ECE 536 – Integrated Optics and Optoelectronics
Lecture 11 – February 22, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

Prof. Umberto Ravaioli

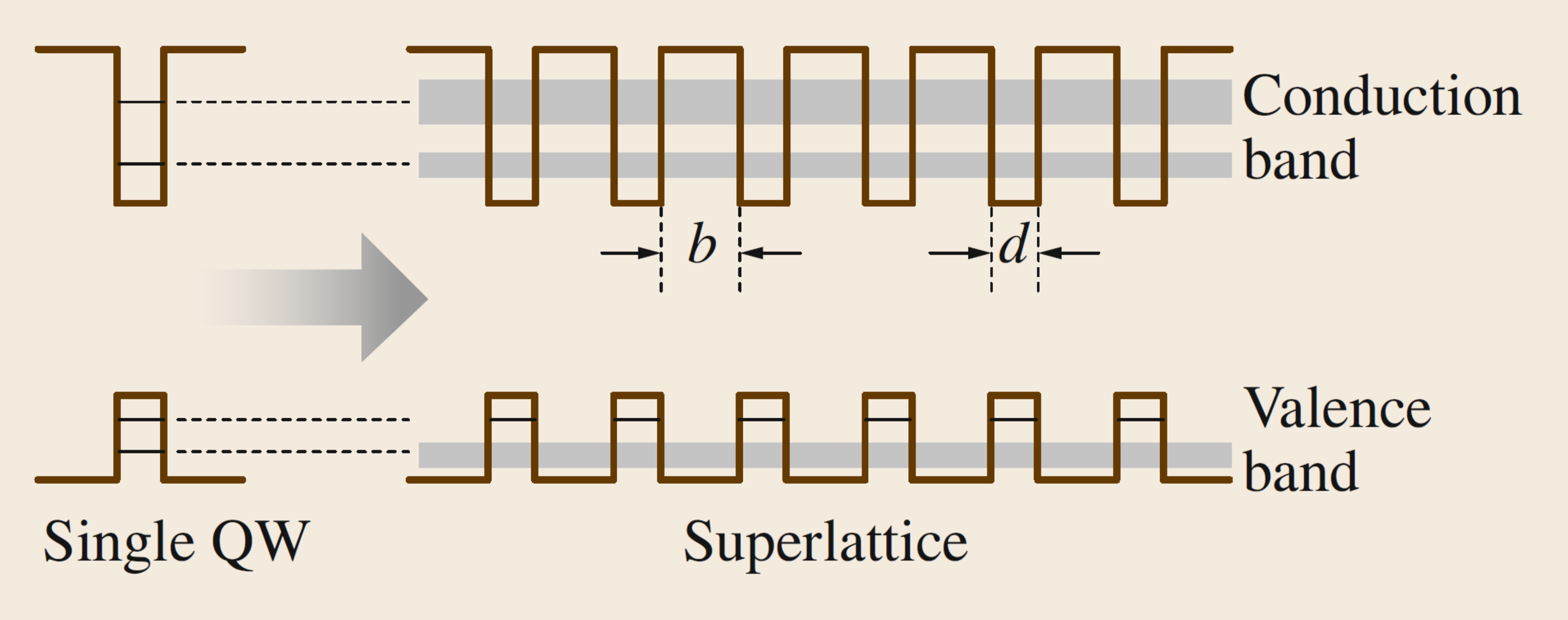
ECE Department, University of Illinois

Lecture 11 Outline

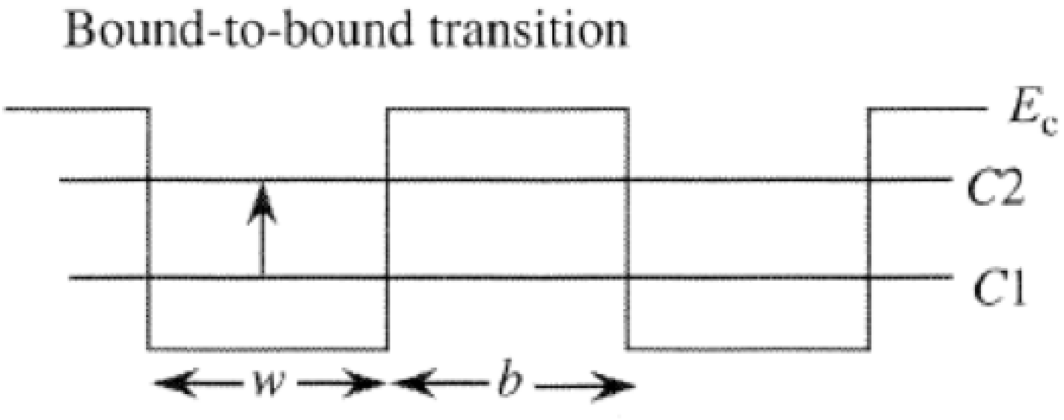
- Transitions in Superlattices
- Quantum cascade
- Laser principles
- Heterojunction behavior
- Semiconductor lasers

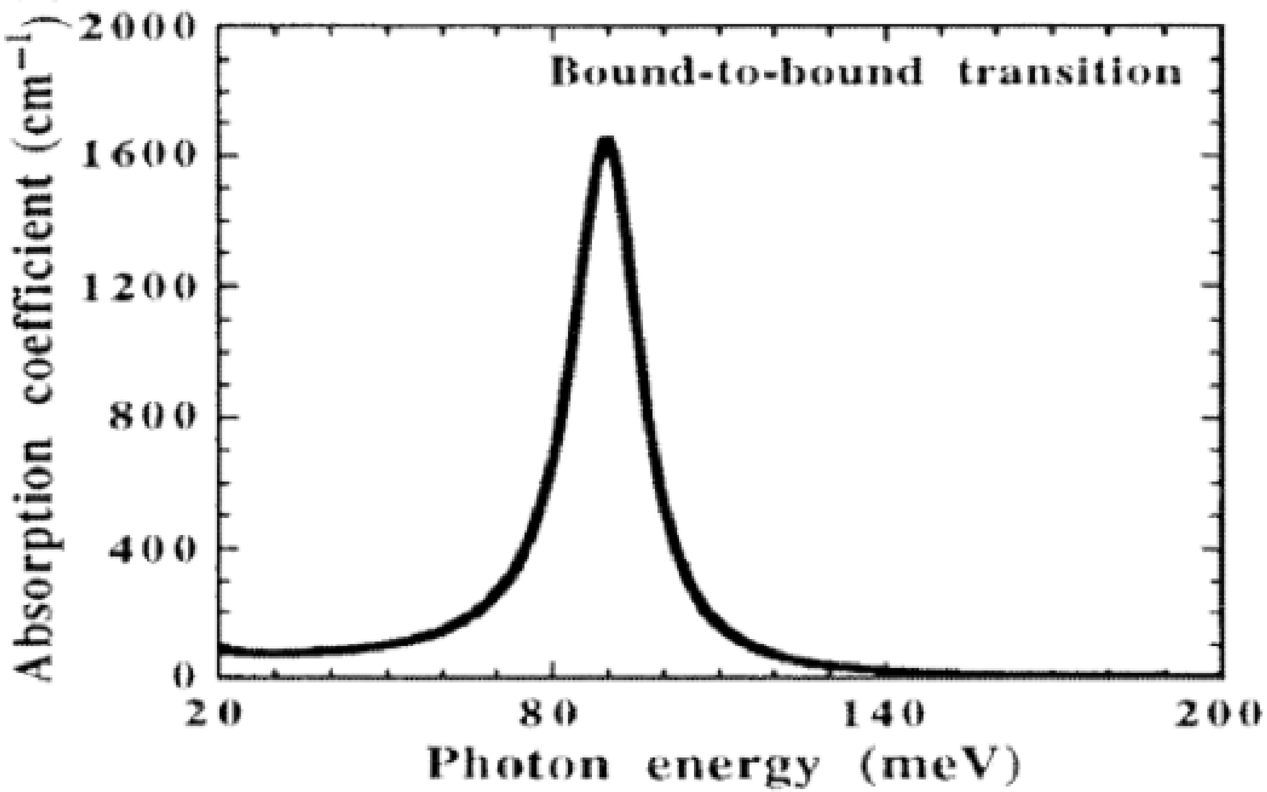
Absorption in Superlattices

Superlattice



Absorption in a Superlattice (GaAs/$Al_{0.3}Ga_{0.7}As$)





