ECE 536 – Integrated Optics and Optoelectronics
Lecture 13 – March 1, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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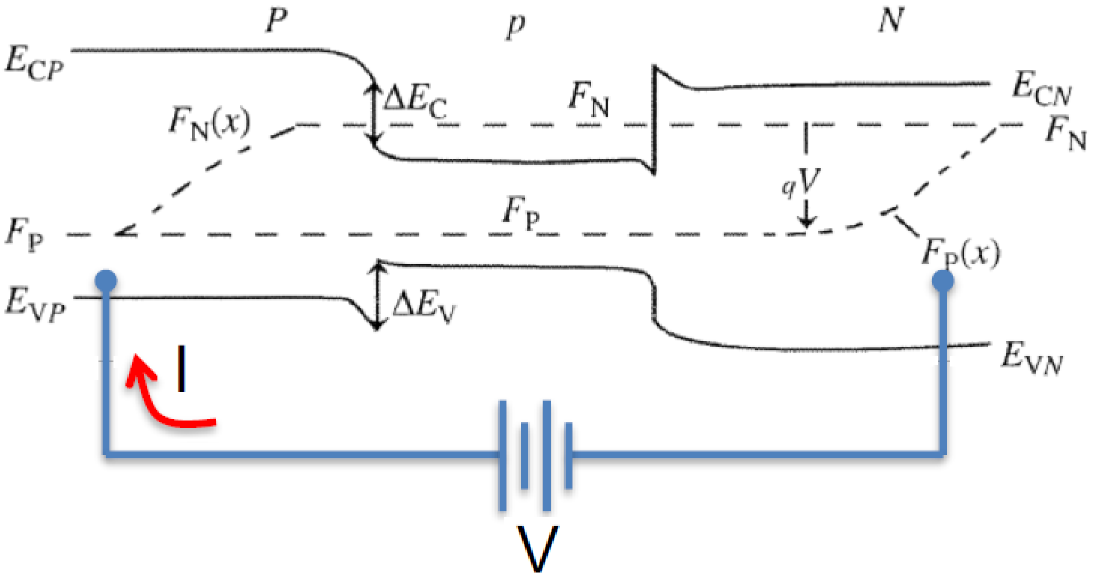
Lecture 13 Outline

- Laser behavior at and above threshold
- Power in lasers
- Measurements

Rate Equation – DH Laser structure

P-InP / p-In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$ / N-InP **structure under forward bias**



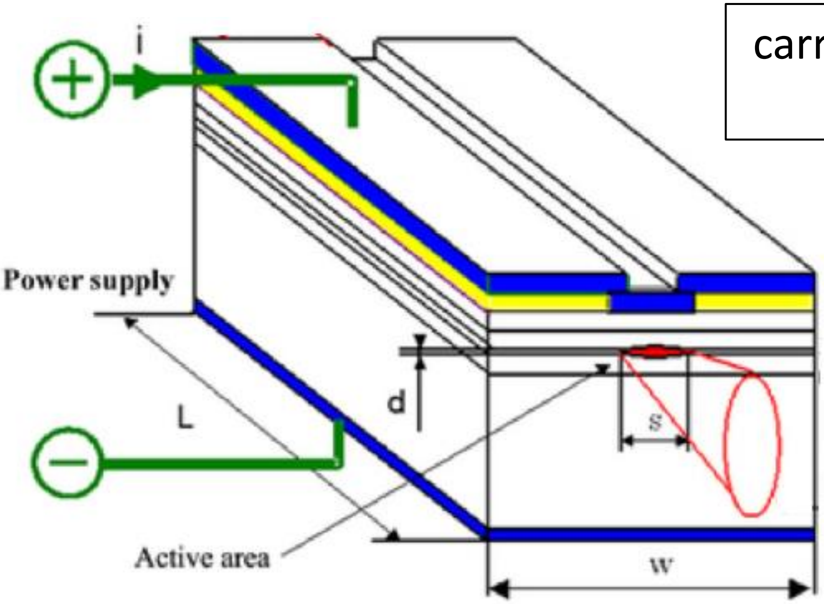
The current concentration is determined by the rate equation

$$\frac{dn(t)}{dt} = G_{gen} - R_{rec}$$

$$G_{gen} = \frac{\eta_i I}{qVol} = \frac{\eta_i I}{qwLd} = \frac{\eta_i J}{qd}$$

carrier collection efficiency

Current Density: $J = \dfrac{I}{wL}$



$$\frac{dn(t)}{dt} = \eta_i \frac{J(t)}{qd} - R(n) - v_g g(n) S(t)$$

Gain from Carrier Injection – DH Laser structure

$$\boxed{\frac{dn(t)}{dt} = \eta_i \frac{J(t)}{qd} - R(n) - v_g g(n) S(t)}$$

$S(t)$ = photon density

Steady-state

$$\frac{dn(t)}{dt} = 0$$

Near threshold

$$v_g g(n) S(t) \simeq 0$$

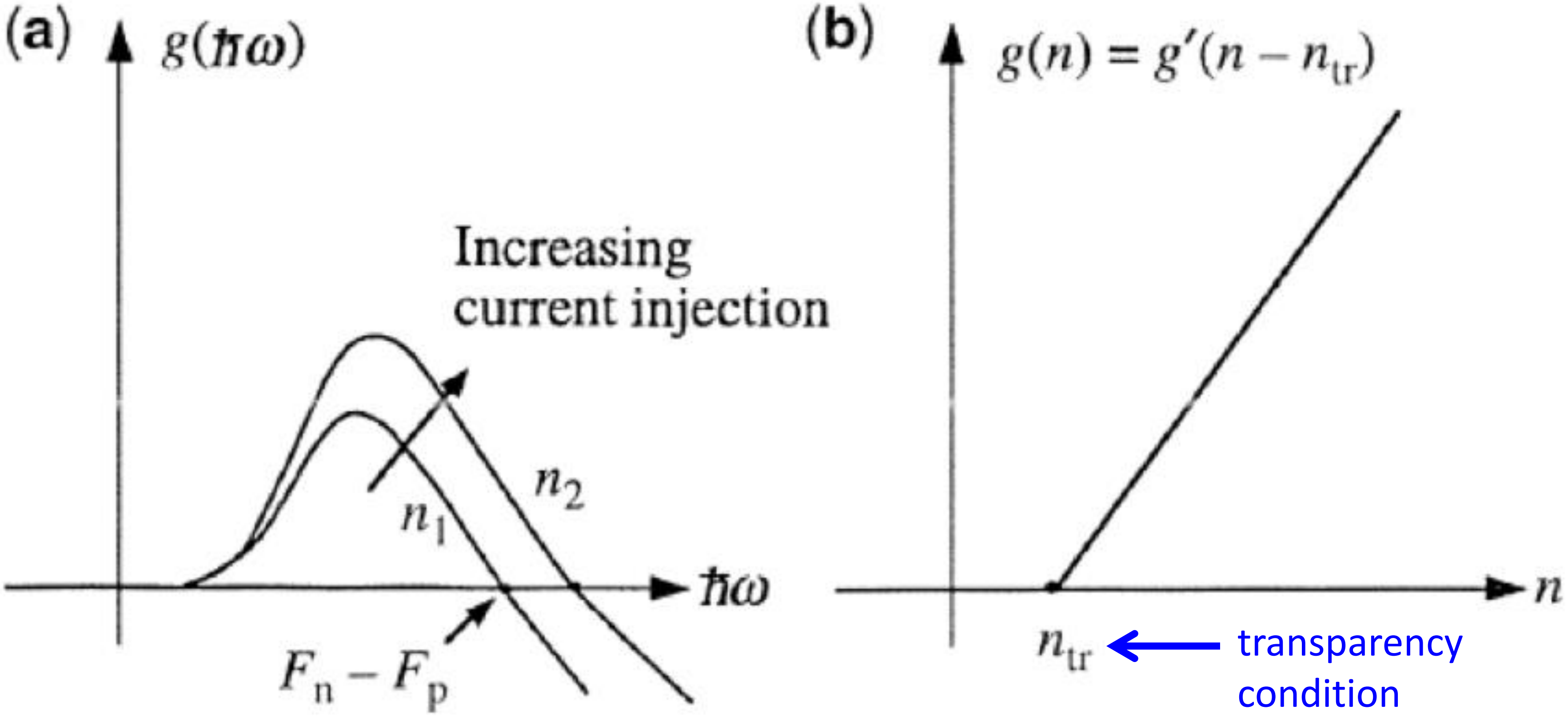
$$\Rightarrow \boxed{\eta_i \frac{J}{qd} = \frac{n}{\tau}}$$

$$\nearrow \quad n = \eta_i \frac{J\tau}{qd}$$

$$\searrow \quad J = \frac{qd}{\eta_i}\left(A + Bn + Cn^2\right)$$

Solve for the carrier concentration n from either

Gain from Carrier Injection



transparency condition

For a bulk semiconductor gain is approximately linear with carrier density n

$$g(n) = g'(n - n_{tr}) \qquad g' \equiv \frac{dg}{dn}$$

differential gain

For a QW laser the empirical relationship is logarithmic.

