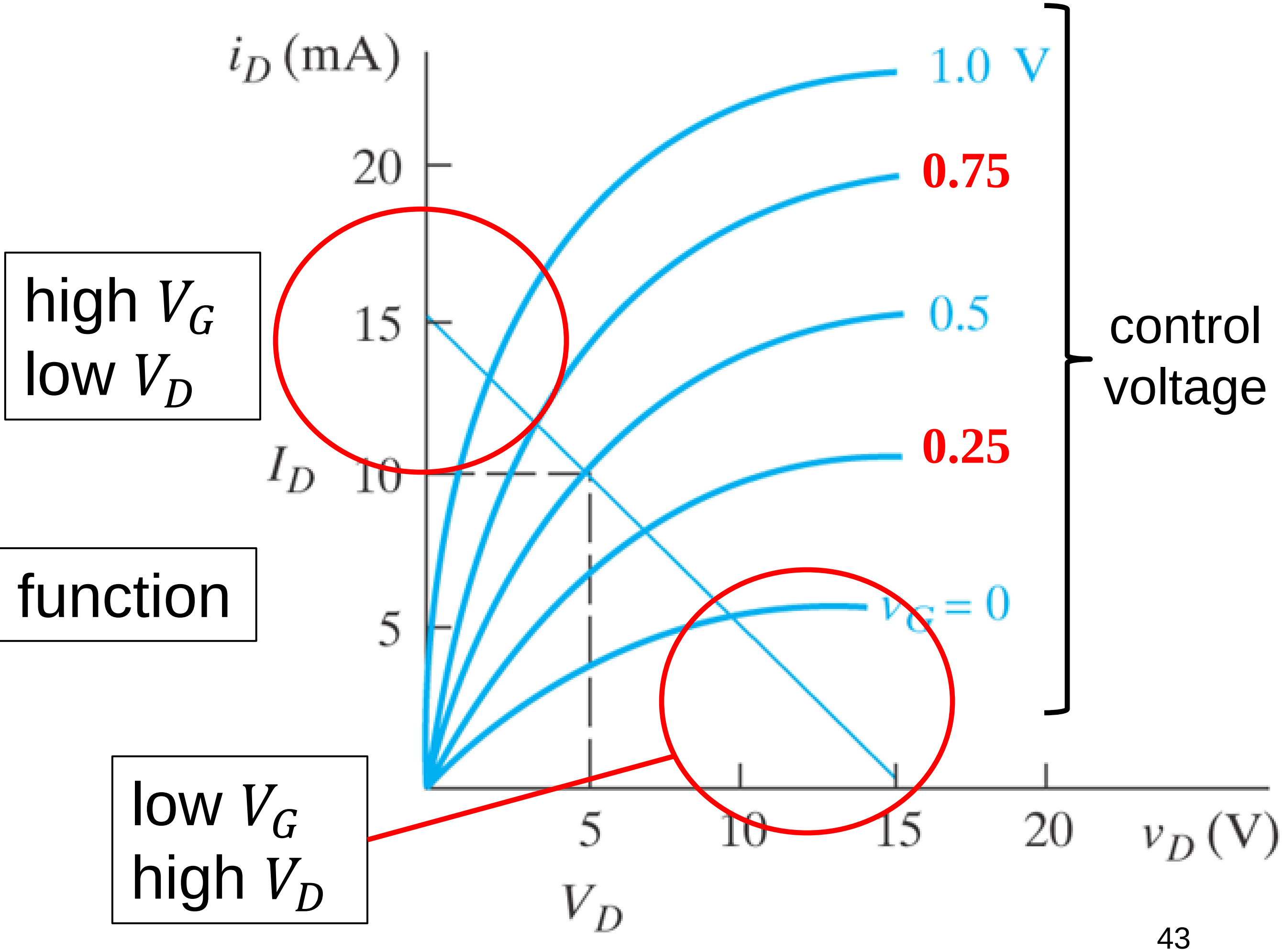
$$|G_V| \approx 8$$



Switching

V_G = input

V_D = output



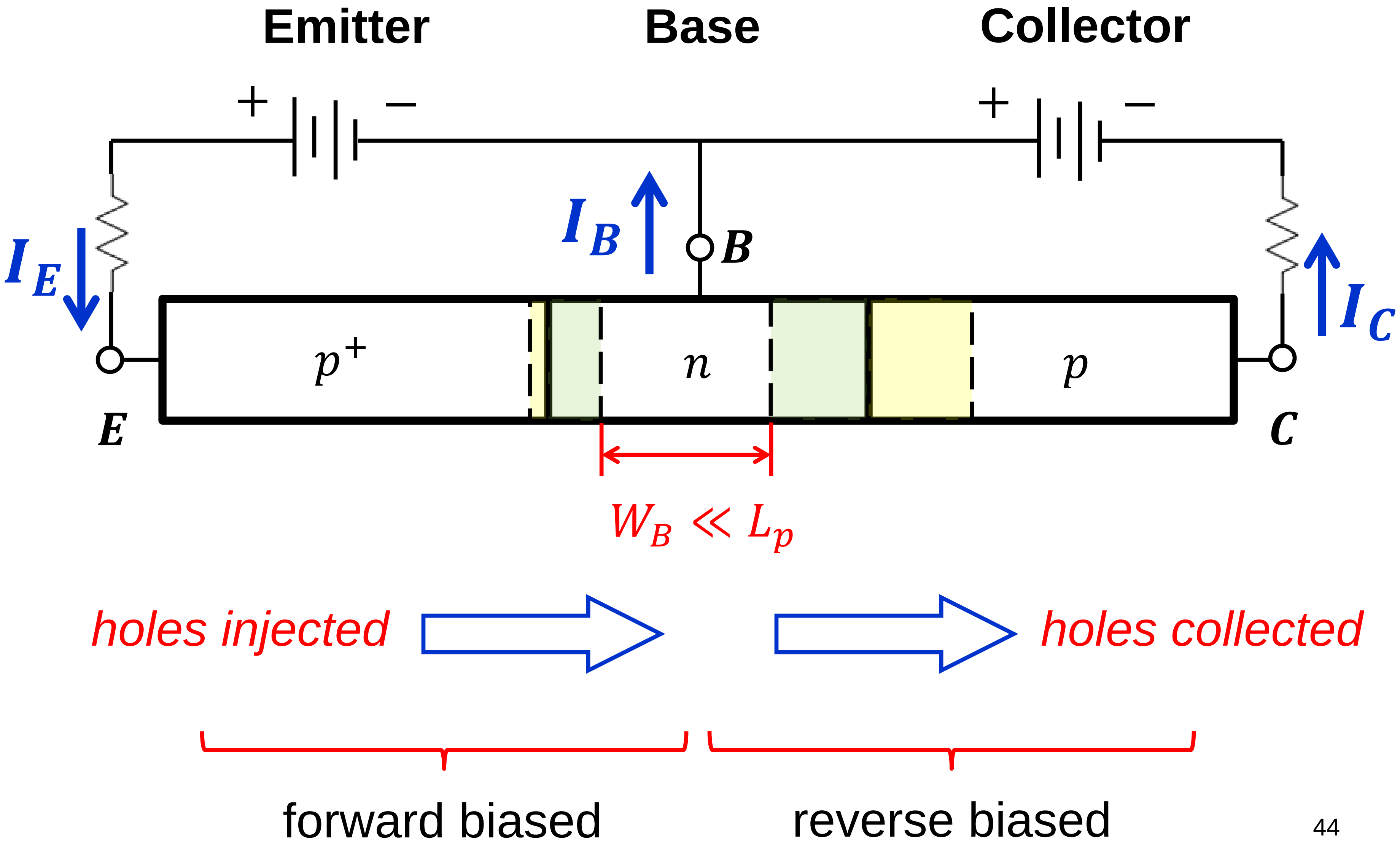
high V_G
low V_D

control voltage

low V_G
high V_D

basic inverter function

Bipolar Junction Transistor (*p-n-p*)



