ECE 536 – Integrated Optics and Optoelectronics
Lecture 7 – February 8, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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Lecture 7 Outline

- Recap of slab optical wave guide
- Refractive index of III-V material. Example of AlGaAs
- Confinement factor (optical power) in slab waveguide
- Electron-Photon interaction
- Optical transitions using Fermi Golden Rule

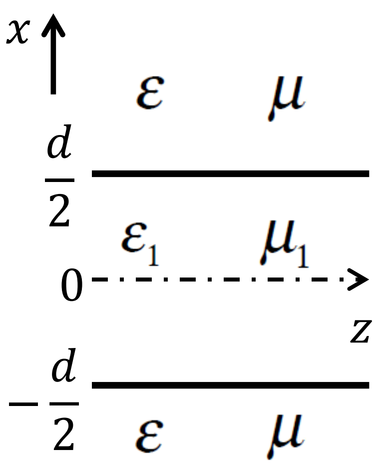
Summary of symmetric slab wave guide

even modes

$$E_y = e^{ik_z z}\begin{cases} C_0 e^{-\alpha(|x|-d/2)} & |x| \geq d/2 \\ C_1 \cos k_x x & |x| \leq d/2 \end{cases}$$

from Faraday's law

$$\boxed{H_z = -i\frac{1}{\omega\mu}\frac{\partial E_y}{\partial x}}$$



continuity of E_y

continuity of H_z

$$C_0 = C_1 \cos\left(k_x \frac{d}{2}\right)$$

$$\frac{\alpha}{\mu}C_0 = C_1 \frac{k_x}{\mu_1}\sin\left(k_x \frac{d}{2}\right)$$

$$\boxed{\alpha = \frac{\mu}{\mu_1}k_x \tan\left(k_x \frac{d}{2}\right)}$$

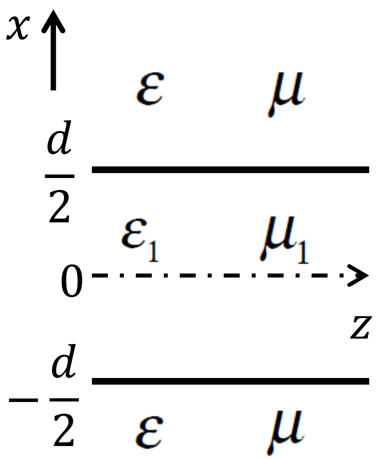
Summary of symmetric slab wave guide

odd modes

$$E_y = e^{ik_z z}\begin{cases} C_0 e^{-\alpha(|x|-d/2)} & x \ge d/2 \\ C_1 \sin k_x x & |x| \le d/2 \\ -C_0 e^{\alpha(|x|+d/2)} & x \le -d/2 \end{cases}$$

from Faraday's law

$$\boxed{H_z = -i\frac{1}{\omega\mu}\frac{\partial E_y}{\partial x}}$$



continuity of E_y

continuity of H_z

$$C_0 = C_1 \sin\left(k_x \frac{d}{2}\right)$$

$$-\frac{\alpha}{\mu}C_0 = C_1 \frac{k_x}{\mu_1}\cos\left(k_x \frac{d}{2}\right)$$

$$\boxed{\alpha = -\frac{\mu}{\mu_1}k_x \cot\left(k_x \frac{d}{2}\right)}$$

Summary of symmetric slab wave guide

Coordinate transformation

$$X = k_x \frac{d}{2} \quad \text{and} \quad Y = \alpha \frac{d}{2}$$

$$\alpha = \frac{\mu}{\mu_1} k_x \tan\left(k_x \frac{d}{2}\right)$$

$$\alpha = -\frac{\mu}{\mu_1} k_x \cot\left(k_x \frac{d}{2}\right)$$

$$\Rightarrow$$

$$\boxed{Y = \begin{cases} \dfrac{\mu}{\mu_1} X \tan X & \text{TE even} \\ -\dfrac{\mu}{\mu_1} X \cot X & \text{TE odd} \end{cases}}$$

$$k_x^2 + k_z^2 = \omega^2 \mu_1 \varepsilon_1$$

$$-\alpha^2 + k_z^2 = \omega^2 \mu \varepsilon$$

$$\Rightarrow$$

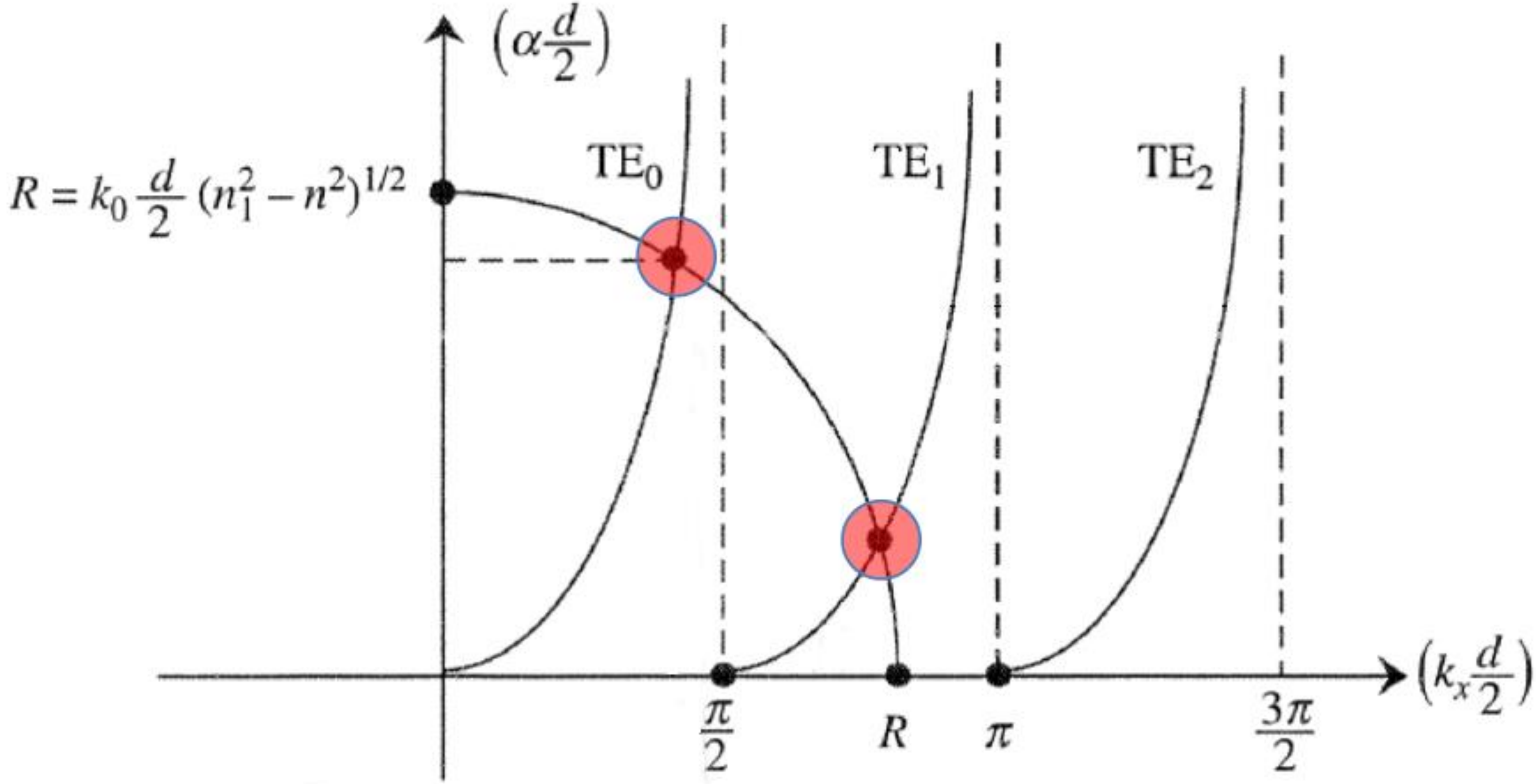
$$k_x^2 + \alpha^2 = \omega^2 \mu_1 \varepsilon_1 - \omega^2 \mu \varepsilon$$

$$\Downarrow$$

$$\boxed{X^2 + Y^2 = R^2}$$

$$R = \omega \left(\frac{d}{2}\right) (\mu_1 \varepsilon_1 - \mu \varepsilon) = \left(k_0 \frac{d}{2}\right) \sqrt{n_1^2 - n^2}$$

Graphical solution



Low Frequency limits

At cut-off condition $\boldsymbol{R = m\pi/2}$ $(m = 0,1,2, \ldots)$

$$k_0 \frac{d}{2}\sqrt{n_1^2 - n^2} = m\frac{\pi}{2}$$

$$\alpha = 0$$

$$\boxed{\begin{aligned} k_x^2 + k_z^2 &= \omega^2 \mu_1 \varepsilon_1 \\ -\alpha^2 + k_z^2 &= \omega^2 \mu \varepsilon \end{aligned}}$$

$$k_x \frac{d}{2} = m\frac{\pi}{2}$$

$$k_z = \omega\sqrt{\mu\varepsilon} \quad (\text{since } \alpha = 0)$$

At low frequency the propagation constant approaches that of the cladding medium outside the waveguide

