ECE 536 – Integrated Optics and Optoelectronics
Lecture 16 – March 10, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

Prof. Umberto Ravaioli

ECE Department, University of Illinois

Lecture 16 Outline

- Effective Index Method
- Quantum Well Lasers

Effective Index Method

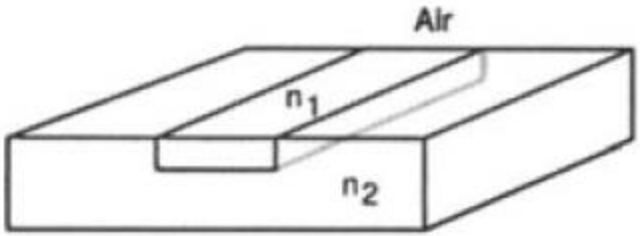
Preamble: Rectangular dielectric waveguides

The rectangular dielectric waveguide is most commonly used in integrated optics, especially in semiconductor diode lasers and in optical components used to process optical signals.

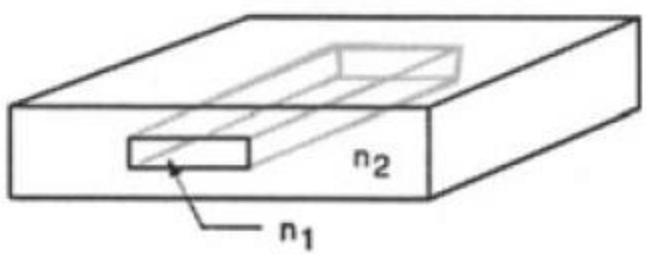
Unlike the planar slab waveguide or the circular fiber, it is in general impossible to find analytical solutions. (Note: in the following we will focus mainly on the fundamental mode).

Examples:

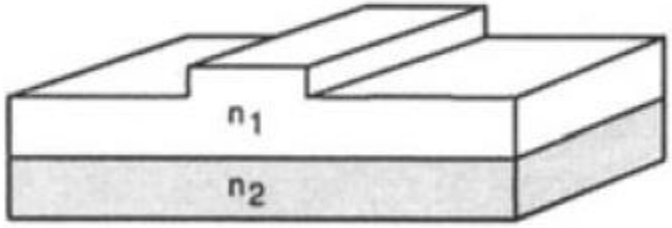
Surface waveguide



Buried waveguide



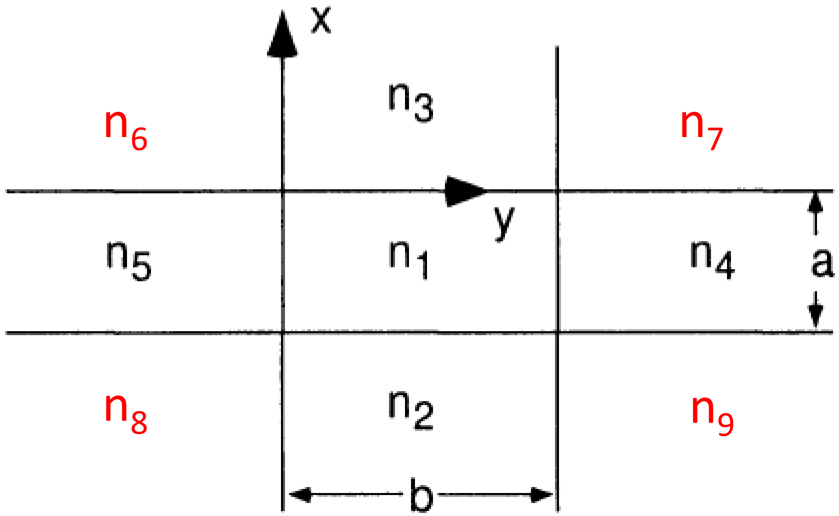
Ridge waveguide



Rectangular dielectric waveguides

The classical method used in metal waveguides decomposes the solution for modes on the cross-section into two orthogonal ones. Because of the simple nature of the (ideal) metal boundaries, this method is exact.

This is not as easy to apply in the case of dielectric waveguides, since nine separate regions are needed.

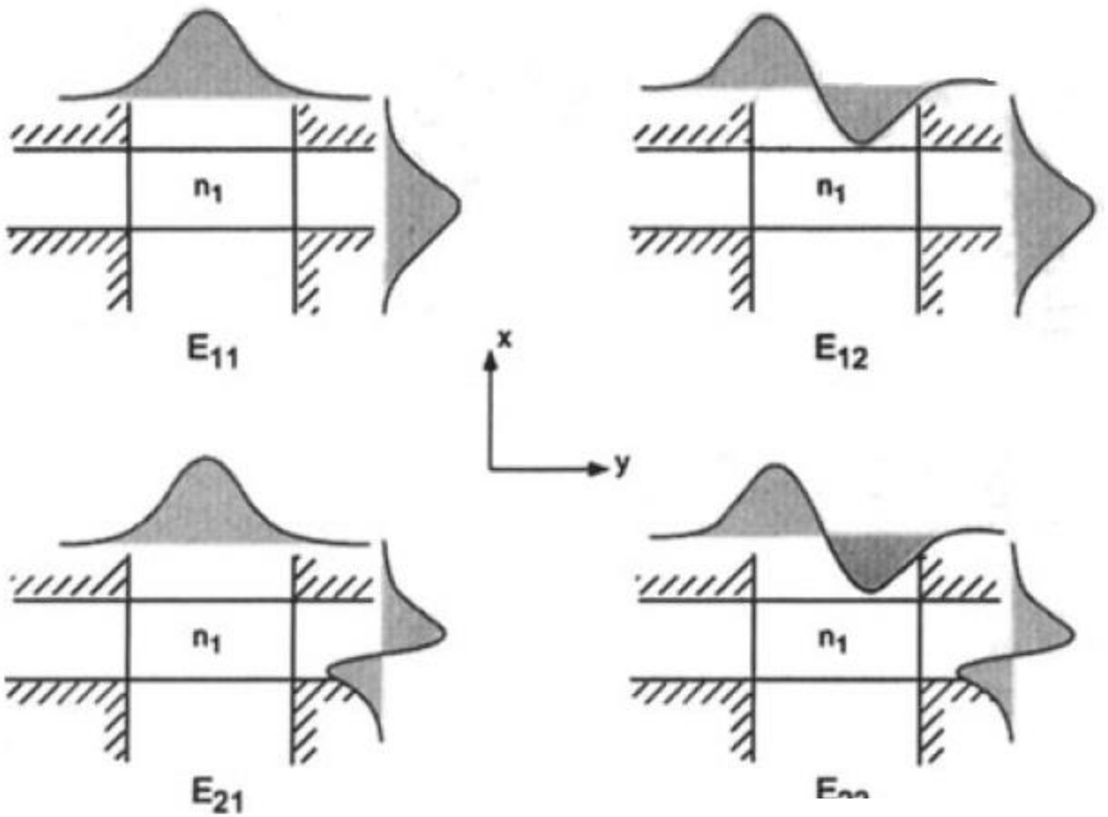


solutions have to be matched at all interfaces

$\exp(-\gamma_3 x)$ $\exp(\gamma_5 y)$	$\cos(\kappa_y y + \Phi_y)$ $\exp(-\gamma_3 x)$ 3	$\exp(-\gamma_3 x)$ $\exp(-\gamma_4 (y-b))$
$\cos(\kappa_x x + \Phi_x)$ $\exp(\gamma_5 y)$ 5	$\cos(\kappa_x x + \Phi_x)$ $\cos(\kappa_y y + \Phi_y)$ 1	$\cos(\kappa_x x + \Phi_x)$ $\exp(-\gamma_4 (y-b))$ 4
$\exp(\gamma_2 (x-a))$ $\exp(\gamma_5 y)$	$\cos(\kappa_y y + \Phi_y)$ $\exp(\gamma_2 (x-a))$ 2	$\exp(-\gamma_4 (y-b))$ $\exp(\gamma_2 (x-a))$

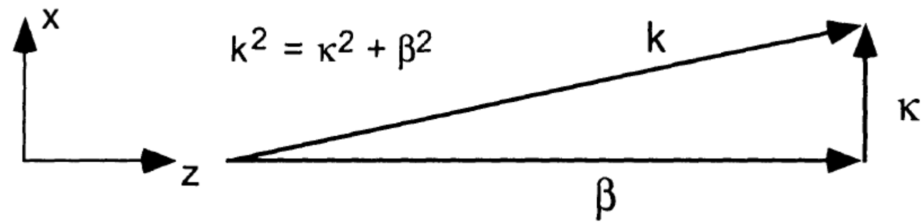
Rectangular dielectric waveguides

Marcatili (1969) solves the problem with two crossing slabs ignoring corner regions



In the core, solutions have the form

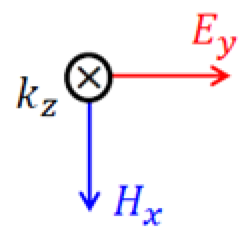
$$
\begin{aligned}
X(x) &= A\cos(\kappa_x x + \phi_x) \\
Y(y) &= B\cos(\kappa_y y + \phi_y)
\end{aligned}
$$

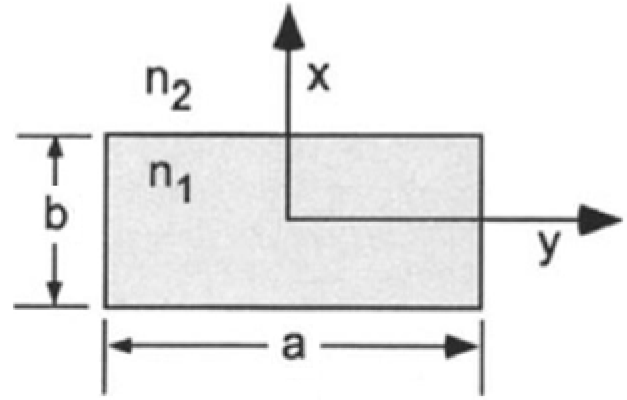


$$\beta^2 = k_0^2 n_1^2 - \kappa_x^2 - \kappa_y^2$$

Rectangular dielectric waveguides

Simplified case





TE characteristic equation

$$\tan\left(\kappa_x b/2\right) = \frac{\gamma_x}{\kappa_x} = \frac{\sqrt{k_0^2(n_1^2 - n_2^2) - \kappa_x^2}}{\kappa_x}$$

TM characteristic equation

$$\tan\left(\kappa_y a/2\right) = \frac{n_1^2}{n_2^2}\frac{\gamma_y}{\kappa_y} = \frac{n_1^2}{n_2^2}\frac{\sqrt{k_0^2(n_1^2 - n_2^2) - \kappa_y^2}}{\kappa_y}$$

$$\beta^2 = k_0^2 n_1^2 - \kappa_x^2 - \kappa_y^2$$

Rectangular dielectric waveguides

Even for the simplest symmetry, this decomposition process involves lengthy and tedious derivations of characteristic equations in order to determine, in the different regions, the propagating and evanescent components of the wave vectors.

The decomposition leads to solutions which are reasonable in the high frequency limit but increasingly inaccurate when approaching the cut-off.