$T = 77\text{K}$

$N_s = 2 \times 10^{11}\text{cm}^{-2}$

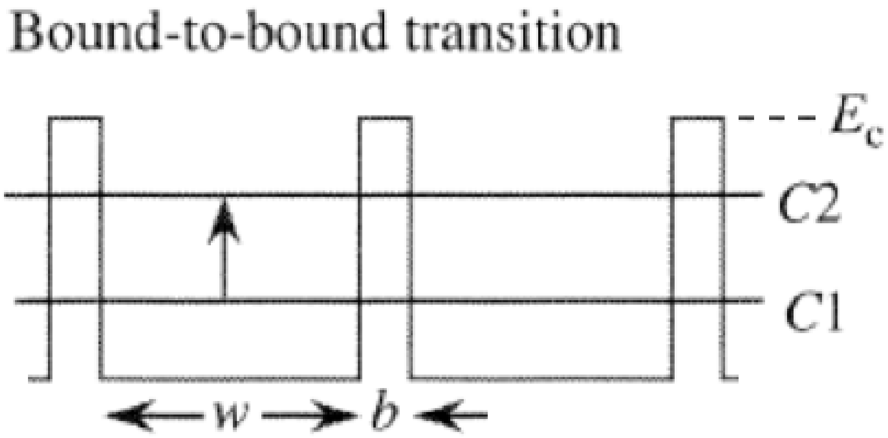
$2\gamma = 15\text{ meV}$

$\boxed{w = b = 100\text{Å}}$

Thick barriers *b*

Small miniband widths

Absorption in a Superlattice


Bound-to-bound transition

$T = 77\text{K}$

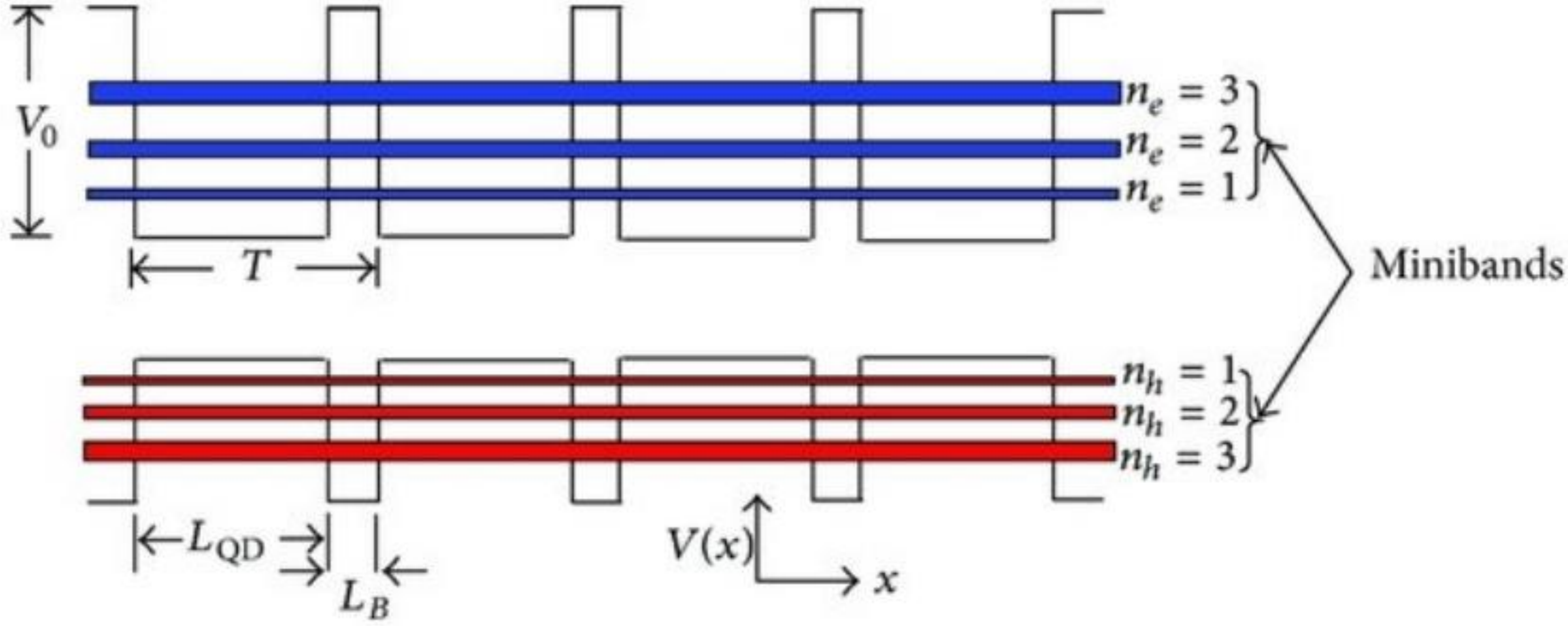
$N_s = 2 \times 10^{11}\text{cm}^{-2}$

$2\gamma = 15\text{ meV}$

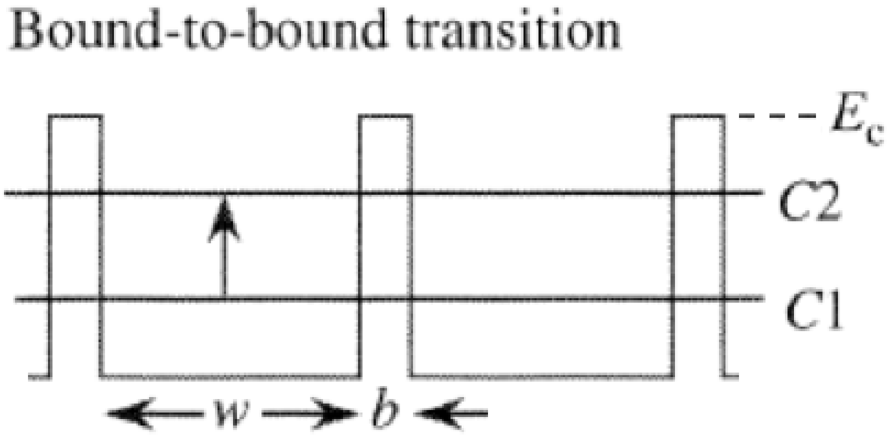
$w = 100\text{Å}$
$b = 20\text{Å}$

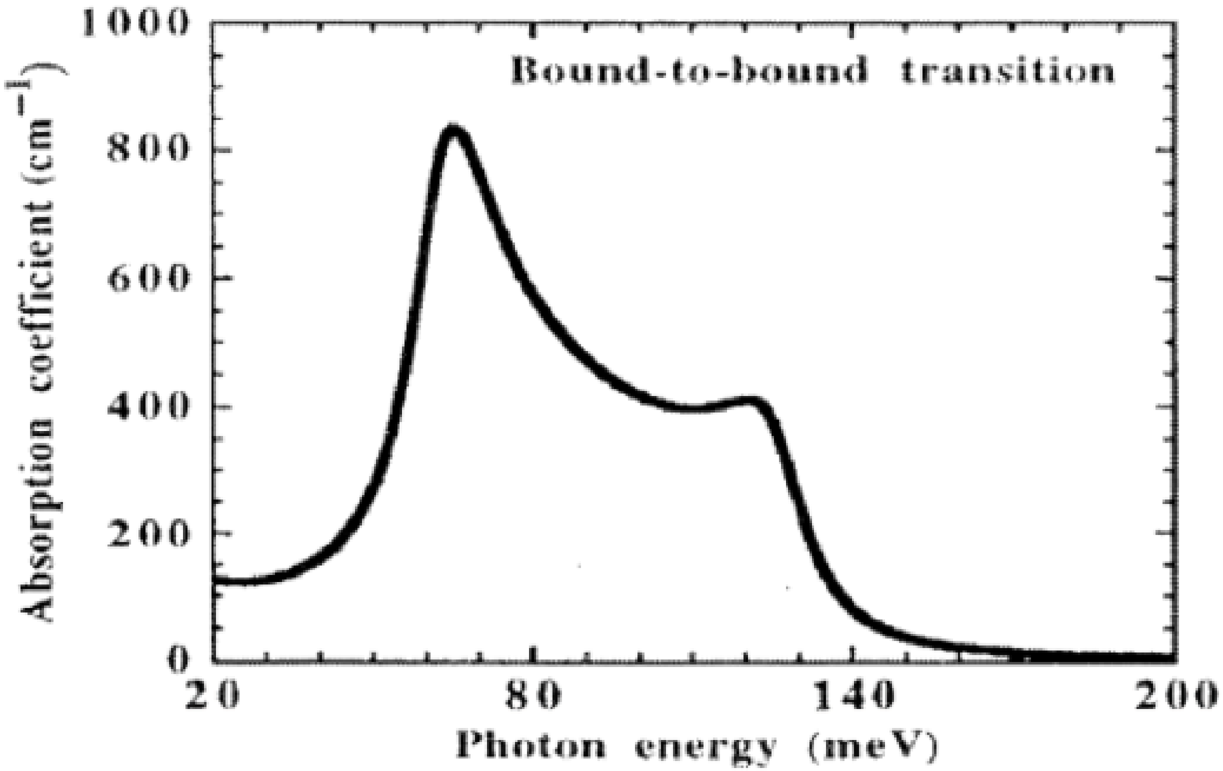
Thin barriers *b*

Large miniband widths



Absorption in a Superlattice





$T = 77\text{K}$

$N_s = 2 \times 10^{11}\text{cm}^{-2}$

$2\gamma = 15 \text{ meV}$

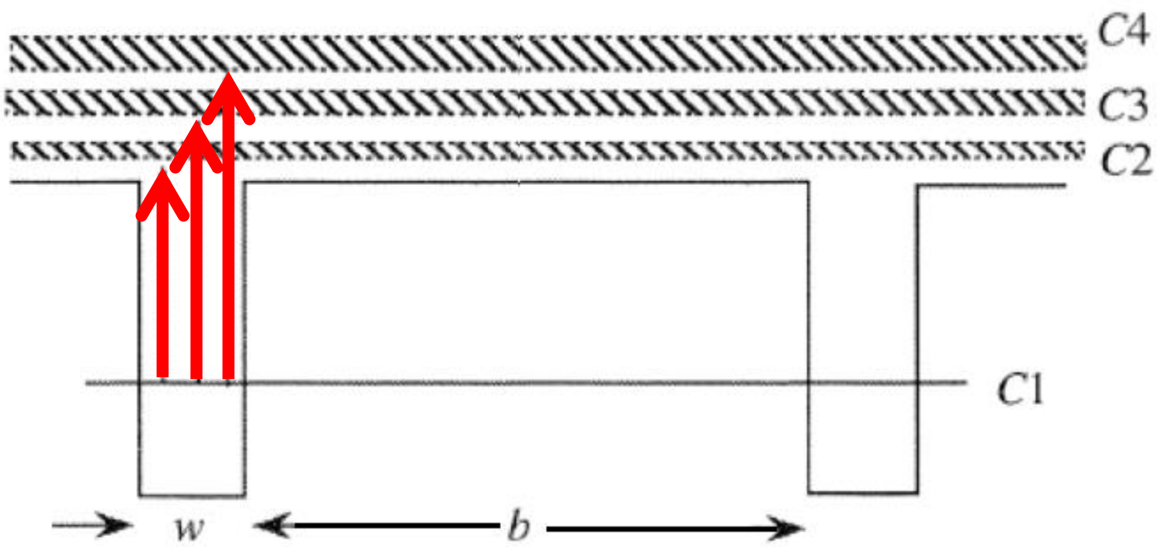
$w = 100\text{Å}$
$b = 20\text{Å}$

Thin barriers *b*

Large miniband widths

Absorption in a Superlattice

Bound-to-continuum miniband transitions

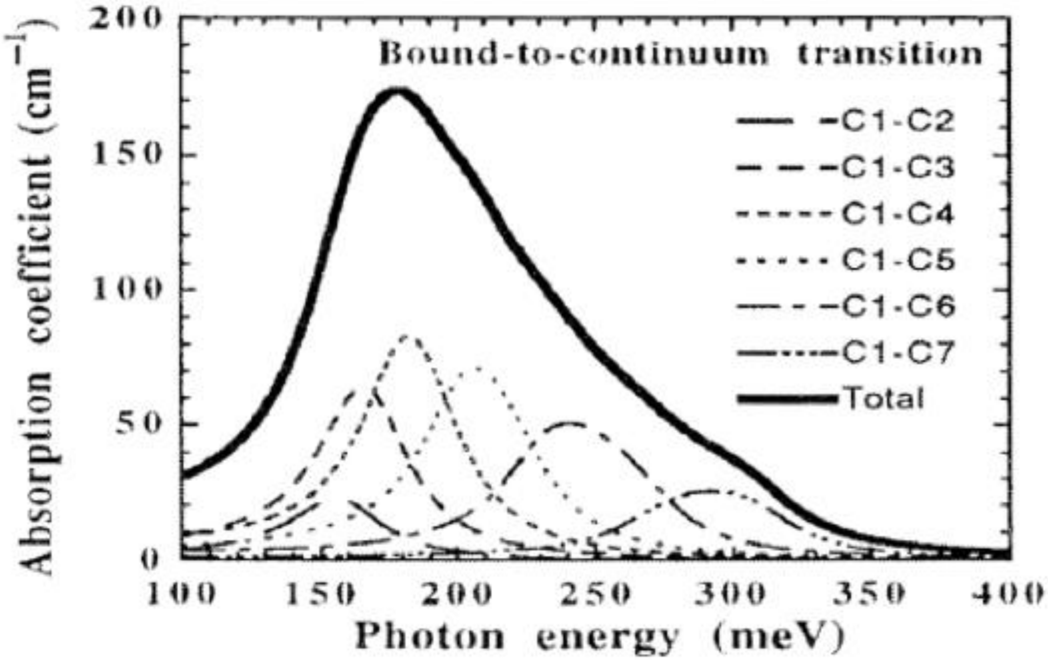


$T = 77\text{K}$

$N_s = 2 \times 10^{11}\text{cm}^{-2}$

$2\gamma = 15\ \text{meV}$

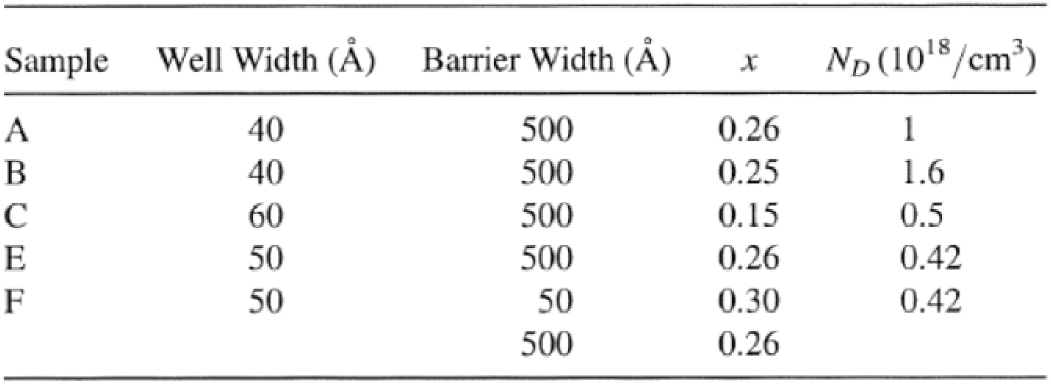
$w = 40\text{Å}$
$b = 300\text{Å}$



Experimental Subband Transition Examples

GaAs QW
$Al_xGa_{1-x}As$ Barrier

[from Levine, *J. Appl. Phys.*, 1993]

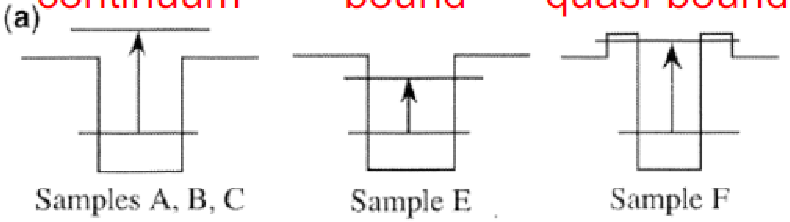


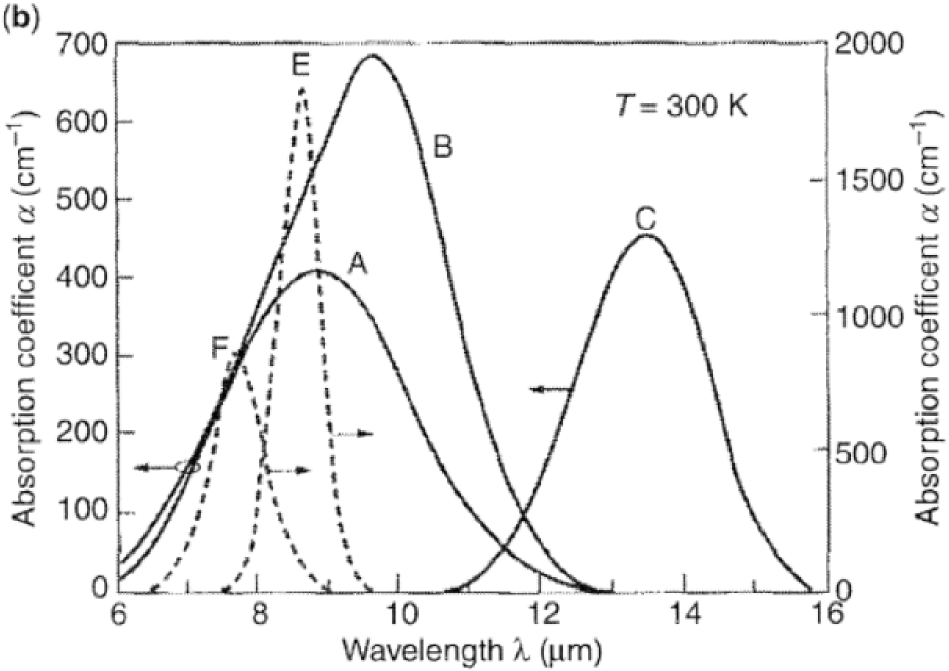
Sample	Well Width (Å)	Barrier Width (Å)	x	N_D ($10^{18}/cm^3$)
A	40	500	0.26	1
B	40	500	0.25	1.6
C	60	500	0.15	0.5
E	50	500	0.26	0.42
F	50	50	0.30	0.42
		500	0.26	

Bound to continuum

Bound to bound

Bound to quasi-bound





A, B, C have weak absorption but good carrier collection

E has the best absorption but doesn't have good carrier collection

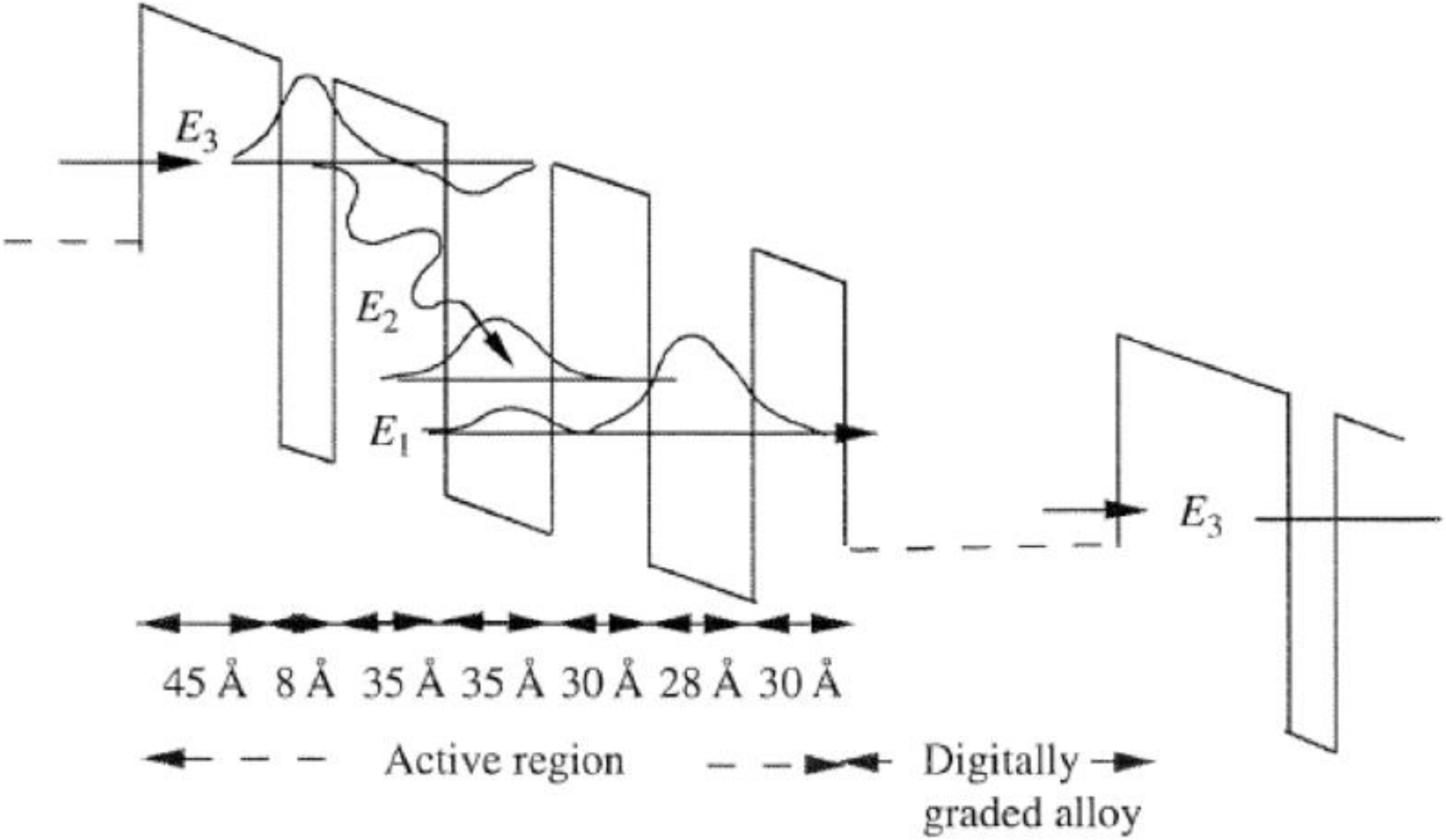
F has good absorption and carrier collection

Quantum Cascade Laser

QCL is an intersubband laser where electrons are injected by tunneling through the barrier into E_3 (t_3=0.2ps).

Small overlap between E_3 and E_2 wavefunctions creates long decay time (t_{32}=4.3ps) and thus a population inversion between states E_3 and E_2 for lasing action.

