# ECE 536 – Integrated Optics and Optoelectronics
# Lecture 15 – March 8, 2022

**Spring 2022**

Tu-Th 11:00am-12:20pm

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# Lecture 15 Outline

- Amplified Spontaneous Emission (ASE)
- Optical Gain Measurements
- Considerations on Lossy/Gain Media
- Gain Guided Laser
- Index Guided Laser

# ASE and Optical Gain Measurements

# Experimental Determination of Optical Gain

Measurements of gain continue to be an important component to validate and refine the design of a laser in order to meet desired performance specifications.

**Optical stripe-length method**

An external laser source is used to excite the sample under investigation. The laser beam is focused as a stripe with a cylindrical lens. The ***amplified spontaneous emission*** (ASE) is measured as a function of the stripe length and the gain is extracted from a fit of the data. This method is applied to a material which has not yet been processed into a complete laser structure.

**Hakki-Paoli method** – the laser is operated below threshold. If the length of the device and the facet reflectivity are known, the gain can be evaluated from maxima and minima of the Fabry-Pérot spectrum recorded with a spectrometer of sufficient resolution.

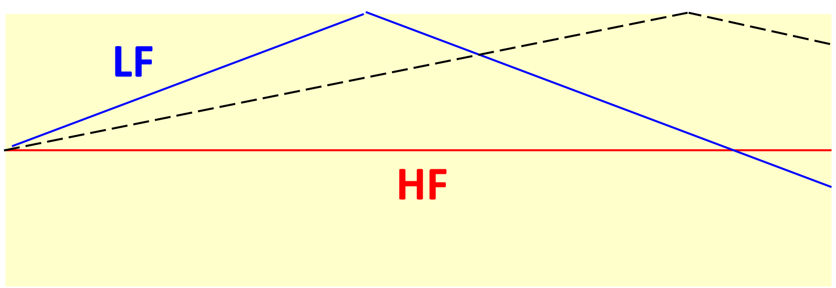
**Transmission method** – Requires a broadband weak light source, covering the spectral region of interest, transmitted through the device (which should operate in the fundamental mode with suppression of the Fabry-Pérot modes by deposition of anti-reflective coatings on the facets). The amplification of this broadband probe light in the diode is measured and it provides directly the gain spectra.

# Review again: effective index $n_e$

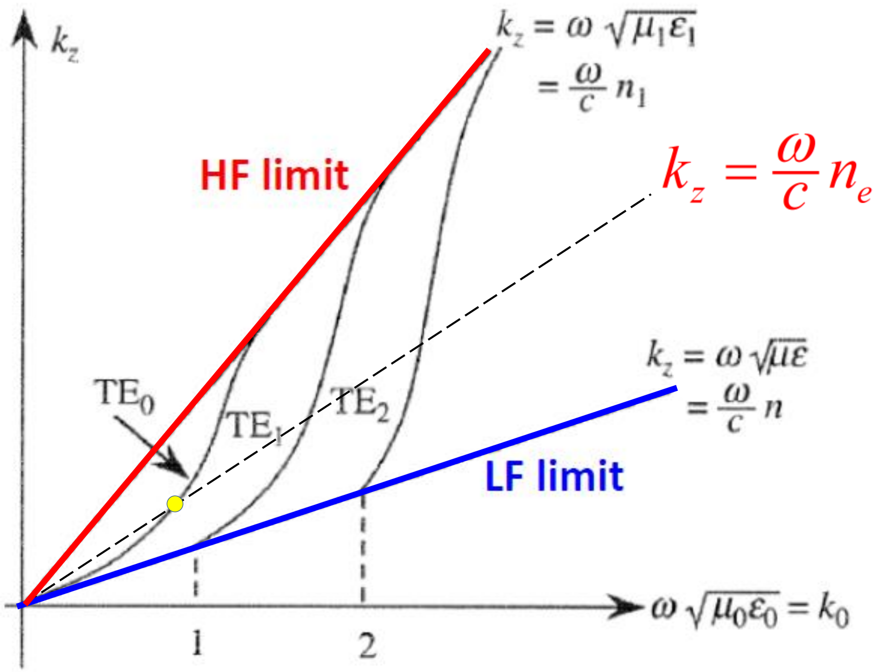
$$k_z = \frac{2\pi}{\lambda_z} = n_e k = n_e \frac{2\pi}{\lambda}$$

$$n_e = \frac{2\pi}{\lambda_z}\frac{\lambda}{2\pi} = \frac{\lambda}{\lambda_z} = \frac{v_p}{f}\frac{f}{v_{pz}} = \frac{v_p}{v_{pz}}$$

$$n < n_e < n_1$$


Strictly, each mode is associated with a specific effective index at a given frequency



# Optical Fields in Cavity with Gain

**Fabry-Perot Cavity**

Initial Field: $E_{sp}(\lambda)$

Field After Single Pass: $E'_{sp}(\lambda) = E_{sp}(\lambda) + E_{sp}(\lambda) r_1 r_2 e^{i2kL}$

General Expression: $E_{ASE}(\lambda) = E_{sp}(\lambda)\left[1 + r_1 r_2 e^{i2kL} + \left(r_1 r_2 e^{i2kL}\right)^2 + \left(r_1 r_2 e^{i2kL}\right)^3 + \ldots\right]$

$$\left[1 + a + a^2 + a^3 + \ldots\right] = \frac{1}{1-a} \quad \longrightarrow \quad \boxed{E_{ASE}(\lambda) = \frac{E_{sp}(\lambda)}{1 - r_1 r_2 e^{i2kL}}}$$

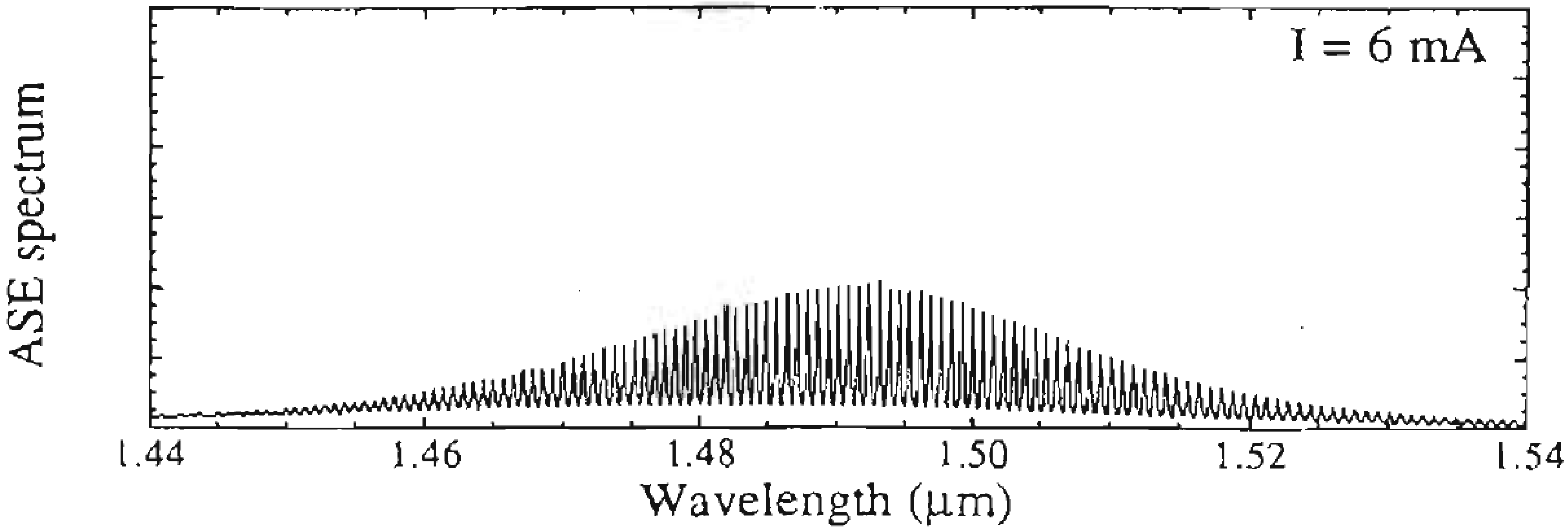
$n_e$ is the effective index

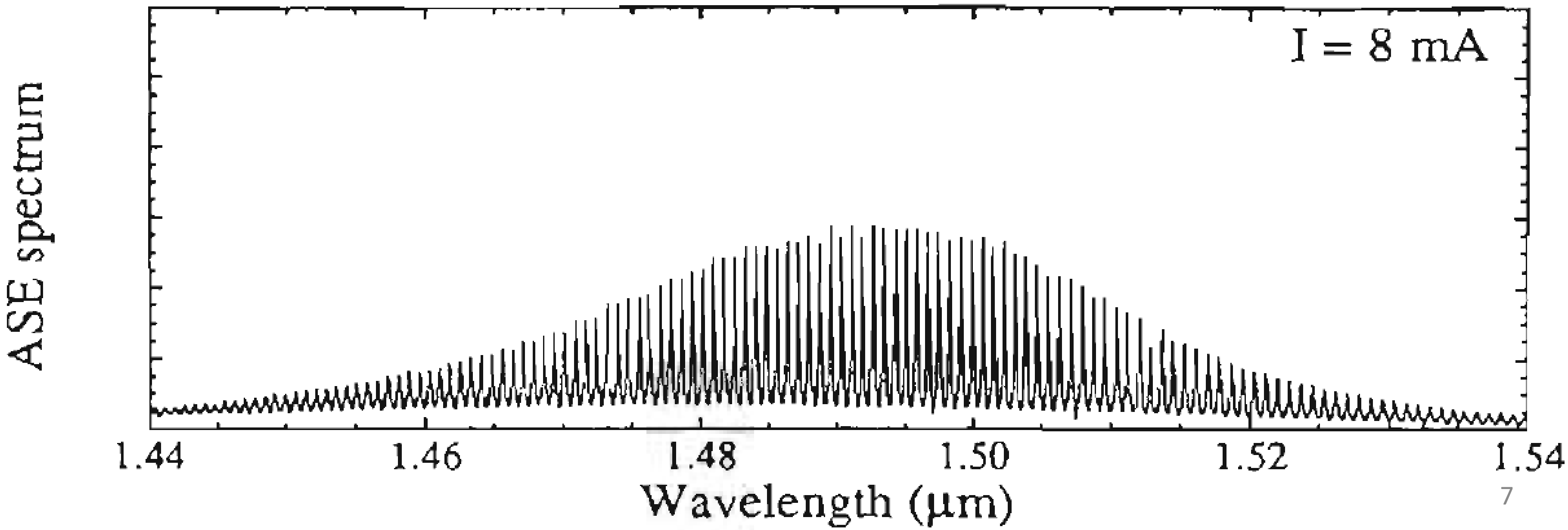
$G_n = \Gamma g - \alpha_i$ is the net modal gain

