ECE 340 Lecture 38
Semiconductor Electronics

Spring 2022
10:00-10:50am
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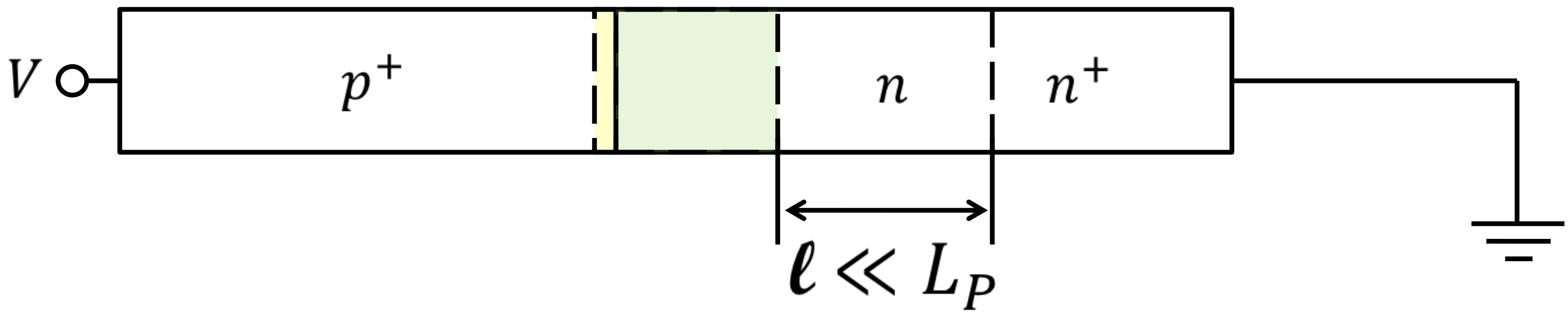
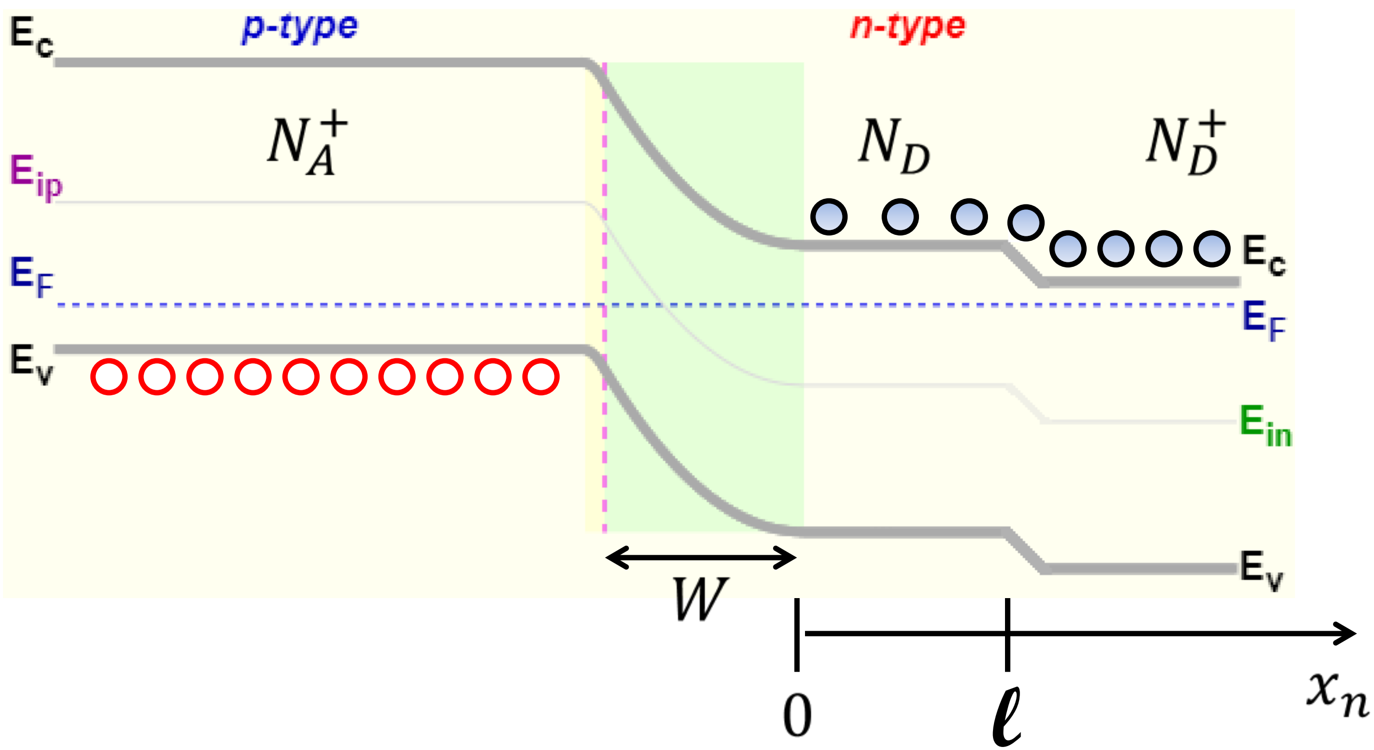
2062 ECE Building

Today’s Discussion

- **Narrow Base Diode**
- **Transistor Operation**
- **Introduction to the Bipolar Junction Transistor (BJT)**

Narrow Base Diode (N-B-D)





Exact Solution (1D diffusion equation) – 1

Assume constant cross sectional area. An exact soution of the diffusion equation is obtained from linear combination of exponentials

$$\delta p(x_n) = \Delta p_n \frac{\exp\left(\dfrac{\ell - x_n}{L_P}\right) - \exp\left(\dfrac{x_n - \ell}{L_P}\right)}{\exp\left(\dfrac{\ell}{L_p}\right) - \exp\left(-\dfrac{\ell}{L_p}\right)}$$

with boundary conditions

$$\delta p(x_n = 0) = \Delta p_n = p_n \left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]$$

$$\delta p(x_n = \ell) \approx 0$$

Exact Solution (1D diffusion equation) – 2

At any point of the *n*-region

$$I_p(x_n) = -qAD_p \frac{d}{dx}\delta p(x_n)$$

$$= qA\frac{D_p}{L_p}\Delta p_n \frac{\exp\left(\dfrac{\ell - x_n}{L_p}\right) - \exp\left(\dfrac{x_n - \ell}{L_p}\right)}{\exp\left(\dfrac{\ell}{L_p}\right) - \exp\left(-\dfrac{\ell}{L_p}\right)}$$

Exact Solution (1D diffusion equation) – 3

At $x_n = 0$

$$I_p(x_n = 0) = qA\frac{D_P}{L_p}\Delta p_n \operatorname{ctnh}\left(\frac{\ell}{L_p}\right)$$

$$= qA\frac{D_P}{\ell}\Delta p_n\left[1 + \frac{\ell^2}{3L_p^2}\right] \qquad \ell \ll L_p$$

using the expansion $\operatorname{ctnh}(y) \sim y^{-1}\left[1 + y^2/3 + \cdots\right]$

For $\ell \gg L_p$ we have $\operatorname{ctnh}(y) \to 1$

we recover the standard diode equation (long base)

Exact Solution (1D diffusion equation) – 4

At $x_n = \ell$

$$I_p(x_n = \ell) = qA\frac{D_p}{L_p}\Delta p_n \operatorname{csch}\left(\frac{\ell}{L_p}\right)$$

$$= qA\frac{D_p}{\ell}\Delta p_n\left[1 - \frac{\ell^2}{6L_p^2}\right] \qquad \ell \ll L_p$$

using the expansion $\operatorname{csch}(y) \sim y^{-1}\left[1 - y^2/6 + \cdots\right]$

slightly less than $I_p(x_n = 0)$

Exact Solution (1D diffusion equation) – 5

majority electron current flowing into the base to offset recombination of holes

$$I_n(\text{recomb}) = I_p(x_n = 0) - I_p(x_n = \ell)$$

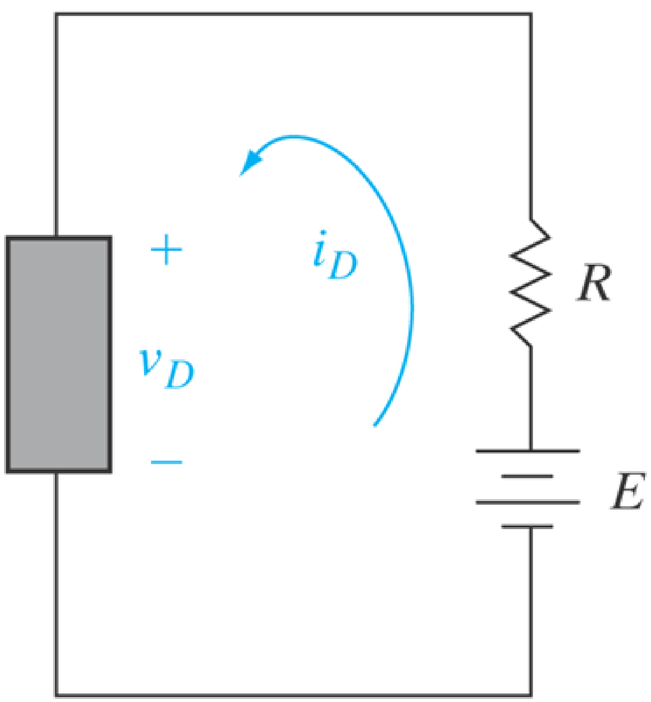
$$= qA\frac{D_p}{L_p}\Delta p_n \tanh\left(\frac{\ell}{2L_p}\right)$$

$$= \underbrace{qA\frac{D_p}{\ell}\Delta p_n}_{I_p(x_n=0)}\left[\frac{\ell^2}{2L_p^2}\right] \qquad \ell \ll L_p$$

- **Transistor Operation**
- **Introduction to the Bipolar Junction Transistor (BJT)**

