ECE 536 – Integrated Optics and Optoelectronics
Lecture 17 – March 25, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

Prof. Umberto Ravaioli

ECE Department, University of Illinois

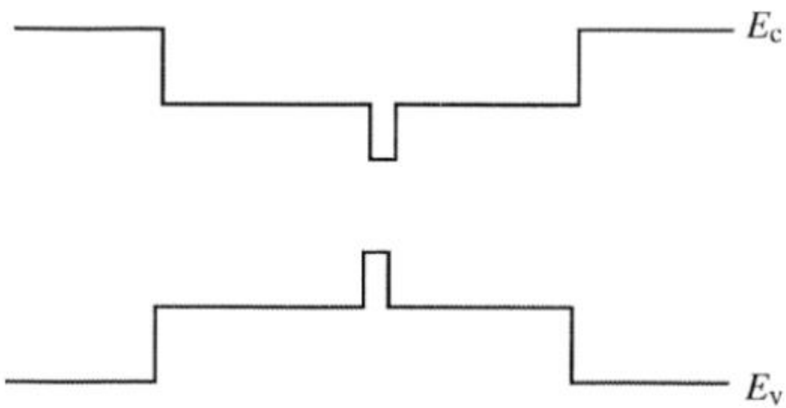
Lecture 17 Outline

- More on Quantum Well Lasers
- Multiple Quantum Wells Lasers
- Scaling Law for Multiple Quantum Wells
- Strain Effects

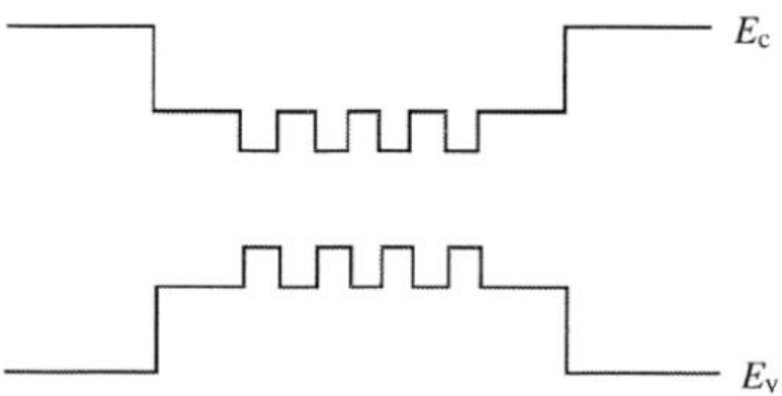
Quantum Well (QW) Lasers

Types of QW Lasers

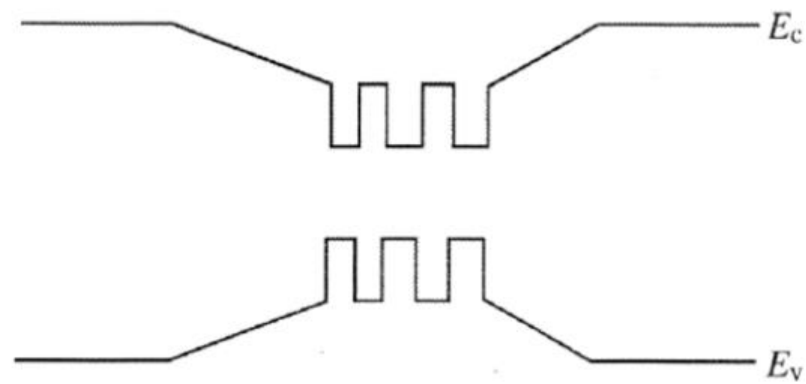
(a) Single-Quantum-Well Separate-Confinement Heterostructure



(b) Multiple-Quantum-Well Separate-Confinement Heterostructure



(c) Graded-Index Separate-Confinement Heterostructure (GRINSCH)



Simplified Gain Model (Chuang – Section 9.4)

(interband transitions between conduction band and valence band)