$$k = k' - i\frac{G_n}{2} = \frac{2\pi}{\lambda} n_e - i\frac{G_n}{2}$$

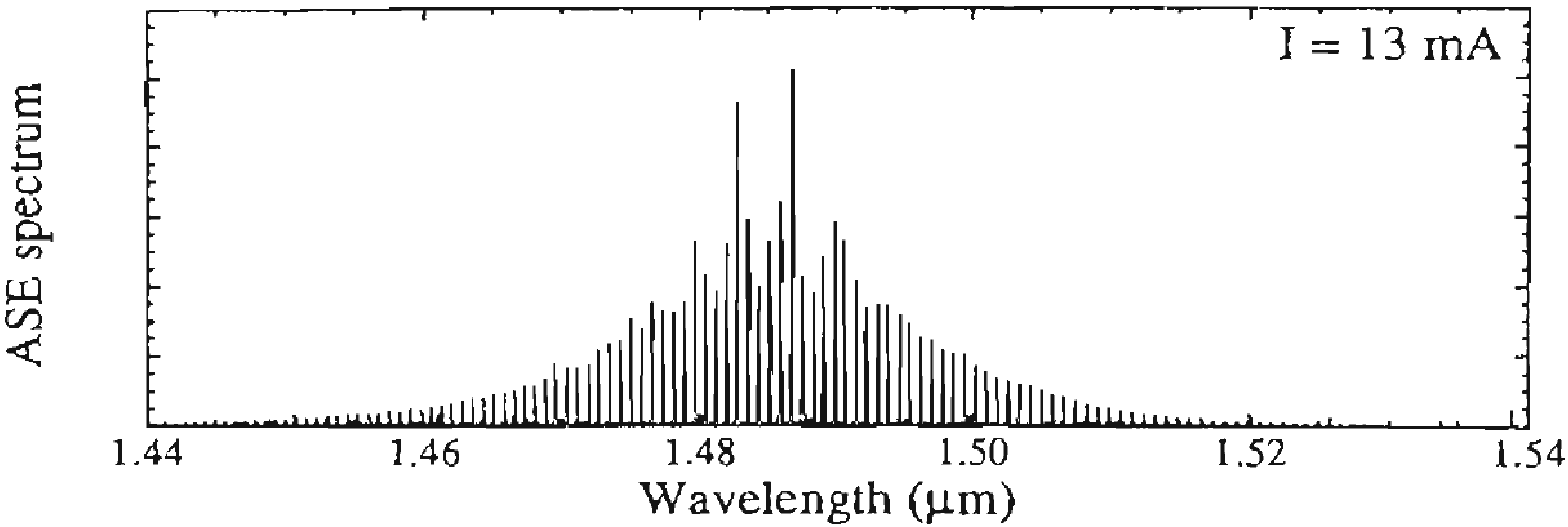
$$G_n = \Gamma g - \alpha_i$$

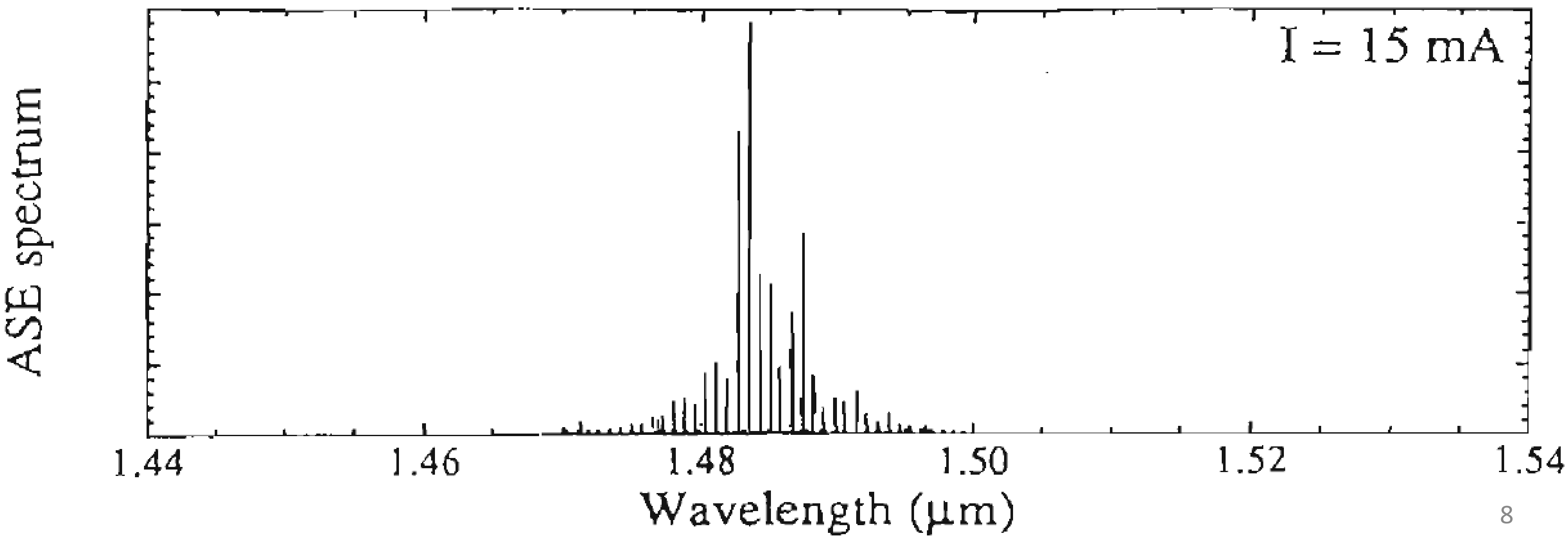
# ASE Behavior below threshold



# ASE Behavior below threshold



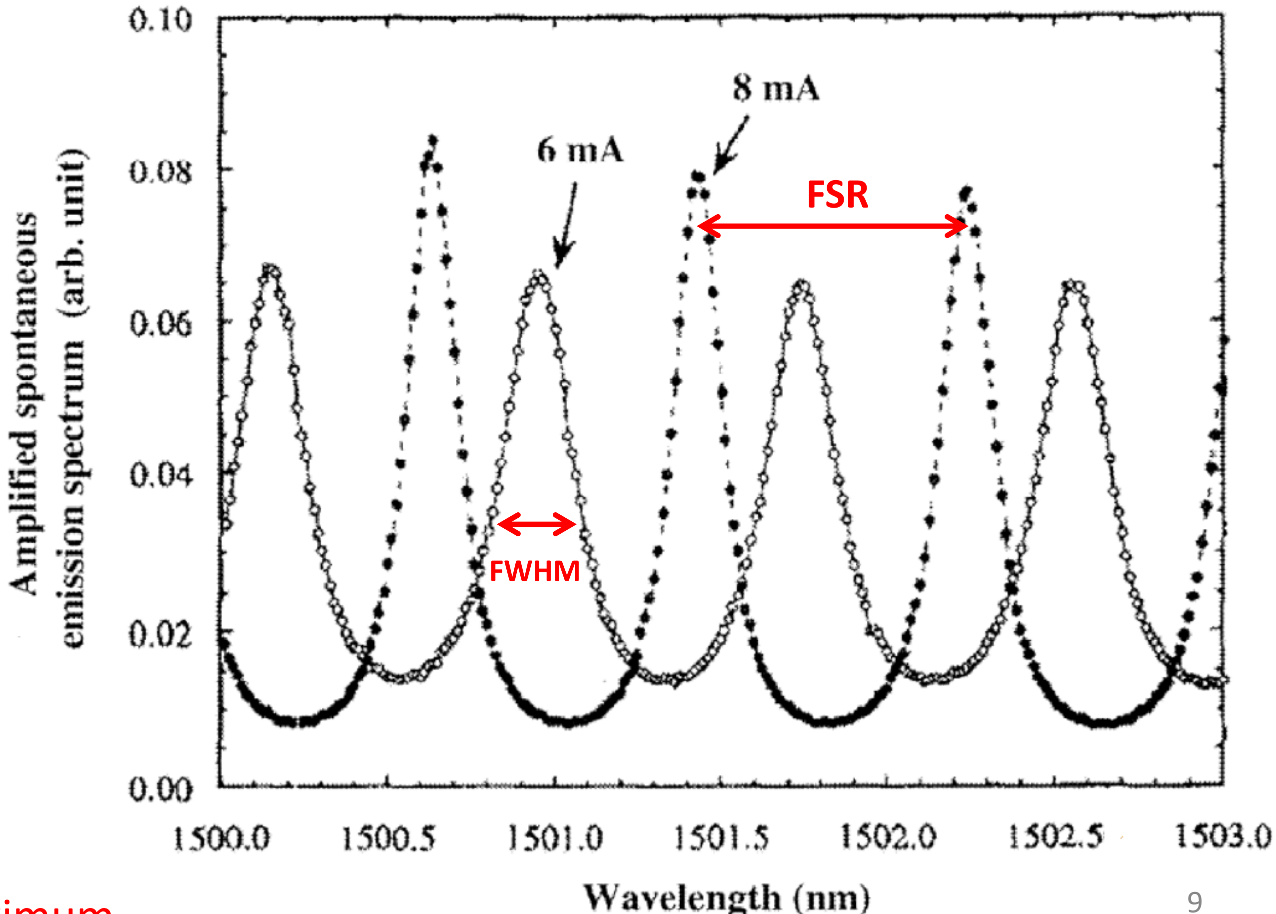
# ASE Power Spectrum

$$P(\lambda) \propto \left|E_{ASE}(\lambda)\right|^2 = \frac{\left|E_{sp}(\lambda)\right|^2}{\left|1 - r_1 r_2 e^{i2kL}\right|^2} = \frac{\left|E_{sp}(\lambda)\right|^2}{(1-A)^2 + 4A\sin^2(k'L)}$$

$$A = \sqrt{R_1 R_2}\, e^{G_n L}$$

$$R_1 = \left|r_1\right|^2$$

$$R_2 = \left|r_2\right|^2$$




FSR = Free Spectral Range

FWHM = Full Width Half Maximum

# Modal Gain from ASE Spectra – 2

Taking the ratio (nearby ASE peaks): $\frac{I_{\max}}{I_{\min}} = \frac{(1+A)^2}{(1-A)^2}$

Solving for A: $A = \frac{\sqrt{I_{\max}/I_{\min}}-1}{\sqrt{I_{\max}/I_{\min}}+1} = \sqrt{R_1 R_2}\, e^{G_n L}$

Solving for G: $G_n = \frac{1}{L}\ln\frac{\sqrt{I_{\max}/I_{\min}}-1}{\sqrt{I_{\max}/I_{\min}}+1} + \frac{1}{2L}\ln\frac{1}{R_1 R_2}$

G is the net modal gain.