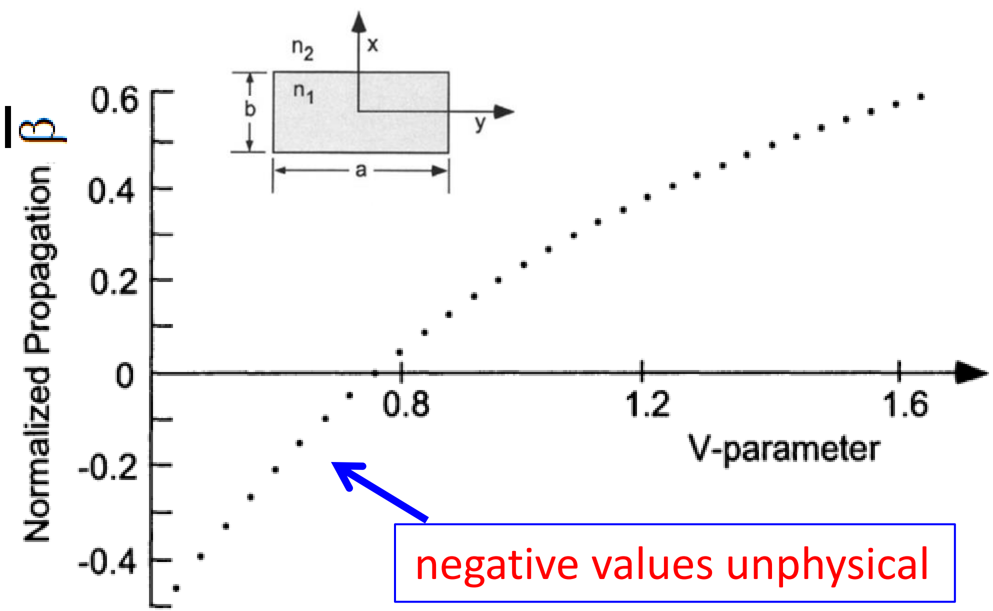
normalized frequency

$$V = k_0 \frac{b}{2} \sqrt{n_1^2 - n_2^2}$$

b = smaller core dimension
n_1 = core index
n_2 = next smaller index

normalized propagation vector

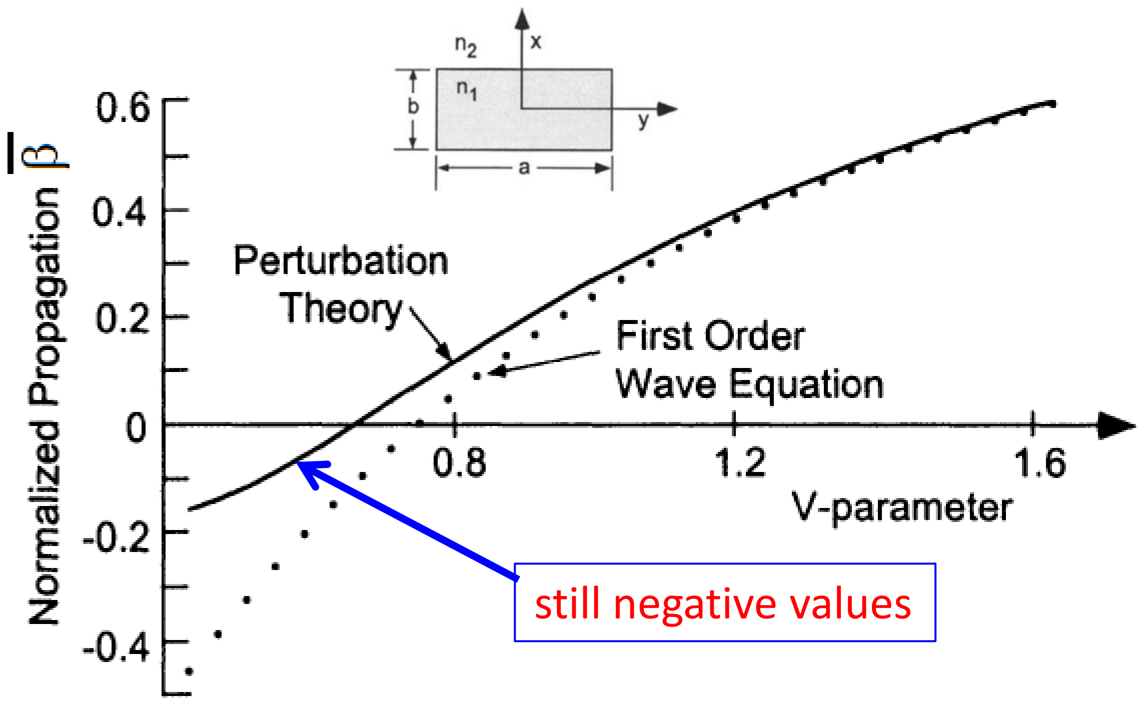
$$\overline{\beta} = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2 n_1^2 - k_0^2 n_2^2}$$



Rectangular dielectric waveguides

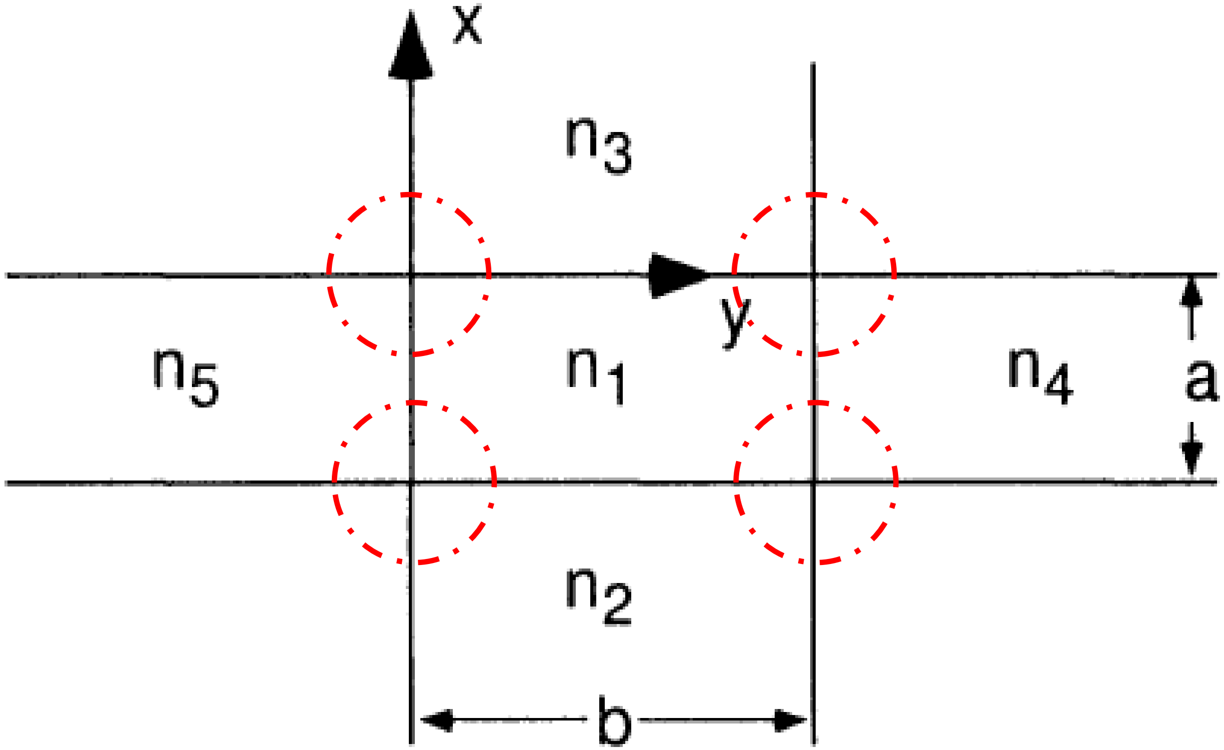
The decomposition method tends to be sufficiently accurate for values of the normalized frequency $V > 2$.

A perturbative correction step could be applied to improve the solution, leading to results that are acceptable for $V > 0.7$



Rectangular dielectric waveguides

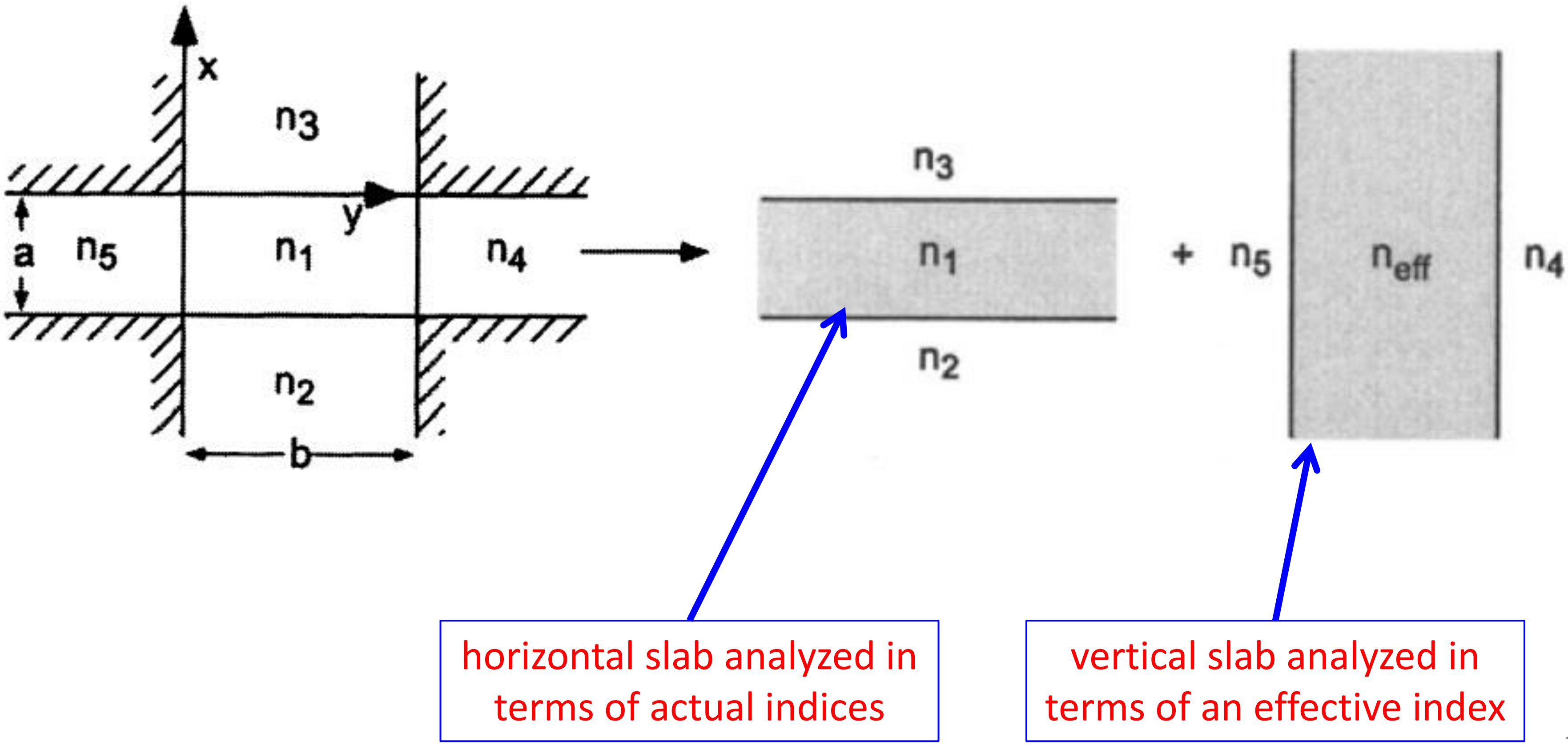
The main problem with the decomposition approach is in the fact that **decoupling** between the x and y orthogonal solutions does not hold well at the **corner regions**, particularly when evanescence is strong at lower frequencies.