High Frequency limit

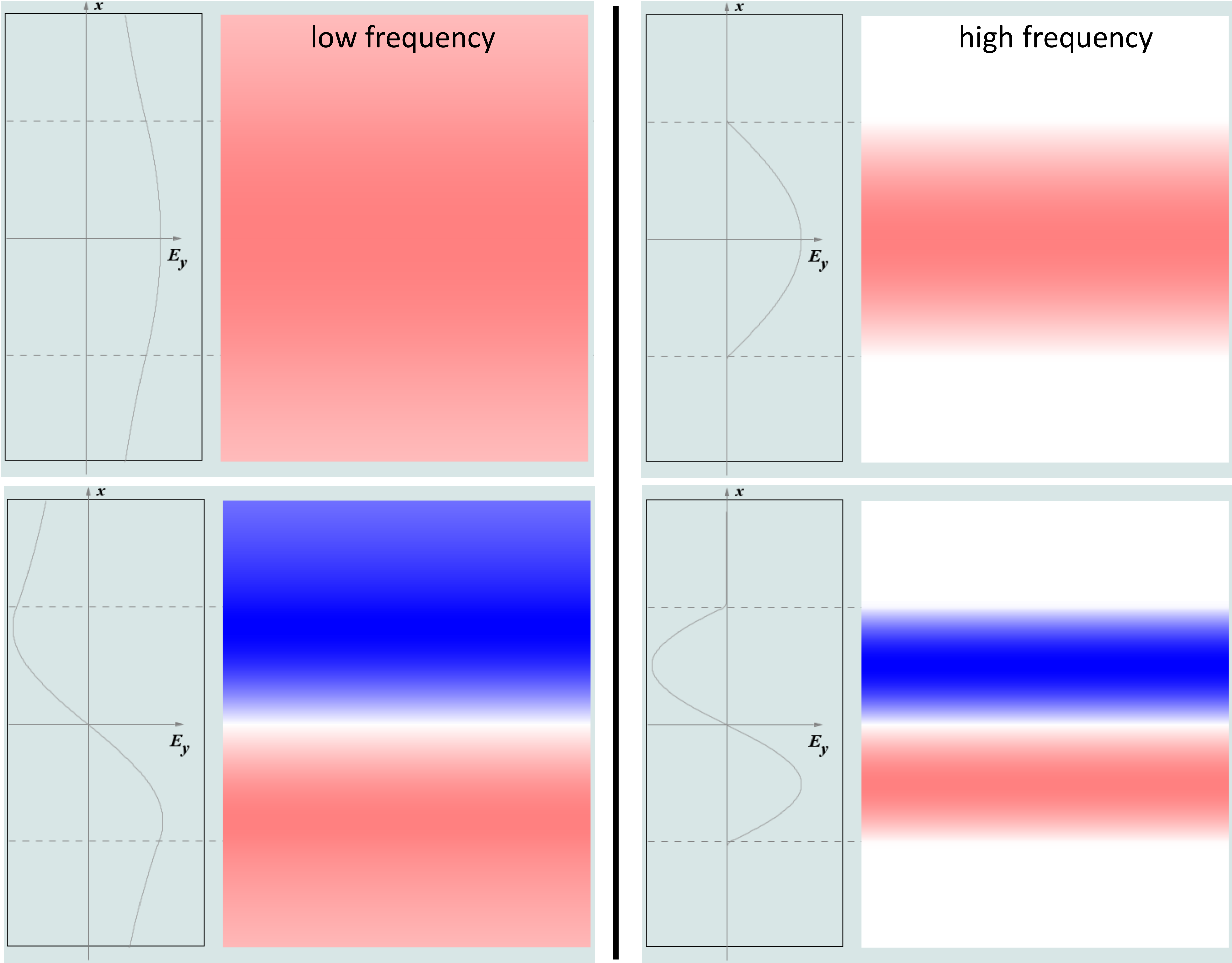
At high frequency $R \rightarrow \infty$

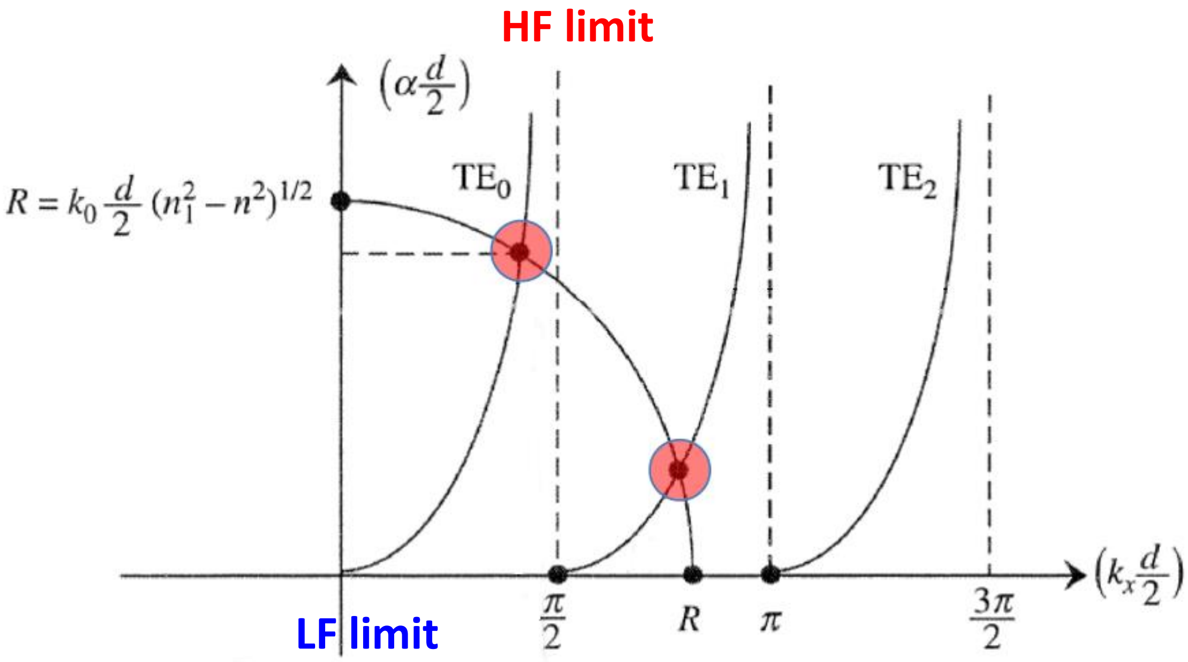
$$k_x^2 + k_z^2 = \omega^2 \mu_1 \varepsilon_1$$
$$-\alpha^2 + k_z^2 = \omega^2 \mu \varepsilon$$

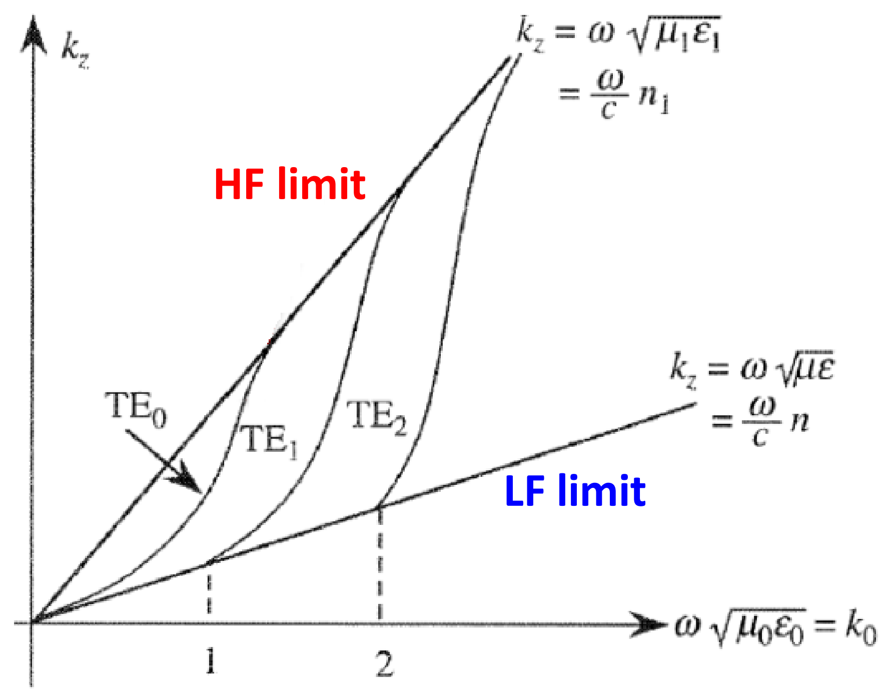
$$k_x \frac{d}{2} \rightarrow (m+1)\frac{\pi}{2}$$

$$k_z \rightarrow \omega\sqrt{\mu_1 \varepsilon_1} = \omega \frac{n_1}{c}$$

At high frequency the propagation constant approaches that of the core medium inside the waveguide. There is very little power in the cladding.


low frequency
high frequency
x
E_y
x
E_y
x
E_y
x
E_y

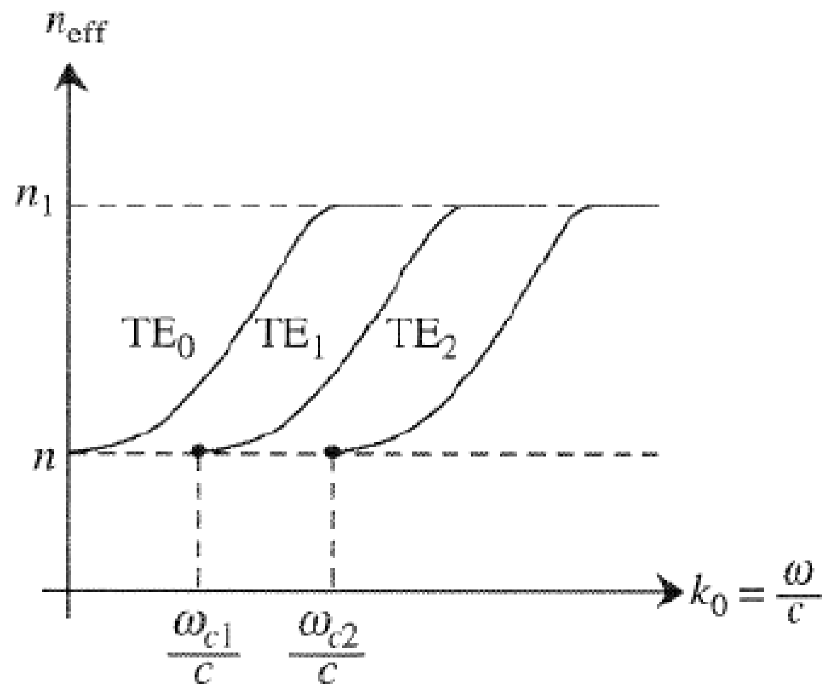

HF limit
$(\alpha \frac{d}{2})$
$R = k_0 \frac{d}{2} (n_1^2 - n^2)^{1/2}$
TE_0
TE_1
TE_2
LF limit
$\frac{\pi}{2}$
R
π
$\frac{3\pi}{2}$
$(k_x \frac{d}{2})$


k_z
$k_z = \omega \sqrt{\mu_1 \varepsilon_1}$
$= \frac{\omega}{c} n_1$
HF limit
TE_0
TE_1
TE_2
$k_z = \omega \sqrt{\mu \varepsilon}$
$= \frac{\omega}{c} n$
LF limit
1
2
$\omega \sqrt{\mu_0 \varepsilon_0} = k_0$