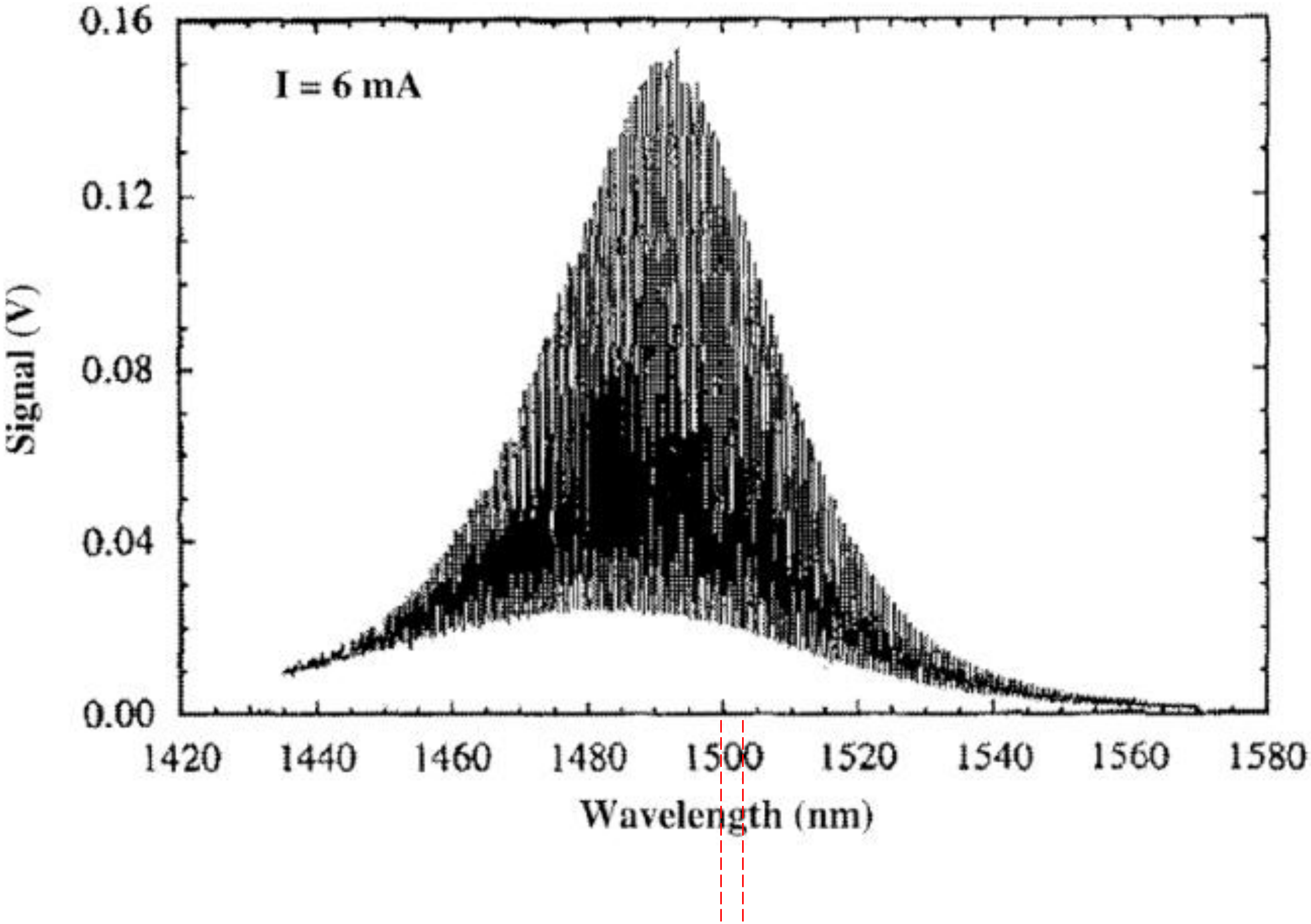
Note: $G_n(\lambda) = \Gamma g(\lambda) - \alpha_i = Q_r + \alpha_m$ where $Q_r = \frac{1}{L}\ln\frac{\sqrt{I_{\max}/I_{\min}}-1}{\sqrt{I_{\max}/I_{\min}}+1}$

# Hakki-Paoli Method for Gain Measurement

- Measure ASE spectrum
- Take the ratio of the magnitude of $I_{max}$ and $I_{min}$ for the peaks near the $m^{th}$ mode $\lambda_m$
- Calculate the mirror reflectivity and either measure the cavity length or calculate it from the measured mode spacing and the effective index $n_e$
- Gain is calculated using