Effective index method

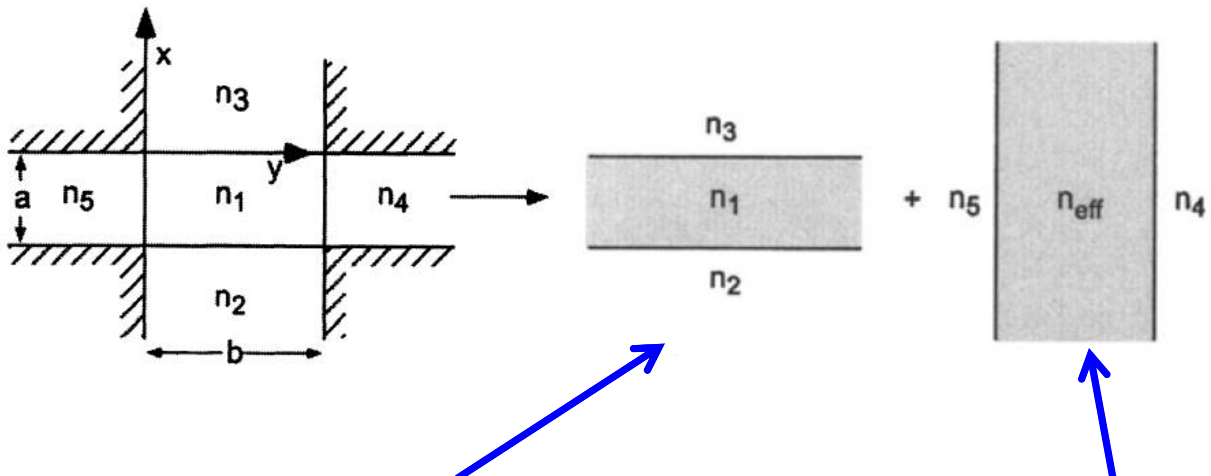
This method attempts to improve accuracy within a simple decomposition framework by including some coupling. Typically the evaluation pertains to the fundamental mode.

Example:



Effective index method

As before, the effective index method transforms a single 2D problem into two 1D problems. It is similar to the previous decomposition method but interaction between the horizontal and vertical waveguides is included.



The 1D horizontal waveguide is analyzed in terms of TE or TM modes to find the allowed propagation vector $\boldsymbol{\beta}'$ for the mode of interest at a given wavelength. When the index difference between core and cladding is very small ($n_1/n_2 \approx 1$) many authors have used the TE dispersion relation as an approximation for the TM polarization one.

Once $\boldsymbol{\beta}'$ is determined, the effective index of the vertical slab is found as

$$n_{eff} = \frac{\beta'}{k_0}$$

(k_0= wave vector in vacuum)

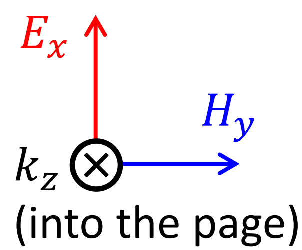
Effective index method

After the effective index is determined, we solve for the modes in the slab waveguide stretched along the x-direction using the effective index n_{eff} instead of the original value n_1.

Best accuracy for aspect ratio $\text{width}/\text{height} \geq 3$. This means the method is not very good for square waveguides. We also need to be careful to use the proper characteristic equation (TE or TM) for each 1D slab waveguide.

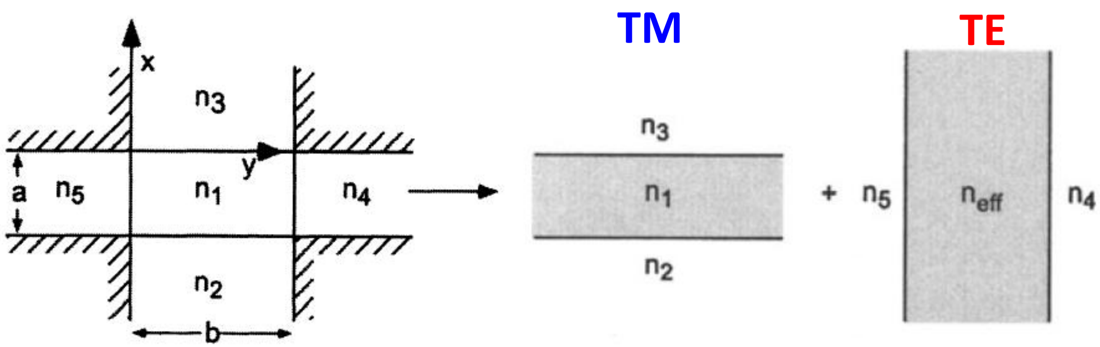
Example:

Electric field polarized along x



Looks like TM

Looks like TE



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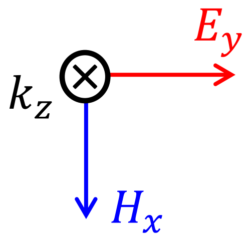
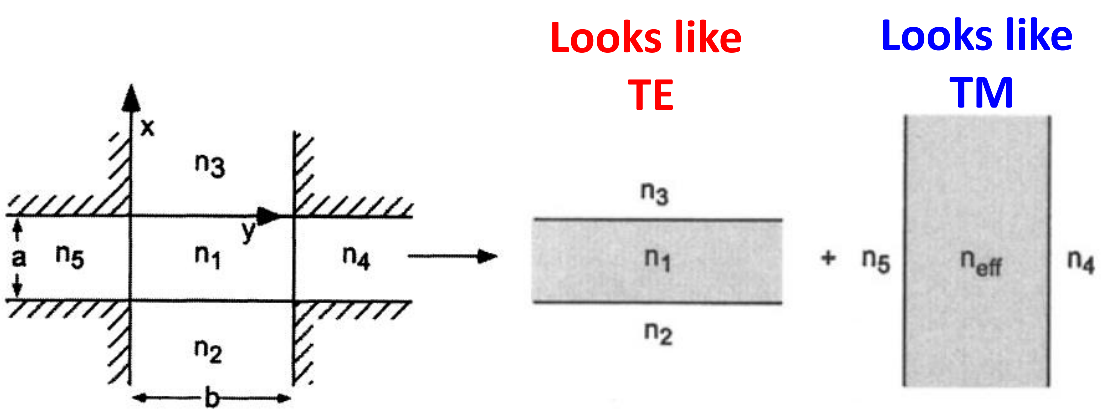
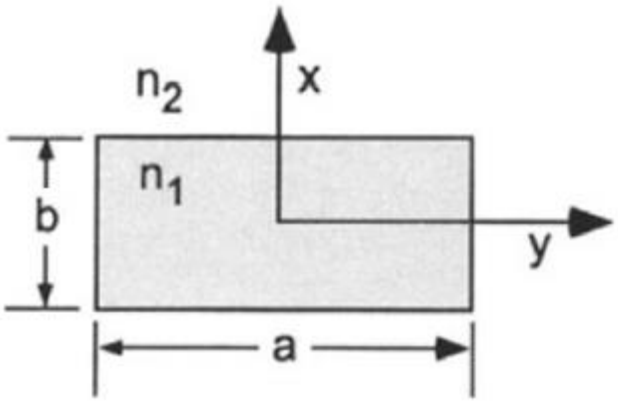


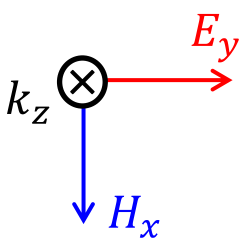


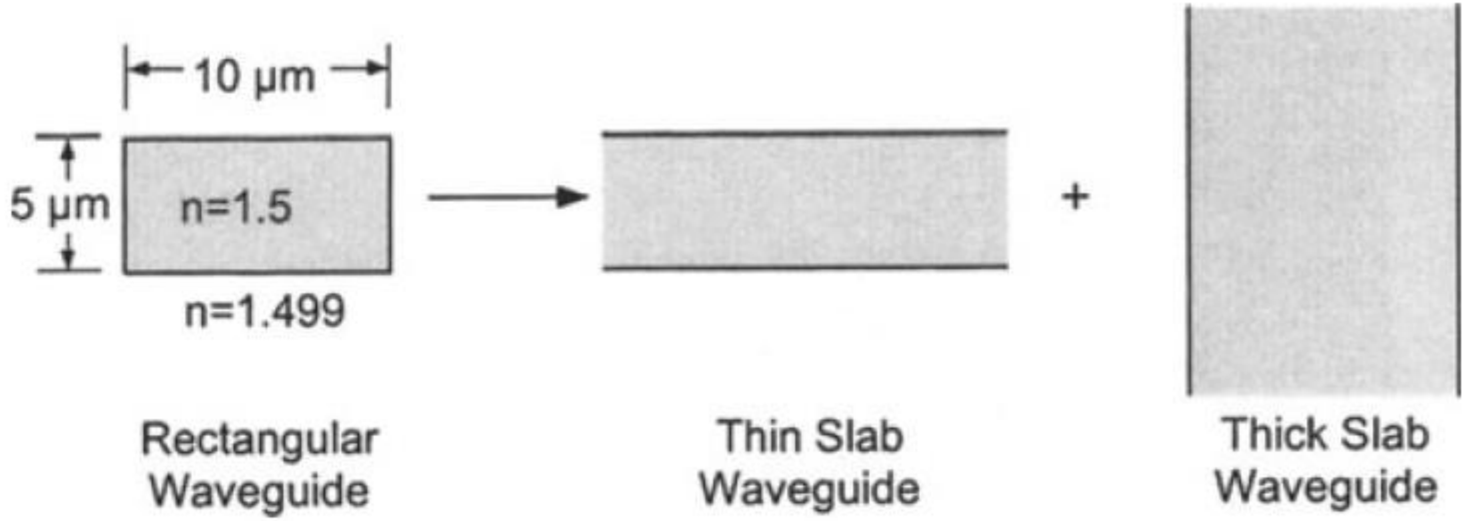
Example of effective index method application

Rectangular buried optical waveguide

Typical range of interest for optical applications $\lambda_0 \in [0.5 \rightarrow 2\mu m]$







Examples of effective index method application

First step: Analyze the structure as if it were a 5μm thick slab waveguide. Since the electric field is oriented along y, the mode can be analyzed as TE for the thin dimension.

The transverse wavevector κ_x is found for the wavelength range of interest using a form of the characteristic equation

$$\tan \kappa_x b/2 = \frac{\sqrt{k_0^2(n_1^2 - n_2^2) - \kappa_x^2}}{\kappa_x}$$

An effective index is assigned to each wavevector value

$$n_{eff} = \sqrt{k_0^2 n_1^2 - \kappa_x^2}/k_0$$

Examples of effective index method application

Second step: Using the values of $\boldsymbol{n_{eff}}$, the transverse wavevectors $\boldsymbol{\kappa_y}$ are found for the second slab waveguide of thickness $\boldsymbol{a}$ using a form of the characteristic equation for TM modes:

transform this

$$\tan \kappa_y a/2 = \frac{n_1^2}{n_2^2} \frac{\sqrt{k_0^2(n_1^2 - n_2^2) - \kappa_y^2}}{\kappa_y}$$

into this

$$\tan \kappa_y a/2 = \frac{n_{eff}^2}{n_2^2} \frac{\sqrt{k_0^2 (n_{eff}^2 - n_2^2) - \kappa_y^2}}{\kappa_y}$$

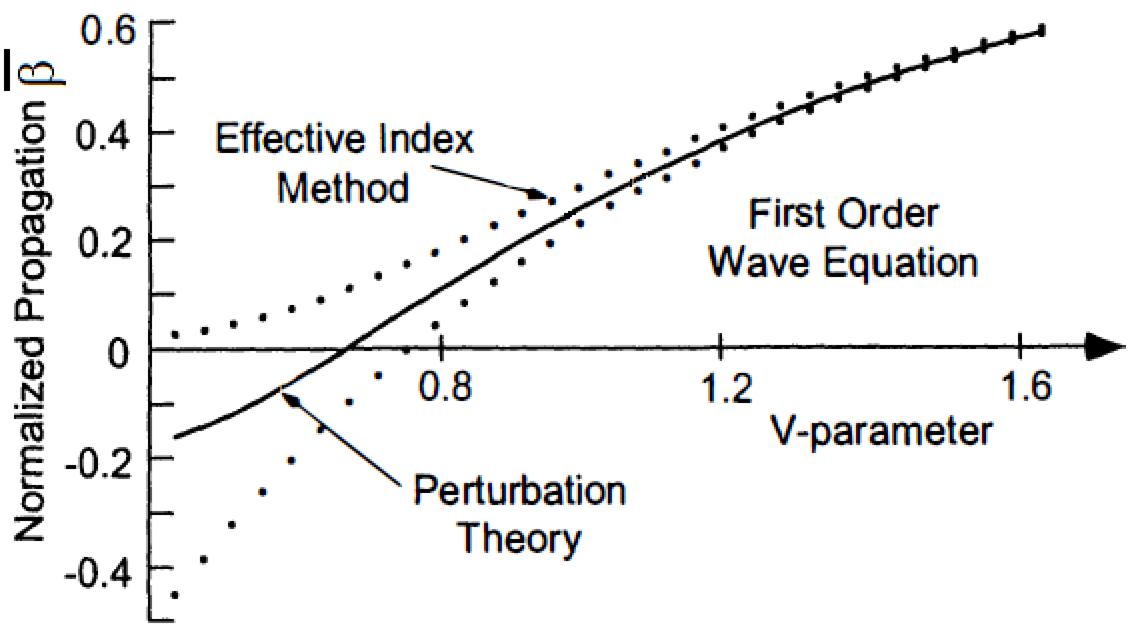
The propagation coefficient along the axis of the waveguide z for this structure is the approximate eigenvalue of the propagation problem for the full structure and it is found from

$$\beta = \sqrt{k_0^2 n_{eff}^2 - \kappa_y^2}$$

Examples of effective index method application

Comparison: Results can be compared to the other approaches illustrated earlier by normalizing the values of β using the actual values of the indices in the waveguide