Lasing Threshold

Intensity grows as

Power coefficient

$$I(z) = I(0)e^{(\Gamma g - \alpha_i)z}$$

Electric field grows as

Factor $\frac{1}{2}$ because it is Field coefficient

$$E(z) = E(0)e^{(\frac{\Gamma g - \alpha_i}{2})z + ik_z z}$$

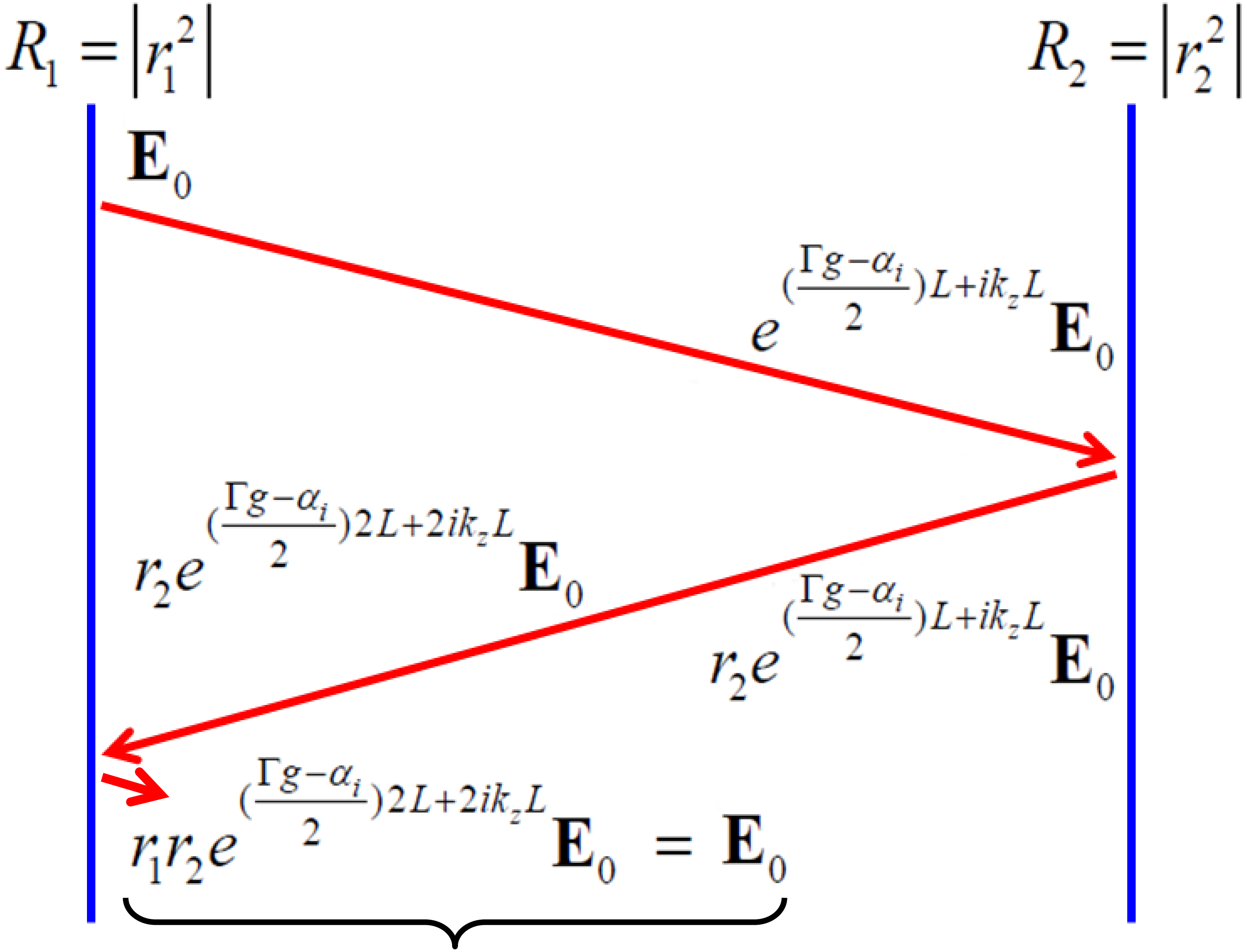
Γ = confinement factor (fraction of power confined to active region)

α_i = intrinsic loss due to absorption inside the guide

$$\alpha_i = \alpha_0\left(1 - \Gamma\right) + \alpha_g \Gamma$$

For laser action we need the E-field to reproduce itself after 1 round trip in the resonant cavity.

$$r_1 r_2 e^{(\Gamma g - \alpha_i)L + 2ik_z L} = 1$$



$R_1 = \left|r_1^2\right|$

$R_2 = \left|r_2^2\right|$

$\mathbf{E}_0$

$e^{(\frac{\Gamma g-\alpha_i}{2})L+ik_zL}\mathbf{E}_0$

$r_2 e^{(\frac{\Gamma g-\alpha_i}{2})2L+2ik_zL}\mathbf{E}_0$

$r_2 e^{(\frac{\Gamma g-\alpha_i}{2})L+ik_zL}\mathbf{E}_0$

$$\underbrace{r_1 r_2 e^{(\frac{\Gamma g-\alpha_i}{2})2L+2ik_zL}\mathbf{E}_0 = \mathbf{E}_0}_{}$$

This requires the amplitude and the phase to match up. In some cases (e.g., metal mirrors) r_1 and r_2 may be complex.

Round-trip Condition (Amplitude)

$$1 = \left| r_1 r_2 e^{(\Gamma g - \alpha_i)L + 2jk_z L} \right|^2 = R_1 R_2 e^{2(\Gamma g_{th} - \alpha_i)L}$$

$$\boxed{\Gamma g_{th} = \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}}$$

Round-trip Condition (Phase)

$$2m\pi = \arg\left(r_1 r_2 e^{(\Gamma g - \alpha_i)L + 2jk_z L} \right) = \arg\left(r_1 r_2 \right) + 2k_z L$$

For r_1 and r_2 real we get $k_z L = m\pi$

$$\boxed{m \frac{\lambda}{n_{eff}} = 2L}$$

integer number of half wavelengths fit in the cavity

Condition for lasing

1. Threshold Condition: Modal Gain equals Loss

$$\Rightarrow \quad \Gamma g_{th} = \alpha_i + \alpha_m \quad \text{where} \quad \alpha_m = \frac{1}{2L} \ln \frac{1}{R_1 R_2}$$

2. Round trip phase is zero:

$$\Rightarrow \quad \arg(r_1 r_2) + 2k_z L = 2m\pi$$

Contributions to α_i can be scattering loss, free carrier absorption, defects.

When threshold condition is reached, gain and carrier concentration are pinned at their threshold values.

Assuming $g(n) = g'(n - n_{tr})$

$$\Rightarrow \begin{cases} g_{th} = \dfrac{(\alpha_i + \alpha_m)}{\Gamma} \\ n_{th} = n_{tr} + \dfrac{(\alpha_i + \alpha_m)}{\Gamma g'} \end{cases}$$

From these expressions

$$J_{th} = \frac{qd}{\eta_i} R(n_{th}) = \frac{qd}{\eta_i}\left(An_{th} + Bn^2{}_{th} + Cn^3{}_{th}\right)$$

$$\Rightarrow \boxed{J_{th} = \frac{qd}{\eta_i}\frac{n_{th}}{\tau_e(n_{th})} \quad \text{with} \quad \tau_e(n_{th}) = \frac{1}{A + Bn_{th} + Cn_{th}^2}}$$

With further increase of current density above threshold

$$J = \frac{qd}{\eta_i}\left(An_{th} + Bn_{th}^2 + Cn_{th}^3\right) + \frac{qd}{\eta_i} R_{st} \underset{\uparrow}{S}$$

photon density

$$= J_{th} + \frac{qd}{\eta_i} v_g g_{th} S$$

where $$R_{st} = v_g g(\underset{\uparrow}{n}) = v_g g_{th}$$

pinned at n_{th} above threshold

Above threshold and steady-state

Photon density

$$S = \frac{\eta_i}{q d v_g g_{th}} (J - J_{th})$$

Photon lifetime τ_p

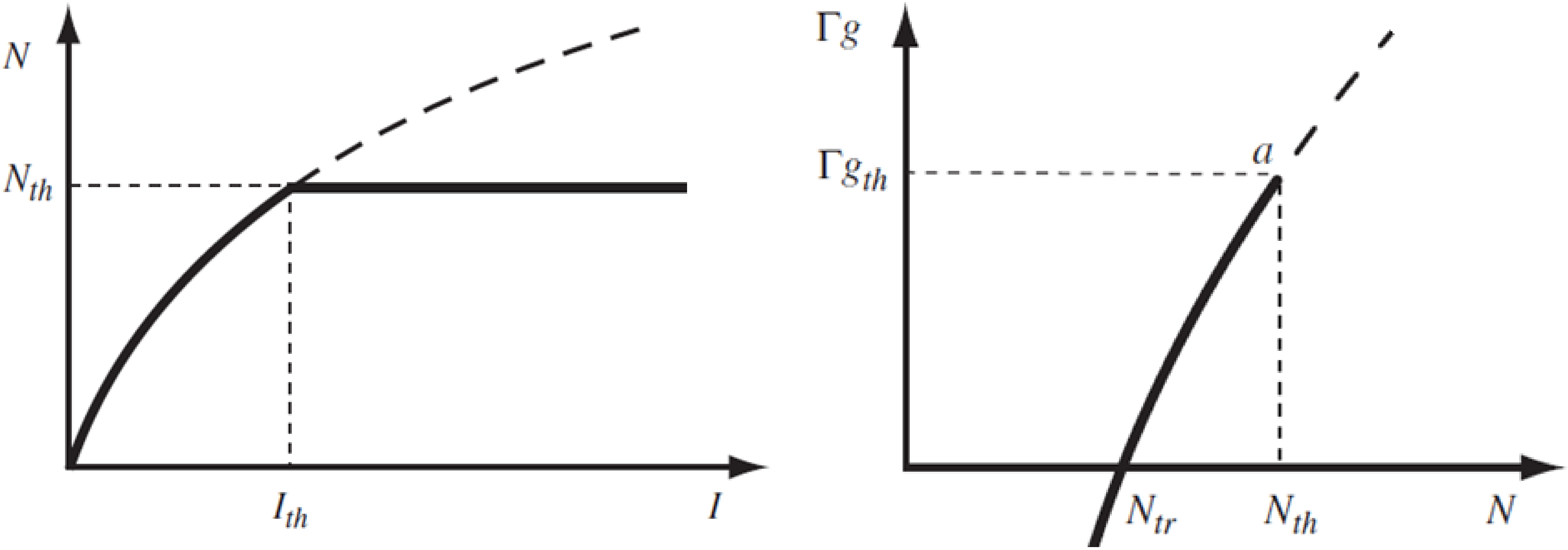
$$\frac{1}{\tau_p} = v_g (\alpha_i + \alpha_m) = v_g \Gamma g_{th}$$