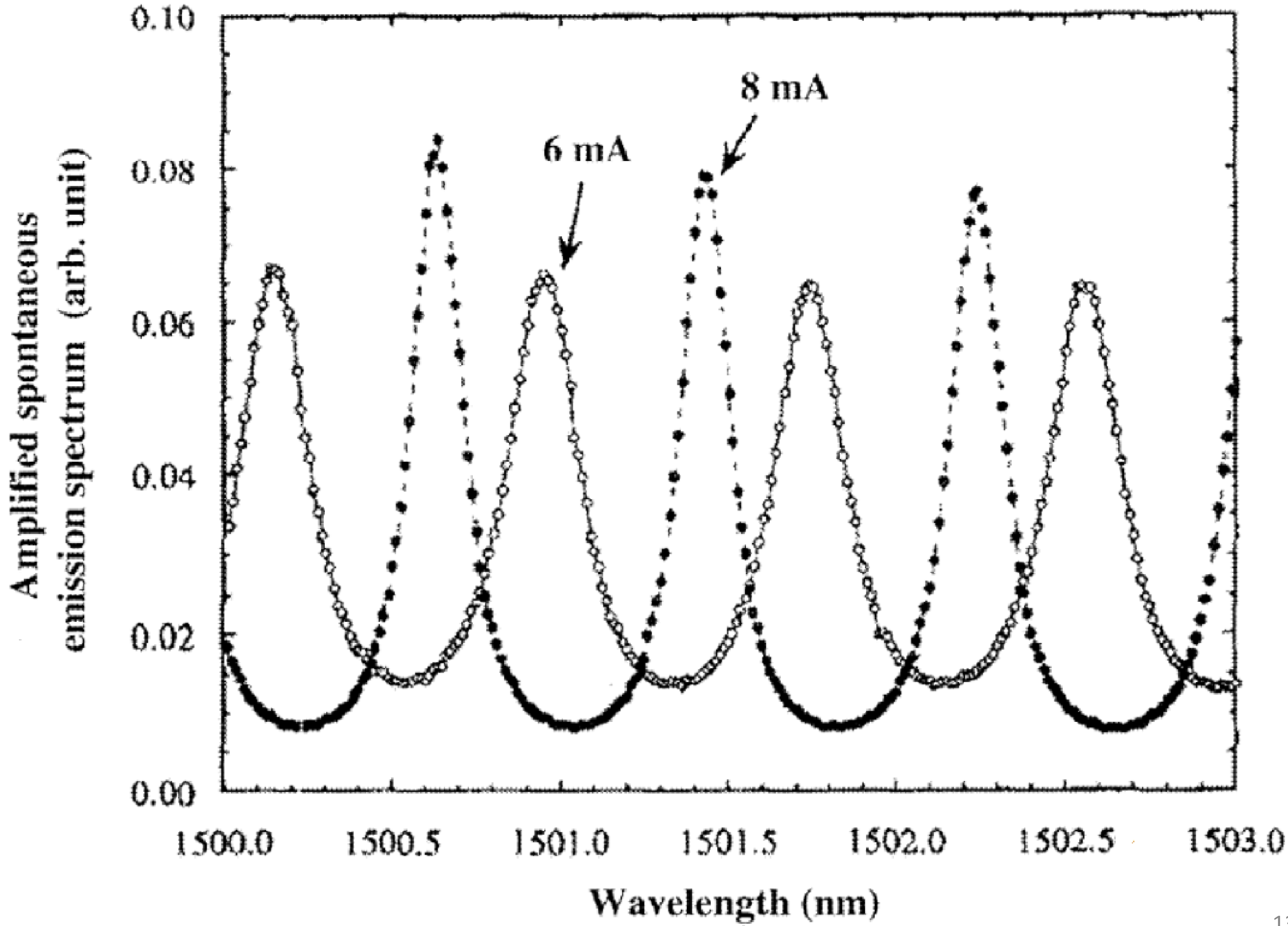
$$G_n(\lambda) = Q_r + \alpha_m$$

# Hakki-Paoli Method for Gain Measurement




spectrum range enlarged in next slide

# Hakki-Paoli Method for Gain Measurement



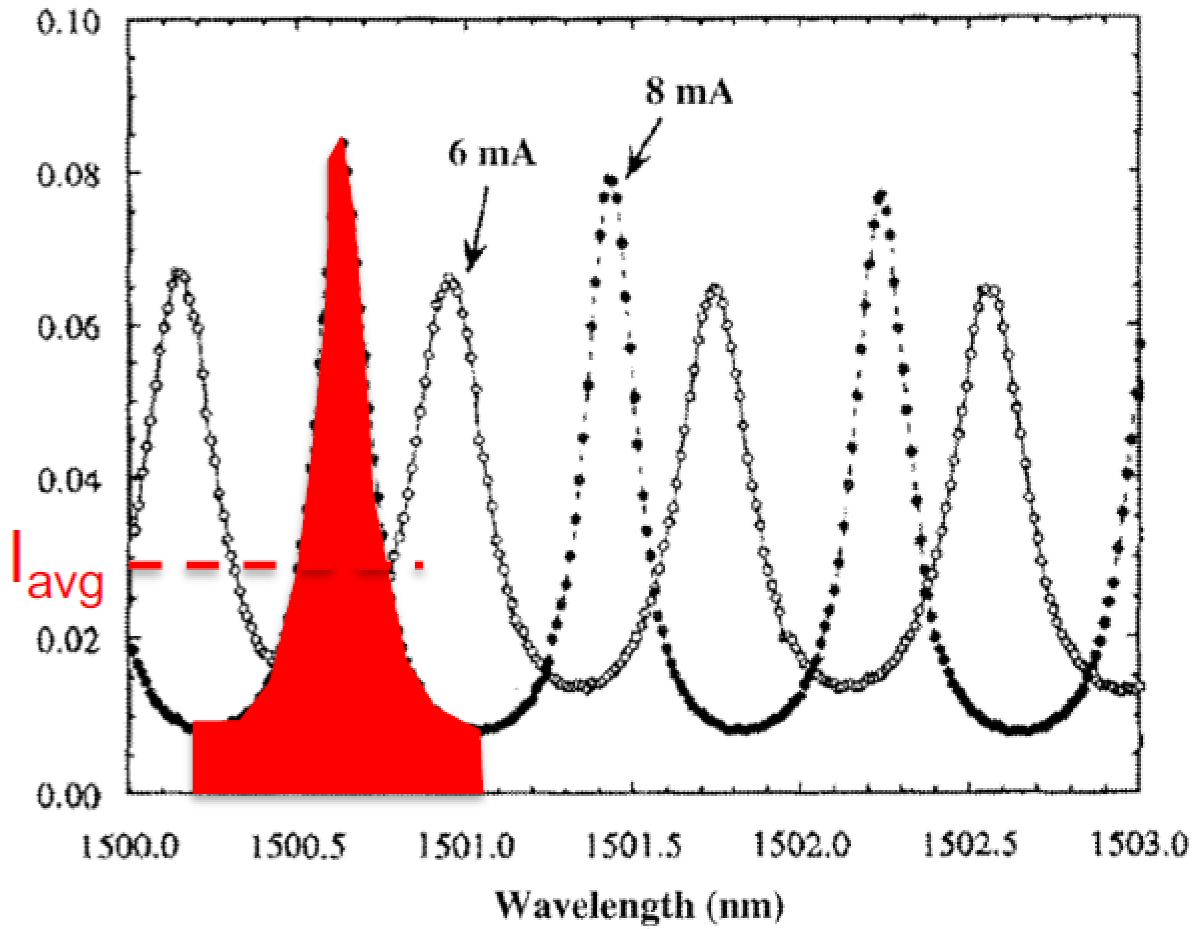
# Cassidy's Method – Variant of Hakki-Paoli Method

Based on the Hakki-Paoli method, except that the ratio: $I_{avg} / I_{min}$ is computed.

$$I_{avg} = \sqrt{I_{max} \cdot I_{min}}$$

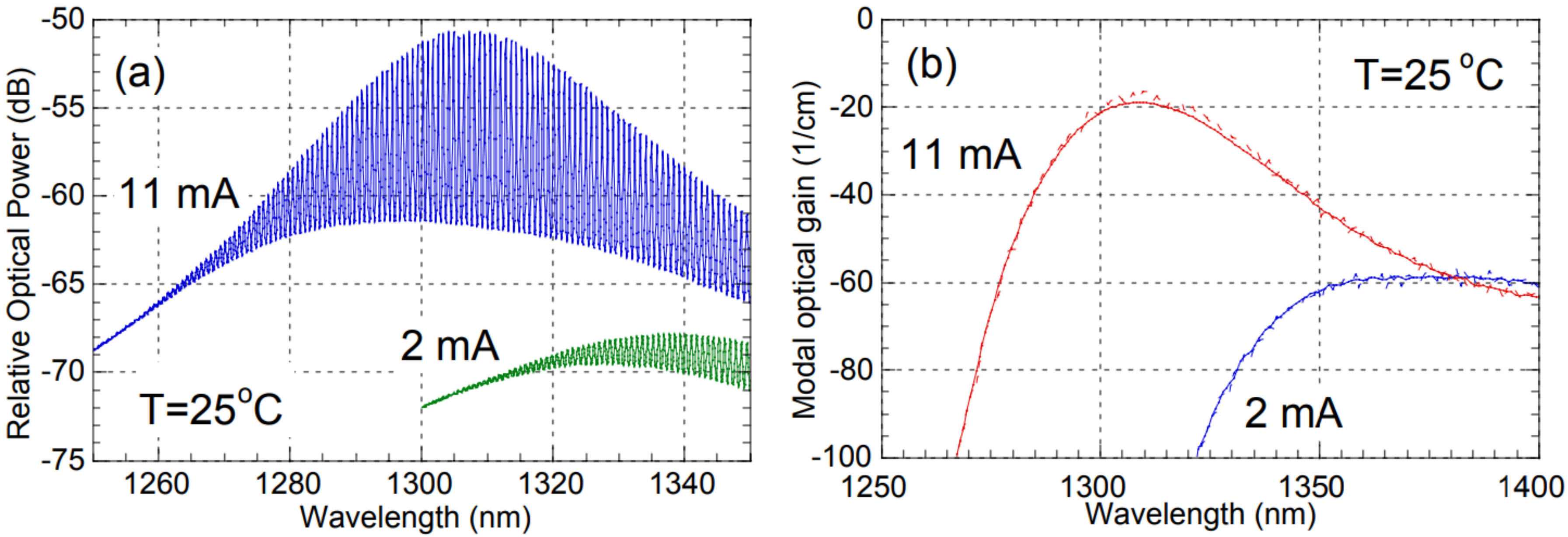
$I_{avg}$ is more accurate to measure than $I_{max}$ which depends on resolution of the instrument.

In alternative one can use instead $I_{max}/I_{avg}$ if data is noisy or $I_{max}$ is more accurate than $I_{min}$.




JAP **56**, 3096 (1984)
JQE **41**, 532 (2005)

# Example

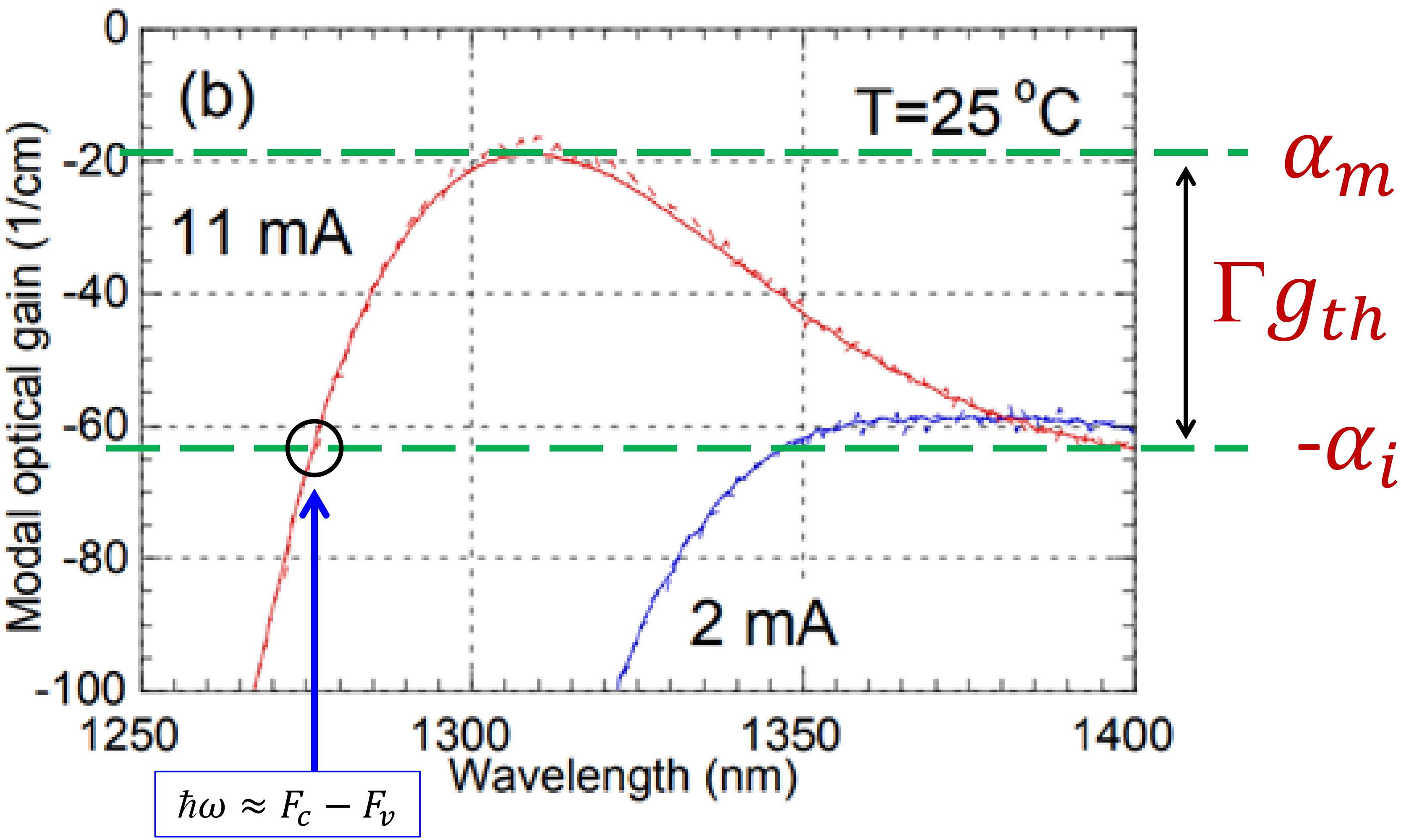



Left: ASE spectra for a 1.3 μm laser with a bulk active region and uncoated mirror facets

Right: Modal optical gain measurements with Hakki-Paoli method (continuous lines) and with Cassidy method (dotted lines)

Shtengel et al., International Journal of High Speed Electronics and Systems (1998)

# Characteristics of the G-λ Plot




Below band edge (longer wavelengths) gain is negligible and the plot $G_n(\lambda)$ approaches -$\alpha_i$. The peak occurs at the laser threshold gain $G_n(\lambda) = \Gamma\, g_{th} - \alpha_i = \alpha_m$.

