ECE 340 Lectures 41
Semiconductor Electronics

Spring 2022
10:00-10:50am
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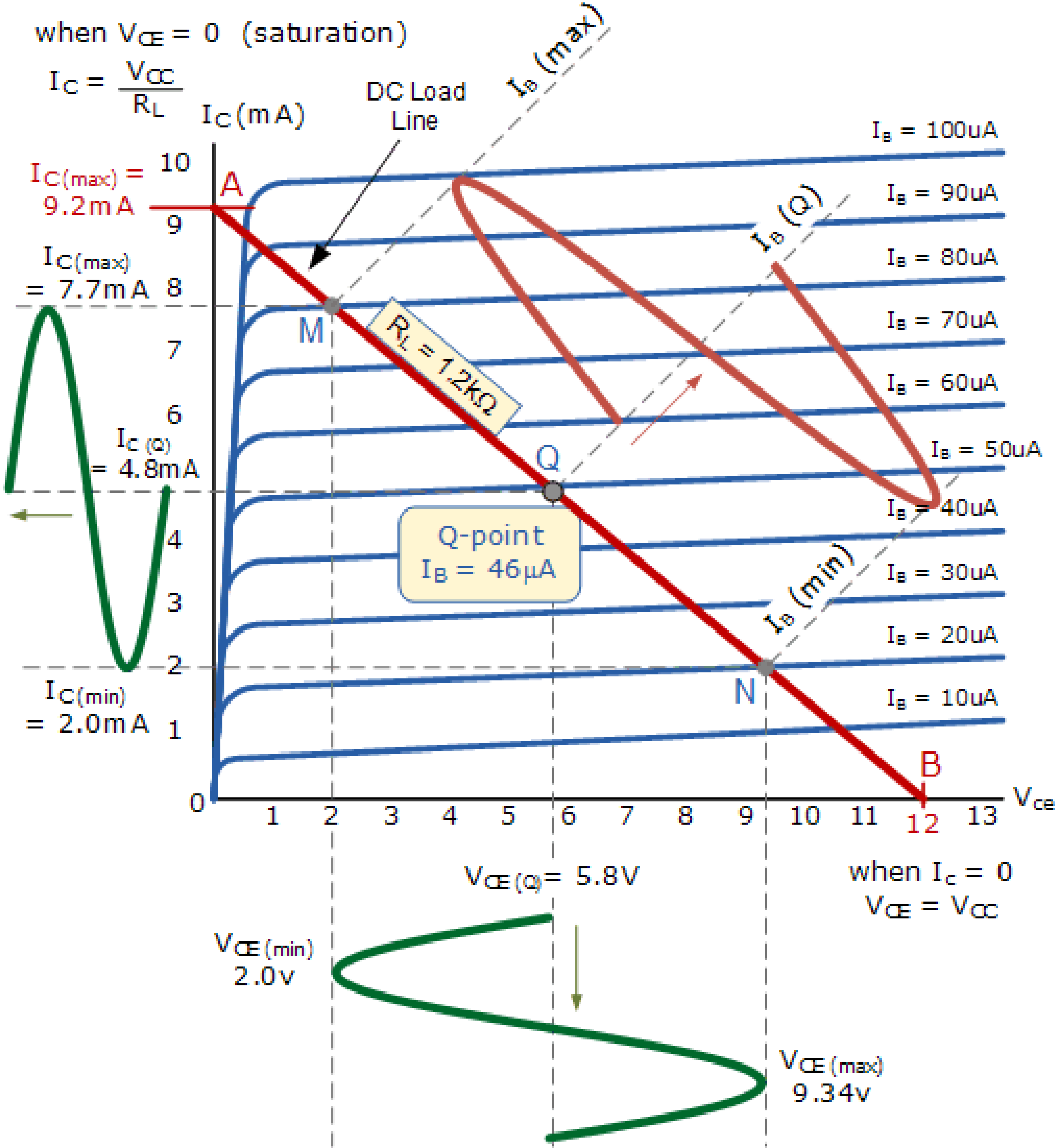
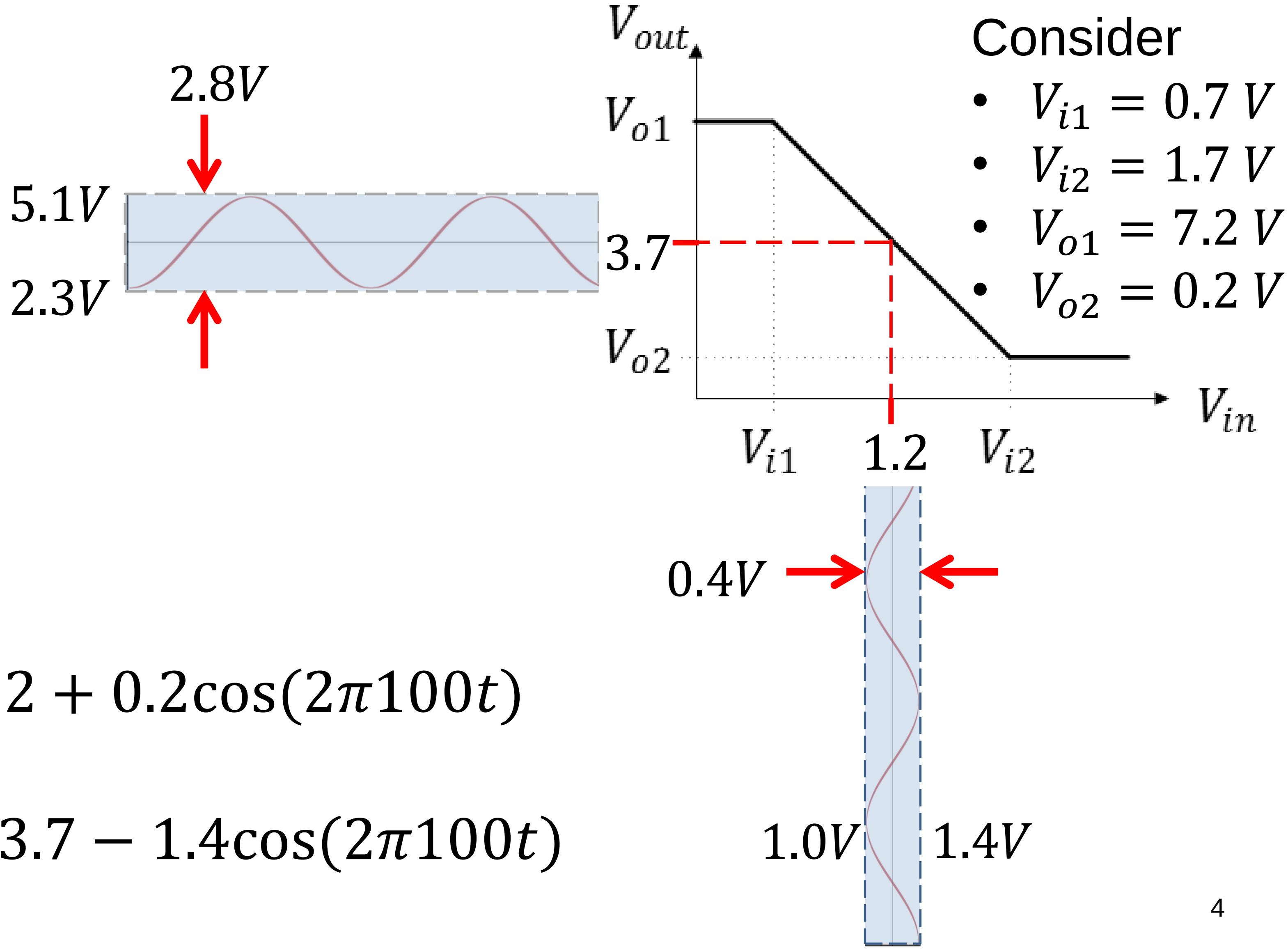
Today’s Discussion