Zero - Linewidth Gain Spectrum :

$$g(\hbar\omega) = C_0 \sum_{m,n} \left|I_{hm}^{en}\right|^2 \left|\hat{e}\bullet\mathbf{p}_{\mathbf{cv}}\right|^2 \left[f_c^n\left(\hbar\omega - E_{hm}^{en}\right) - f_v^m\left(\hbar\omega - E_{hm}^{en}\right)\right] \rho_r^{2D} H\left(\hbar\omega - E_{hm}^{en}\right)$$

$$= \sum_{m,n} g_{\max} \left[f_c^n\left(\hbar\omega - E_{hm}^{en}\right) - f_v^m\left(\hbar\omega - E_{hm}^{en}\right)\right] H\left(\hbar\omega - E_{hm}^{en}\right)$$

where $g_{\max} = C_0 \left|\hat{e}\bullet\mathbf{M}\right|^2 \left|I_{hm}^{en}\right|^2 \rho_r^{2D} \simeq C_0 \left|\hat{e}\bullet\mathbf{M}\right|^2 \rho_r^{2D} \delta_{nm}$

$\left|\hat{e}\bullet\mathbf{M}\right|^2 = \left|\hat{e}\bullet\mathbf{p}_{\mathbf{cv}}\right|^2$ and $C_0 = \dfrac{\pi e^2}{n_r c \varepsilon_0 m_0^2 \omega}$ and $\rho_r^{2D} = \dfrac{m_r^*}{\pi \hbar^2 L_z}$

Interband Momentum Matrix Element (Chuang – Section 9.5)

Momentum Matrix Elements, TE Polarization $(\hat{e} = \hat{x} \text{ or } \hat{y})$:

$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-hh} \right|^2 \right\rangle_{TE} = \frac{3}{4}\left(1 + \cos^2\theta\right) M_b^2 \quad \text{c-hh}$$

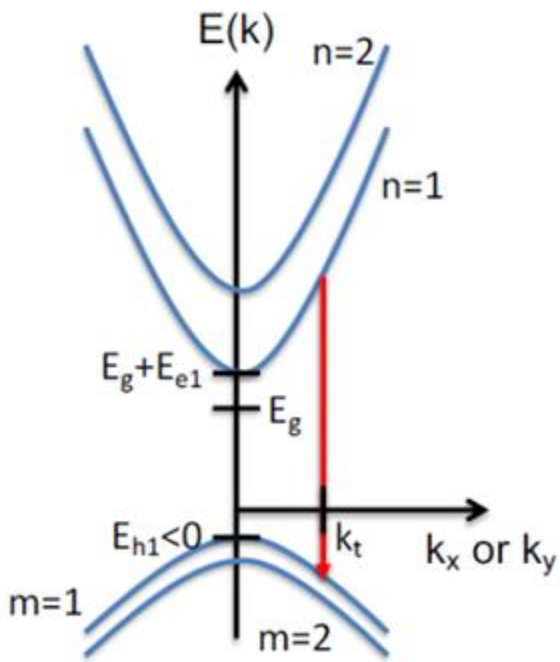
$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-lh} \right|^2 \right\rangle_{TE} = \left(\frac{5}{4} - \frac{3}{4}\cos^2\theta\right) M_b^2 \quad \text{c-lh}$$

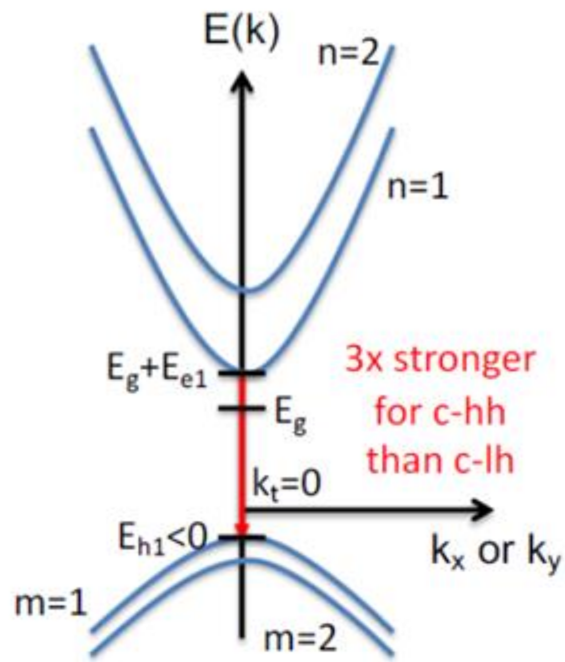
At each sub-band edge: $\theta = 0°$

$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-hh} \right|^2 \right\rangle_{TE} = \frac{3}{2} M_b^2$$

$$\left\langle \left| \hat{e} \cdot \mathbf{M}_{c-lh} \right|^2 \right\rangle_{TE} = \frac{1}{2} M_b^2$$

$$\cos^2(\theta) \approx \frac{E_{en} + \left|E_{hm}\right|}{E_{en} + \left|E_{hm}\right| + \frac{\hbar^2 k_t^2}{2m_r^*}}$$





