ECE 340 Lectures 42
Solid State Electronic Devices

Spring 2022
10:00-10:50am
Professor Umberto Ravaioli
Department of Electrical and Computer Engineering
2062 ECE Building

Today's Discussion

- **General Review**

Topics covered

- **p-n junctions**
- **Photodetectors/Solar Cells/Lasers**
- **metal-semiconductor junction**
- **MOS Capacitor**
- **Bipolar junction transistor**

p-n junction

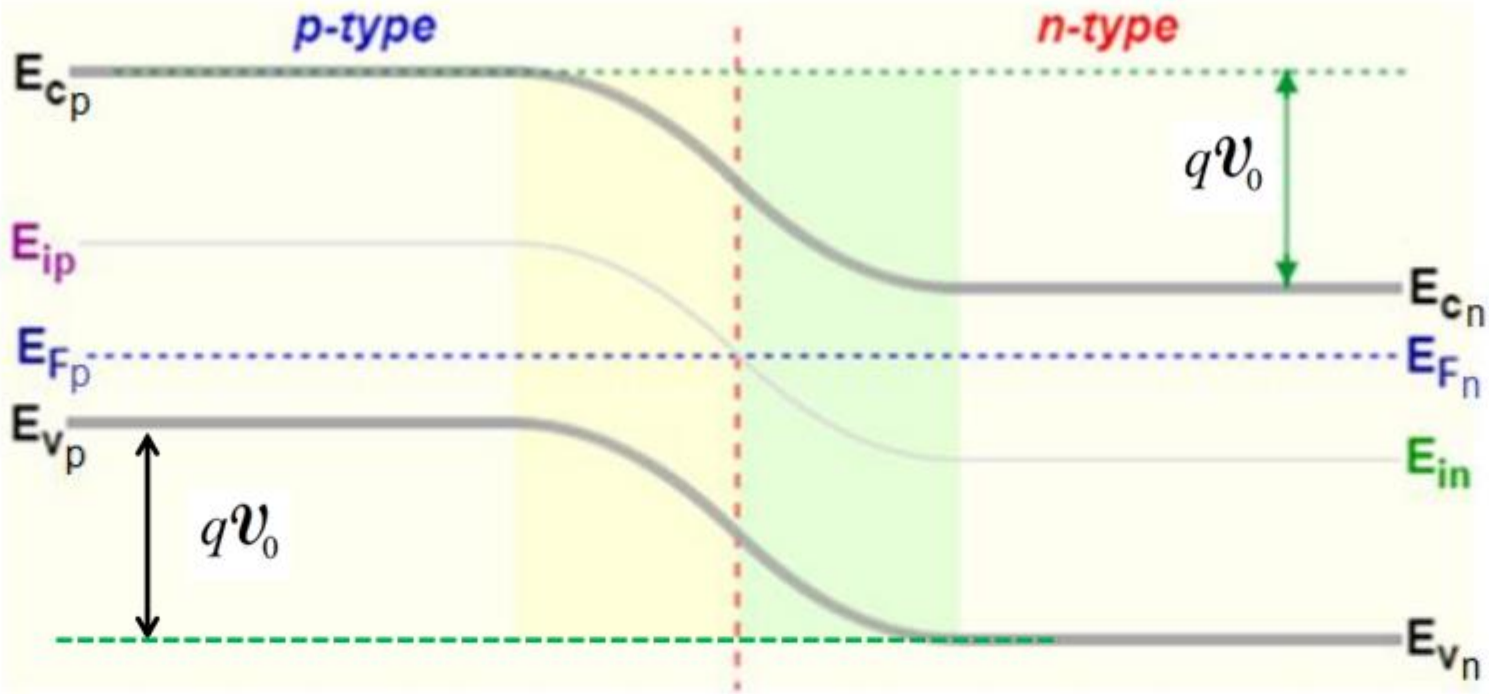
(equilibrium)

$$qV_0 = E_{Vp} - E_{Vn}$$

$$qV_0 = E_{Cp} - E_{Cn}$$

$$qV_0 = E_{ip} - E_{in}$$

$$E_{Fp} = E_{Fn}$$



p-n junction

(equilibrium)

Step junction: N_A on p-side & N_D on n-side

$$v_0 = \frac{k_B T}{q} \ln \frac{p_p}{p_n} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$\boxed{p_p \approx N_A} \qquad \boxed{p_n \approx \frac{n_i^2}{N_D}}$$

In the regions far away from the junction

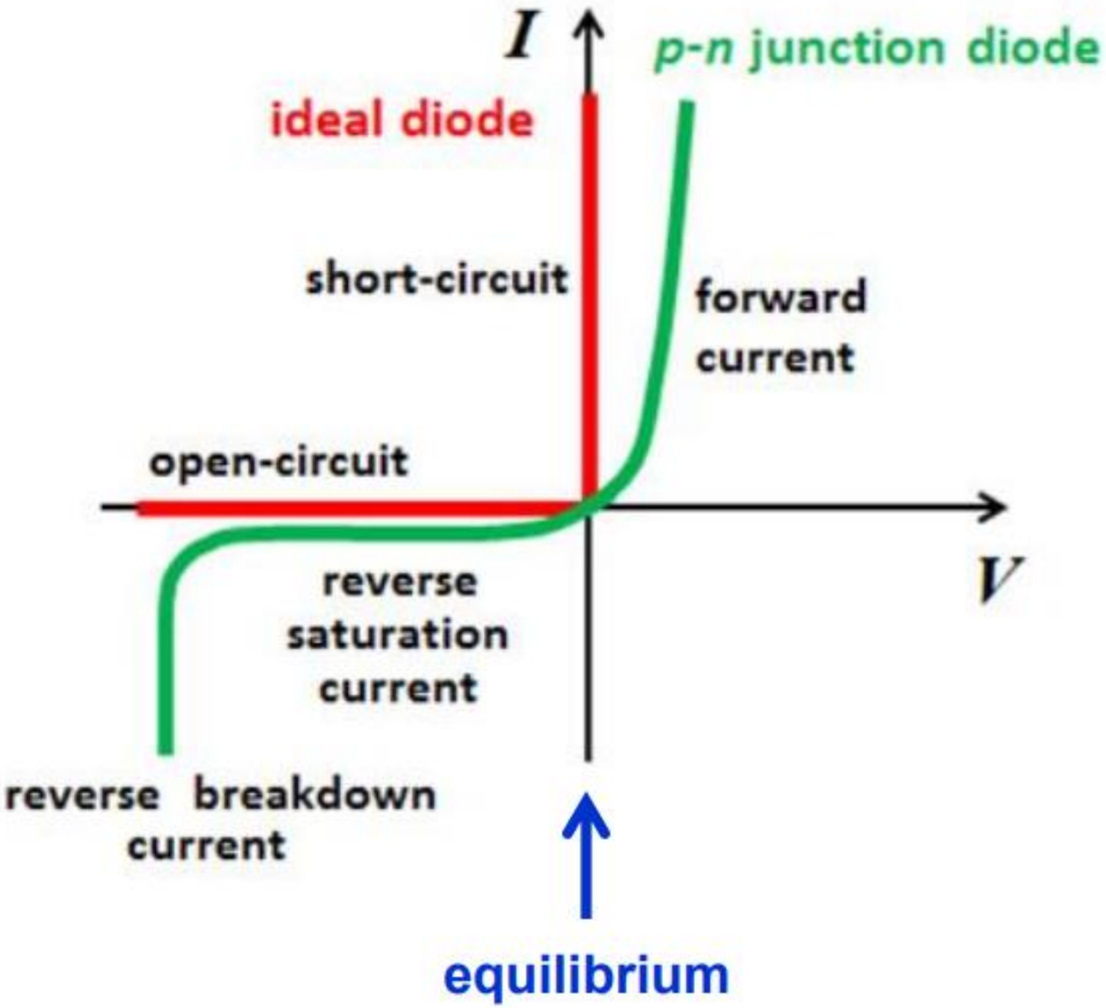
$$\boxed{n_p = \frac{n_i^2}{p_p}} \qquad \boxed{n_n = \frac{n_i^2}{p_n}}$$

Also:

$$\boxed{\begin{aligned} \frac{p_p}{p_n} &= \exp\left(\frac{q v_0}{k_B T}\right) \\ \frac{n_n}{n_p} &= \exp\left(\frac{q v_0}{k_B T}\right) \end{aligned}}$$

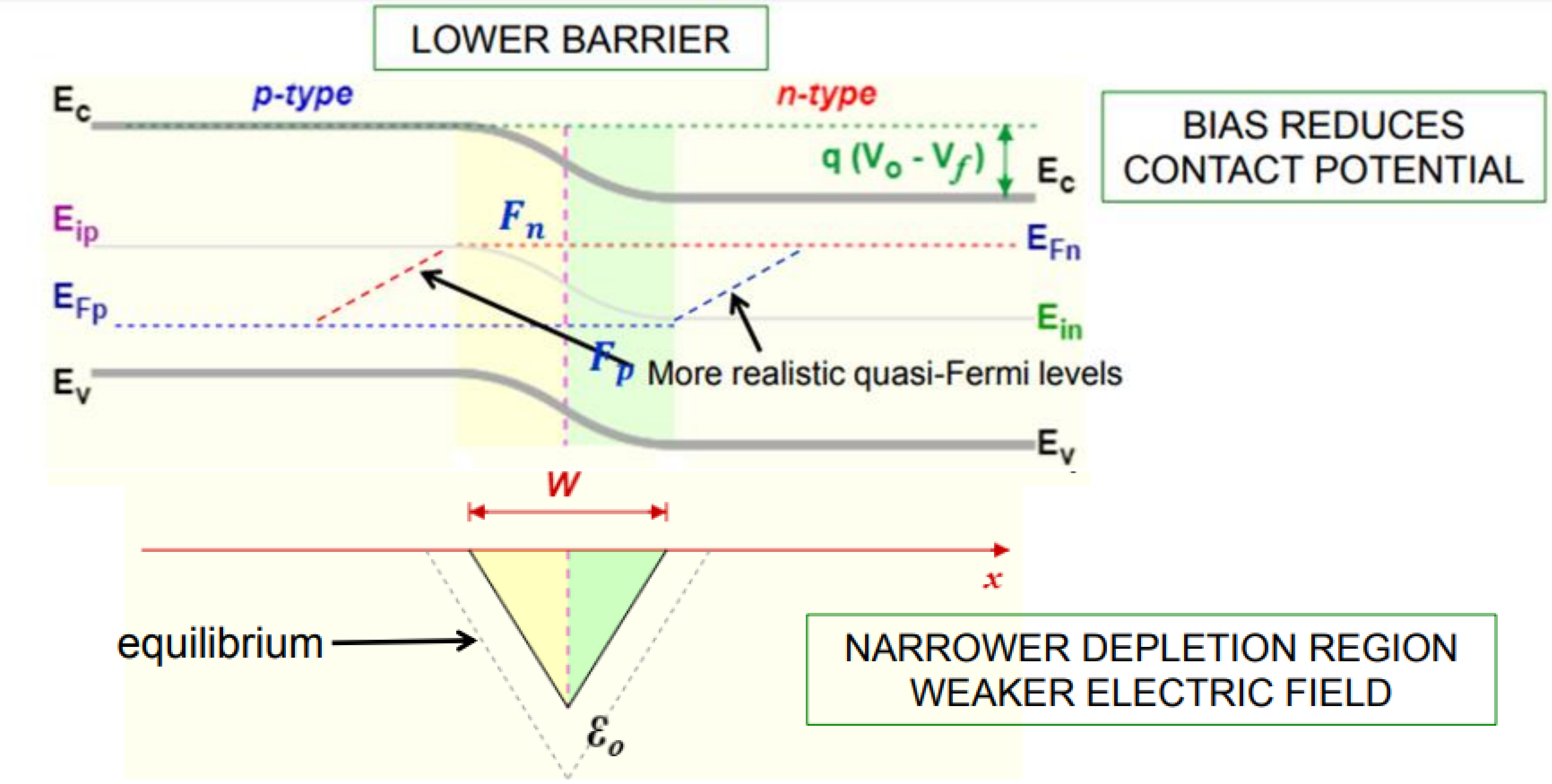
p-n junction

(out of equilibrium)



p-n junction

Forward bias



LOWER BARRIER

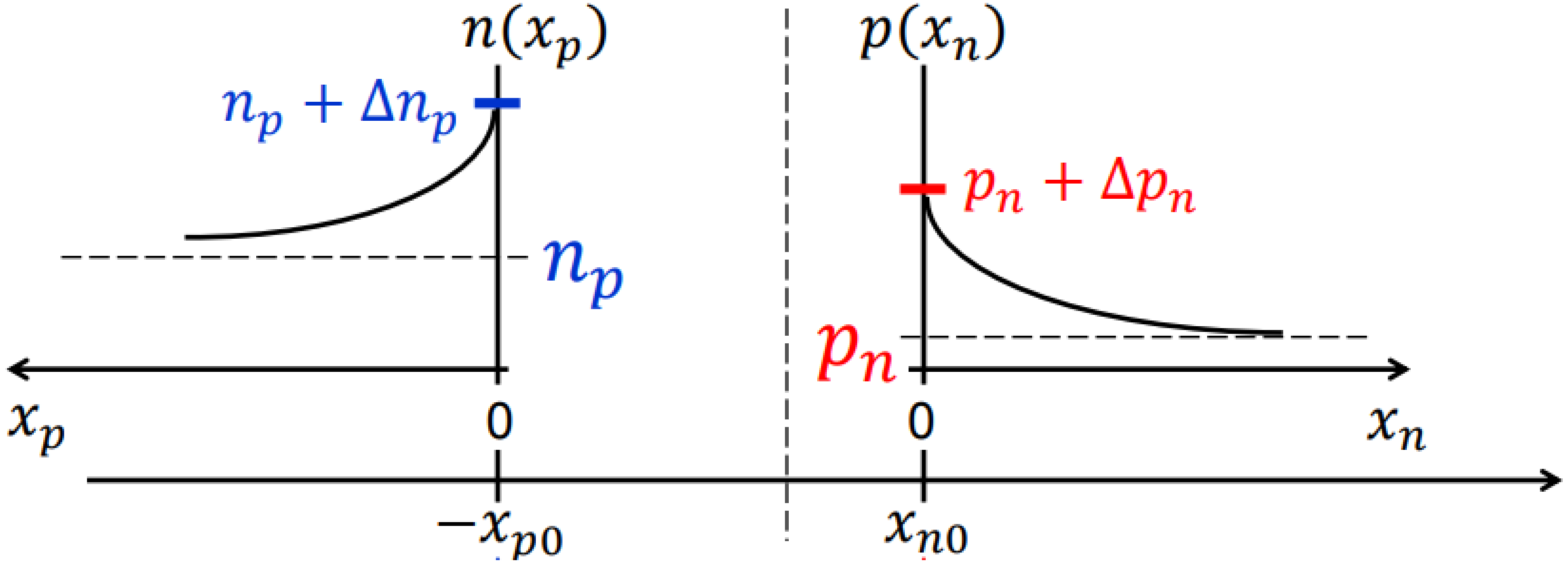
BIAS REDUCES CONTACT POTENTIAL

NARROWER DEPLETION REGION
WEAKER ELECTRIC FIELD

More realistic quasi-Fermi levels

p-n junction

Forward bias



$$\boxed{\Delta p_n = p_n\left[\exp\left(\frac{qV}{k_B T}\right) - 1\right]}$$

p-n junction

Forward bias

$$I = I_p(x_n = 0) - I_n(x_p = 0)$$

$$= qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] + qA\frac{D_n}{L_n}n_p\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

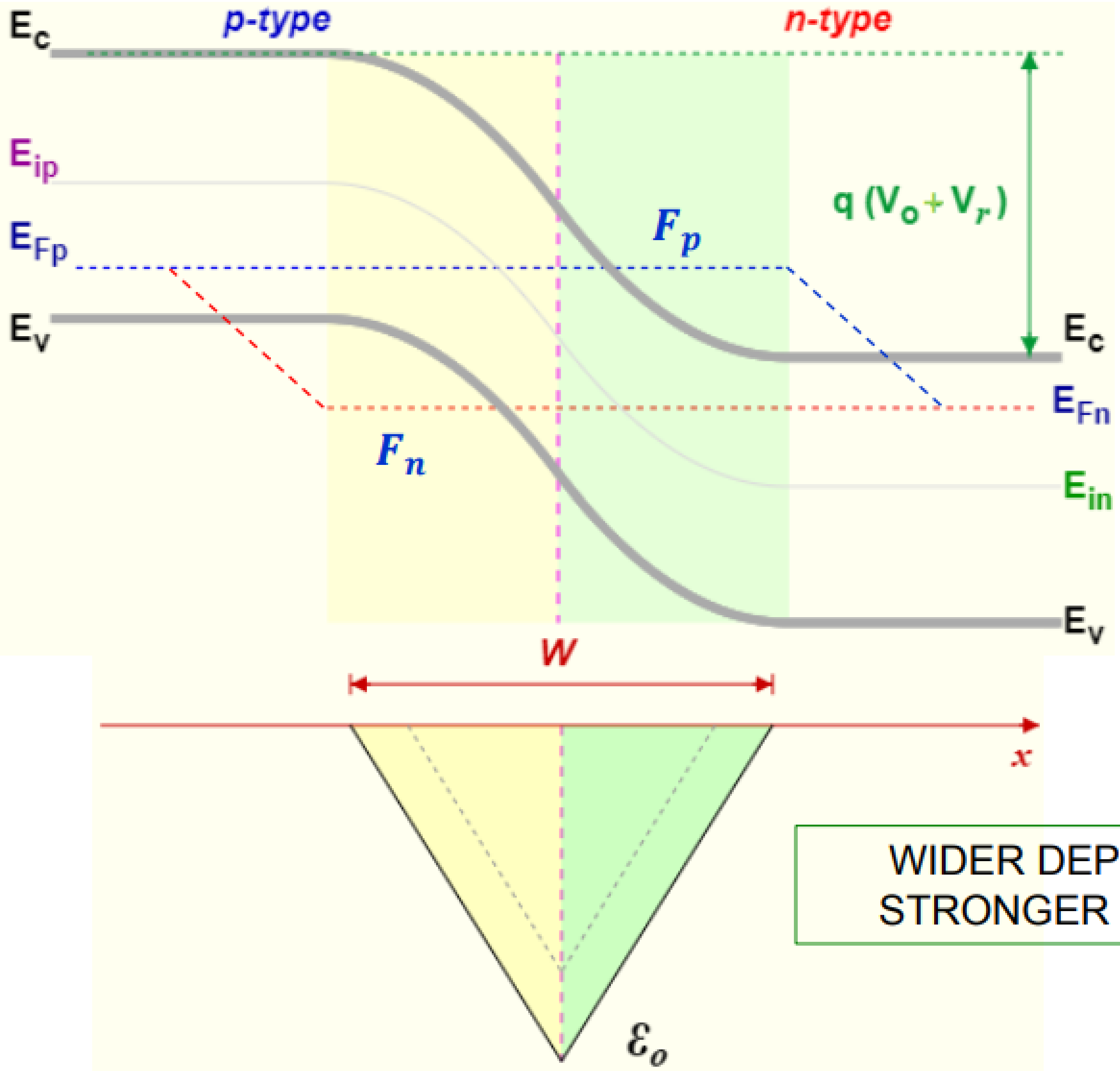
$$= \underbrace{qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)}_{I_0}\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$I = I_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

p-n junction

Reverse bias

HIGHER BARRIER



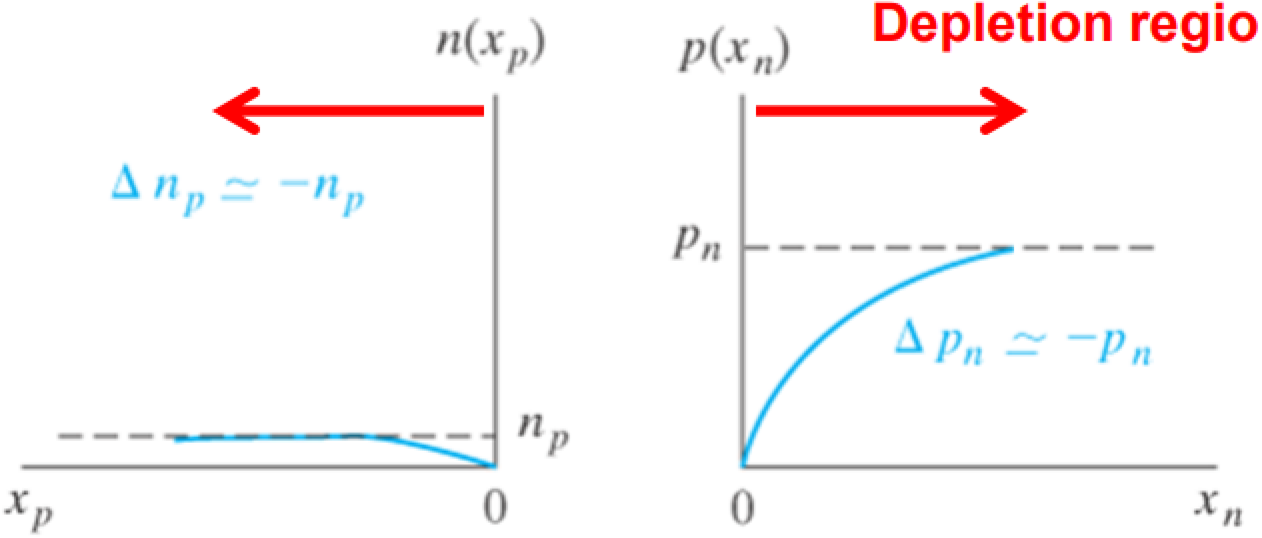
BIAS INCREASES
CONTACT POTENTIAL

WIDER DEPLETION REGION
STRONGER ELECTRIC FIELD

p-*n* junction

Reverse bias

$$V = -V_r$$



Depletion region widens

p-*n* junction

Reverse bias

Reverse bias $V = -V_r$ with $V_r \gg k_B T/q$

$$\Delta p_n = p_n \left[\exp\left(-\frac{qV_r}{k_B T}\right) - 1\right] \approx -p_n$$

$$\Delta n_p = n_p \left[\exp\left(-\frac{qV_r}{k_B T}\right) - 1\right] \approx -n_p$$

p-n junction

Reverse bias

Reverse bias $V = -V_r$

$$I = I_0 \left[\exp\left(-\frac{qV_r}{k_B T} \right) - 1 \right]$$

$$I = -I_0 = -qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right)$$

p-n junction

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

Equilibrium

$$W = \sqrt{\frac{2\varepsilon(V_0 - V_f)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

Forward Bias

$$W = \sqrt{\frac{2\varepsilon(V_0 + V_r)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