τ_p accounts for loss rate of phonons in laser cavity, due to absorptions and transmissions at the end facets

Total number of photons leaving the cavity per second

$$\frac{S}{\tau_p} = \frac{\eta_i \Gamma}{q d} (J - J_{th})$$

The carrier density clamps at threshold causing the gain to clamp also



Confinement factor in Double heterostructure

In a symmetric dielectric slab waveguide, we can define a normalized wave guide thickness (it has the same expression as what is called the normalized frequency $\boldsymbol{V}$ in optical wave guides)

$$V = \frac{2\pi}{\lambda_0} d \sqrt{n_1^2 - n_2^2}$$

An approximate relation for the confinement factor is

$$\Gamma \approx \frac{V^2}{2 + V^2}$$

(Appendix A3 of Corzine, Coldren and Mašanović)

Threshold density N_{th}

For linear gain dependence with density we had

$$g(n) = g'(n - n_{tr}) \qquad g' \equiv \frac{dg}{dn}$$

differential gain

If we can measure a *total loss per pass* "γ" such that

$$\frac{\gamma}{L} = \alpha_i + \alpha_m$$

we can determine the threshold density from

$$n_{th} = n_{tr} + \frac{(\alpha_i + \alpha_m)}{\Gamma g'}$$

Power in Laser

Light Output Intensity

$$P_{out} = \begin{bmatrix} \text{Photon} \\ \text{Energy} \end{bmatrix} \times \begin{bmatrix} \text{Photon} \\ \text{Density} \end{bmatrix} \times \begin{bmatrix} \text{Effective Volume} \\ \text{of Optical Mode} \end{bmatrix} \times \begin{bmatrix} \text{Photon} \\ \text{Escape Rate} \end{bmatrix}$$

$$= [\hbar\omega] \times [S] \times [wL\, d_{op}] \times [v_g\, \alpha_m]$$

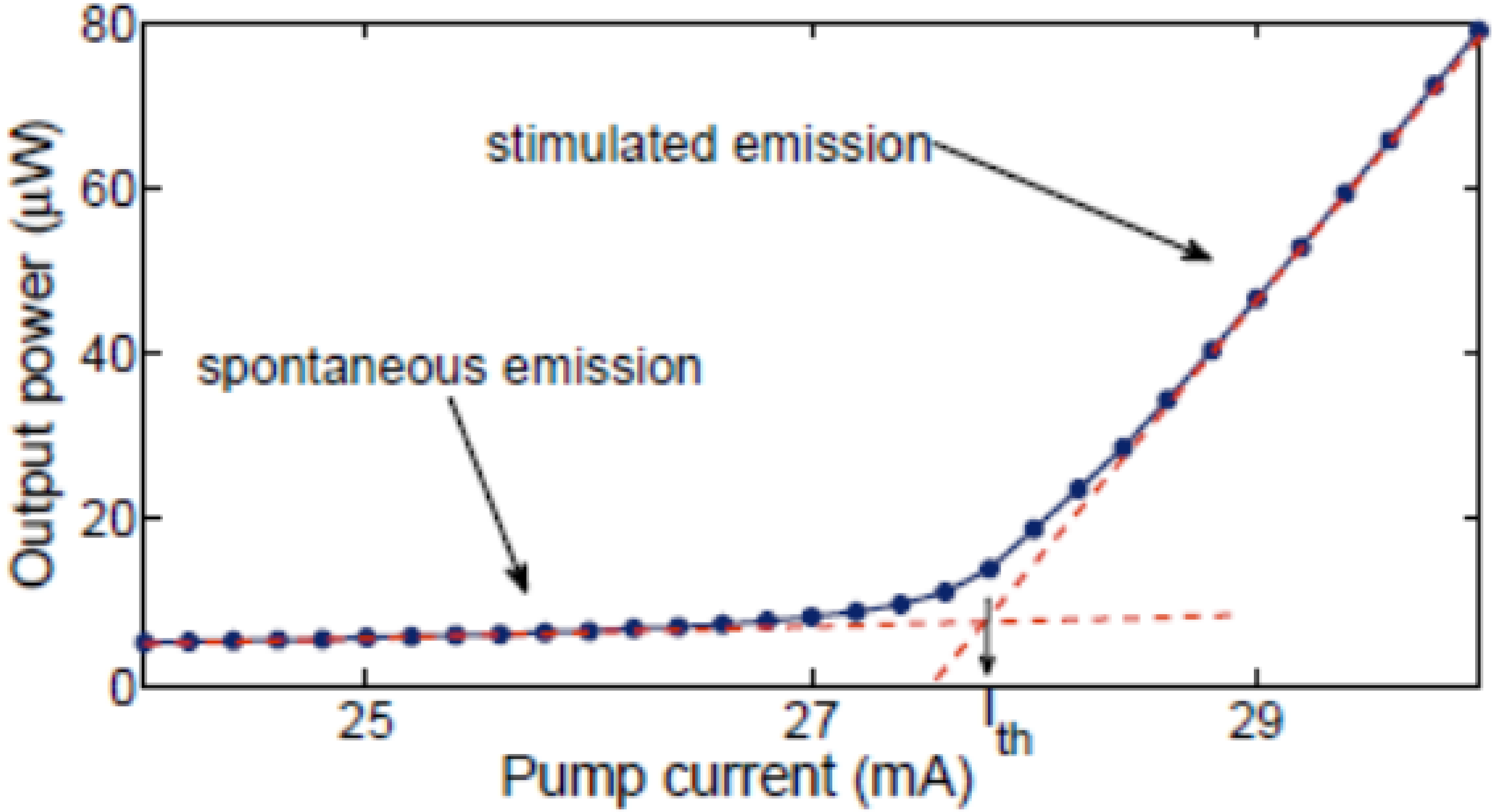
Since

$$\boxed{d_{op} = \frac{d}{\Gamma}}$$

$$\boxed{I = w\, L\, \boldsymbol{J}}$$

$$\boxed{P_{out} = \eta_i \frac{\hbar\omega}{q} \frac{\alpha_m}{\alpha_i + \alpha_m} (I - I_{th})}$$

Example of measured power



Hitachi Laser Diode HL6724MG

External Quantum Efficiency

$$\boxed{\eta_e = \frac{dP_{out}/dI}{\hbar\omega/q} = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m} = \eta_i \frac{\ln(1/R)}{\alpha_i L + \ln(1/R)}}$$

also $$\boxed{\eta_e^{-1} = \eta_i^{-1}\left[1 + \frac{\alpha_i L}{\ln(1/R)}\right]}$$