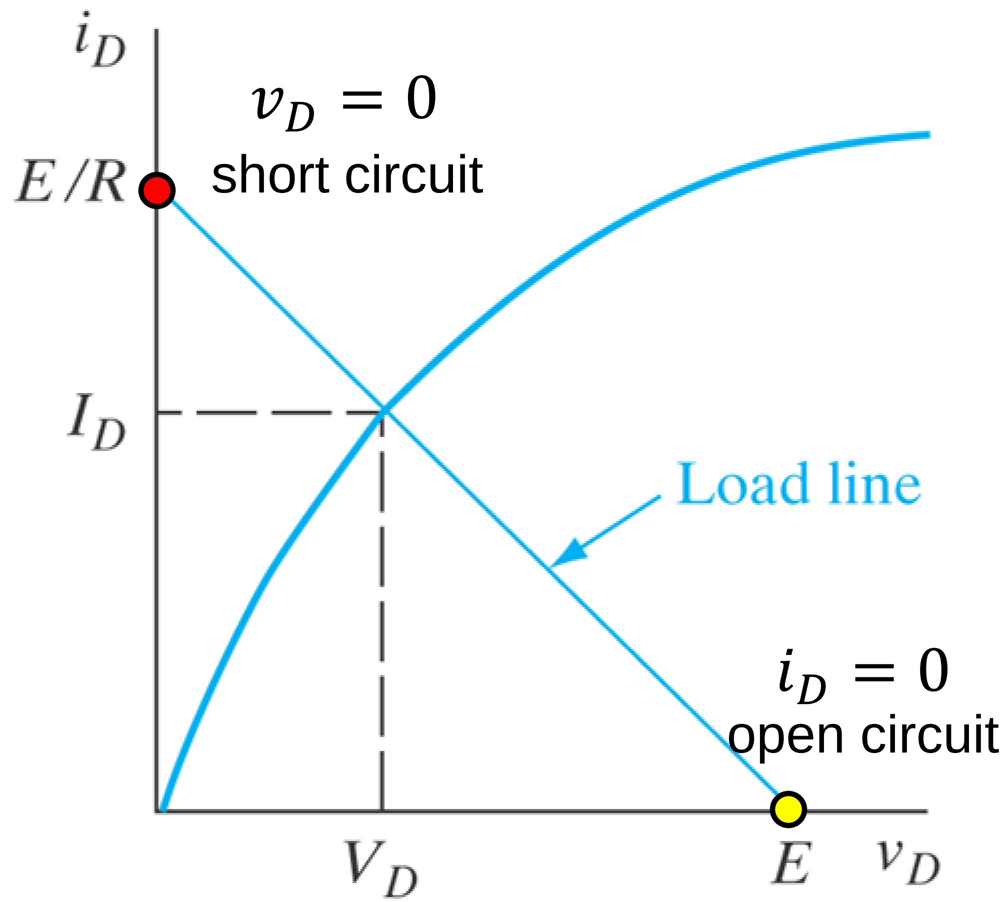
Graphical solution of circuit

- **The load line in a two-terminal device**



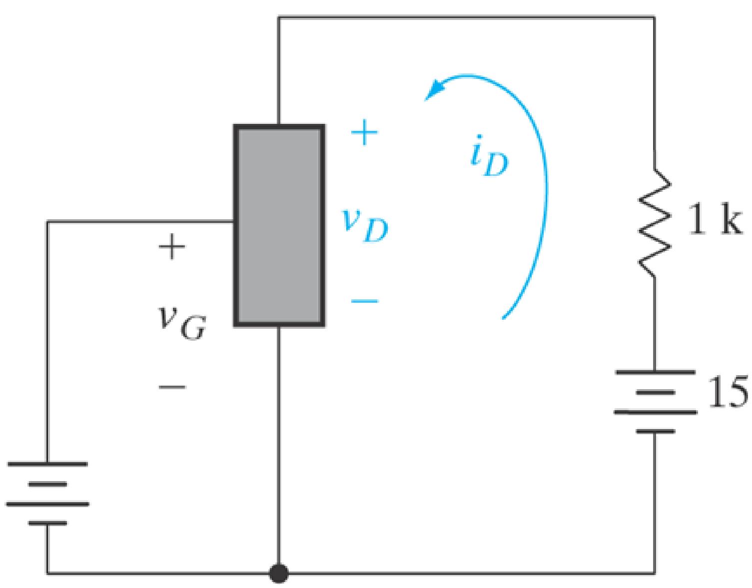
$$E = i_D R + v_D$$

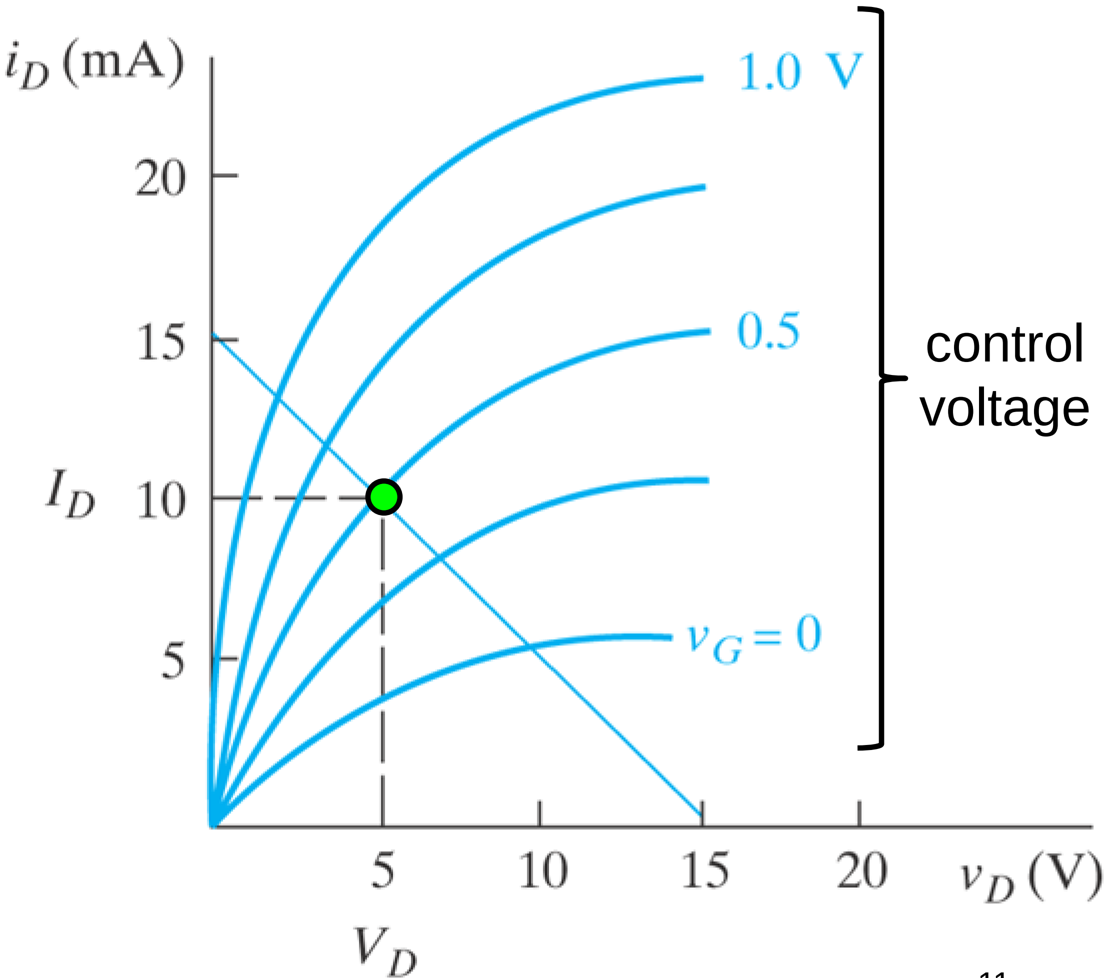
load line equation



Graphical solution of circuit

- **The load line in a three-terminal device**





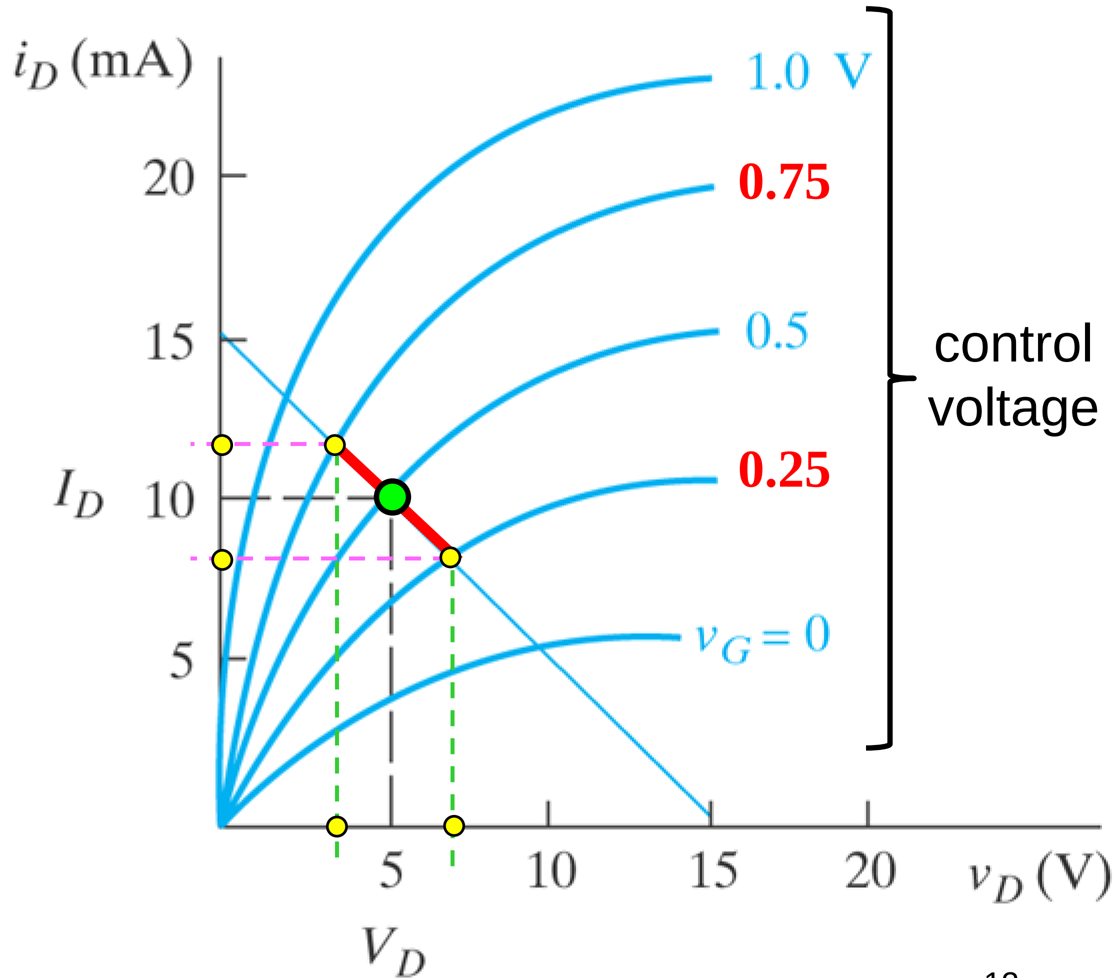
$\boxed{V_G = 0.5\ \text{V}}$

$V_D = 5\ \text{V}$

$I_D = 10\ \text{mA}$

Amplification

a.c. component added to the control input generates large variation of the output