Reverse Bias

p-n junction

Junction Capacitance

$$|Q| = qAx_{n0}N_D = qAx_{p0}N_A$$

$$x_{n0} = \frac{N_A}{N_A + N_D}W \qquad x_{p0} = \frac{N_D}{N_A + N_D}W$$

$$|Q| = qA\frac{N_D N_A}{N_A + N_D}W$$

p-n junction

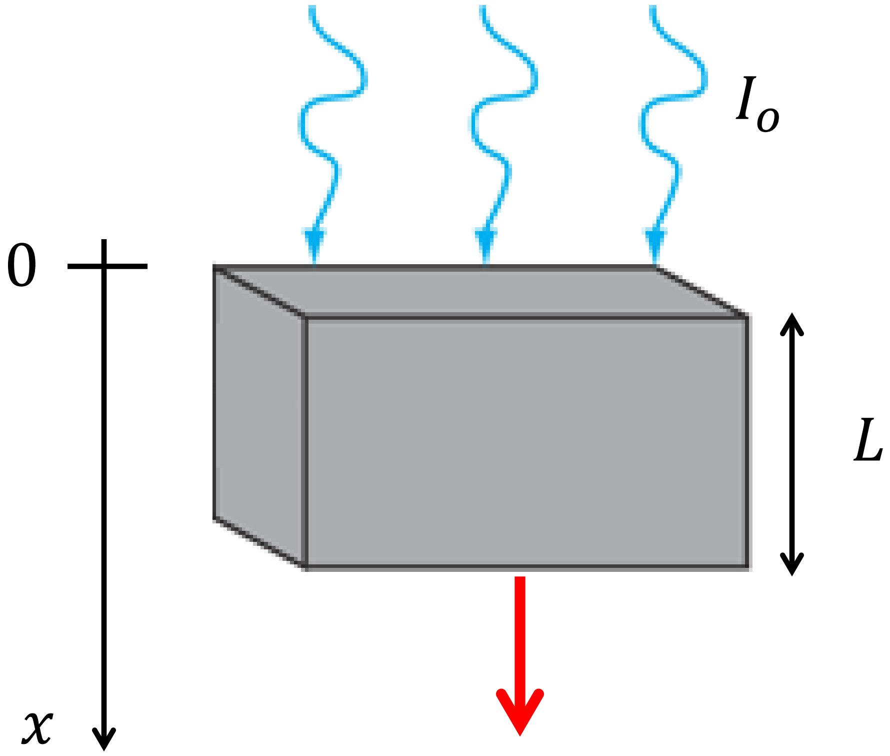
Junction Capacitance

$$|Q| = qA\frac{N_D N_A}{N_A + N_D}W =$$

$$= \epsilon A\sqrt{\frac{2q}{\epsilon}(V_0 - V)\frac{N_D N_A}{N_A + N_D}}$$

$$C_j = \left|\frac{dQ}{d(V_0 - V)}\right| = \epsilon A\underbrace{\sqrt{\frac{q}{2\epsilon(V_0 - V)}\frac{N_D N_A}{N_A + N_D}}}_{W^{-1}}$$

Absorption of light in material



$$I(x) = I_o e^{-\alpha x}$$

$$I(L) = I_o e^{-\alpha L}$$

Absorption of light in material

Quantum efficiency

ratio of the number of electrons in the external circuit produced by an incident photon of a given wavelength

Internal Quantum Efficiency (IQE)

Considers only photons which have been able to enter the material

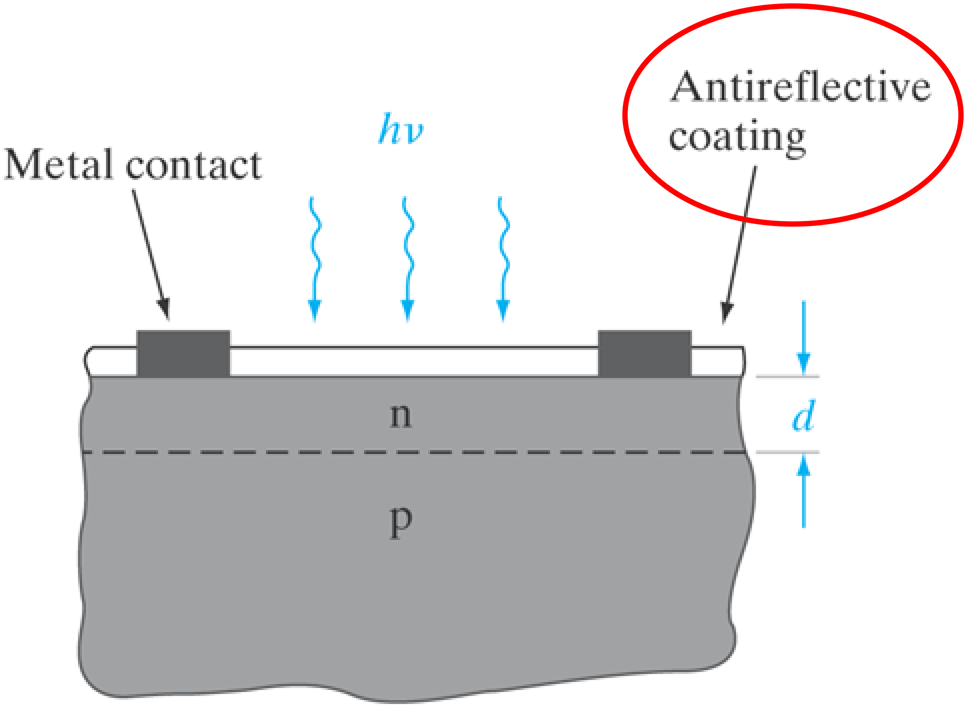
External Quantum Efficiency (EQE)

Considers all impinging photons, including those who are not able to enter the material (for instance, photons which are reflected away from the material surface)

Solar Cell

Solar Cell – is made of a p-n junction

Both electrons and holes contribute to power generation

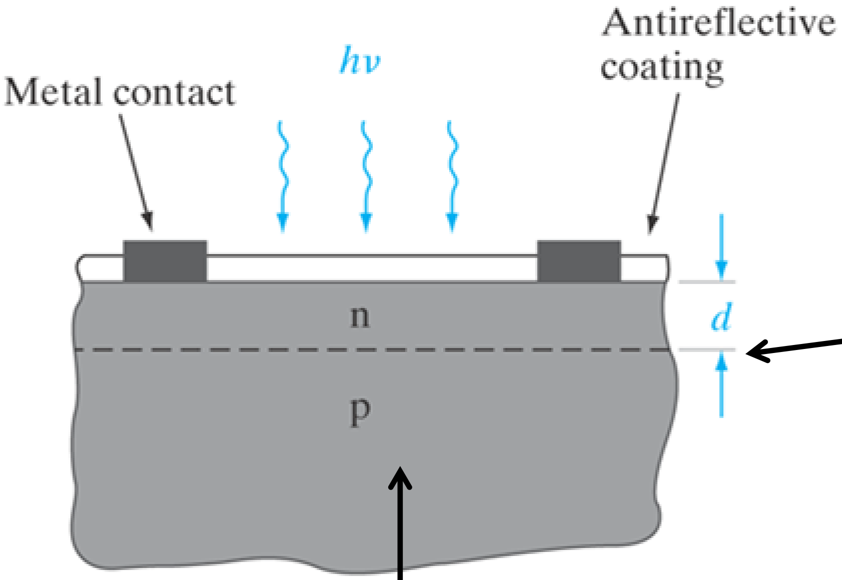


Improves EQE

Also, mirrors could be used to improve the EQE

Solar Cell

Design of Solar Cell is a compromise



Junction depth should be close to surface so intensity of incident light is as high as possible

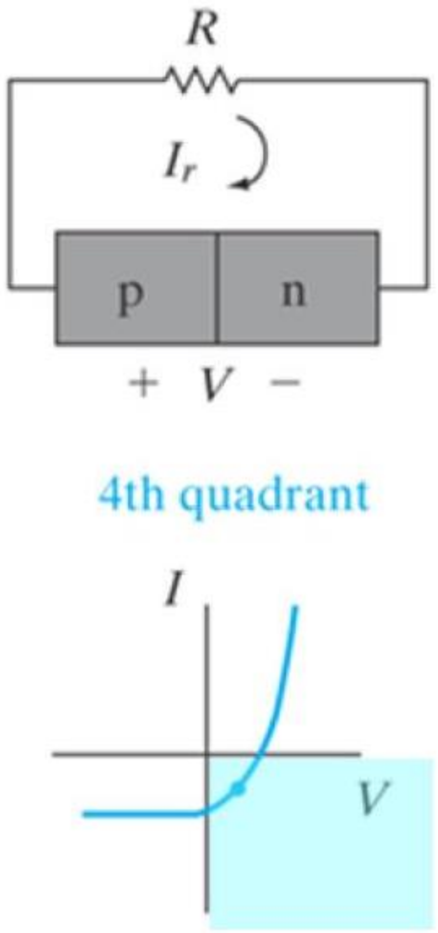
Junction depth should be less than L_p to allow holes generated near the surface to reach the junction

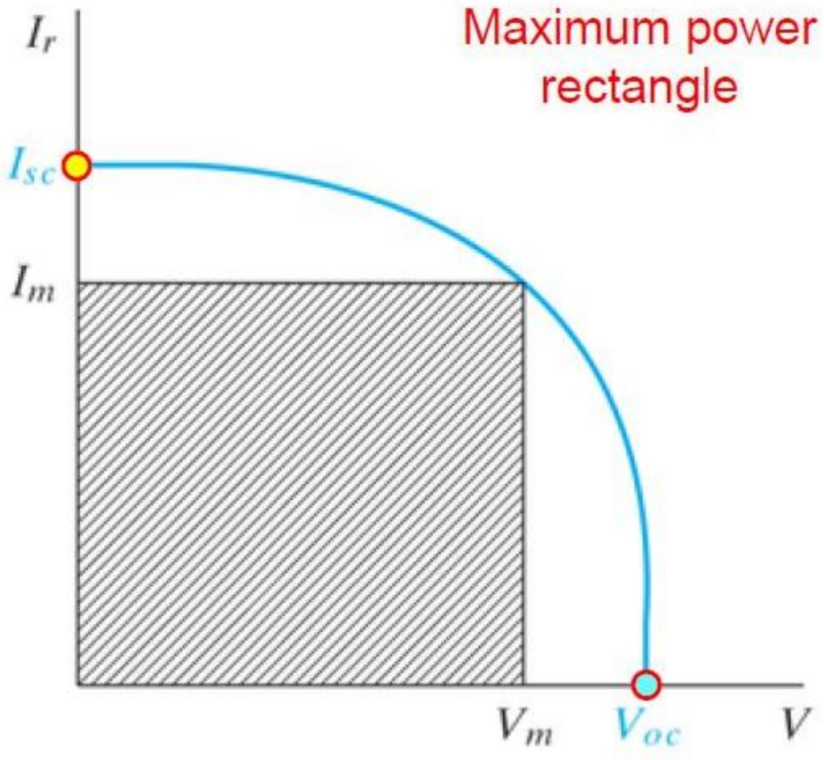
Thickness of p-region: we want also generated electrons to be able to diffuse to the depletion region.

Could make p-region thin and place a mirror at the bottom to send light back inside the device

Optoelectronic devices – from Lectures 28

External resistor should not be too high or too low, so that one can approach the optimal maximum power rectangle

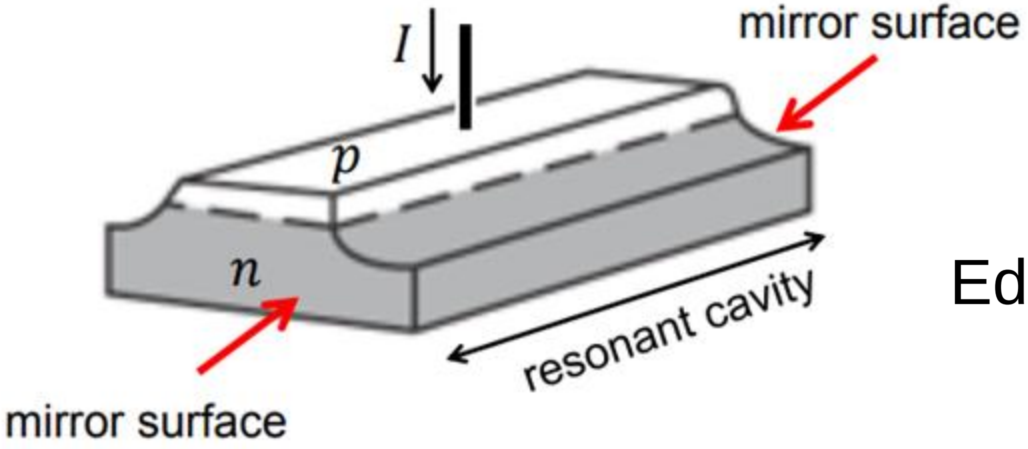




Optoelectronic devices

Semiconductor lasers

- **Simple *p-n* junction (e.g., GaAs)**



Edge emitting laser

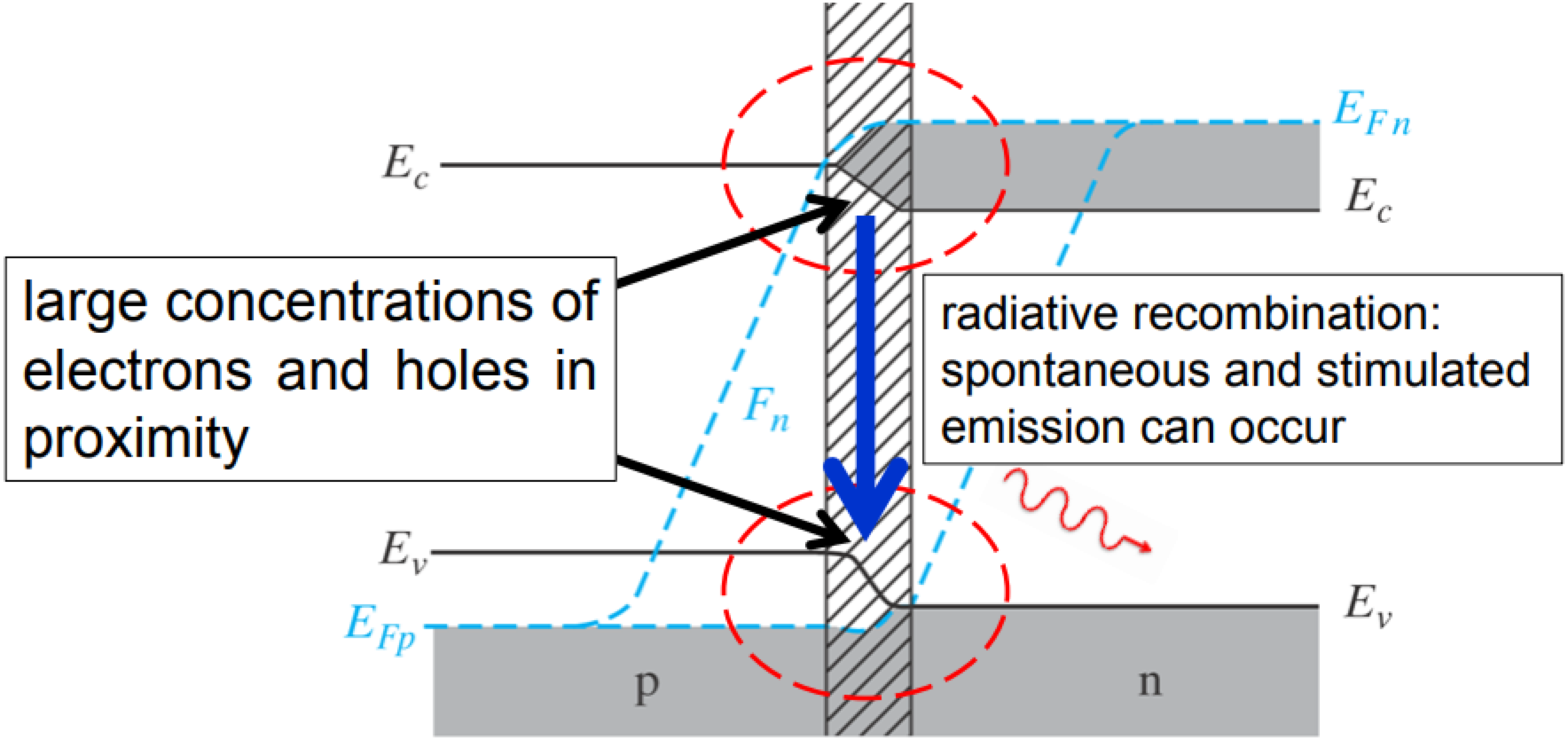
Two ingredients are needed to make a laser:

- population inversion (stable population of excited states)
- resonant cavity to build up a coherent photon population for stimulated emission to occur (coherence)

Optoelectronic devices

Semiconductor lasers

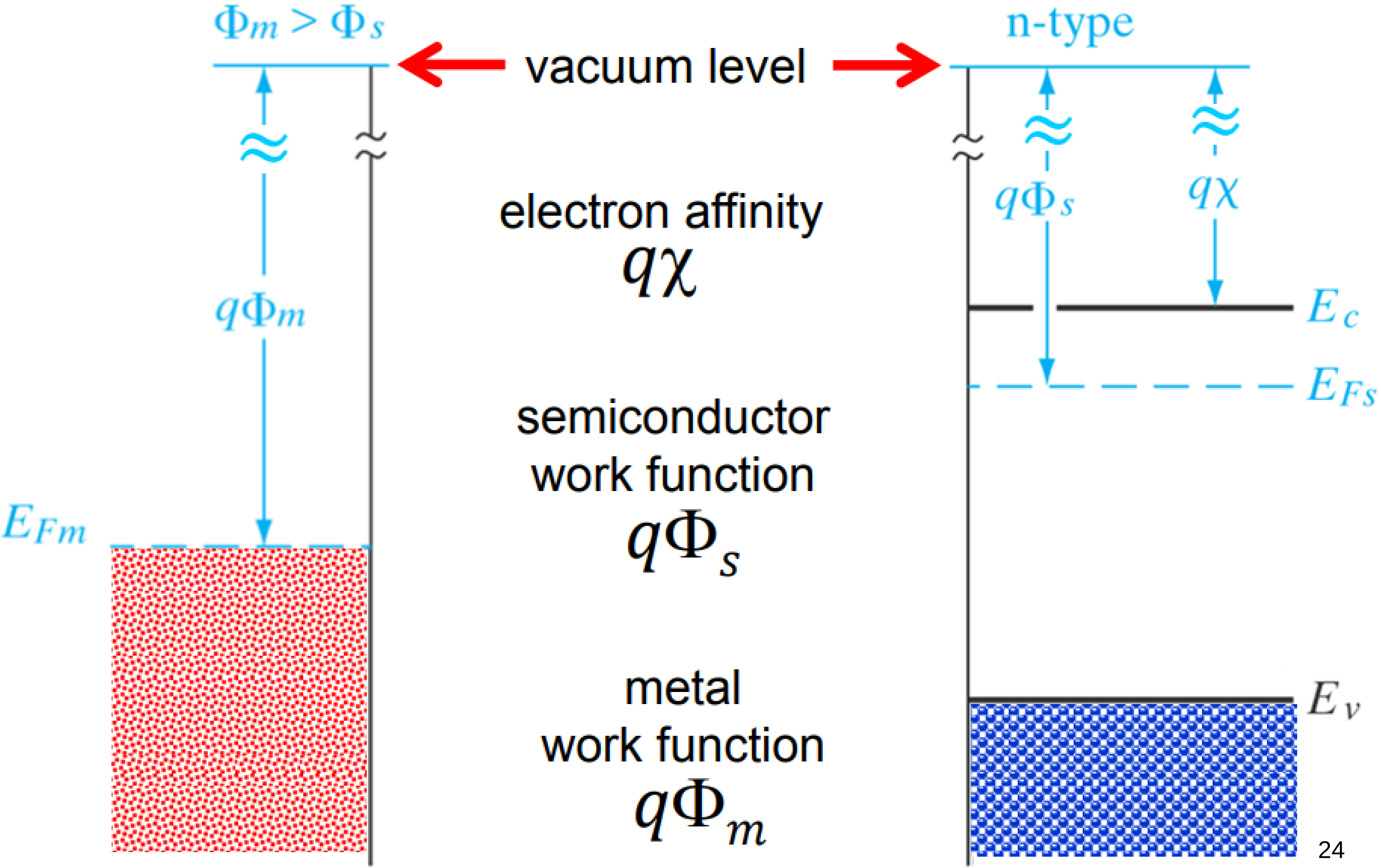
- **Heavily doped *p-n* junction in forward bias**