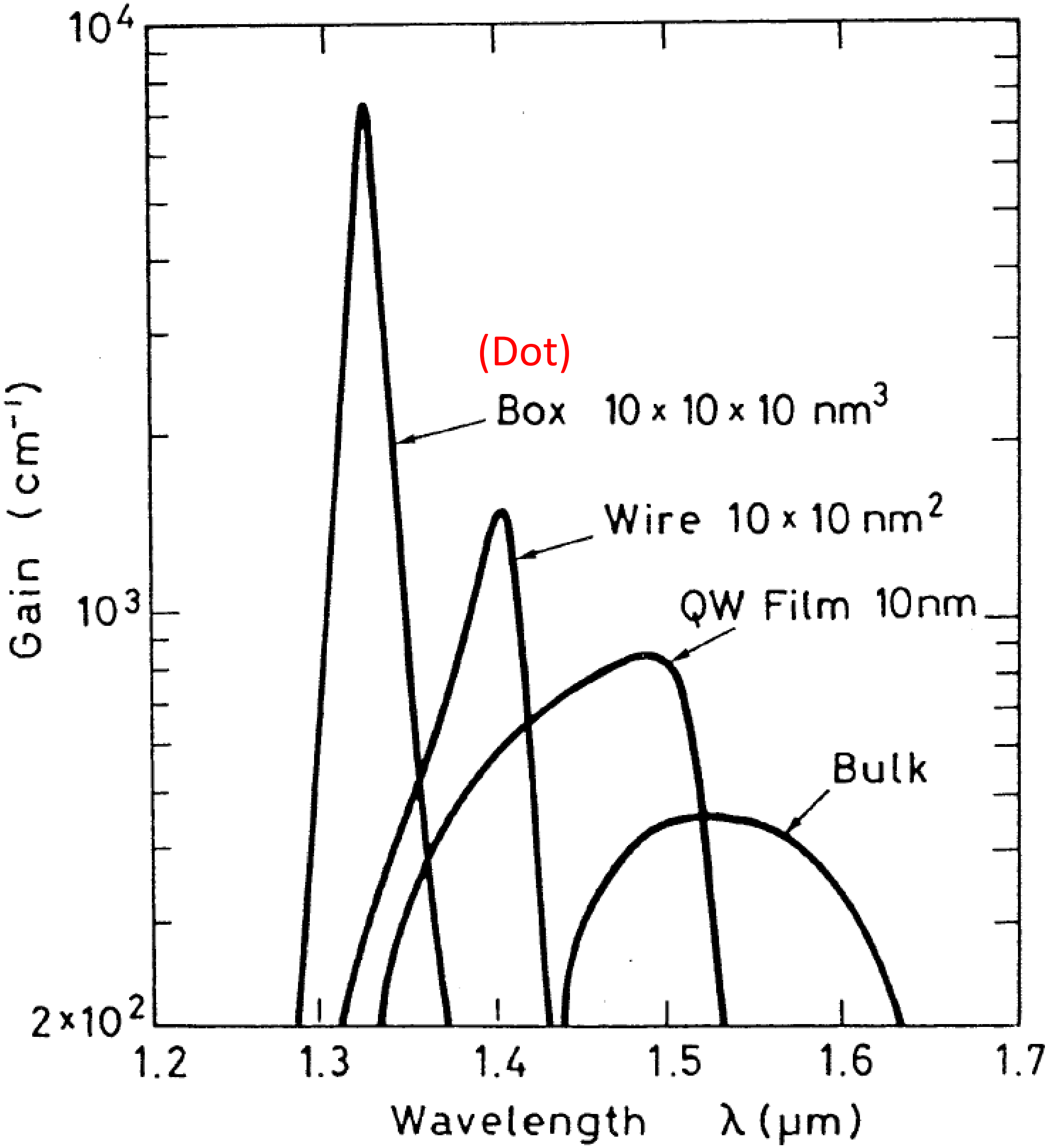
Quick decay from E_2 to E_1 (t_{21}=0.6ps).



Gain in Semiconductors

Calculated gain coefficient

$Ga_{047}In_{053}$.As/InP
$T = 300$ K
$\tau_{in} = 1 \times 10^{-13}$ s
$N = 3 \times 10^{18}$ cm^{-3}



M. Asada, Y. Miyamoto, and Y. Suematsu, Gain and Threshold of Three-Dimensional Quantum-Box Lasers, *IEEE J. Quantum Electron* **QE–22**, 1915–1921 (1986).

Final Summary on Gain

- **The stimulated events must exceed the absorption events for gain. At any transition frequency, the maximum gain coefficient g_{max} is attained at *T* = 0K and $g_{max} = \alpha_0$.**

(Adapted from textbook by Corzine, Coldren, Mašanović)

$$|H'_{21}|^2 = \left(\frac{eA_0}{2m_0}\right)^2 |M_T|^2 \qquad |M_T|^2 = |\langle u_c|\hat{e}\cdot\mathbf{p}|u_v\rangle|^2\, |\langle\varphi_2|\varphi_1\rangle|^2$$

Material gain per unit length under pumping/injection conditions

$$g_{21} = g_{max}(E_{21})\,[f_c(E_2) - f_v(E_1)]$$

$$g_{max}(E_{21}) = \frac{\pi e^2\hbar}{n\varepsilon_0 c m_0^2}\,\frac{1}{\hbar\omega_{21}}|M_T(E_{21})|^2\,\rho_r(E_{21})$$

Same as $\alpha_0(\hbar\omega)$

Final Summary on Gain

Adapted from textbook by Corzine, Coldren, Mašanović

bulk case

$$|M_T|^2 = \frac{1}{3}\,|M|^2$$

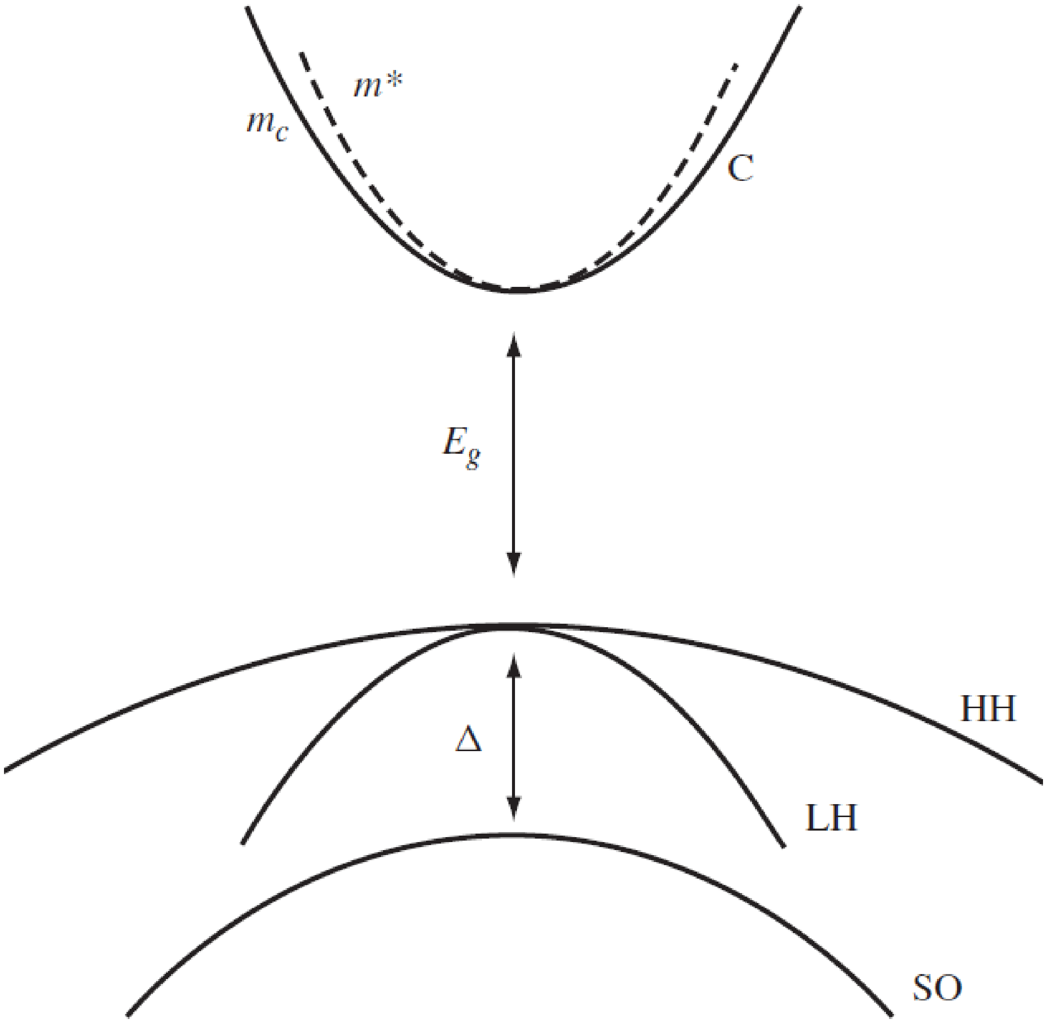
for both TE and TM polarizations

$$|M|^2 = \frac{1}{2}\left(\frac{m_0}{m^*} - 1\right)\frac{\left(E_g + \Delta\right)}{\left(E_g + \frac{2}{3}\Delta\right)} \times m_0 E_g$$

Δ=spin-orbit split gap

- Yan, Corzine, Coldren, and Suemune, “Corrections to the Expression for Gain in GaAs,” IEEE Journal of Quantum Electronics, vol. 26, p.213, 1990 (posted on website)
- Textbook by Corzine, Coldren and Mašanović, Section 4.3, Appendix Section A8.3
- Textbook by Chuang Section 9.5

[from textbook by Corzine, Coldren, Mašanović]



The effective mass m^* should be a bit smaller than the value m_e^* measured in the conduction band, due to more complete contributions of the band structure

For GaAs: $m^* = 0.053$ and $m_e^* = 0.0065\, m_0$ which gives $|M_{avg}|^2 \approx 3.38 m_0 E_g$

Established Experimental Values

Use these values whenever possible. Analytical results provide useful guideline and reasonable order of magnitude, but are unavoidably approximate.

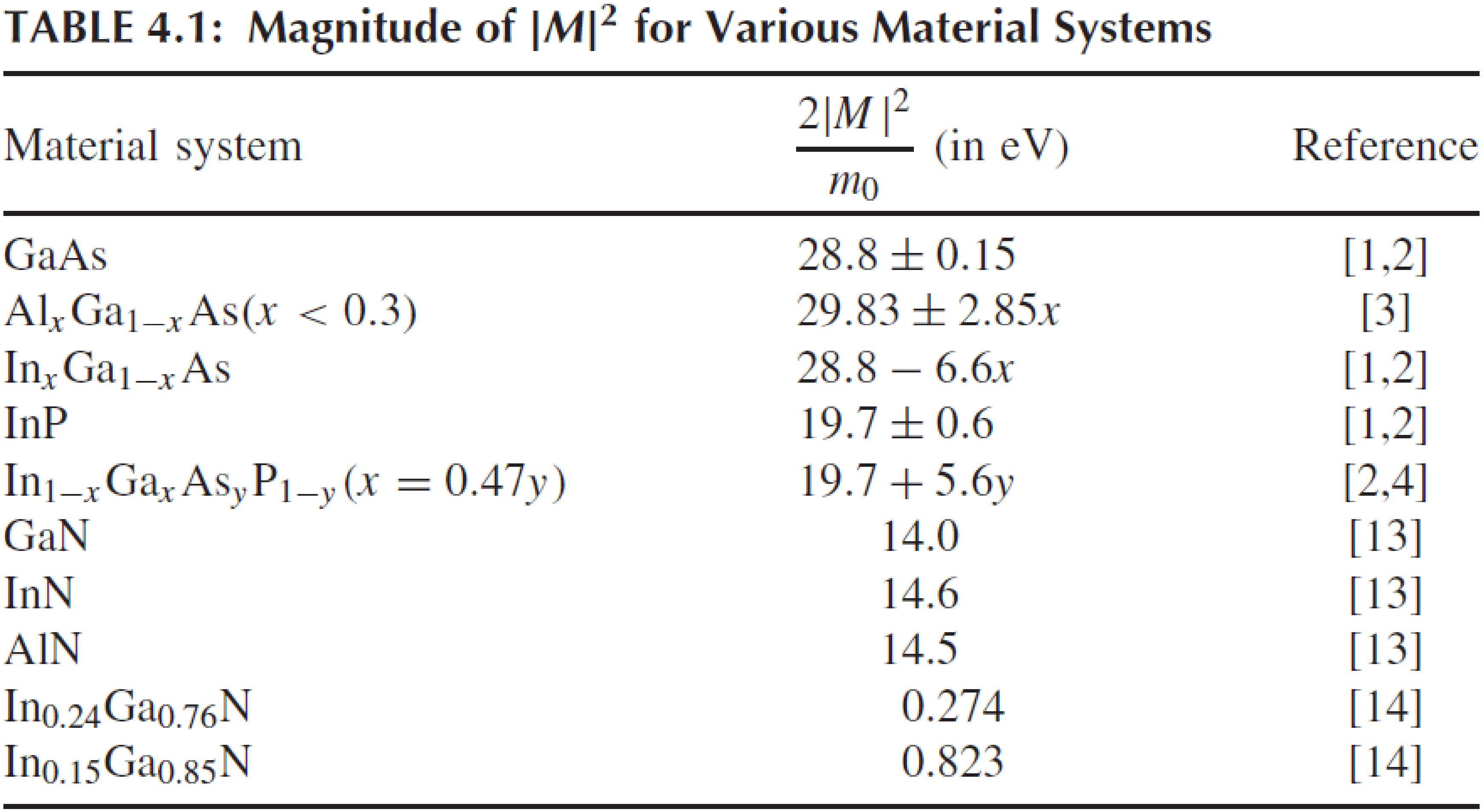


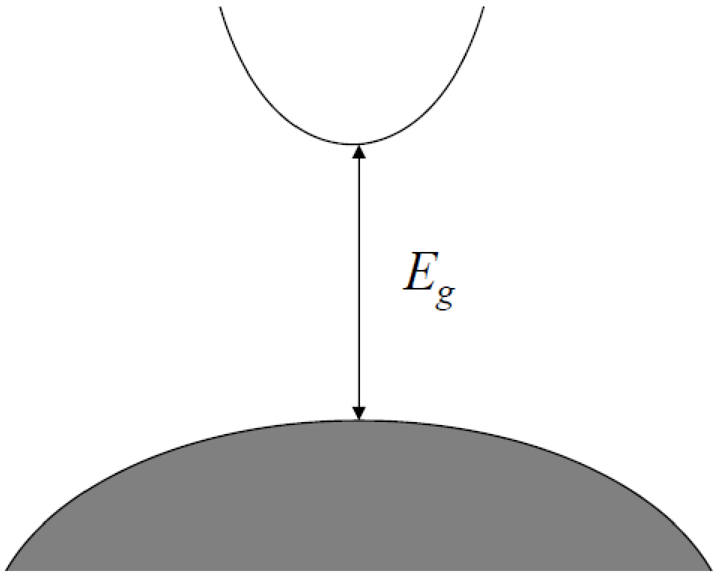
TABLE 4.1: Magnitude of $|M|^2$ for Various Material Systems

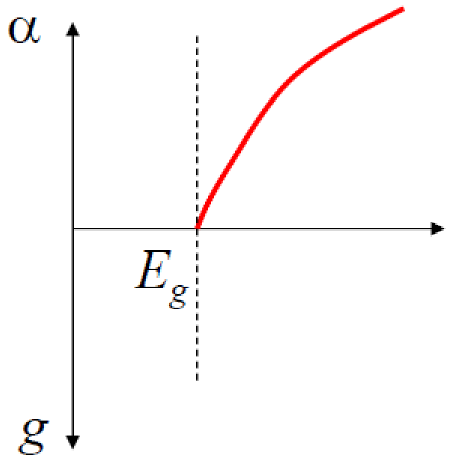
Material system	$\frac{2\|M\|^2}{m_0}$ (in eV)	Reference
GaAs	28.8 ± 0.15	[1,2]
$Al_xGa_{1-x}As(x < 0.3)$	$29.83 \pm 2.85x$	[3]
$In_xGa_{1-x}As$	$28.8 - 6.6x$	[1,2]
InP	19.7 ± 0.6	[1,2]
$In_{1-x}Ga_xAs_yP_{1-y}(x = 0.47y)$	$19.7 + 5.6y$	[2,4]
GaN	14.0	[13]
InN	14.6	[13]
AlN	14.5	[13]
$In_{0.24}Ga_{0.76}N$	0.274	[14]
$In_{0.15}Ga_{0.85}N$	0.823	[14]