- **Transistor as amplifier**
- **Small signals circuit**
- **Exercises**


when $V_{CE} = 0$ (saturation)
$I_C = \frac{V_{CC}}{R_L}$
I_C (mA)
DC Load Line
I_B (max)
$I_{C(max)} =$ 9.2mA
A
$I_{C(max)}$ = 7.7mA
M
R_L = 1.2kΩ
I_B (Q)
Q
$I_{C(Q)}$ = 4.8mA
Q-point
I_B = 46µA
I_B (min)
N
$I_{C(min)}$ = 2.0mA
B
I_B = 100uA
I_B = 90uA
I_B = 80uA
I_B = 70uA
I_B = 60uA
I_B = 50uA
I_B = 40uA
I_B = 30uA
I_B = 20uA
I_B = 10uA
10
9
8
7
6
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
V_{ce}
$V_{CE(Q)}$ = 5.8V
when $I_c = 0$
$V_{CE} = V_{CC}$
$V_{CE(min)}$ 2.0v
$V_{CE(max)}$ 9.34v

Amplification Example



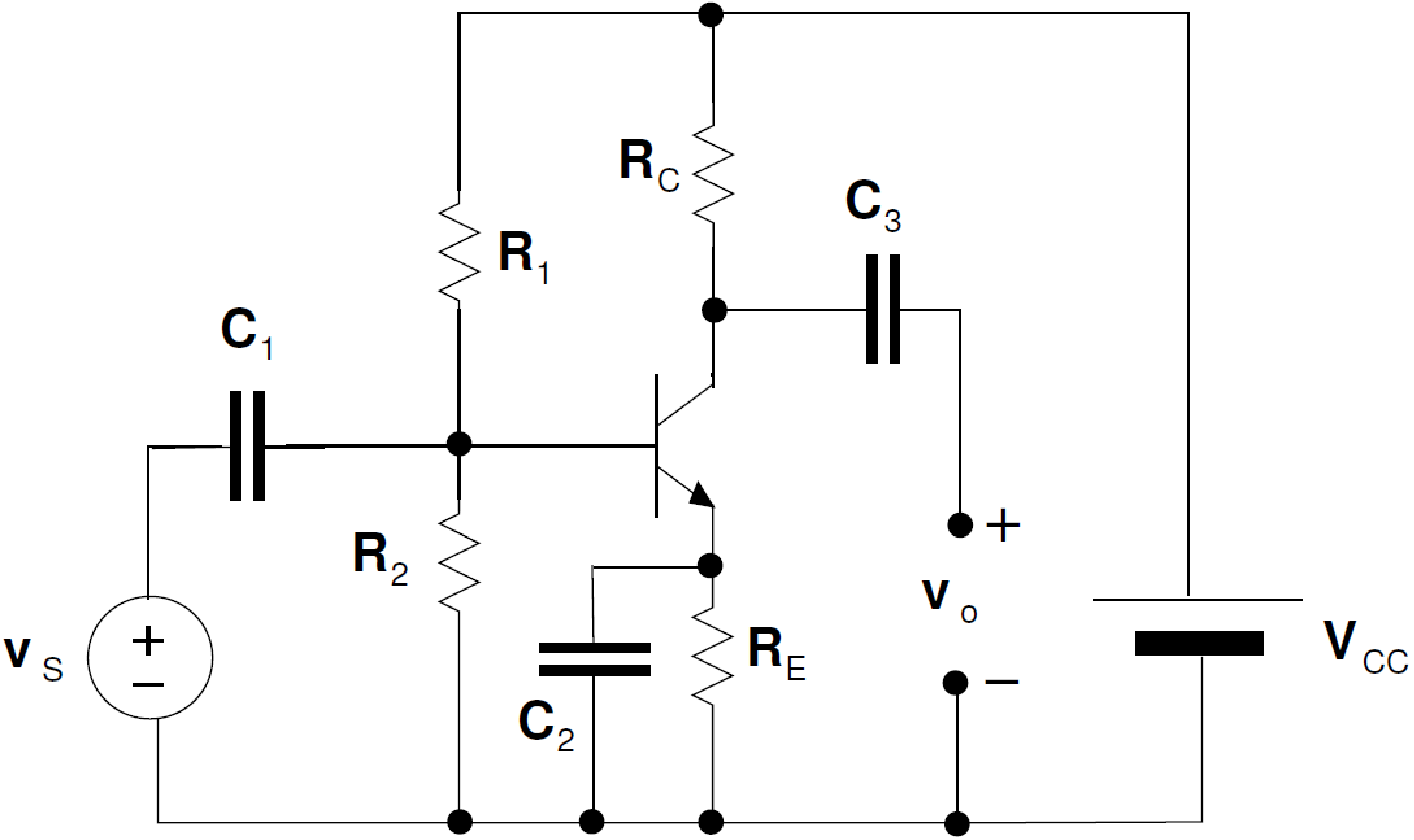
Consider

- $V_{i1} = 0.7\ V$
- $V_{i2} = 1.7\ V$
- $V_{o1} = 7.2\ V$
- $V_{o2} = 0.2\ V$

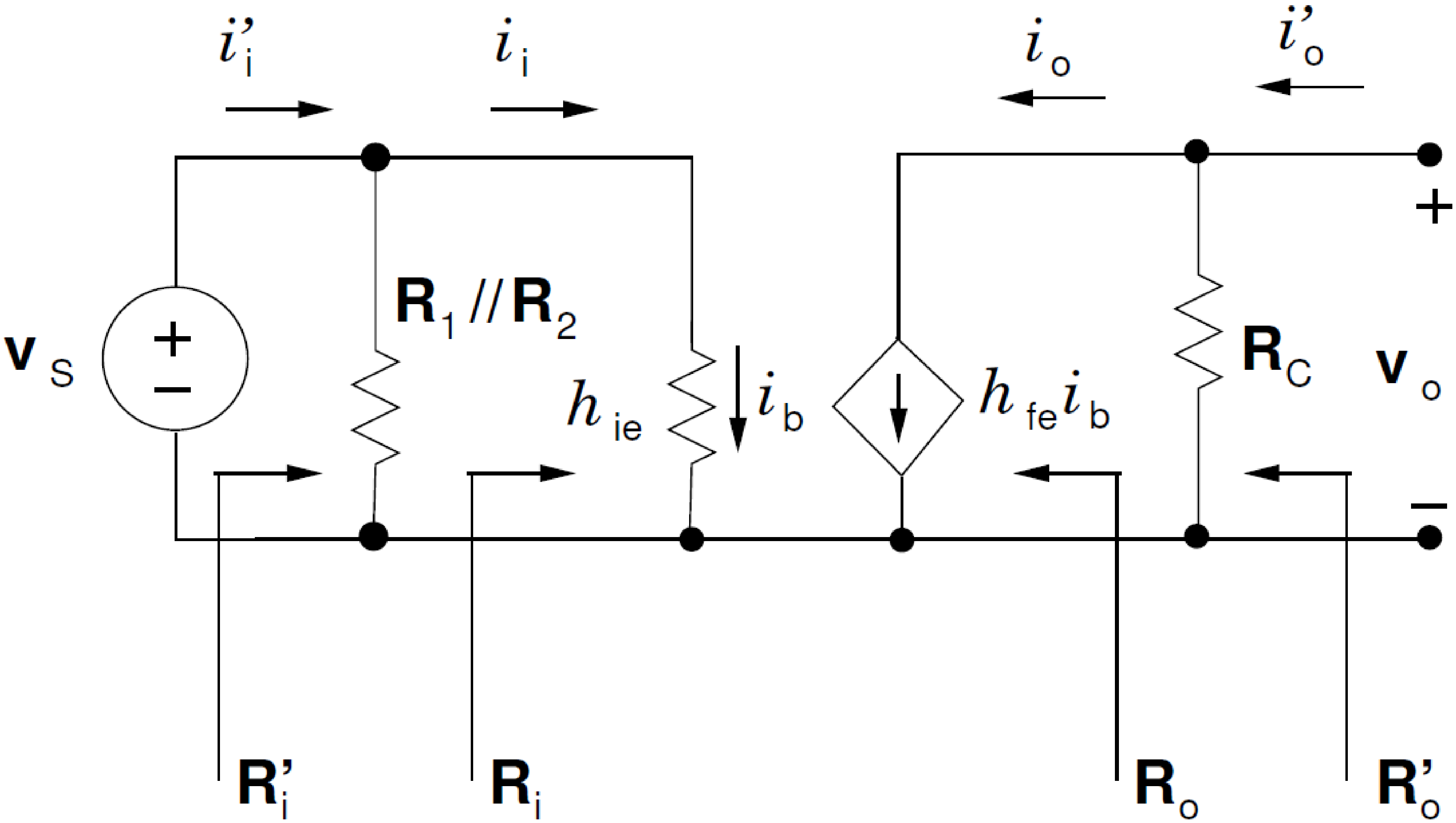
$$V_{IN} = 1.2 + 0.2\cos(2\pi 100t)$$

$$V_{OUT} = 3.7 - 1.4\cos(2\pi 100t)$$

Common Emitter Amplifier Stage



Small signals equivalent circuit



Gain – Input & Output Resistance

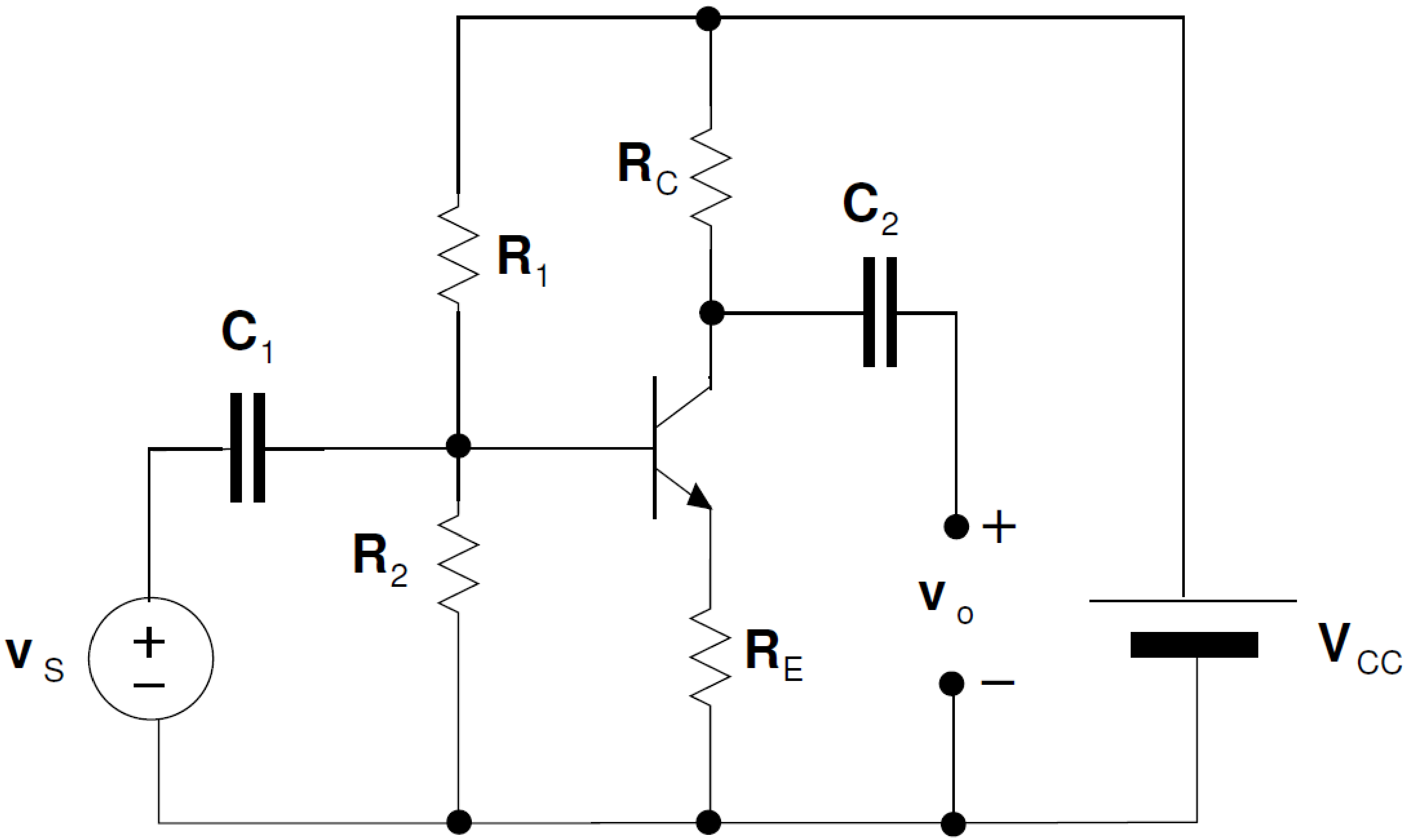
$$A_i = \frac{i_o}{i_i} = \frac{h_{fe} i_b}{i_b} = h_{fe}$$

$$A_v = \frac{v_o}{v_i} = \frac{-h_{fe} i_b R_C}{h_{ie} i_b} = -\frac{h_{fe} R_C}{h_{ie}}$$