large concentrations of electrons and holes in proximity

radiative recombination: spontaneous and stimulated emission can occur

Metal-Semiconductor Junction



vacuum level

electron affinity
$q\chi$

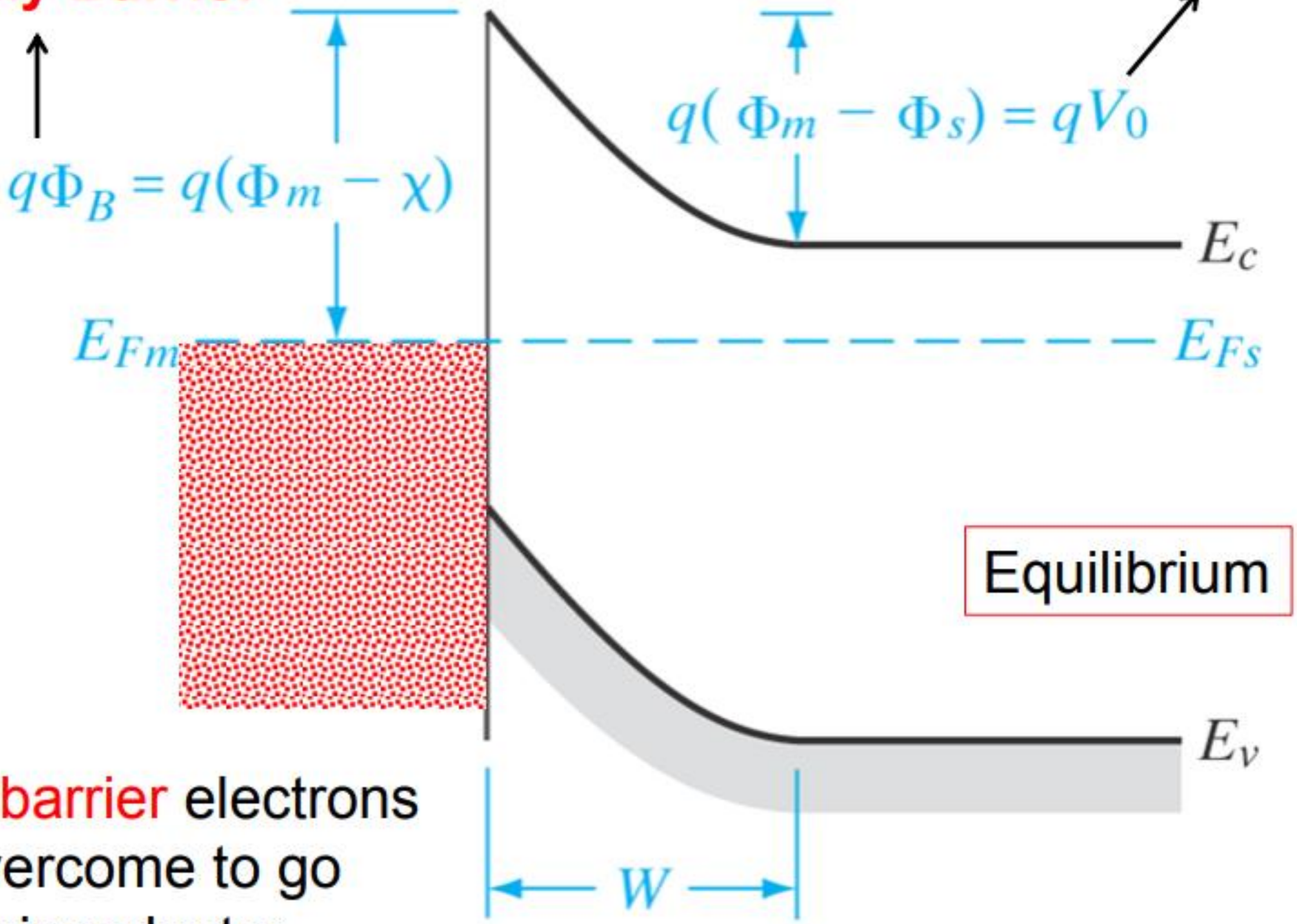
semiconductor
work function
$q\Phi_s$

metal
work function
$q\Phi_m$

Metal-Semiconductor Junction

Schottky Barrier

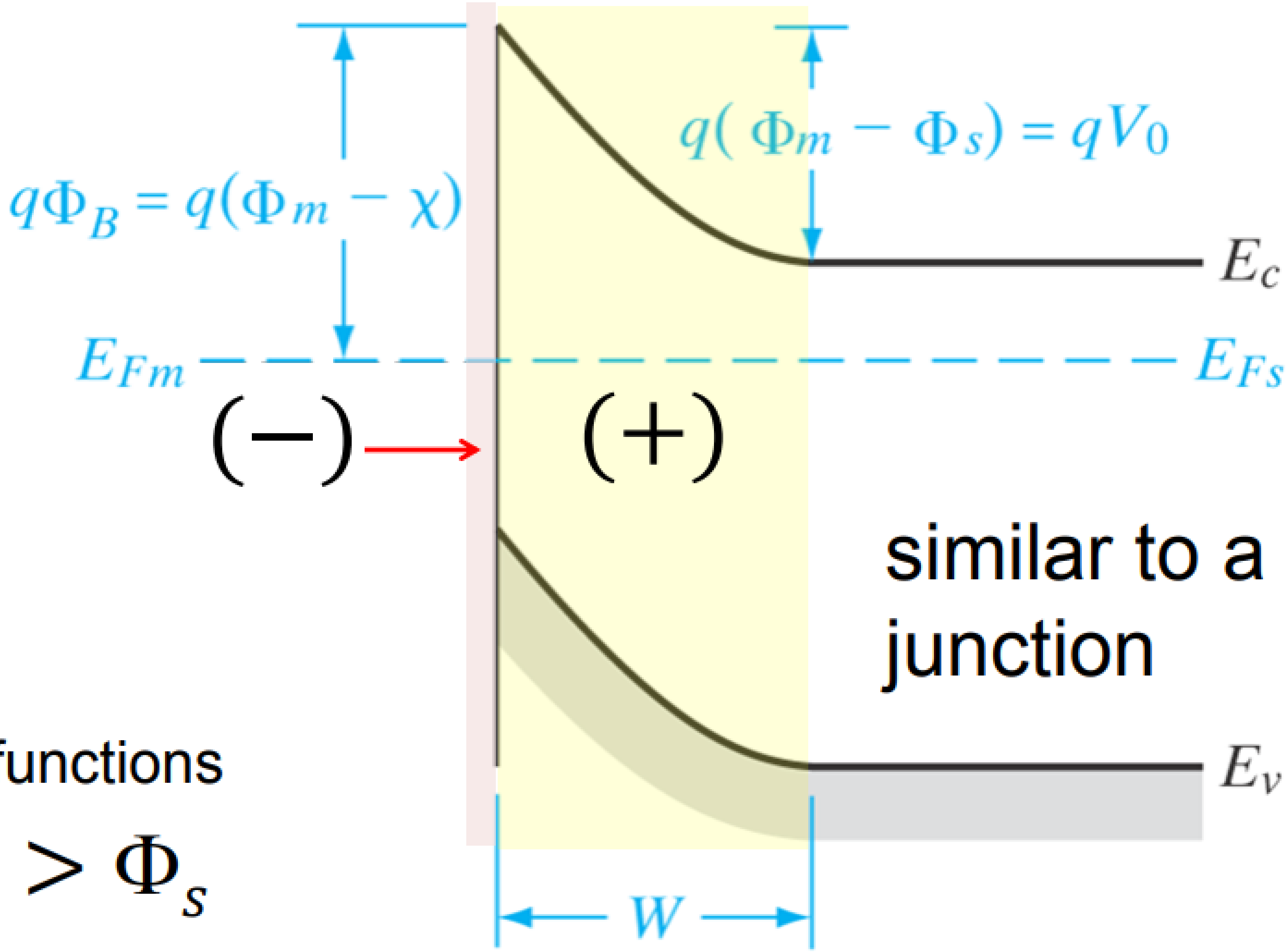
built-in potential



Equilibrium

Energy barrier electrons must overcome to go into semiconductor

Metal-Semiconductor Junction

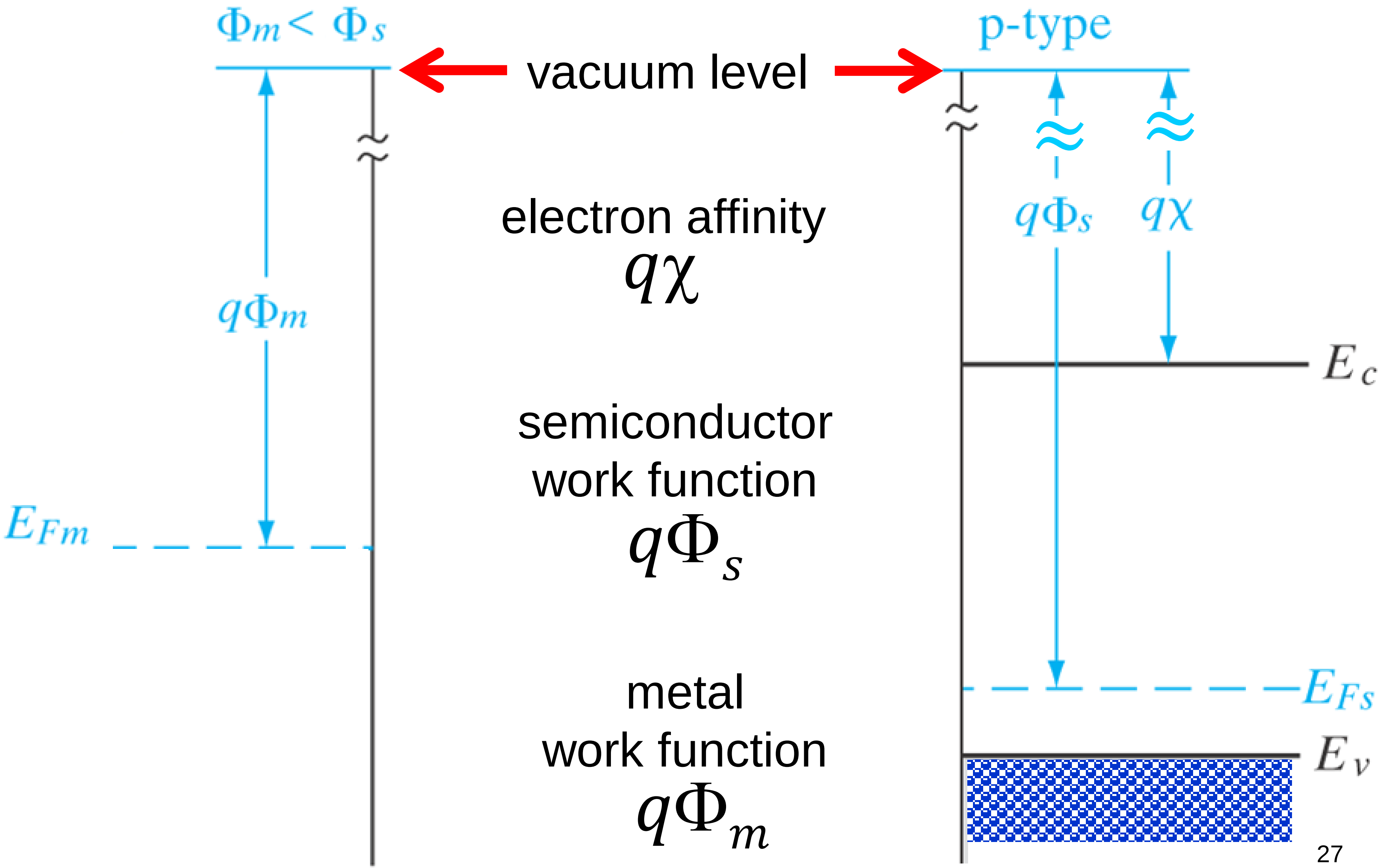


similar to a p^+-n junction

Workfunctions

$$\Phi_m > \Phi_s$$

Metal-Semiconductor Junction



vacuum level

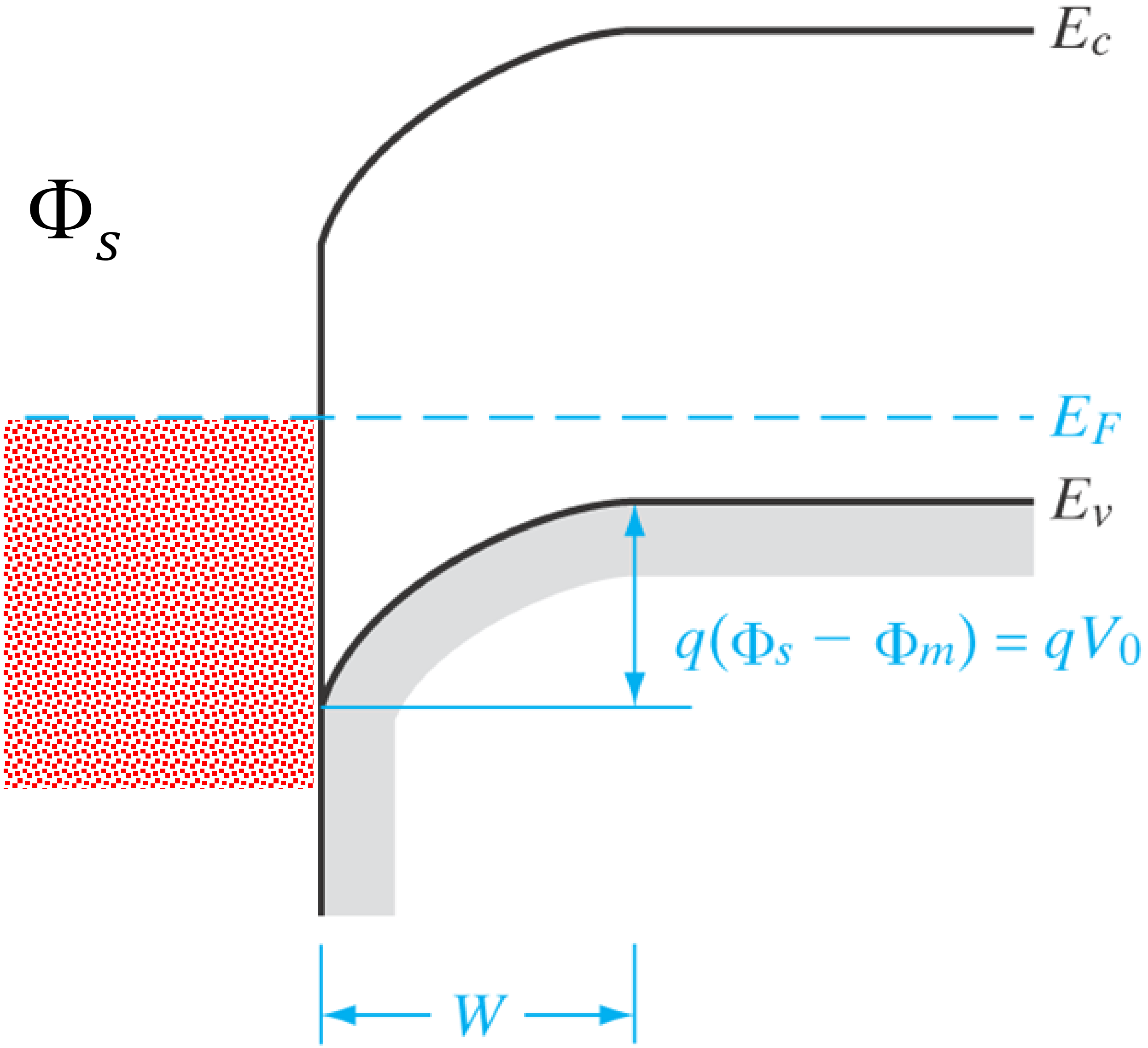
electron affinity
$q\chi$

semiconductor
work function
$q\Phi_s$

metal
work function
$q\Phi_m$