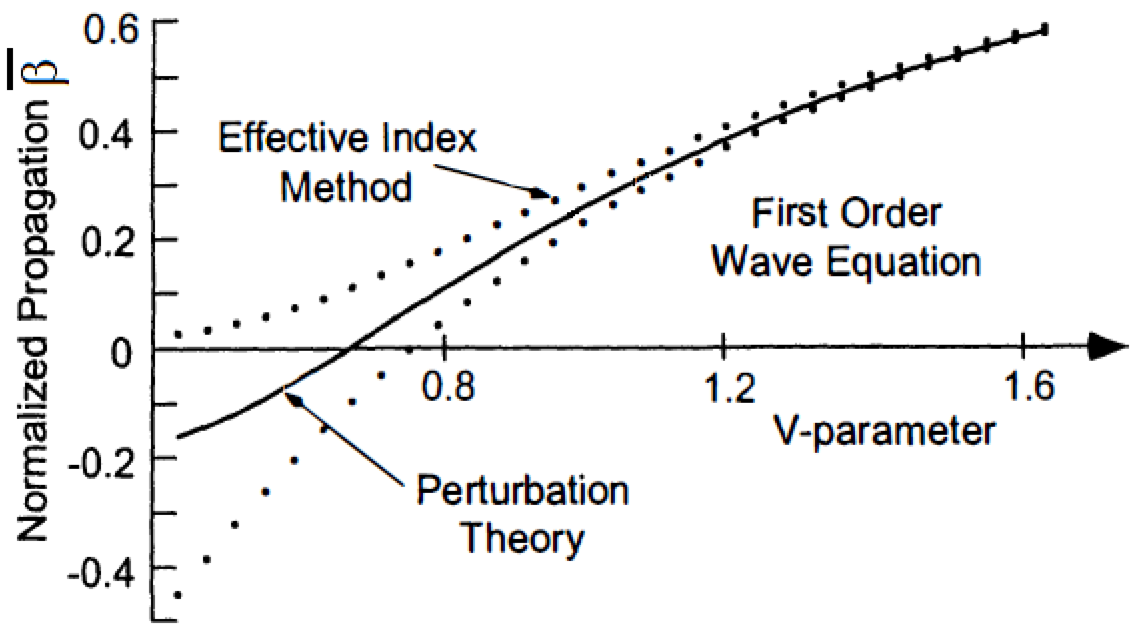
$$\overline{\beta} = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2 n_1^2 - k_0^2 n_2^2}$$



Examples of effective index method application

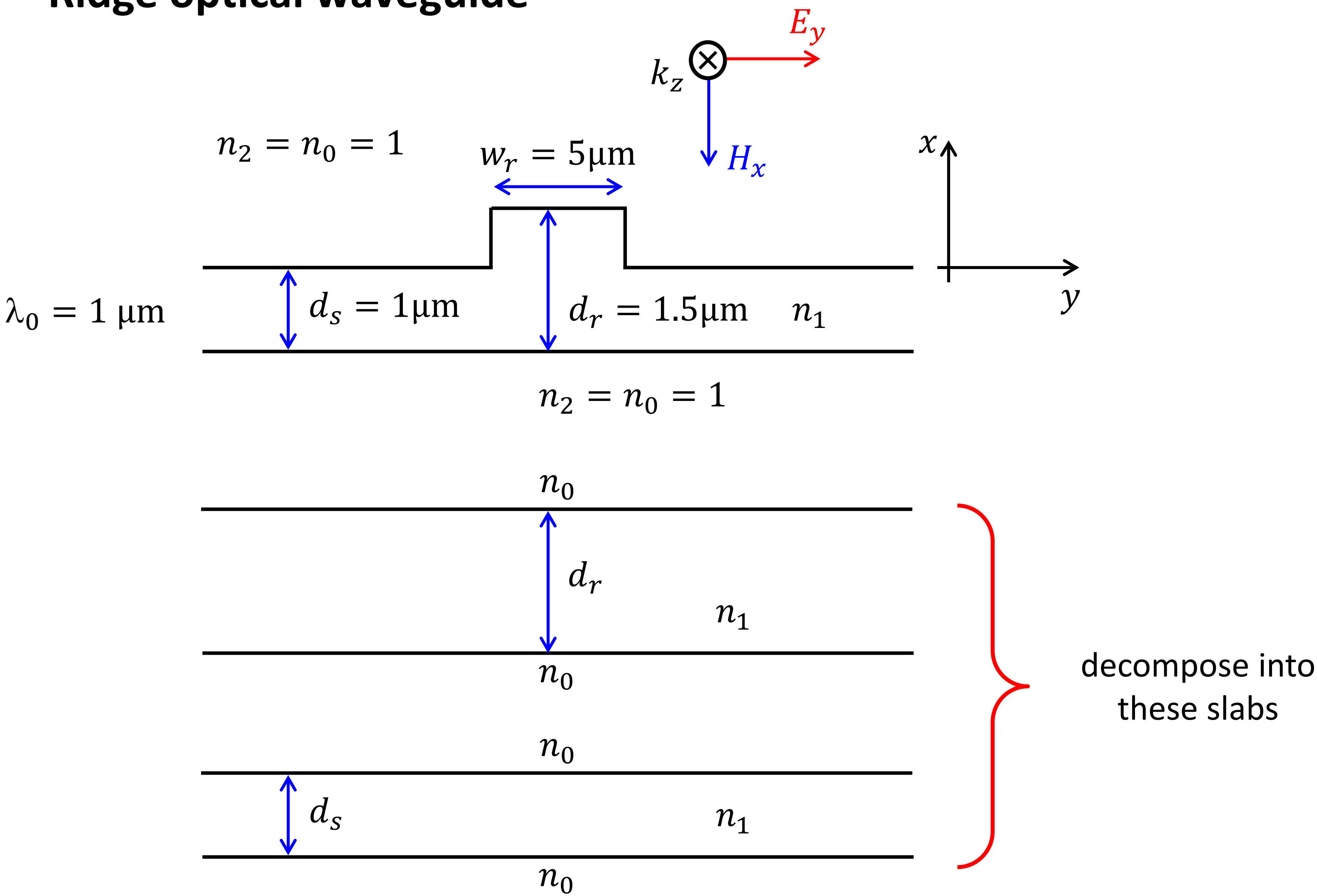
Comparison: For this case the effective index method is the only one which predicts the existence of a mode at low values of V. Its application, however, is stretching the limits since the aspect ratio of the waveguide is only 2 (at least 3 is needed for good accuracy) and β is slightly overestimated.

Around $V \geq 1$ the perturbation theory correction would give the best results.



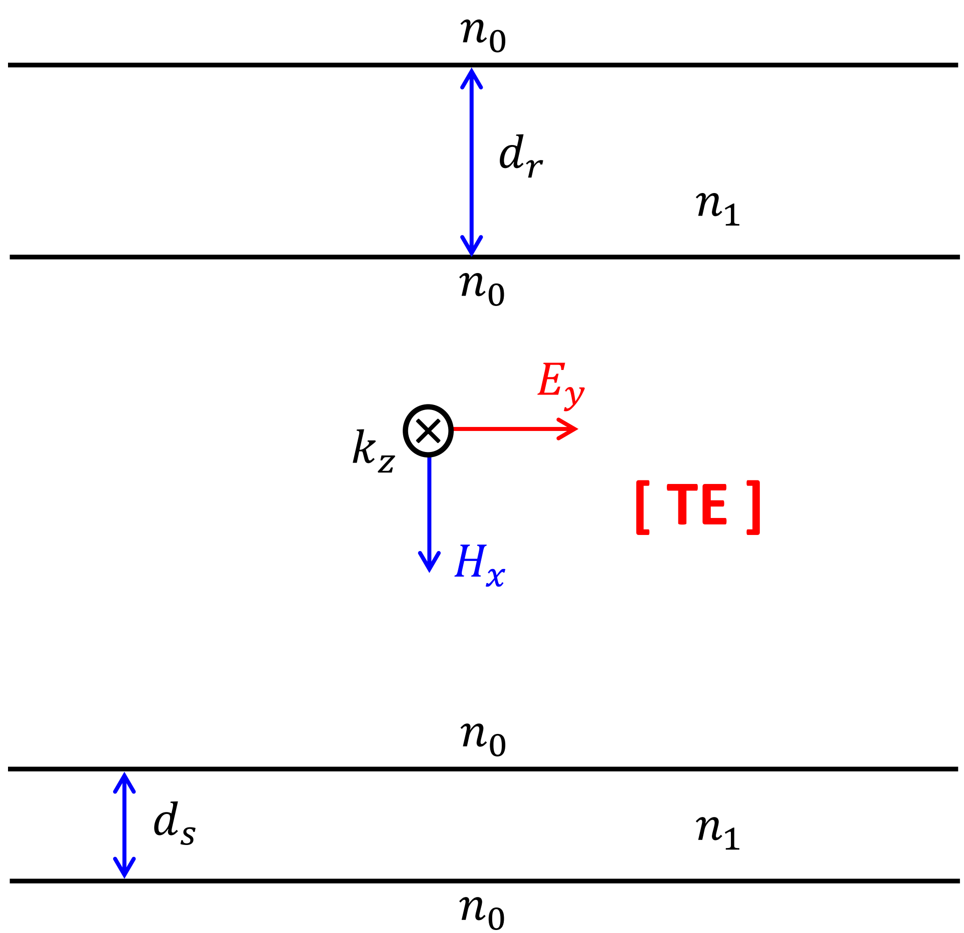
Example of effective index method application

Ridge optical waveguide



Example of effective index method application

Ridge optical waveguide



If w_r is large enough one may ignore that the ridge has finite extent and consider it as this slab, which has modes with propagation constant β_r associated with effective refractive index $n_{r,eff} = \beta_r / k_0$