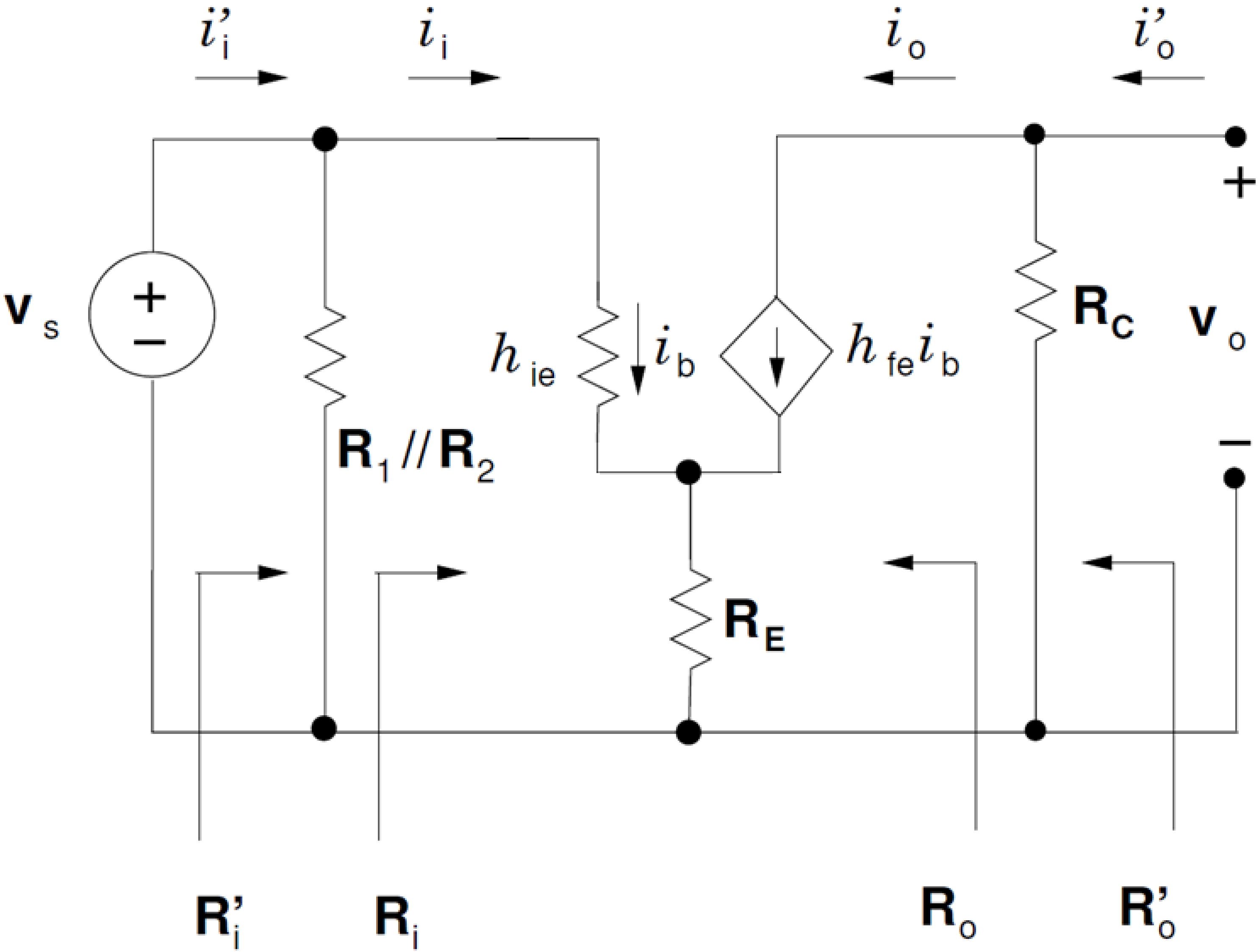
$$R_i = \frac{v_s}{i_i} = h_{ie} \qquad R_o = \left.\frac{v_o}{i_o}\right|_{v_s=0} = \infty$$

$$R_i' = R_1 // R_2 // h_{ie} \qquad R_o' = R_o // R_C = R_C$$

Common Emitter with Emitter Resistance



Small signals equivalent circuit



Gain – Input & Output Resistance

$$v_s = h_{ie} i_b + (h_{fe} + 1) i_b R_E$$

$$v_o = -h_{fe} i_b R_C$$

$$A_v = \frac{v_o}{v_s} = \frac{-h_{fe} R_C}{h_{ie} + (h_{fe} + 1) R_E}$$

$$R_i = h_{ie} + (h_{fe} + 1) R_E$$

$$R_o = \infty$$

$$R_i' = R_1 // R_2 // [h_{ie} + (h_{fe} + 1) R_E]$$

$$R_o' = R_C$$

BJT parameters

$$\gamma = \frac{i_{Ep}}{i_E} = \frac{i_{Ep}}{i_{Ep} + i_{En}}$$

emitter injection efficiency

$$i_C = \boldsymbol{B}\, i_{Ep}$$

base transport factor

$$\frac{\boldsymbol{i_C}}{\boldsymbol{i_E}} = B\gamma = \boldsymbol{\alpha}$$

current transfer ratio

$$\frac{i_C}{i_B} = \frac{\alpha}{1-\alpha} = \beta$$

amplification factor

Summary – general solution for currents

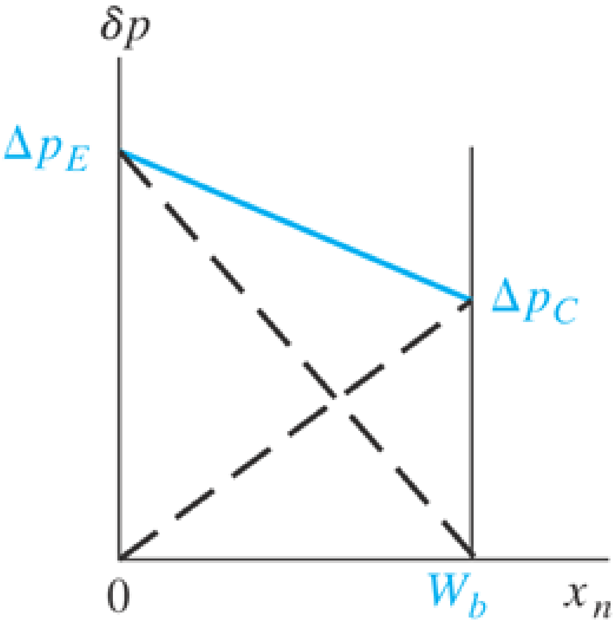
$$I_{Ep} = qA\frac{D_p}{L_p}\left[\Delta p_E \operatorname{ctnh}\frac{W_B}{L_P} - \Delta p_C \operatorname{csch}\frac{W_B}{L_P}\right]$$

$$I_C = qA\frac{D_p}{L_p}\left[\Delta p_E \operatorname{csch}\frac{W_B}{L_P} - \Delta p_C \operatorname{ctnh}\frac{W_B}{L_P}\right]$$