$W_B \ll L_p$

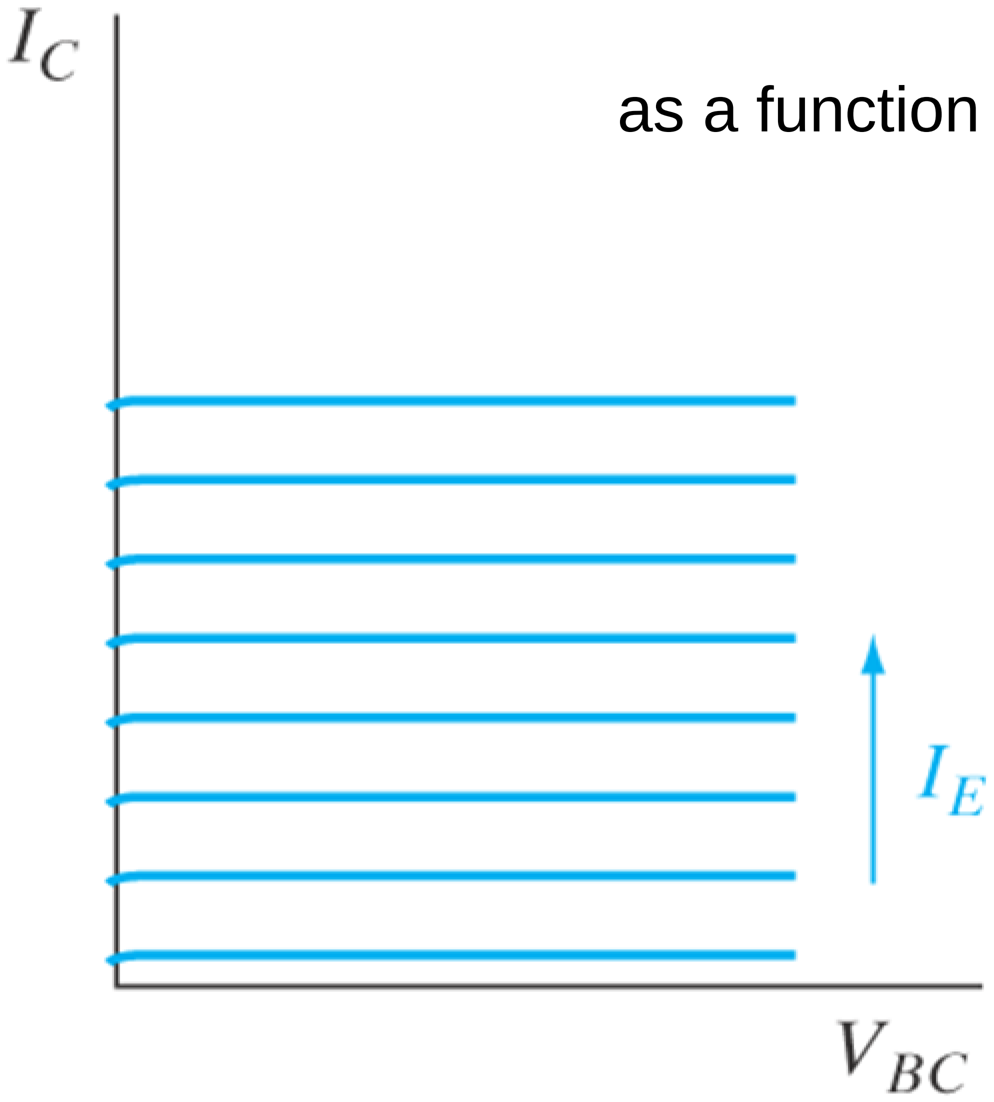
holes injected

holes collected

forward biased

reverse biased

I-V curves of the reverse-biased junction