Effective index for the guided modes

$$n_{eff} = \frac{k_z}{k_0}$$

$$n_{eff} = \frac{2\pi}{\lambda_z}\frac{\lambda_0}{2\pi} = \frac{f}{v_{pz}}\frac{c}{f} = \frac{c}{v_{pz}}$$



$$n_{eff} = n_1 \sin\theta$$
$$n \leq n_{eff} \leq n_1$$

$$\lambda_z = \frac{\lambda_0}{n_{eff}}$$

Normalized frequency

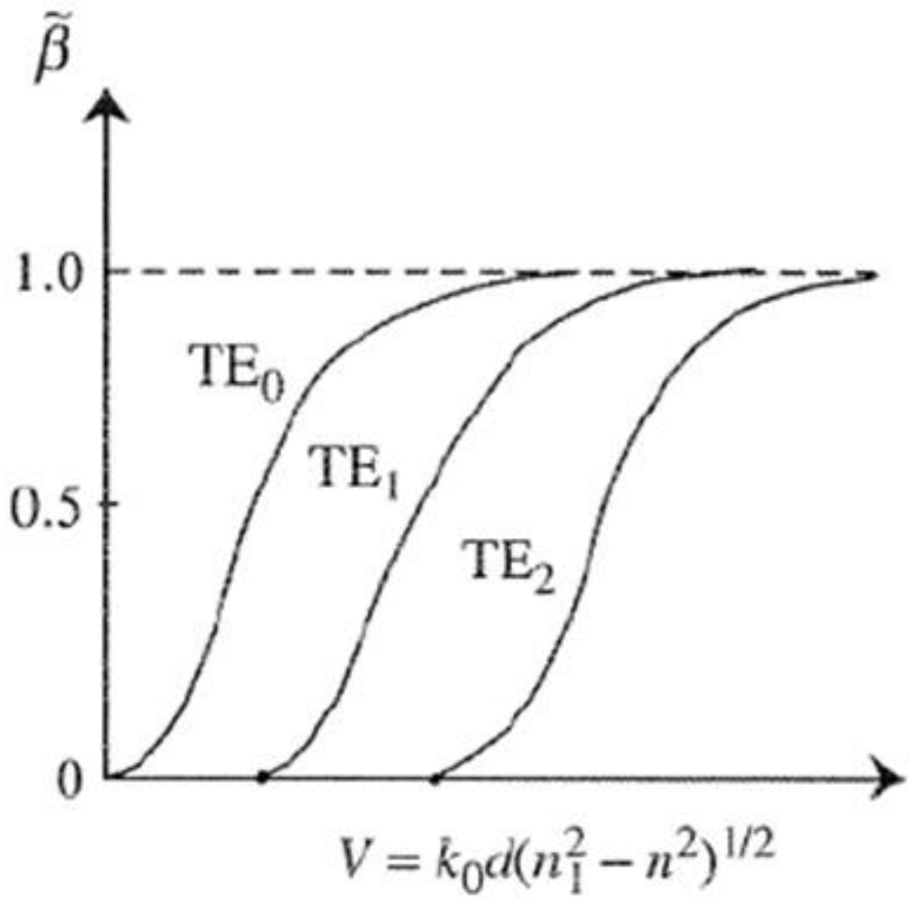
$$V = k_0 d\sqrt{n_1^2 - n^2}$$

$$V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n^2} = f\,\underbrace{\frac{2\pi}{c} d\sqrt{n_1^2 - n^2}}$$

contains all the information on wave guide geometry and materials

Normalized propagation parameter

$$\tilde{\beta} = \frac{k_z^2 - \omega^2\mu\varepsilon}{\omega^2\mu_1\varepsilon_1 - \omega^2\mu\varepsilon} = \frac{n_{eff}^2 - n^2}{n_1^2 - n^2}$$



Validity of refraction index model

The theoretical analysis provides always good insight but it has assumed that the refractive indices are independent of frequency.

This is true only in ideal dielectrics. In real media, particularly in semiconductors, this is generally not true.

Normalization constant for the optical mode

Assume total guided power to be unity

$$P = \frac{1}{2}\mathrm{Re}\int_{-\infty}^{\infty}\left(\mathbf{E}\times\mathbf{H}^*\right)\bullet\hat{\mathbf{z}}\,dx = 1$$

For the TE modes, $H_x = -\dfrac{k_z}{\omega\mu}E_y$

$$P = -\frac{1}{2}\int E_y H_x^* dx = \frac{1}{2}\int_{-\infty}^{\infty}\frac{k_z}{\omega\mu}\left|E_y\right|^2 dx$$

$$C_1 = \left[\frac{4\omega\mu}{k_z\left(d+\dfrac{2}{\alpha}\right)}\right]^{1/2}$$

(for $\mu = \mu_1$)

Even Solution (cos functions):

$$E_y = e^{ik_z z}\begin{cases} C_0 e^{-\alpha(|x|-d/2)} & |x| \ge d/2 \\ C_1 \cos k_x x & |x| \le d/2 \end{cases}$$

Odd Solution (sin functions):

$$E_y = e^{ik_z z}\begin{cases} C_0 e^{-\alpha(|x|-d/2)} & x \ge d/2 \\ C_1 \sin k_x x & |x| \le d/2 \\ -C_0 e^{\alpha(|x|+d/2)} & x \le -d/2 \end{cases}$$

Confinement factor

Fraction of optical power guided in the core of the waveguide

$$\Gamma = \frac{\frac{1}{2}\int_{core} \mathrm{Re}\left(\mathrm{E}\times\mathrm{H}^*\right)\cdot\hat{z}\,dx}{\frac{1}{2}\int_{all} \mathrm{Re}\left(\mathrm{E}\times\mathrm{H}^*\right)\cdot\hat{z}\,dx} = \frac{\frac{k_z}{2\omega\mu_1}\int_{|x|<d/2}\left|E_y\right|^2 dx}{\frac{k_z}{2\omega\mu_1}\int_{|x|<d/2}\left|E_y\right|^2 dx + \frac{k_z}{2\omega\mu}\int_{|x|>d/2}\left|E_y\right|^2 dx}$$

Even Modes

$$\Gamma = \left[1+\left(\frac{\mu_1}{\mu}\right)\left(\frac{2}{\alpha d}\right)\frac{\cos^2\left(\frac{k_x d}{2}\right)}{\left(1+\frac{\sin k_x d}{k_x d}\right)}\right]^{-1}$$

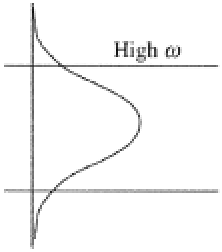
Odd Modes

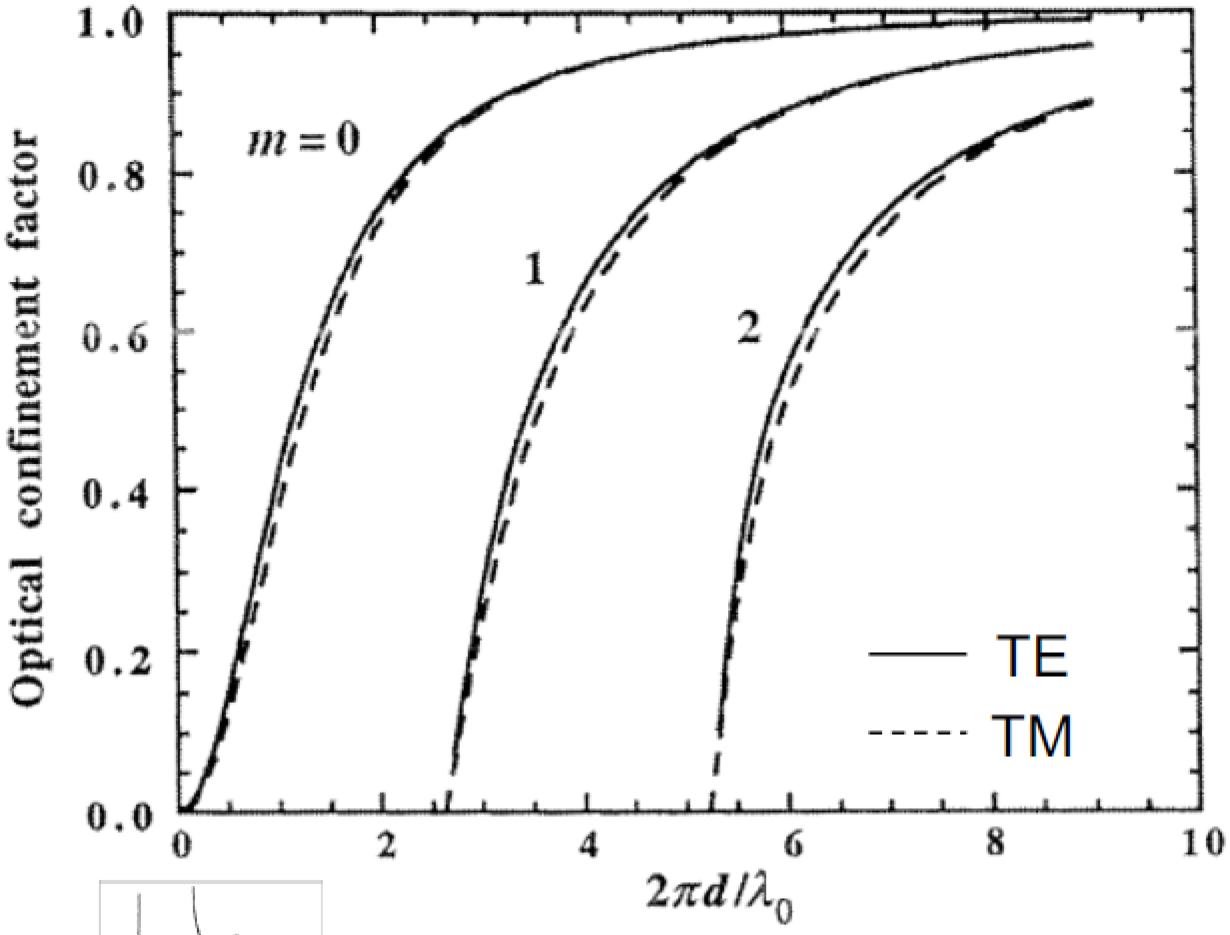
$$\Gamma = \left[1+\left(\frac{\mu_1}{\mu}\right)\left(\frac{2}{\alpha d}\right)\frac{\sin^2\left(\frac{k_x d}{2}\right)}{\left(1-\frac{\sin k_x d}{k_x d}\right)}\right]^{-1}$$

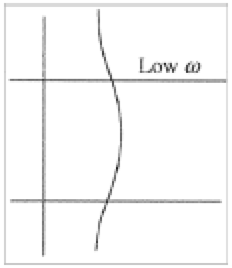
In the limit of thin waveguide (very small d) and $\mu = \mu_1$ it can be shown that the TE0 confinement factor is

$$\Gamma(\mathrm{TE}_0) \approx 2\left(\frac{\pi d}{\lambda_0}\right)^2\left(n_1^2 - n^2\right)$$

Confinement factor







From TE to TM solutions – Duality Principle

Equivalent solutions can be obtained by making the following replacements:

$$\mathbf{E} \rightarrow \mathbf{H} \qquad \mu \rightarrow \varepsilon$$

$$\mathbf{H} \rightarrow -\mathbf{E} \qquad \varepsilon \rightarrow \mu$$

TM) $\mathbf{H} = H_y \hat{\mathbf{y}}$ $$\mathbf{E} = \frac{1}{i\omega\varepsilon_i}\left(ik_z H_y \hat{\mathbf{x}} - \frac{\partial}{\partial x} H_y \hat{\mathbf{z}} \right)$$

$\varepsilon_i = \varepsilon_1$ inside

$\varepsilon_i = \varepsilon$ outside

Eigenequations:

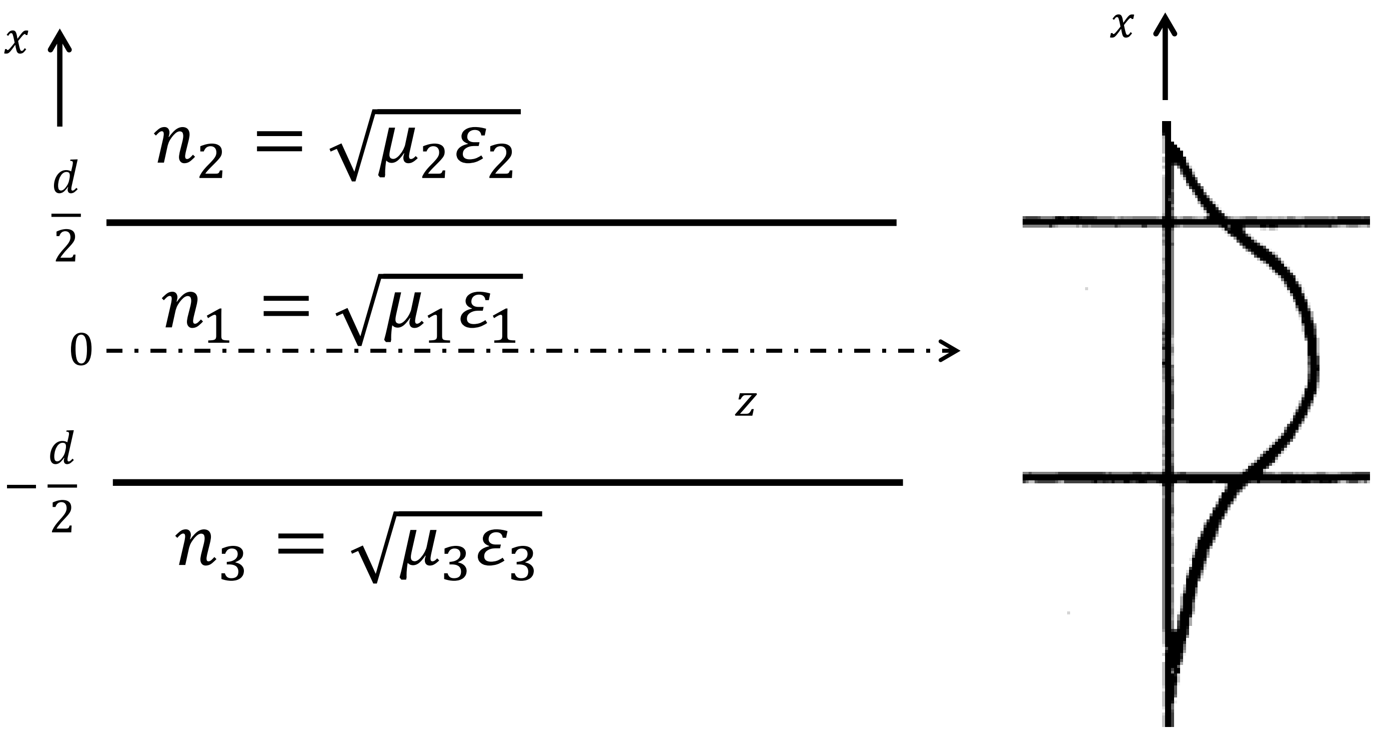
TM Even: $$\alpha \frac{d}{2} = \frac{\varepsilon}{\varepsilon_1}\left(k_x \frac{d}{2} \right) \tan\left(k_x \frac{d}{2} \right)$$

TM Odd: $$\alpha \frac{d}{2} = -\frac{\varepsilon}{\varepsilon_1}\left(k_x \frac{d}{2} \right) \cot\left(k_x \frac{d}{2} \right)$$

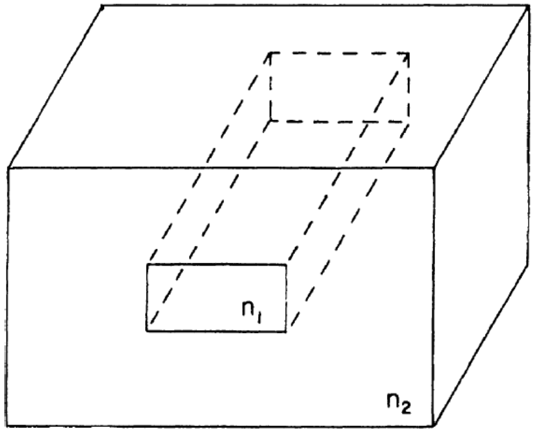
Asymmetric wave guide

In an asymmetric wave guide, usually $n_1 > n_3 > n_2$

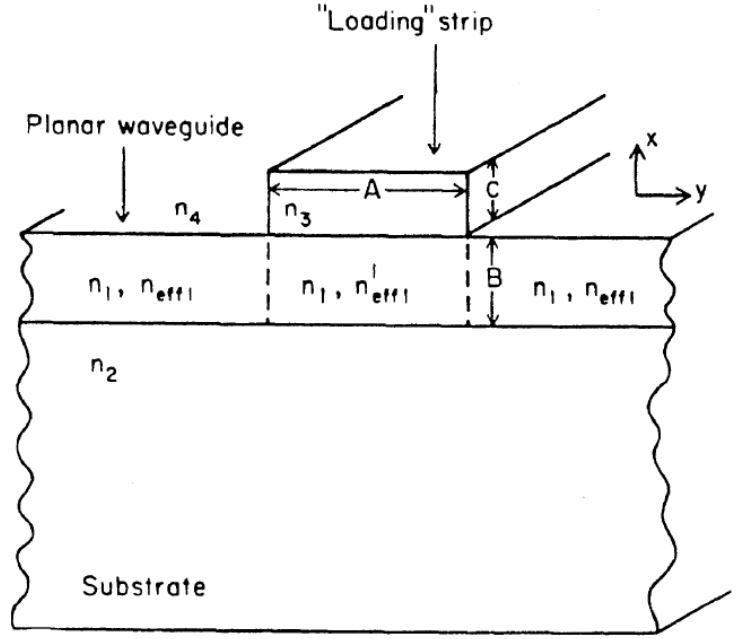
The top layer of the guide (material 2) is often air, but in some cases it can also be replaced by a metallization



Various possible geometries

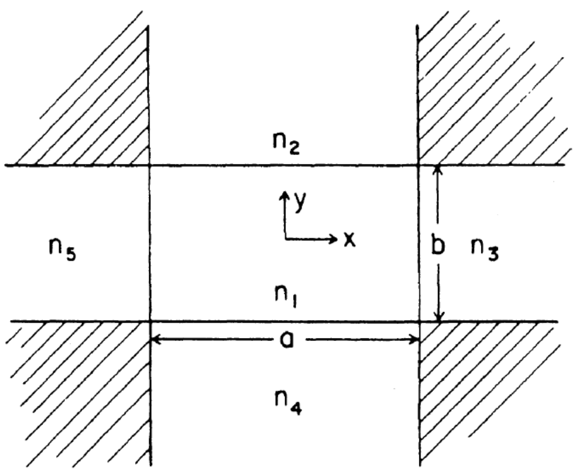


$n_1 > n_2$

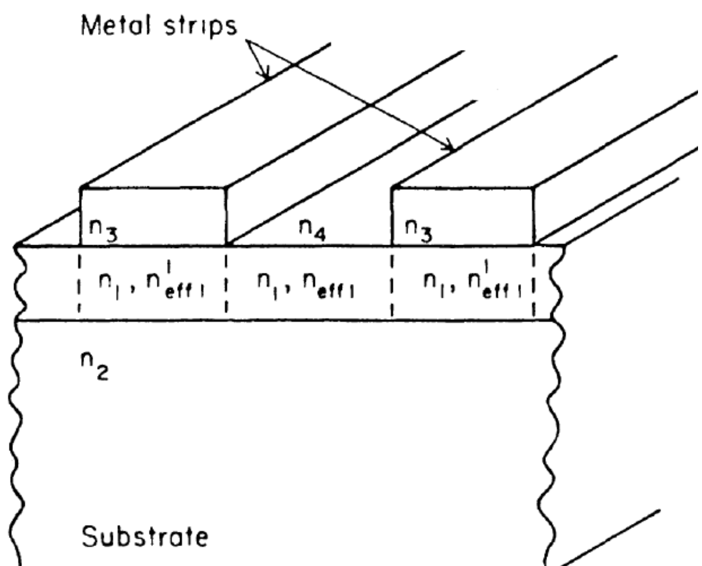


$n_1 > n_2 \geq n_3 > n_4$

or $n_1 > n_3 \geq n_2 > n_4$

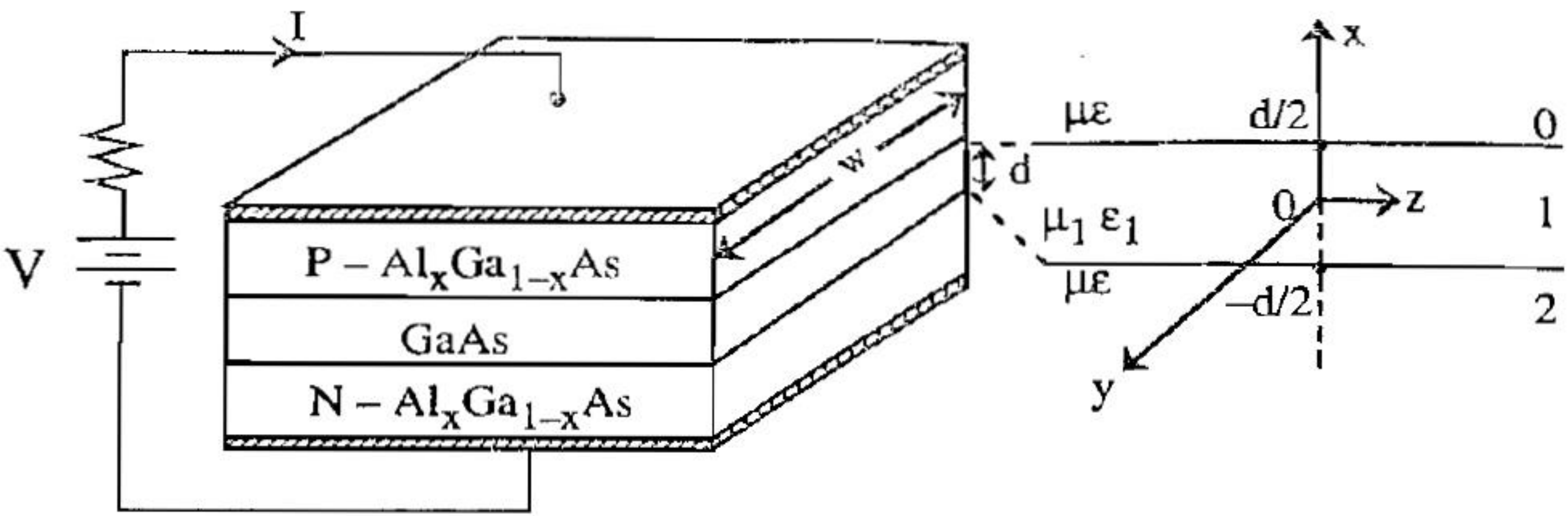


$n_1 > n_2, n_3, n_4, n_5$



$n_1 > n_2$

Guiding structures may be realized as part optoelectronic devices realized with III-V semiconductors.