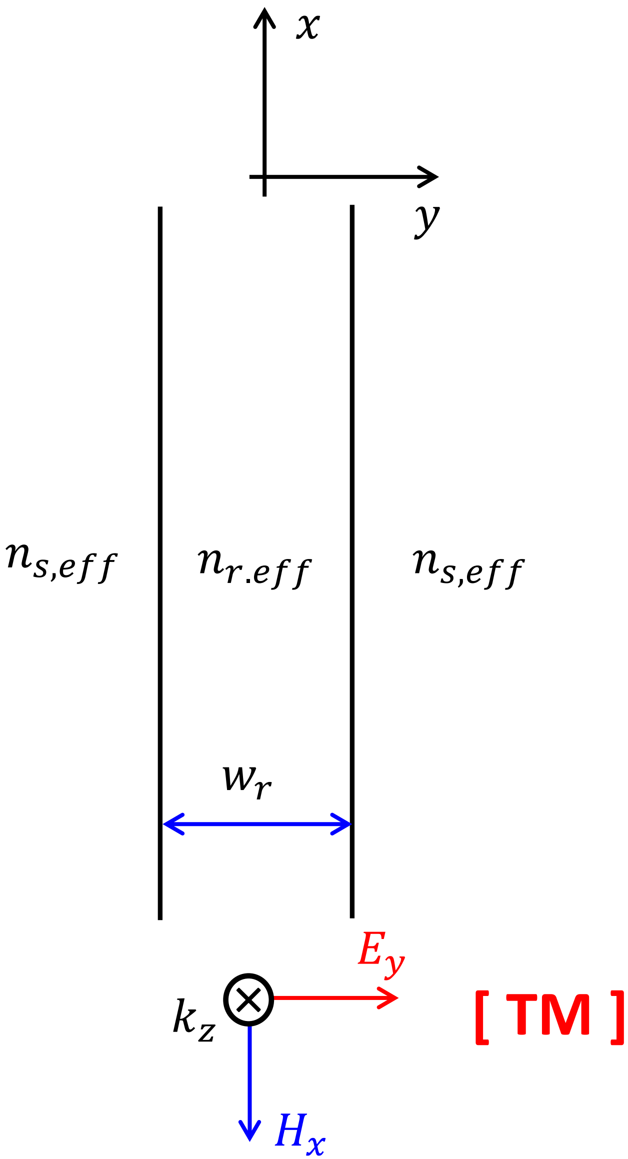
$$\tan\left(\kappa_x \frac{d_r}{2}\right) = \frac{\sqrt{k_0^2(n_1^2 - n_0^2) - \kappa_x^2}}{\kappa_x}$$

$$\beta_r = \sqrt{k_0^2 n_1^2 - \kappa_x^2}$$

The smaller slab is calculated in the same way ignoring the ridge, and it has propagation constant β_s associated with effective refractive index $n_{s,eff} = \beta_s / k_0 < n_{r,eff}$

Example of effective index method application

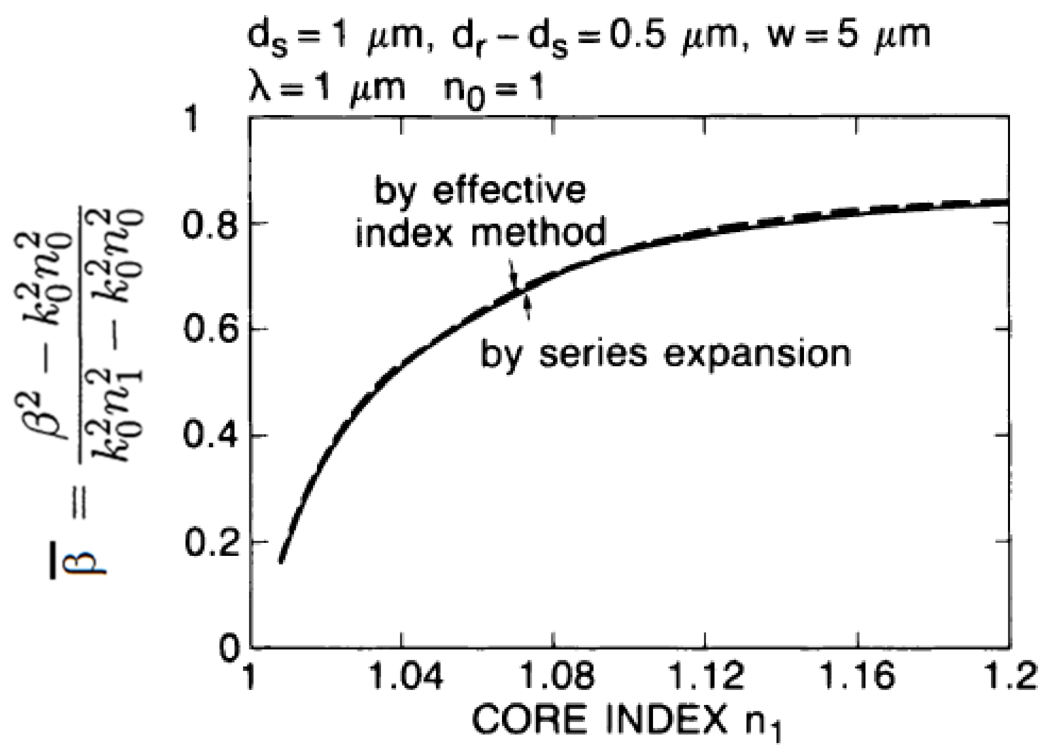
Ridge optical waveguide



Finally, we obtain the equivalent slab used for computing the ridge waveguide.

From this slab solution we get the transverse propagation constant κ_y which is substituted together with the value κ_x found earlier into

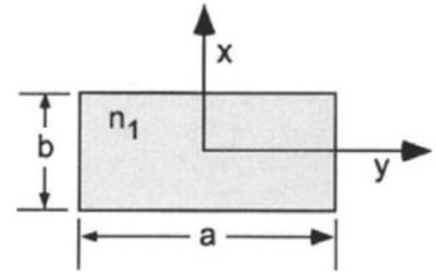
$$\beta = \sqrt{k_0^2\, n_1^2 - \kappa_x^2 - \kappa_y^2}$$

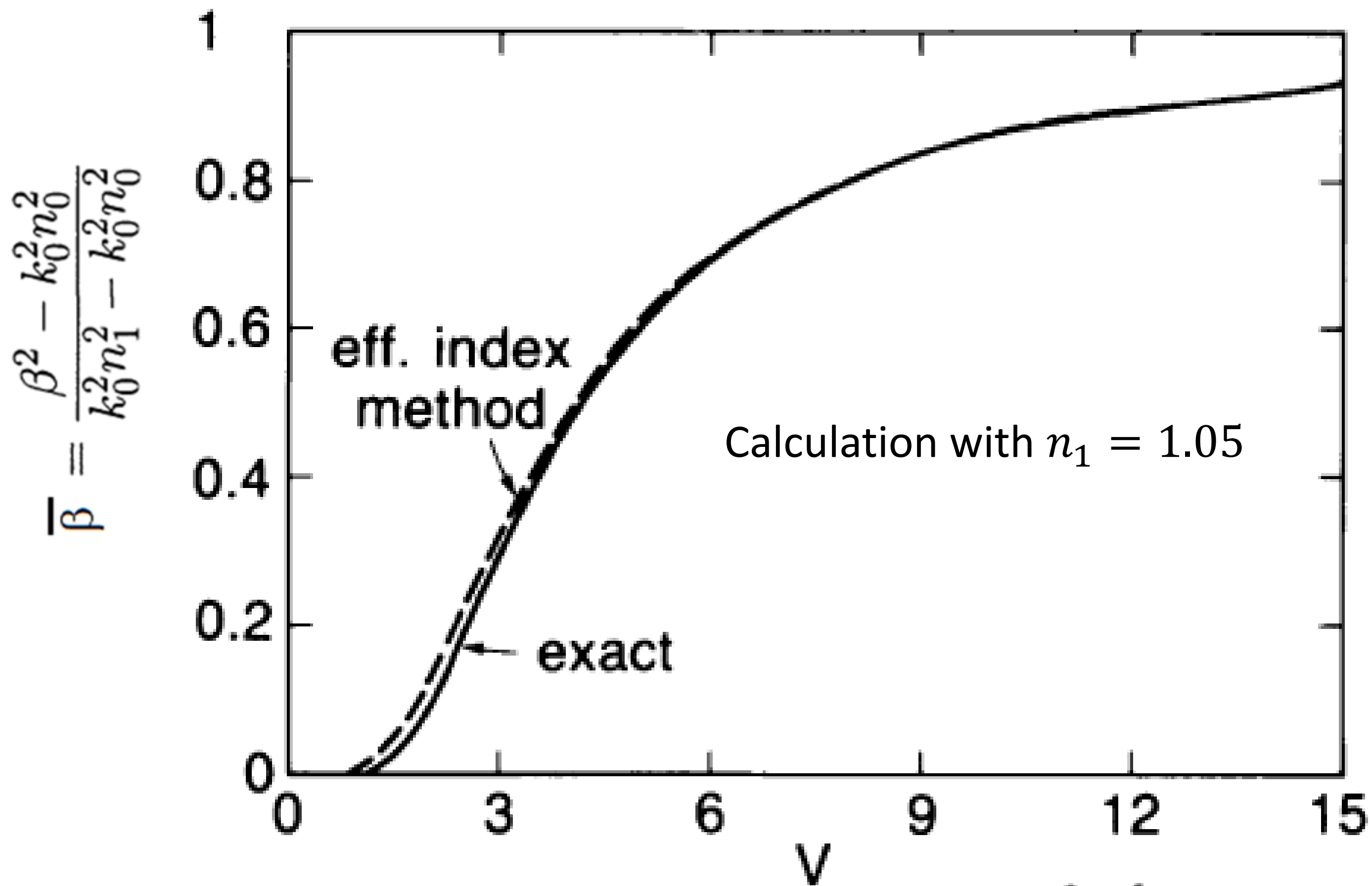


Example of effective index method application

With $d_s \rightarrow 0$ we recover the rectangular waveguide

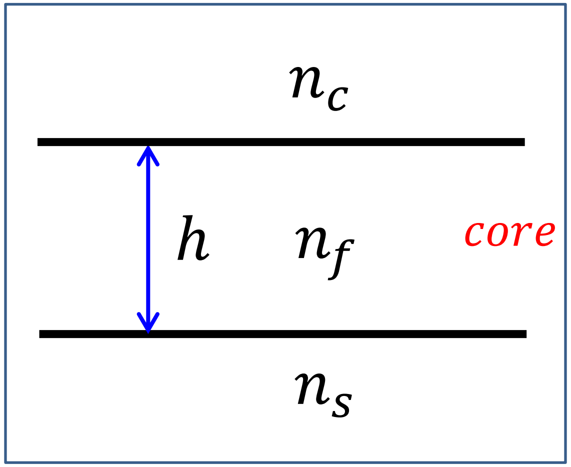
$$a = w_r = 5\ \mu\text{m}$$
$$b = d_r = d = 1.5\ \mu m$$





$$V = \frac{2\pi d}{\lambda}\sqrt{{n_1}^2 - {n_0}^2}$$

In case of an asymmetric slab configuration, the methodology remains essentially the same, but the asymmetric versions of the dispersion relations are used.



$$\beta = \sqrt{k_0^2 n_f^2 - \kappa_f^2}$$

TE

$$\tan(h\kappa_f) = \frac{\gamma_c + \gamma_s}{\kappa_f\left[1 - \frac{\gamma_c\,\gamma_s}{\kappa_f^2}\right]}$$

TM

$$\tan(h\kappa_f) = \kappa_f\,\frac{\frac{n_f^2}{n_c^2}\gamma_c + \frac{n_f^2}{n_s^2}\gamma_s}{\left[\kappa_f^2 - \frac{n_f^4}{n_c^2 n_s^2}\gamma_c\,\gamma_s\right]}$$

$$\gamma_s = \sqrt{\beta^2 - k_0^2 n_s^2} = \sqrt{k_0^2\left(n_f^2 - n_s^2\right) - \kappa_f^2}$$

$$\gamma_c = \sqrt{\beta^2 - k_0^2 n_c^2} = \sqrt{k_0^2\left(n_f^2 - n_c^2\right) - \kappa_f^2}$$

κ_f = **transverse wave vector in core layer**

Basic references

E. A. J. Marcatili, "Dielectric Rectangular Waveguide and Directional Coupler for Integrated Optics," *Bell System Technical Journal*, 48, 2071-2102 (1969)

G.B. Hocker and W.K. Burns, “Mode dispersion in diffused channel waveguides by the effective index method,” Applied Optics, 16, 113 (1977)

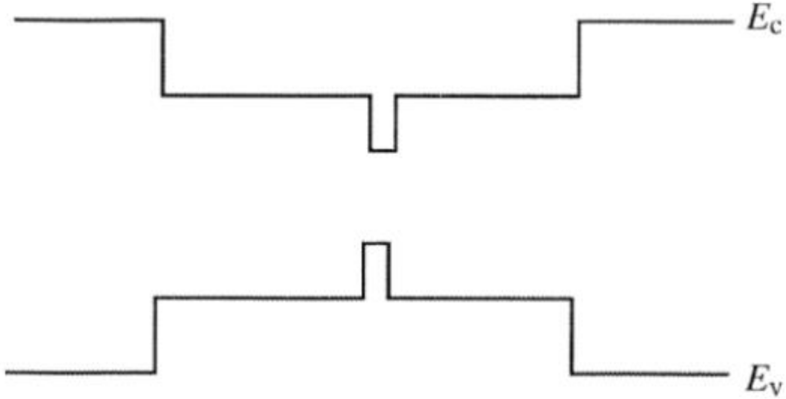
Buus, J. , "Application of the effective index method to non-planar structures, " IEEE J. Quantum Electron. QE-20 , 1106-1109 (1984).