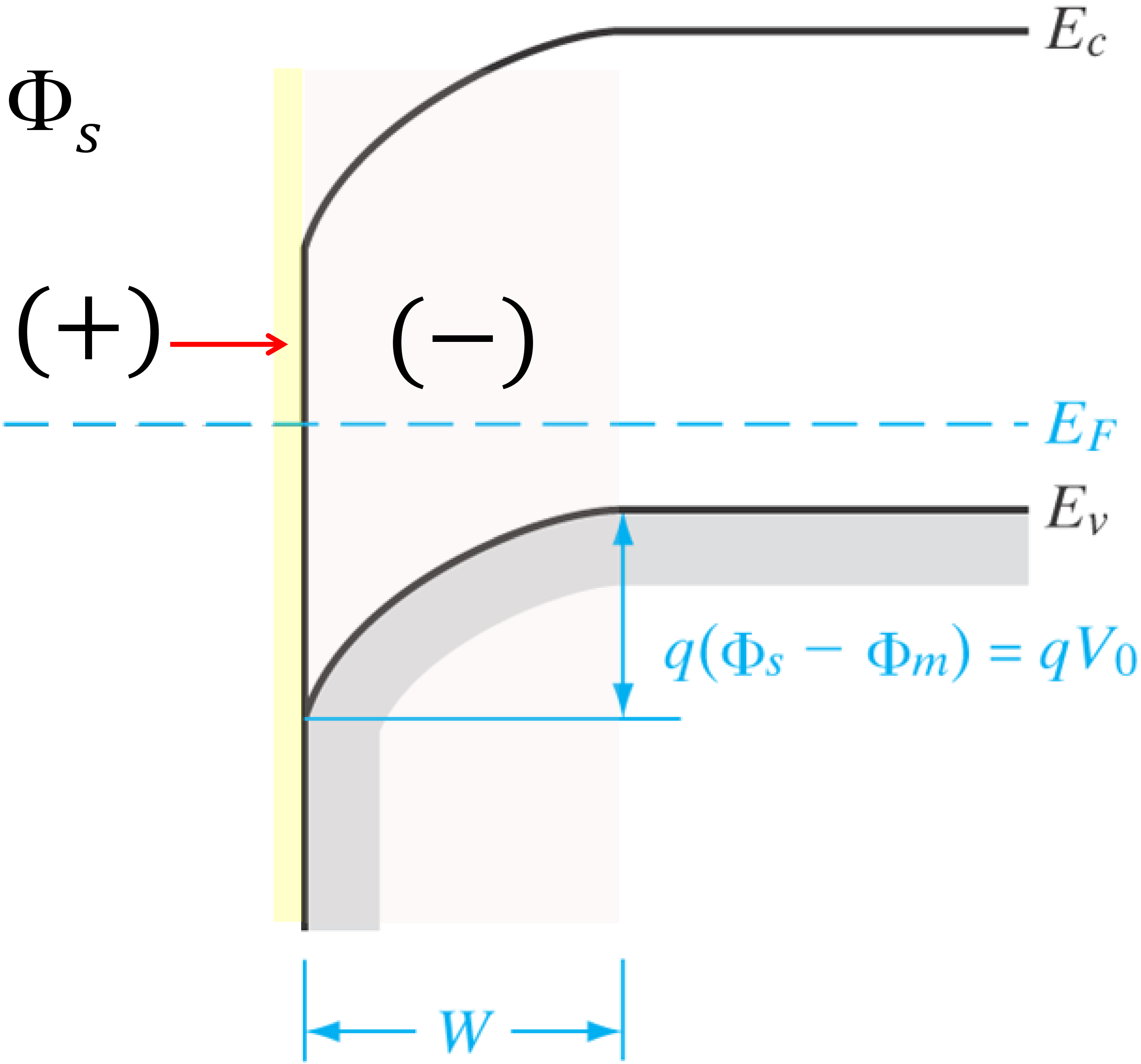
Metal-Semiconductor Junction

$$\Phi_m < \Phi_s$$



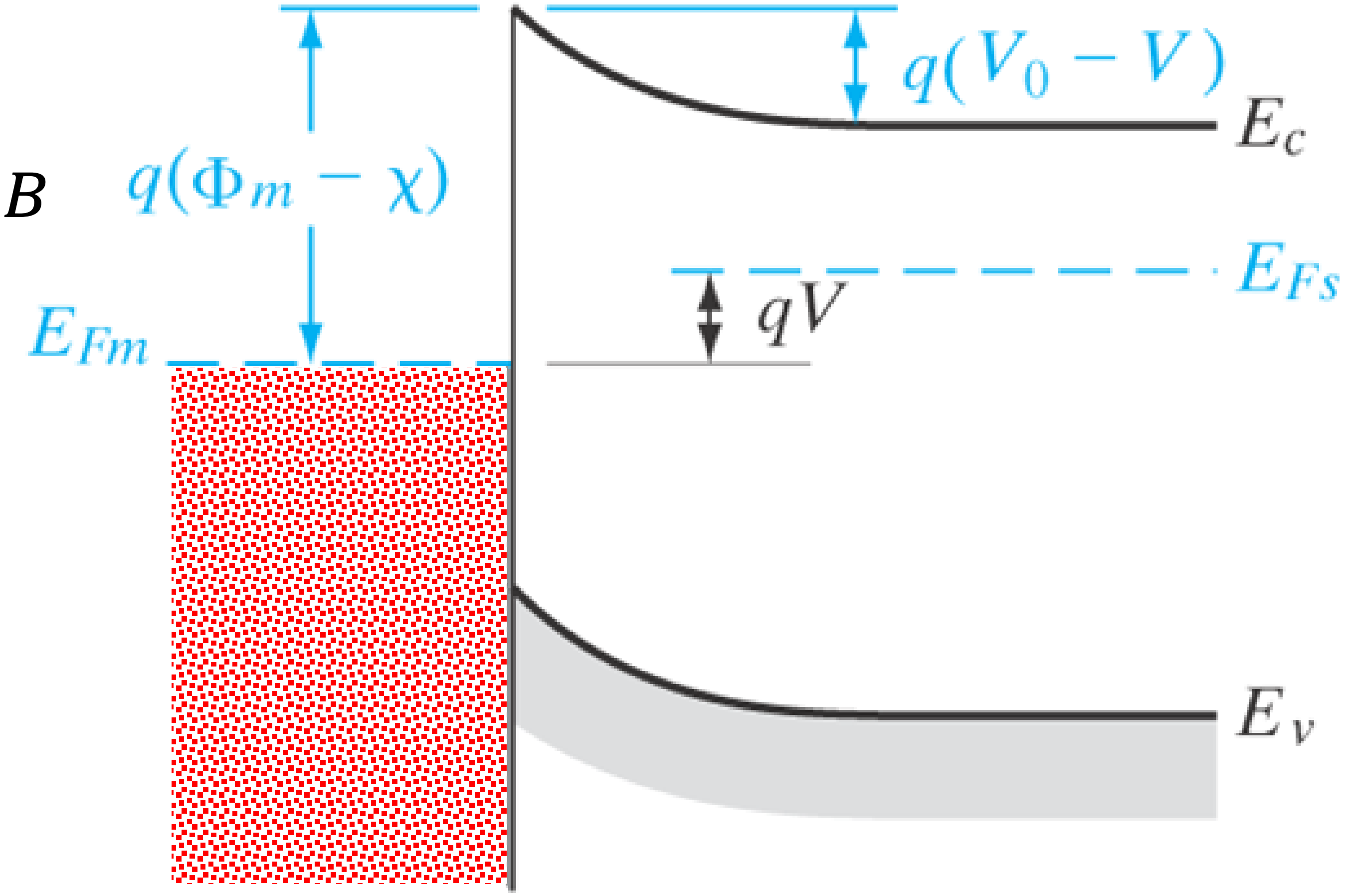
Metal-Semiconductor Junction

$\Phi_m < \Phi_s$



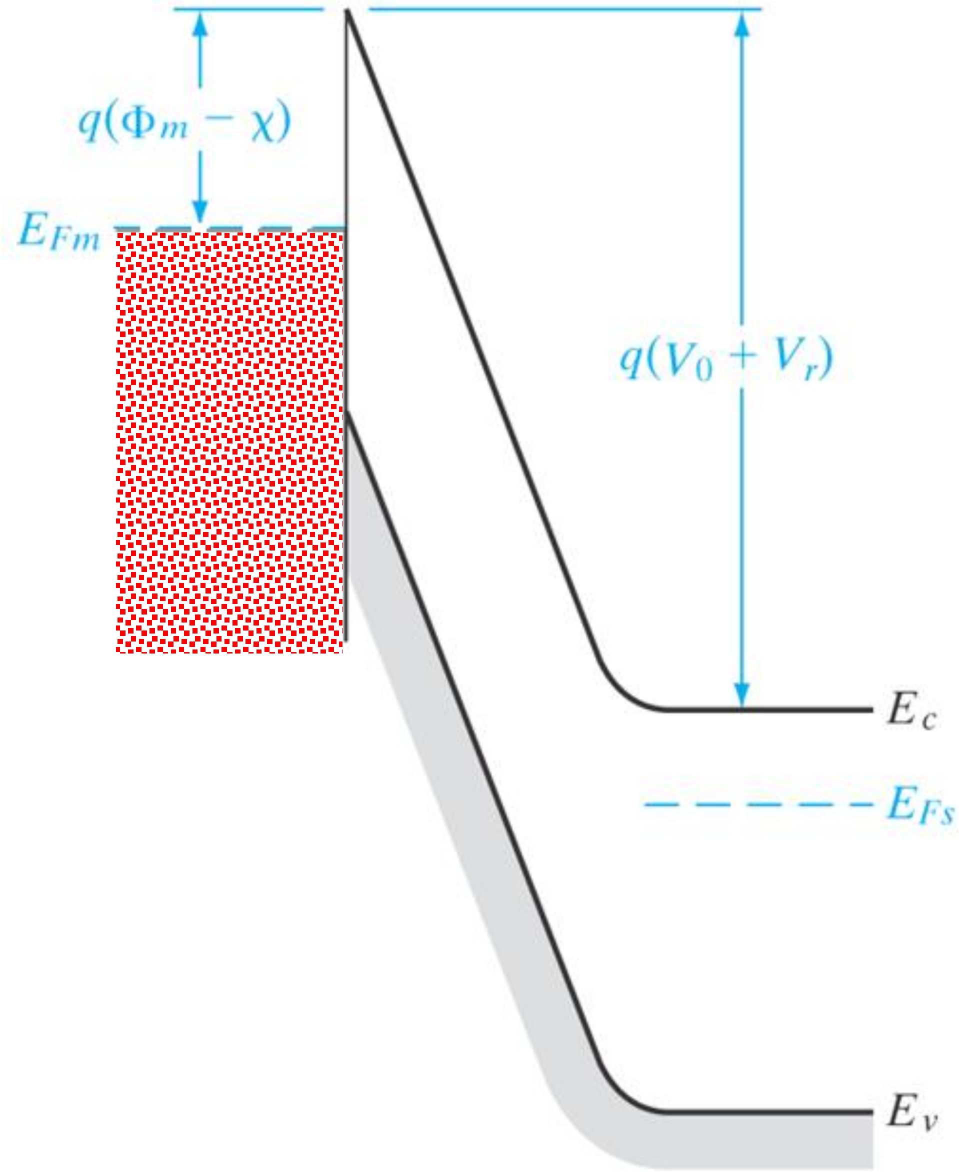
Forward Bias

$q\Phi_B$



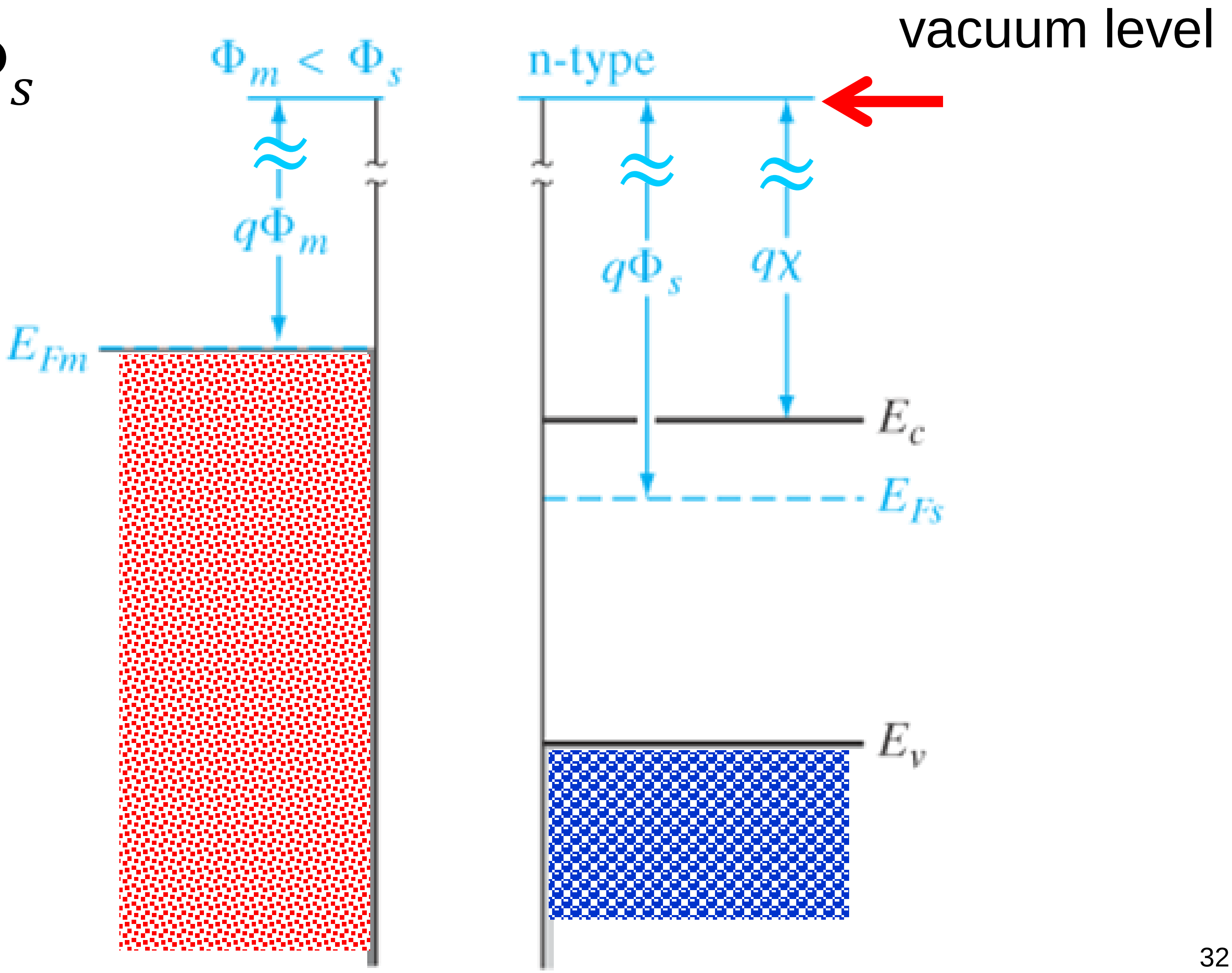
Reverse Bias

$q\Phi_B$



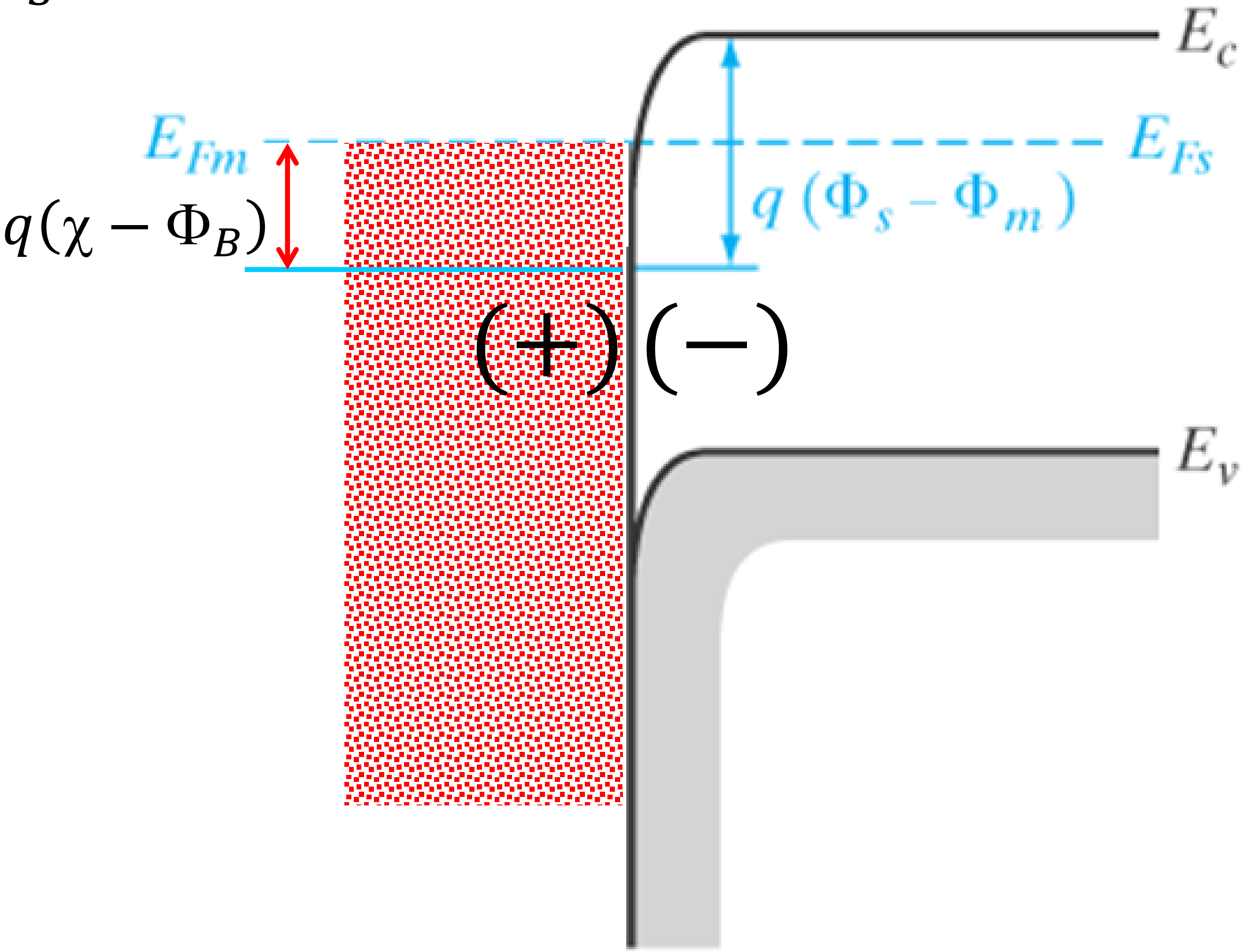
Ohmic contact

$\Phi_m < \Phi_s$



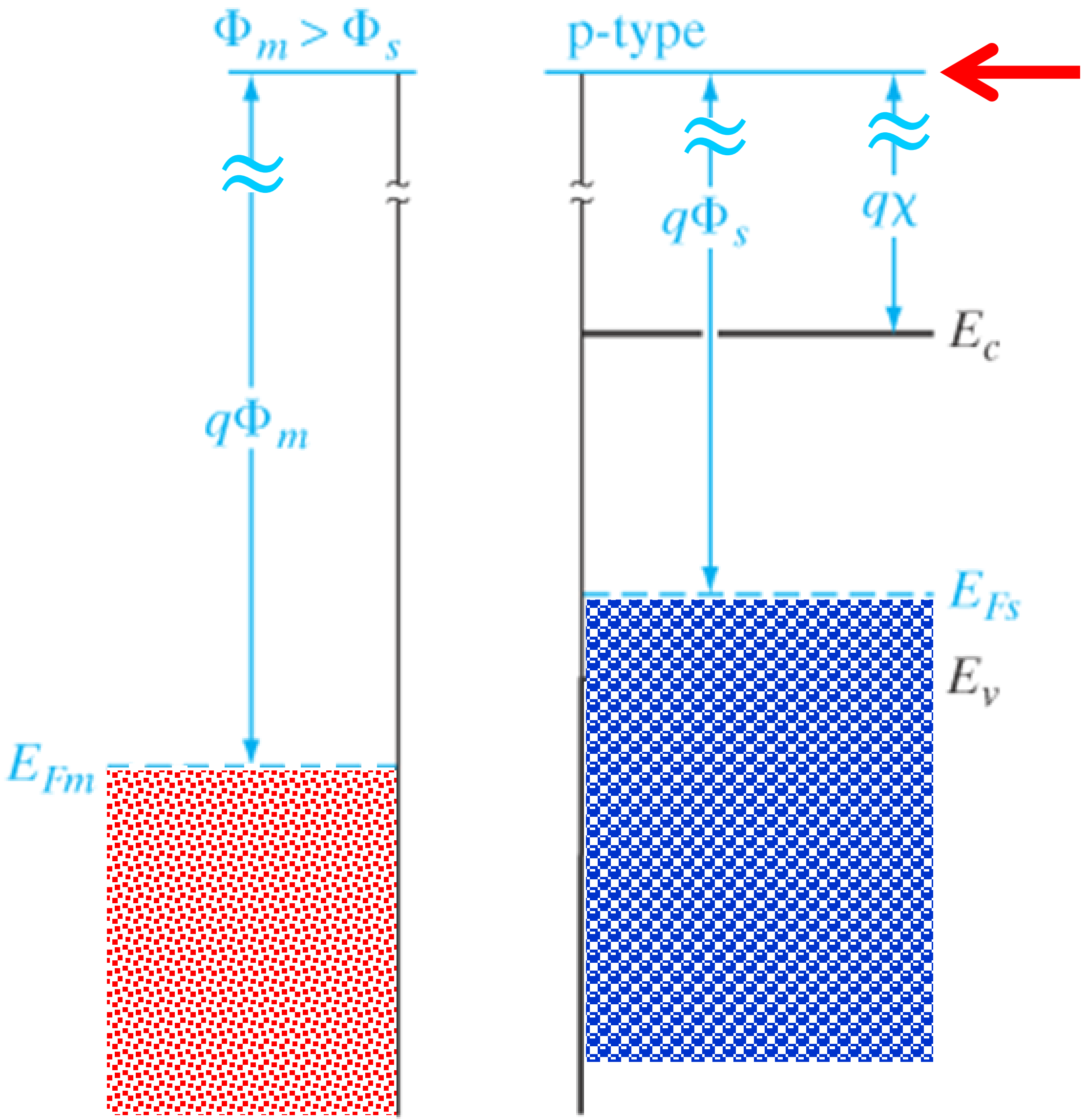
Ohmic contact

$\Phi_m < \Phi_s$



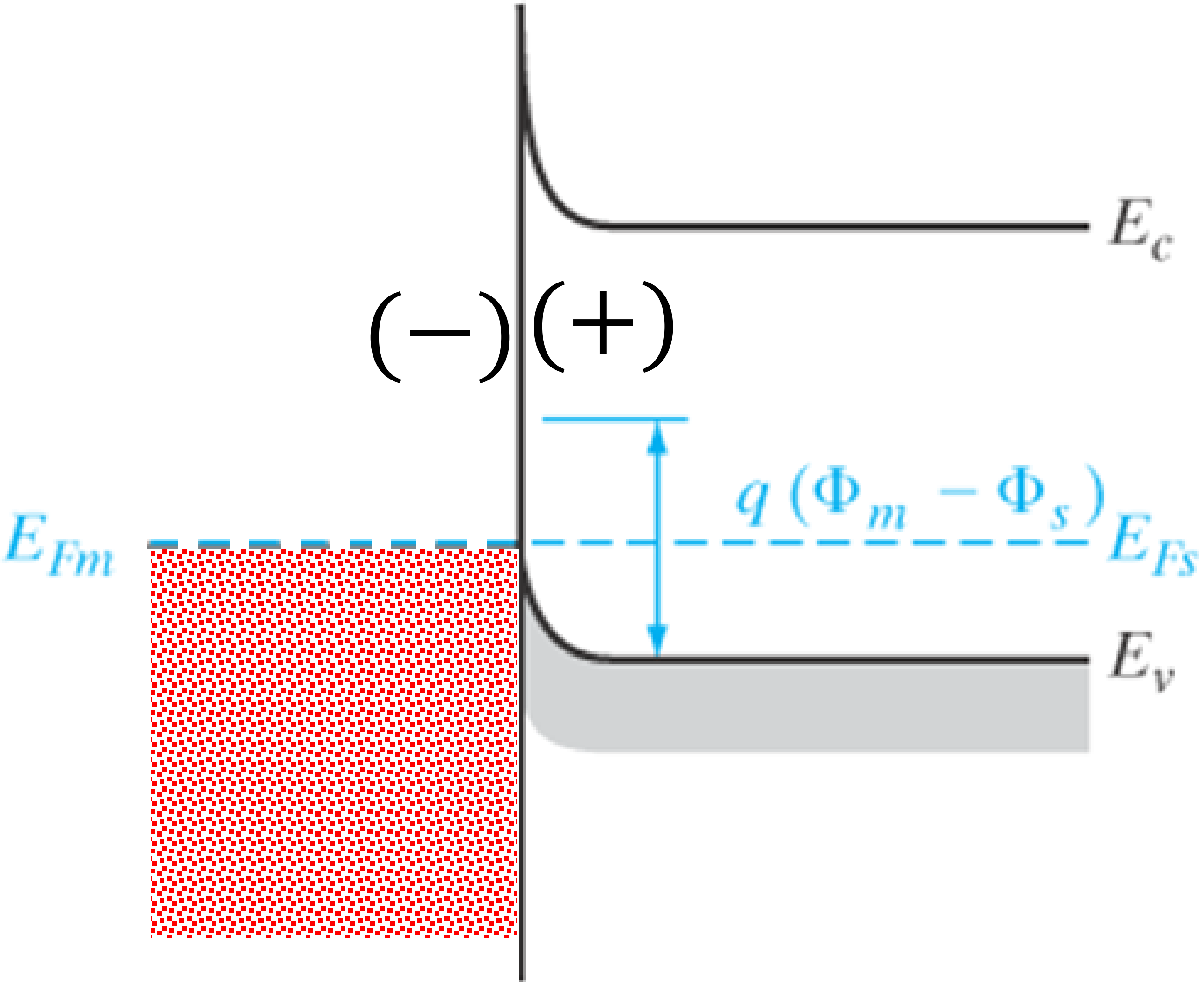
Ohmic contact

$\Phi_m > \Phi_s$

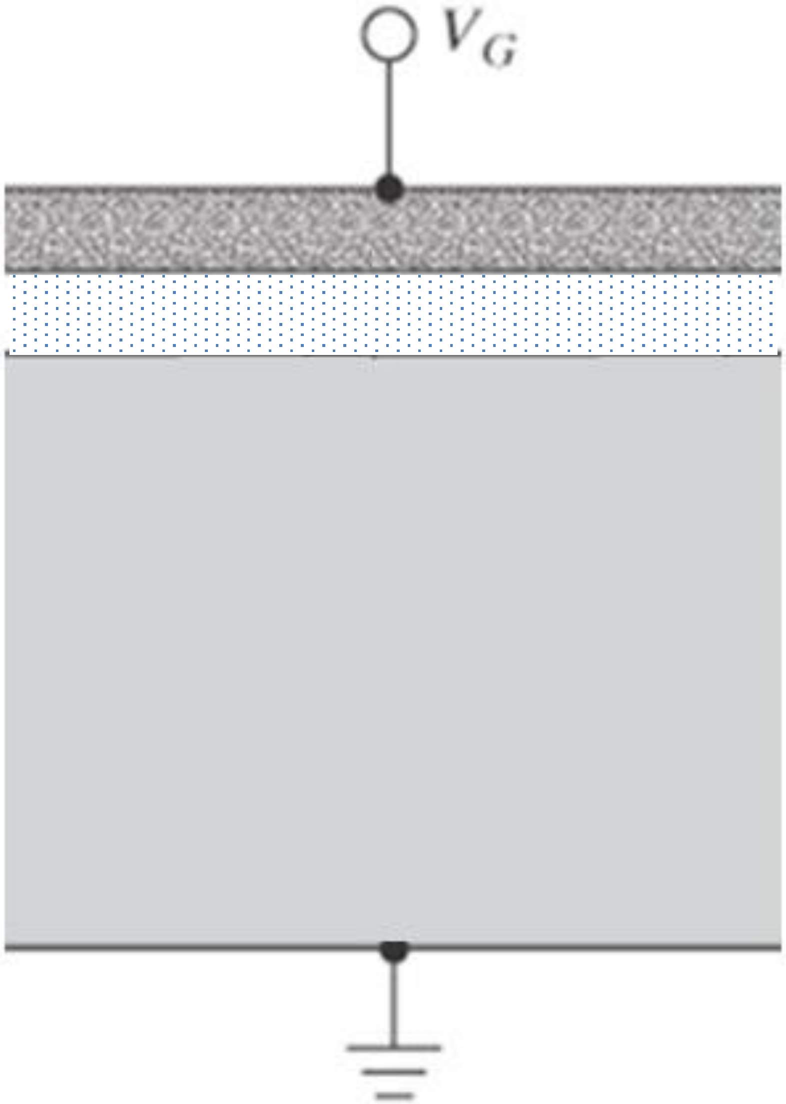


Ohmic contact

$\Phi_m > \Phi_s$



Ideal MOSFET Capacitor



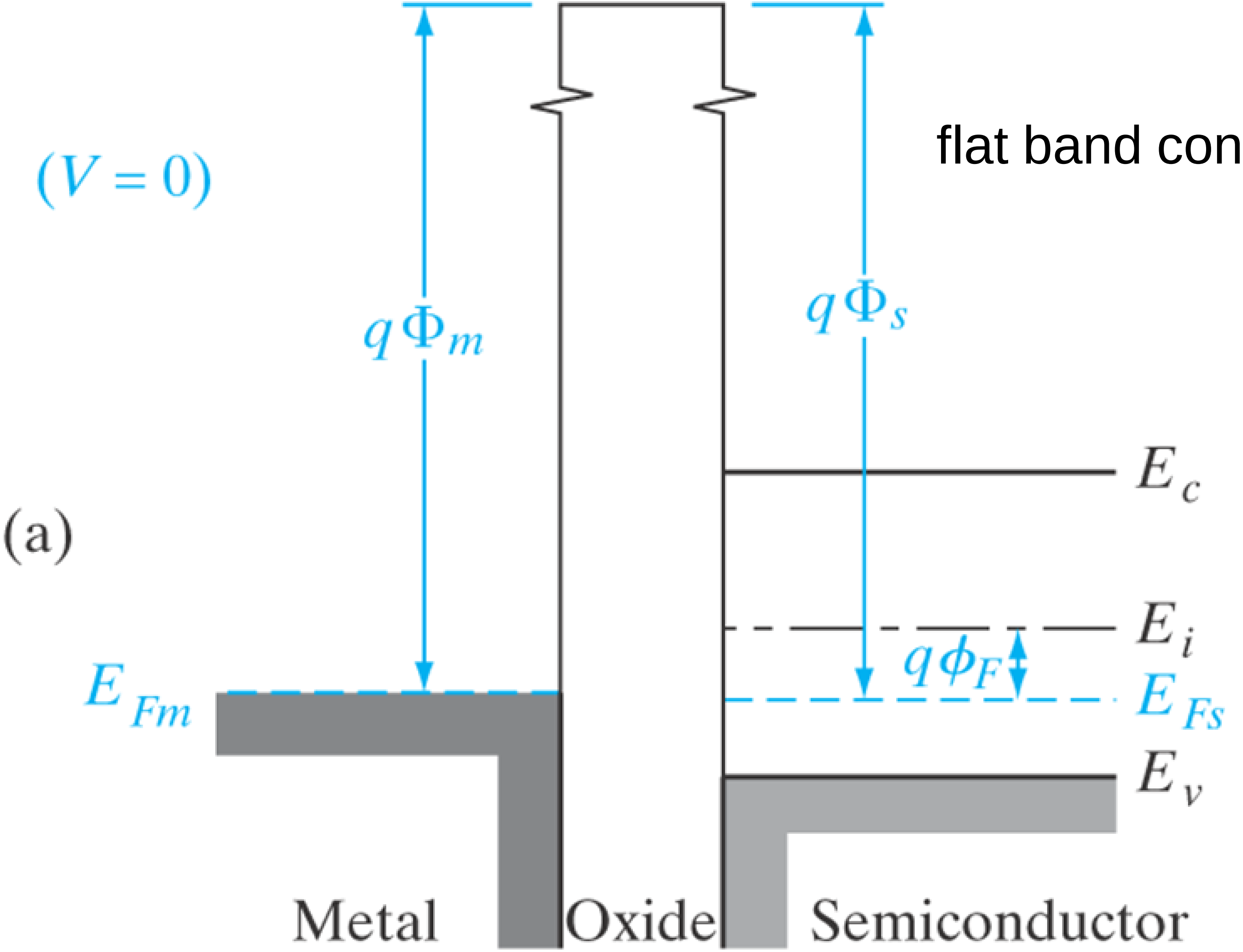
gate contact

oxide

Silicon bulk

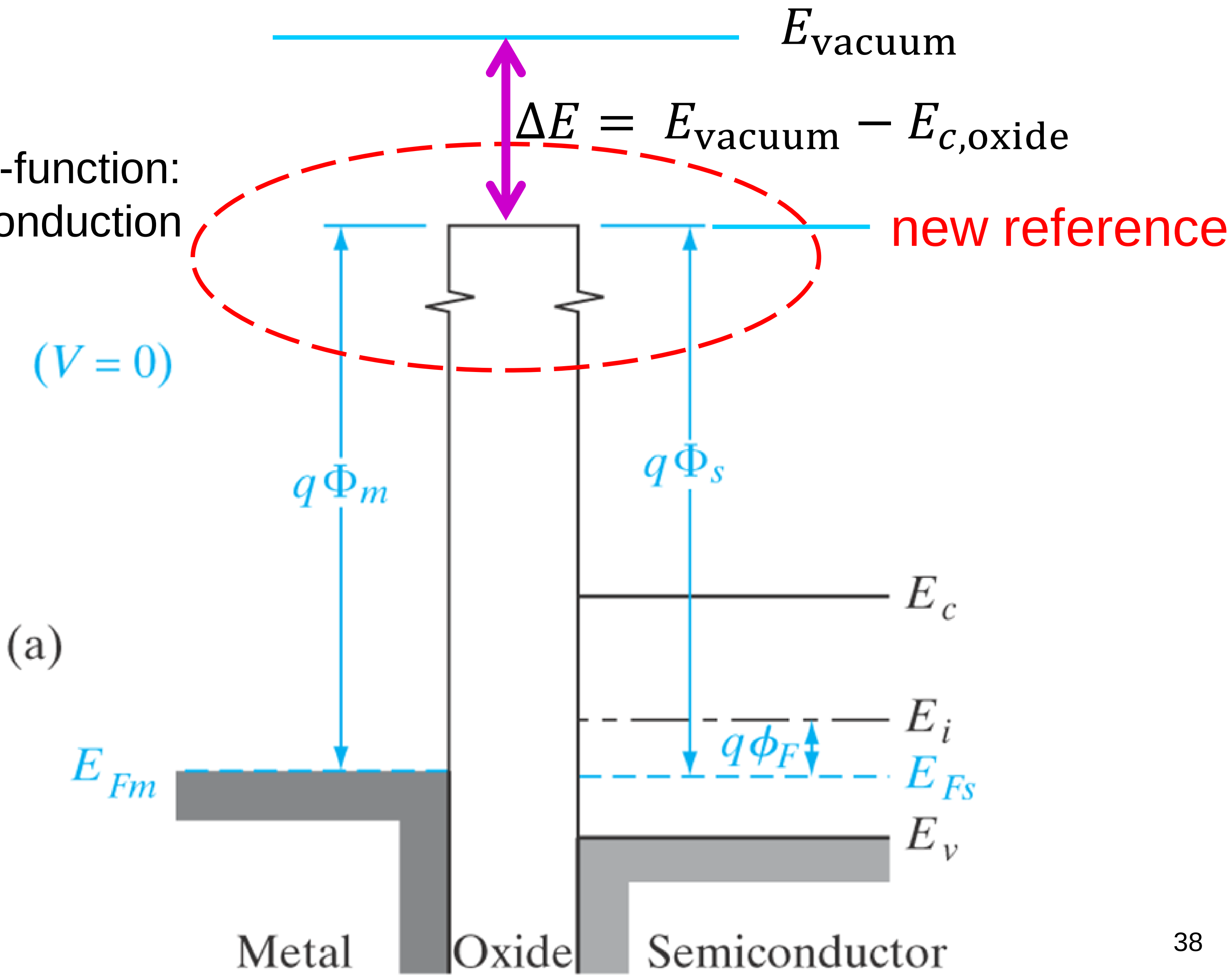
substrate
contact

Ideal MOSFET Capacitor (Equilibrium)