Refractive index of $\mathbf{Al_xGa_{1-x}As}$

Refractive index "below the band gap" for many III-V compound materials is with good approximation (imaginary part is considered to be negligible below the bandgap)

$$n(\omega) \simeq \sqrt{\frac{\varepsilon'(\omega)}{\varepsilon_o}}$$

Real part of material permittivity

The most popular models are based on semi-empirical fitting of measurements (Adachi, 1985). More complete model based on the Kramers-Kronig relations (Adachi, 1988) addresses the full dielectric function $\epsilon(\omega)=\epsilon_1(\omega)+i\epsilon_2(\omega)$ for the complete energy range.

$$\epsilon_2(\omega)=-\frac{2}{\pi}\int_0^\infty \frac{\epsilon_1(\omega')}{(\omega')^2-\omega^2}d\omega' ,$$

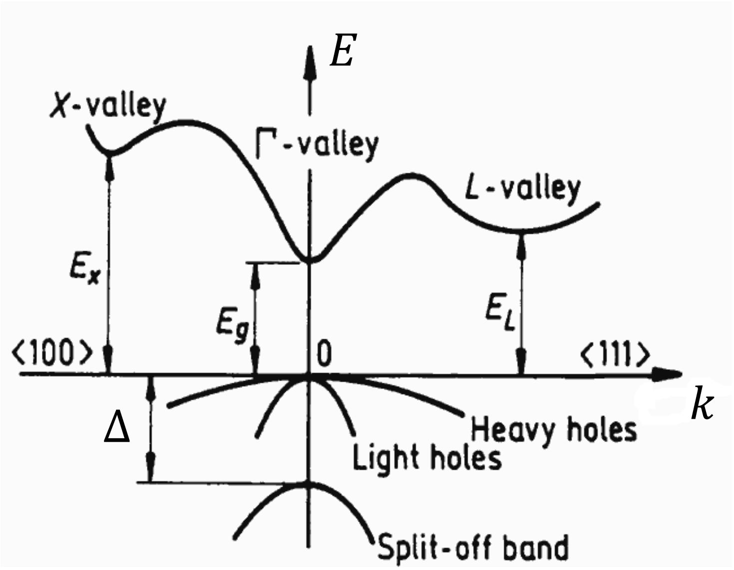
$$\epsilon_1(\omega)-1=\frac{2}{\pi}\int_0^\infty \frac{\omega'\epsilon_2(\omega')}{(\omega')^2-\omega^2}d\omega' .$$

Refractive index of $Al_xGa_{1-x}As$

The approximate model for refractive index below the band gap is mainly related to properties of the band structure

$$\frac{\varepsilon'}{\varepsilon} \simeq A(x)\left\{ f(y) + \frac{1}{2}\left[\frac{E_g(x)}{E_g(x)+\Delta(x)}\right]^{3/2} f(y_{so}) \right\} + B(x)$$

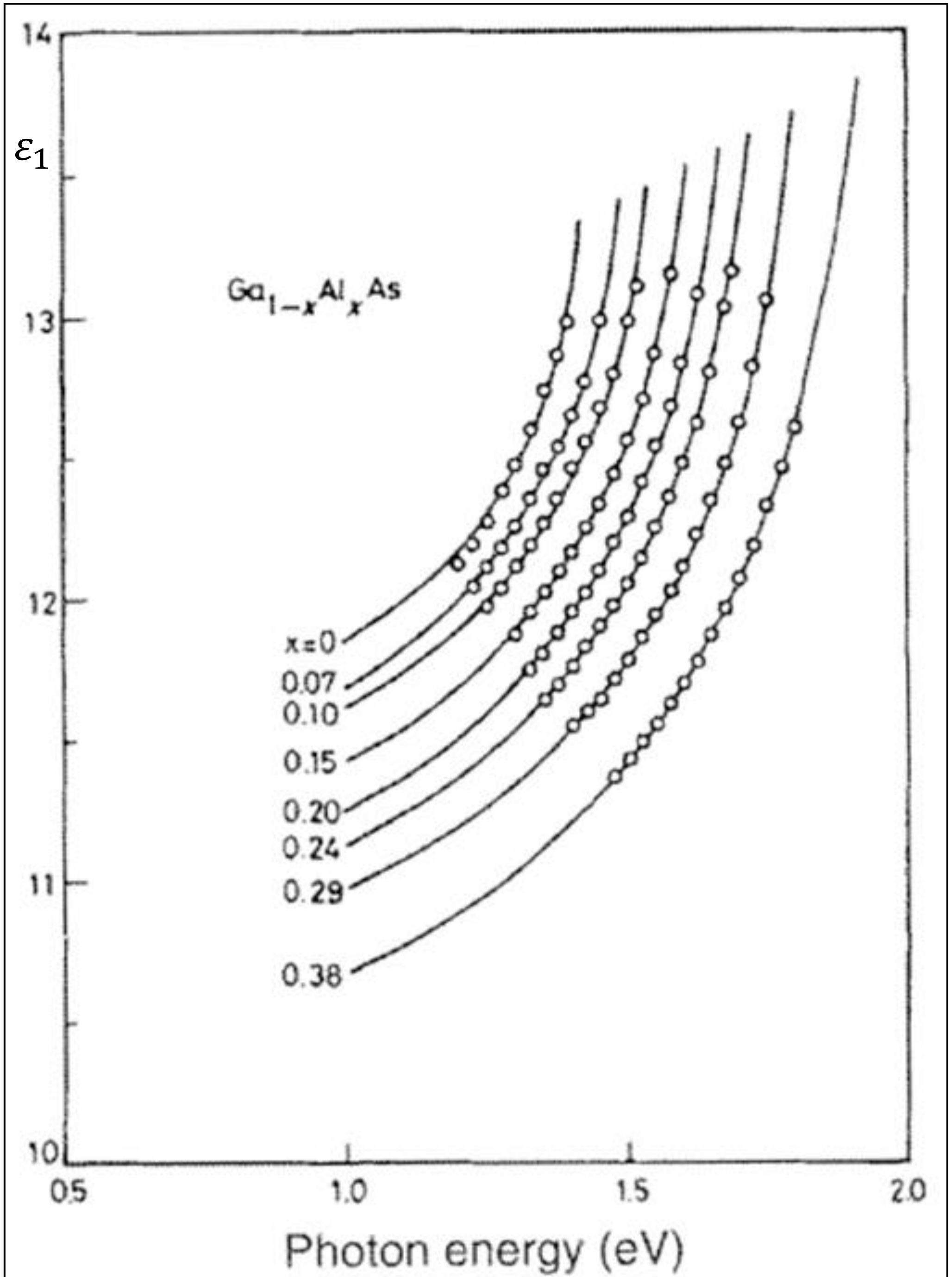
$$\begin{cases} f(y) = \dfrac{1}{y^2}\left[2-(1+y)^{1/2}-(1-y)^{1/2}\right] \\ y = \dfrac{\hbar\omega}{E_g(x)} \\ y_{so} = \dfrac{\hbar\omega}{\left[E_g(x)+\Delta(x)\right]} \end{cases}$$

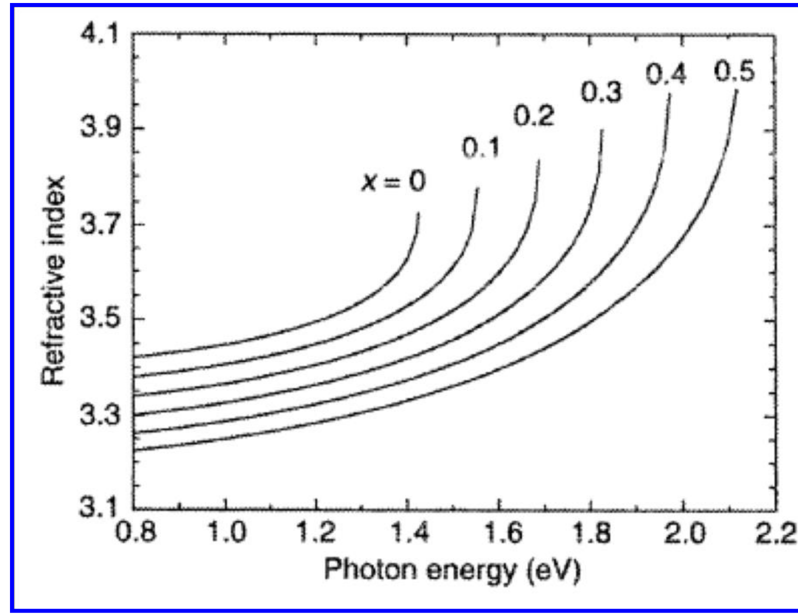


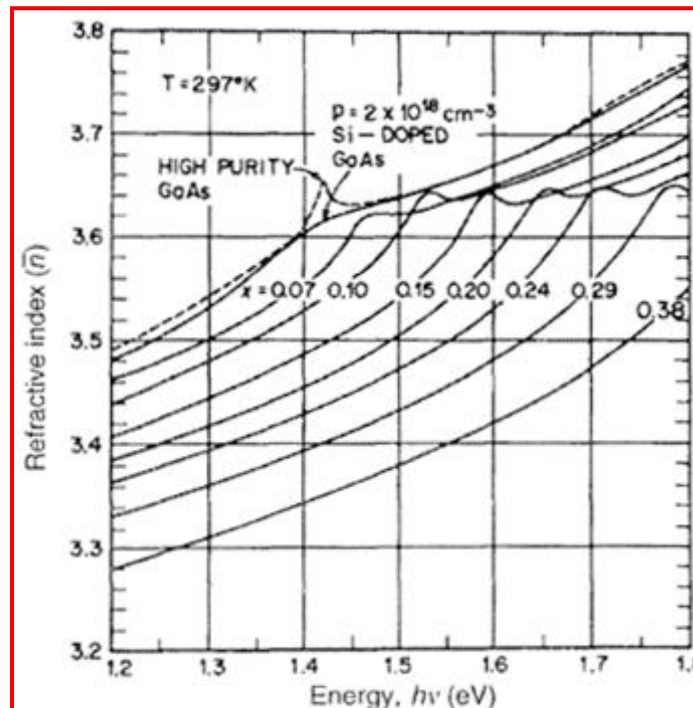
Parameters used in Chuang's book

$$\boxed{\begin{aligned} A(x) &= 6.64 + 16.92x \\ B(x) &= 9.20 - 9.22x \end{aligned}}$$

$$\boxed{\begin{aligned} E_g(x) &= 1.424 + 1.266x + 0.26x^2 \\ \Delta(x) &= 0.34 - 0.5x \end{aligned}}$$