From the graph

$\Delta V_G = 0.75 - 0.25 = 0.5$ V

$\Delta V_D \approx 7.0 - 3.0 = 4$ V

$\Delta I_D \approx 12.0 - 8.0 = 4$ mA

Gain

$$|G_V| \approx 8$$

Switching

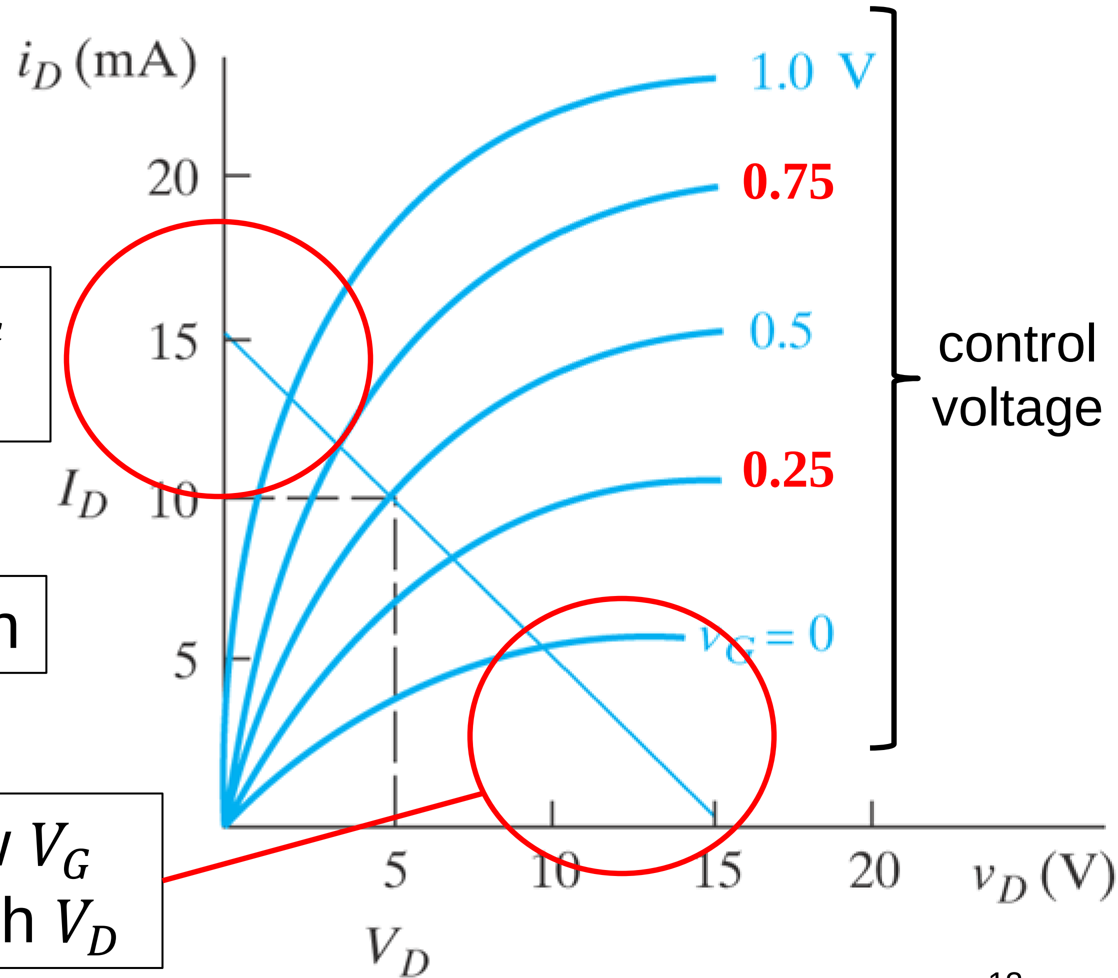
V_G = input

V_D = output

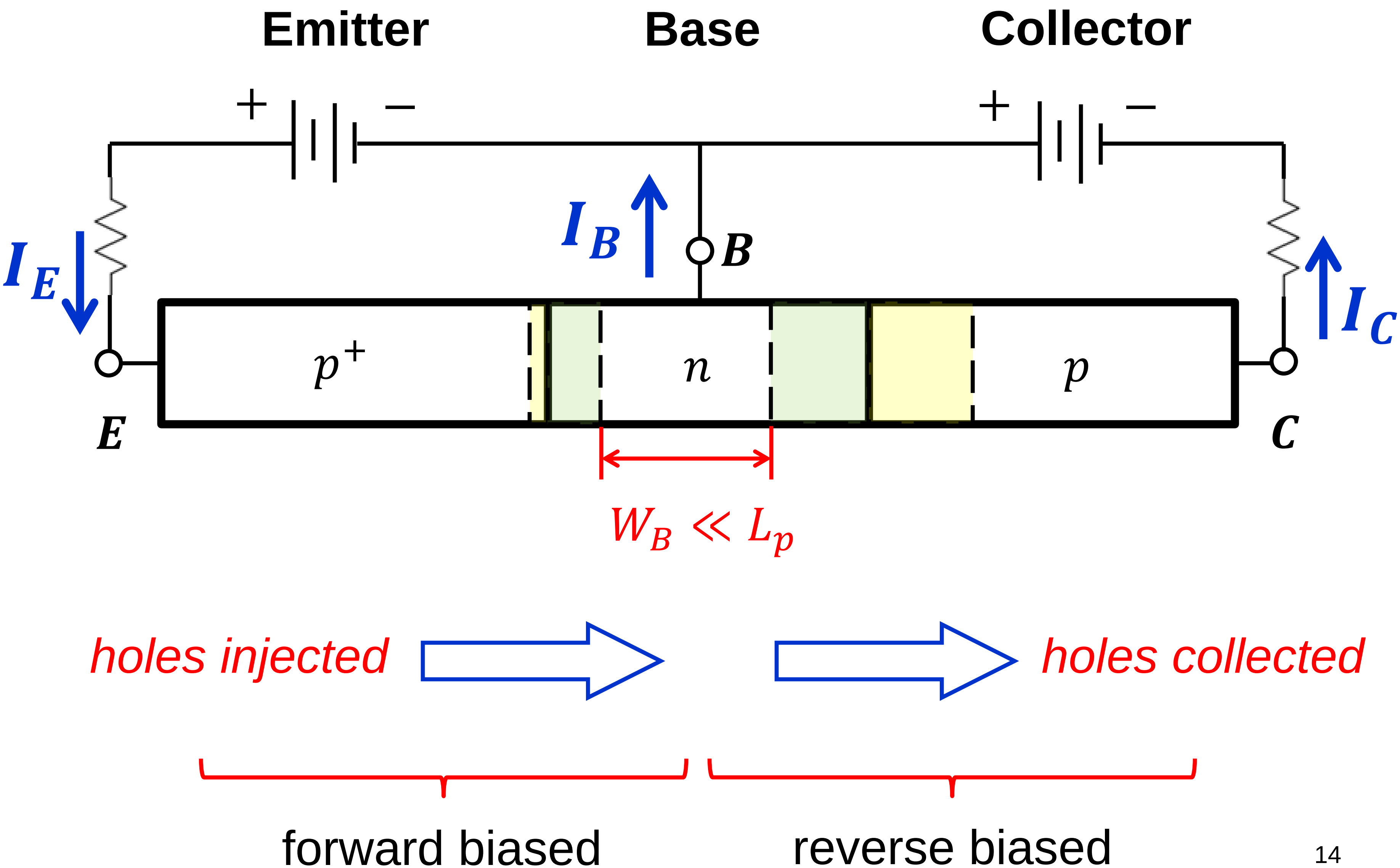
high V_G
low V_D

basic inverter function

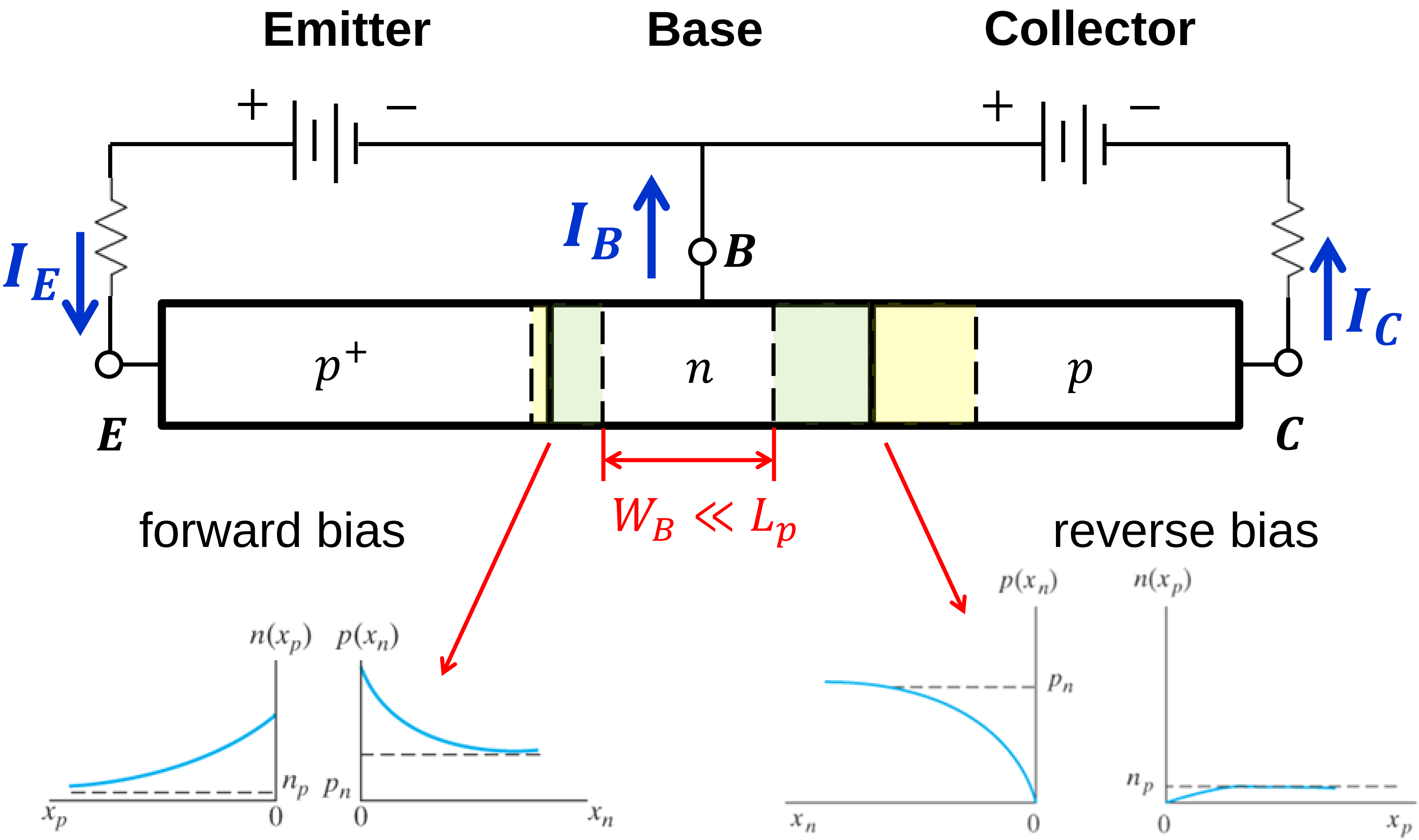
low V_G
high V_D



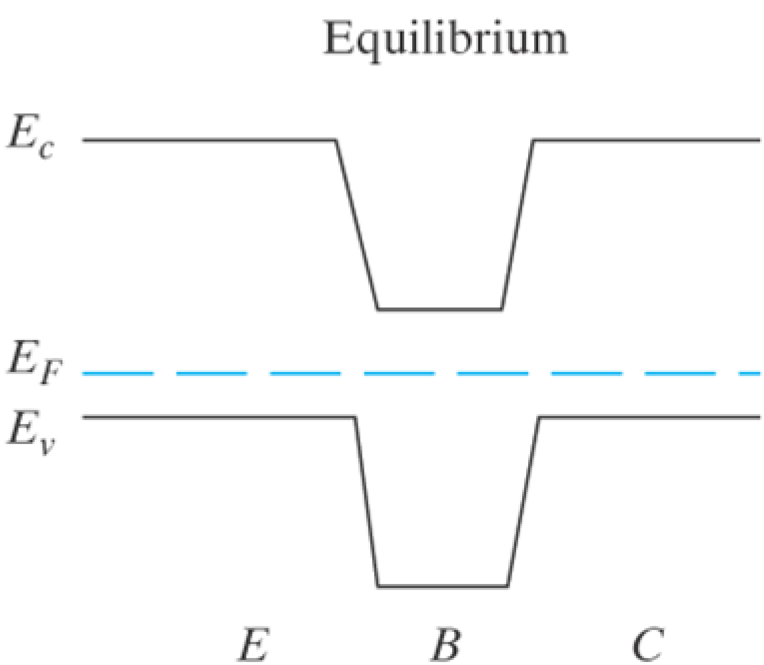
Bipolar Junction Transistor (*p-n-p*)

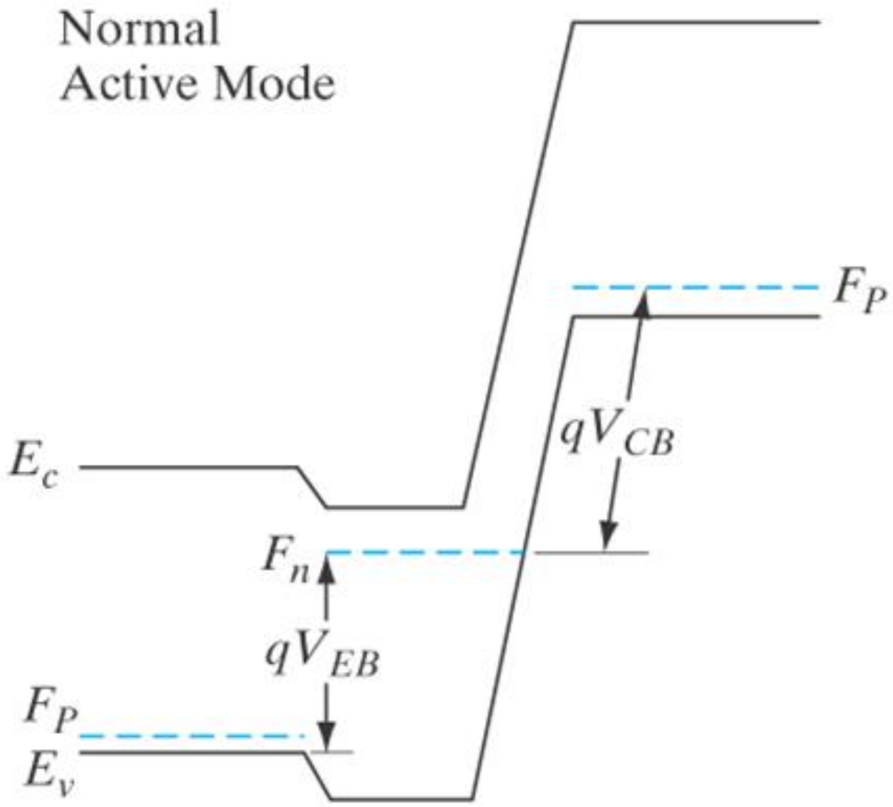


Bipolar Junction Transistor (*p-n-p*)



Bipolar Junction Transistor (*p-n-p*)





Boundary conditions for the neutral base

$$\Delta p_E = p_n \left[\exp\left(\frac{qV_{EB}}{k_B T}\right) - 1\right]$$ emitter side

$$\Delta p_C = p_n \left[\exp\left(\frac{qV_{CB}}{k_B T}\right) - 1\right]$$ collector side

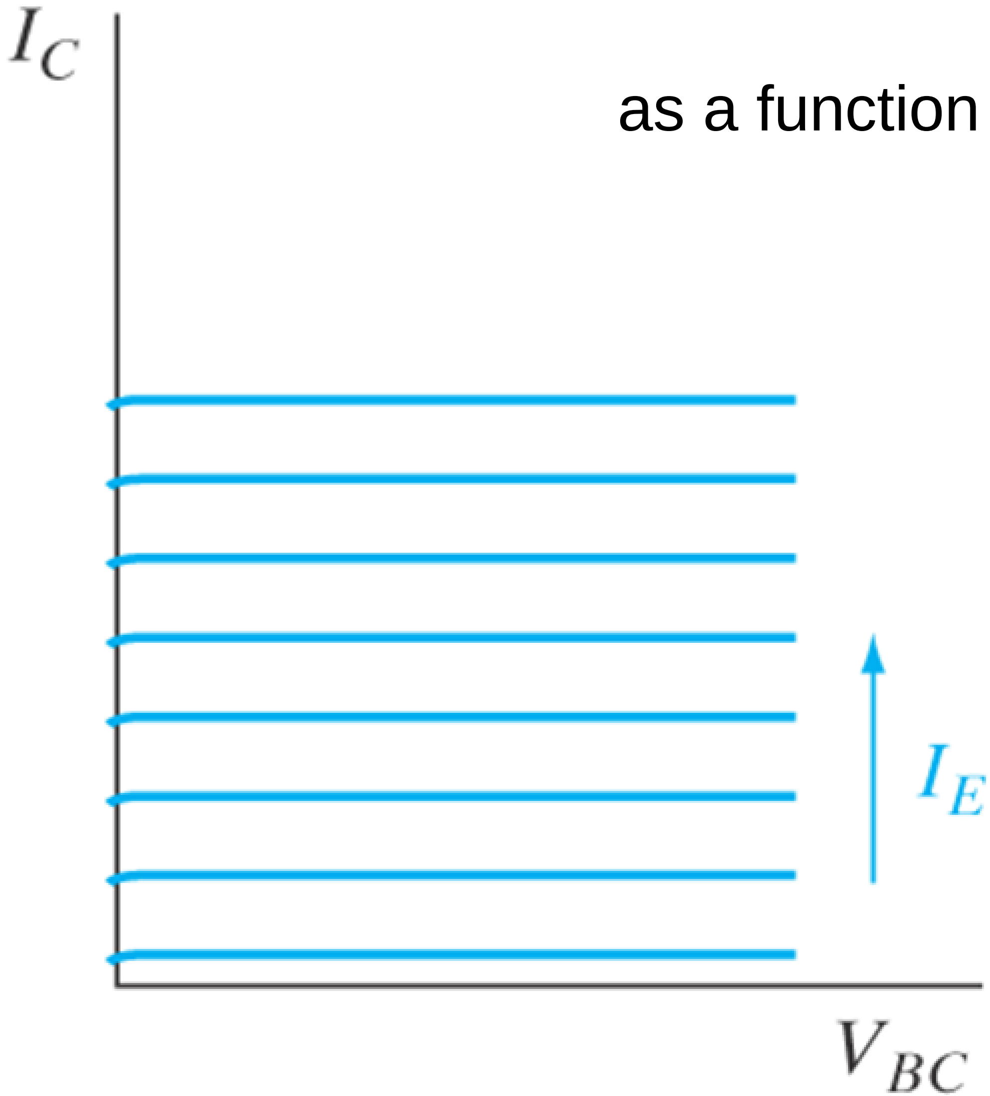
emitter junction
forward biased $V_{EB} \gg k_B T$