From textbook by Corzine, Coldren and Mašanović

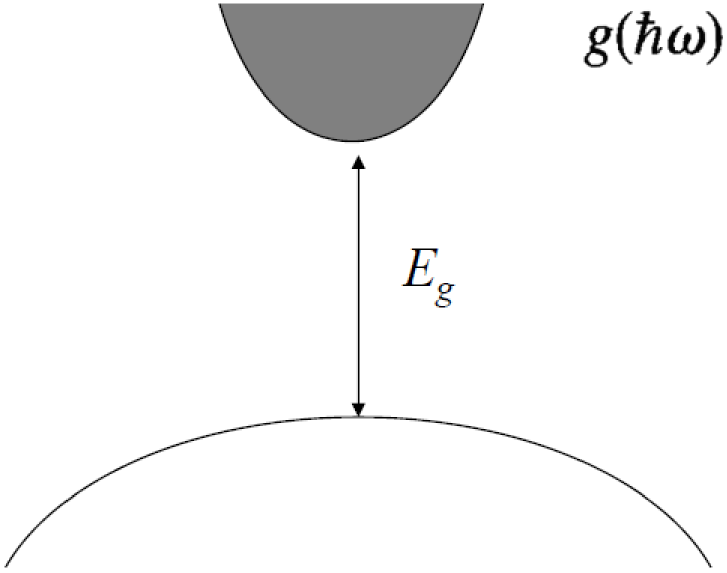
Summary of gain concepts

Absorption

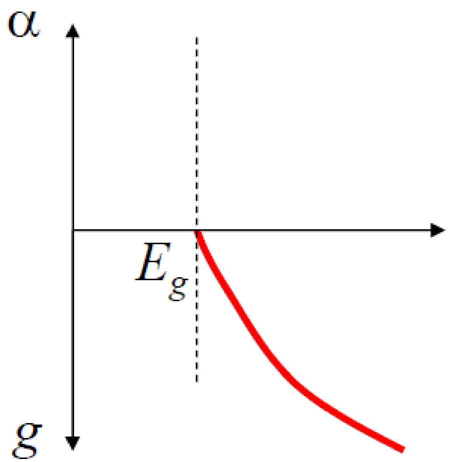




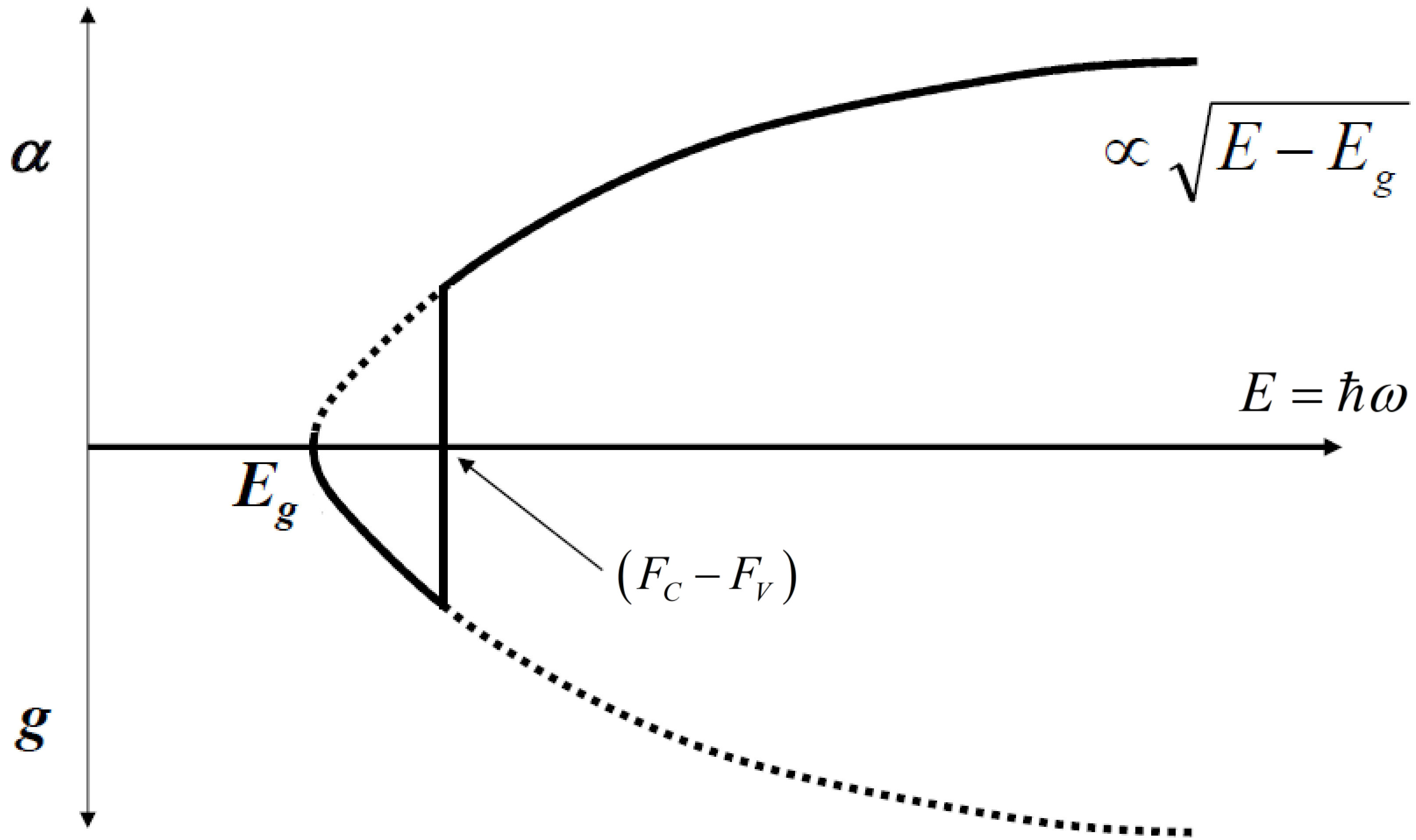
Gain



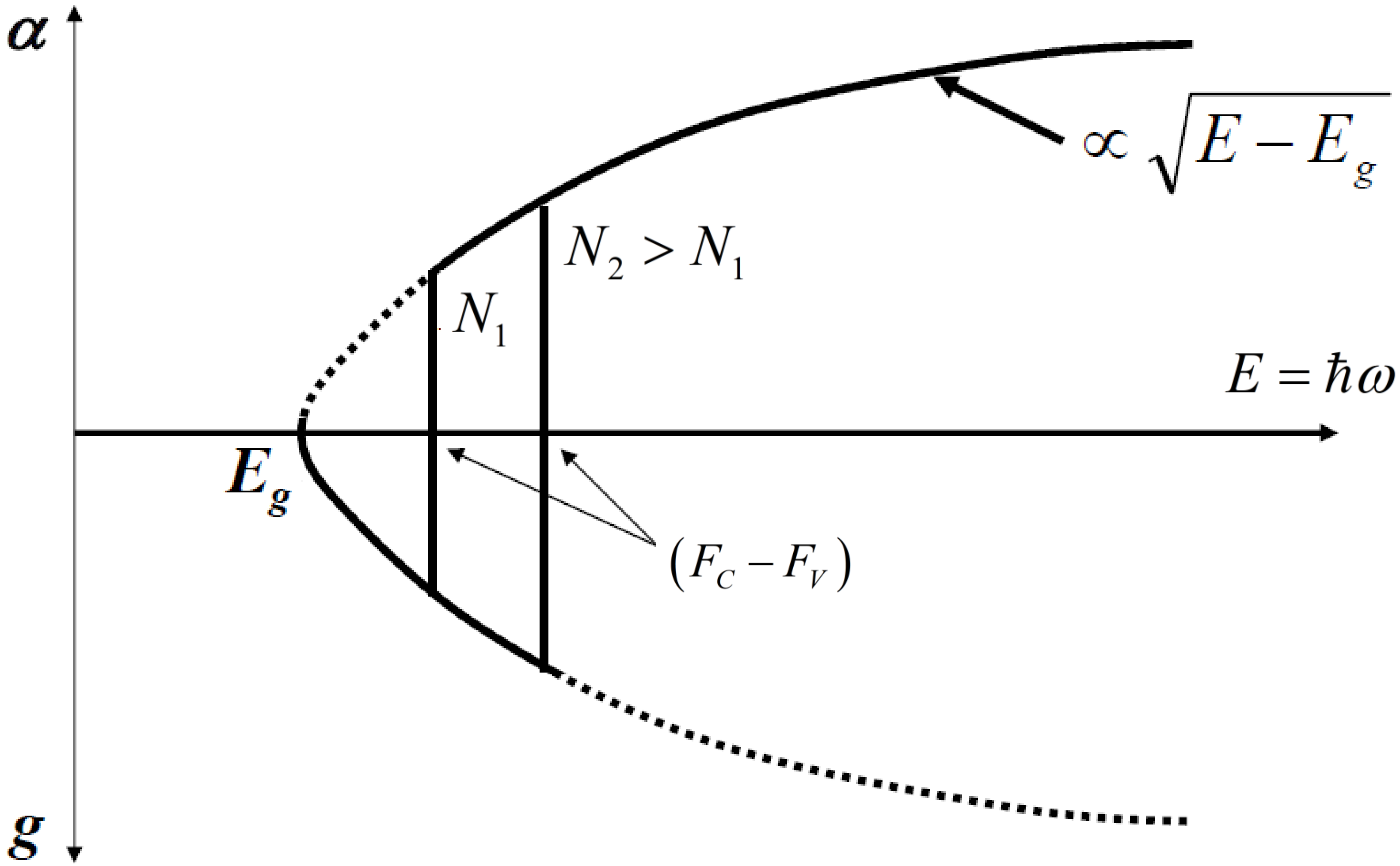
$$g(\hbar\omega) = -\alpha(\hbar\omega)$$



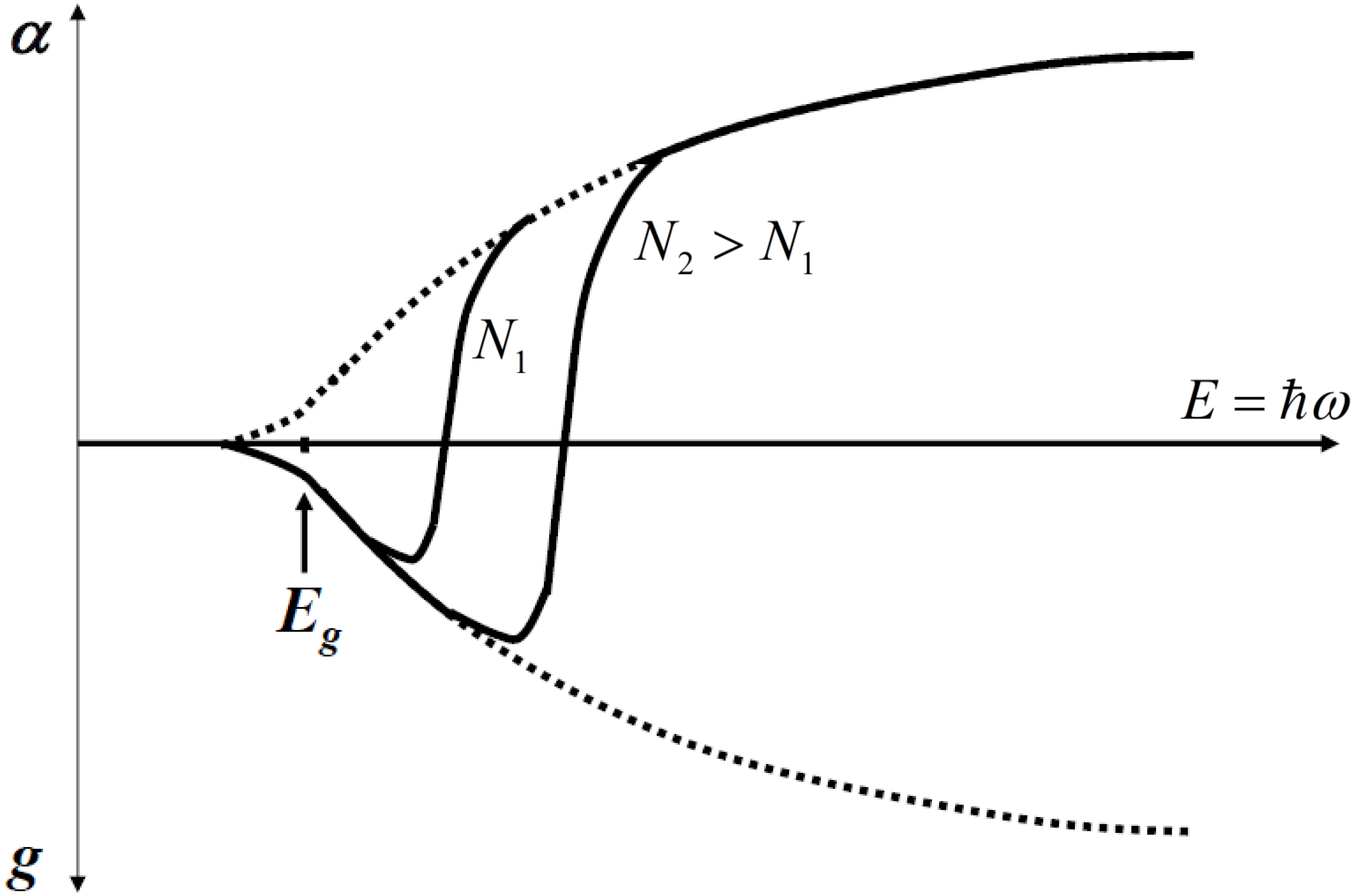
Gain and Loss at T = 0K



Gain and Loss at T = 0K **Different Pumping Rates**



Gain and Loss at T > 0K



Semiconductor Lasers

General Laser Principles

- **Stimulated emission causes amplification of light.**
- **Introduction of a feedback in an amplifying medium can create a light oscillator.**
- **Light may also be absorbed by this medium. We need to create the conditions for efficient amplification.**
- **Besides feedback, we need "population inversion": there must be more "systems" in the upper energy level that cause emission processes than there are "systems" in the lower energy level causing absorption processes.**

Semiconductor Laser Materials

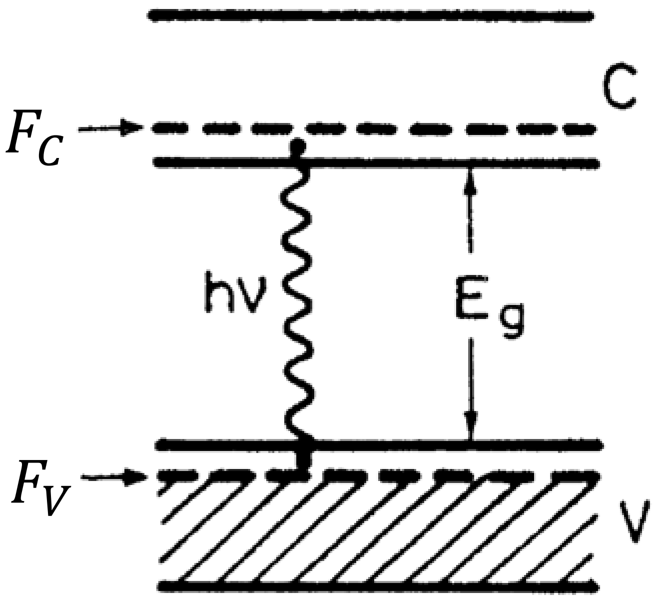
- Semiconductor lasers require a direct gap material for the active medium, most commonly III-V binary (e.g., Al, Ga, or In with N, P, As, or Sb), ternary (e.g., AlGAAs, InGaAs, InGaN) and quaternary compounds (e.g., InGaAsP).