Interband Momentum Matrix Element (Chuang – Section 9.5)

Momentum Matrix Elements, TM Polarization $(\hat{e}=\hat{z})$:

$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-hh} \right|^2 \right\rangle_{TM} = \frac{3}{2}\sin^2\theta M_b^2 \quad \text{c-hh}$$

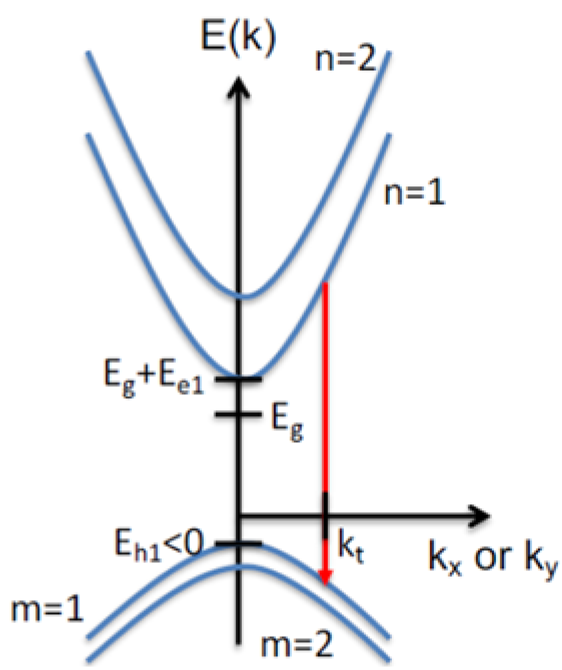
$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-lh} \right|^2 \right\rangle_{TM} = \frac{1}{2}\left(1+3\cos^2\theta\right) M_b^2 \quad \text{c-lh}$$

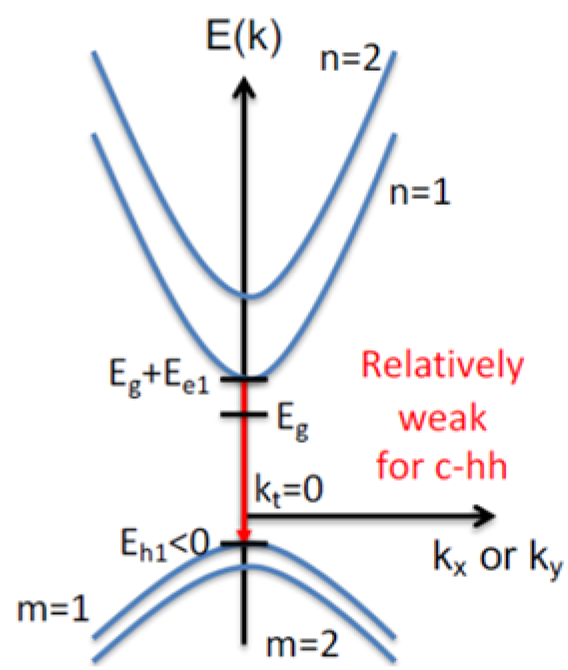
At each sub-band edge: $\theta=0°$

$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-hh} \right|^2 \right\rangle_{TM} = 0$$

$$\left\langle \left| \hat{e}\cdot\mathbf{M}_{c-lh} \right|^2 \right\rangle_{TM} = 2M_b^2$$