collector junction
reverse biased $V_{CB} \ll 0$

$$\Delta p_E \approx p_n \exp\left(\frac{qV_{EB}}{k_B T}\right)$$

$$\Delta p_C \approx -p_n$$

I-V curves of the reverse-biased junction

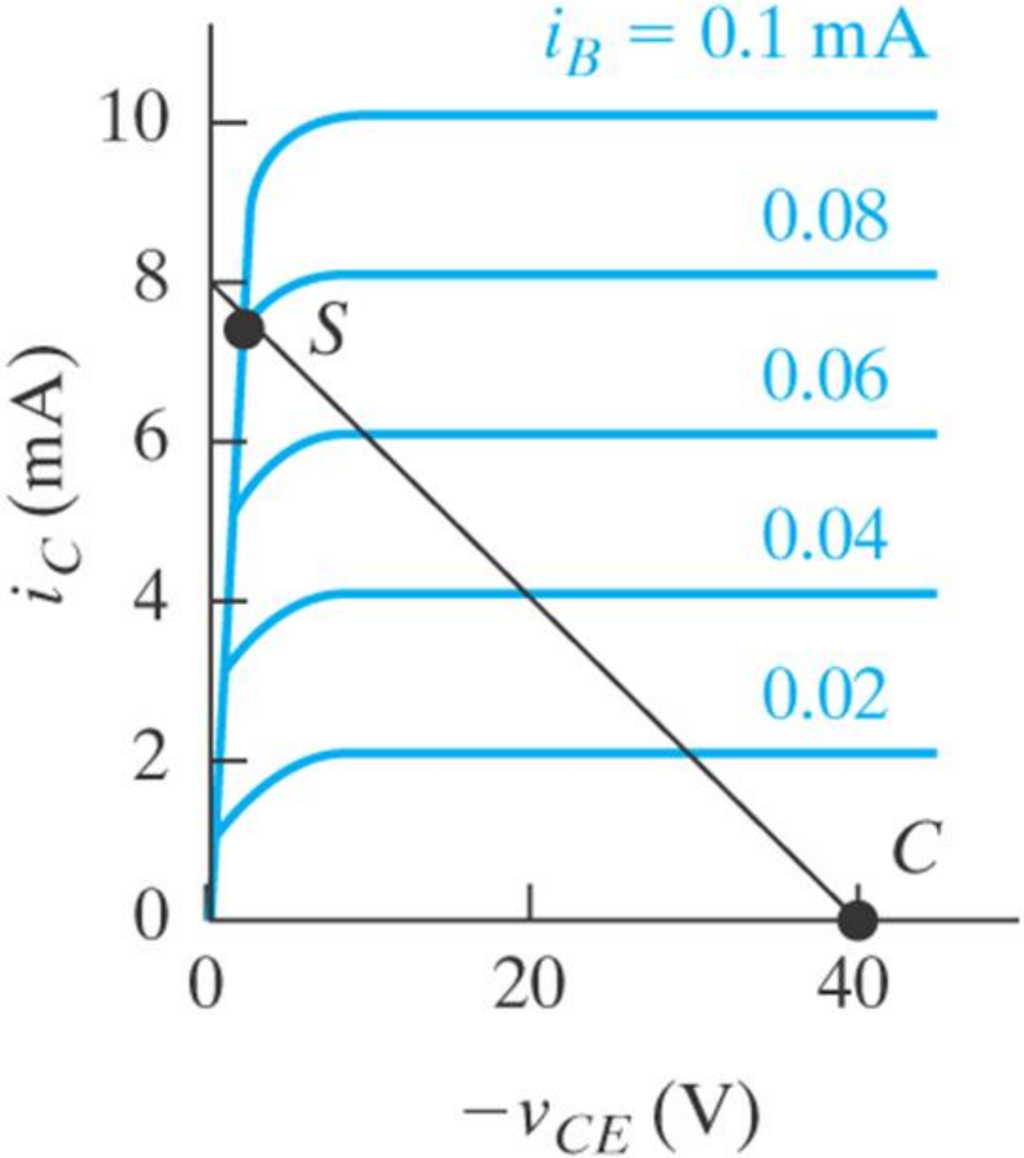


as a function of emitter current

resembles behavior
of the photodiode

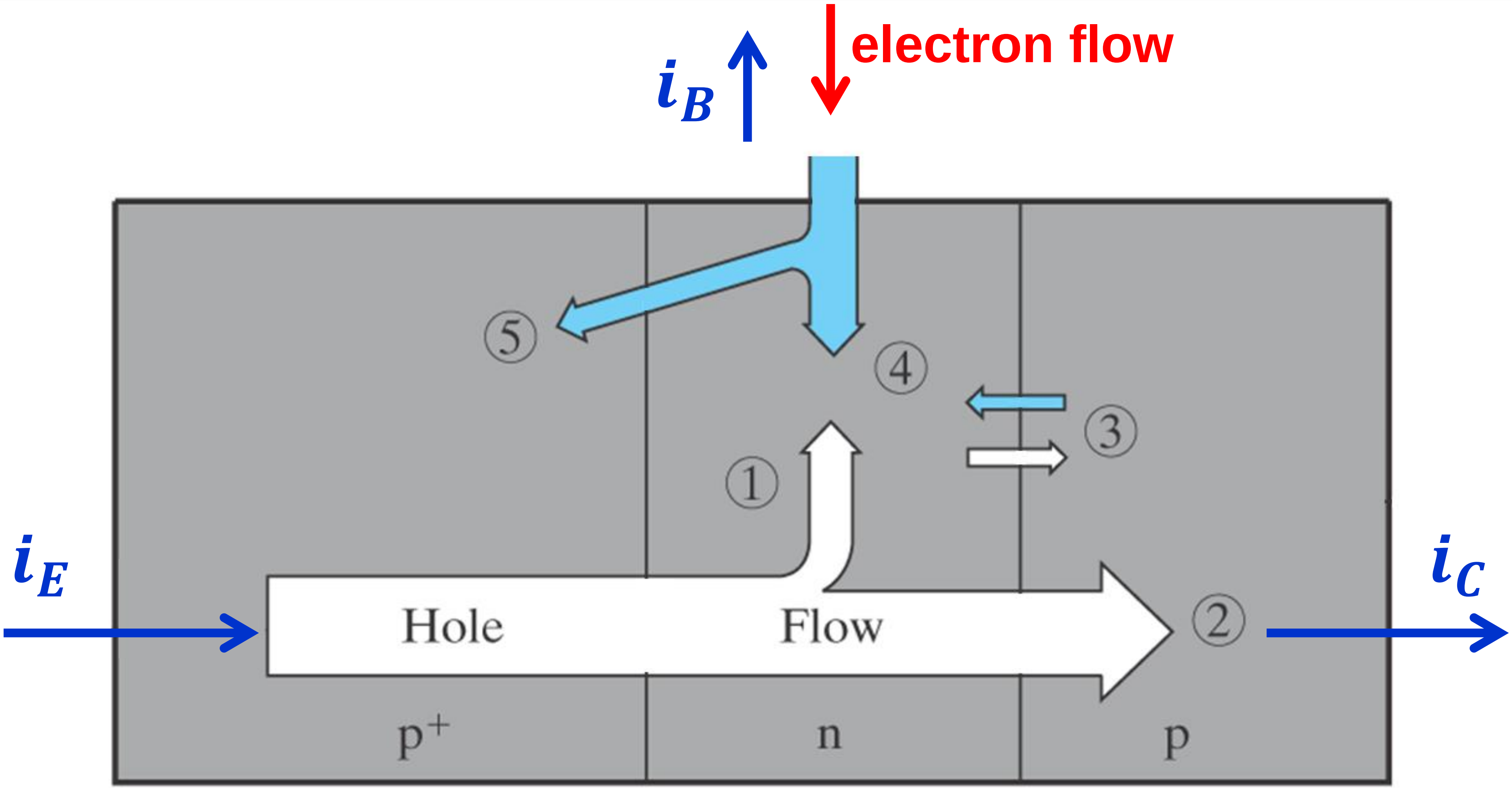
current generator

BJT transistor I-V curves



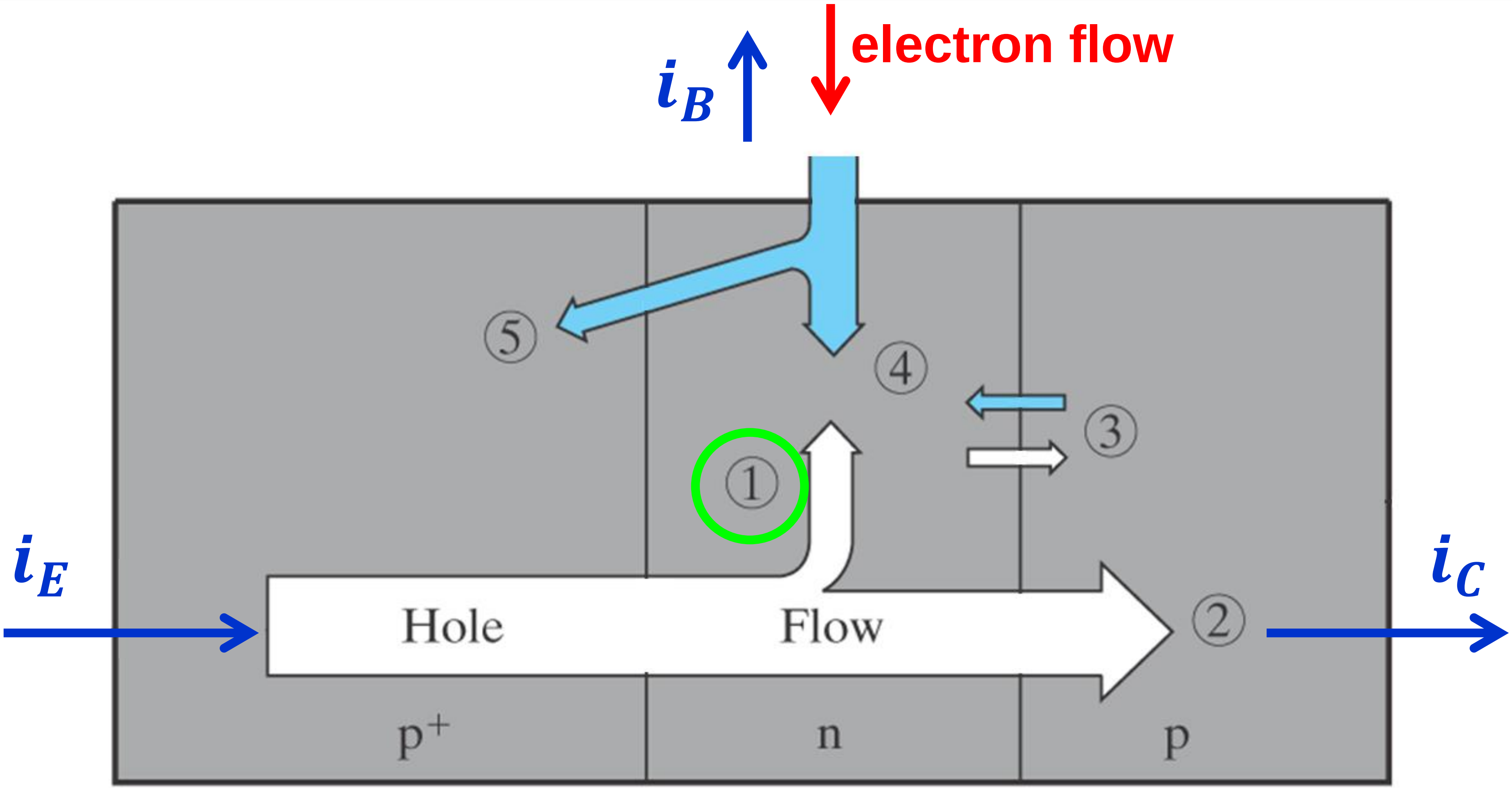
The control input is the base current i_B