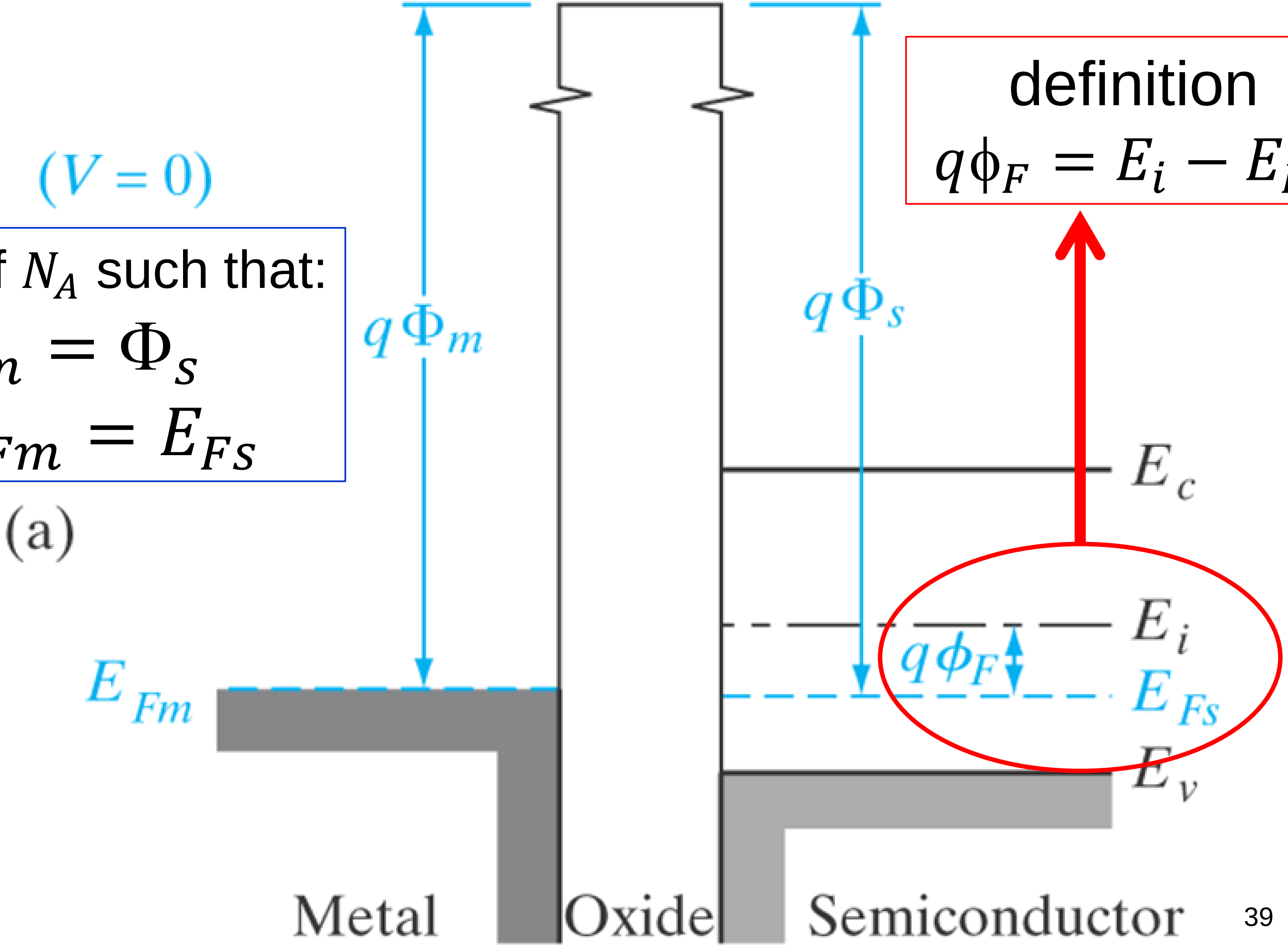
flat band condition

Ideal MOSFET Capacitor (Equilibrium)



Modified work-function: reference is conduction band of oxide

Ideal MOSFET Capacitor (Equilibrium)



choice of N_A such that:

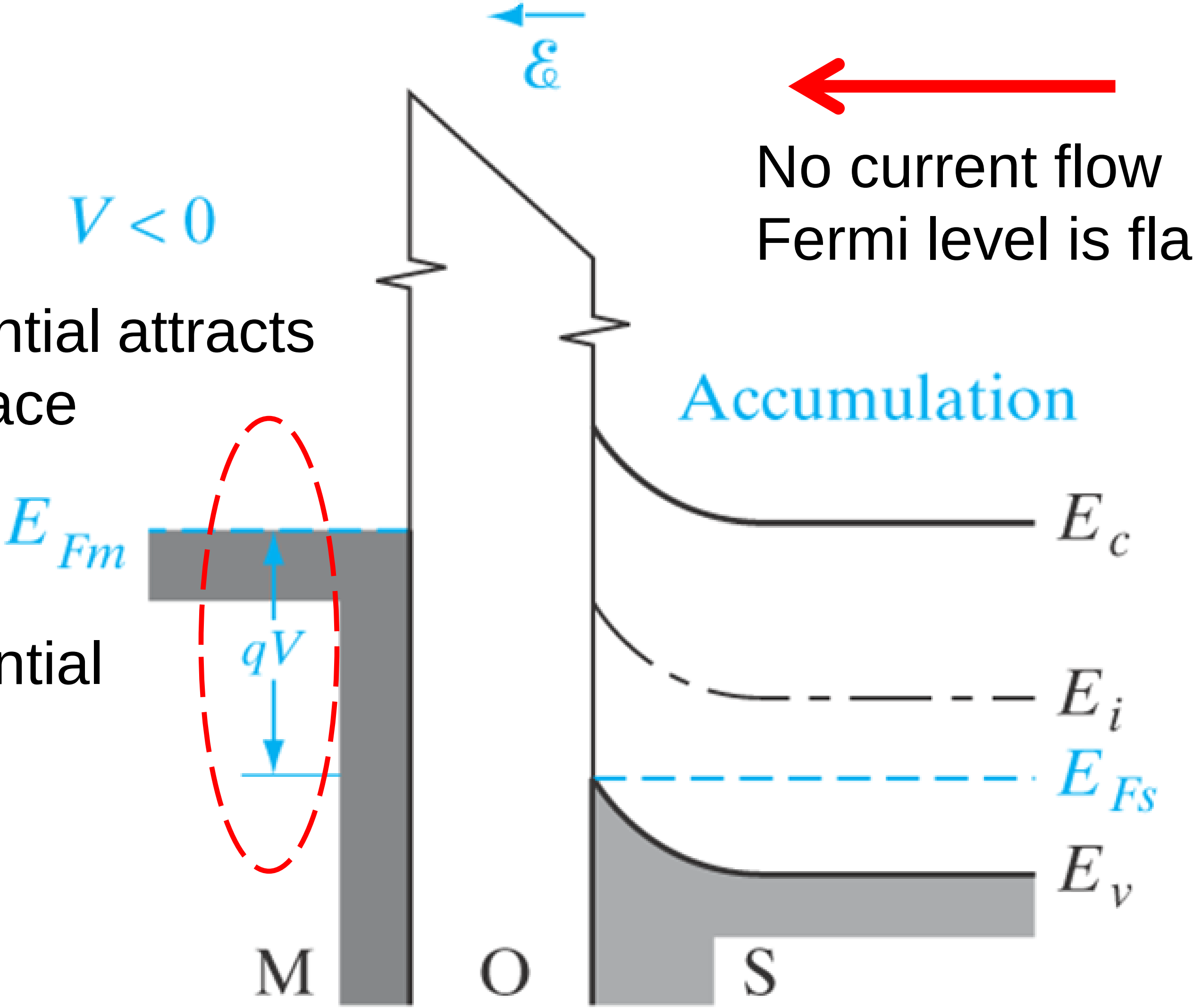
$$\Phi_m = \Phi_s$$

$$\rightarrow E_{Fm} = E_{Fs}$$

definition

$$q\phi_F = E_i - E_{Fs}$$

Ideal MOSFET Capacitor (Accumulation)

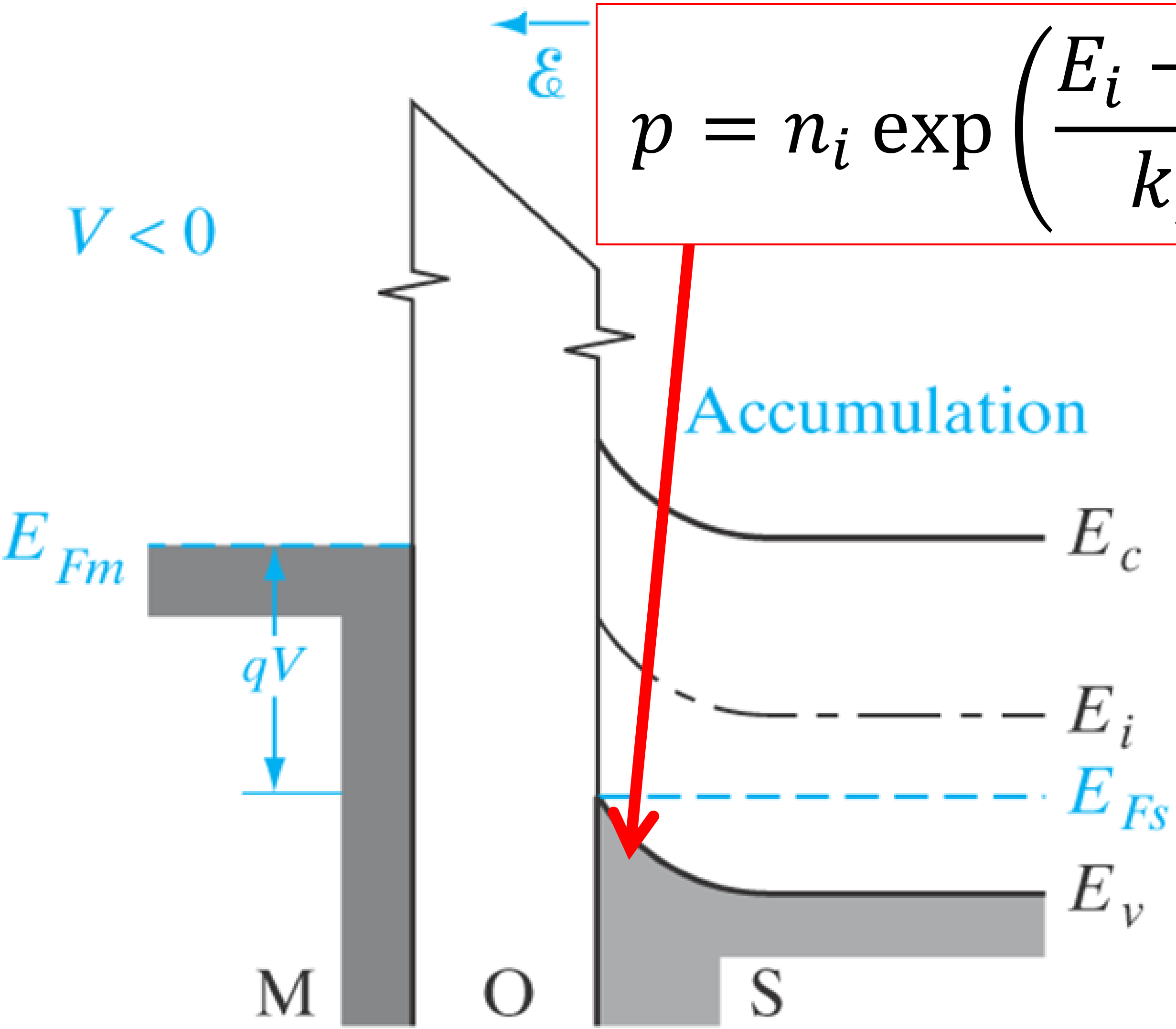


No current flow
Fermi level is flat

Negative potential attracts holes to interface

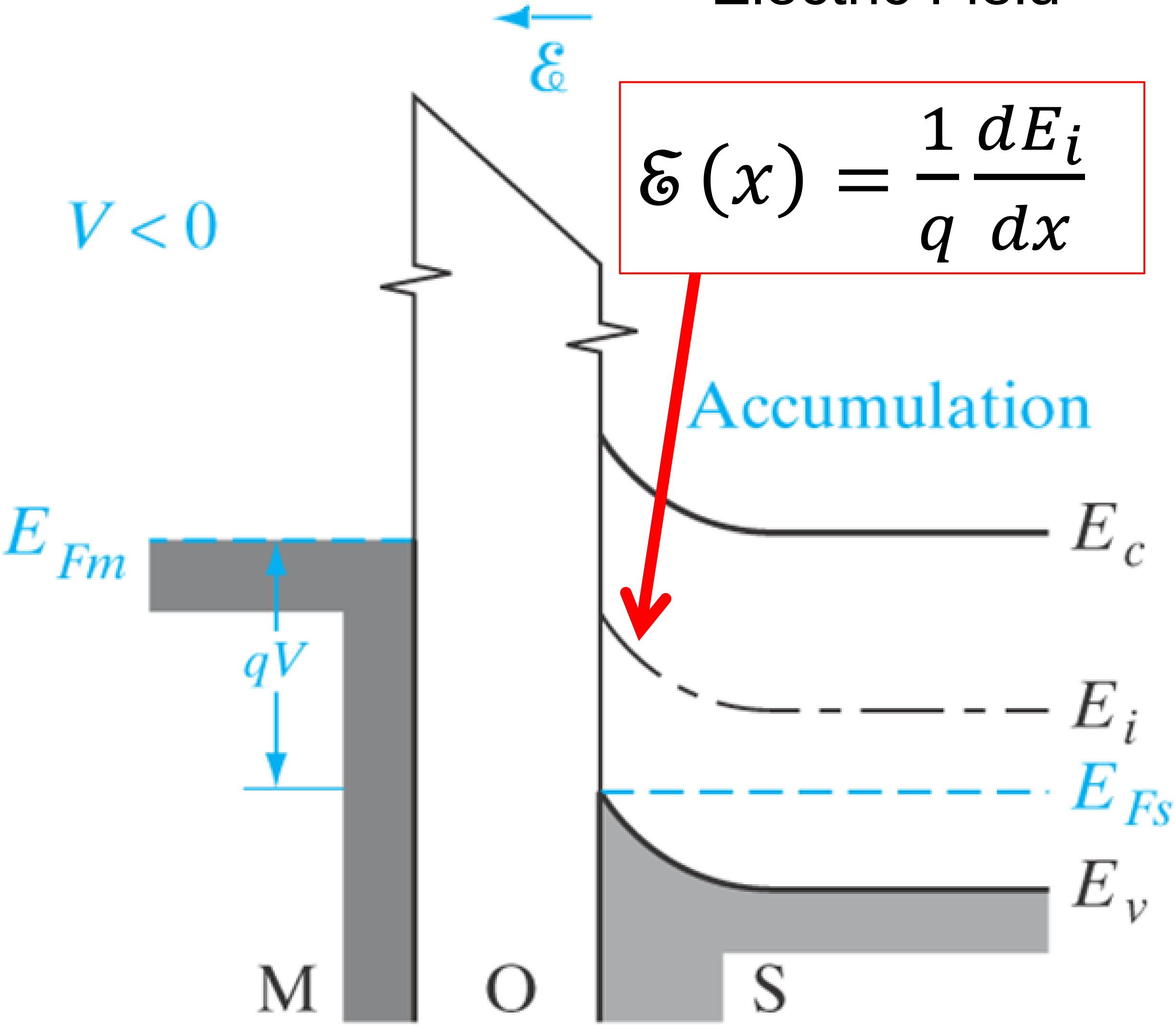
positive potential energy

Ideal MOSFET Capacitor (Accumulation)



$$p = n_i \exp\left(\frac{E_i - E_{Fs}}{k_B T}\right)$$

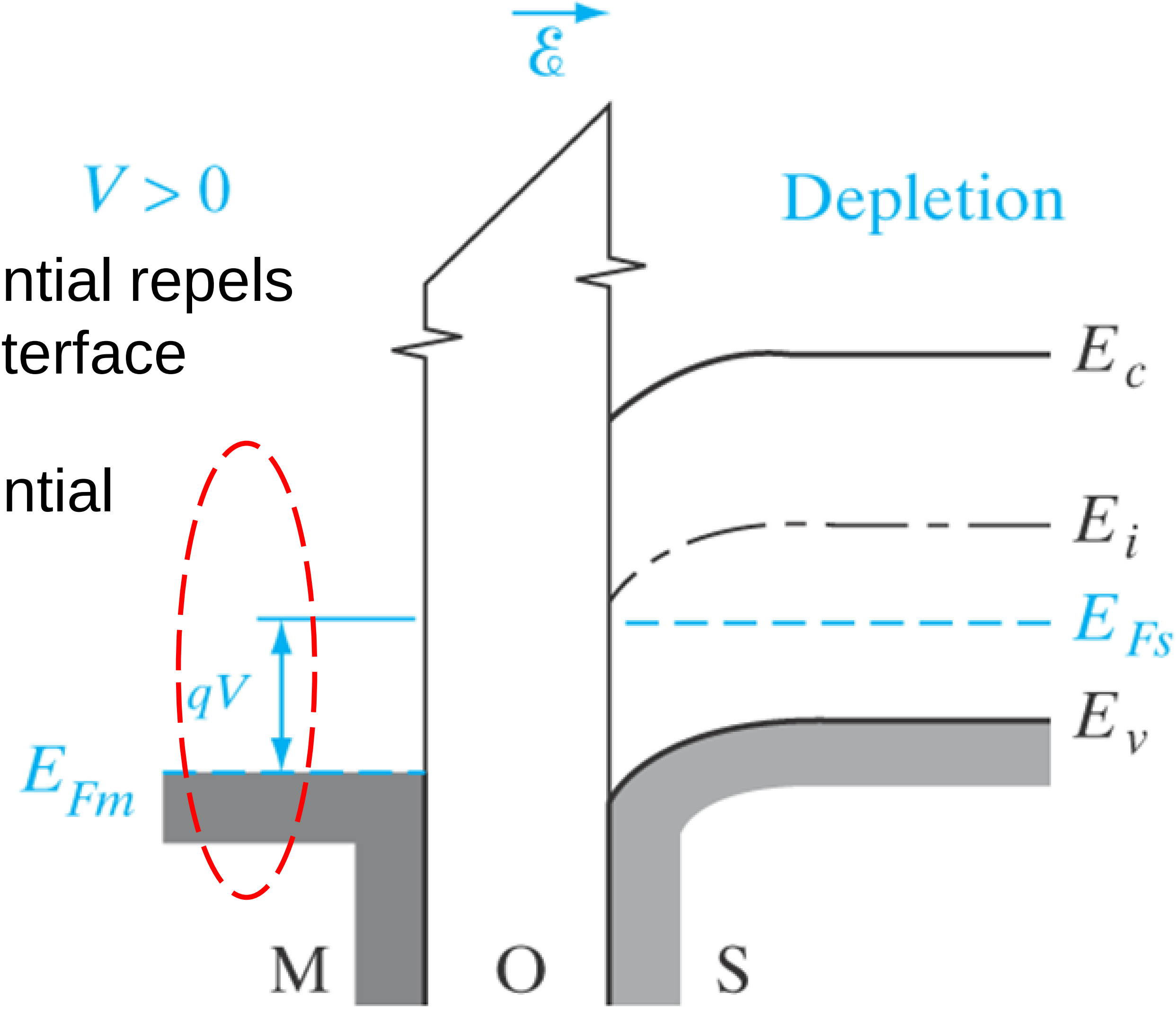
Ideal MOSFET Capacitor (Accumulation)



Electric Field

$$\mathcal{E}(x) = \frac{1}{q}\frac{dE_i}{dx}$$

Ideal MOSFET Capacitor (Depletion)

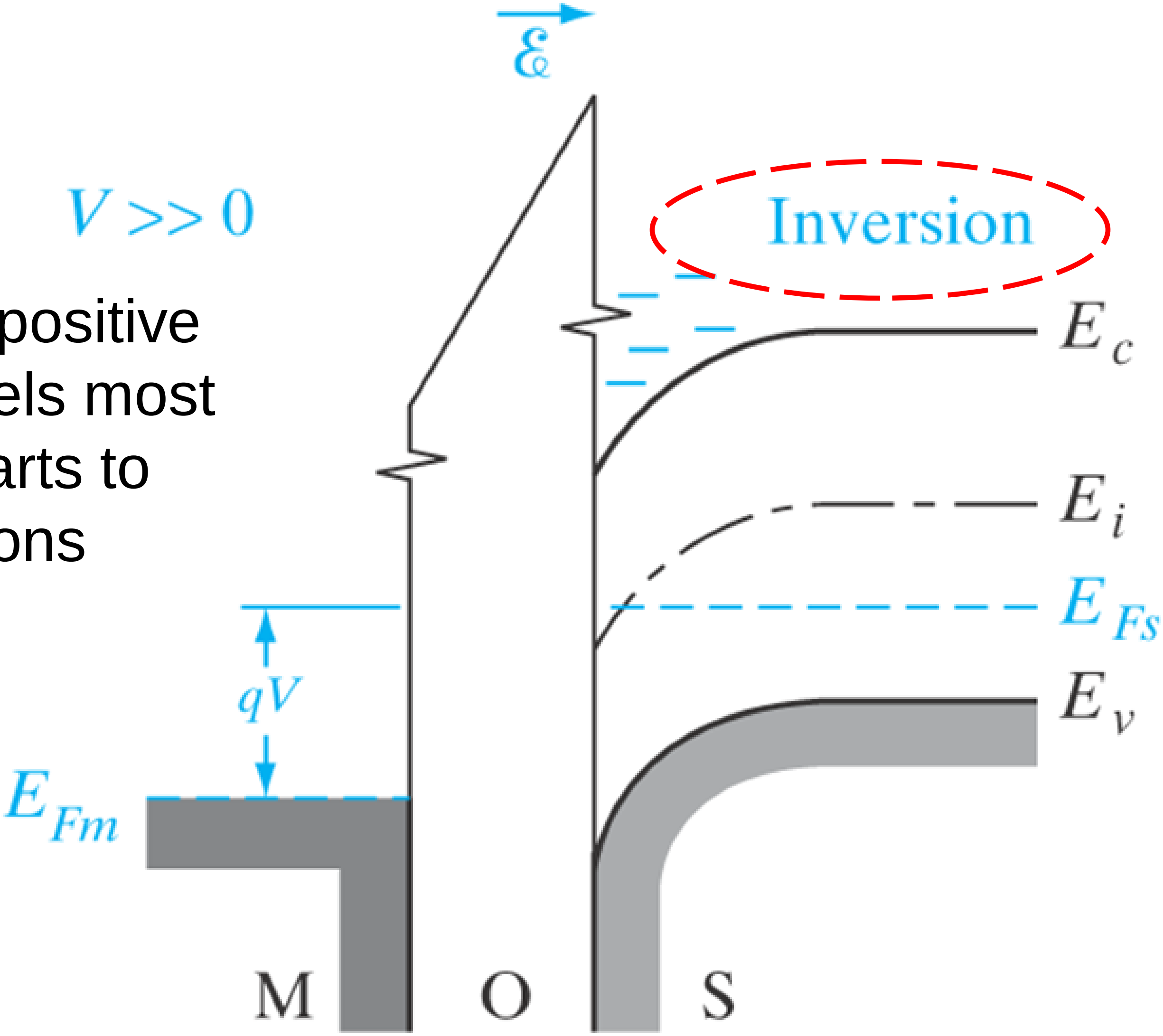


Positive potential repels holes from interface

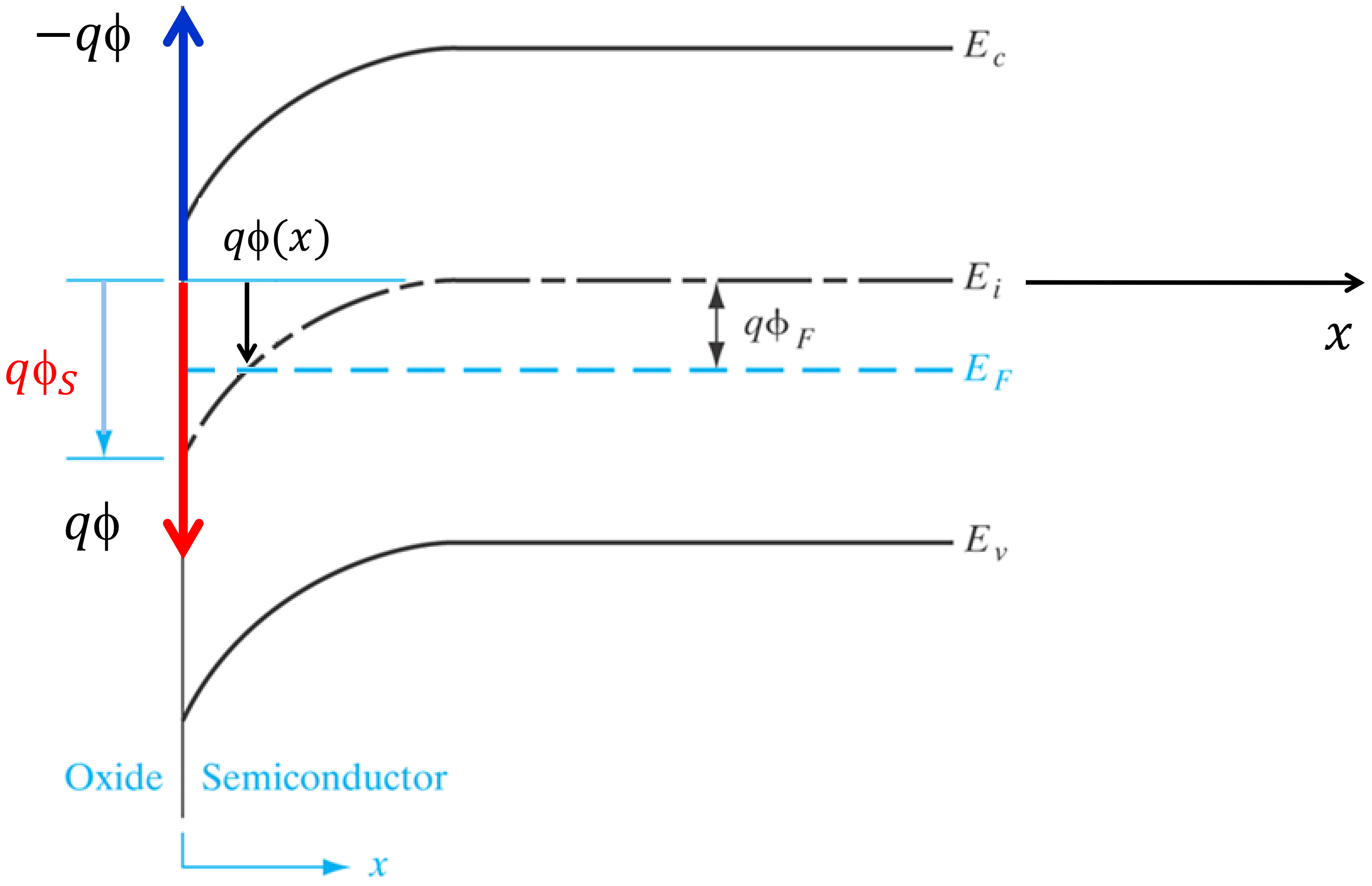
negative potential energy

Ideal MOSFET Capacitor (Inversion)