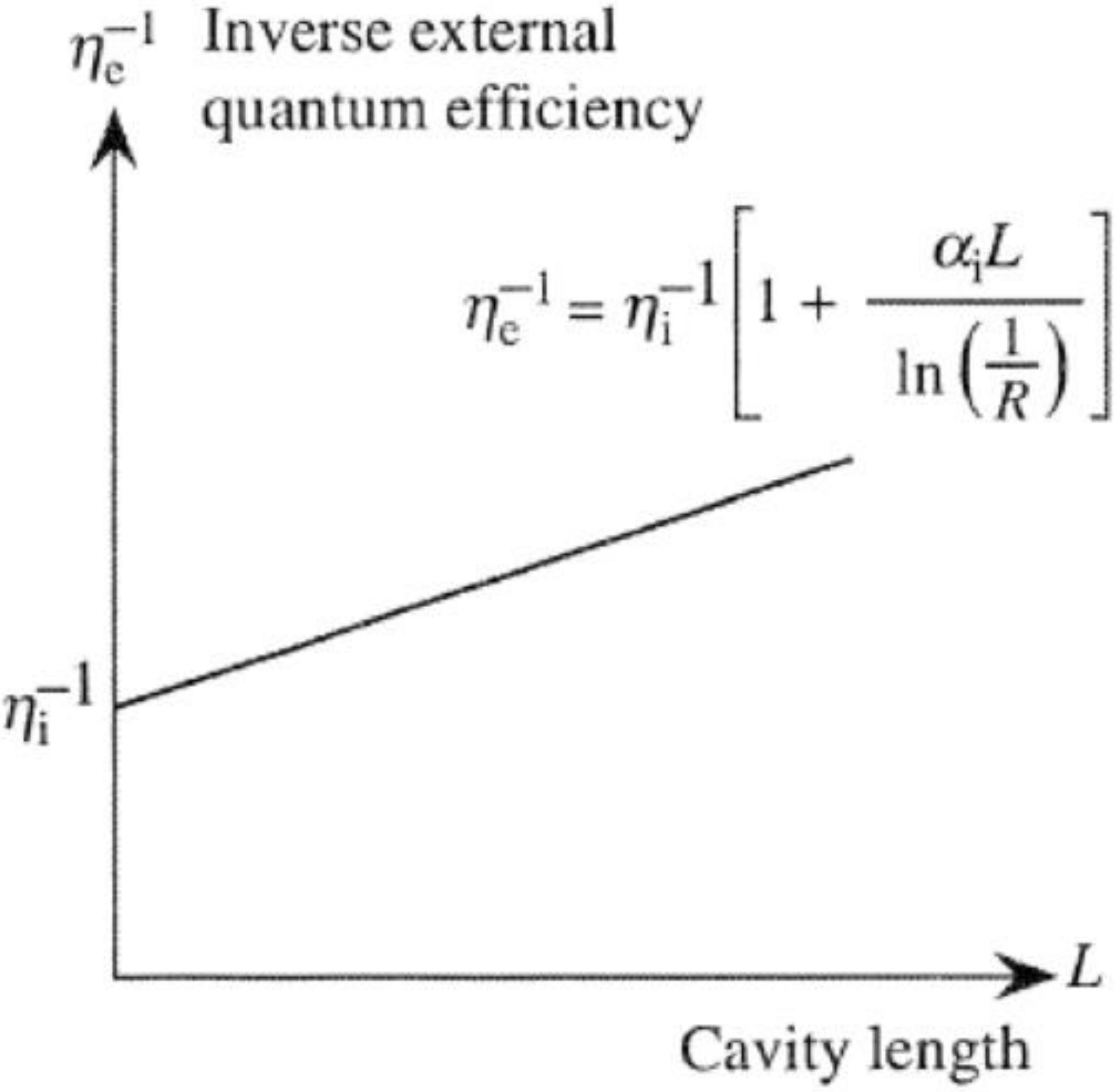
$$\alpha_m = \frac{1}{2L}\ln\frac{1}{R_1 R_2}$$

Plotting η_e^{-1} versus L gives a line with a y-intercept of η_i^{-1}

The slope divided by the y-intercept is

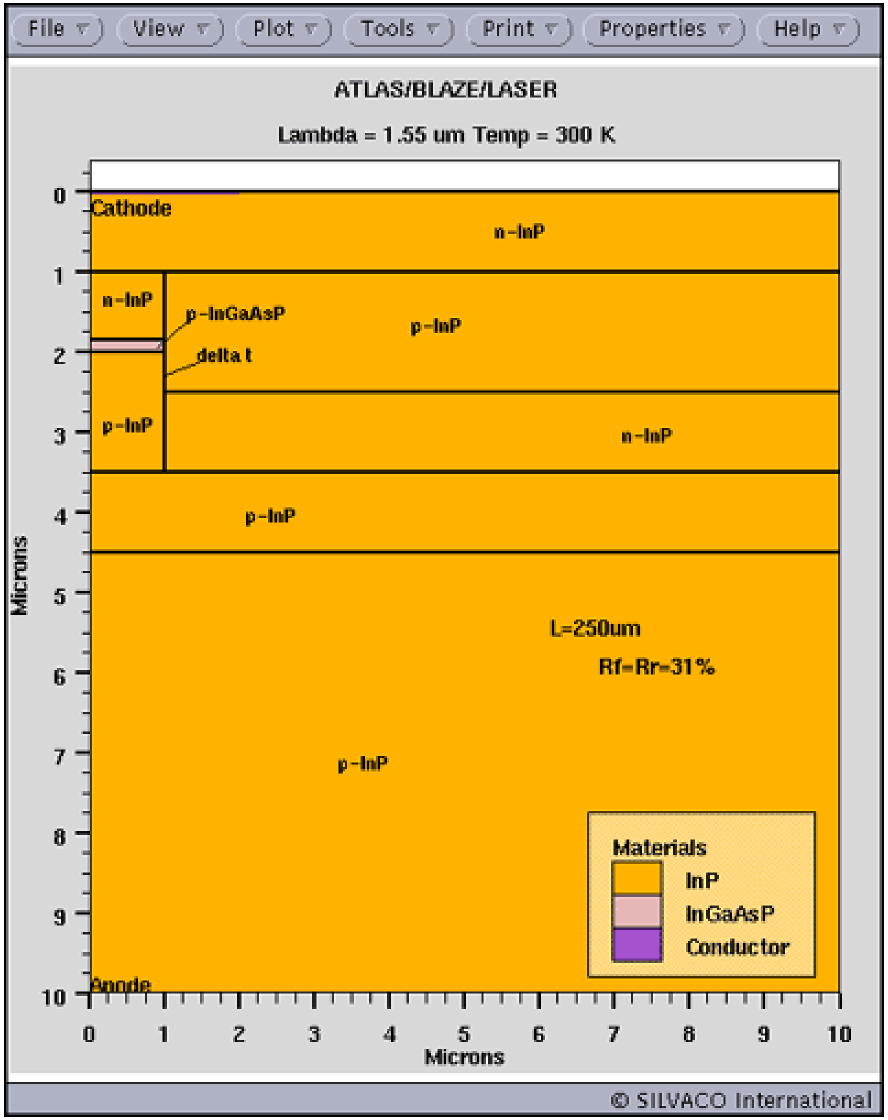
$\dfrac{\alpha_i}{\ln(1/R)}$ and can determine α_i