Carrier flow



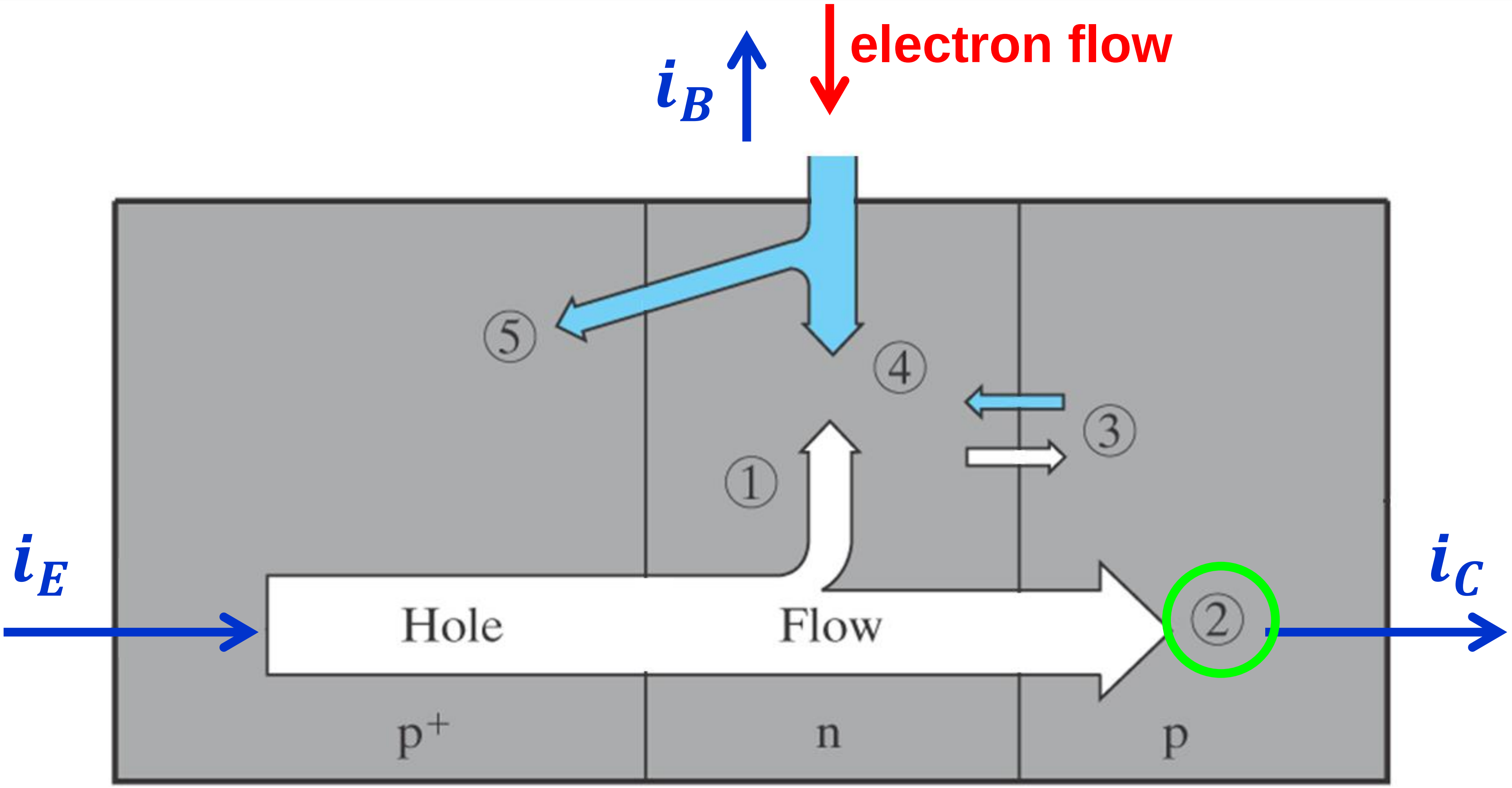
$$\overrightarrow{i_E} = \begin{cases} \overrightarrow{\quad\quad} \; i_{Ep} \\ \rightarrow \; i_{En} \end{cases} \qquad\qquad \overrightarrow{i_C}$$

Carrier flow



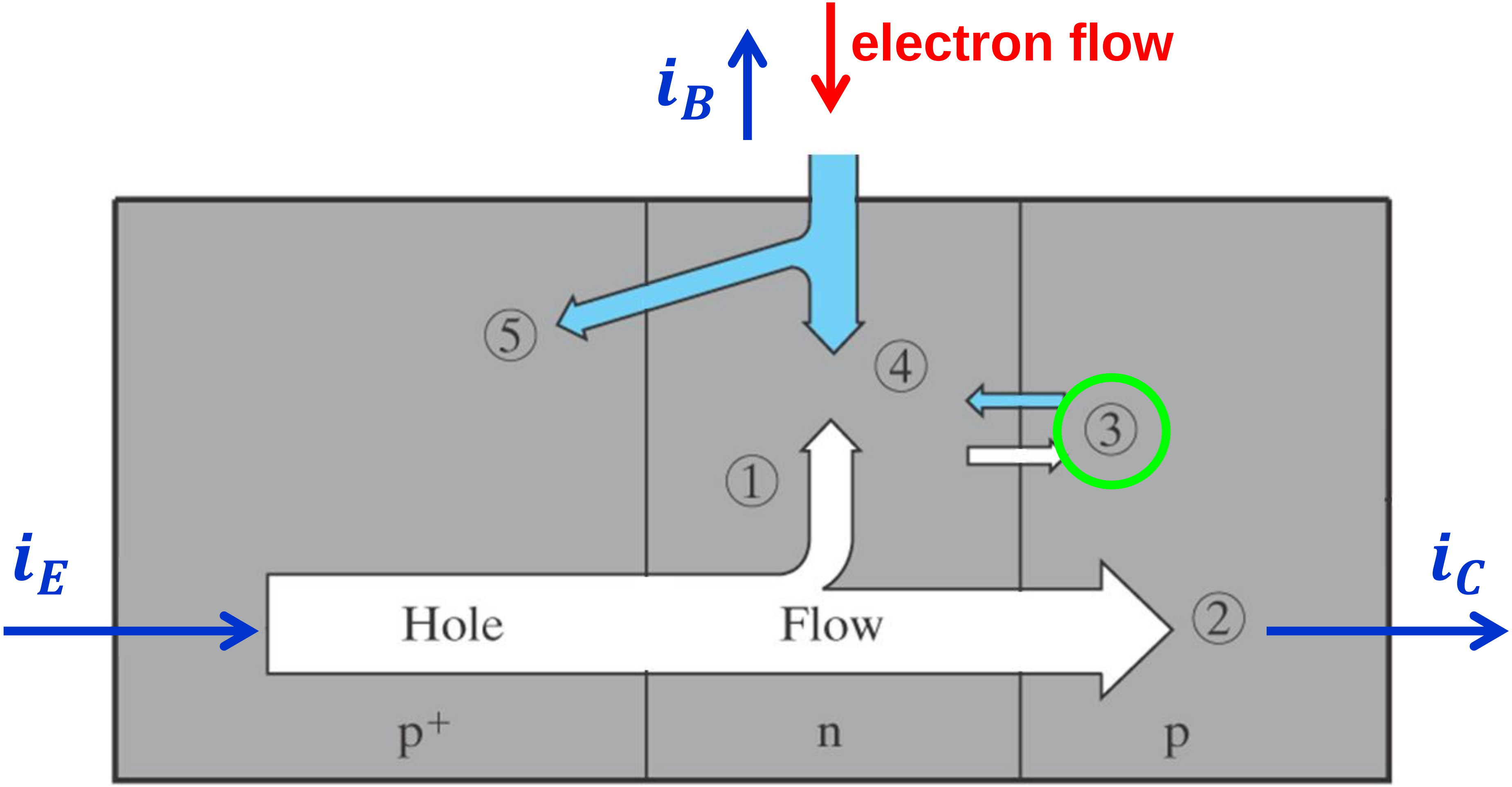
1 injected holes lost to recombination in the base

Carrier flow



2 injected holes reaching reverse-biased collector junction

Carrier flow



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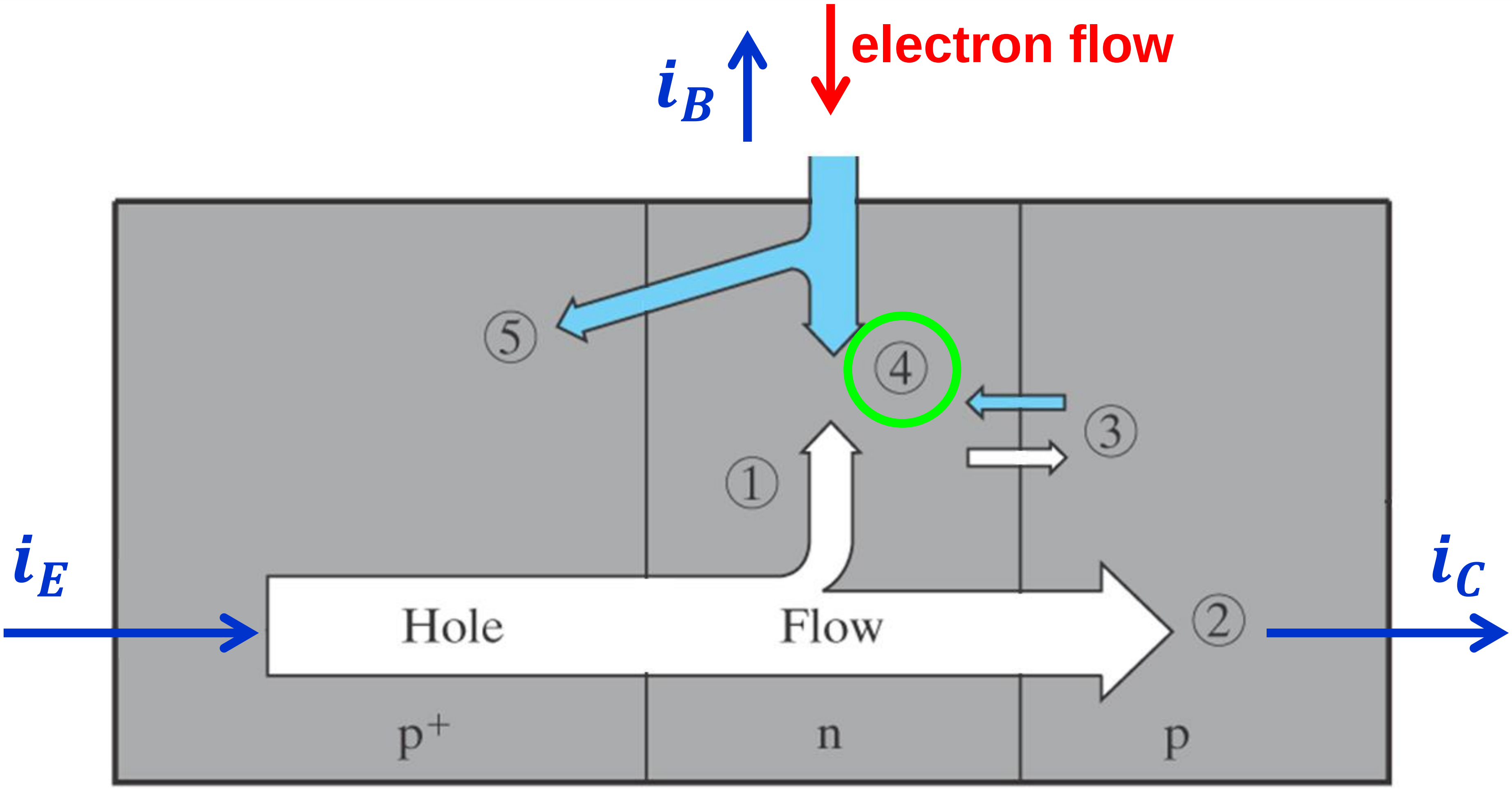
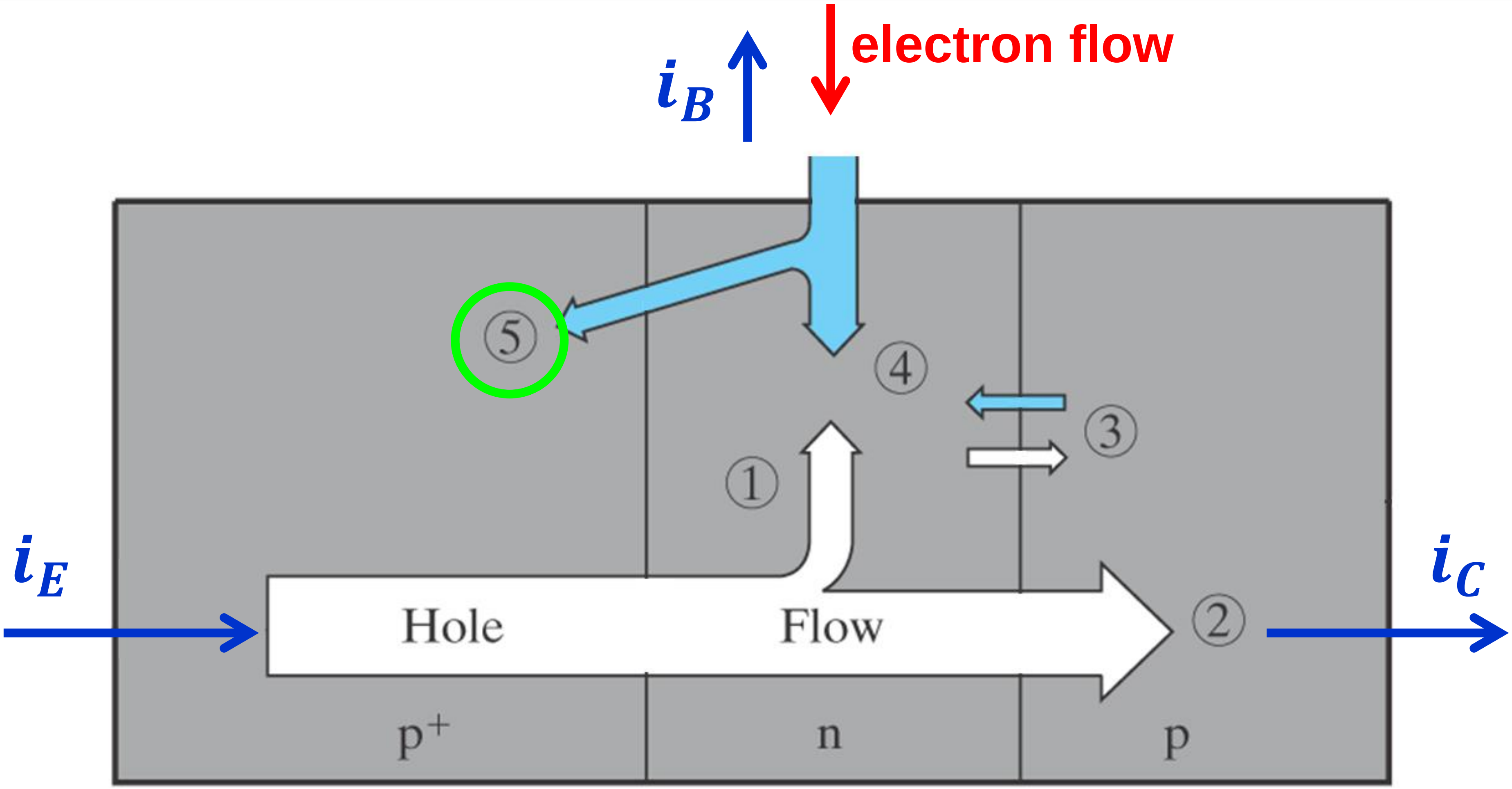
thermally generated electrons and holes
(reverse saturation current of collector junction)
which is part of the collector current