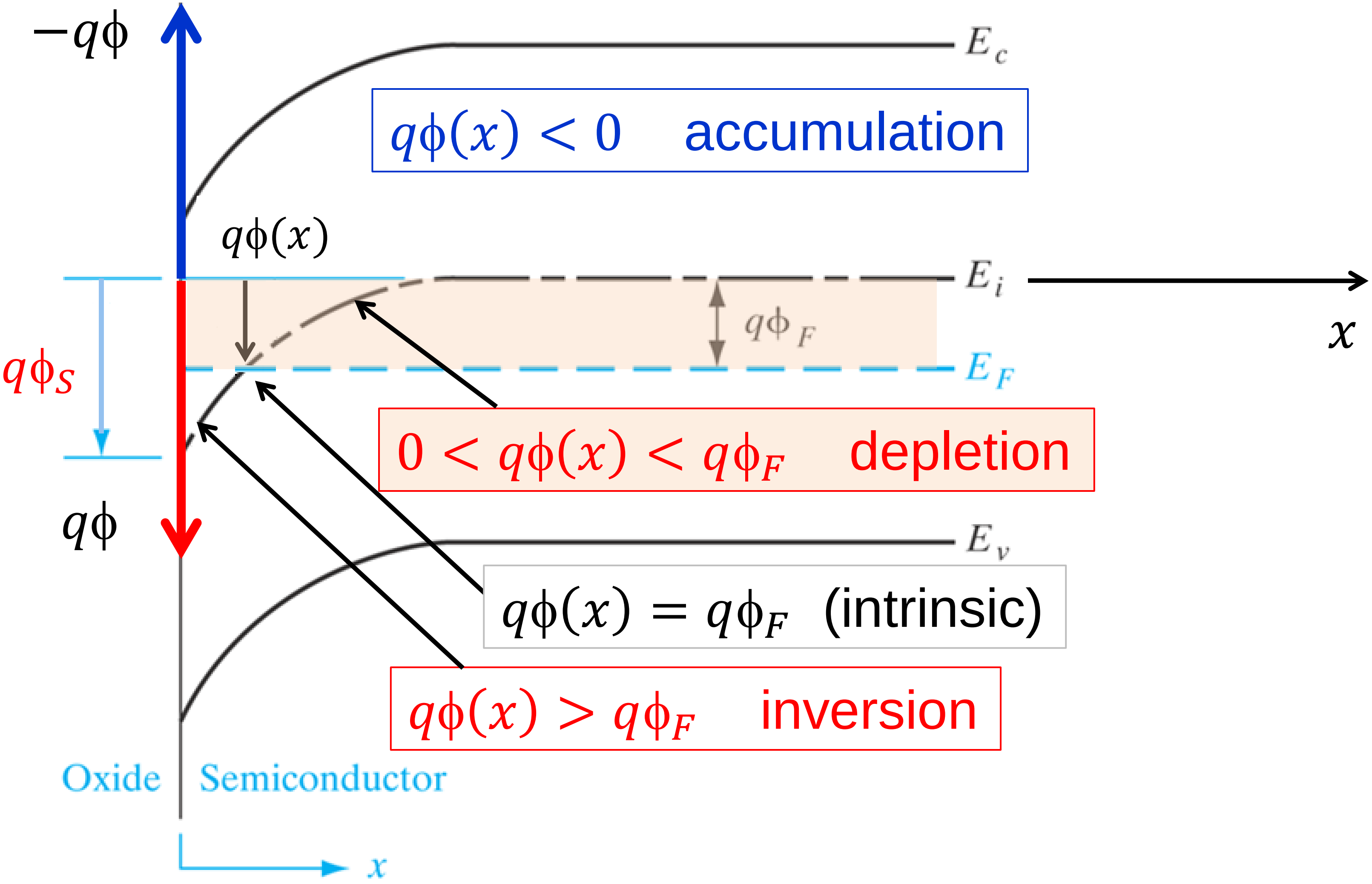
Even higher positive potential repels most holes and starts to attract electrons



Potential energy system of reference



Potential energy system of reference



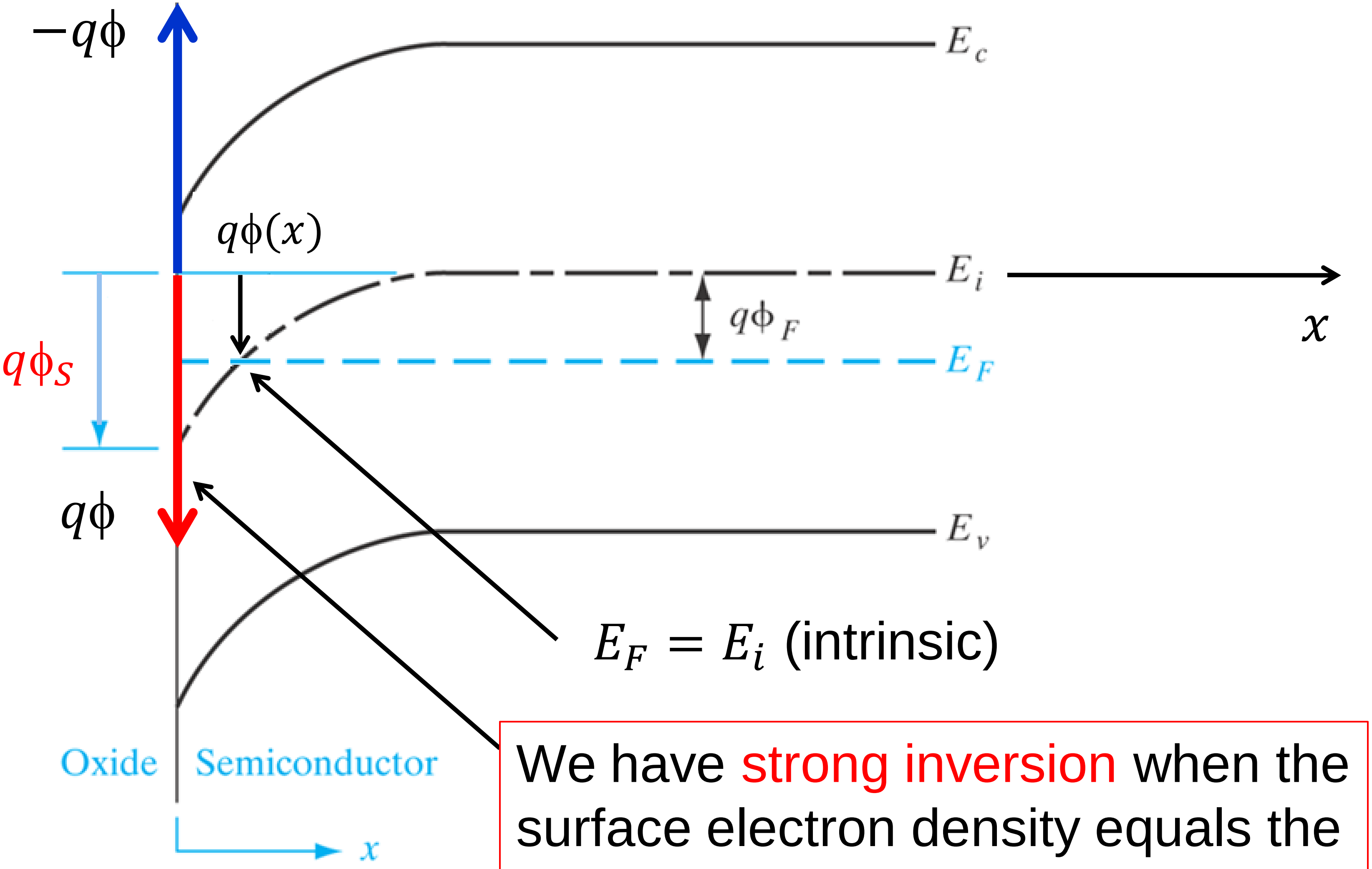
$q\phi(x) < 0$ accumulation

$0 < q\phi(x) < q\phi_F$ depletion

$q\phi(x) = q\phi_F$ (intrinsic)

$q\phi(x) > q\phi_F$ inversion

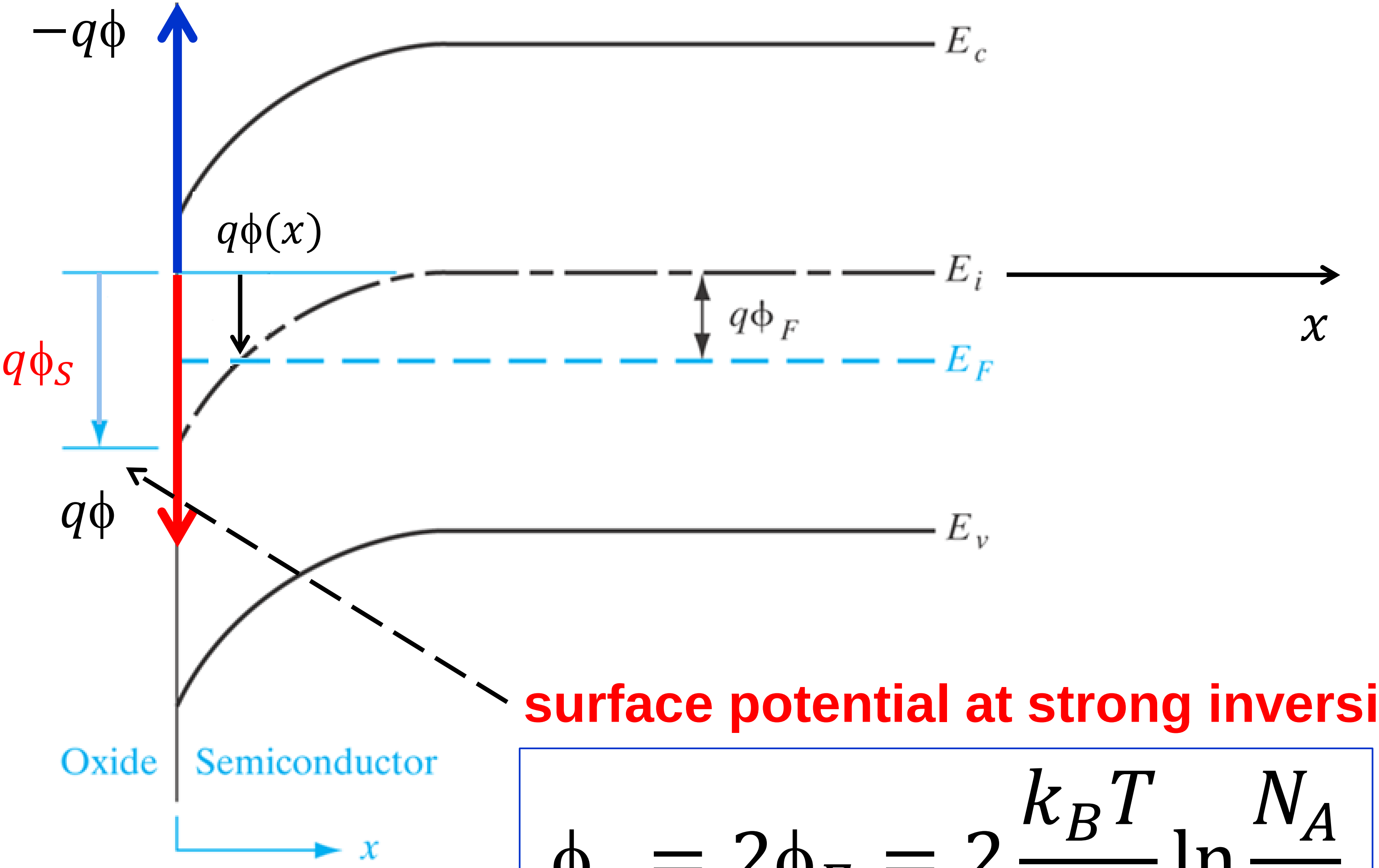
Strong inversion condition (definition)



$E_F = E_i$ (intrinsic)

We have strong inversion when the surface electron density equals the hole density in the bulk

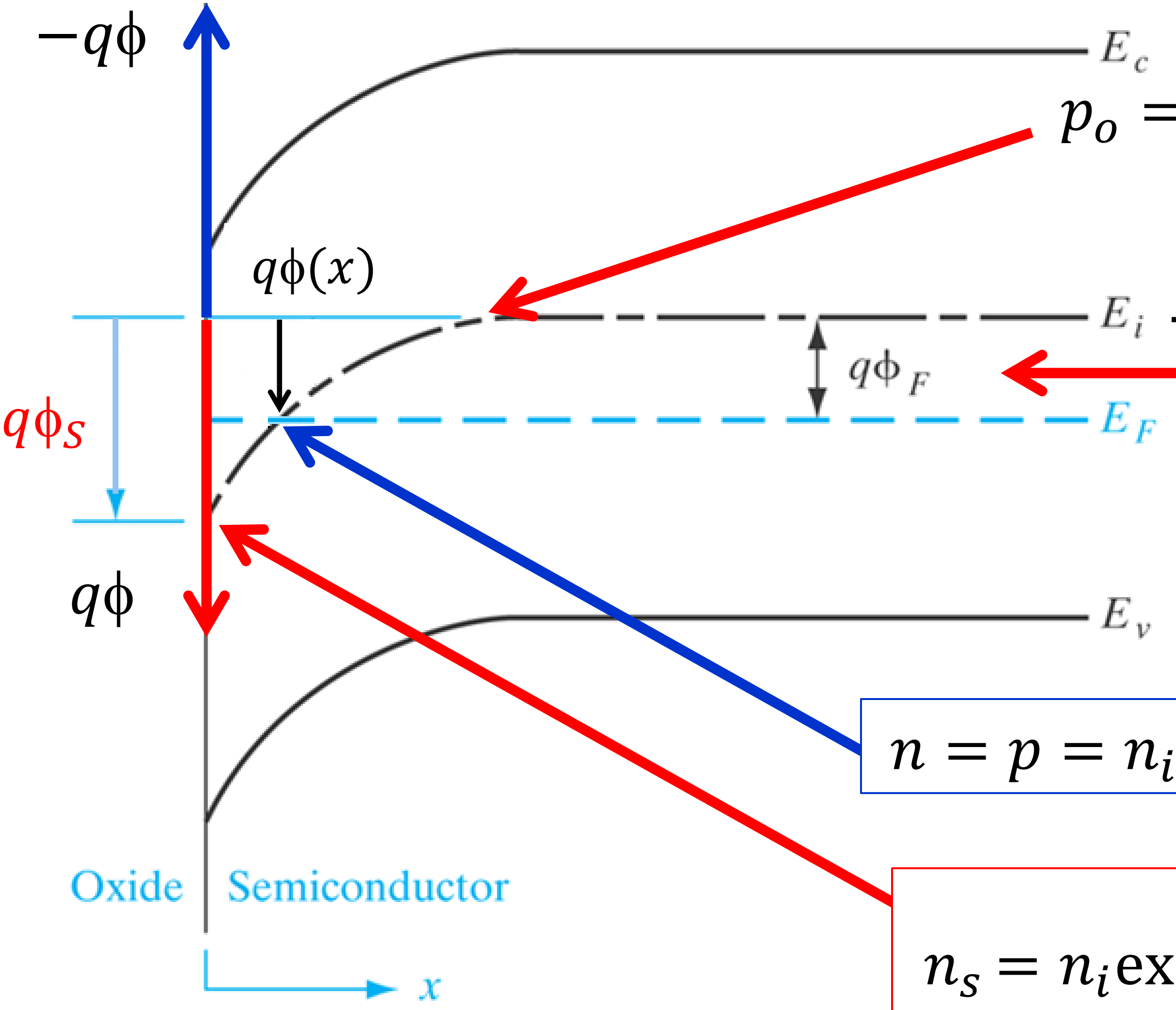
Strong inversion condition (definition)



surface potential at strong inversion

$$\boxed{\phi_s = 2\phi_F = 2\frac{k_B T}{q}\ln\frac{N_A}{n_i}}$$

Strong inversion condition



$$p_o = N_A = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)$$

$$q\phi_F = (E_i - E_F) = \\ = k_B T \ln \frac{N_A}{n_i}$$

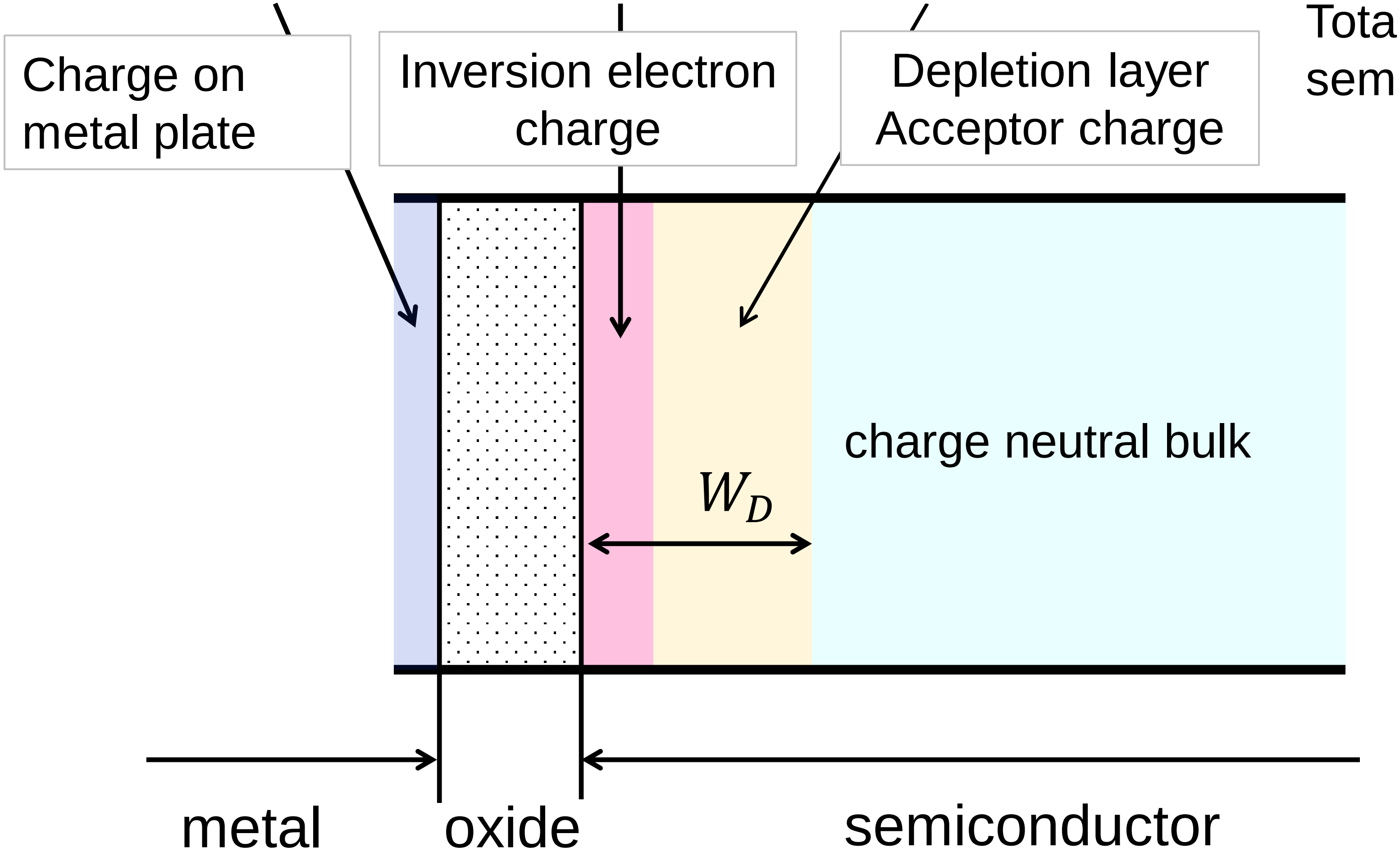
$$n = p = n_i$$

$$n_s = n_i \exp\left(\frac{E_F - E_i(0)}{k_B T}\right) = p_o$$

Charge density distribution

$$Q_M = -(Q_n + qN_A W_D) = -Q_S$$

Total charge in semiconductor



Charge on metal plate

Inversion electron charge

Depletion layer Acceptor charge

charge neutral bulk

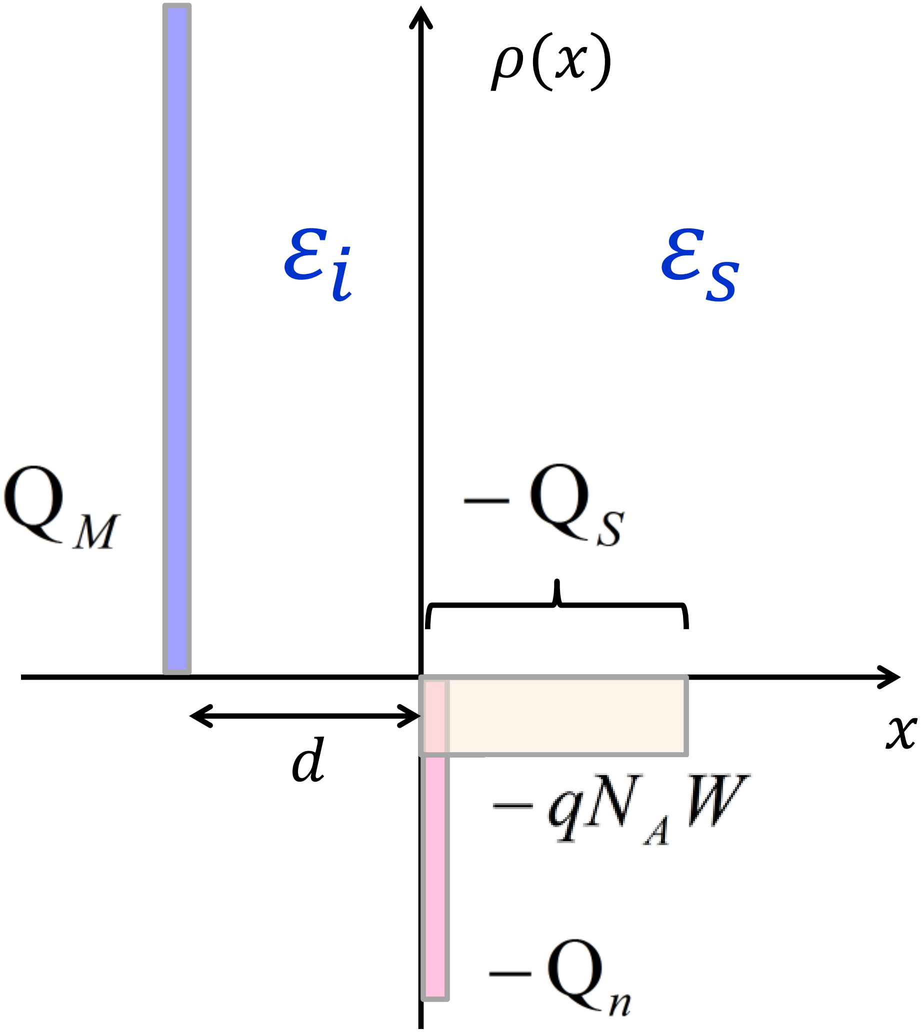
W_D

metal

oxide

semiconductor

Charge density distribution



d = thickness of oxide

Oxide capacitance (unit area)

$$C_i = \frac{\varepsilon_i}{d}$$

Voltage across oxide

$$V_i = \frac{-Q_s}{C_i} = \frac{-Q_s d}{\varepsilon_i}$$

Applied voltage

$$V = V_i + \phi_s$$

Threshold Voltage (ideal case)