- There are also II-VI compounds (e.g., Cd and Zn with S and Se) for the blue-green end of the spectrum and IV-VI compounds for the mid infrared (e.g., Pb salts of S, Se, and Te).

Principles of operation

- Suppose some electrons are raised from the valence band to the conduction band by a suitable pumping mechanism.
- After a very short time (~ 1ps) electrons in the conduction band drop to the lowest unoccupied levels in this band. Any electron near the top of the valence band drops to the lowest unoccupied valence states. Steady-state can be described by introducing the quasi-Fermi levels F_C and F_V .
- Light emission can take place by spontaneous radiative recombination as in an LED.
- In conditions of stimulated emission a generated photon can be amplified rather than just being reabsorbed

$$E_g \leq \hbar\omega \leq F_C - F_V$$



Principles of operation

- **The quasi-Fermi levels F_C and F_V are functions of the density of generated electrons.** $[\ \uparrow N \quad \uparrow F_C(N) \quad \downarrow F_V(N)\ ]$
- **There is therefore a critical (transparency) density at which**

$$E_g = F_C(N_{tr}) - F_V(N_{tr})$$

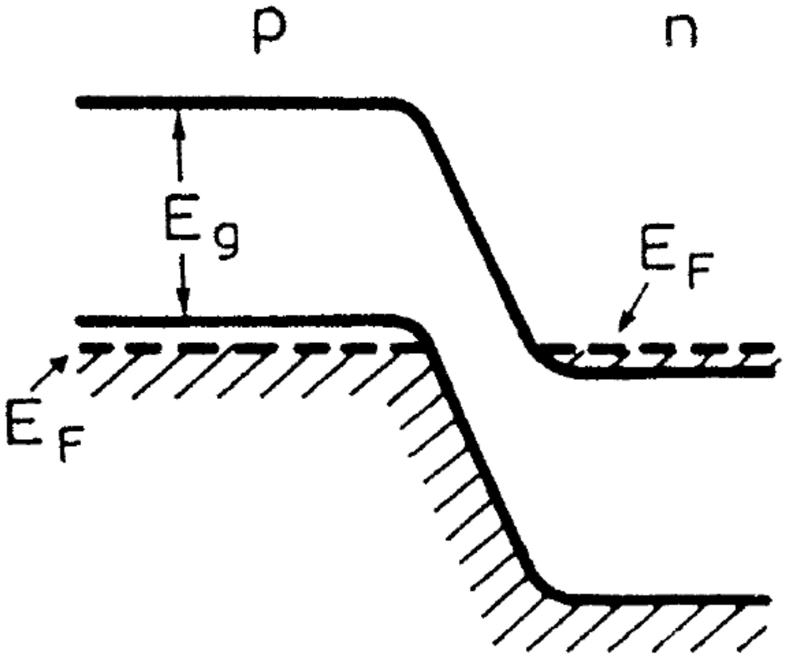
- **This is the *transparency condition*, at which the gain is zero.**

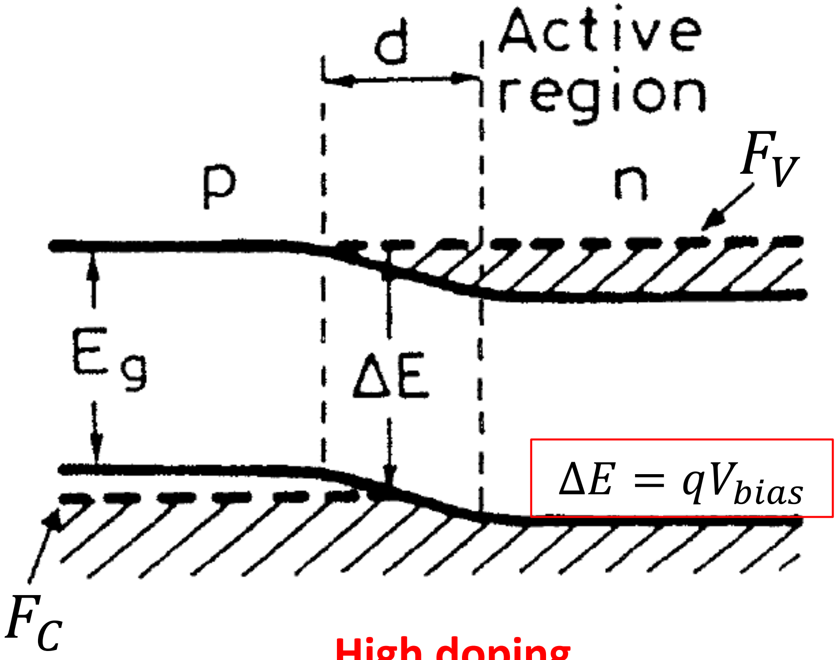
- **If the injected carrier density is larger than N_{tr} the semiconductor exhibits a net gain. When this active medium is placed in an appropriate cavity, laser action can occur.**

- **In order to obtain laser action, the injected carrier density must reach a threshold value $N_{th} > N_{tr}$ by a sufficient margin to overcome cavity losses.**

Homojunction laser

- **Although there are various viable pumping mechanisms, the most convenient one is to inject current in a diode structure.**
- **Laser action was first observed in 1962 in *p-n* junctions made of the same material (homojunction laser).**



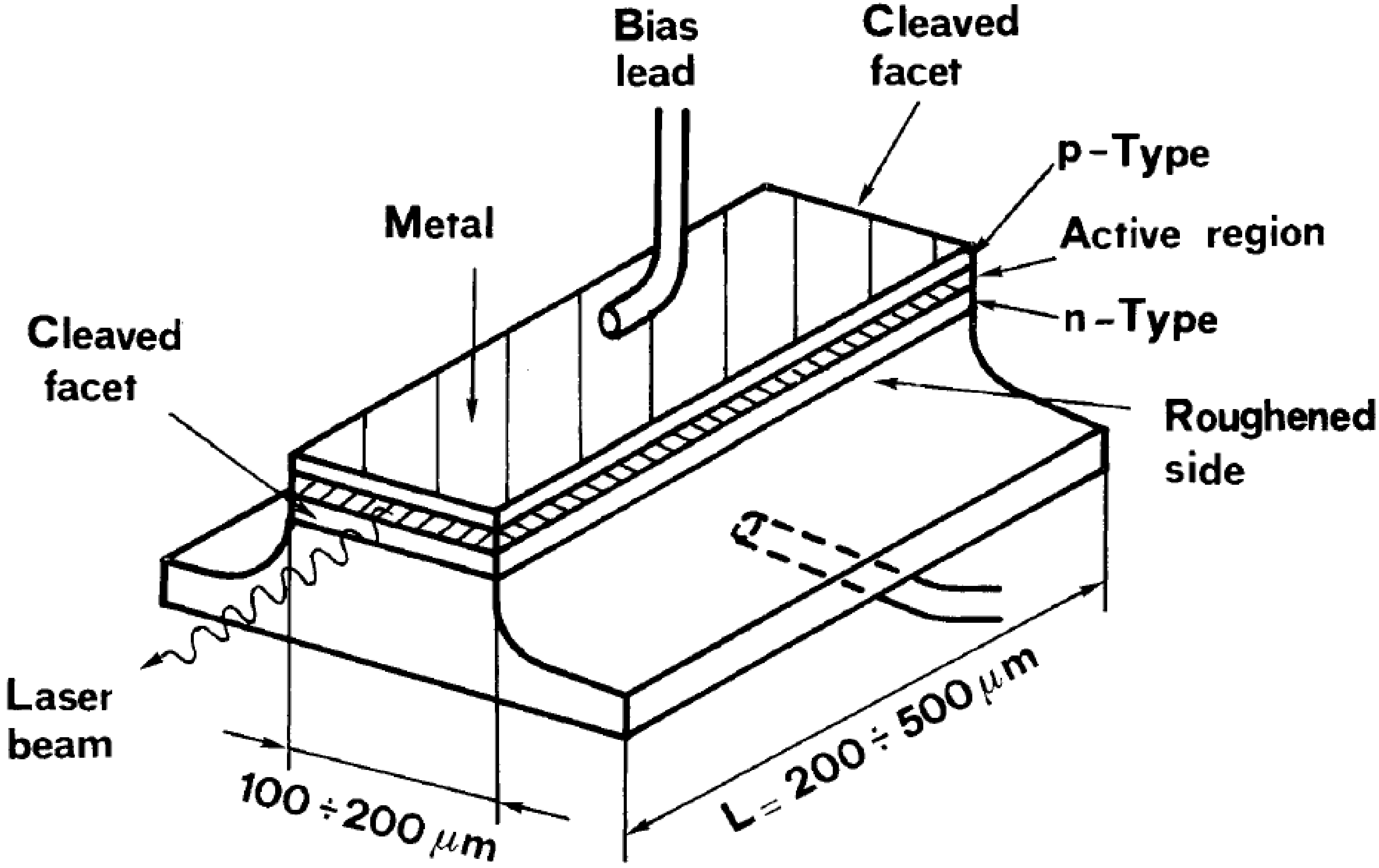


$\Delta E = qV_{bias}$

High doping

$(\approx 10^{18}\ \text{atoms/cm}^3)$

Semiconductor lasers

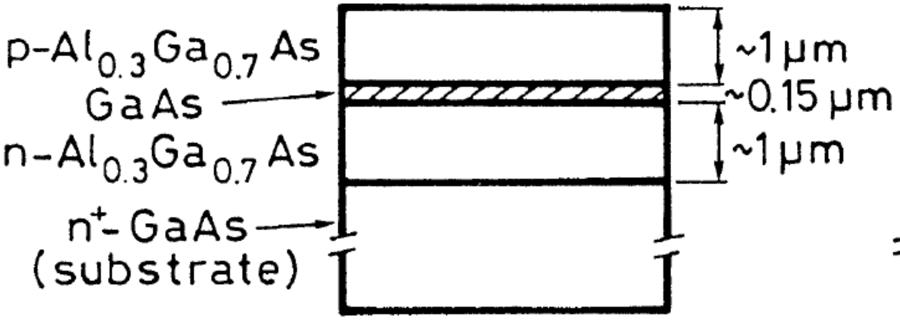


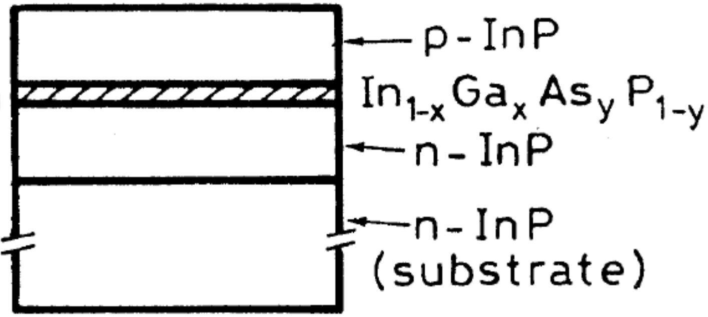
Typical broad-area *p-n* homojunction laser

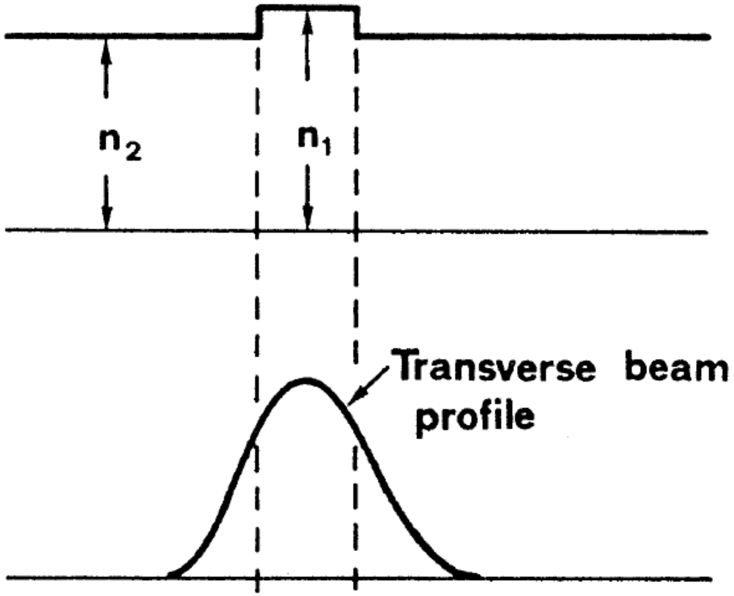
Main Limitations:

- This device can work in continuous wave only at cryogenic temperatures (T = 77K) due to very high threshold current at room temperature ($\mathrm{J_{th}} \approx 10^5\ \mathrm{A/cm^2}$).
- Electrons encounter a very small potential barrier and can penetrate the p-side where they become minority carriers. The penetration depth is $d = \sqrt{D\tau}$ from diffusion theory. In GaAs $D = 10\ cm^2/s$ and $\tau \cong 3\ \mathrm{ns}$ which gives $d \approx 1.7\ \mu\mathrm{m}$ while the depletion layer is $W \approx 0.1\ \mu\mathrm{m}$. The active region is limited by the diffusion length rather than by the thickness of the depletion layer.
- The threshold current is proportional to the volume of the active medium, hence proportional to the thickness d.
- The laser beam has comparatively large transverse dimension ($\approx 5\mu\mathrm{m}$) and it strongly absorbed outside the active region.

Double Heterostructure Laser







$n_1 = 3.6$ for GaAs

$E_{g_1} \cong 1.5\,\text{eV}$

$n_2 = 3.4$ for $Al_{0.3}Ga_{0.7}As$

$E_{g_2} \cong 1.8\,\text{eV}$

Double Heterostructure Laser – general properties

The room temperature current density at threshold is reduced by about two orders of magnitude ($\mathrm{J}_{\mathrm{th}} \approx 10^3\ \mathrm{A/cm^2}$).

- **The refractive index of the active layer is sufficiently larger than the one in the cladding layers. The laser beam is mostly confined to the active layer.**
- **The bandgap of the active layer is significantly smaller than that of the cladding layers forming barriers at the heterojunctions which confine the carriers. For a given current density the concentration of holes and electrons in the active region is higher, thus increasing the gain.**
- **Since the gap E_{g2} in the cladding layers is larger, the laser beam with $\omega \cong E_{g1}/\hbar$ is less strongly absorbed in the tails of the beam profile by the cladding layers.**

Lattice match

To form good heterostructures the lattice constants should be within 0.1% on the two sides. Otherwise interface strain results in misfit dislocations, active as very effective non-radiative recombination centers.

For the GaAs/AlGaAs structure we have *a*(GaAs) = 5.64 Å and *a*(AlAs) = 5.66 Å.

The radius of Ga is smaller than that of In, so that the lattice constant of $In_{1-x}Ga_xP$ is smaller than that of InP. The radius of As is larger than the radius of P, so for a certain ratio ($y \approx 2.2x$) the alloy $In_{1-x}Ga_xAs_yP_{1-y}$ can be lattice matched to InP.

By changing x while keeping y/x equal to the lattice match value, the emission wavelength of $In_{1-x}Ga_xAs_yP_{1-y}$ can vary between 1,150nm and 1,670nm, covering the second and third transmission windows in silica optical fibers.

Quantum Well Double Heterostructure Laser

If the thickness of the core region is sufficiently thin, there is quantization of the energy forming a quantum well or quasi-2D electron gas.