Carrier flow



4 electrons supplied by the base contact which recombine with holes in the base neutral region

Carrier flow



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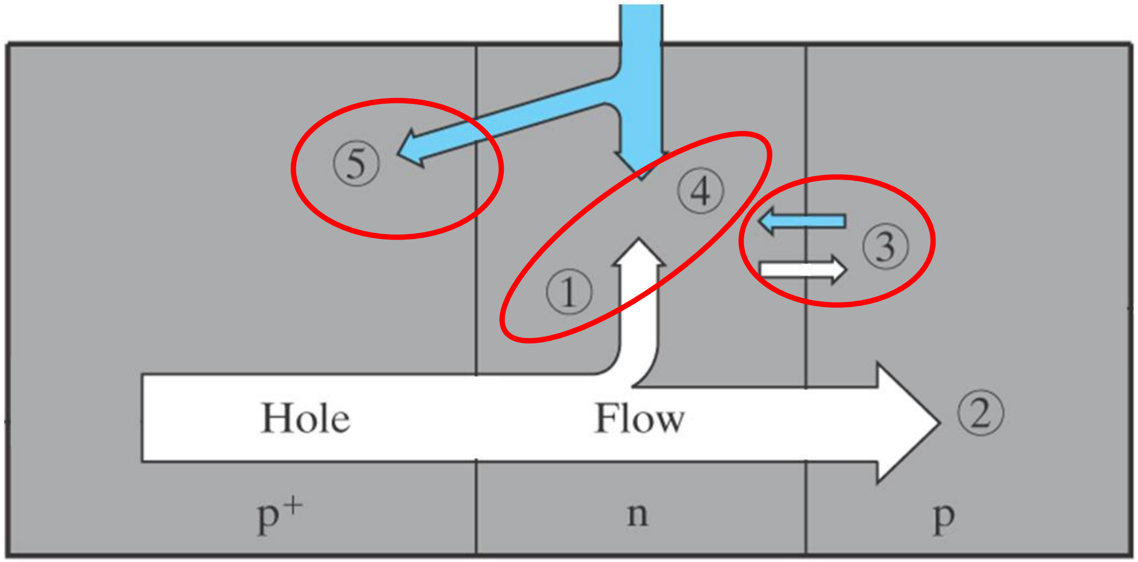
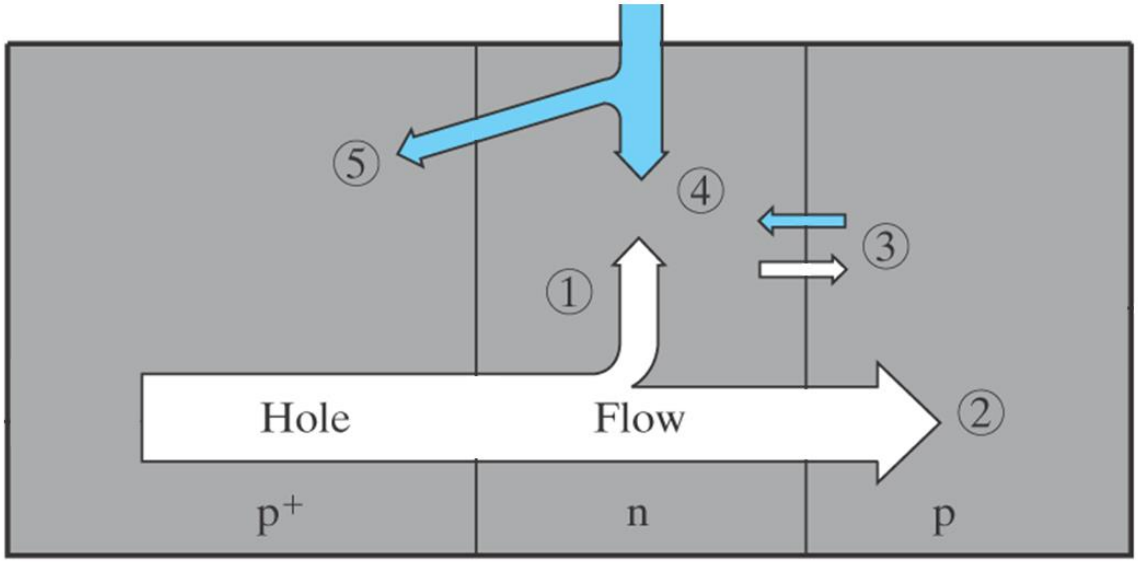
electrons supplied by the base contact which are injected across the forward biased junction and are part of the emitter current

Base current physical mechanisms



- recombination of injected holes even if $W_B \ll L_p$
- injection of electrons into the emitter
- thermally generated electrons in the reverse biased junction are swept into the base reducing the supply from the base contact

For a well-designed BJT

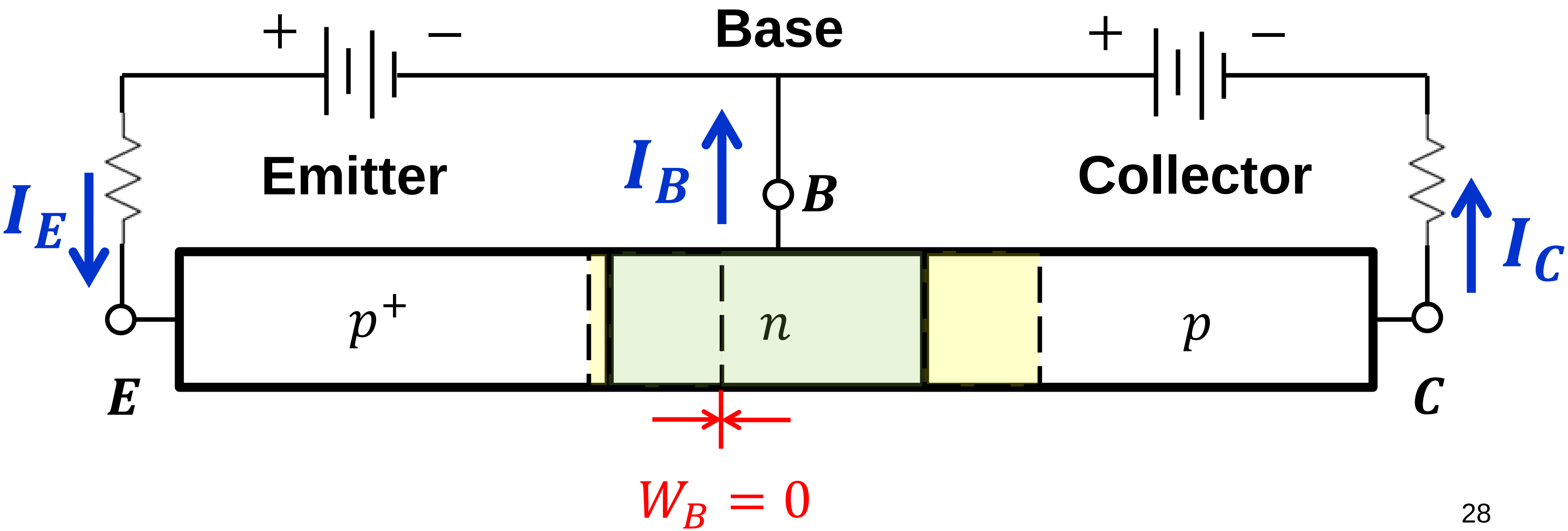


- Holes injected by the emitter into the base are collected as much as possible ($I_E - I_C$ very small) $\rightarrow$ we need base with narrow width and long hole lifetime so that $W_B \ll L_p = \sqrt{D_p \tau_p}$
- Current crossing the emitter should consist almost entirely of holes $\rightarrow$ we need high doping in emitter with respect to base doping (e.g., p^+-n emitter-base junction)

Base limitations: punch-through

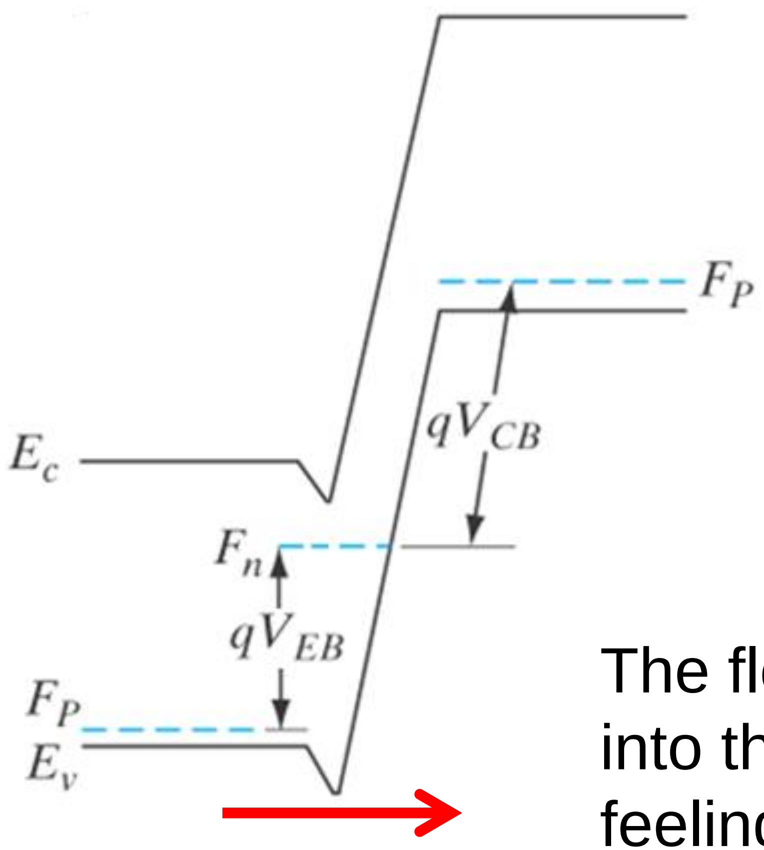
- If the base region is too thin and if the base doping is too light, at the desired voltages the depletions from the two junctions may meet, resulting in "**punch-through**".

- In such conditions, the base current is no longer able to control the emitter current.



Base limitations: punch-through

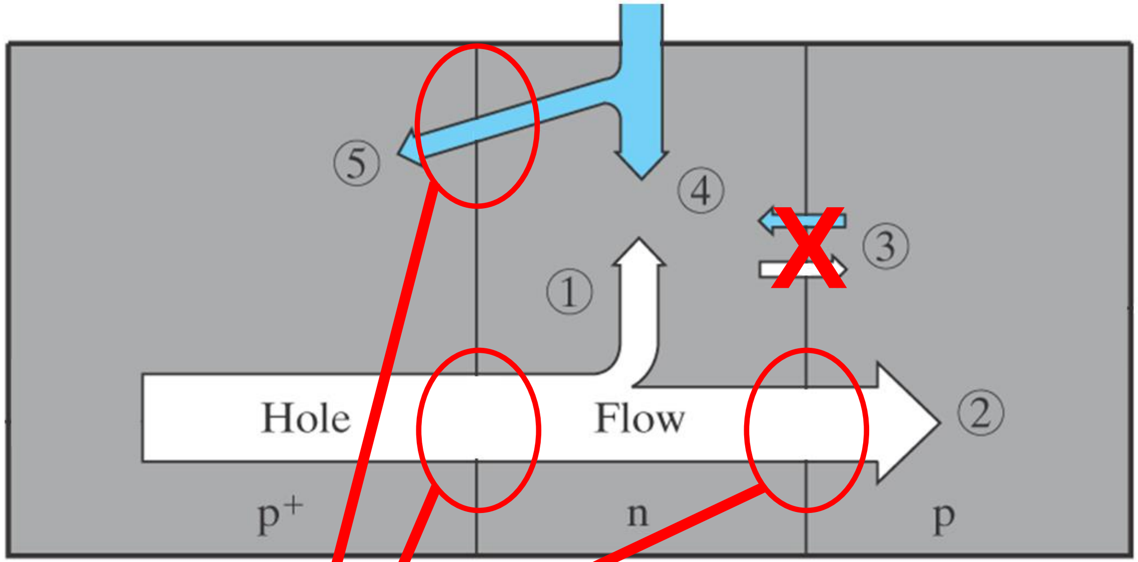
- The band diagram might look approximately like this:



The flow of holes goes directly into the collector region without feeling the influence of the base