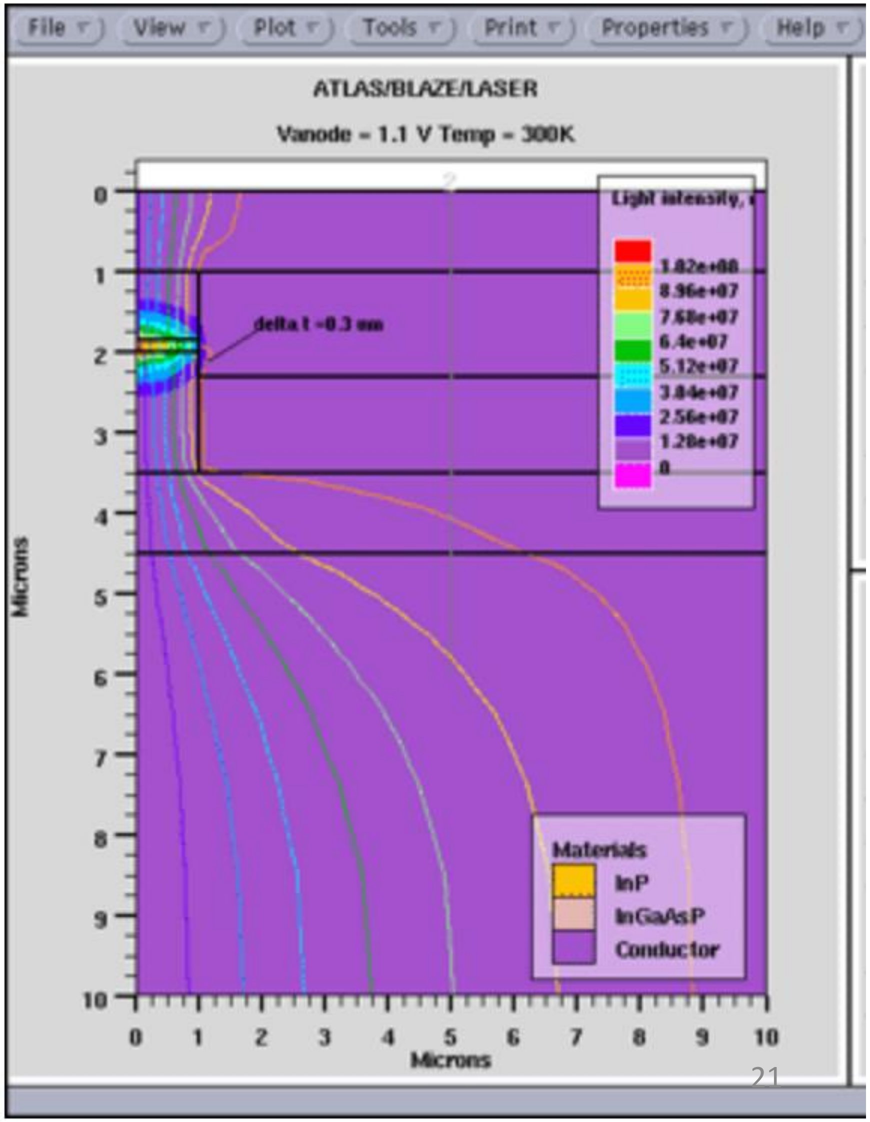
External Quantum Efficiency



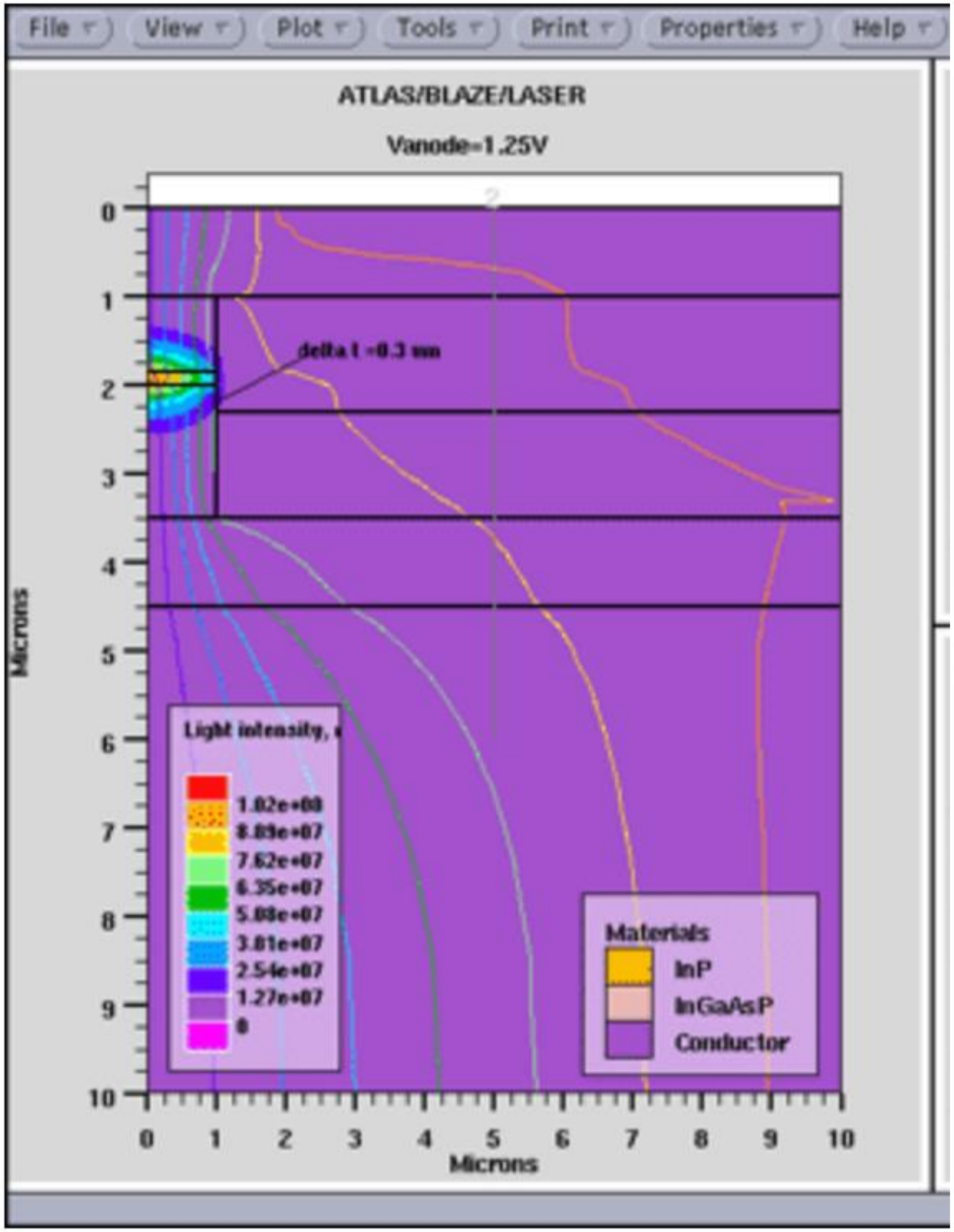
Leakage Current

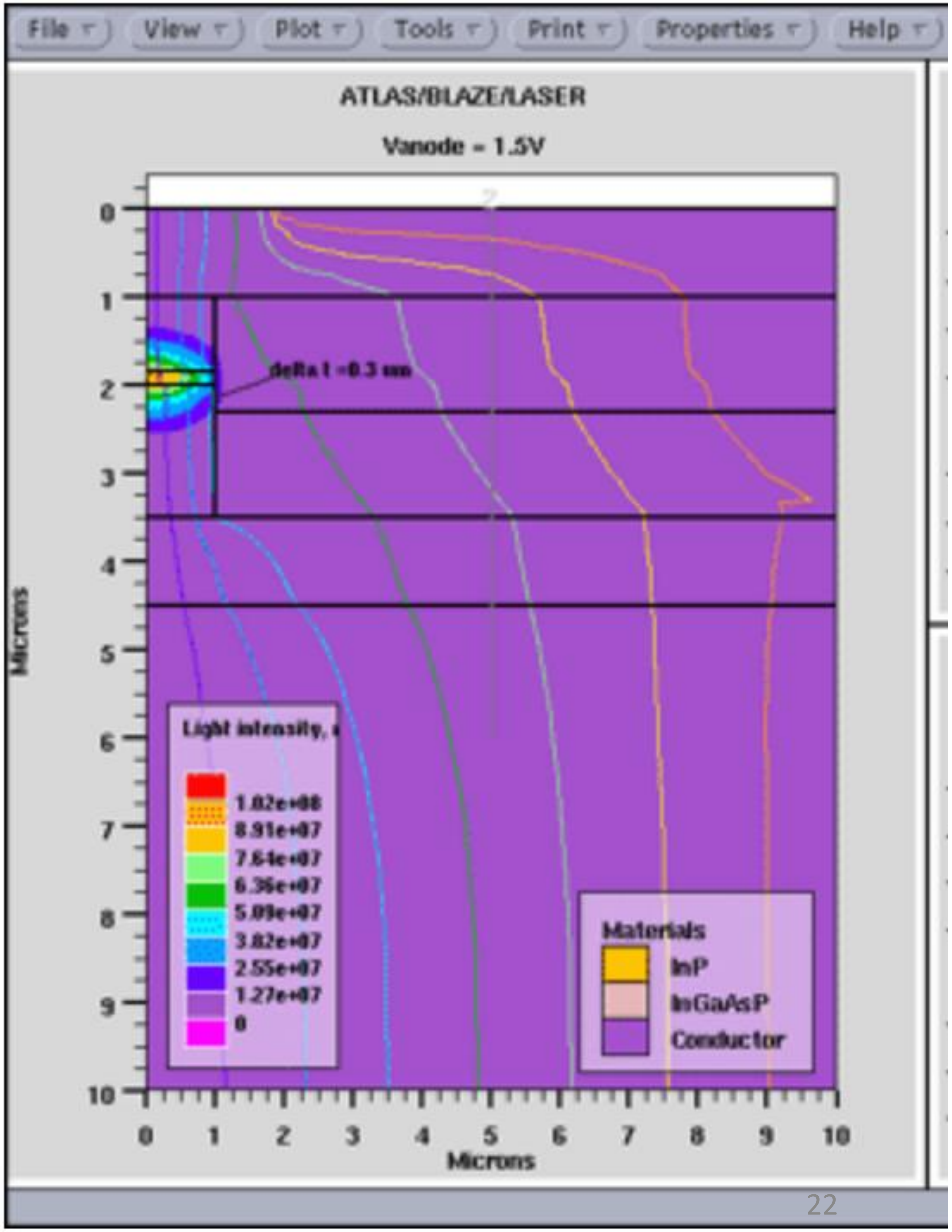
Current spread may let carrier flow in regions where stimulated emission is weaker.





Leakage Current





Leakage Current

$$I = I_A + I_L = JwL + I_L$$

$$I_{th} = J_{th} wL + I_{L@th} = \frac{qn_{th}\left(wLd\right)}{\eta_i \tau_e\left(n_{th}\right)} + I_{L@th}$$

Revised Expression for P_{out}

$$\boxed{P_{out} = \eta_i \frac{\hbar\omega}{q}\frac{\alpha_m}{\alpha_m + \alpha_i}\left(I - I_{th} - \Delta I_L\right)}$$