# Lossy Media Waveguides

# Waveguide Realized with Lossy Media

**In a material with loss or gain, the permittivity is complex**

$$\varepsilon = \varepsilon' + i\varepsilon''$$

$$\begin{cases} \varepsilon'' > 0 & \text{loss} \\ \varepsilon'' < 0 & \text{gain} \end{cases}$$

**Propagation constant**

$$k = \omega\sqrt{\mu(\varepsilon' + i\varepsilon'')} = \frac{\omega}{c}(n + i\kappa) = k' + ik''$$

## Consider a waveguide with complex permittivity everywhere

| | | |
|---|---|---|
| **cladding** | $\varepsilon' + i\varepsilon''$ | $\varepsilon'' > 0$ loss |
| **core** | $\varepsilon_1' + i\varepsilon_1''$ | $\varepsilon_1'' < 0$ gain |
| **cladding** | $\varepsilon' + i\varepsilon''$ | $\varepsilon'' > 0$ loss |

## Gain Region (core)

expand $\sqrt{\ }$ assuming $\varepsilon_1'' << \varepsilon_1'$

$$k_1 = \omega\sqrt{\mu\left(\varepsilon_1' + i\varepsilon_1''\right)} = \omega\sqrt{\mu\varepsilon_1'}\sqrt{1 + i\frac{\varepsilon_1''}{\varepsilon_1'}} \simeq \omega\sqrt{\mu\varepsilon_1'}\left(1 + i\frac{\varepsilon_1''}{2\varepsilon_1'}\right) = k_1' - i\frac{g}{2}$$

$$k_1' = \omega\sqrt{\mu\varepsilon_1'} = k_0 n_1$$

gain coefficient

$$g = -k_1'\frac{\varepsilon_1''}{\varepsilon_1'} \qquad \left[\text{for } \varepsilon_1'' < 0\right]$$

## Loss Region (cladding)

$$k = \omega\sqrt{\mu\left(\varepsilon' + i\varepsilon''\right)} \simeq \omega\sqrt{\mu\varepsilon'}\left(1 + i\frac{\varepsilon''}{2\varepsilon'}\right) = k' + i\frac{\alpha}{2}$$

$$k' = \omega\sqrt{\mu\varepsilon_1'} = k_0 n$$

$$\alpha = k'\frac{\varepsilon''}{\varepsilon'} \qquad \left[\text{decay constant}\right]$$

# Solution to Wave Equation (TE Mode)

$$\mathbf{E} = \hat{y}E_y = \hat{y}\,\phi(x)\,e^{ik_z z}$$

$$\left[\frac{d^2}{dx^2} - k_z^2 + \omega^2 \mu \varepsilon(x)\right]\phi(x) = 0$$

## We can solve with the variational method to find $k_z$

$$k_z^2 = \frac{\int_{-\infty}^{\infty} \phi^*(x)\left[\frac{d^2}{dx^2} + \omega^2 \mu \varepsilon(x)\right]\phi(x)\,dx}{\int_{-\infty}^{\infty} \phi^*(x)\phi(x)\,dx}$$

$$\varepsilon(x) = \varepsilon^{(0)}(x) + \Delta\varepsilon(x) = \begin{cases} \varepsilon_1' + i\varepsilon_1'' & \text{core} \\ \varepsilon' + i\varepsilon'' & \text{cladding} \end{cases}$$

treat as perturbation

## Equation for the unperturbed case (lossless waveguide)

$$\left[\frac{d^2}{dx^2} - k_z^{(0)2} + \omega^2 \mu \varepsilon(x)\right]\phi^{(0)}(x) = 0$$

## Solving for $k_z$ using unperturbed wave envelopes

$$k_z^2 = k_z^{(0)2} + i\omega^2 \mu \varepsilon_1'' \Gamma + i\omega^2 \mu \varepsilon'' (1-\Gamma)$$

**with** $$\Gamma = \frac{\int_{inside} \left|\phi^{(0)}(x)\right|^2 dx}{\int_{-\infty}^{\infty} \left|\phi^{(0)}(x)\right|^2 dx}$$

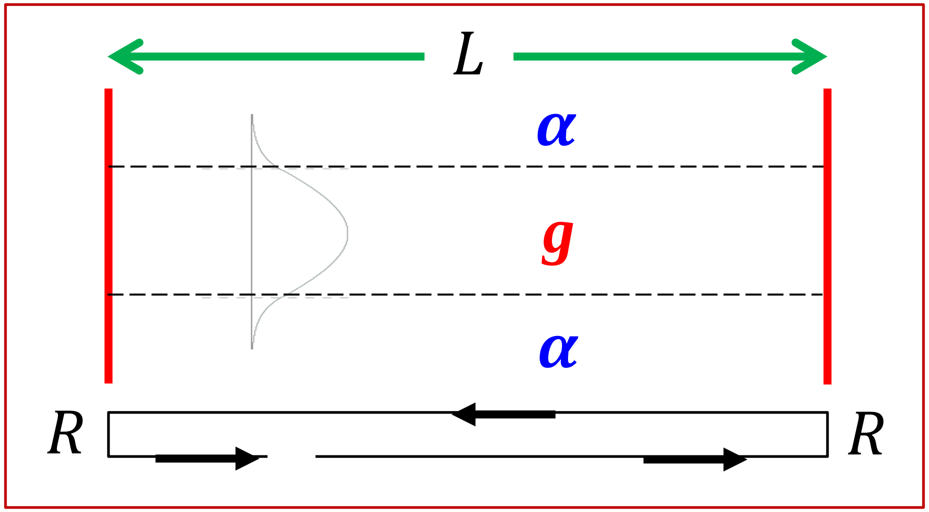
# Phase and Threshold Gain Conditions (case $\varepsilon_1' \sim \varepsilon'$ )

$$k_z = \sqrt{k_z^{(0)2} + i\omega^2\mu\varepsilon_1''\Gamma + i\omega^2\mu\varepsilon''(1-\Gamma)}$$

$$\simeq k_z^{(0)} - i\Gamma\frac{g}{2} + i(1-\Gamma)\frac{\alpha}{2}$$

**Consider a laser cavity with field reflectivity $r$ and length $L$**

$$r^2 e^{ik_z 2L} = 1$$




$$r^2 e^{i\left(k_z^{(0)} - i\Gamma\frac{g}{2} + i(1-\Gamma)\frac{\alpha}{2}\right)2L} = R\cdot e^{i2k_z^{(0)}L}e^{\Gamma gL-(1-\Gamma)\alpha L} = 1$$

$$r^2 e^{i\left(k_z^{(0)} - i\Gamma\frac{g}{2} + i(1-\Gamma)\frac{\alpha}{2}\right)2L} = R \cdot e^{i2k_z^{(0)}L} e^{\Gamma g L - (1-\Gamma)\alpha L} = 1$$

$$\Downarrow$$

**Phase condition**

$$2k_z^{(0)}L = 2m\pi$$

$$\boxed{k_z^{(0)} \simeq \frac{\omega}{c} n_e}$$

**Threshold Gain Condition**

$$R \cdot e^{\Gamma g L - (1-\Gamma)\alpha L} = 1 \quad \Rightarrow \quad \Gamma g = (1-\Gamma)\alpha + \frac{1}{L}\ln\frac{1}{R}$$

# Fabry-Pérot Mode Spacing

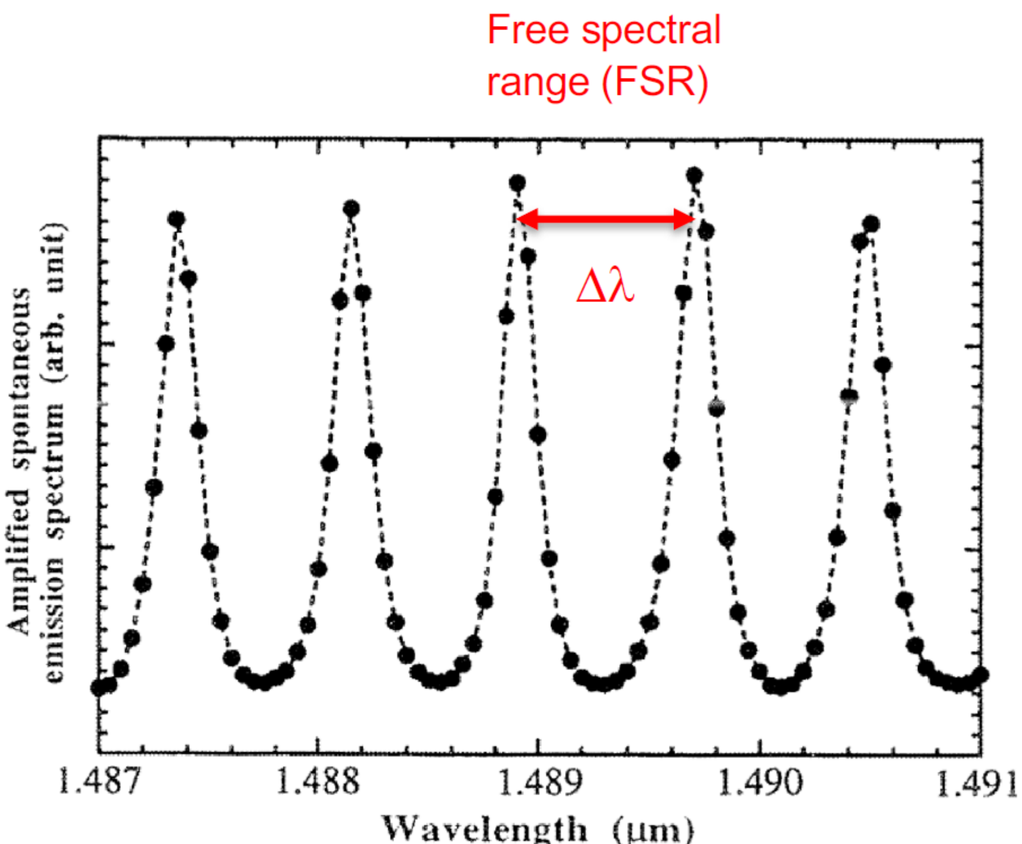
$$\left.\begin{aligned} 2k_z^{(0)}L &= 2m\pi \\ k_z^{(0)} &\simeq \frac{\omega}{c}n_e \end{aligned}\right\} \Rightarrow \frac{\omega}{c}n_e L = \frac{2\pi f_m}{c}n_e L = m\pi$$