Amplification with BJT

- The control input is the base current i_B



no current loss by recombination

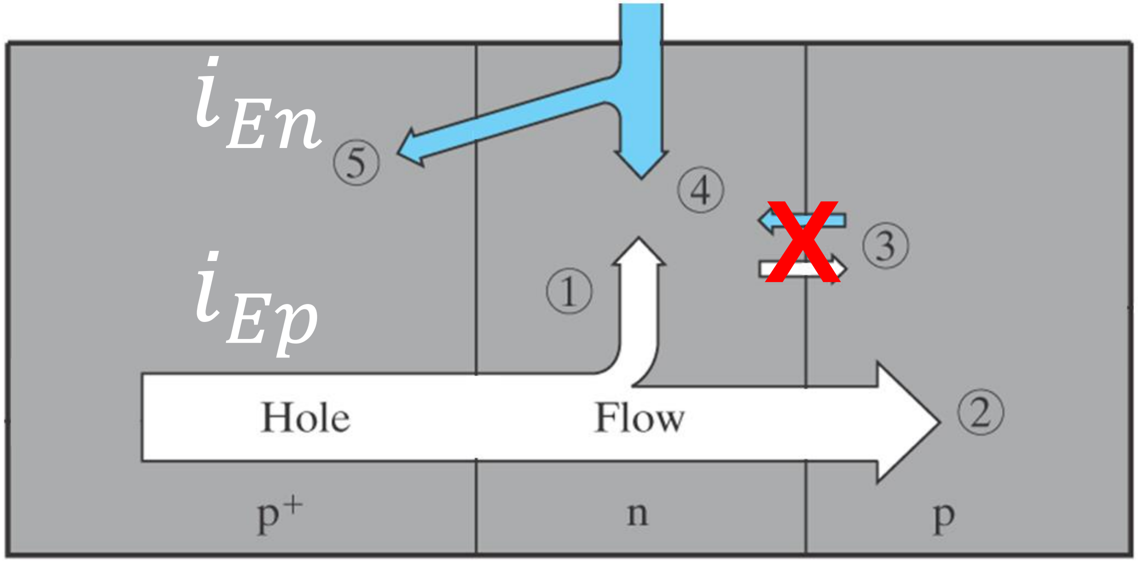
- Simplifying Assumptions:
  - Neglect reverse saturation current at collector
  - Neglect recombination in the depletion regions

Amplification with BJT – injection efficiency

$$i_E = i_{Ep} + i_{En}$$

$$\gamma = \frac{i_{Ep}}{i_E} = \frac{i_{Ep}}{i_{Ep} + i_{En}}$$

emitter injection efficiency



Amplification with BJT – base transport factor

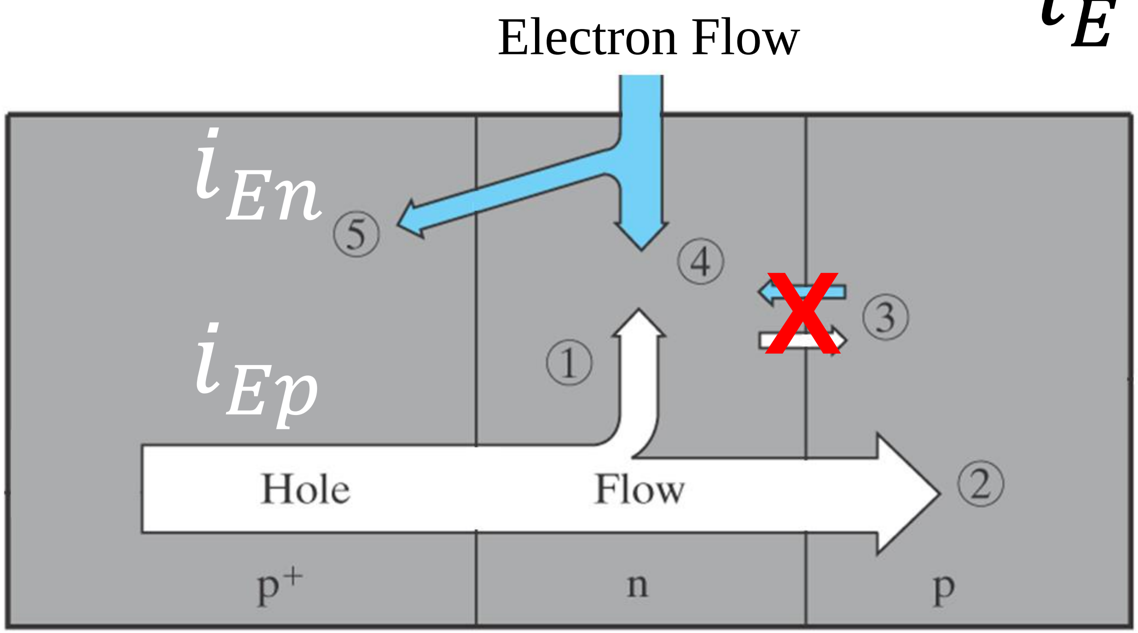
- Collector current

$$i_C = B\, \boldsymbol{i_{Ep}}$$

base transport factor

hole component of emitter current

$$i_E = \boldsymbol{i_{Ep}} + i_{En}$$



For an efficient transistor – 1

- **Emitter injection efficiency → unity**

  emitter current is mostly holes

$$\gamma = \frac{i_{Ep}}{i_E} = \frac{i_{Ep}}{i_{Ep} + i_{En}} = \frac{1}{1 + \frac{i_{En}}{i_{Ep}}}$$

$$\gamma \rightarrow 1$$

$$\frac{i_{En}}{i_{Ep}} \rightarrow 0$$

We need

$$\boldsymbol{i_{En} \ll i_{Ep}}$$

For an efficient transistor – 2

- **Base transport factor → unity**

  most of the holes make it to the collector

Minimize recombination in the base so that

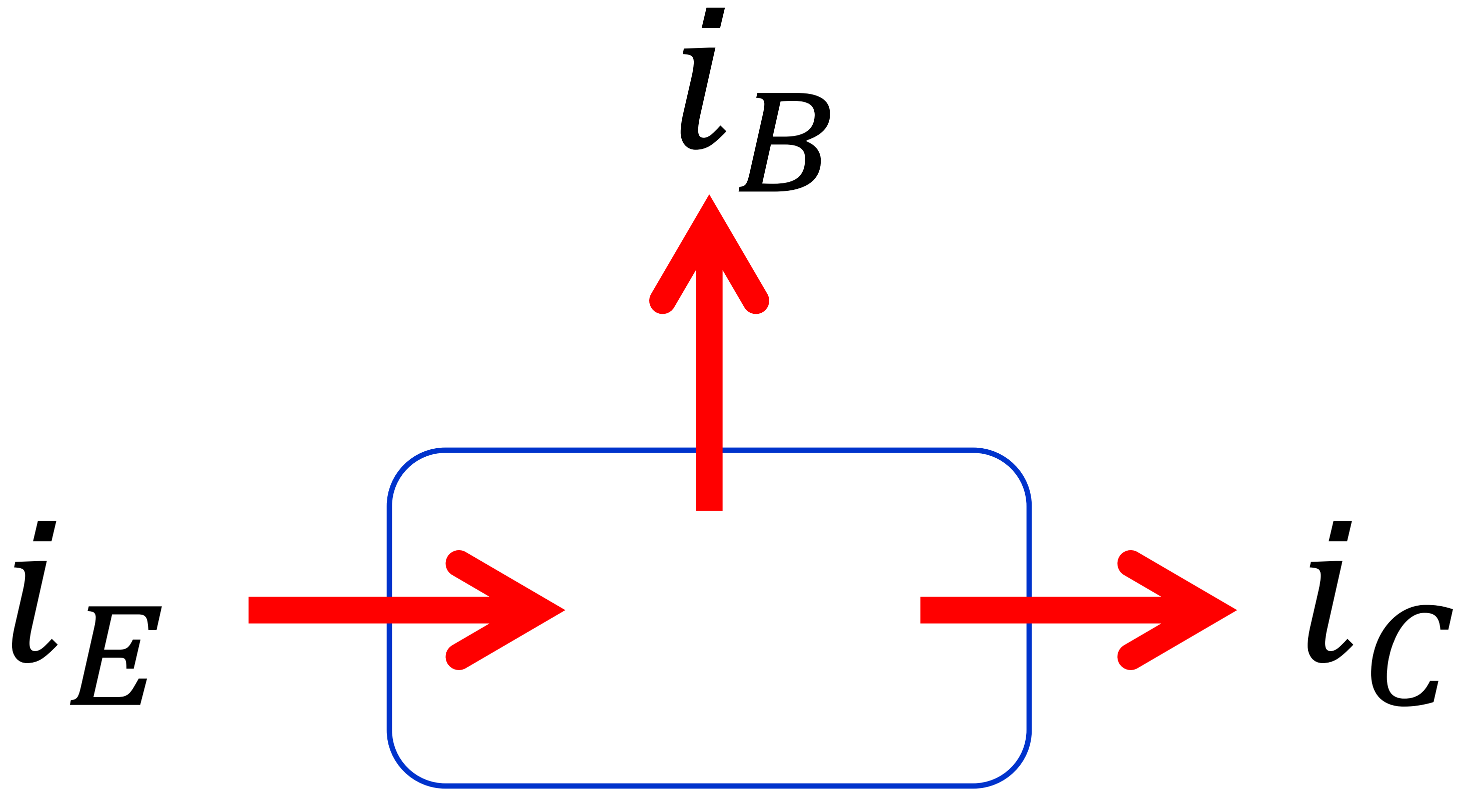
$$i_C \approx \boldsymbol{i_{Ep}}$$

Putting it all together

$$\frac{\boldsymbol{i_C}}{\boldsymbol{i_E}} = \frac{B\, i_{Ep}}{i_{Ep} + i_{En}} = B\gamma = \boldsymbol{\alpha}$$

current transfer ratio

Base-to-Collector Amplification



$$i_E = i_B + i_C$$

Base-to-Collector Amplification

$$\frac{i_C}{i_B} = \boldsymbol{\beta}$$

base-to-collector
current amplification factor

$$i_B = i_E - i_C = i_{En} + i_{Ep} - B i_{Ep}$$
$$i_B = i_{En} + i_{Ep}(1 - B)$$

$\boldsymbol{B}$ = fraction of holes that make it across the base

$(\mathbf{1} - \boldsymbol{B})$ is the fraction of holes that recombine

Base-to-Collector Amplification

$$\frac{i_C}{i_B} = \frac{B\, i_{Ep}}{i_{En} + i_{Ep}(1-B)} =$$

multyply by $\frac{(i_{En} + i_{Ep})}{(i_{En} + i_{Ep})} = 1$

$$= \frac{B \boxed{\frac{i_{Ep}}{i_{En} + i_{Ep}}}}{\frac{i_{En}}{i_{En} + i_{Ep}} + \frac{i_{Ep}}{i_{En} + i_{Ep}}(1-B)} =$$

$= \gamma$

$= 1$

$$= \frac{B\gamma}{\boxed{\frac{i_{En}}{i_{En} + i_{Ep}} + \frac{i_{Ep}}{i_{En} + i_{Ep}}} - B\boxed{\frac{i_{Ep}}{i_{En} + i_{Ep}}}} =$$

$= \gamma$

Base-to-Collector Amplification Factor β

$$\frac{i_C}{i_B} = \frac{\underbrace{B\gamma}_{= \alpha}}{1 - B\gamma} = \frac{\alpha}{1-\alpha} = \beta$$

Since α is close unity, β can be quite large

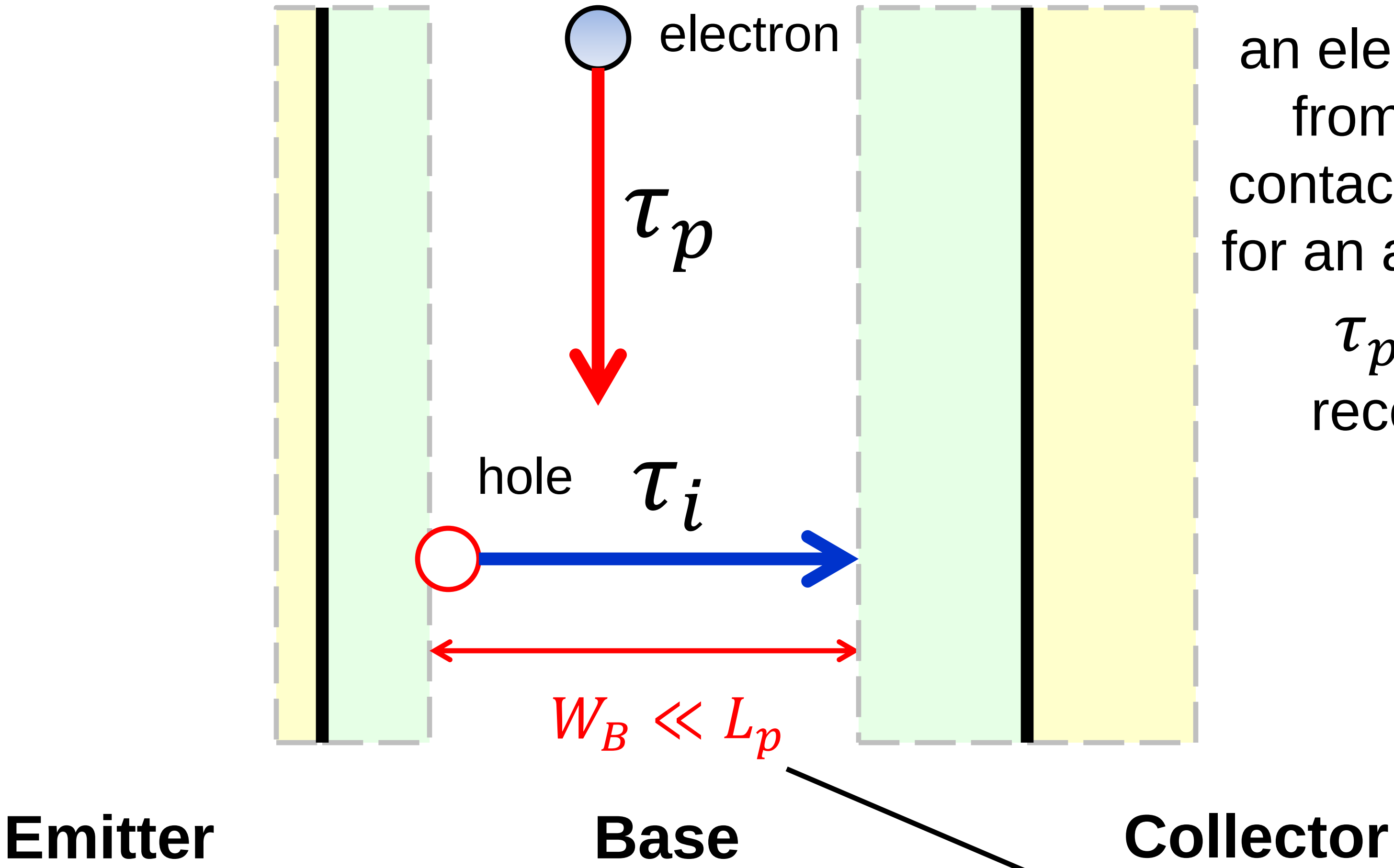
$$\alpha = 0.9 \quad \rightarrow \quad \beta = 9$$