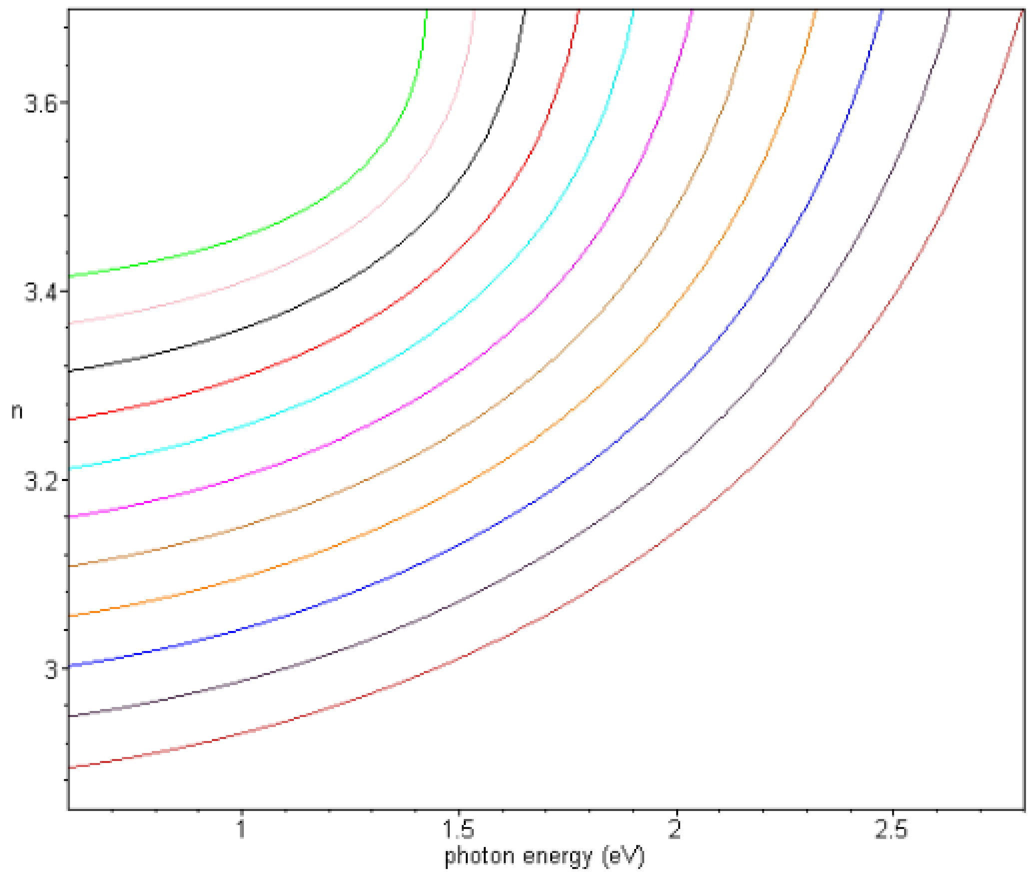

ε_1
$Ga_{1-x}Al_xAs$
x=0
0.07
0.10
0.15
0.20
0.24
0.29
0.38
14
13
12
11
10
0.5
1.0
1.5
2.0
Photon energy (eV)


Refractive index
Photon energy (eV)
x = 0
0.1
0.2
0.3
0.4
0.5
4.1
3.9
3.7
3.5
3.3
3.1
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2


T = 297°K
p = 2 x 10^18 cm^-3
Si – DOPED
GaAs
HIGH PURITY
GaAs
x = 0.07
0.10
0.15
0.20
0.24
0.29
0.38
Refractive index ($\bar{n}$)
Energy, hν (eV)
3.8
3.7
3.6
3.5
3.4
3.3
3.2
1.2
1.3
1.4
1.5
1.6
1.7
1.8

Another parameter set – Adachi (1985)

$$A(x) = 6.3 + 19.0x$$
$$B(x) = 9.4 - 10.2x$$
$$E_g(x) = 1.425 + 1.155\,x + 0.37\,x^2 \text{ [eV]}$$
$$\Delta(x) = 0.34 \text{ [eV]}$$

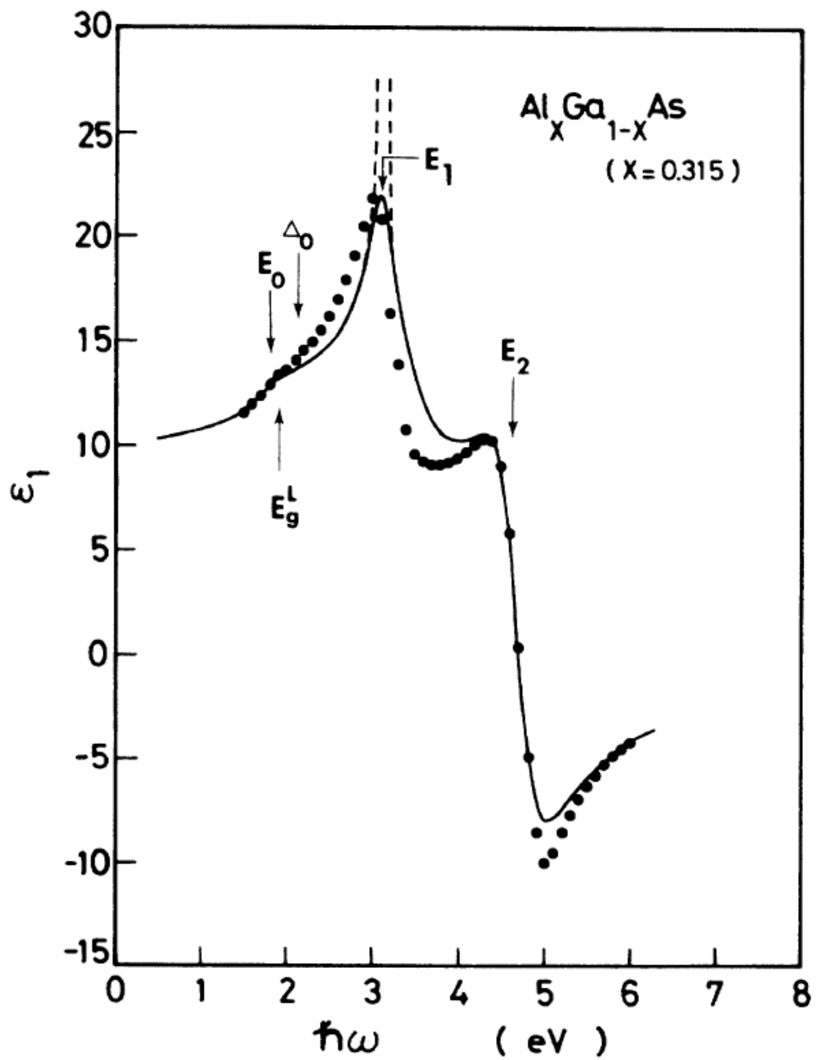


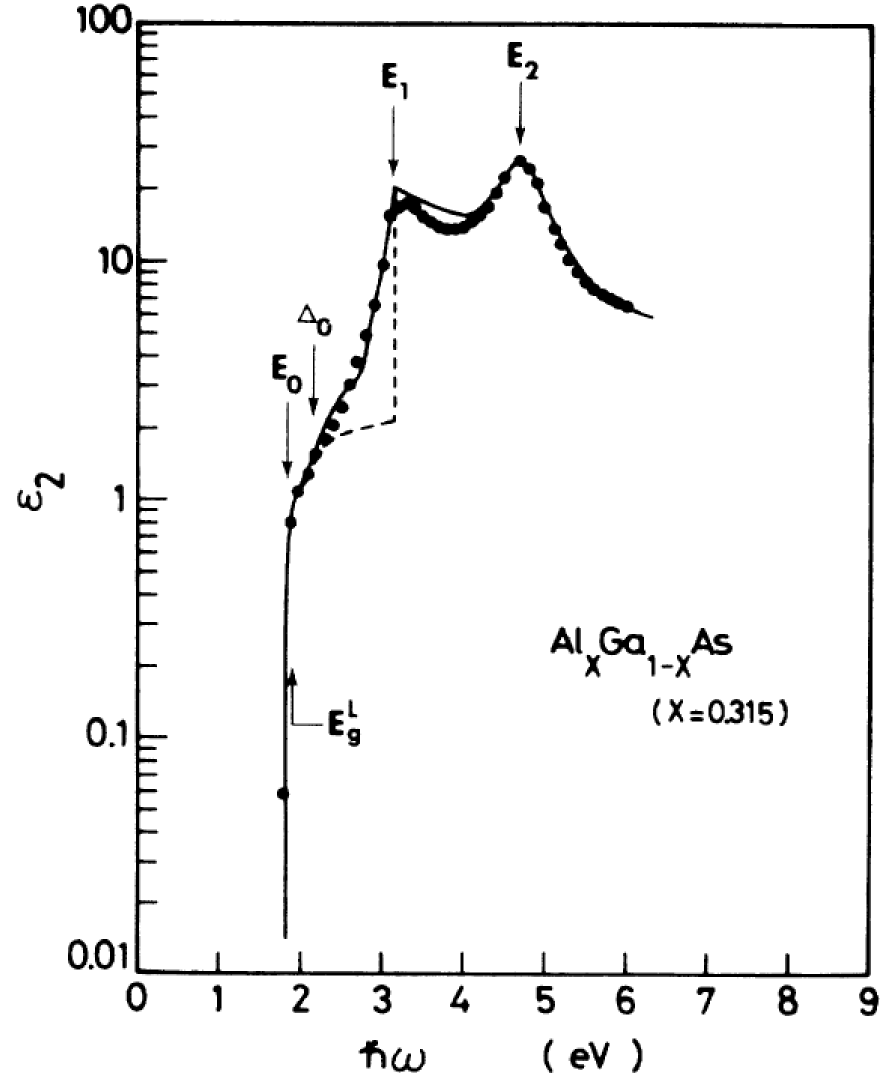
Another parameter set – Adachi (1985)

Calculated real part of refractive index $n(Al_xGa_{1-x}As)$ at T = 300 K

λ (nm)	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
800	-	-	3.551	3.468	3.396	3.332	3.265	3.208	3.141	3.085	3.023
850	-	3.572	3.491	3.423	3.362	3.305	3.240	3.186	3.119	3.061	2.995
900	3.597	3.519	3.455	3.396	3.336	3.277	3.217	3.164	3.106	3.047	2.991
950	3.544	3.486	3.428	3.370	3.315	3.260	3.203	3.148	3.081	3.036	2.973
1000	3.516	3.463	3.410	3.356	3.305	3.245	3.192	3.137	3.068	3.011	2.972
1050	3.497	3.445	3.393	3.342	3.286	3.231	3.178	3.128	3.062	3.002	2.972
1100	3.483	3.434	3.382	3.335	3.273	3.225	3.167	3.114	3.062	3.001	2.968
1150	3.470	3.425	3.372	3.320	3.268	3.217	3.160	3.120	3.055	3.000	2.957
1200	3.464	3.415	3.366	3.314	3.265	3.206	3.154	3.100	3.045	2.994	2.938
1250	3.457	3.407	3.356	3.308	3.253	3.200	3.156	3.089	3.044	2.982	2.932
1300	3.450	3.401	3.354	3.303	3.245	3.202	3.149	3.089	3.034	2.972	2.926
1500	3.434	3.385	3.332	3.289	3.230	3.184	3.133	3.099	3.011	2.951	2.937
2000	3.418	3.366	3.311	3.268	3.223	3.173	3.127	3.056	2.965	2.951	2.915

Example of complete analysis – Adachi (1988)





measurements ●●● D. E. Aspnes, S. M. Kelso, R. A. Logan, and R. Bhat, J. Appl. Phys. **60**, 754 (1986).