**Frequency Spectrum**

$$f_m = m\frac{c}{2n_e L}$$

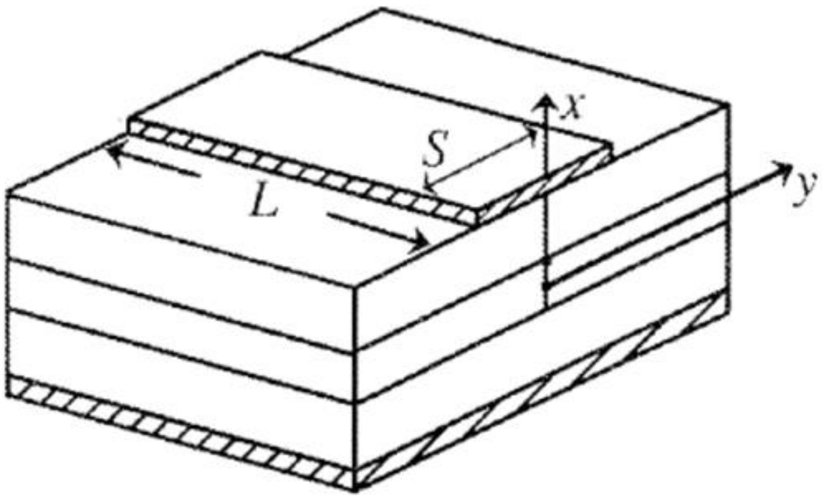
**Fabry-Pérot Frequency Spacing**

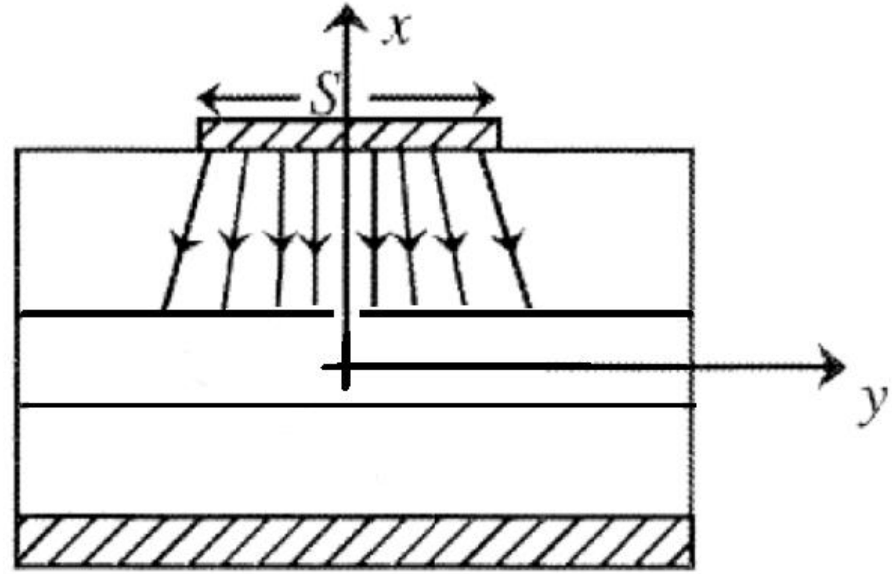
$$\Delta f = \frac{c}{2n_g L} \equiv FSR$$

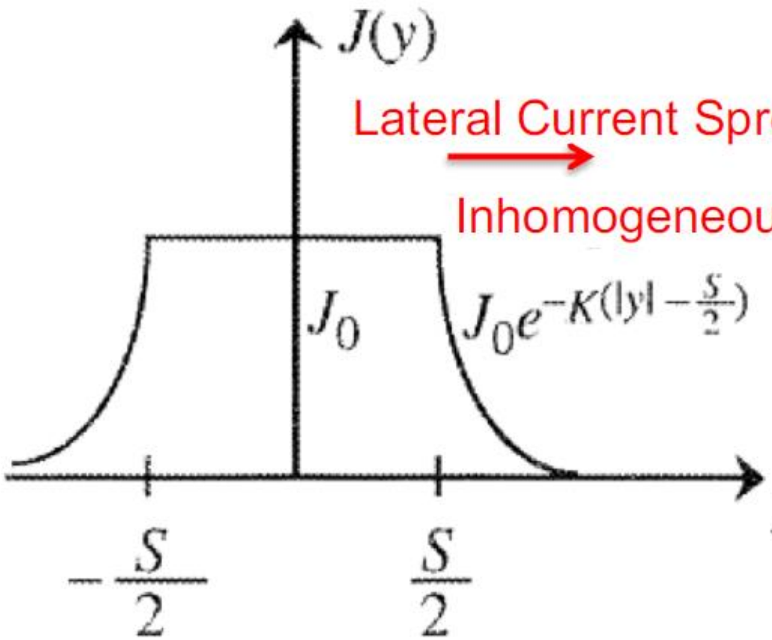


# Gain-Guided Lasers

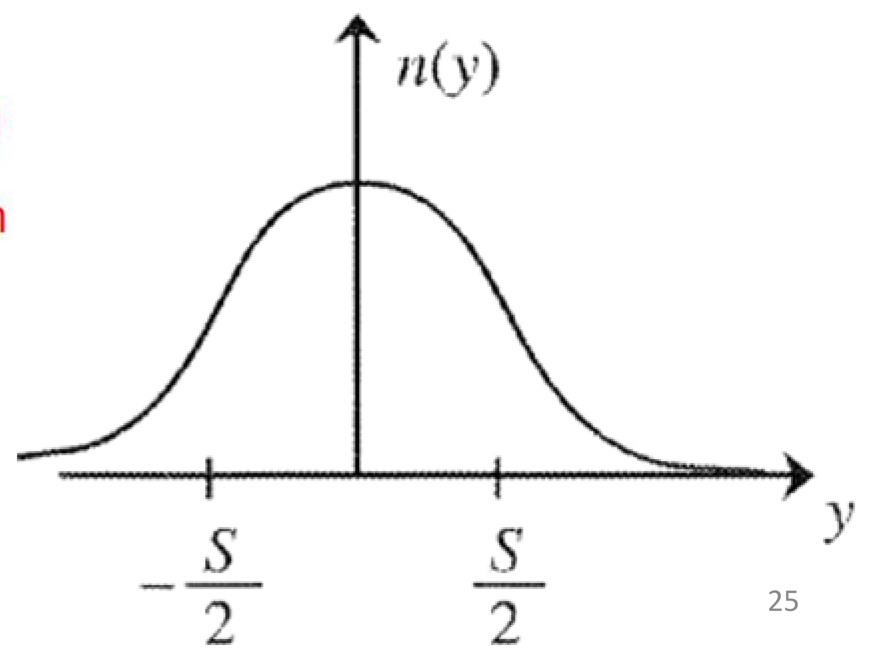
**Stripe-Geometry Laser**




Lateral Current Spreading → Inhomogeneous Gain



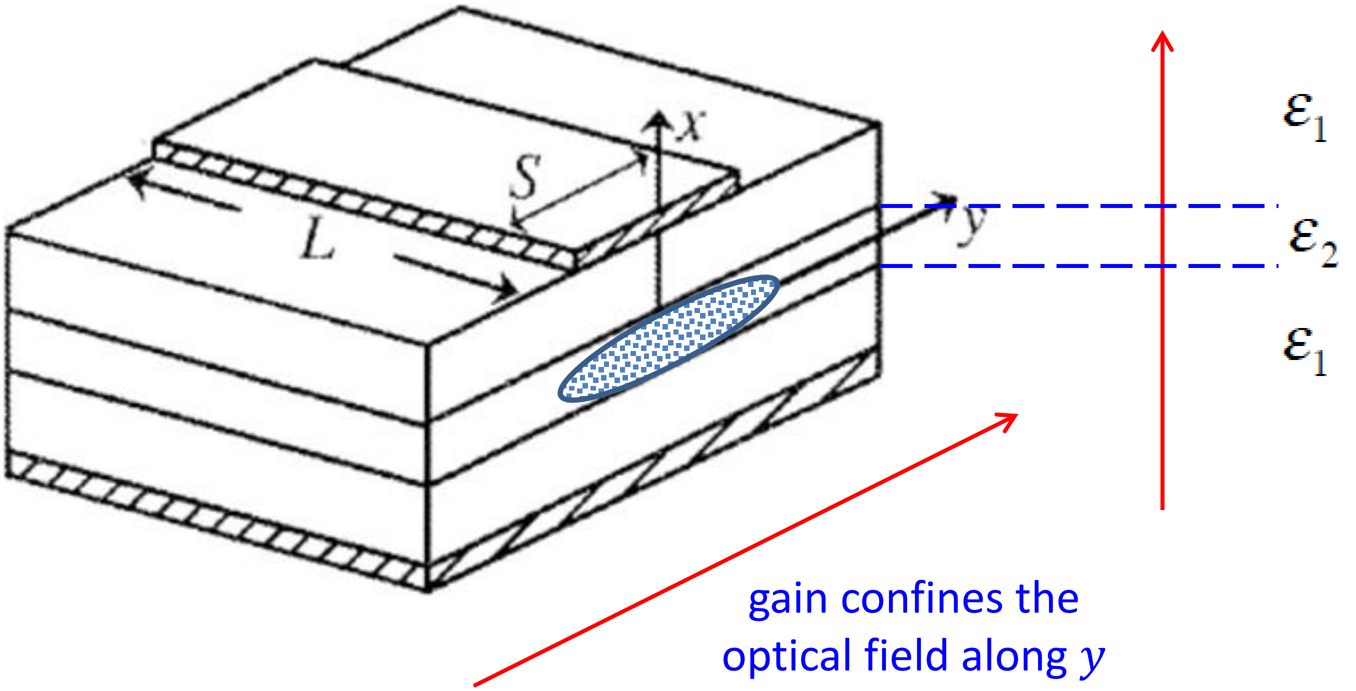
# Gain-Guided Lasers

**Gain guided lasers have no internal lateral structure. The sole lateral variation along $y$ is the charge density distribution caused by current flow from the conducting stripe and the lateral carrier diffusion within the active region.**

**At threshold, charge density is maximum beneath the stripe center, hence, most light is generated at the center where gain is greatest. For this reason the light is said to be gain guided.**

**In such a structure without lateral confinement, light tends to "leak" radially. Additionally, the charge within the active region acts to depress the real refractive index at the lasing frequency further enhancing the lateral radiation losses. There is competition between "guiding" and "antiguiding"**

# Gain-Guided Lasers



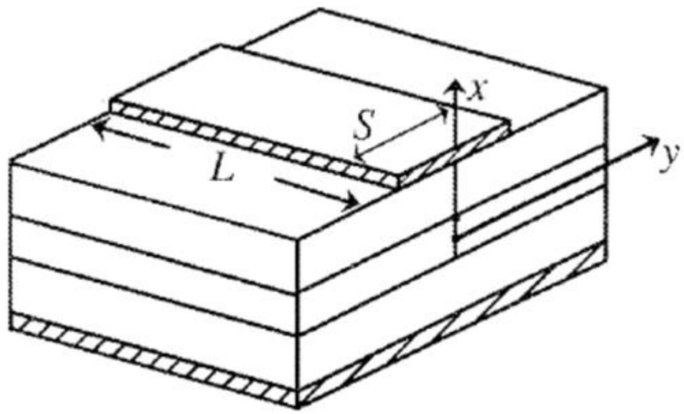

index of refraction confines the optical field along $x$

gain confines the optical field along $y$

$$\varepsilon(x,y)=\begin{cases}\varepsilon_1 & x>d/2\\ \varepsilon_2(0)-a^2y^2 & |x|\le d/2\\ \varepsilon_1 & x<-d/2\end{cases}$$