$$\alpha = 0.95 \quad \rightarrow \quad \beta = 19$$

$$\alpha = 0.99 \quad \rightarrow \quad \beta = 99$$

$$\alpha = 0.999 \quad \rightarrow \quad \beta = 999$$

Hole transit time in the base



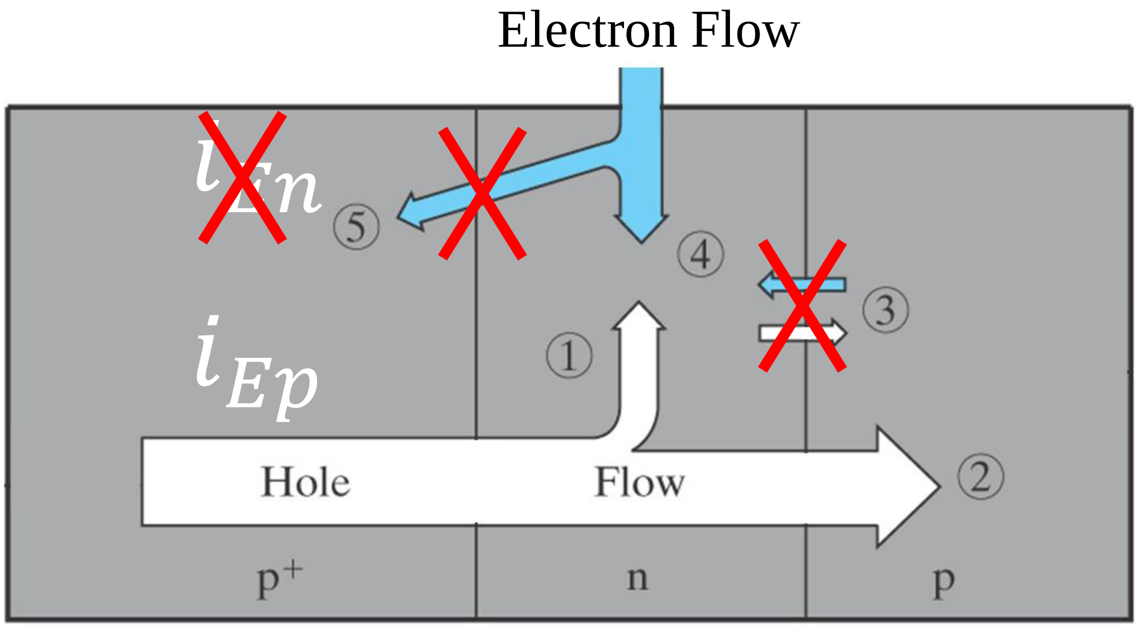
an electron enters from the base contact and lingers for an average time τ_p before recombining

Average time spent by hole in the narrow base $\tau_i \ll \tau_p$

Charge storage in the base

At steady-state there are excess electrons and holes in the base and for charge neutrality $Q_n = Q_p$

Assume $\tau_n = \tau_p$, perfect emitter injection efficiency $(\gamma = 1)$ and negligible saturation current



Charge storage in the base

$$\boxed{i_C = \frac{Q_p}{\tau_i}}$$

$$\boxed{i_B = \frac{Q_n}{\tau_n} = \frac{Q_n}{\tau_p}}$$

$$\boxed{Q_p = i_C \tau_i}$$

$$\boxed{Q_n = i_B \tau_p}$$

$$Q_n = Q_p$$

$$\boxed{i_C \tau_i = i_B \tau_n}$$

Amplification factor – physical interpretation

For each electron entering from the base contact, a number τ_p/τ_i of holes goes from emitter to collector maintaining charge neutrality

$$\boxed{\frac{i_C}{i_B} = \frac{\tau_p}{\tau_i} = \beta}$$

Mathematical analysis of the *p-n-p* BJT

- **Some simplifying assumptions are necessary in order to develop a manageable model which is general and valid for general bias conditions:**

1. **Negligible drift in the base region (holes move by diffusion)**
2. **Emitter injection efficiency $\gamma = 1$ (emitter is highly doped *p*+)**
3. **Reverse saturation current at the collector is negligible**
4. **Uniform cross-sectional area A (1-D model)**
5. **Steady-state conditions**

Diffusion equation in the neutral base

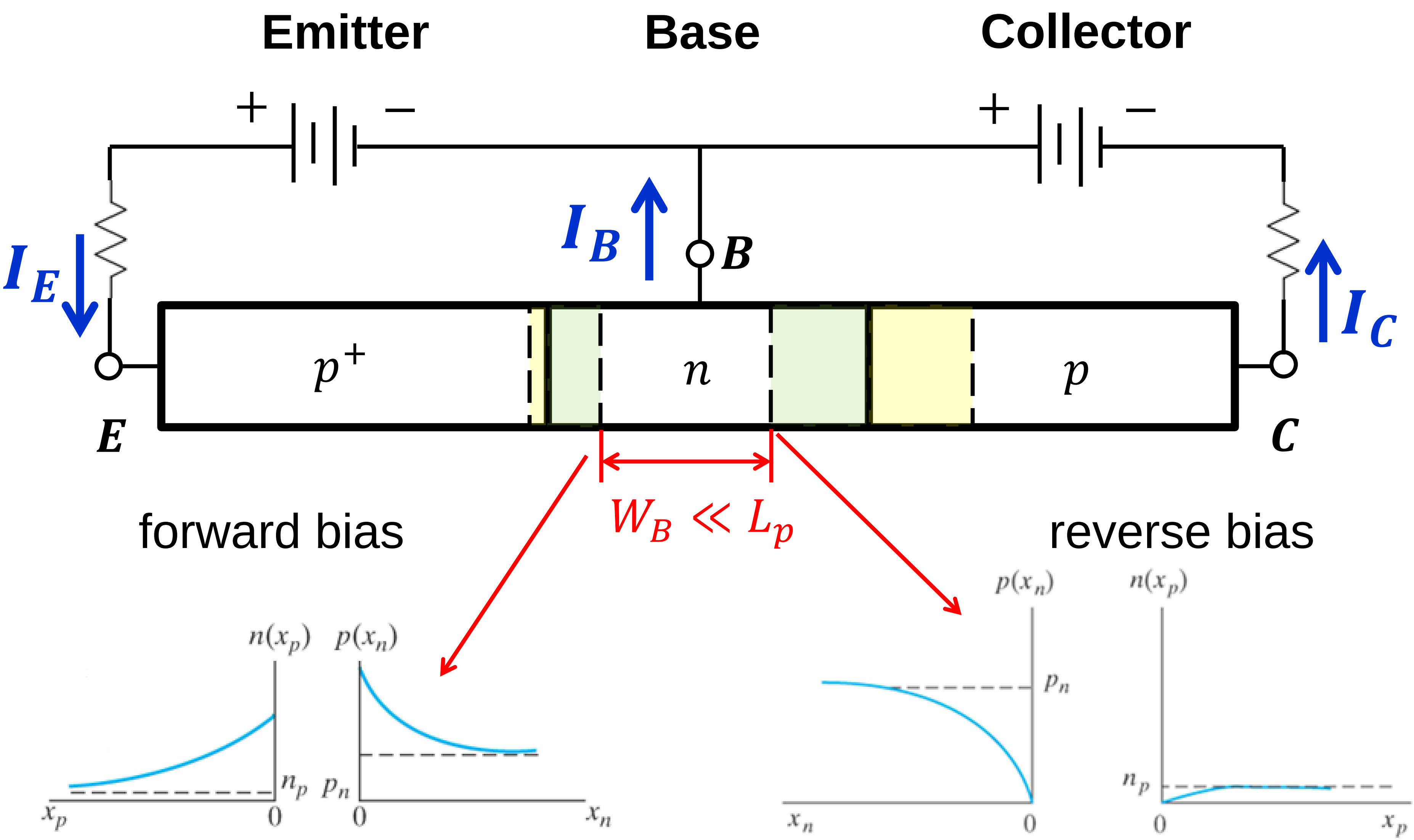
$$\frac{d^2 \delta p(x_n)}{dx^2} = \frac{\delta p(x_n)}{L_p^2}$$

general solution

$$\delta p(x_n) = C_1 \exp\left(\frac{x_n}{L_p}\right) + C_2 \exp\left(-\frac{x_n}{L_p}\right)$$

Remember: $W_B \ll L_p$

Base n-region boundary conditions



Diffusion equation in the neutral base

$$\delta p(x_n) = C_1 \exp\left(\frac{x_n}{L_p}\right) + C_2 \exp\left(-\frac{x_n}{L_p}\right)$$

At the boundaries of the neutral base

$$\delta p(x_n = 0) = C_1 + C_2 = \Delta p_E$$

$$\delta p(x_n = W_B) = C_1 \exp\left(\frac{W_B}{L_p}\right) + C_2 \exp\left(-\frac{W_B}{L_p}\right)$$

$$= \Delta p_C$$

Solve for C_1 and C_2

$$C_1 = \frac{\Delta p_C - \Delta p_E \exp\left(-\frac{W_B}{L_p}\right)}{\exp\left(\frac{W_B}{L_p}\right) - \exp\left(-\frac{W_B}{L_p}\right)}$$

$$C_2 = \frac{\Delta p_E \exp\left(\frac{W_B}{L_p}\right) - \Delta p_C}{\exp\left(\frac{W_B}{L_p}\right) - \exp\left(-\frac{W_B}{L_p}\right)}$$

Evaluate terminal currents $(x_n = 0)$

We obtained earlier this result for the diffusion current

$$I_p(x_n) = -qAD_p \frac{d\,\delta p(x_n)}{dx_n}$$

Evaluate at $x_n = 0$

$$I_{Ep} = I_p(x_n = 0) = qA\frac{D_p}{L_p}(C_2 - C_1)$$

Evaluate terminal currents $(x_n = W_B)$

We obtained earlier this result for the diffusion current

$$I_p(x_n) = -qAD_p \frac{d\,\delta p(x_n)}{dx_n}$$

Evaluate at $x_n = W_B$

$$I_C = I_p(x_n = W_B) =$$

$$qA\frac{D_p}{L_p}\left(C_2 \exp\left(-\frac{W_B}{L_P}\right) - C_1 \exp\left(\frac{W_B}{L_P}\right)\right)$$

Terminal currents (Emitter and Collector)

$$I_{Ep} = qA\frac{D_p}{L_p}\left[\frac{\Delta p_E\left(\exp\left(\frac{W_B}{L_P}\right)+\exp\left(-\frac{W_B}{L_P}\right)\right)-2\Delta p_c}{\exp\left(\frac{W_B}{L_P}\right)-\exp\left(-\frac{W_B}{L_P}\right)}\right]$$

Using hyperbolic functions

$$I_{Ep} = qA\frac{D_p}{L_p}\left[\Delta p_E \operatorname{ctnh}\frac{W_B}{L_P} - \Delta p_C \operatorname{csch}\frac{W_B}{L_P}\right]$$

$$I_C = qA\frac{D_p}{L_p}\left[\Delta p_E \operatorname{csch}\frac{W_B}{L_P} - \Delta p_C \operatorname{ctnh}\frac{W_B}{L_P}\right]$$

Terminal currents (Base)

$$I_B = I_E - I_C$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\left(\operatorname{ctnh}\frac{W_B}{L_P} - \operatorname{csch}\frac{W_B}{L_P}\right)\right]$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_B}{2L_P}\right]$$

Summary – general solution for currents

$$I_{Ep} = qA\frac{D_p}{L_p}\left[\Delta p_E \operatorname{ctnh}\frac{W_B}{L_P} - \Delta p_C \operatorname{csch}\frac{W_B}{L_P}\right]$$

$$I_C = qA\frac{D_p}{L_p}\left[\Delta p_E \operatorname{csch}\frac{W_B}{L_P} - \Delta p_C \operatorname{ctnh}\frac{W_B}{L_P}\right]$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_B}{2L_P}\right]$$

For the narrow base diode

$$I_p(x_n = 0) = qA\frac{D_p}{L_p}\Delta p_n \operatorname{ctnh}\frac{\boldsymbol{\ell}}{L_P}$$

$$I_p(x_n = \boldsymbol{\ell}) = qA\frac{D_p}{L_p}\Delta p_n \operatorname{csch}\frac{\boldsymbol{\ell}}{L_P}$$

$$I_n(\mathrm{recomb}) = qA\frac{D_p}{L_p}\Delta p_n \tanh\frac{\boldsymbol{\ell}}{2L_P}$$

With $\Delta p_c \approx 0$ essentially the same result obtained for BJT

For the curious ones:

- Video by Bill Hammack on the first transistor invented by Bardeen and Brattain at Bell Labs (point-contact transistor)

https://www.youtube.com/watch?v=RdYHljZi7ys

The book by Shockley contains an extensive description of the point-contact transistor, based on metal-semiconductor junctions rather than p-n junctions
https://archive.org/details/ElectronsAndHolesInSemiconductors

- AT&T Archives video: Genesis of the Transistor:
  https://www.youtube.com/watch?v=WiQvGRjrLnU