Experimental measurements

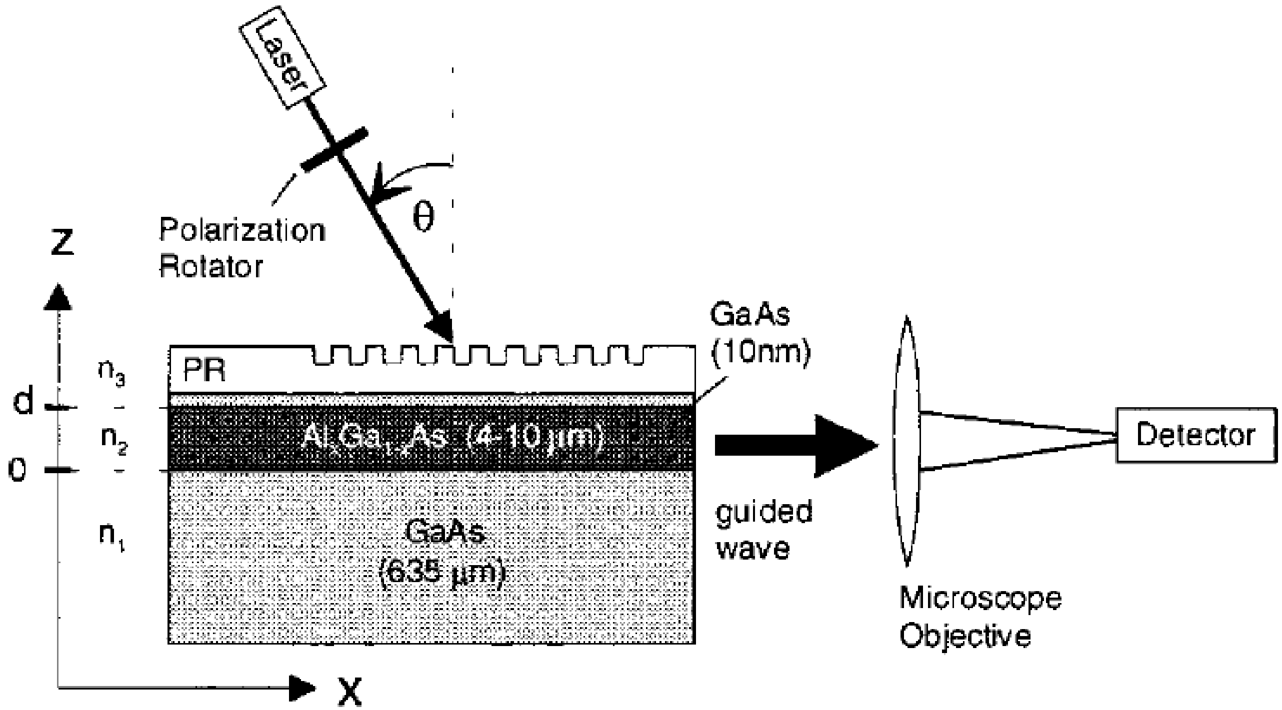


FIG. 1. Schematic sample structure and grating coupling experiment. The thickness of the layers is indicated by the number in brackets. The laser is a single frequency titanium sapphire laser operating in the 730–830 nm range. The detector, either a CCD camera or a photodiode, is positioned at the real image of the cleaved sample. PR: photoresist layer.

S. Gehrsitz, F. K. Reinhart, C. Gourgon, N. Herres, A. Vonlanthen and H. Sigg, "The refractive index of $Al_xGa_{1-x}As$ below the band gap: Accurate determination and empirical modeling," Journal of Applied Physics, vol. 87, p. 7825, 2000.

Experimental measurements

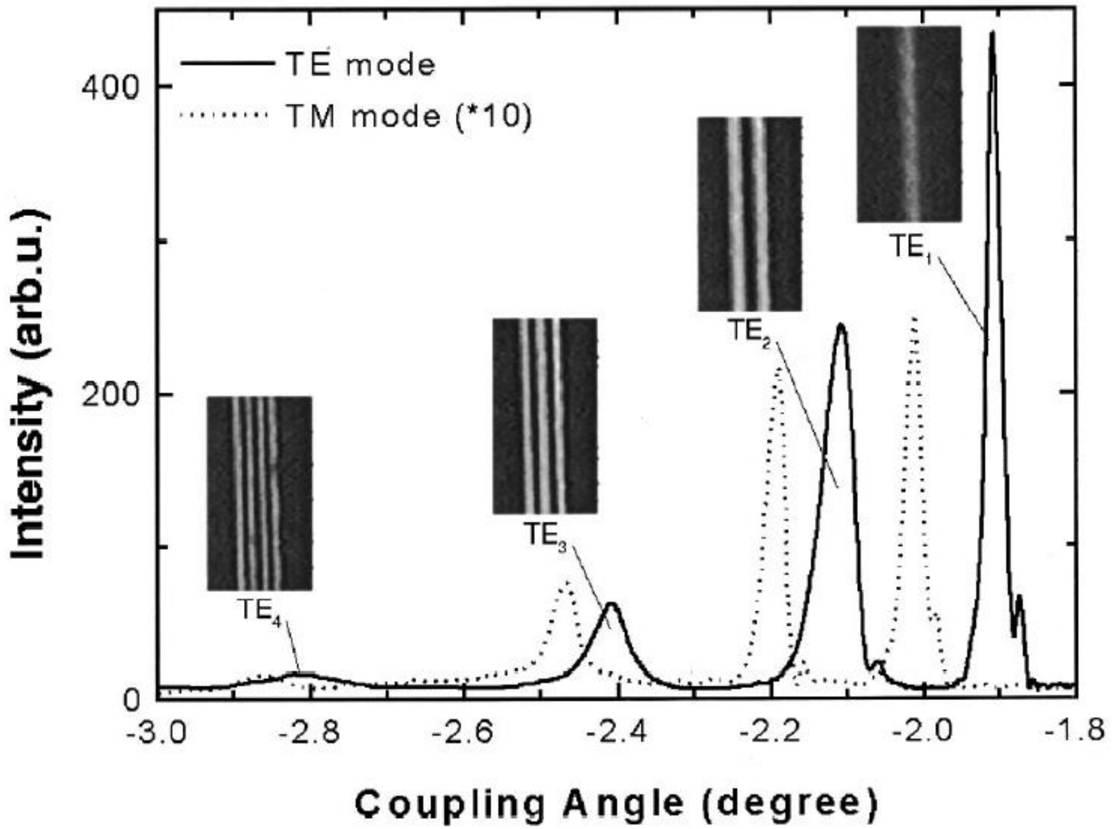


FIG. 2. Angular scan of sample with $x=0.753$ for the TE and TM modes at $\lambda=0.793$ μm and $T=23$ °C. The oscillations on the right hand side of the first and second order peak may be attributed to the leaky character of the modes and/or the limited aperture of the detection system. The insets show the corresponding near field images of the TE modes.

S. Gehrsitz *et al*. (2000)

The Electron-Photon Interaction Hamiltonian

Representative crystal momentum for electron in a crystal

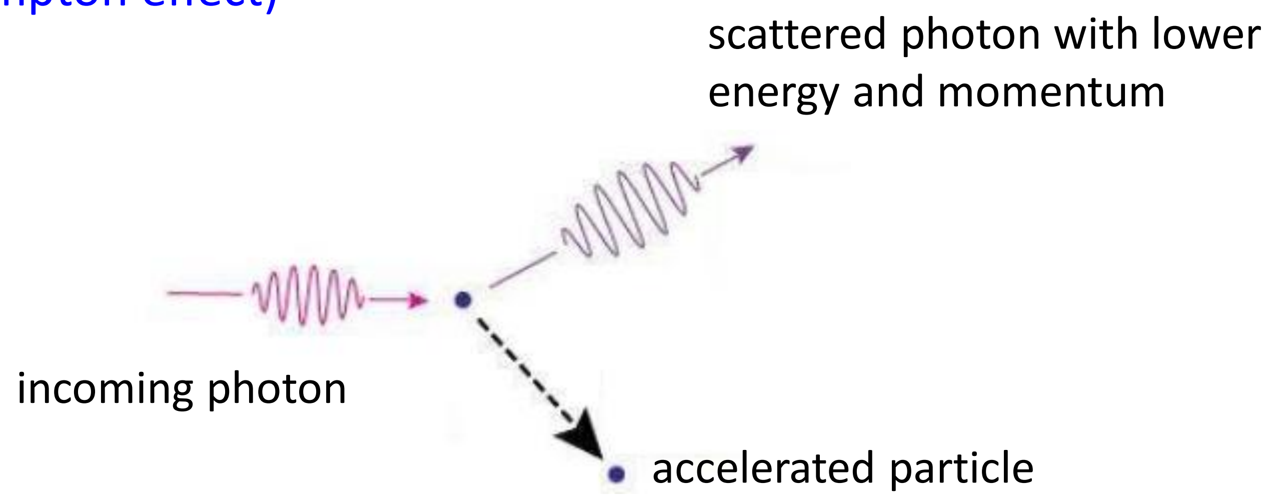
$$\mathbf{p} = \hbar k \approx \frac{\hbar\pi}{a}. \quad \text{With } a_o \approx 5.5\ \text{Å},$$

$$\mathbf{p} \approx \frac{\pi \times 1.054 \times 10^{-34}}{5.5 \times 10^{-10}} = 6.02 \times 10^{-25} \quad \mathrm{J \cdot s/m}$$

Momentum of a photon at 1μm wavelength

$$\hbar k_{opt} \approx \frac{\hbar 2\pi}{\lambda} \approx \frac{2\pi \times 1.054 \times 10^{-34}}{10^{-6}} = 6.6 \times 10^{-28} \quad \mathrm{J \cdot s/m}$$

In free space (Compton effect)



Energy in light wave

$$\mathbf{U_{wave}} = \varepsilon E^2$$

There is no obvious dependence on frequency

$$\mathbf{U_{photons}} = \underset{\substack{\uparrow \\ \text{Number of} \\ \text{photons/m}^3}}{N} \cdot \hbar\omega$$

There is an obvious dependence on frequency

Since we must have

$$\mathbf{U_{wave}} = \mathbf{U_{photons}}$$

the number of photons per unit volume N must be proportional to E^2 with constant of proportionality which depends on the wave frequency

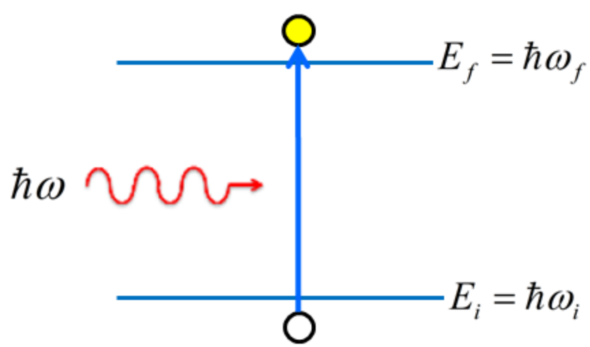
We have seen earlier the Fermi Golden Rule