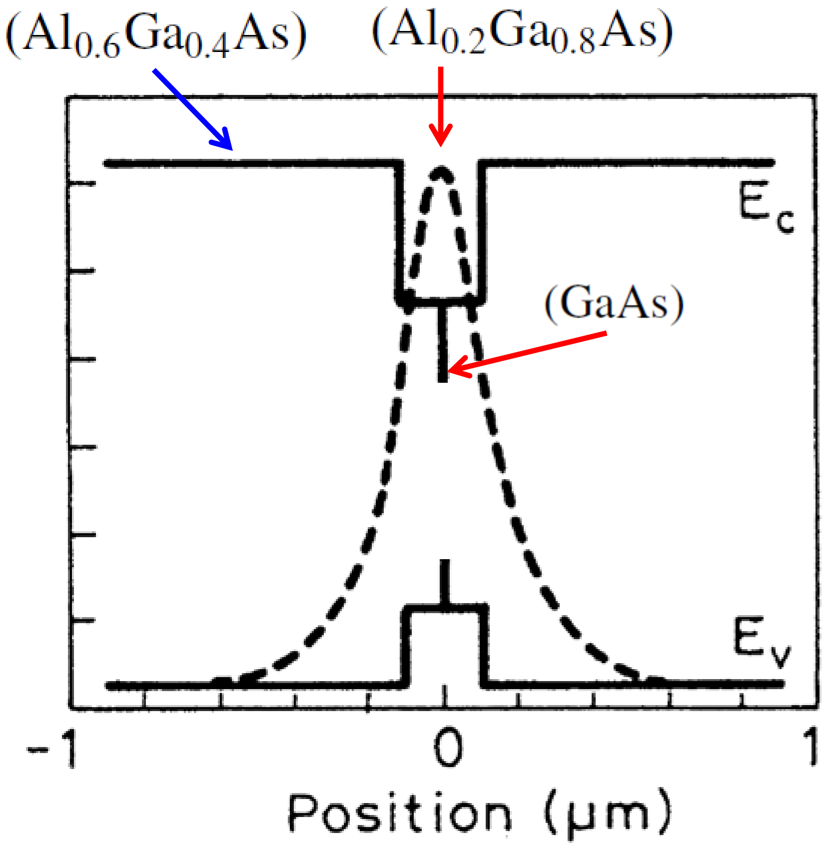
This structure takes advantage of the more favorable optical properties of a quantum well with respect to bulk, in particular the increased differential gain, as well as decreased dependence of this gain on temperature.

These properties are essentially related to the completely different density of states arising from 2D confinement with respect to bulk. The room temperature current density at threshold is further reduced ($\mathrm{J_{th}} \approx 200\ \mathrm{A/cm^2}$).

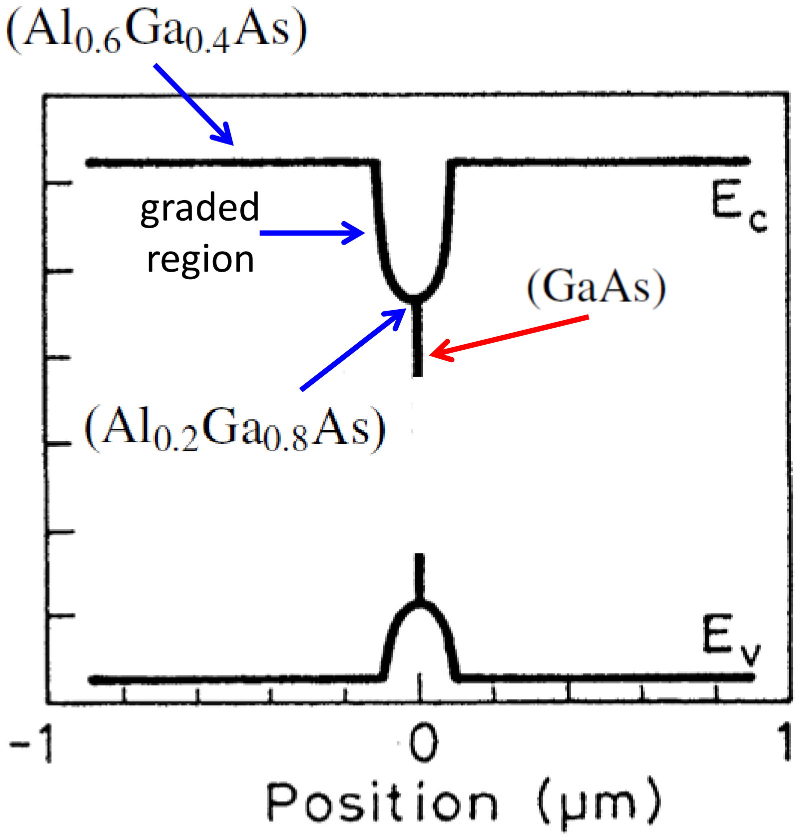
A potential problem is the reduction in confinement factor, due to the reduced layer thickness, which is addressed by using a separate confinement structure.

Quantum Well Double Heterostructure Laser

Examples of Separate confinement QW heterostructures



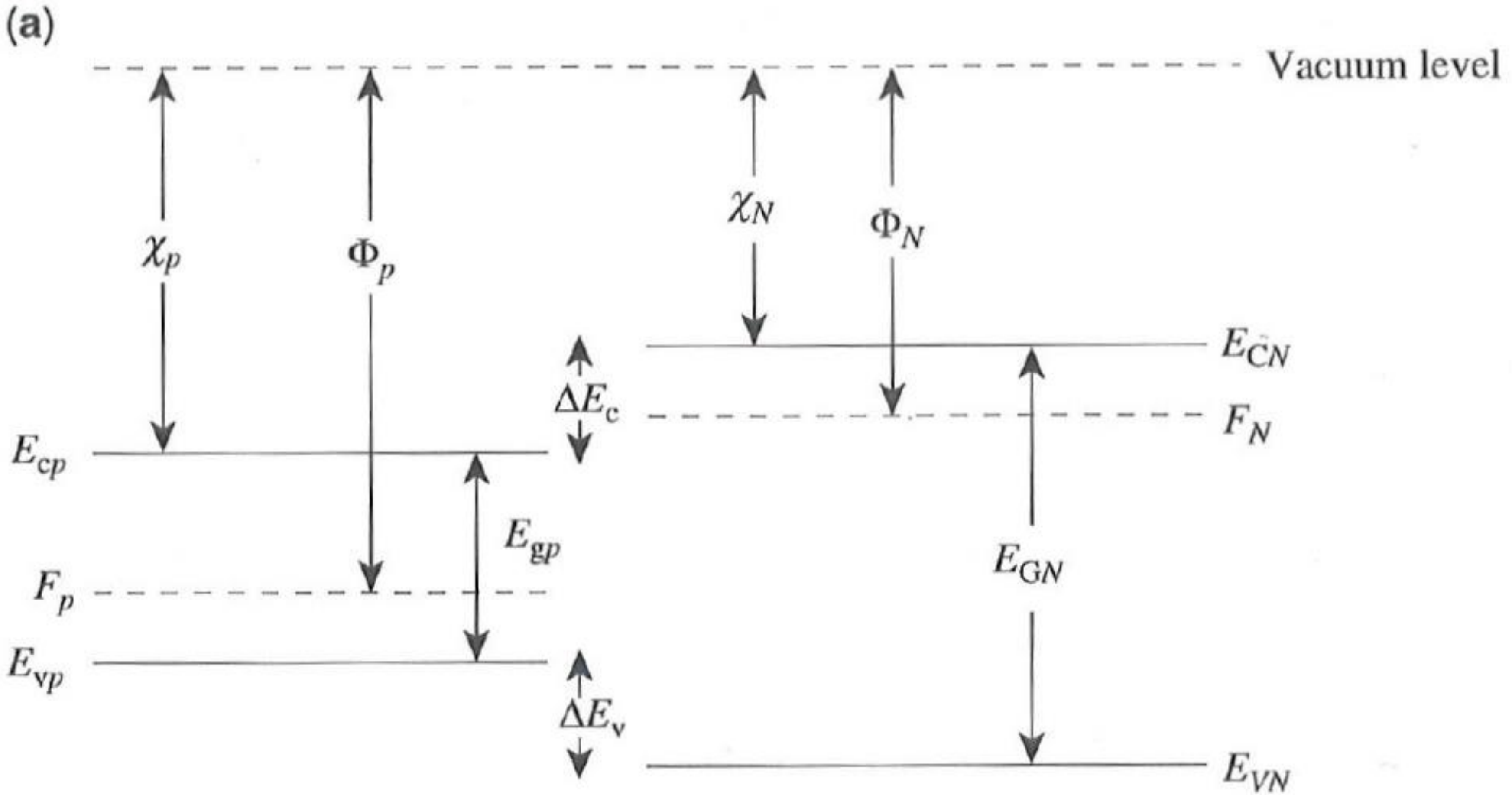
Step index



Graded index (GRINSCH)

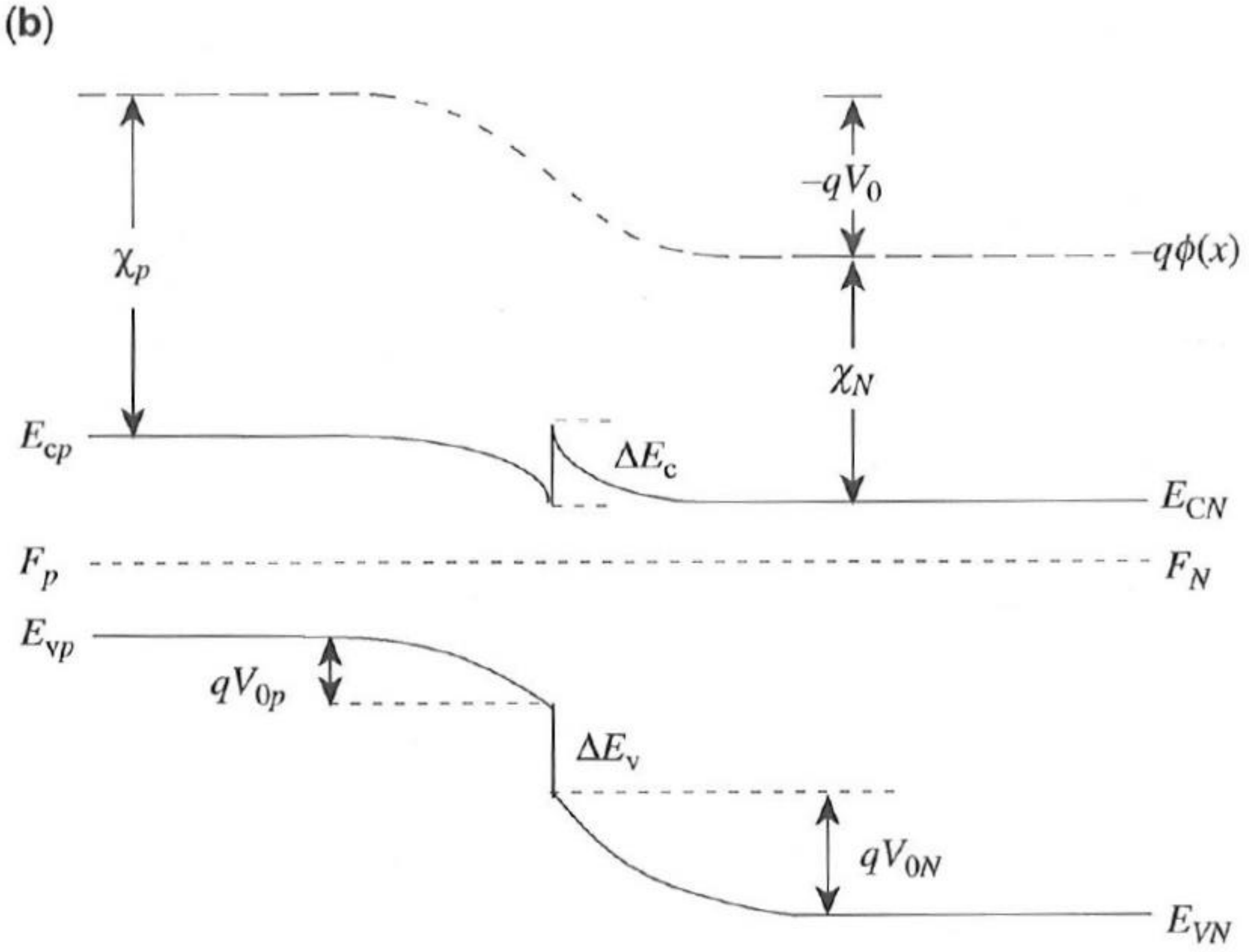
Band alignments for a p-n heterojunction

Anderson model – before joining material



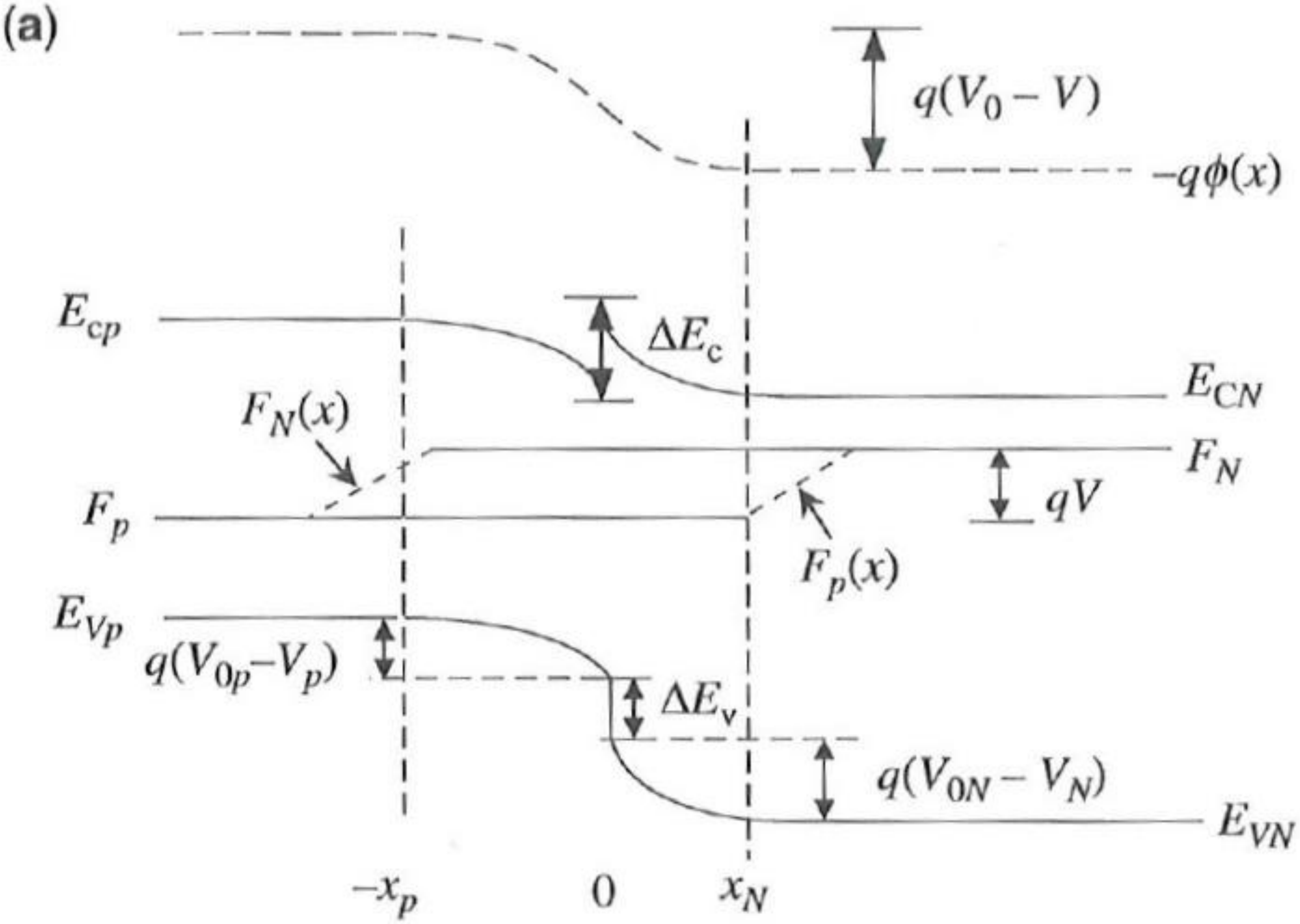
Band alignments for a p-n heterojunction