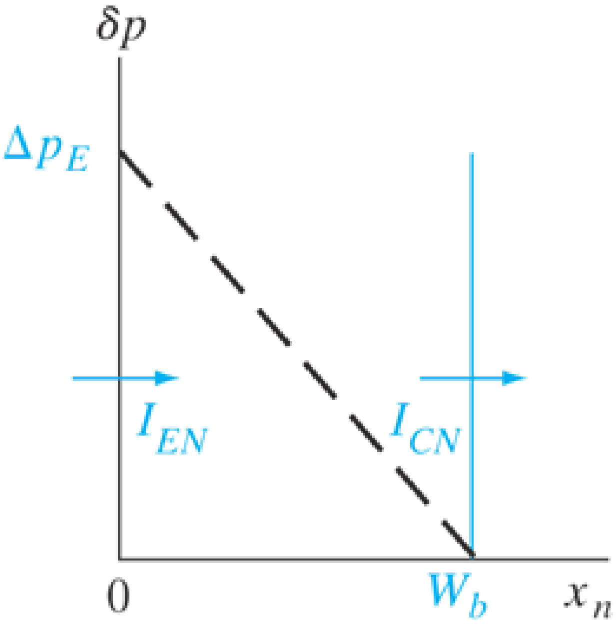
$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_B}{2L_P}\right]$$

These expressions apply to all bias conditions

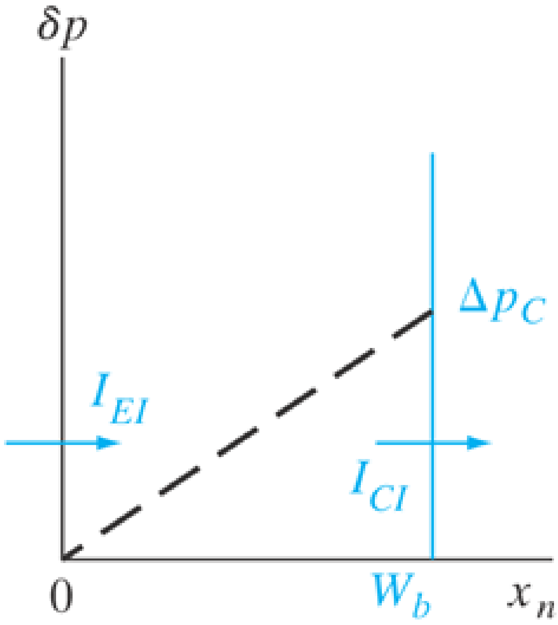
Excess minority carrier distribution in the base



both junctions forward biased



normal mode



inverted mode

Current Transfer Ratio α

To evaluate α we need to be more accurate with γ instead of approximating $\gamma = 1$.

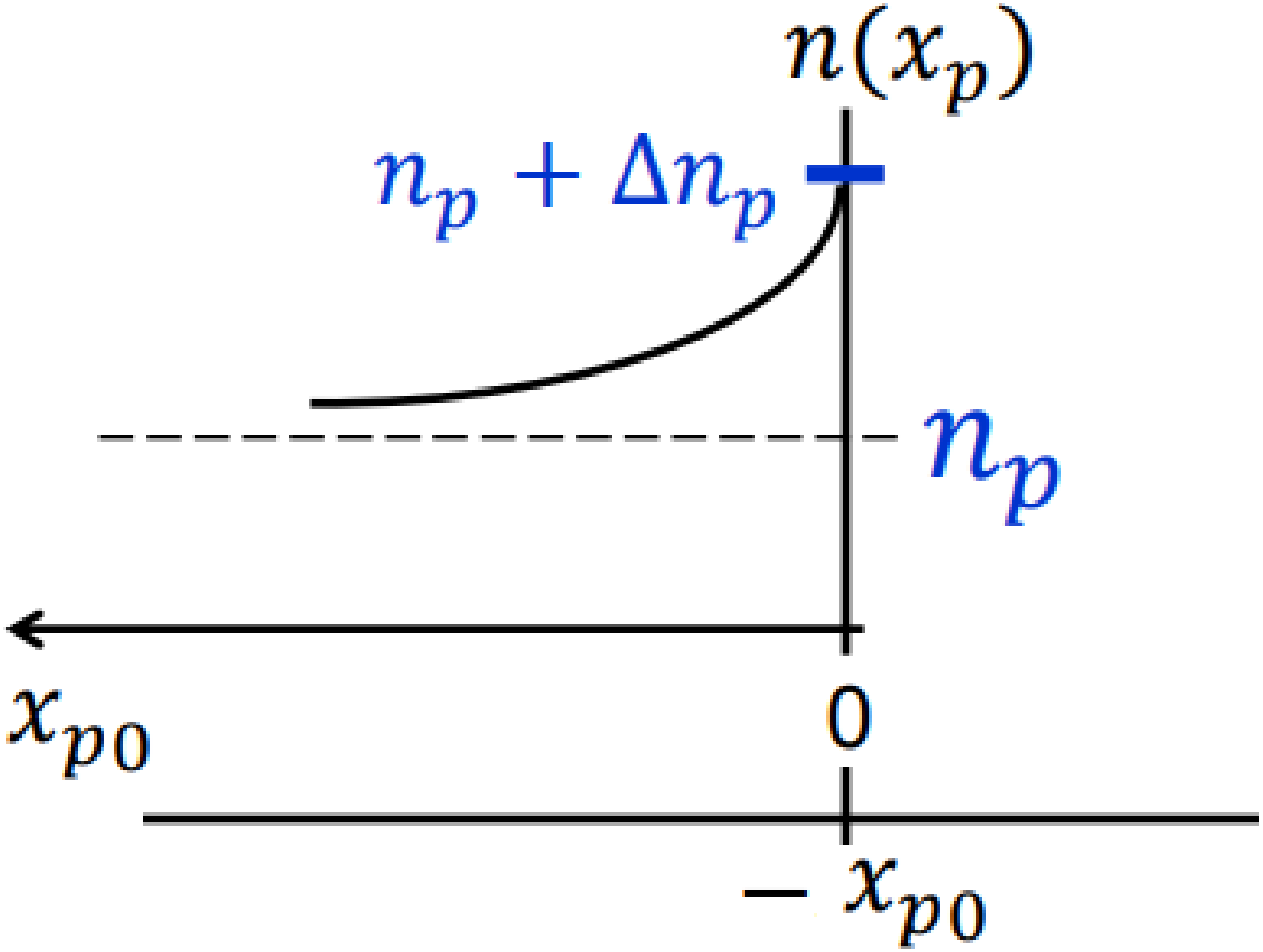
The electron current from the base must be included for the emitter injection efficiency.