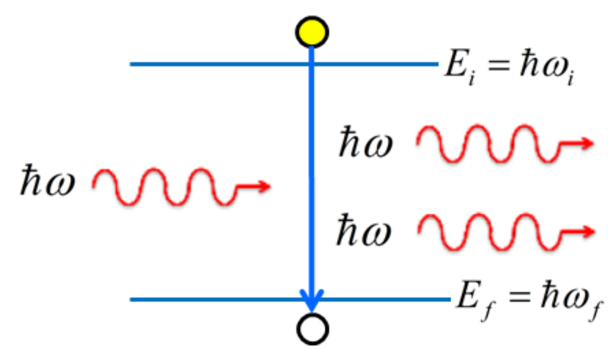
Given a time-harmonic optical perturbation applied at *t=0*

$$H'(\mathbf{r},t) = \begin{cases} H'(\mathbf{r})e^{-i\omega t} + H'^{+}(\mathbf{r})e^{+i\omega t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Transition rate of a particle from state "i" to state "f"

$$W_{if} = \frac{d}{dt}\left|a_f^{(1)}(t)\right|^2 = \underbrace{\frac{2\pi}{\hbar}\left|H'_{fi}\right|^2 \delta\left(E_f - E_i - \hbar\omega\right)}_{\textbf{Absorption}} + \underbrace{\frac{2\pi}{\hbar}\left|H'^{+}_{fi}\right|^2 \delta\left(E_f - E_i + \hbar\omega\right)}_{\textbf{Stimulated Emission}}$$





$$H'_{mn}(t) = \langle m | H'(\mathbf{r},t) | n \rangle = \int_V \phi_m^*(\mathbf{r})\left(H'(\mathbf{r})e^{-i\omega t} + H'^{+}(\mathbf{r})e^{+i\omega t}\right)\phi_n(\mathbf{r})dV$$

Optical Transitions Using Fermi Golden Rule

Hamiltonian describing the electron-photon interaction in a semiconductor

$$H = \frac{1}{2m_0}\left(\mathbf{p} - e\mathbf{A}\right)^2 + V\left(\mathbf{r}\right)$$

periodic crystal potential

EM vector potential

electron charge

momentum variable

Expansion of the Hamiltonian

Hamiltonian for interaction with light: $H = \frac{1}{2m_0}(\mathbf{p} - e\mathbf{A})^2 + V(\mathbf{r})$

Expand the squared term

$$H = \underbrace{\frac{p^2}{2m_o} + V(\mathbf{r})}_{\substack{H_0 \\ \text{Unperturbed} \\ \text{Hamiltonian}}} \underbrace{- \frac{e}{2m_o}(\mathbf{p} \bullet \mathbf{A} + \mathbf{A} \bullet \mathbf{p}) + \frac{e^2 A^2}{2m_o}}_{\substack{H' \\ \text{Perturbed} \\ \text{Hamiltonian}}}$$

Simplifications:

- In practical cases $|eA| \ll |\mathbf{p}|$ so the term $\frac{e^2 A^2}{2m_0}$ is neglected
- Choice of Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ such that $\mathbf{p} \cdot \mathbf{A} = \mathbf{A} \cdot \mathbf{p}$

$$\Rightarrow \quad \boxed{H' \simeq -\frac{e}{m_o}\mathbf{A} \bullet \mathbf{p}}$$

Expansion of the Hamiltonian

Hamiltonian for interaction with light: $H = \frac{1}{2m_0}(\mathbf{p} - e\mathbf{A})^2 + V(\mathbf{r})$

Expand the squared term

$$H = \underbrace{\frac{p^2}{2m_o} + V(\mathbf{r})}_{H_0} \underbrace{- \frac{e}{2m_o}(\mathbf{p}\cdot\mathbf{A} + \mathbf{A}\cdot\mathbf{p}) + \frac{e^2A^2}{2m_o}}_{H'}$$

Unperturbed Hamiltonian

Perturbed Hamiltonian

$$\boxed{\left(\hat{\vec{p}}\cdot\vec{A} - \vec{A}\cdot\hat{\vec{p}}\right) f(\vec{r}) = \frac{\hbar}{i}\left(\vec{A}\cdot\nabla f + f\,\nabla\cdot\vec{A} - \vec{A}\cdot\nabla f\right) = \frac{\hbar}{i} f\,\nabla\cdot\vec{A}}$$

- Choice of Coulomb gauge $\nabla\cdot\mathbf{A} = 0$ such that $\mathbf{p}\cdot\mathbf{A} = \mathbf{A}\cdot\mathbf{p}$

$$\Rightarrow \quad \boxed{H' \simeq -\frac{e}{m_o}\mathbf{A}\cdot\mathbf{p}}$$

$\mathbf{A}$ is the vector potential

ϕ is the scalar potential

Coulomb Gauge $\nabla \bullet \mathbf{A} = 0$

Lorenz Gauge $\nabla \bullet \mathbf{A} = -\mu\varepsilon \frac{\partial \phi}{\partial t}$

The number of possible gauges is infinite. We are free to some extent to choose the gauge which simplifies solution of a problem, as long as the underlining physics is not affected.

Coulomb gauge is often used when no sources are present.

The Lorenz gauge is necessary when the action of "retarded potentials" has to be considered.

Optical Fields

We define the perturbing field through the magnetic vector potential

$$\mathbf{A} = \hat{e} A_o \cos\left(\mathbf{k_{op}} \bullet \mathbf{r} - \omega t\right) = \hat{e}\frac{A_o}{2} e^{i\mathbf{k_{op}}\bullet\mathbf{r}} e^{-i\omega t} + \hat{e}\frac{A_o}{2} e^{-i\mathbf{k_{op}}\bullet\mathbf{r}} e^{+i\omega t}$$

unit vector (direction of Electric Field with assumption below)

Electric Field (V/m)

$$\mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{A}}{\partial t} \boxed{-\nabla\phi}$$

vanishes for optical field

$$= -\frac{\partial \mathbf{A}}{\partial t} = -\hat{e}\omega A_o \sin\left(\mathbf{k_{op}} \bullet \mathbf{r} - \omega t\right)$$

Magnetic Field (A/m)

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{\mu}\nabla \times \mathbf{A} = -\frac{1}{\mu}\mathbf{k_{op}} \times \hat{e} A_o \sin\left(\mathbf{k_{op}} \bullet \mathbf{r} - \omega t\right)$$

Assumption

$$\mu = \mu_o$$

Poynting Vector

$$\mathbf{P}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t) = \hat{k}\ k_{op} \frac{\omega A_o^2}{\mu} \sin^2\left(\mathbf{k_{op}} \bullet \mathbf{r} - \omega t\right)$$

direction of power flow

Time Average Poynting Flux

$$\left|\left\langle \mathbf{P}(\mathbf{r},t) \right\rangle\right| = \frac{\omega A_o^2}{2\mu} k_{op}$$

We have used $\boxed{\left|\left\langle \sin^2(x) \right\rangle\right| = \frac{1}{2}}$

$$\boxed{k_{op} = \frac{2\pi}{\lambda} = \frac{2\pi f}{\mathrm{v}_{ph}} = \frac{\omega\sqrt{\mu_r \varepsilon_r}}{c} = \frac{\omega\sqrt{\varepsilon_r}}{c} = \frac{\omega n_r}{c}}$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \Rightarrow \quad \mu_0 = \frac{1}{c^2 \varepsilon_0}}$$

$$\boxed{\mu = \mu_o}$$

$$\left|\left\langle \mathbf{P}(\mathbf{r},t) \right\rangle\right| = \frac{n_r c \varepsilon_o \omega^2 A_o^2}{2}$$

Interaction Hamiltonian

Substitute the assumed magnetic vector potential into the expanded Hamiltonian

$$\mathbf{A} = \hat{e} A_o \cos\left(\mathbf{k}_{\mathbf{op}} \bullet \mathbf{r} - \omega t\right) = \hat{e}\frac{A_o}{2} e^{i\mathbf{k}_{\mathbf{op}} \bullet \mathbf{r}} e^{-i\omega t} + \hat{e}\frac{A_o}{2} e^{-i\mathbf{k}_{\mathbf{op}} \bullet \mathbf{r}} e^{+i\omega t}$$

$$\boxed{H' \simeq -\frac{e}{m_o}\mathbf{A} \bullet \mathbf{p}}$$

$$H'(\mathbf{r}, t) = -\frac{e A_o e^{i\mathbf{k}_{\mathbf{op}} \bullet \mathbf{r}}}{2 m_o}\left(\hat{e} \bullet \mathbf{p}\right) e^{-i\omega t} - \frac{e A_o e^{-i\mathbf{k}_{\mathbf{op}} \bullet \mathbf{r}}}{2 m_o}\left(\hat{e} \bullet \mathbf{p}\right) e^{+i\omega t}$$

Interaction Hamiltonian

$$H'(\mathbf{r},t) = \underbrace{-\frac{eA_o e^{i\mathbf{k}_{op}\cdot\mathbf{r}}}{2m_o}(\hat{e}\cdot\mathbf{p})}_{H'(\mathbf{r})} e^{-i\omega t} \underbrace{-\frac{eA_o e^{-i\mathbf{k}_{op}\cdot\mathbf{r}}}{2m_o}(\hat{e}\cdot\mathbf{p})}_{H'^{+}(\mathbf{r})} e^{+i\omega t}$$

$$\boxed{H'(\mathbf{r},t) = H'(\mathbf{r})e^{-i\omega t} + H'^{+}(\mathbf{r})e^{+i\omega t}}$$

$$\left\{\begin{aligned} H'(\mathbf{r}) &= -\frac{eA_o e^{i\mathbf{k}_{op}\cdot\mathbf{r}}}{2m_o}(\hat{e}\cdot\mathbf{p}) \\ H'^{+}(\mathbf{r}) &= -\frac{eA_o e^{-i\mathbf{k}_{op}\cdot\mathbf{r}}}{2m_o}(\hat{e}\cdot\mathbf{p}) \end{aligned}\right.$$

Hermitian adjoint (conjugate) of $H'(\mathbf{r})$

Hermitian Conjugate Operator: $\langle H^{+}\varphi | \psi \rangle = \langle \varphi | H\psi \rangle$

Reading Assignments:

Chapter 7 of Chuang's book
Section 9.1 of Chuang's book

Chapter 7 of Coldren & Corzine's book (supplemental)
Appendix 3 (optical waveguides)
Appendix 9 (Fermi Golden rule)

References downloadable from the digital library:

S. Adachi, "Properties of Semiconductor Alloys: Group IV, III-V and II-VI Semiconductors," Wiley, 2009

S. Adachi, "Physical Properties of III-V Semiconductor Compounds – InP, InAs, GaAs, GaP, InGaAs, and InGaAsP," Wiley, 1992