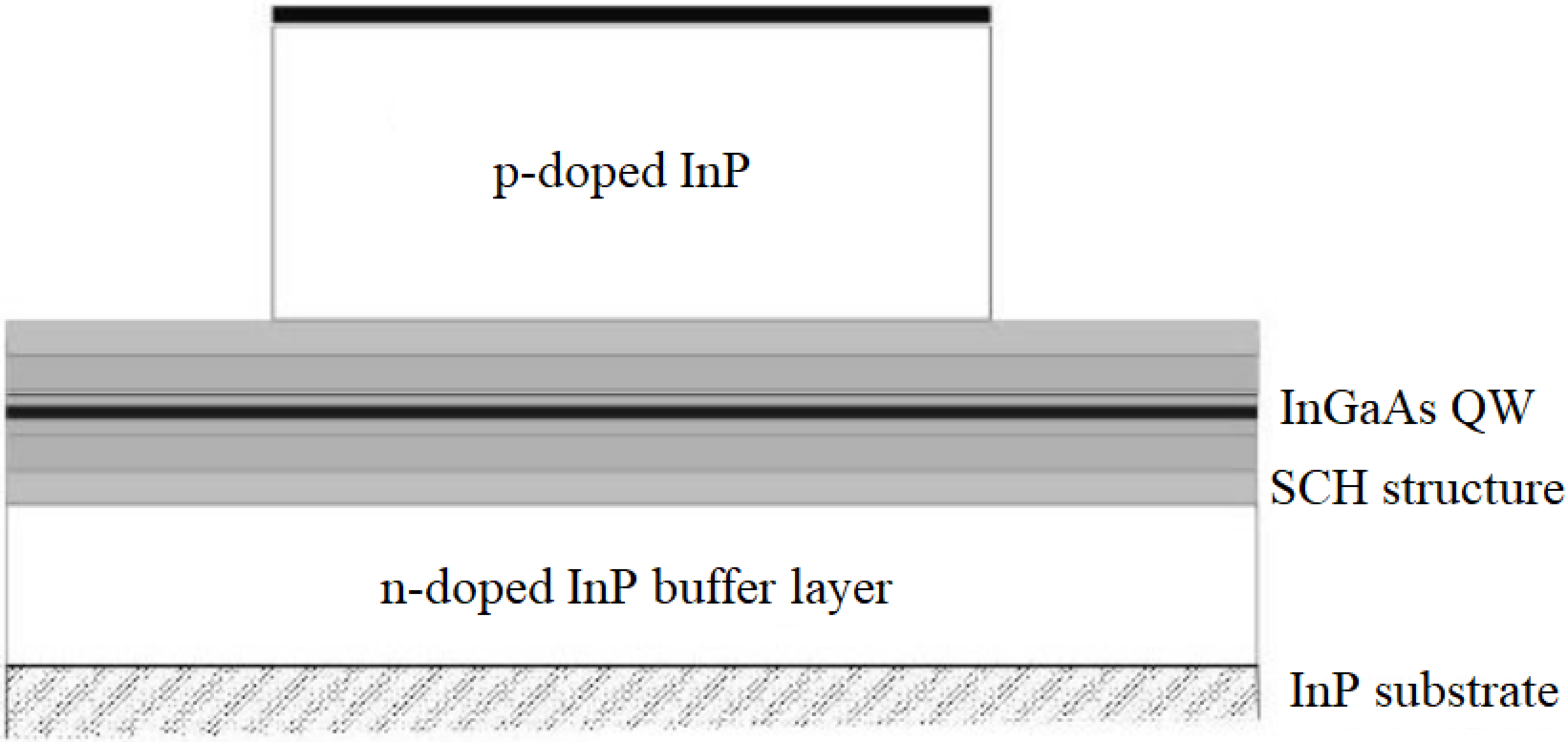
$$\cos^2(\theta) \approx \frac{E_{en}+\left|E_{hm}\right|}{E_{en}+\left|E_{hm}\right|+\dfrac{\hbar^2 k_t^2}{2m_r^*}}$$





Gain Spectrum in a QW Laser(Chuang – Section 9.8)