From p-n junction theory

$$I_{En} = qA\frac{D_n^p}{L_n^p} n_p \exp\left(q\frac{V_{EB}}{k_B T}\right) \qquad \text{for} \quad V_{EB} \gg k_B T/q$$

Current Transfer Ratio α

On the *p*-side of the forward biased junction



Current Transfer Ratio α

From the general solution with $\Delta p_C = 0$

At $x_n = 0$

$$I_{Ep} = qA\frac{D_p^n}{L_p^n}\Delta p_E \operatorname{ctnh}\frac{W_B}{L_p^n}$$

$$= qA\frac{D_p^n}{L_p^n} p_n \operatorname{ctnh}\frac{W_B}{L_p^n}\exp\left(q\frac{V_{EB}}{k_B T}\right)$$

$$I_E = I_{Ep} + I_{En} =$$

$$= qA\left[\frac{D_p^n}{L_p^n} p_n \operatorname{ctnh}\frac{W_B}{L_p^n} + \frac{D_n^p}{L_n^p} n_p\right]\exp\left(q\frac{V_{EB}}{k_B T}\right)$$

Current Transfer Ratio α

$$\gamma = \frac{i_{Ep}}{i_E} = \frac{i_{Ep}}{i_{Ep} + i_{En}} = \left[1 + \frac{i_{En}}{i_{Ep}}\right]^{-1}$$

$$\gamma = \left[1 + \frac{qA\dfrac{D_n^p}{L_n^p} n_p \exp\left(q\dfrac{V_{EB}}{k_B T}\right)}{qA\dfrac{D_p^n}{L_p^n} p_n \operatorname{ctnh}\dfrac{W_B}{L_P} \exp\left(q\dfrac{V_{EB}}{k_B T}\right)}\right]^{-1}$$

Current Transfer Ratio α

$$\gamma = \left[1 + \frac{\frac{D_n^p}{L_n^p} n_p}{\frac{D_p^n}{L_p^n} p_n} \tanh \frac{W_B}{L_p^n} \right]^{-1}$$

$$\boxed{\frac{D_n^p}{D_p^n} = \frac{\mu_n^p}{\mu_p^n}}$$

$$\boxed{n_p p_p = n_n p_n}$$

from Einstein’s relations

in the neutral regions

Current Transfer Ratio α

Finally:

$$\gamma = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n} \tanh \frac{W_B}{L_p^n}\right]^{-1} \approx \left[1 + \frac{\cancel{L_p^n} n_n \mu_n^p}{L_n^p p_p \mu_p^n} \frac{W_B}{\cancel{L_p^n}}\right]^{-1}$$

At first order $\tanh y \approx y$

L_p^n = hole diffusion length in the n-region
L_n^p = electron diffusion length in the p-region
μ_p^n = hole mobility in the n-region
μ_n^p = electron mobility in the p-region

Current Transfer Ratio α

The base transport factor is

$$B = \frac{I_C}{I_{Ep}} = \frac{\operatorname{csch}\frac{W_B}{L_p}}{\operatorname{ctnh}\frac{W_B}{L_p}} = \operatorname{sech}\frac{W_B}{L_p} \approx 1 - \frac{1}{2}\left(\frac{W_B}{L_p}\right)^2$$

and

$$\alpha = B\gamma \approx \left[1 + \frac{W_B n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1} \left[1 - \frac{1}{2}\left(\frac{W_B}{L_p}\right)^2\right]$$