**with** $\boxed{a=a_r+ia_i}$ **complex**

# Consider a TE mode in the gain guided laser




**Form of the Electric Field**

$$\mathbf{E} \simeq \hat{y} E_y(x,y,z) \simeq \hat{y} F(x) G(y) e^{ik_z z}$$

space decomposition

**Wave Equation**

$$\nabla^2 E_y + \omega^2 \mu \varepsilon(x,y) E_y = 0$$

A solution can be found analytically from perturbation theory in terms of Hermite polynomials (see section 3.3 of the textbook)

$$E_y(x,y,z) = E_y^{(0)}(x) H_n\left( (k_0 a)^{1/2} \left( \frac{\Gamma}{\varepsilon_0} \right)^{1/4} y \right) e^{-k_0 a \left(\frac{\Gamma}{\varepsilon_0}\right)^{1/2} y^2/2} e^{ik_z z}$$

unperturbed Electric Field

**with** $a = a_r + i a_i$ **complex**

**and** $k_z^2 = \beta_z^2 - \left(\omega^2 \mu a^2 \Gamma\right)^{1/2} (2n+1)$

# Phase Wavefront

$$E_y(x,y,z) = E_y^{(0)}(x) H_n\left( (k_0 a)^{1/2} \left( \frac{\Gamma}{\varepsilon_0} \right)^{1/4} y \right) e^{-k_0 a \left( \frac{\Gamma}{\varepsilon_0} \right)^{1/2} y^2/2} e^{ik_z z}$$

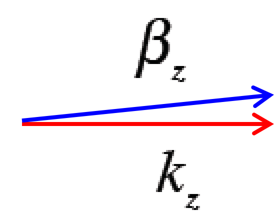
**Consider the case** $n = 0$

$$k_z^2 = \beta_z^2 - \left( \omega^2 \mu a^2 \Gamma \right)^{1/2}$$

$$e^{-k_0 a \left( \frac{\Gamma}{\varepsilon_0} \right)^{1/2} y^2/2} \quad e^{ik_z z}$$

$$a = a_r + i a_i$$

$$k_z \sim \beta_z$$




phase propagators

$$e^{-i a_i k_0 \left( \frac{\Gamma}{\varepsilon_0} \right)^{1/2} y^2/2}$$

$$e^{i \operatorname{Re}(k_z) z}$$

# Phase Wavefront

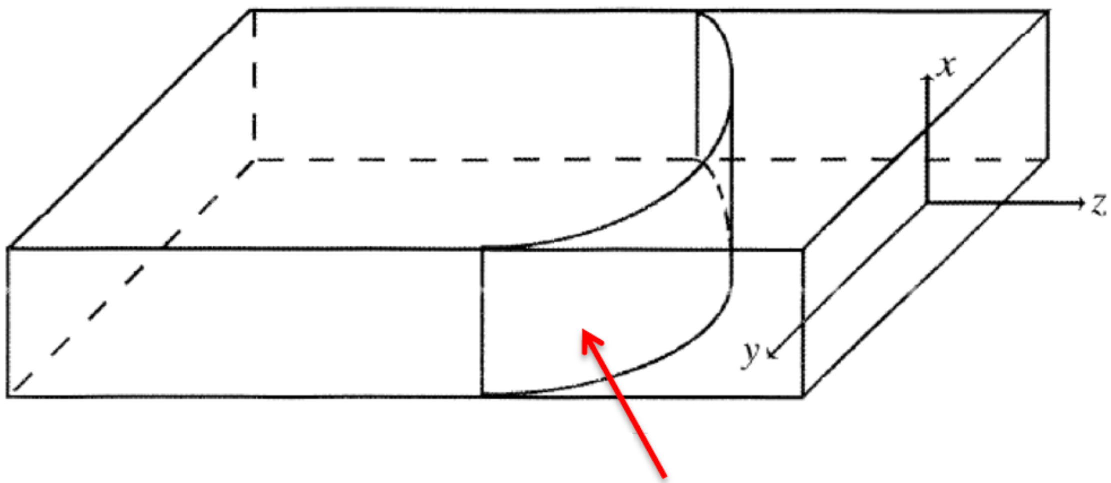
phase propagators

$$e^{-i a_i k_0 \left(\frac{\Gamma}{\varepsilon_0}\right)^{1/2} y^2/2}$$

$$e^{i\,\mathrm{Re}(k_z) z}$$

## Surface of constant phase (wavefront) occurs where

$$\mathrm{Re}(k_z) z - k_0 a_i \left(\frac{\Gamma}{\varepsilon_0}\right)^{1/2} \frac{y^2}{2} = \text{constant}$$




Constant Phase Surface

(cylindrical parabolic)

## Wave vector

$$k_z^2 - \beta_z^2 = -\left(\omega^2 \mu a^2 \Gamma\right)^{1/2}$$

$$\left(k_z + \beta_z\right)\left(k_z - \beta_z\right) = -\left(\omega^2 \mu a^2 \Gamma\right)^{1/2}$$

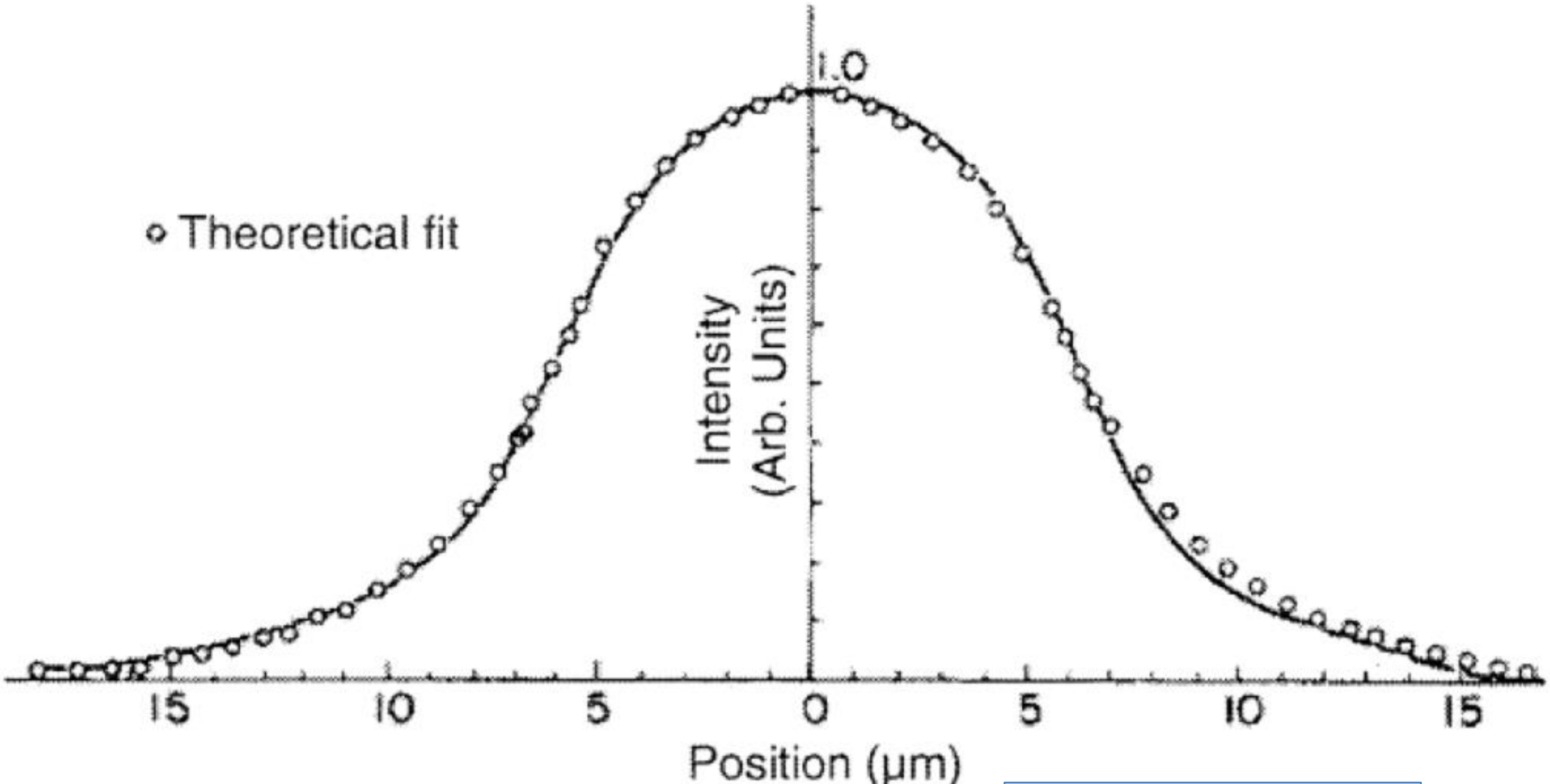
$$k_z \simeq \beta_z$$

$$2\beta_z\left(k_z - \beta_z\right) \simeq -\left(\omega^2 \mu a^2 \Gamma\right)^{1/2}$$

$$k_z \simeq \beta_z - \frac{1}{2\beta_z}\left(\omega^2 \mu a^2 \Gamma\right)^{1/2} = \beta_z - \frac{k_0 a}{2\beta_z}\left(\frac{\Gamma}{\varepsilon_0}\right)^{1/2}$$

# Carrier Density Profile

The carrier density may be mapped by measuring the spontaneous emission intensity profile