Temperature Dependence

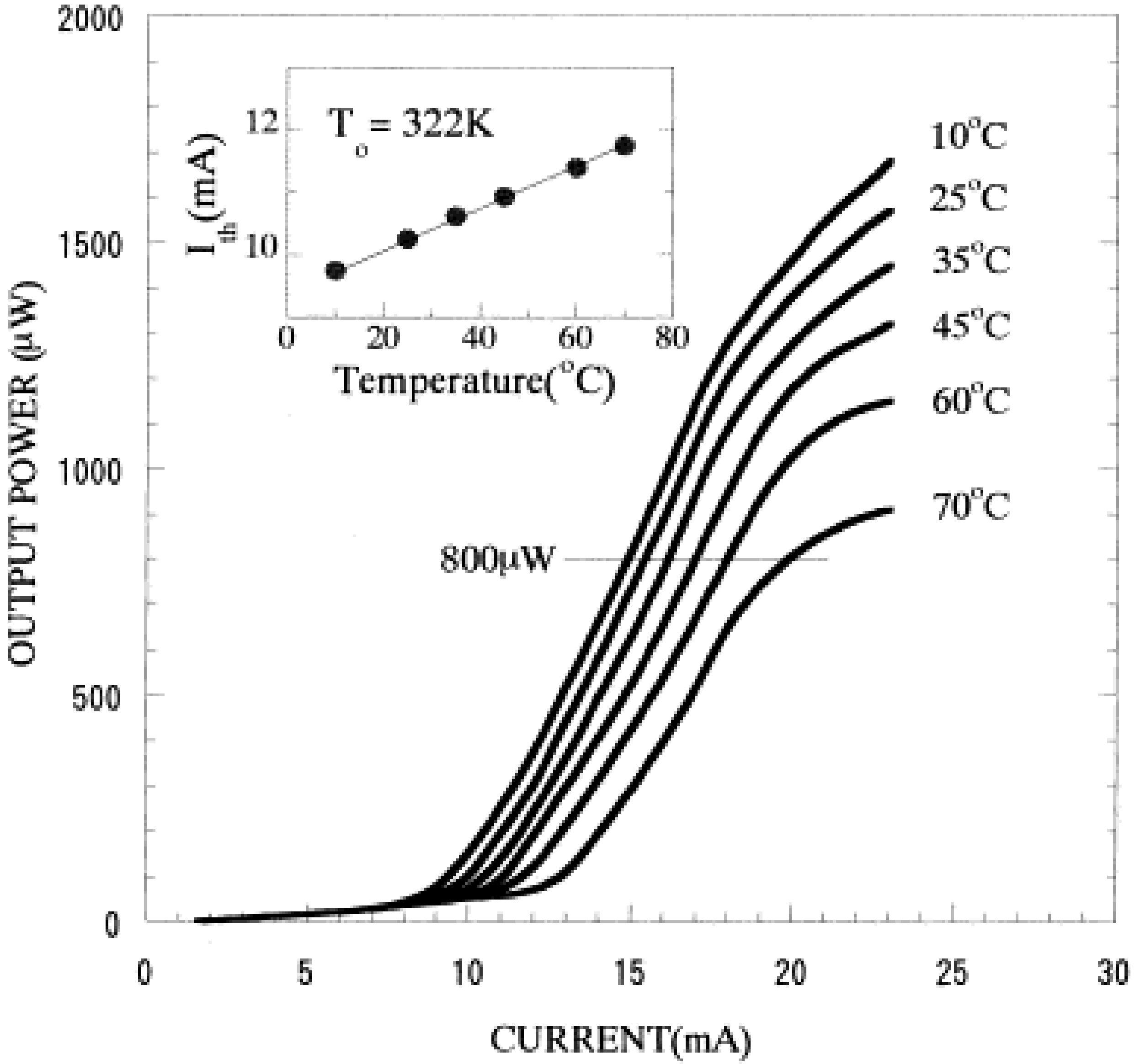
Laser threshold and efficiency vary with temperature since $g(\hbar\omega)$, Auger recombination, and other processes are temperature dependent

$$\boxed{\uparrow T \qquad \uparrow I_{th}(T) \qquad \downarrow \eta_e(T)}$$

$$I_{th}(T) = \{\text{constant}\}\, e^{T/T_0} = I_{th}(T_a)\, e^{(T-T_a)/T_0}$$

$$\eta_e(T) = \{\text{constant}\}\, e^{-T/T_1} = \eta_e(T_a)\, e^{-(T-T_a)/T_1}$$

Temperature Dependence



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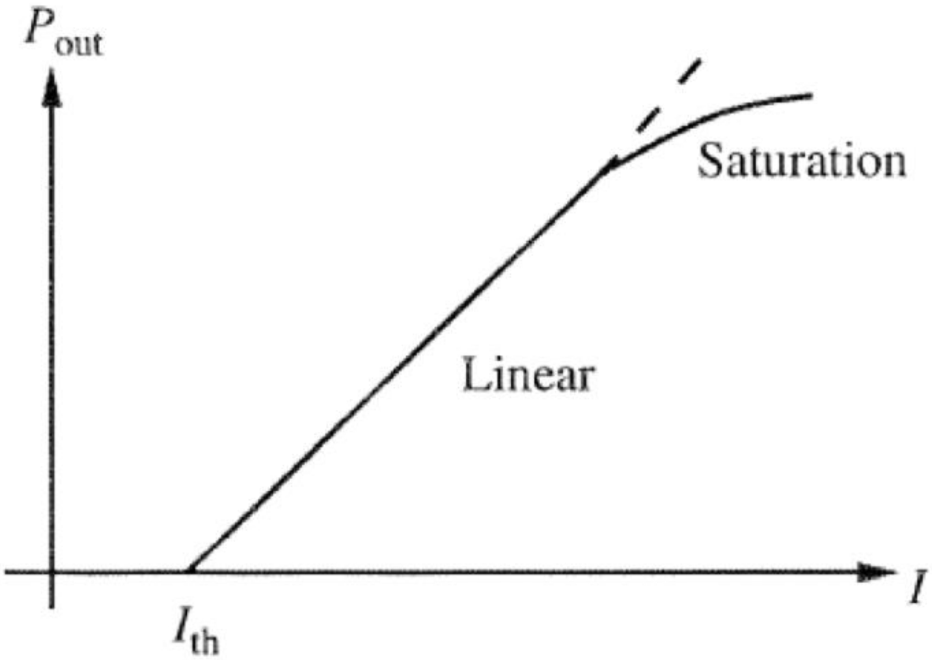
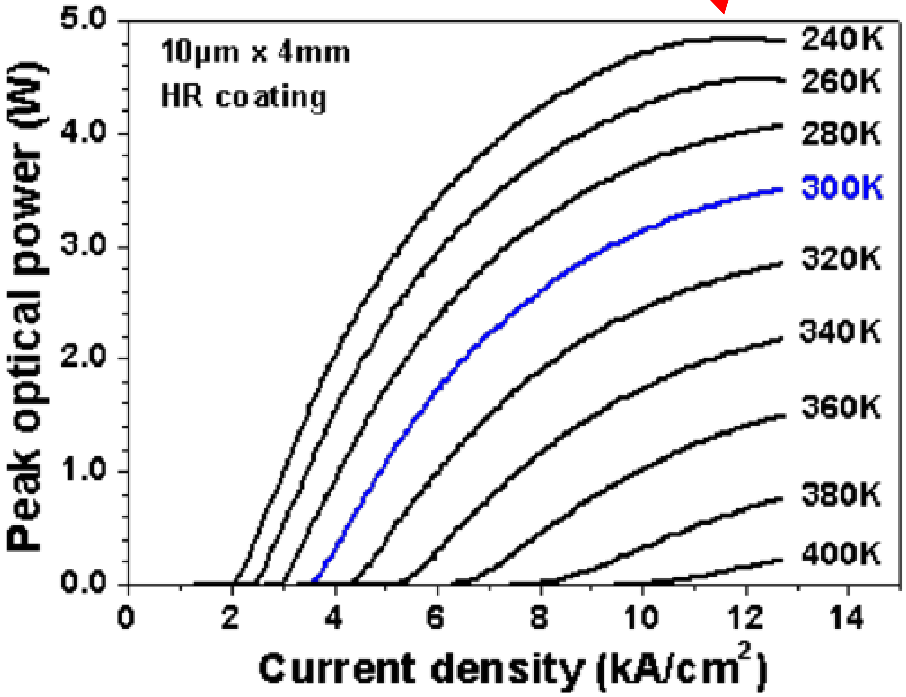
Saturation of Laser Output Power

Possible causes are:

- increasing leakage current
- Junction heating
- Increasing internal absorption α_i

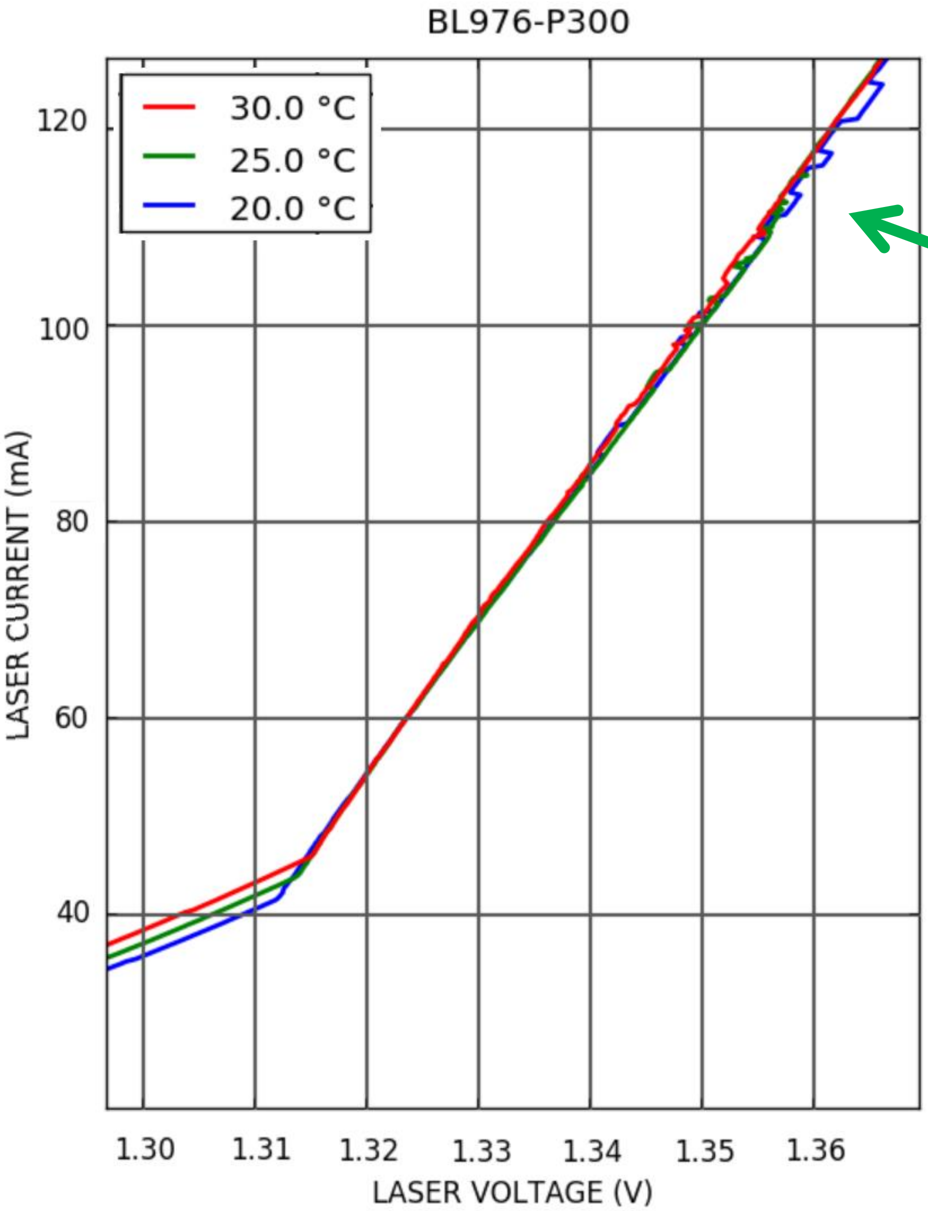
In continuous wave (CW) regime there is typically a maximum power $P_{max}(T)$