Anderson model – after joining materials



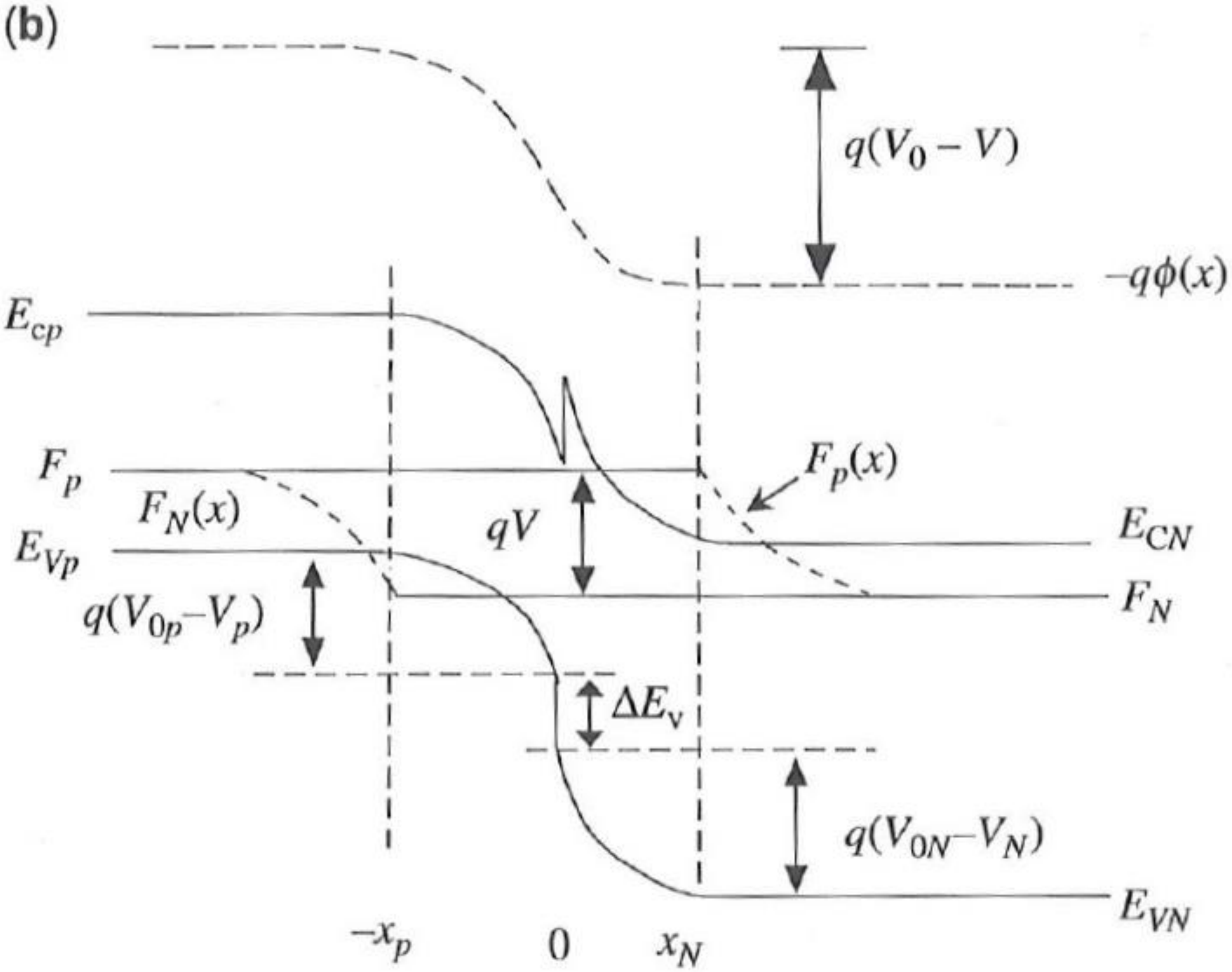
Band alignments for a p-n heterojunction

Anderson model – forward bias



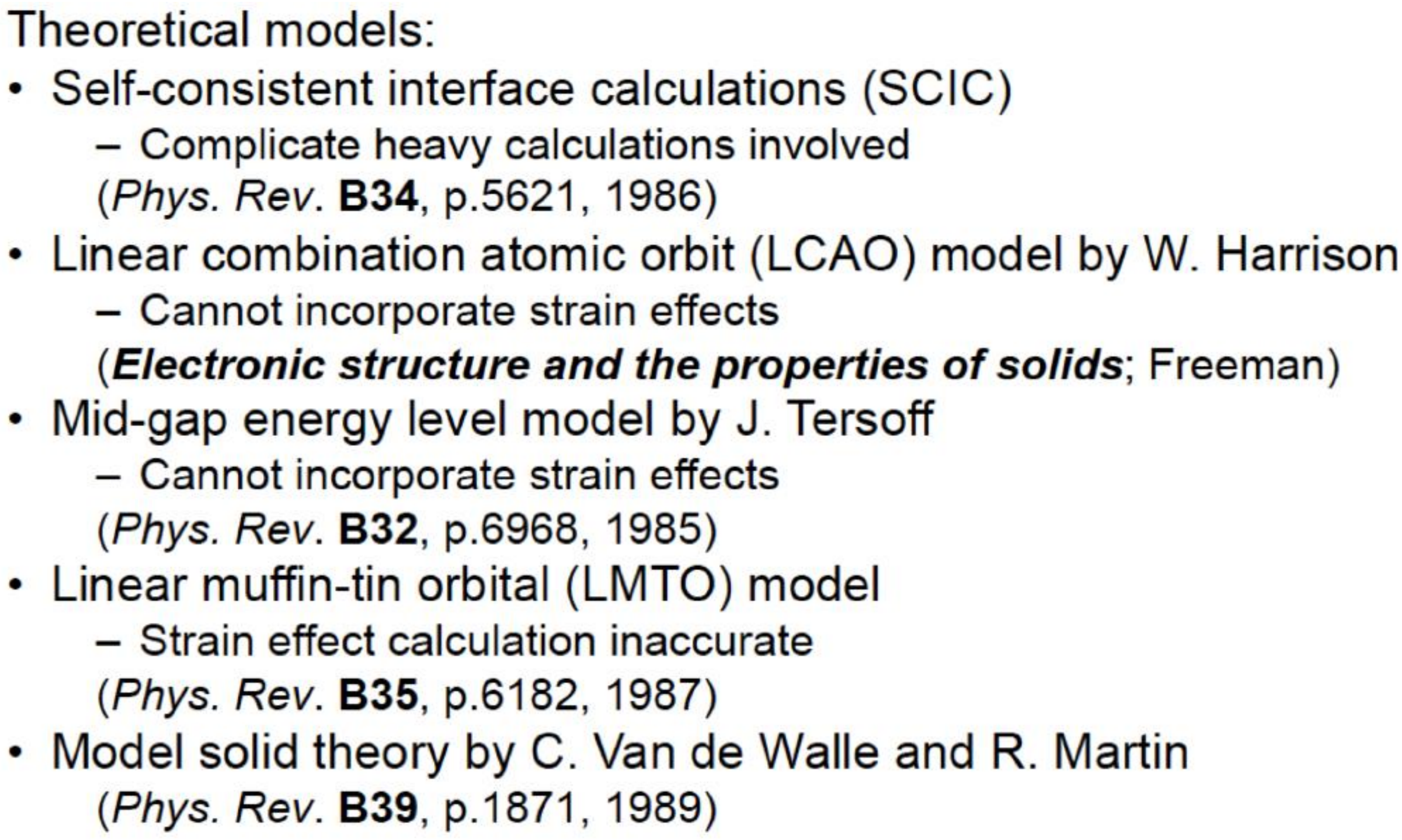
Band alignments for a p-n heterojunction

Anderson model – reverse bias



Band alignments for a p-n heterojunction

Anderson model is idealized and only takes electron affinity into account. There are various more advanced model but experimental data remain necessary to calibrate the distribution of band gap difference at the interface between conduction and valence band



Theoretical models:

- Self-consistent interface calculations (SCIC)
  - Complicate heavy calculations involved

  (*Phys. Rev.* **B34**, p.5621, 1986)
- Linear combination atomic orbit (LCAO) model by W. Harrison
  - Cannot incorporate strain effects

  (***Electronic structure and the properties of solids***; Freeman)
- Mid-gap energy level model by J. Tersoff
  - Cannot incorporate strain effects

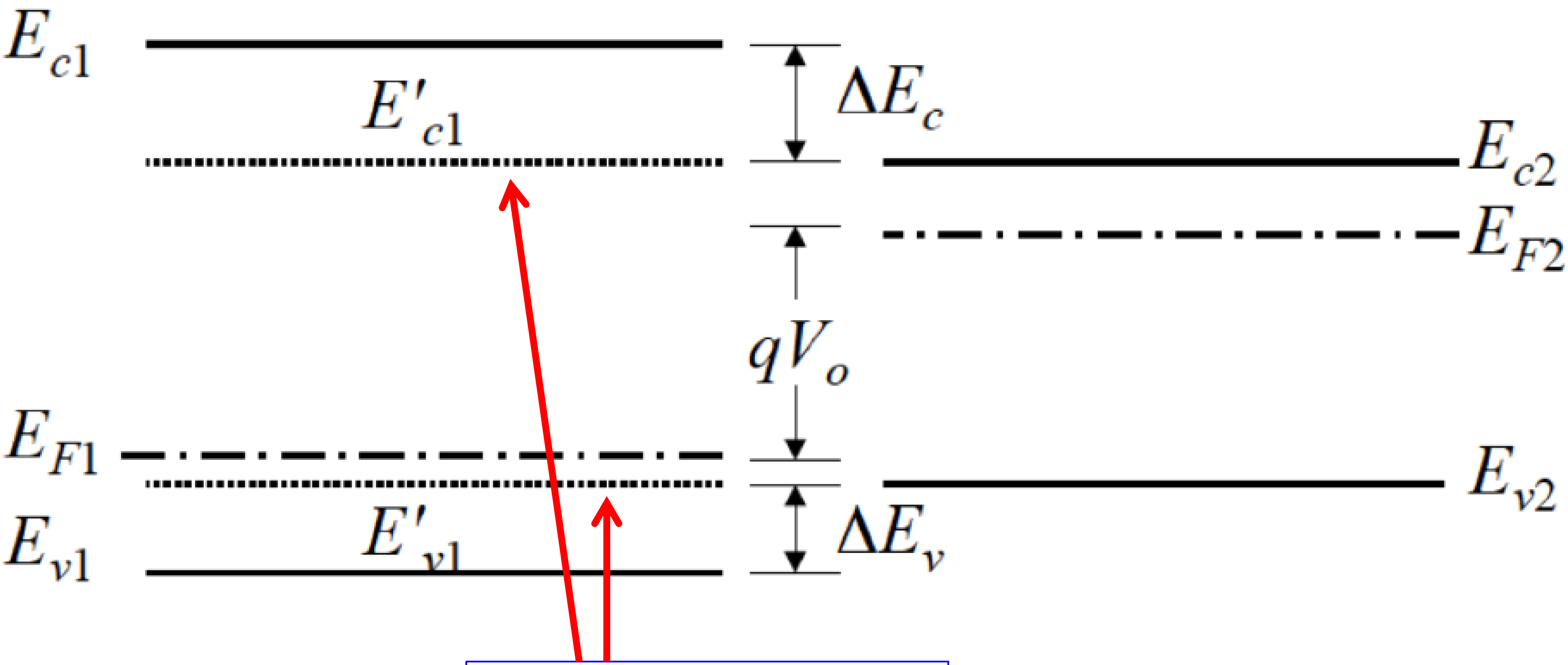
  (*Phys. Rev.* **B32**, p.6968, 1985)
- Linear muffin-tin orbital (LMTO) model
  - Strain effect calculation inaccurate

  (*Phys. Rev.* **B35**, p.6182, 1987)
- Model solid theory by C. Van de Walle and R. Martin

  (*Phys. Rev.* **B39**, p.1871, 1989)

Constructing band diagrams for heterojunctions

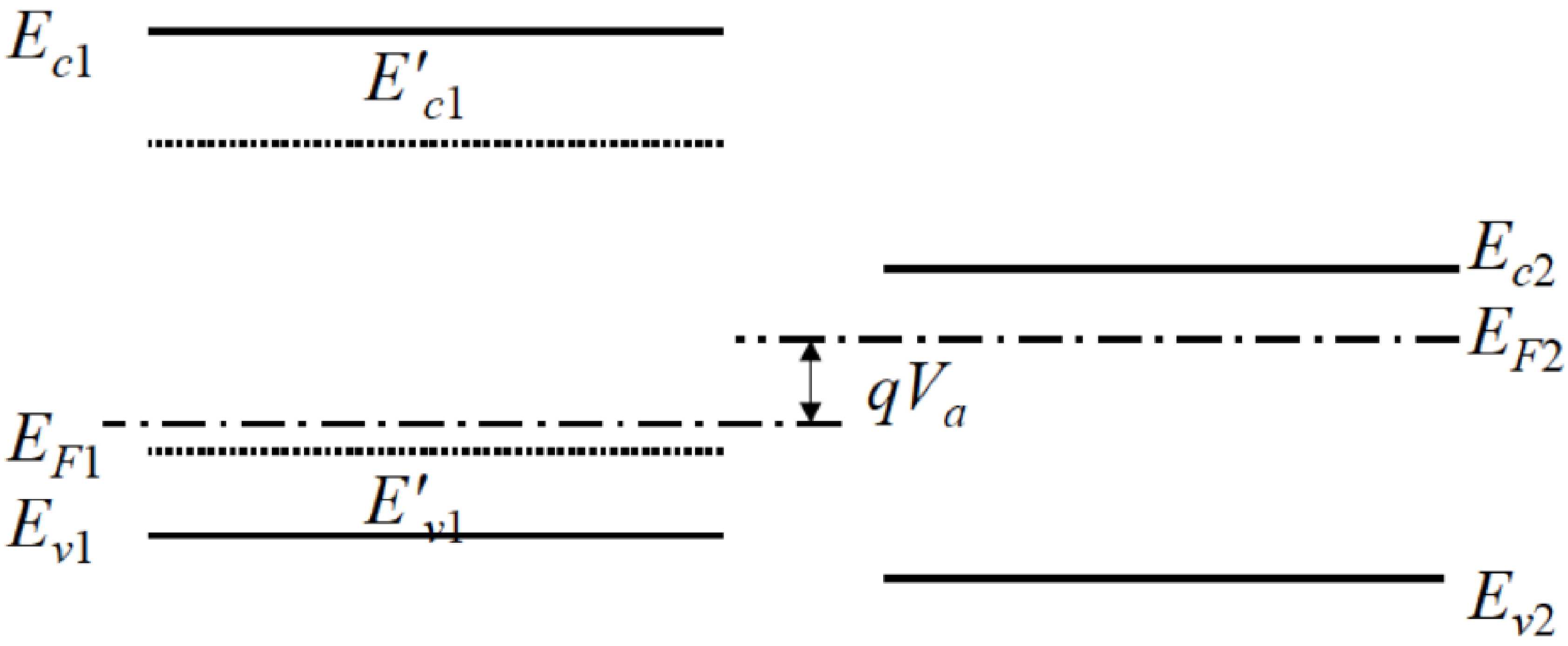
Start with flat bands on each side and align with the discontinuities in the conduction band and the valence band at the interface, as inferred by experiments