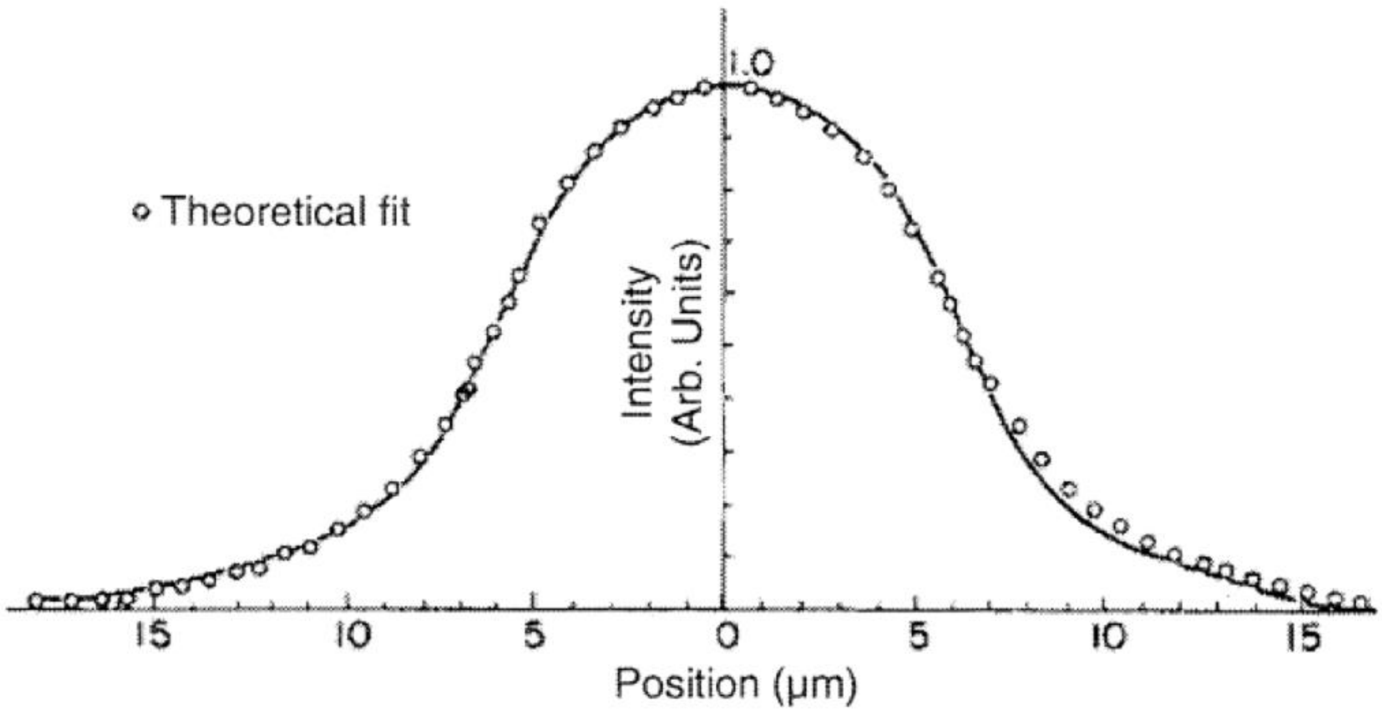



Nash, IEEE JQE, 1977

# Carrier Density Profile

If we neglect current spreading down to the depth of the active layer from the stripe contact, a purely diffusive model yields:

$$\delta n(y) = \begin{cases} Ae^{-y/L_n} + Be^{y/L_n} + G_0\tau_n & |y| \le S/2 \\ Ce^{-\left(|y|-\frac{S}{2}\right)/L_n} & |y| \ge S/2 \end{cases}$$



# Index of refraction

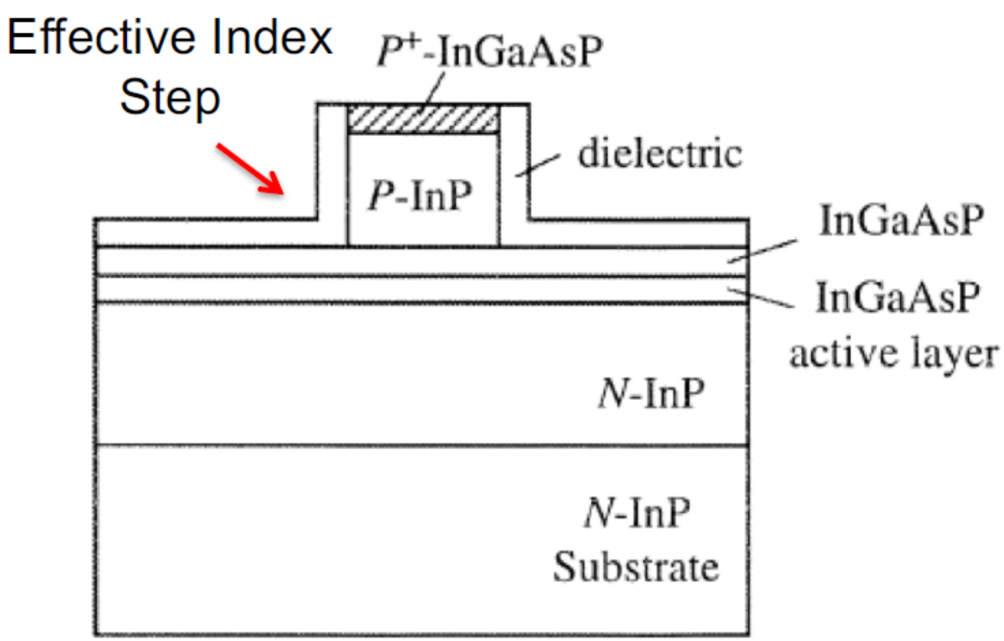
The model obtained for the index of refraction has the form

$$n_r(y) \simeq \left(\frac{\varepsilon_{2r}(0)}{\varepsilon_0}\right)^{1/2}\left[1 - \frac{a_r^2 - a_i^2}{2\varepsilon_{2r}(0)}y^2\right]$$

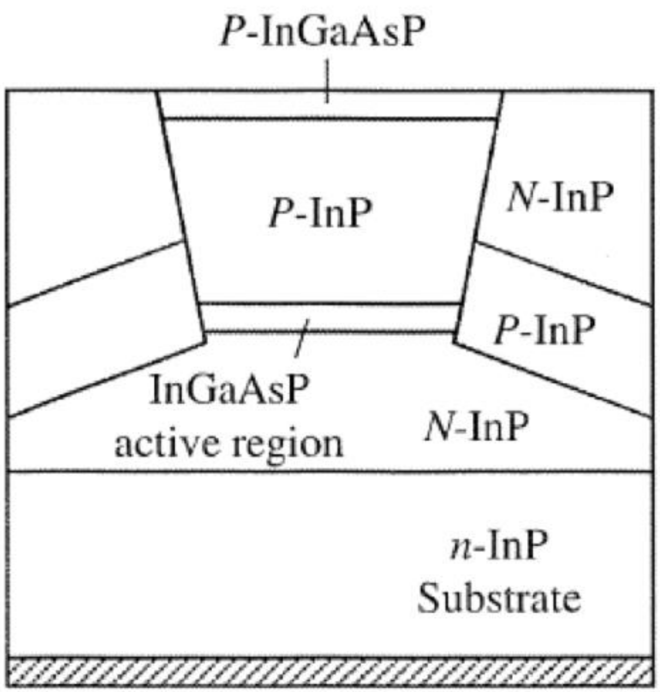
$$n_i(y) \simeq \left(\frac{\varepsilon_{2r}(0)}{\varepsilon_0}\right)^{1/2}\left[\frac{\varepsilon_{2i}(0) - 2a_r a_i y^2}{2\varepsilon_{2r}(0)}\right]$$

# Index-Guided Lasers

## Ridge-Waveguide Laser




## Buried Heterostructure



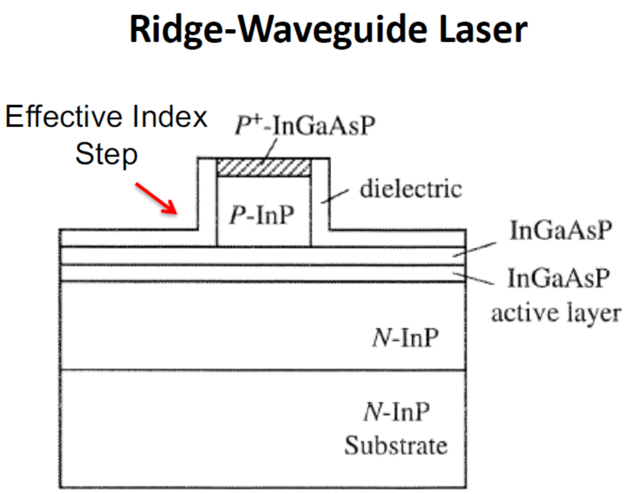

**ADVANTAGES:**

- **Lower threshold current density**
- **Higher differential quantum efficiency**
- **Better optical confinement and mode control**

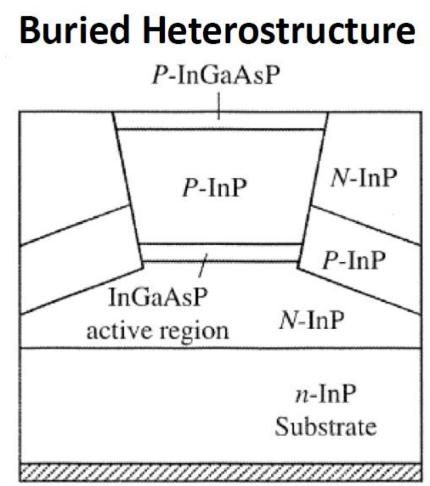
# Confinement factor

**The effective index can be determined by solving the slab waveguide problem under the ridge and outside the ridge.**

**Confinement along $y$ is provided by the effective index step.**




**Confinement is created by a change of material properties along $x$ and along $y$.**



# Confinement factor

$$\mathbf{E} \simeq \hat{y} E_y(x,y,z) \simeq \hat{y} F(x) G(y) e^{ik_z z}$$

Optical Confinement Factor:

$$\Gamma = \frac{\int\limits_{active\ region}\int \frac{1}{2}\mathrm{Re}\left(\mathbf{E}\times\mathbf{H}^*\right)\cdot\hat{z}\ dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2}\mathrm{Re}\left(\mathbf{E}\times\mathbf{H}^*\right)\cdot\hat{z}\ dxdy}$$

using $\frac{1}{2}\mathrm{Re}\left(\mathbf{E}\times\mathbf{H}^*\right)\cdot\hat{z} \simeq \frac{1}{2}\mathrm{Re}\left(-E_y H_x^*\right) = \frac{k_z \left|E_y\right|^2}{2\omega\mu}$

$$\Gamma = \frac{\int\limits_{active\ region}\int \left|E_y(x,y)\right|^2 dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|E_y(x,y)\right|^2 dxdy} \simeq \Gamma_x \Gamma_y$$