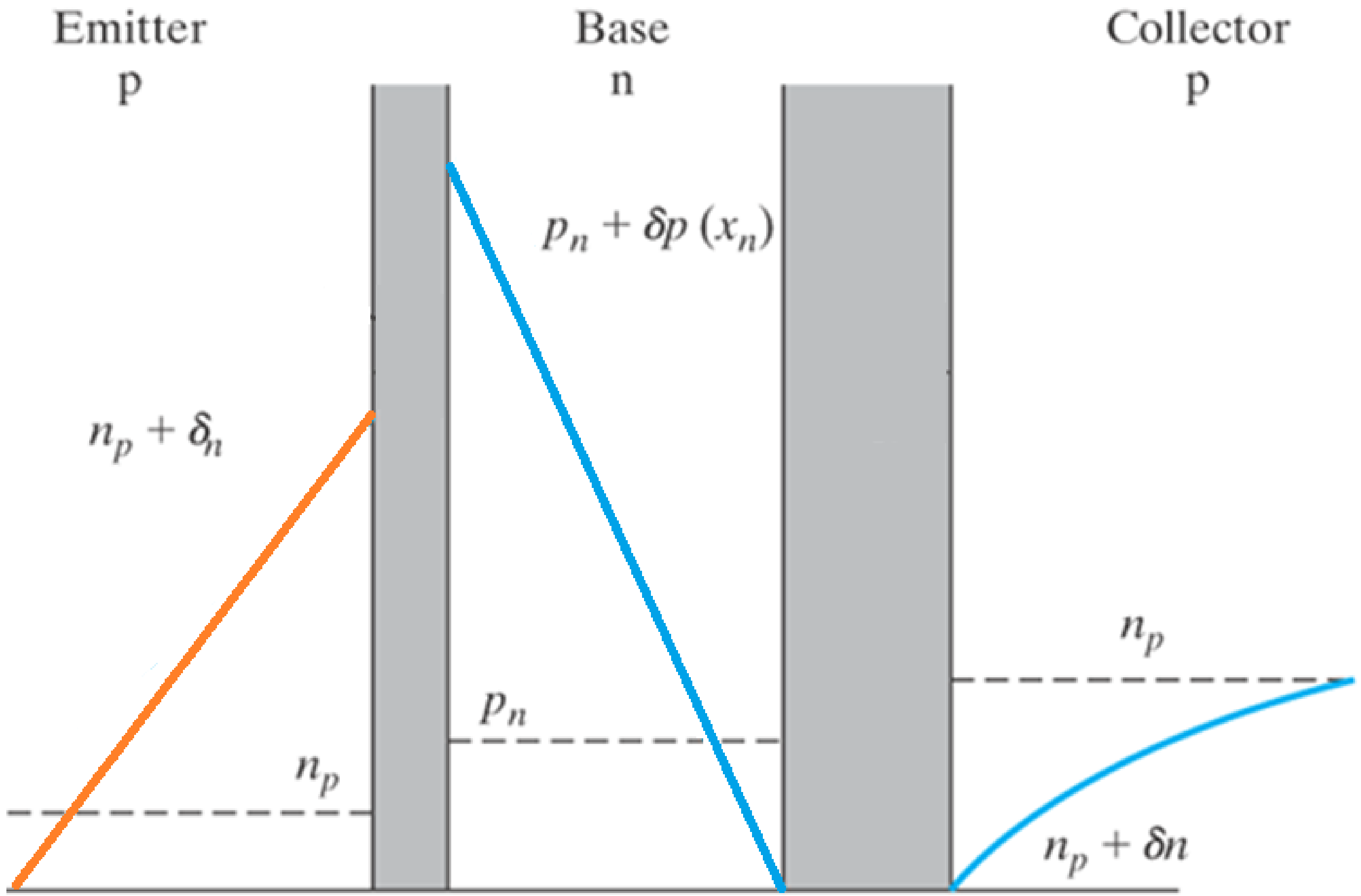
Exercise on γ and B

- Consider a BJT.
- Emitter doping is 100 times the doping in the base region.
- Neutral Base width $W_B \ll L_p$
- Neutral Emitter width $W_n \ll L_n$

Question:
Estimate emitter injection efficiency γ and base transport factor B

Exercise on γ and B

- Emitter and base width are much smaller than L_p, L_n
  $\rightarrow$ carrier concentration profiles vary linearly



Exercise on γ and B

From narrow base diode lecture

$$I_p(x_n) \approx A\, J_p(\mathrm{diff}) = -q\, A\, \frac{D_p}{\ell}\, p_n\left[\exp\left(\frac{qV}{k_B T}\right)-1\right]$$

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_n} \qquad V = V_{BE}$$

Similar for emitter with narrow width. Emitter is highly doped diffusivities of minority carriers on the two sides do not differ much

$$I_{E_n} \propto \frac{n_i^2}{N_E \cdot W_E} \cdot e^{\frac{q \cdot V_{BE}}{kT}} \qquad I_{E_p} \propto \frac{n_i^2}{N_B \cdot W_B} \cdot e^{\frac{q \cdot V_{BE}}{kT}}$$

Exercise on γ and B

$$N_E = 100 \cdot N_B \text{ and } W_E = 0.1 \cdot W_B$$

$$I_{E_n} \propto \frac{1}{N_E \cdot W_E} \qquad I_{E_p} \propto \frac{1}{N_B \cdot W_B}$$

$$\gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}} = \frac{\frac{1}{N_B \cdot W_B}}{\frac{1}{N_B \cdot W_B} + \frac{1}{N_E \cdot W_E}}$$

Exercise on γ and B

$$N_E = 100 \cdot N_B \text{ and } W_E = 0.1 \cdot W_B$$

$$\gamma = \frac{1}{1 + \frac{N_B \cdot W_B}{N_E \cdot W_E}} = \frac{1}{1 + \frac{N_B \cdot W_B}{100 \cdot N_B \cdot 0.1 \cdot W_B}} =$$

$$= \frac{1}{1 + 0.1} = 0.91$$

Base carrier profile is linear so $B = 1$

Exercise on γ and B

- Suppose $L_p = L_n = L$
- Neutral Base width $W_B \gg L_p$
- Neutral Emitter width $W_n \gg L_n$

- Now carrier concentrations decay exponentially, not linearly

$$I_{E_n} \propto \frac{1}{N_E \cdot L} \qquad I_{E_p} \propto \frac{1}{N_B \cdot L}$$

Exercise on γ and B

$$N_E = 100 \cdot N_B$$

$$\gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}} = \frac{\frac{1}{N_B \cdot L}}{\frac{1}{N_B \cdot L} + \frac{1}{N_E \cdot L}} =$$

$$= \frac{1}{1 + \frac{N_B}{N_E}} = \frac{1}{1 + \frac{N_B}{100 \cdot N_B}} = \frac{1}{1 + 0.01} = 0.99$$

Exercise on γ and B

However, the base transport factor B would not be good for transistor operation, since only holes within a diffusion length from the edge of the depletion layer (on the base-collector side) would contribute to collector current.

The rest of the injected holes would recombine in the base.

So, we want the emitter to be long enough not to be considered “narrow”, but the base region should be narrow to have B as close as possible to 1.