D. Marcuse, *Theory of Dielectric Waveguides, 2nd ed.,* Academic Press, San Diego, (1991) [downloadable from digital library – recommended]

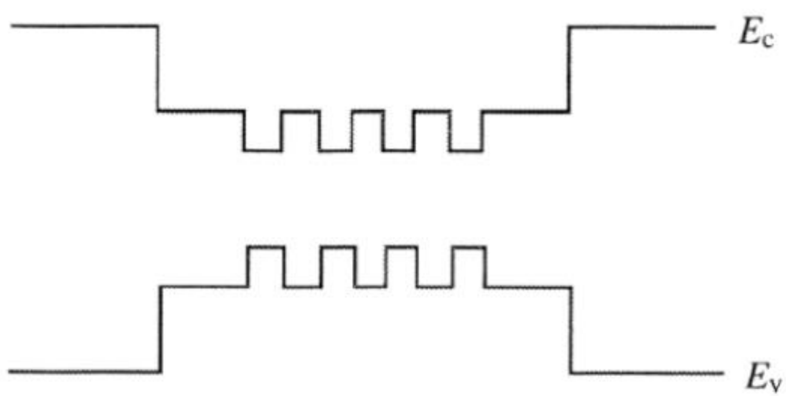
Quantum Well Lasers

Types of QW Lasers

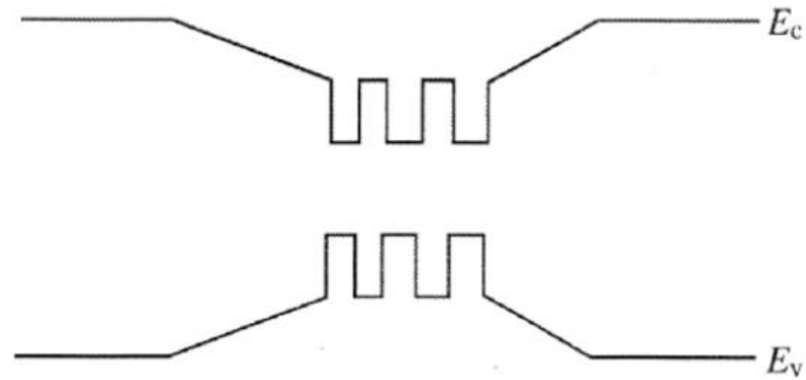
(a) Single-Quantum-Well Separate-Confinement Heterostructure



(b) Multiple-Quantum-Well Separate-Confinement Heterostructure



(c) Graded-Index Separate-Confinement Heterostructure (GRINSCH)



(parabolic bands)

Simplified Gain Model (Chuang – Section 9.4)

Zero - Linewidth Gain Spectrum :

$$g(\hbar\omega) = C_0 \sum_{m,n} \left|I_{hm}^{en}\right|^2 \left|\hat{e}\cdot\mathbf{p}_{\mathbf{cv}}\right|^2 \left[f_c^n\left(\hbar\omega - E_{hm}^{en}\right) - f_v^m\left(\hbar\omega - E_{hm}^{en}\right)\right] \rho_r^{2D} H\left(\hbar\omega - E_{hm}^{en}\right)$$

Simplified Gain Model (Chuang – Section 9.4)

(interband transitions between conduction band and valence band)

Zero - Linewidth Gain Spectrum :

$$g(\hbar\omega) = C_0 \sum_{m,n} \left|I_{hm}^{en}\right|^2 \left|\hat{e}\cdot\mathbf{p}_{\mathbf{cv}}\right|^2 \left[f_c^n\left(\hbar\omega - E_{hm}^{en}\right) - f_v^m\left(\hbar\omega - E_{hm}^{en}\right)\right] \rho_r^{2D} H\left(\hbar\omega - E_{hm}^{en}\right)$$

- $\left|I_{hm}^{en}\right|^2$: overlap integral
- $\left|\hat{e}\cdot\mathbf{p}_{\mathbf{cv}}\right|^2$: momentum matrix element
- f_c^n, f_v^m: probability of state occupation
- ρ_r^{2D}: reduced density of states
- $H\left(\hbar\omega - E_{hm}^{en}\right)$: step function

Simplified Gain Model (Chuang – Section 9.4)

(interband transitions between conduction band and valence band)

Zero - Linewidth Gain Spectrum :

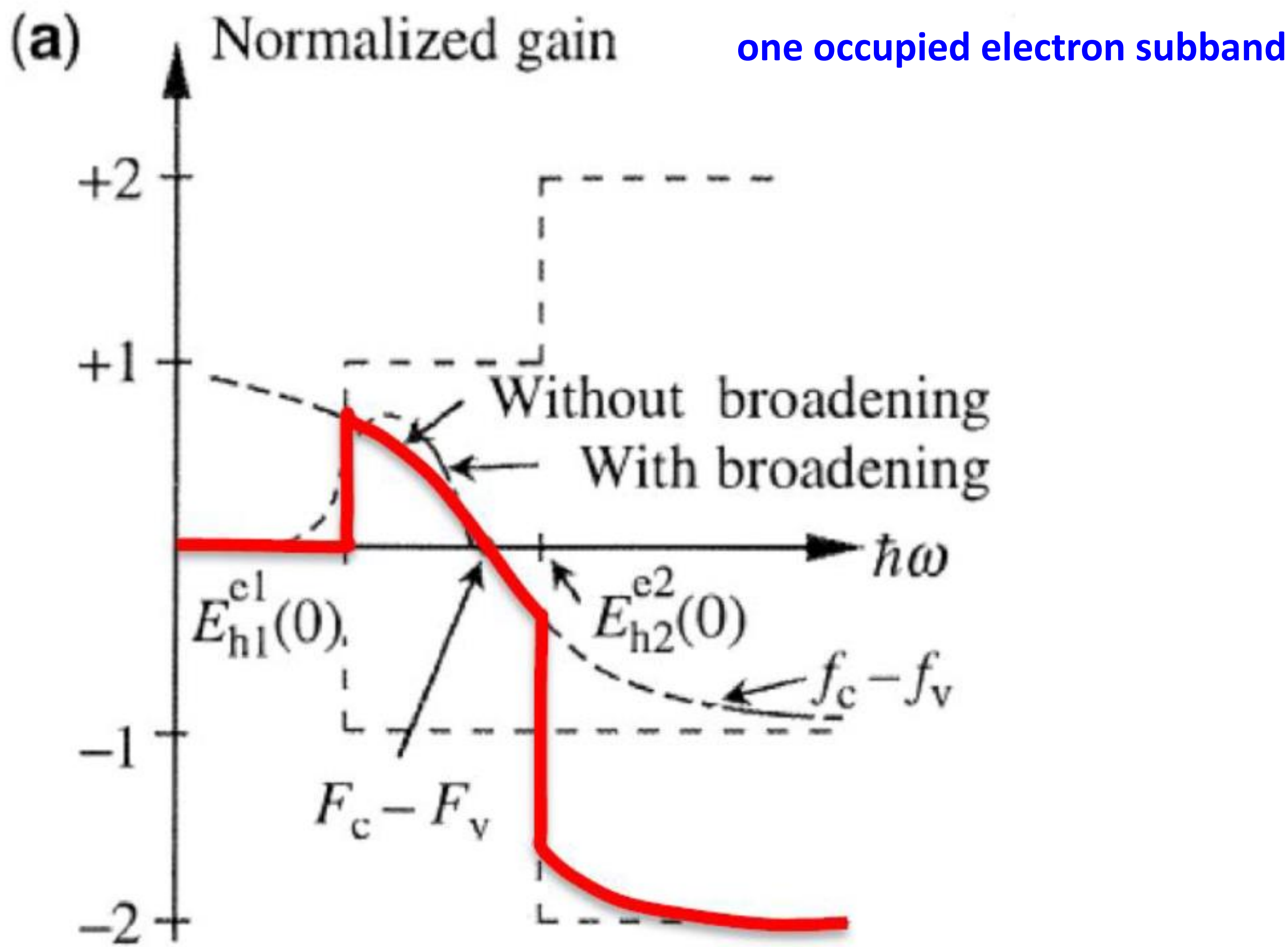
$$g(\hbar\omega) = C_0 \sum_{m,n} \left| I_{hm}^{en} \right|^2 \left| \hat{e} \bullet \mathbf{p}_{cv} \right|^2 \left[f_c^n \left(\hbar\omega - E_{hm}^{en} \right) - f_v^m \left(\hbar\omega - E_{hm}^{en} \right) \right] \rho_r^{2D} H\left(\hbar\omega - E_{hm}^{en} \right)$$

$$= \sum_{m,n} g_{\max} \left[f_c^n \left(\hbar\omega - E_{hm}^{en} \right) - f_v^m \left(\hbar\omega - E_{hm}^{en} \right) \right] H\left(\hbar\omega - E_{hm}^{en} \right)$$

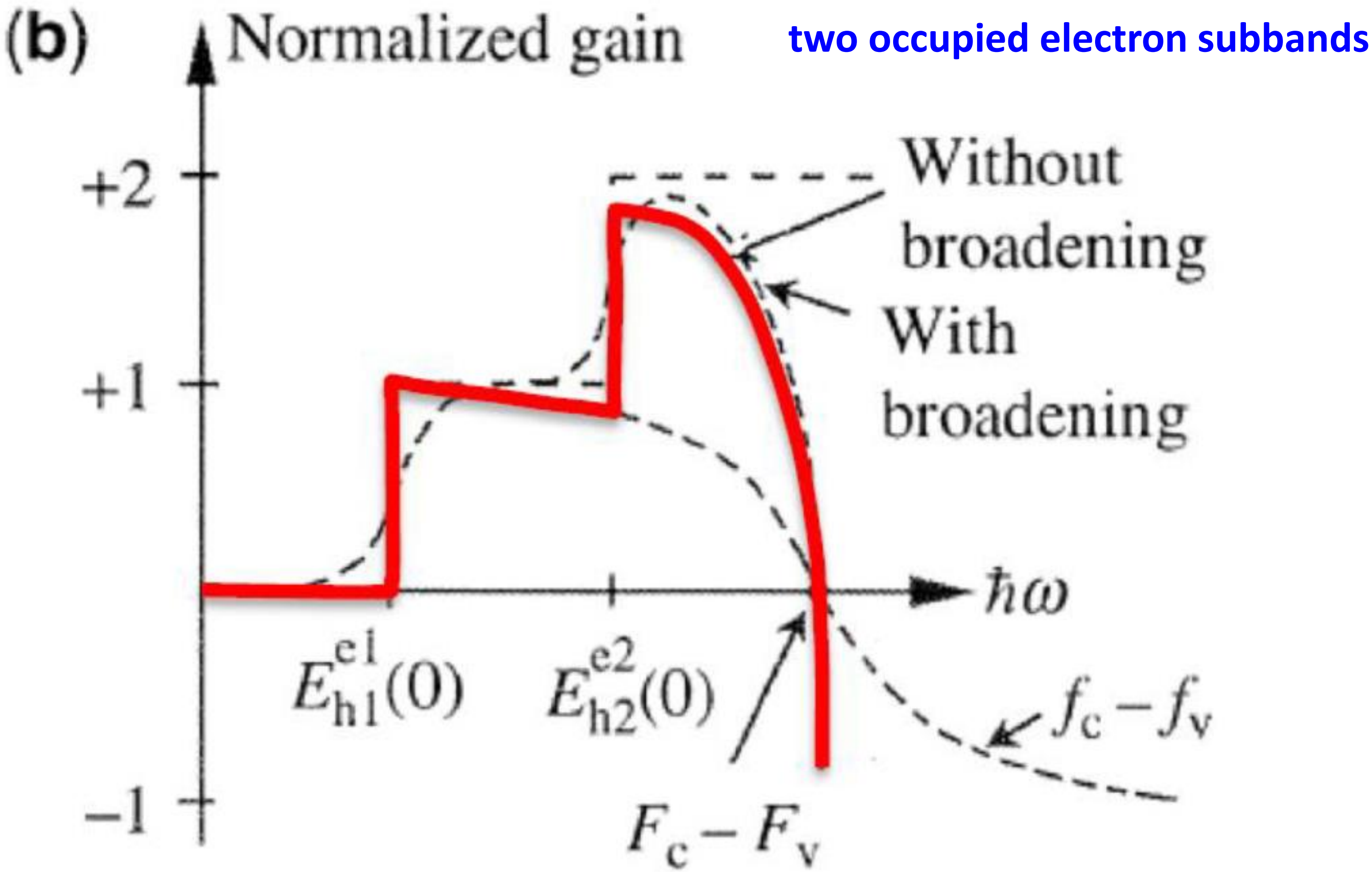
where $g_{\max} = C_0 \left| \hat{e} \bullet \mathbf{M} \right|^2 \left| I_{hm}^{en} \right|^2 \rho_r^{2D} \simeq C_0 \left| \hat{e} \bullet \mathbf{M} \right|^2 \rho_r^{2D} \delta_{nm}$