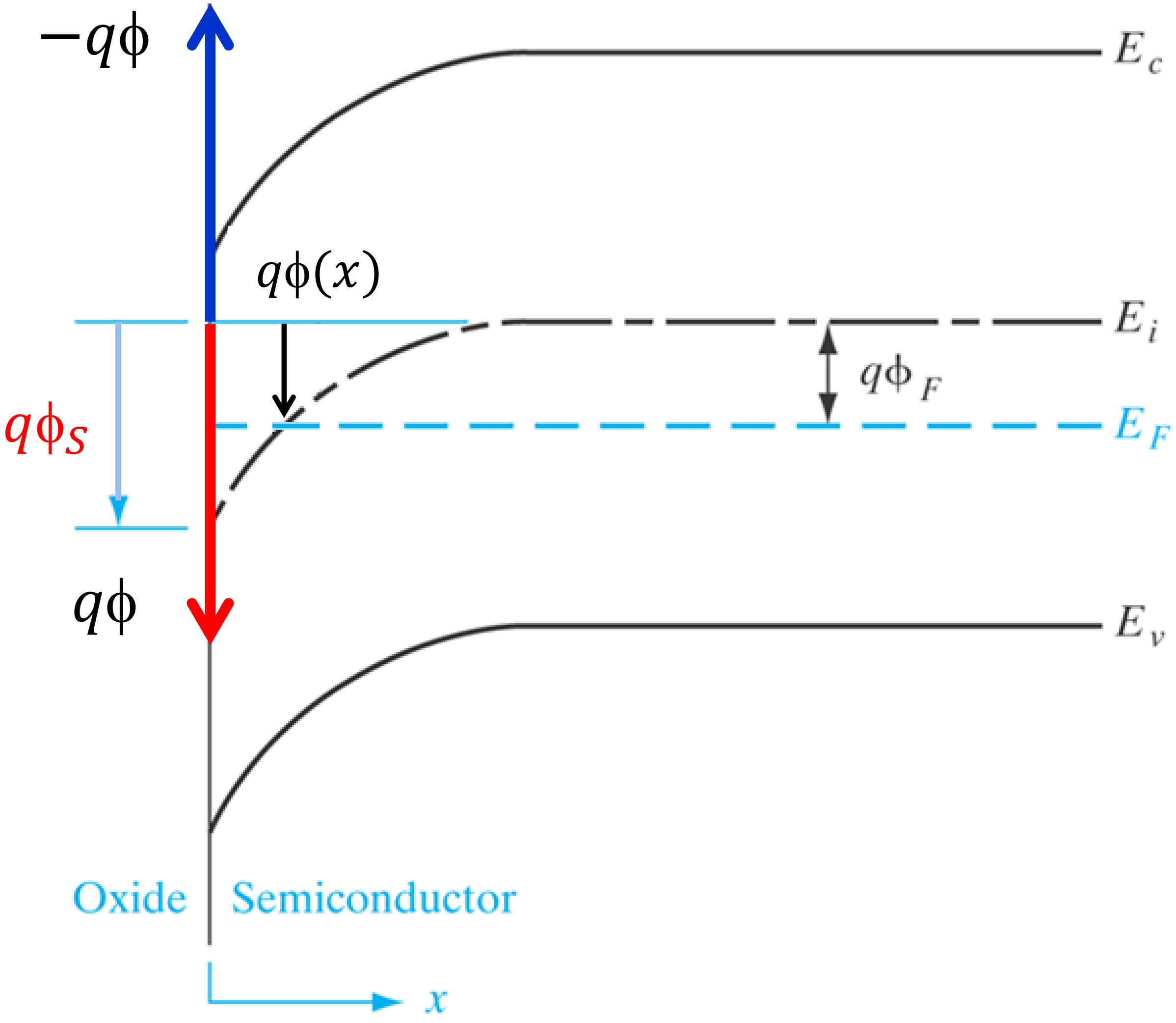
$$\boxed{Q_D = -qN_AW = 2\sqrt{q\epsilon_s N_A \phi_F}}$$

maximum value

$$V_T = \underbrace{-\frac{\overbrace{Q_d}^{\text{Depletion layer charge}}}{C_i}}_{\text{Voltage drop across oxide}} + \underbrace{2\phi_F}_{\text{Strong inversion condition}}$$

(Assuming that depletion charge dominates Q_s at threshold)

Summary of conditions – surface potential



accumulation
$\phi_S < 0$

flat-band
$\phi_S = 0$

depletion
$0 < \phi_S < \phi_F$

intrinsic
$\phi_S = \phi_F$

weak inversion
$2\phi_F > \phi_S > \phi_F$

strong inversion
$\phi_S \geq 2\phi_F$

Real Surface effects – Work function difference

- **We assumed in the previous analysis for simplicity**

$$\boxed{\Phi_m = \Phi_s}$$

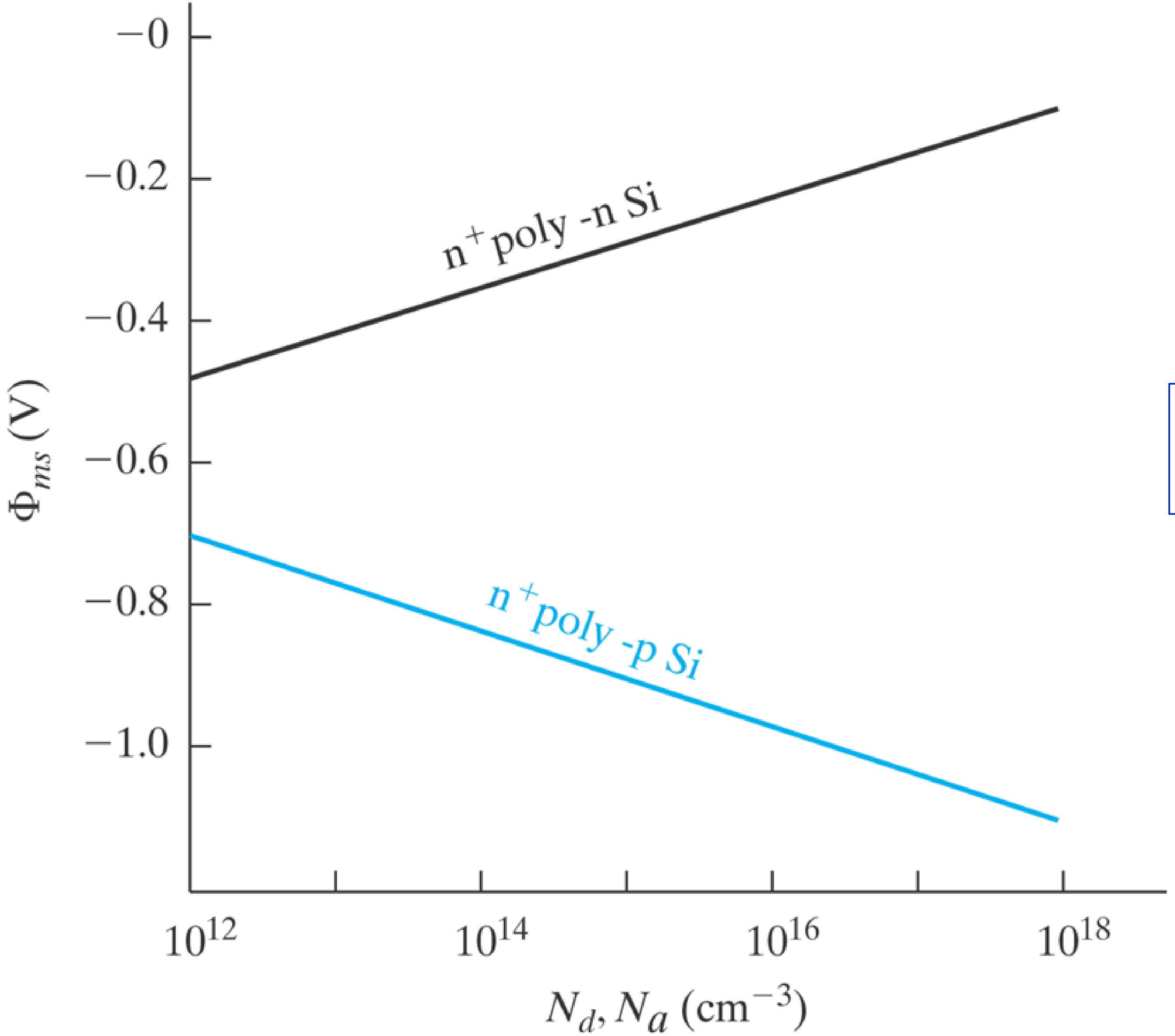
- **In general, we are limited in the choice of metal by technological constraints and**

$$\boxed{\Phi_m \neq \Phi_s}$$

- **It is convenient to define the quantity**

$$\boxed{\Phi_{ms} = \Phi_m - \Phi_s}$$

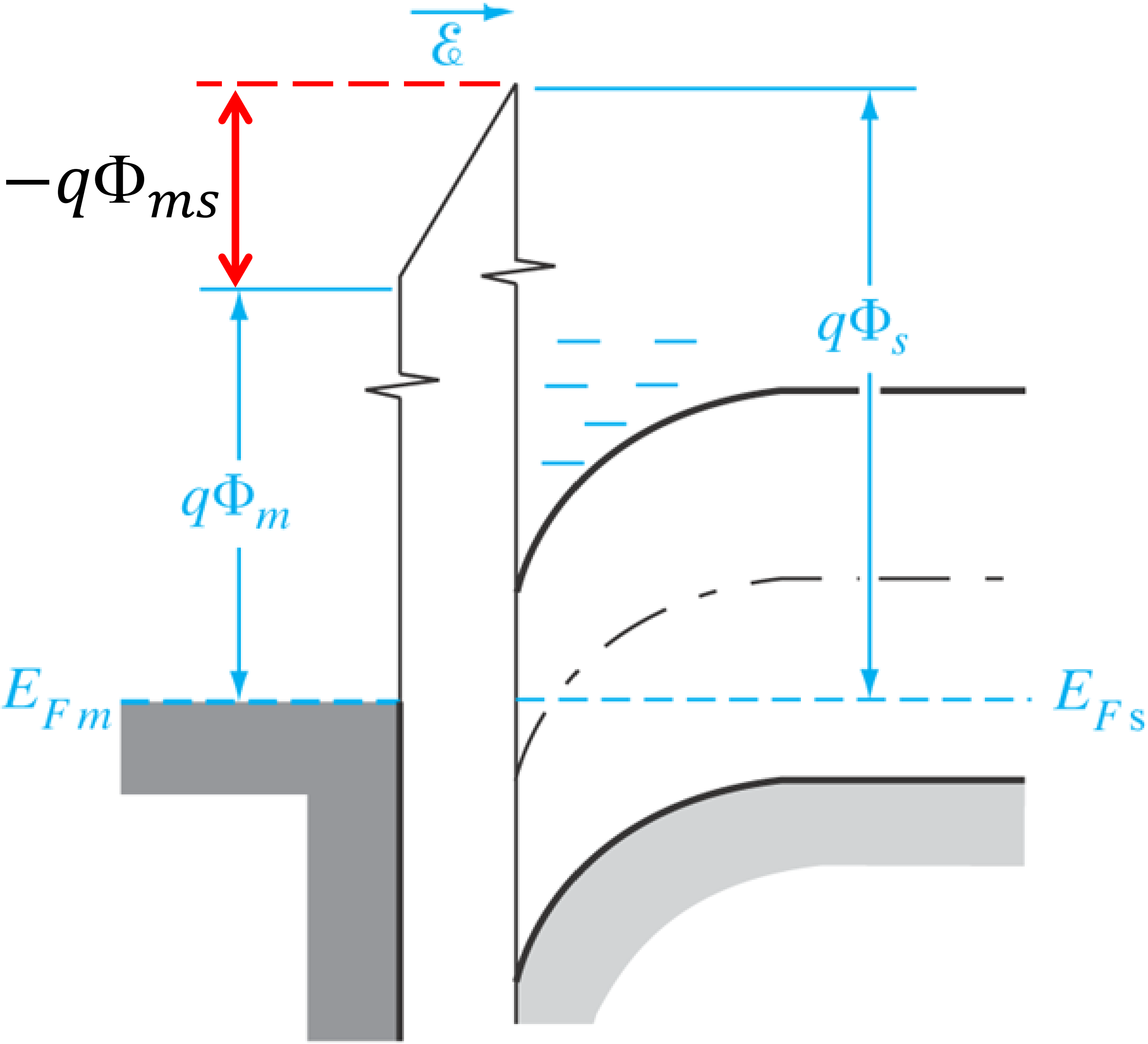
n+ plus polysilicon for gate electrode



common choice instead of metal

$$\Phi_{ms} < 0$$

Effect of negative workfunction difference

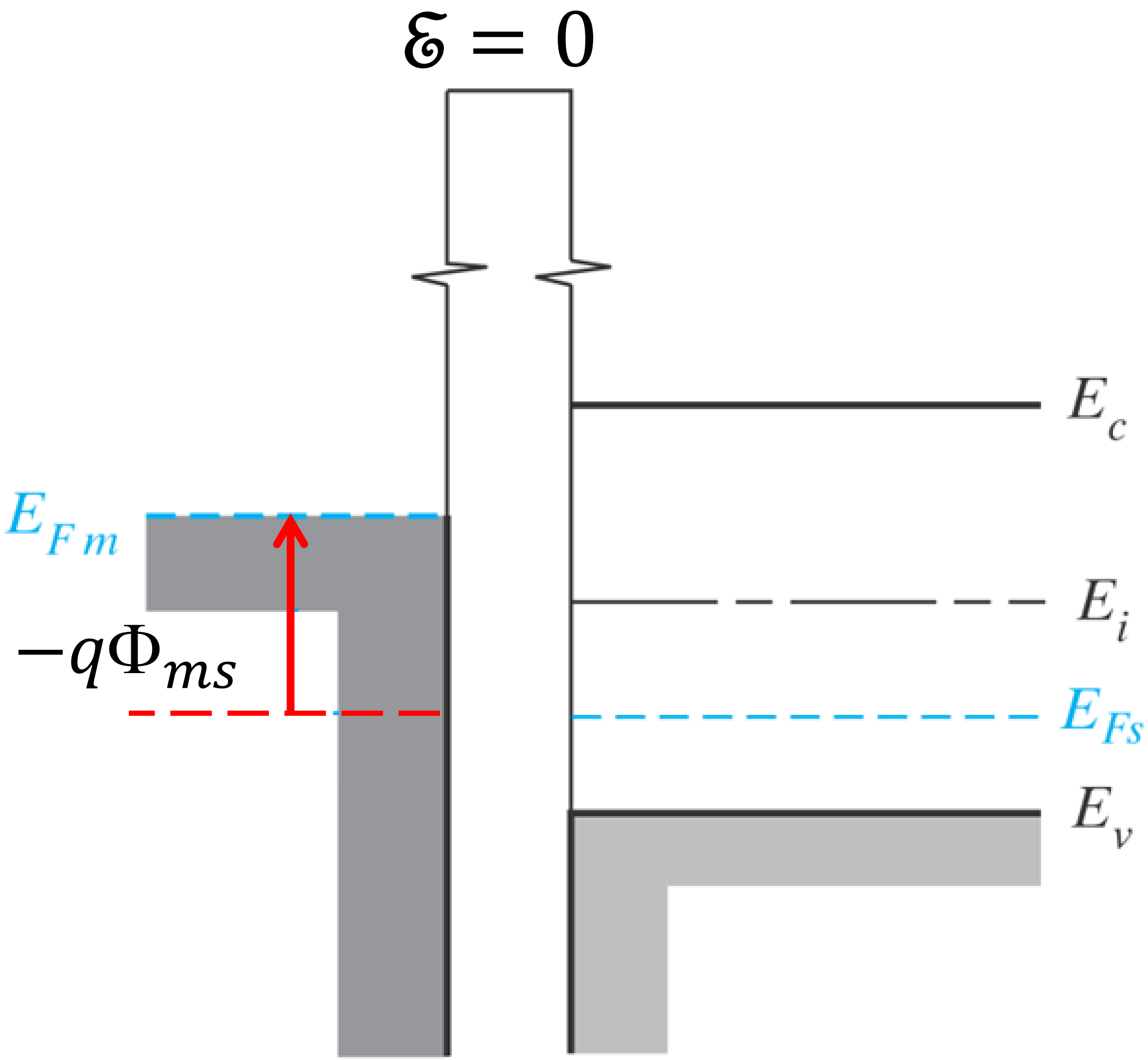


EQUILIBRIUM

$\boldsymbol{V = 0}$

$$\boxed{\Phi_{ms} < 0}$$

Apply $V_{FB} = \Phi_{ms}$ to obtain flat band



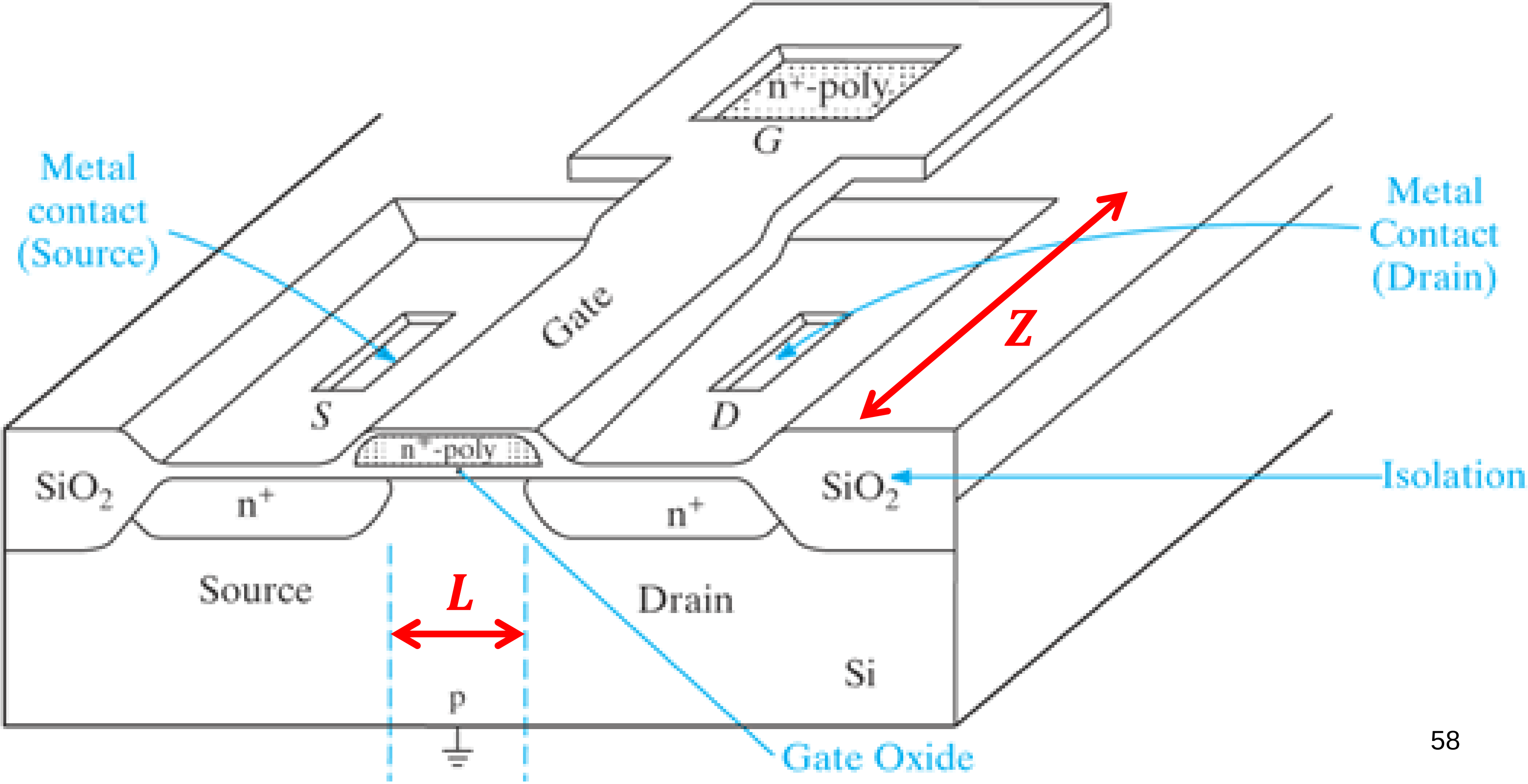
FLAT BAND

$V = V_{FB} = \Phi_{ms}$

$$\boxed{\Phi_{ms} < 0}$$

The MOSFET

When an inversion layer is formed under the gate, current can flow from drain to sourse (n-channel device)