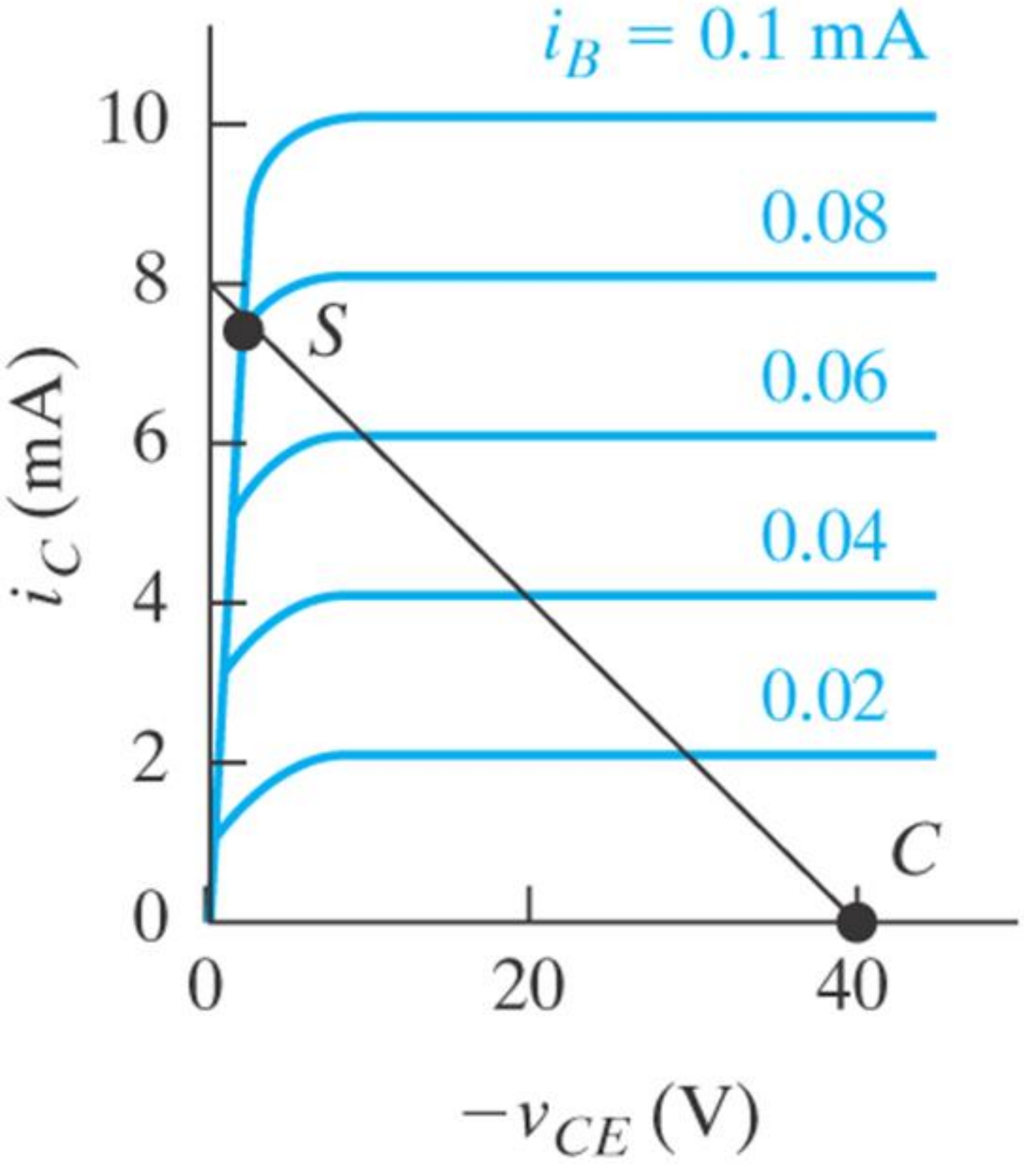


as a function of emitter current

resembles behavior
of the photodiode

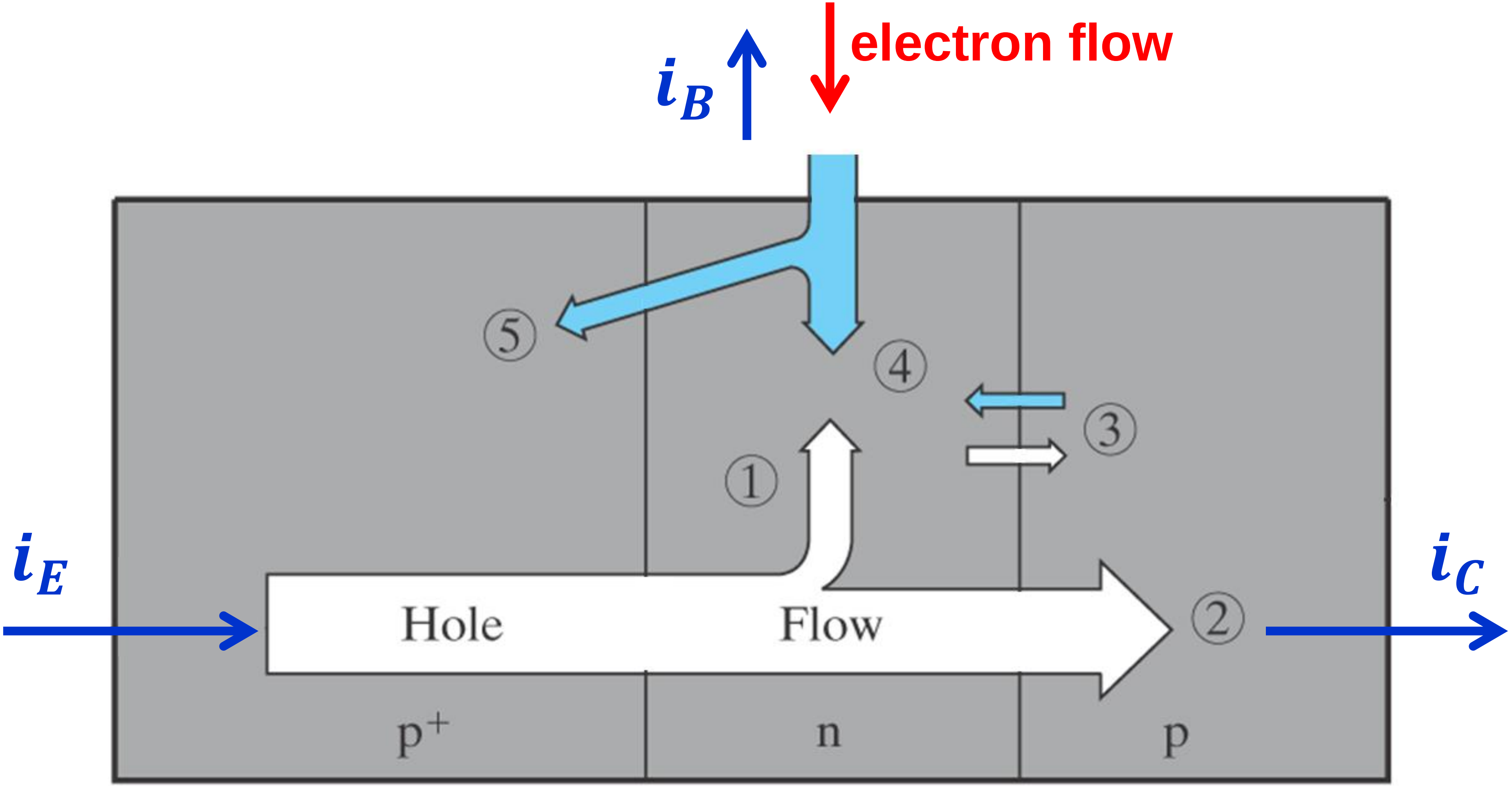
current generator

BJT transistor I-V curves



The control input is the base current i_B

Carrier flow



$$\xrightarrow{i_E} = \begin{cases} \longrightarrow i_{Ep} \\ \rightarrow i_{En} \end{cases} \qquad \xrightarrow{i_C}$$