$\left| \hat{e} \bullet \mathbf{M} \right|^2 = \left| \hat{e} \bullet \mathbf{p}_{cv} \right|^2$ and $C_0 = \dfrac{\pi e^2}{n_r c \varepsilon_0 m_0^2 \omega}$ and $\rho_r^{2D} = \dfrac{m_r^*}{\pi \hbar^2 L_z}$

Simplified Gain Model (Chuang – Section 9.4)



Simplified Gain Model (Chuang – Section 9.4)



Interband Momentum Matrix Element (Chuang – Section 9.5)

For bulk : $\left\langle \left|\hat{e}\bullet\mathbf{M}_{c-hh}\right|^2 \right\rangle = \left\langle \left|\hat{e}\bullet\mathbf{M}_{c-lh}\right|^2 \right\rangle = M_b^2$ (a parameter of the material)

This quantity is independent of the polarization of the light.

$$M_b^2 = \frac{m_0}{6} E_p$$

For a quantum well there is polarization dependence for the gain.

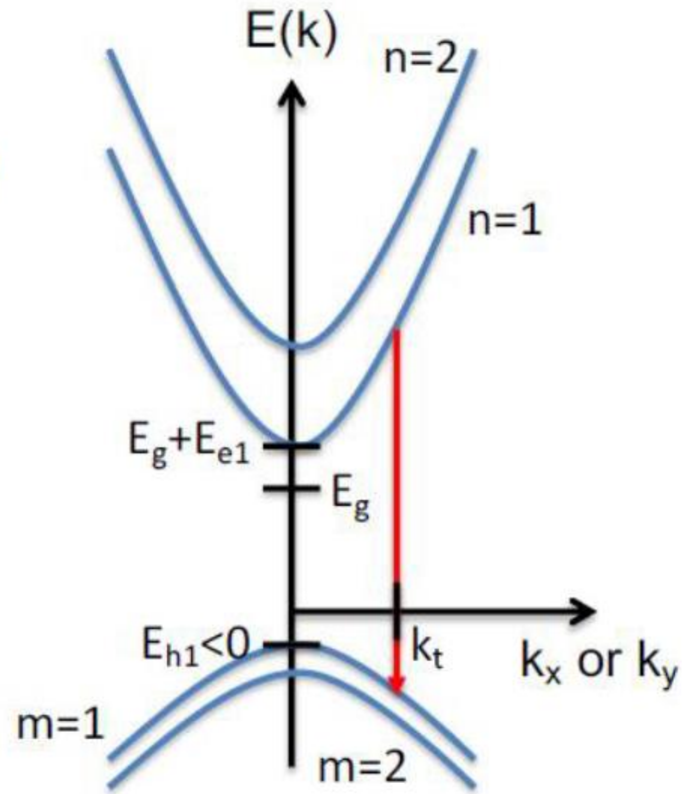
Electron wave vector in spherical coordinates

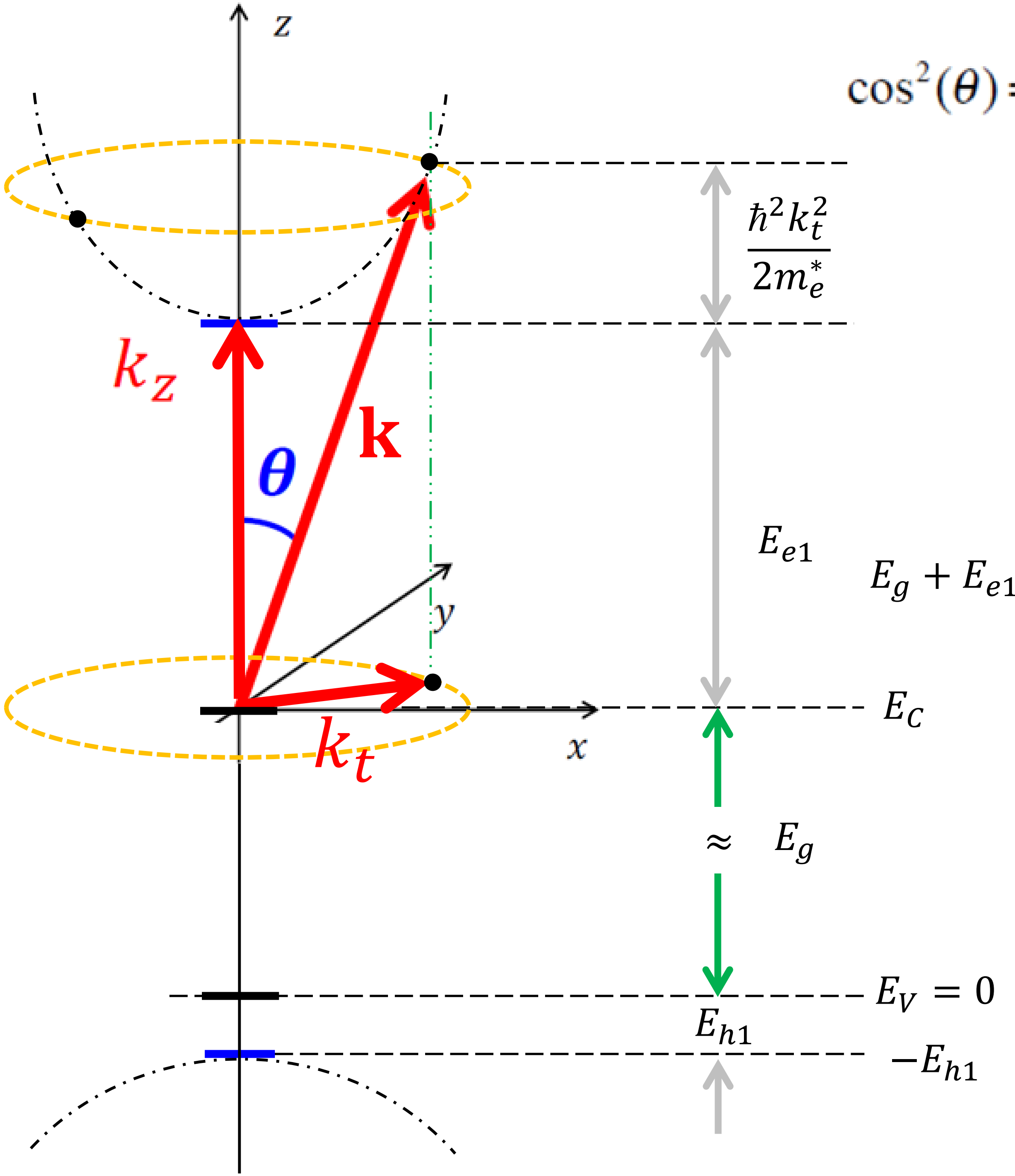
$$\mathbf{k} = k(\sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\theta)\hat{z})$$

$$\cos^2(\theta) = \left(\frac{k_z}{k}\right)^2 = \frac{k_z^2}{k_x^2 + k_y^2 + k_z^2} = \frac{E_{en}}{E_{en} + \dfrac{\hbar^2 k_t^2}{2m_e^*}}$$

$$\approx \frac{E_{en} + |E_{hm}|}{E_{en} + |E_{hm}| + \dfrac{\hbar^2 k_t^2}{2m_r^*}}$$

good approximation





$$\cos^2(\theta) = \left(\frac{k_z}{k}\right)^2 \approx \frac{E_{en} + |E_{hm}|}{E_{en} + |E_{hm}| + \frac{\hbar^2 k_t^2}{2m_r^*}}$$

At the subband edge

$$k_x = k_y = 0$$

$\mathbf{k} = k_z$ and $\theta = 0$

Interband Momentum Matrix Element (Chuang – Section 9.5)

Momentum Matrix Elements, TE Polarization $(\hat{e} = \hat{x} \text{ or } \hat{y})$:

$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-hh} \right|^2 \right\rangle_{TE} = \frac{3}{4}\left(1+\cos^2\theta\right) M_b^2$$ c-hh

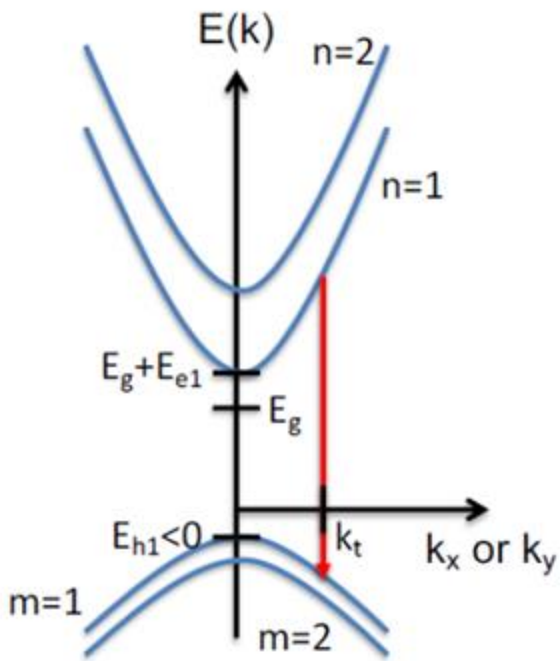
$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-lh} \right|^2 \right\rangle_{TE} = \left(\frac{5}{4} - \frac{3}{4}\cos^2\theta\right) M_b^2$$ c-lh

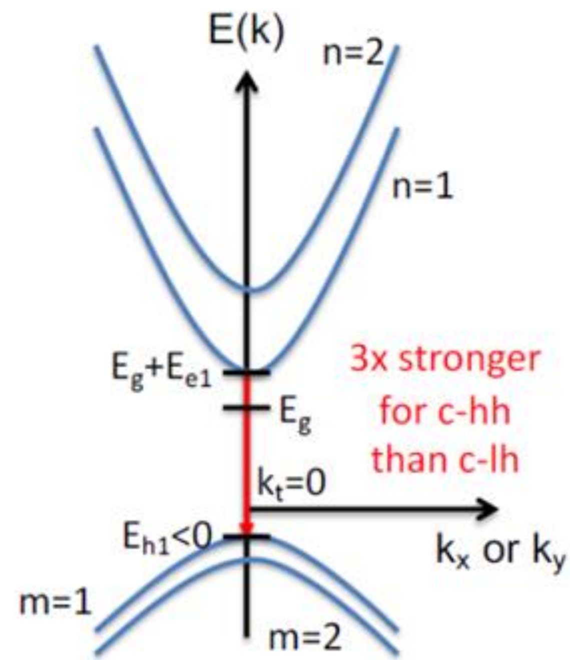
At each sub-band edge: $\theta = 0°$

$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-hh} \right|^2 \right\rangle_{TE} = \frac{3}{2} M_b^2$$