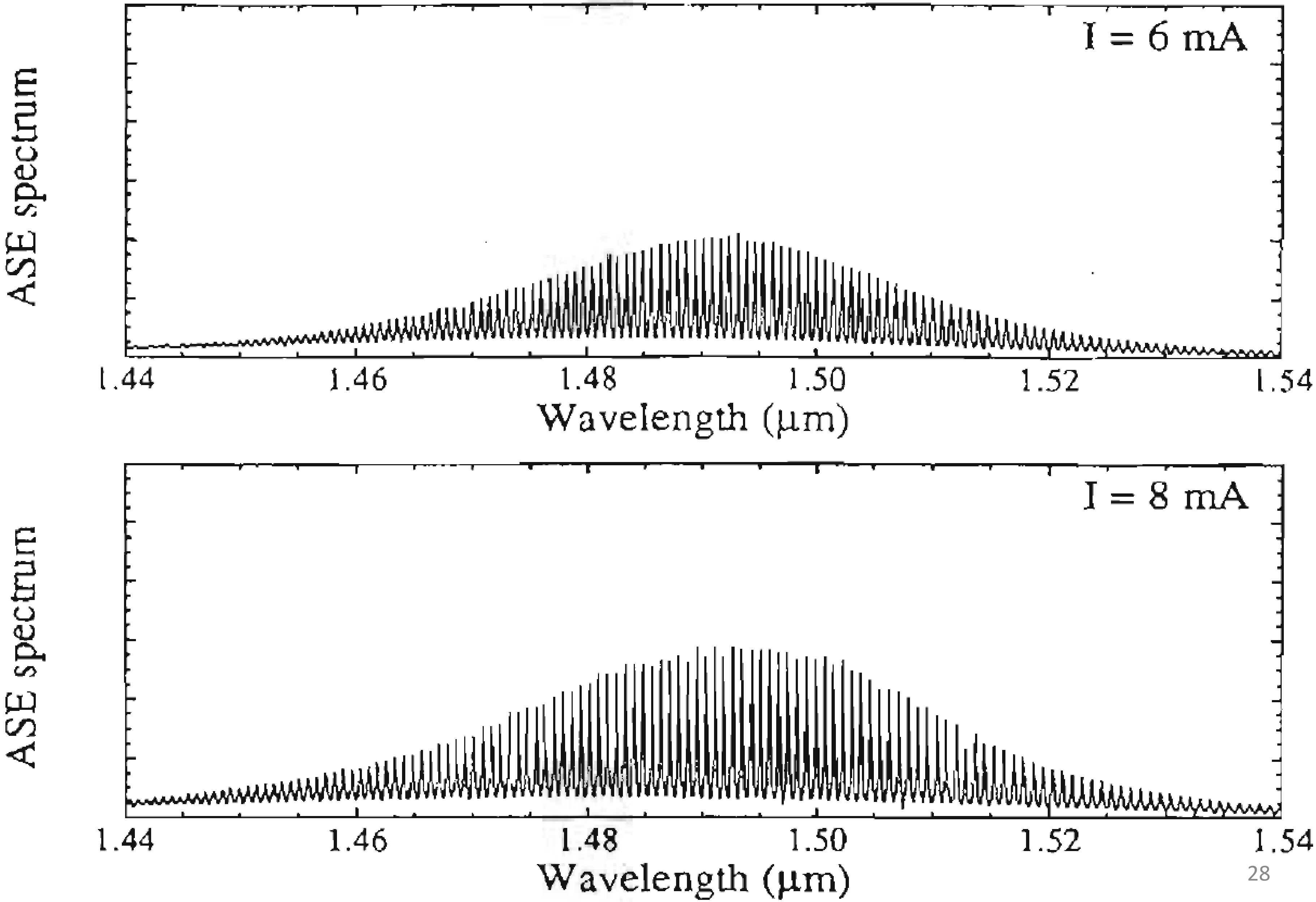
[University of Sheffield]

Measured I-V curve – Mode hopping

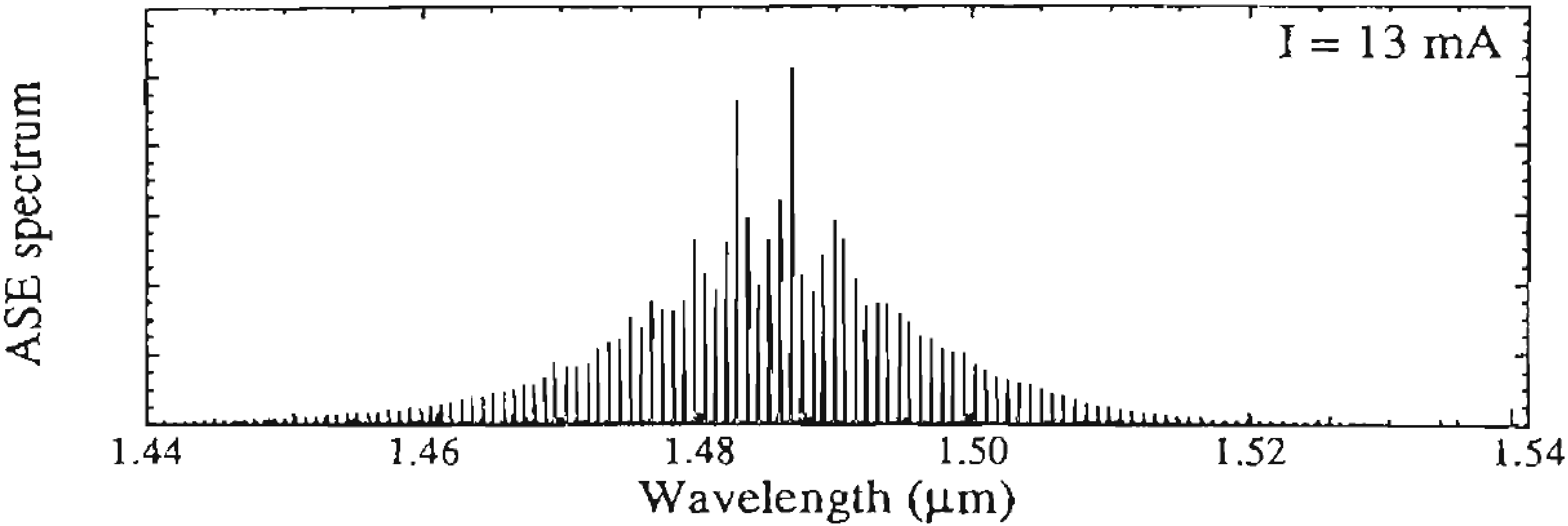


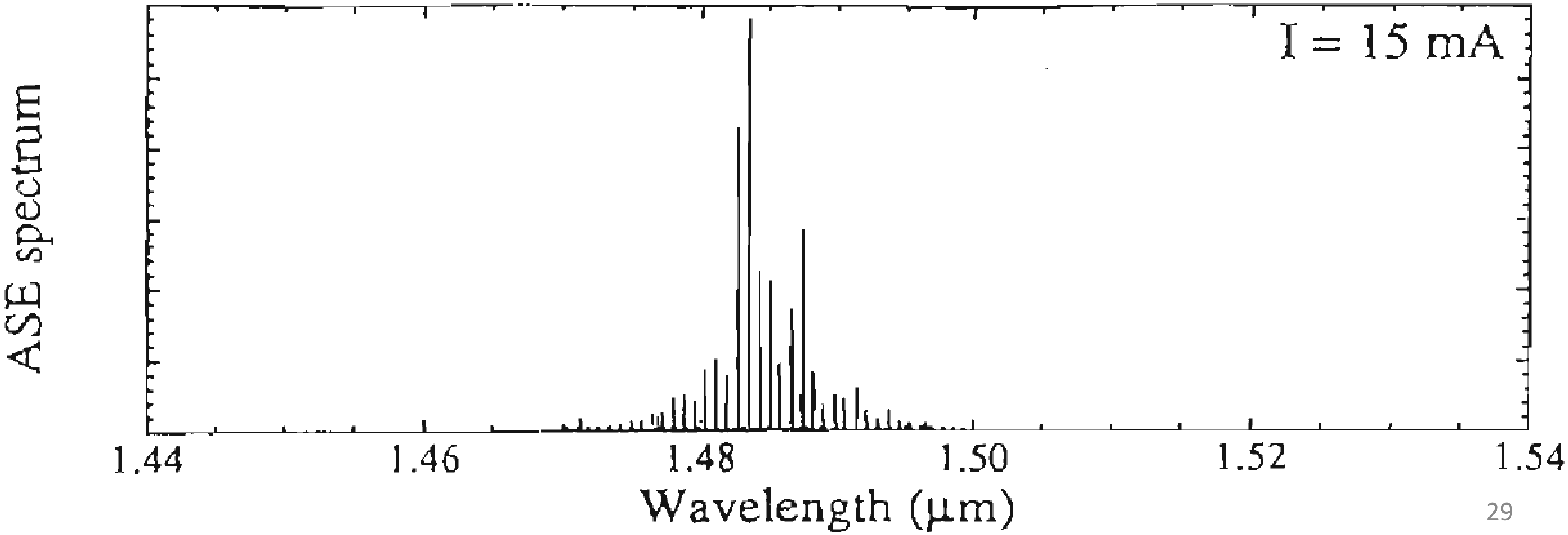
mode hopping

ASE Behavior below threshold



ASE Behavior below threshold





Saturation of Laser Output Power

$$\boxed{P_{in} = IV} \qquad \boxed{P_{out} = P_{in}\,\eta_{wp}}$$

$$\eta_{wp} = \frac{P_{out}}{P_{in}} = \frac{\frac{\hbar\omega}{q}\,\frac{\alpha_m}{\alpha_i + \alpha_m}\,\eta_i (I - I_{th})}{IV}$$