draw these reference lines

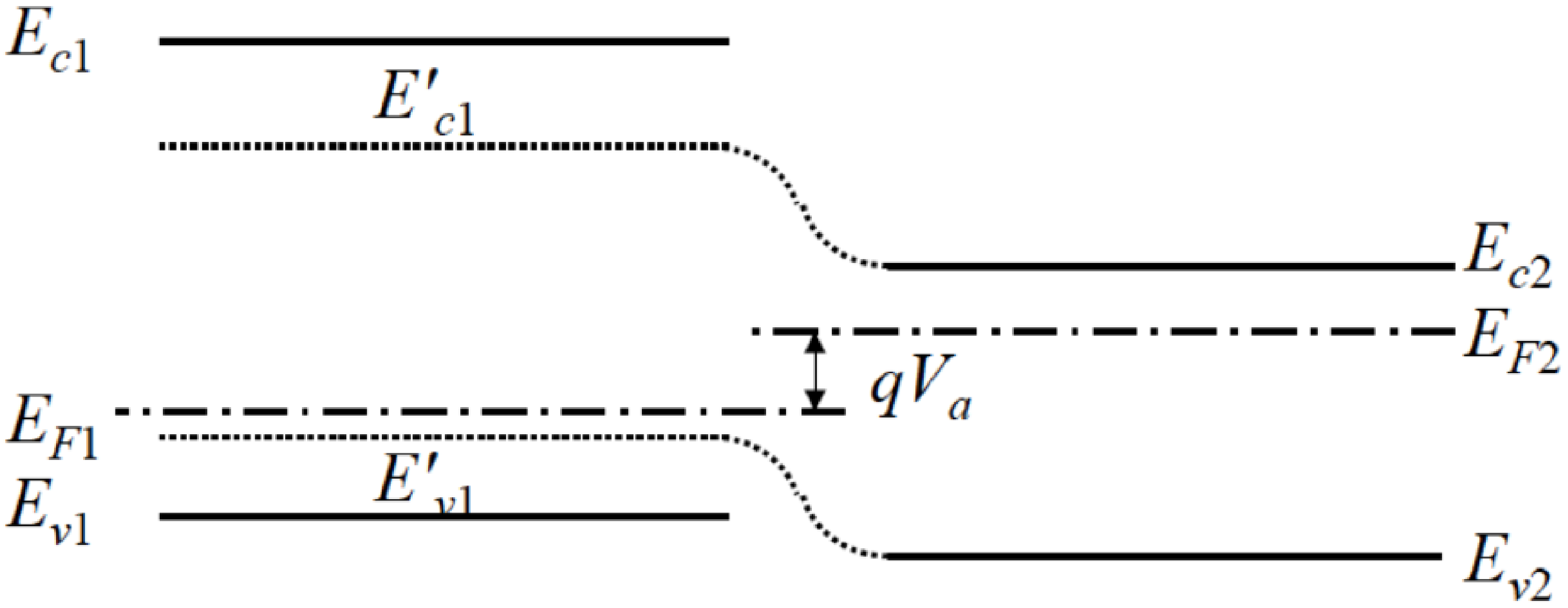
Constructing band diagrams for heterojunctions

Align Fermi levels. If there is a bias applied, the difference in the quasi-Fermi levels far away from the junction is qV_a



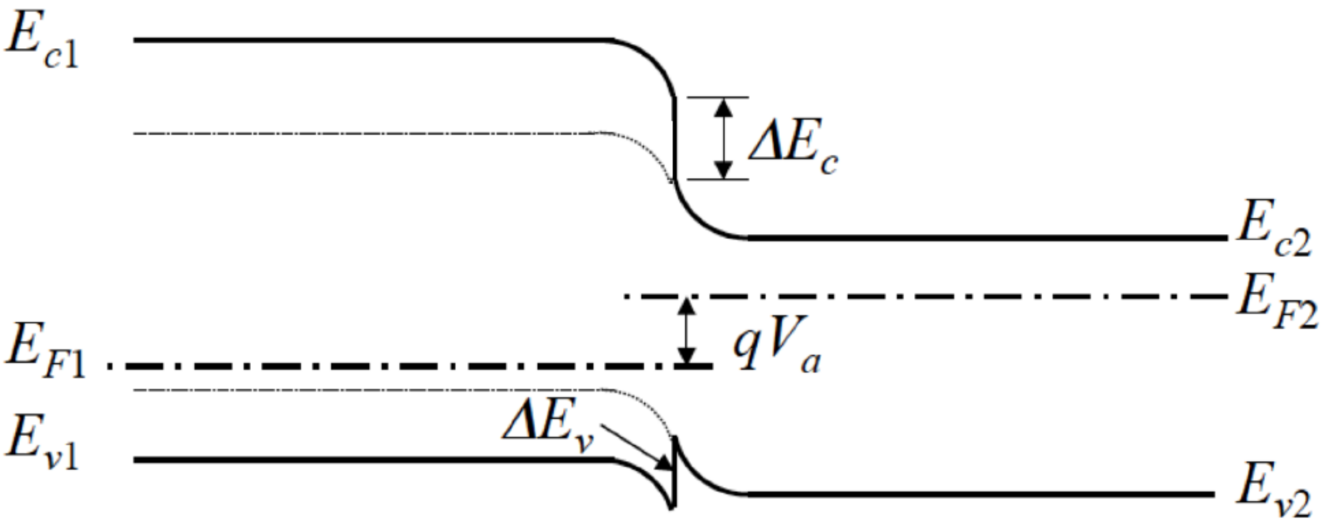
Constructing band diagrams for heterojunctions

Join the reference line with parallel curves (which follow the electrostatic potential as if a homojunction) to draw band lines on the right side.

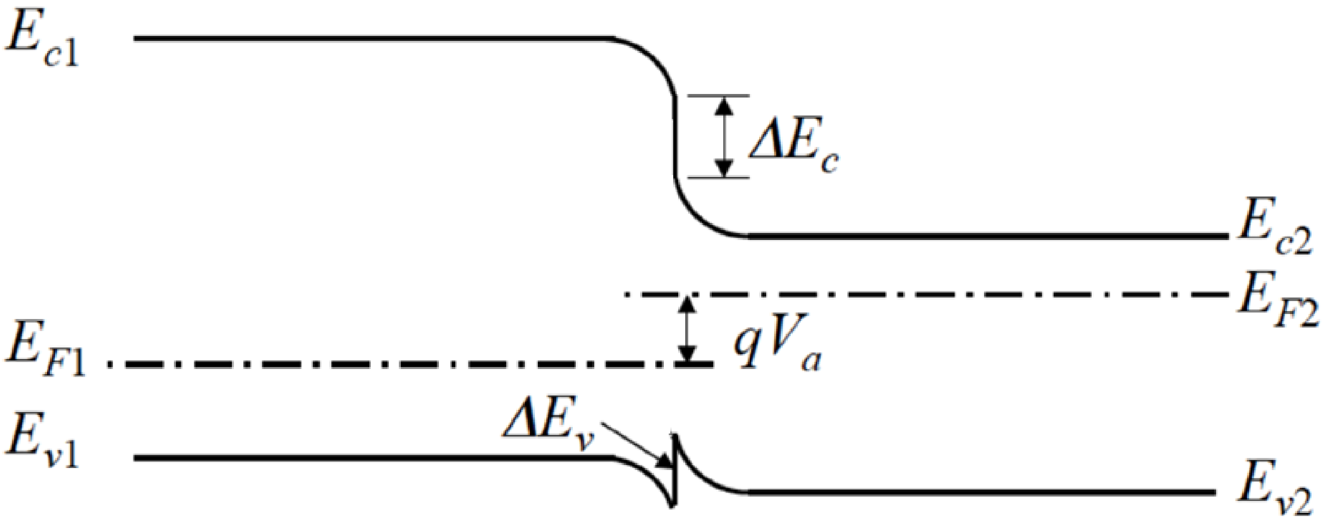


Constructing band diagrams for heterojunctions

Apply the discontinuities T the interface and connect with the bands on the left side



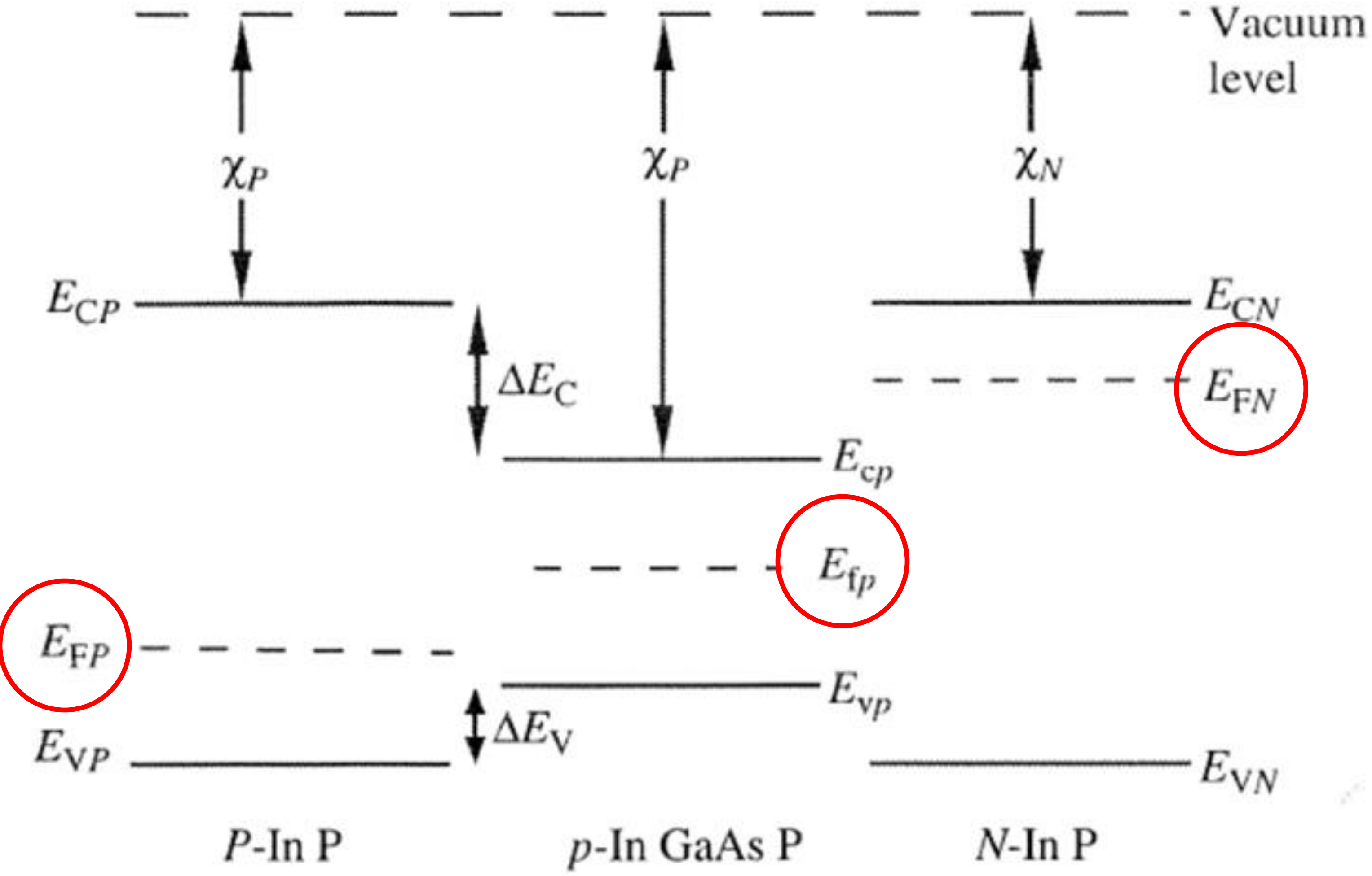
Remove the reference lines



Built-in Potentials – DH Laser Structure

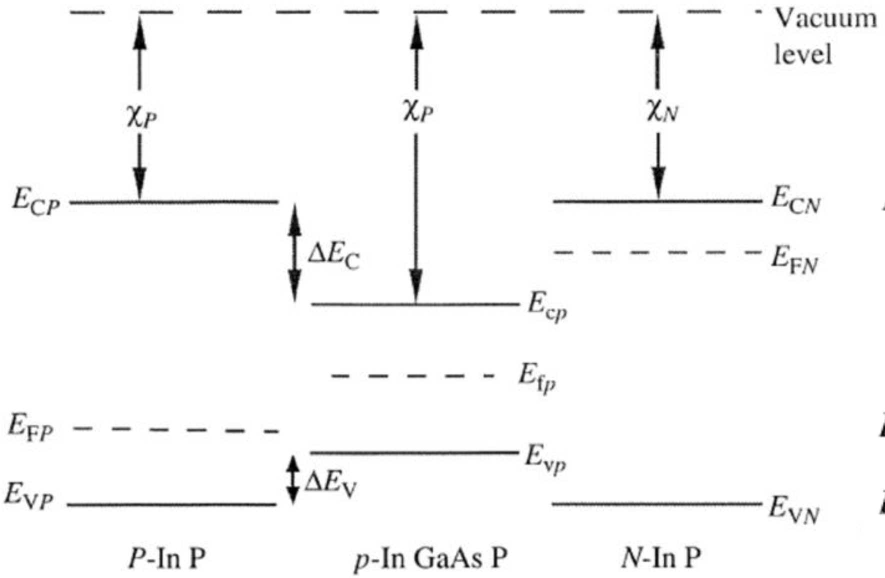
Consider a P-InP / p-$In_{1-x}Ga_xAs_{1-y}P_y$ / N-InP **double heterojunction structure**

$$\boxed{\Delta E_V = E_{vp} - E_{VP} = E_{vp} - E_{VN}}$$

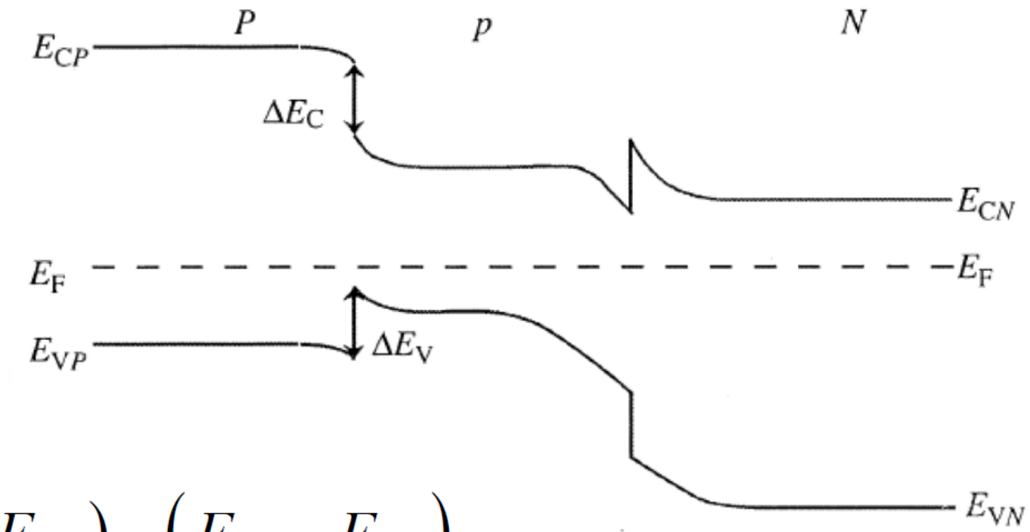


Built-in Potentials – DH Laser Structure

Consider a P-InP / p-$In_{1-x}Ga_xAs_{1-y}P_y$ / N-InP double heterojunction structure
The built-in potentials are determined by the Fermi levels



$$\boxed{\Delta E_V = E_{vp} - E_{VP} = E_{vp} - E_{VN}}$$



$$-qV_D^{P-p} = E_{FP} - E_{fp} = \left(E_{FP} - E_{VP}\right) - \left(E_{fp} - E_{VP}\right)$$

$$= \left(E_{FP} - E_{VP}\right) - \Delta E_V - \left(E_{fp} - E_{vp}\right)$$

$$-qV_D^{p-N} = E_{fp} - E_{FN} = \left(E_{fp} - E_{VN}\right) - \left(E_{FN} - E_{VN}\right)$$

$$= \left(E_{fp} - E_{vp}\right) + \Delta E_V - \left(E_{FN} - E_{VN}\right)$$