$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-lh} \right|^2 \right\rangle_{TE} = \frac{1}{2} M_b^2$$

$$\cos^2(\theta) \approx \frac{E_{en} + \left|E_{hm}\right|}{E_{en} + \left|E_{hm}\right| + \frac{\hbar^2 k_t^2}{2m_r^*}}$$





Interband Momentum Matrix Element (Chuang – Section 9.5)

Momentum Matrix Elements, TM Polarization $(\hat{e}=\hat{z})$:

$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-hh} \right|^2 \right\rangle_{TM} = \boxed{\frac{3}{2}\sin^2\theta M_b^2}$$ **c-hh**

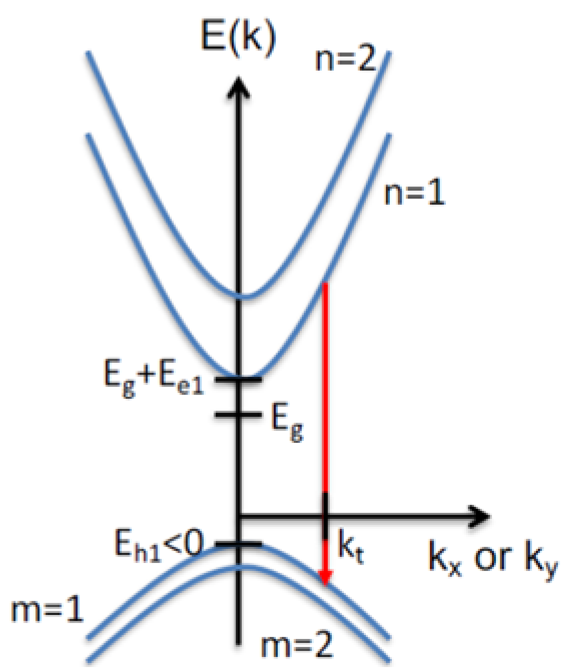
$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-lh} \right|^2 \right\rangle_{TM} = \boxed{\frac{1}{2}\left(1+3\cos^2\theta\right) M_b^2}$$ **c-lh**

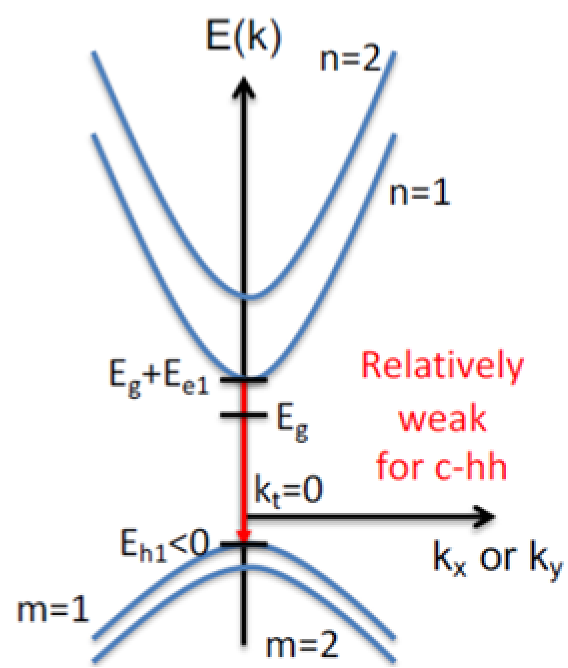
At each sub-band edge: $\theta=0°$

$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-hh} \right|^2 \right\rangle_{TM} = 0$$

$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-lh} \right|^2 \right\rangle_{TM} = 2M_b^2$$

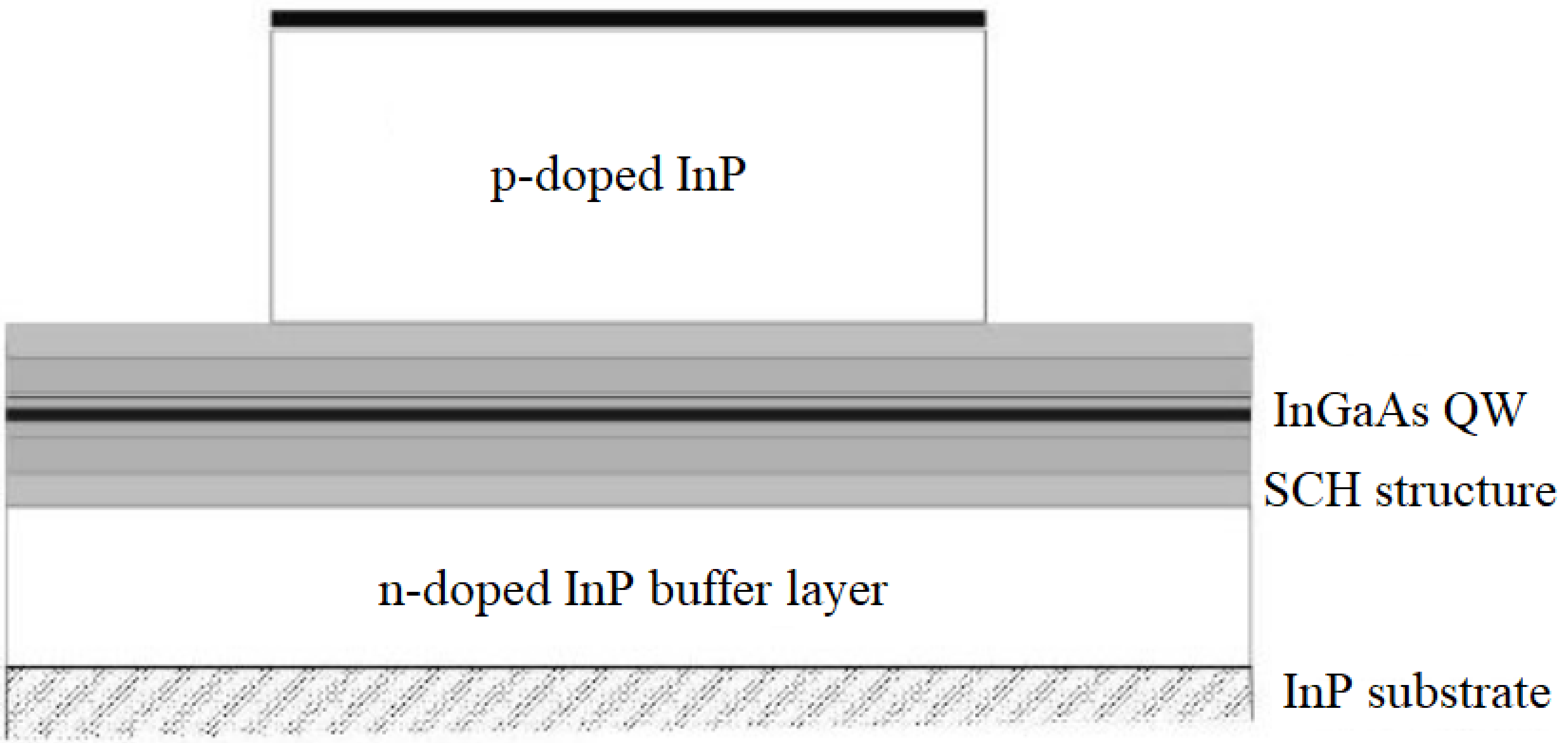
$$\cos^2(\theta) \approx \frac{E_{en}+\left|E_{hm}\right|}{E_{en}+\left|E_{hm}\right|+\dfrac{\hbar^2 k_t^2}{2m_r^*}}$$





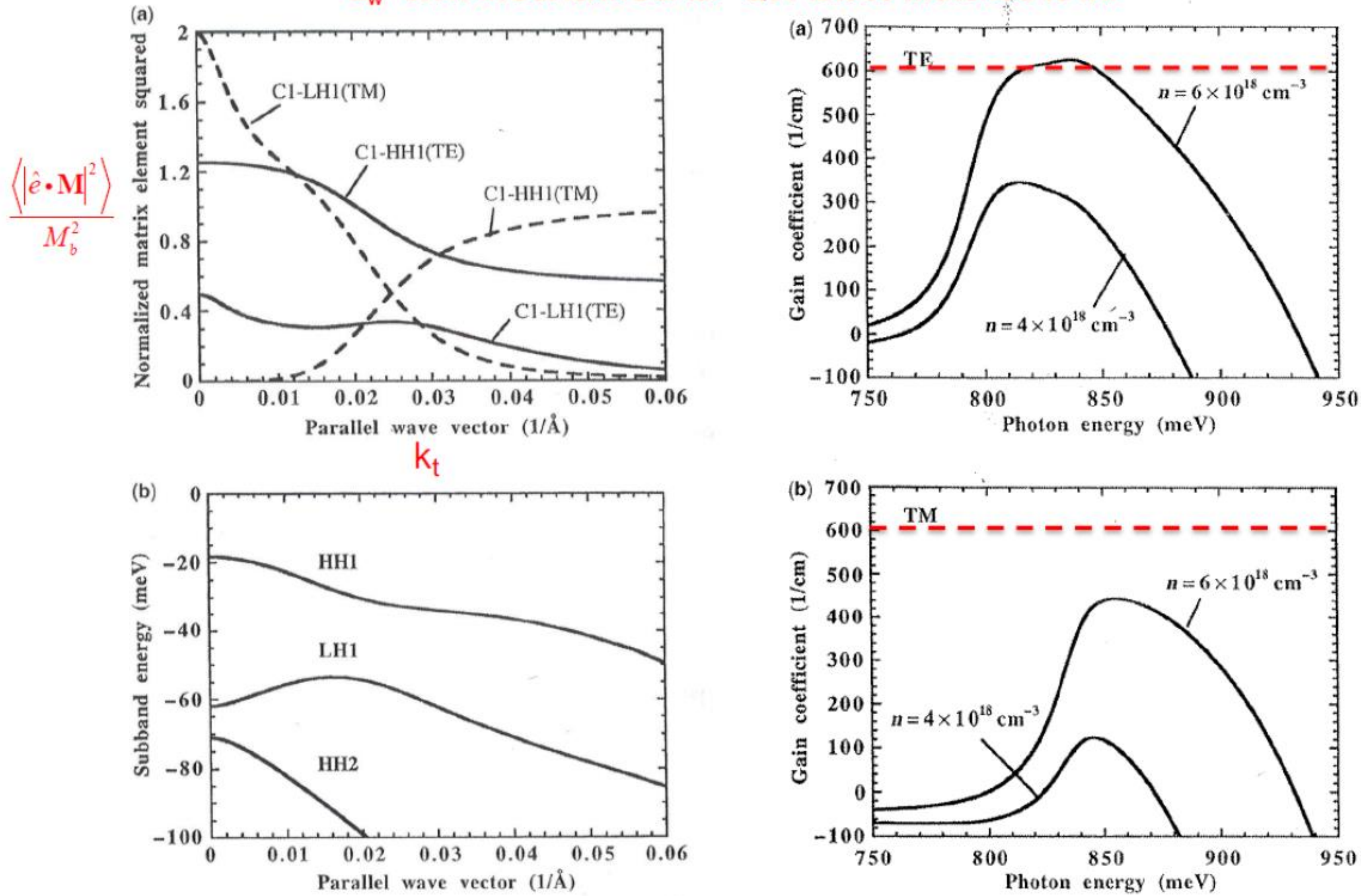
Gain Spectrum in a QW Laser(Chuang – Section 9.8)

L_w=6nm InGaAs/InGaAsP QW lattice matched to InP



Gain Spectrum in a QW Laser(Chuang – Section 9.8)

L_w=6nm InGaAs/InGaAsP QW lattice matched to InP



Reading Assignments:

Sections 10.1 and 10.2 of Chuang's book