Power conversion efficiency is high in lasers when compared to other light emitter, but it typically does not exceed 60%. The power which is not converted into light is dissipated as heat, which needs to be removed by a heatsink.

The heatsink is characterized by a ***thermal resistance*** R_T

$$R_{\mathrm{T}} = \frac{\Delta T}{P_{in} - P_{out}}$$

$$\Delta T = R_T(P_{in} - P_{out}) = R_T\left(1 - \eta_{wp}\right)P_{in}$$

$\Delta T =$ ***laser overheating***

Improving P_{max}

To decrease overheating ΔT

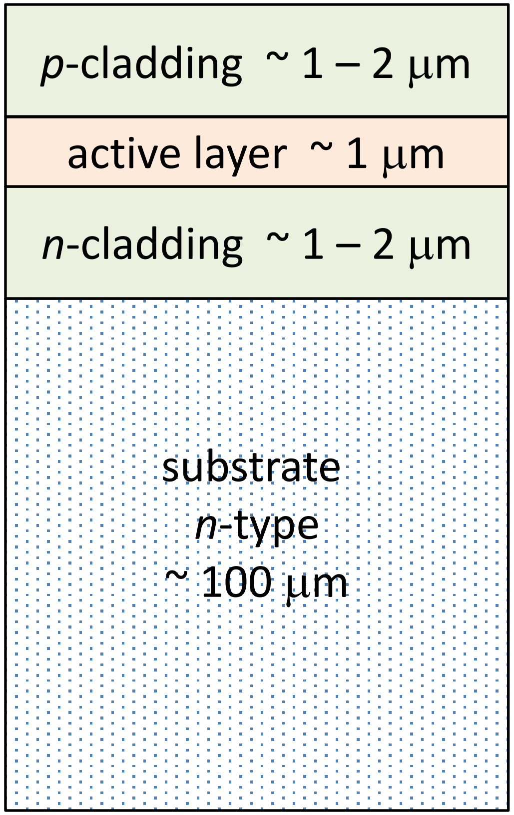
- Develop better heatsink and laser mounting to reduce R_T
- Improve laser "*wall-plug efficiency*"
    - Reduce voltage drop across heterostructure
    - Reduce internal loss α_i
    - Increase injection efficiency η_i
    - Reduce threshold current I_{th}

To decrease temperature sensitivity

- Suppress carrier leakage which affects external quantum efficiency
- Minimize non-radiative recombination which reduces the concentration at threshold

Improving P_{max}

Laser substrate is thick and has high thermal resistance. High power lasers are mounted upside down with the *p*-layer in contact with the heatsink.



p-cladding ~ 1 – 2 μm

active layer ~ 1 μm

n-cladding ~ 1 – 2 μm

substrate
n-type
~ 100 μm

Improving P_{max}

The thermal resistance is inversely proportional to the cavity length. For high power operation, a long cavity is better as long as laser efficiency is not affected too much.

$$\boxed{\alpha_m = \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)}$$

$$\boxed{\eta_e = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m}}$$

$$\boxed{\uparrow L \qquad \downarrow \alpha_m \qquad \downarrow \eta_e}$$

As mirror losses are lower with increasing cavity length, the relative role of internal loss increases. This causes quantum efficiency to drop.

Internal loss α_i

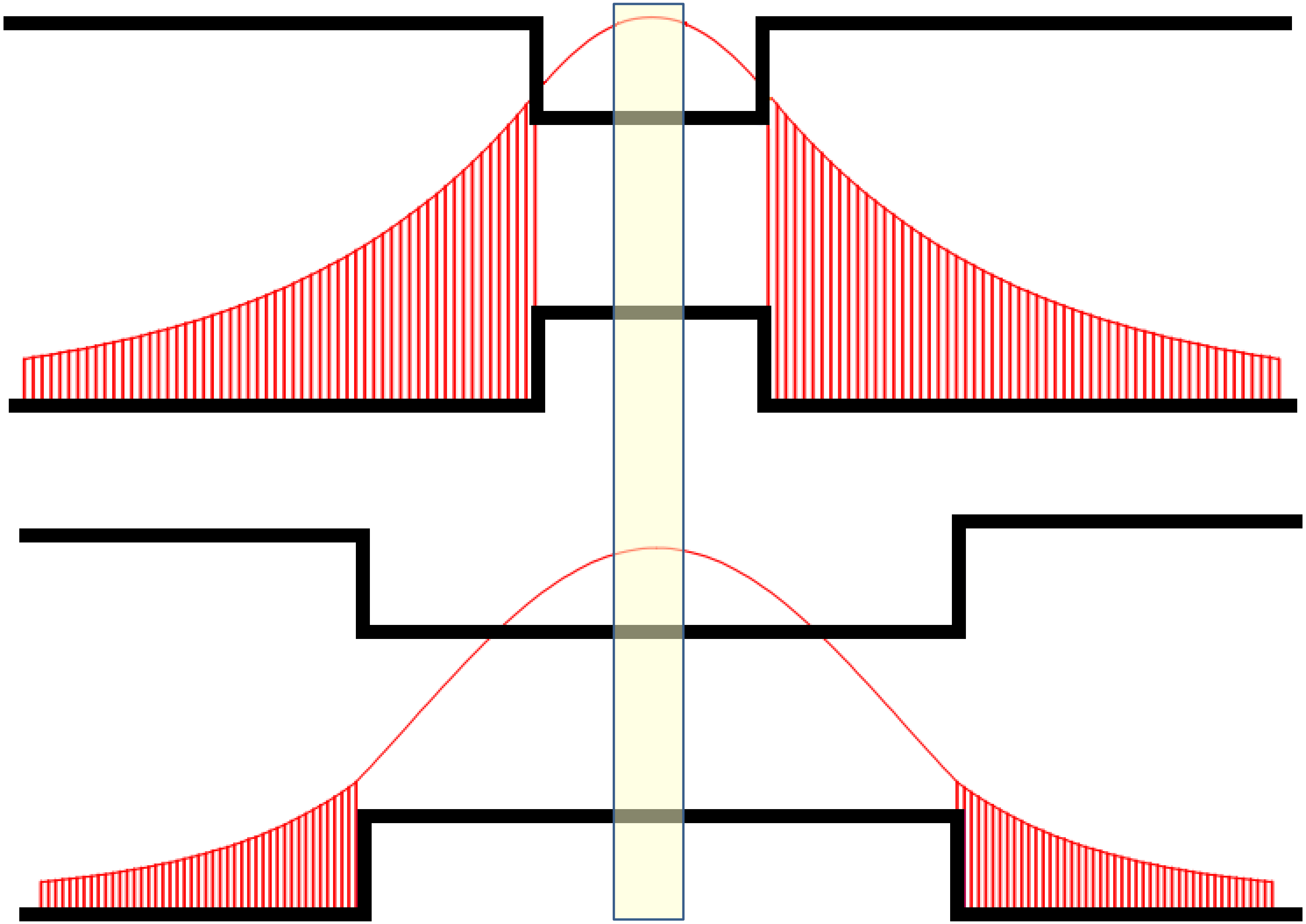
Much of the internal loss is due to free carrier absorption. Transitions between HH and SH valence bands, known as "intervalence band absorption" are an important component.

The main contribution to net internal loss is absorption in highly doped cladding regions.

The confinement factor is somewhat reduced in a *broadened waveguide* but losses in the cladding regions are greatly reduced, leading to a reduced threshold current density.

Efficiency of the broadened waveguide is less sensitive to cavity length, so this structure is suitable for power applications, increasing maximum CW power.

Broadened waveguide



active quantum
region

Minimize threshold current to increase power output

$$\boxed{P_{out} = \eta_i \frac{\hbar}{\omega} \frac{\alpha_m}{\alpha_i + \alpha_m} (I - I_{th})}$$

$$I_{th} = I_{tr} + I_{loss}$$

To minimize I_{tr}

- Reduce number of quantum wells to a minimum
- Compressive strain in QW reduces difference in effective mass between C and HH bands

To minimize I_{loss}

- Compressive strain in QW reduces difference in effective masses and increases differential gain

Minimize threshold current to increase power output