MOSFET – channel conductance at x

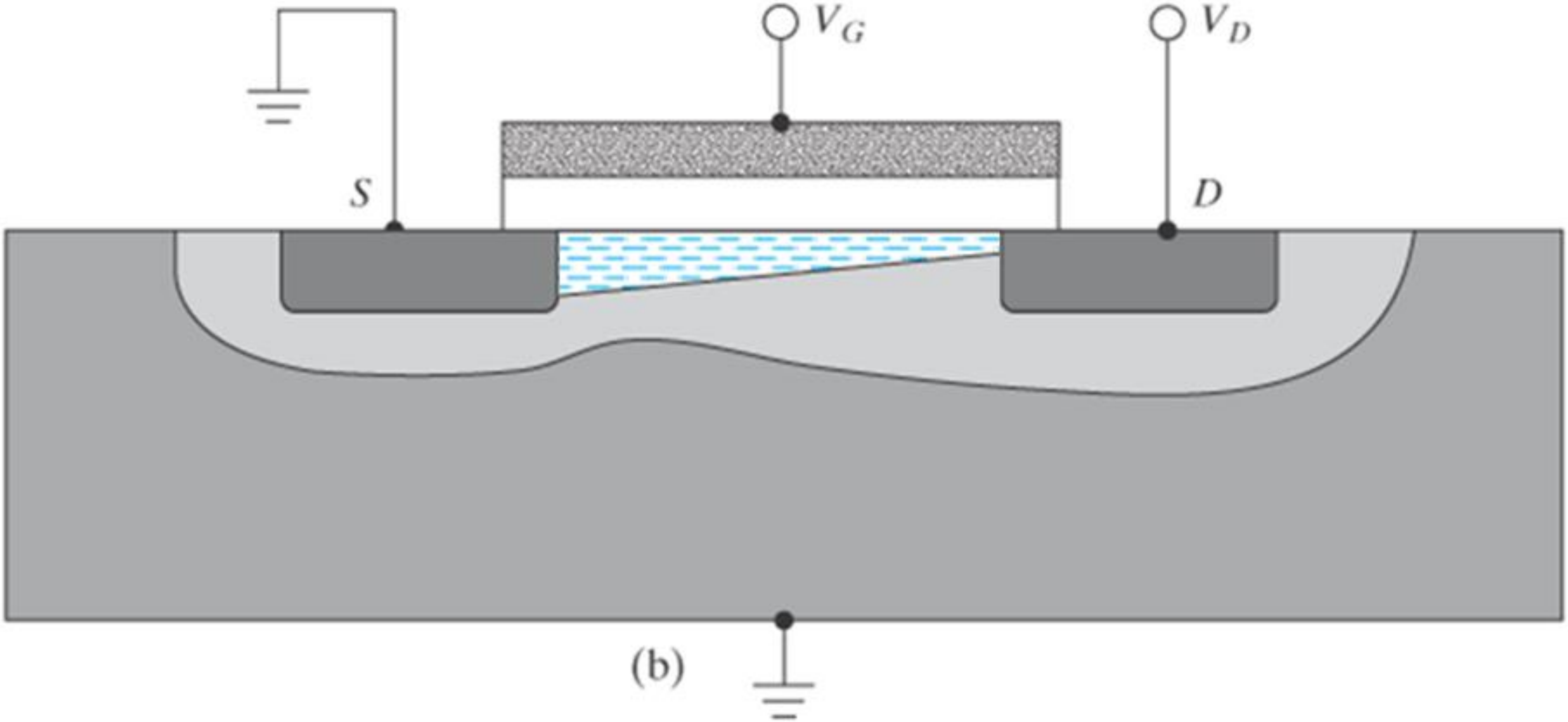
$$\boxed{\sigma(x) = \overline{\mu_n} \mathrm{Q}_n(x)\, Z / dx}$$

$\overline{\mu_n}$ surface electron mobility

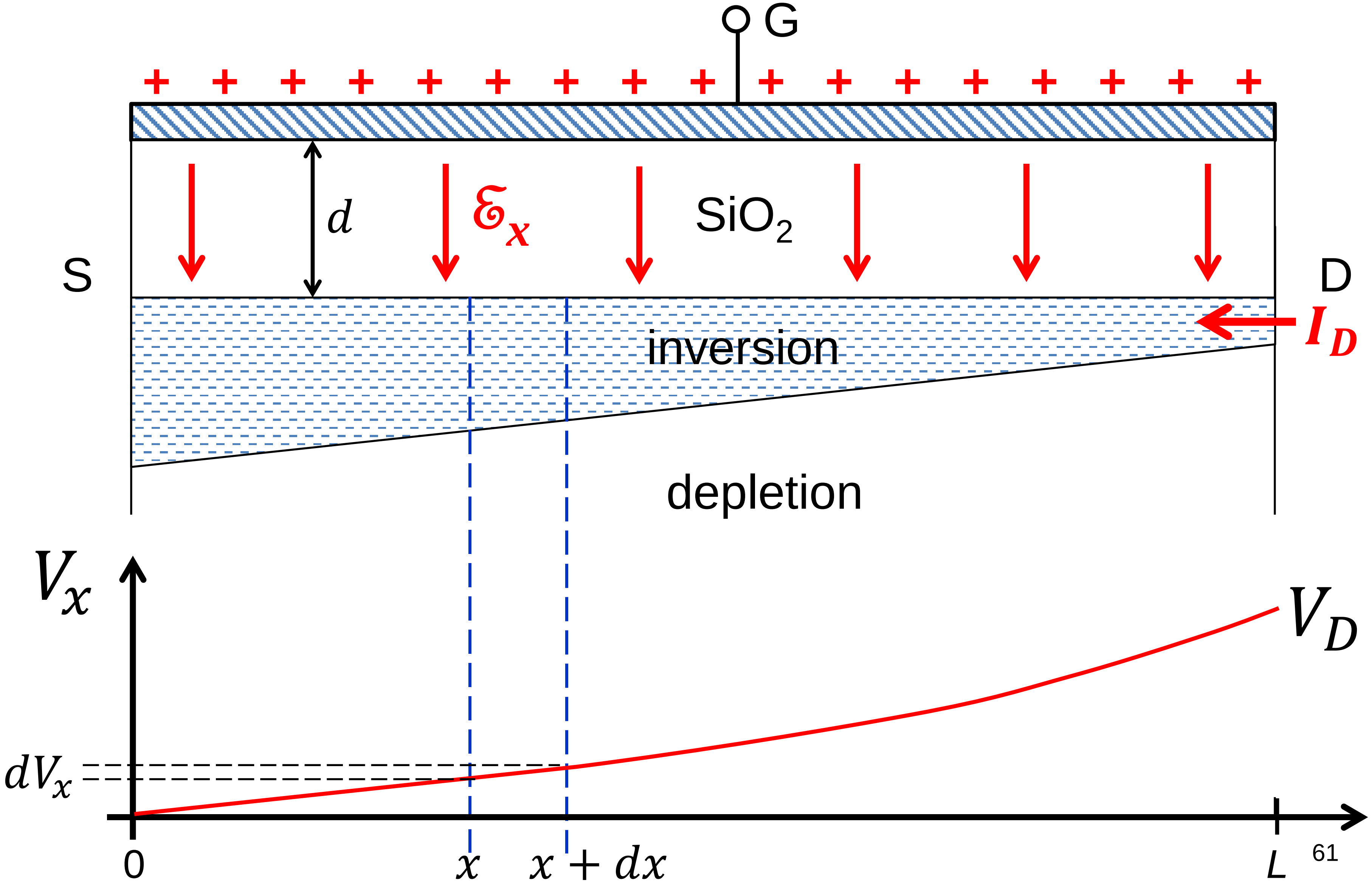
mobility in the narrow layer close to the surface is much lower than in the free bulk due to surface roughness irregularities and quantum effects

Z width of the channel

MOSFET – channel current at x



MOSFET – channel current at x



MOSFET – Drain Current

$$I_D\,dx = \overline{\mu_n}\,Z\,|Q_n(x)|\,dV_x$$

$$Q_n(x) = -C_i[V_G - V_T - V_x]$$

$$\boxed{\int_0^L I_D\,dx = \overline{\mu_n} Z C_i \int_0^{V_D} (V_G - V_T - V_x)\;dV_x}$$

$$I_D = \underbrace{\boxed{\frac{\overline{\mu_n} Z C_i}{L}}}_{k_N}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

MOSFET – Conductance of channel

linear region

$$I_D = \frac{\overline{\mu_n} Z C_i}{L}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

$V_D \ll (V_G - V_T)$ (linear region)
$V_G > V_T$ (channel condition)

$$g = \frac{\partial I_D}{\partial V_D} = \frac{\overline{\mu_n} Z C_i}{L}(V_G - V_T)$$

$$= k_N(\mathrm{lin.})\,(V_G - V_T)$$

MOSFET – Conductance of channel

linear region

$$I_D = \frac{\overline{\mu_n} Z C_i}{L}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

$V_D \ll (V_G - V_T)$ (linear region)
$V_G > V_T$ (channel condition)

For small drain voltage, you can use this approximation

$$I_D = \frac{\overline{\mu_n} Z C_i}{L}[(V_G - V_T)V_D]$$

MOSFET – Saturation

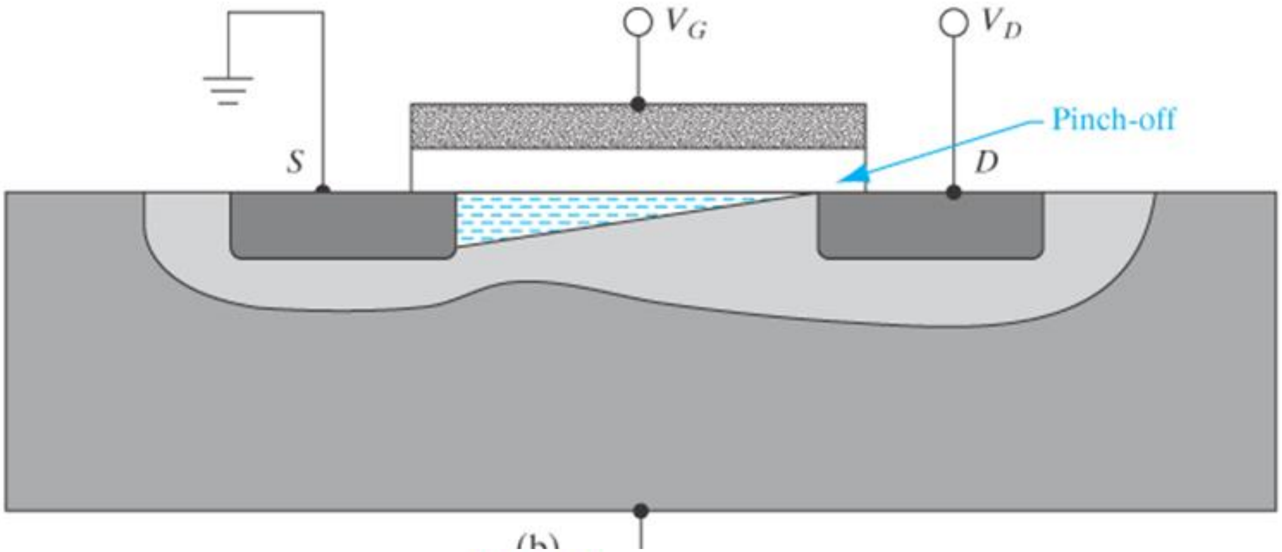
As the drain voltage is increased, the voltage across the oxide decreases near the drain as does Q_S.

The channel goes in “pinch off” at the drain and current saturates

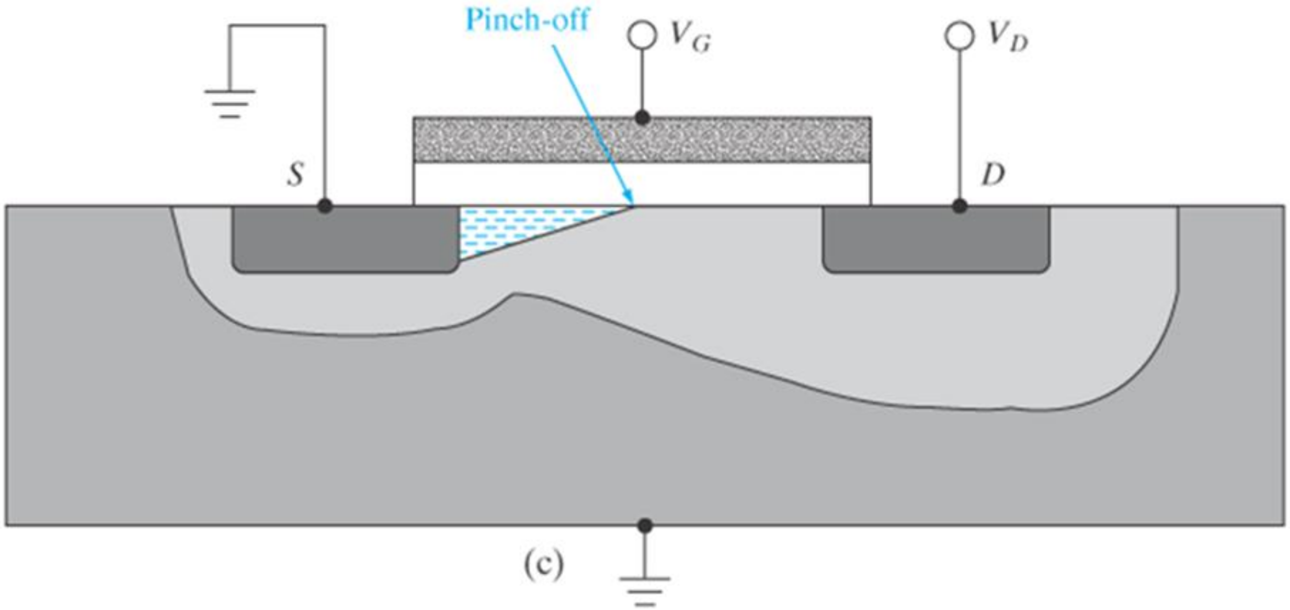
in saturation condition we have approximately

$$\boxed{V_D(\text{sat.}) \approx (V_G - V_T)}$$

MOSFET – Saturation



onset of saturation



deep saturation

MOSFET – Example

n-channel MOSFET

$$d = 10\text{ nm} = 10^{-6}\text{ cm}$$

$$V_T = 0.6\text{V}$$

$$Z = 25\mu\text{m}$$

$$L = 1\ \mu\text{m}$$

$$\overline{\mu_n} = 200\text{ cm}^2/\text{V}\cdot\text{s}$$

$$C_i = \frac{\epsilon_i}{d} = \frac{3.9 \times 8.85 \times 10^{-14}}{10^{-6}}$$

$$= 3.45 \times 10^{-7}\ \text{F/cm}^2$$ (unit area)

$$k_N = \frac{\overline{\mu_n} Z C_i}{L} = \frac{200 \times 25 \times 10^{-4} \times 3.45 \times 10^{-7}}{10^{-4}} = .001725$$

$$\boxed{V_G = 5\text{ V}}$$

$$\boxed{V_D = 0.1\text{ V}}$$

$$\boxed{V_D = 0.1\text{ V} < (V_G - V_T) = 4.4\text{ V}}$$

linear region

$$I_D = \frac{\overline{\mu_n} Z C_i}{L}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right] =$$

$$= \underbrace{0.001725 \times ((5-0.6) \times 0.1}_{7.59 \times 10^{-4}} - 0.5 \times 0.1^2) = 7.5 \times 10^{-4}\text{A}$$

MOSFET – Example

n-channel MOSFET

$$d = 10\text{ nm} = 10^{-6}\text{ cm}$$

$$V_T = 0.6\text{V}$$

$$Z = 25\mu\text{m}$$

$$L = 1\ \mu\text{m}$$

$$\overline{\mu_n} = 200\text{ cm}^2/\text{V}\cdot\text{s}$$

$$\boxed{V_G = 3\text{ V}}\quad \boxed{V_D = 5\text{ V}}$$

$$\boxed{V_D = 5\text{ V} > (V_G - V_T) = 2.4\text{ V}}$$

$$\boxed{V_D(\text{sat.}) = 2.4\text{ V}}$$ saturation region

$$k_N = \frac{\overline{\mu_n} Z C_i}{L} = .001725$$

$$I_D = \frac{\overline{\mu_n} Z C_i}{L}\left[(V_G - V_T)V_D(\text{sat.}) - \frac{1}{2}V_D^2(\text{sat.})\right] =$$

$$= 0.001725 \times ((3 - 0.6) \times 2.4 - 0.5 \times 2.4^2)$$

$$= 4.968 \times 10^{-3}\text{A}$$

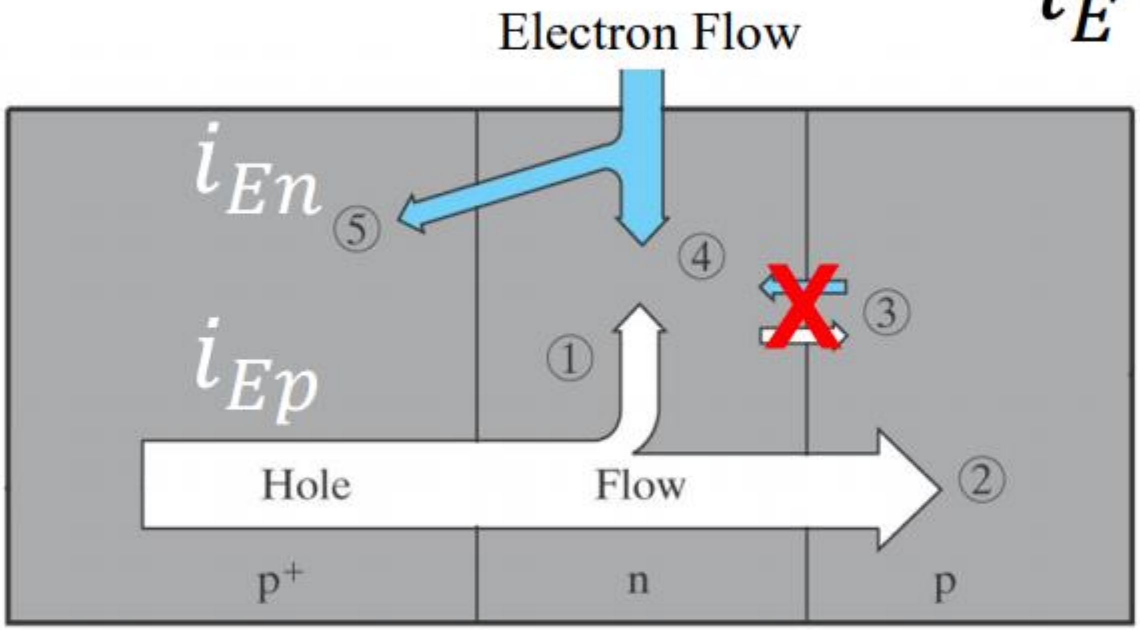
$\boxed{V_D = 7\text{ V} > (V_G - V_T) \ \rightarrow\ V_D(\text{sat.}) = 2.4\text{ V} \ \rightarrow\ I_D \text{ is the same}}$

Bipolar Junction Transistor (BJT)

Here is material with just a few essentials

Amplification parameters are usually defined assuming that the reverse saturation current of the reverse biased base-collector junction is negligible, but not the minority current (in this case electrons) across the forward biased emitter-base junction.

$$i_E = i_{Ep} + i_{En}$$



Bipolar Junction Transistor (BJT)

emitter injection efficiency

$$\gamma = \frac{i_{Ep}}{i_{Ep} + i_{En}}$$

base transport factor

B = fraction of holes that make it across the base without recombining

$$B = \frac{i_C}{i_{Ep}}$$

current transfer ratio

$$\frac{i_C}{i_E} = \frac{B\, i_{Ep}}{i_{Ep} + i_{En}} = B\gamma = \alpha$$

Base-to-Collector Amplification Factor β

$$\frac{i_C}{i_B} = \frac{\overset{= \alpha}{\boxed{B\gamma}}}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} = \beta$$

Since α is close unity, β can be quite large

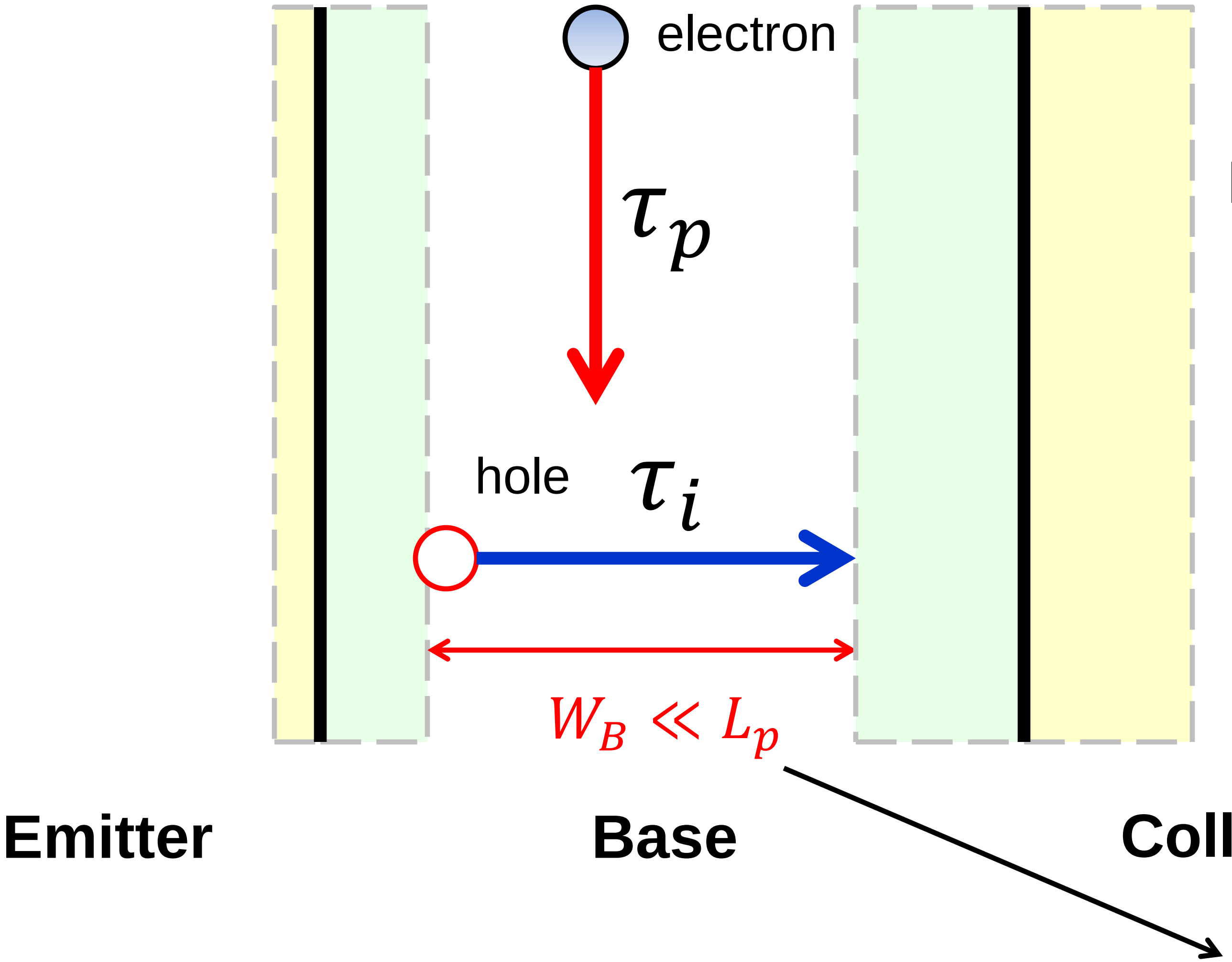
$$\alpha = 0.9 \quad \rightarrow \quad \beta = 9$$

$$\alpha = 0.95 \quad \rightarrow \quad \beta = 19$$

$$\alpha = 0.99 \quad \rightarrow \quad \beta = 99$$

$$\alpha = 0.999 \quad \rightarrow \quad \beta = 999$$

Hole transit time in the base



an electron enters from the base and lingers for average time τ_p before recombining

Average time spent by hole in the narrow base $\tau_i \ll \tau_p$

Charge storage model in the base

At steady-state there are excess electrons and holes in the base and for charge neutrality

$$Q_n = Q_p$$

Assuming $\tau_n = \tau_p$ and $\gamma = 1$ and negligible saturation current

$$i_C = \frac{Q_p}{\tau_i} \qquad i_B = \frac{Q_n}{\tau_n} = \frac{Q_n}{\tau_p}$$

$$\boxed{Q_p = i_C \tau_i} \qquad \boxed{Q_n = i_B \tau_p} \qquad \boxed{i_C \tau_i = i_B \tau_n}$$

Amplification factor again

For each electron entering from the base contact, a number τ_p/τ_i of holes goes from emitter to collector maintaining charge neutrality

$$\boxed{\frac{i_C}{i_B} = \frac{\tau_p}{\tau_i} = \beta}$$