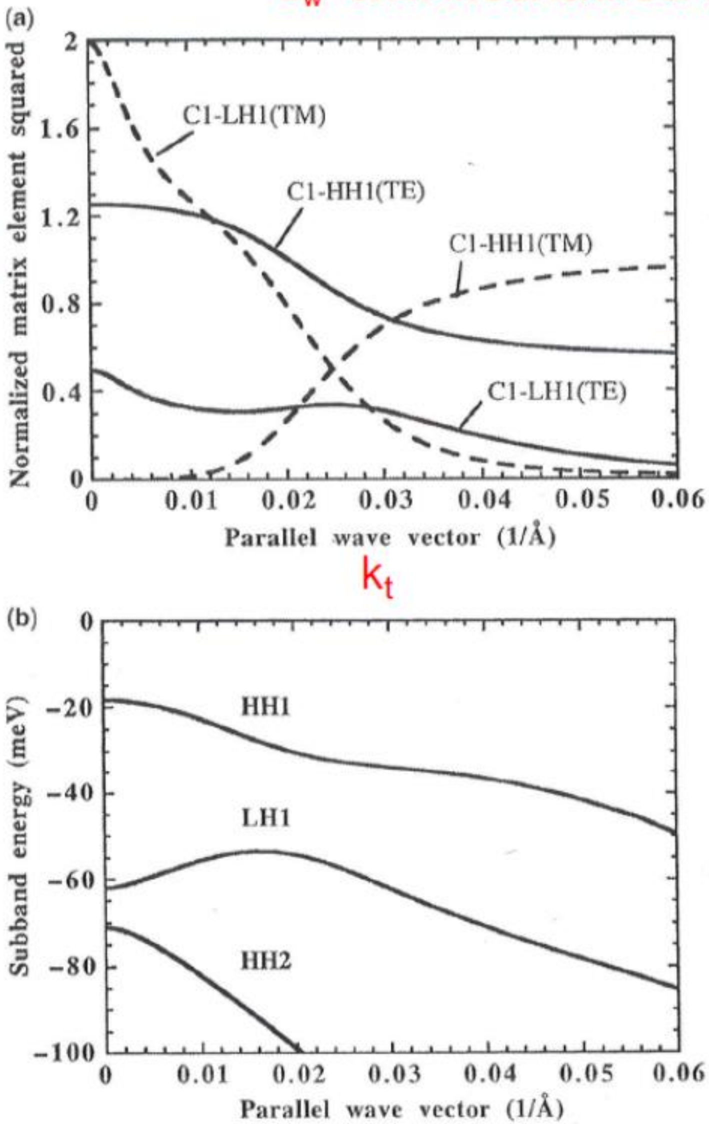
L_w=6nm InGaAs/InGaAsP QW lattice matched to InP

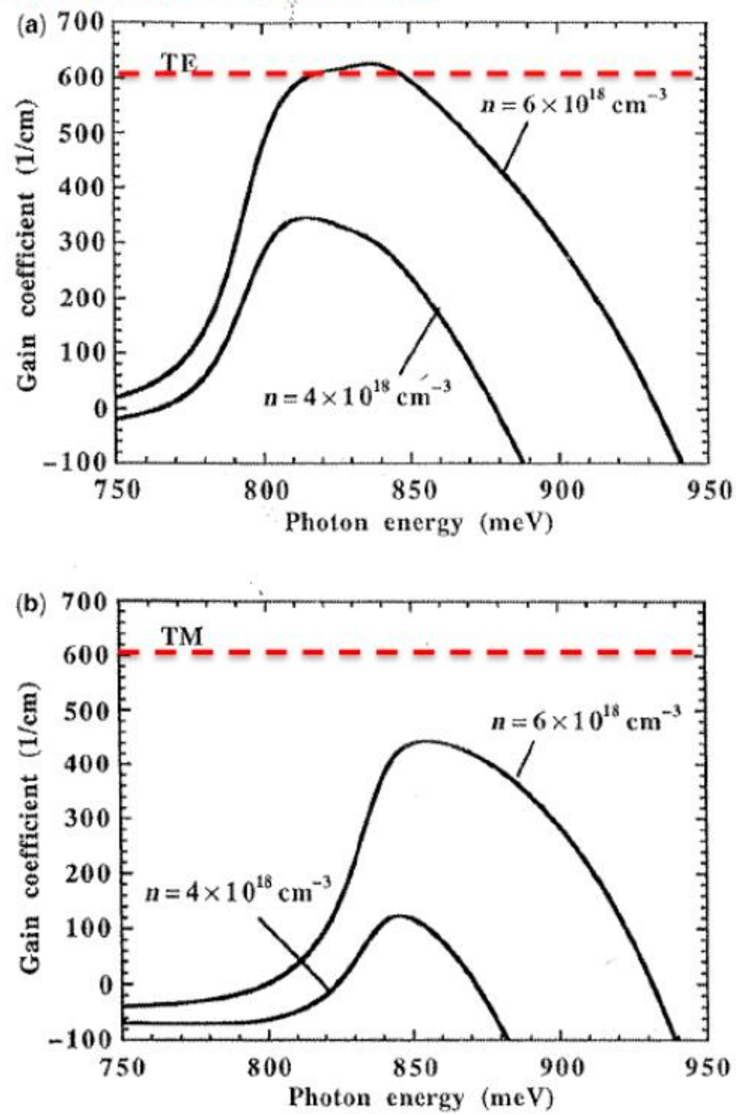


Gain Spectrum in a QW Laser(Chuang – Section 9.8)

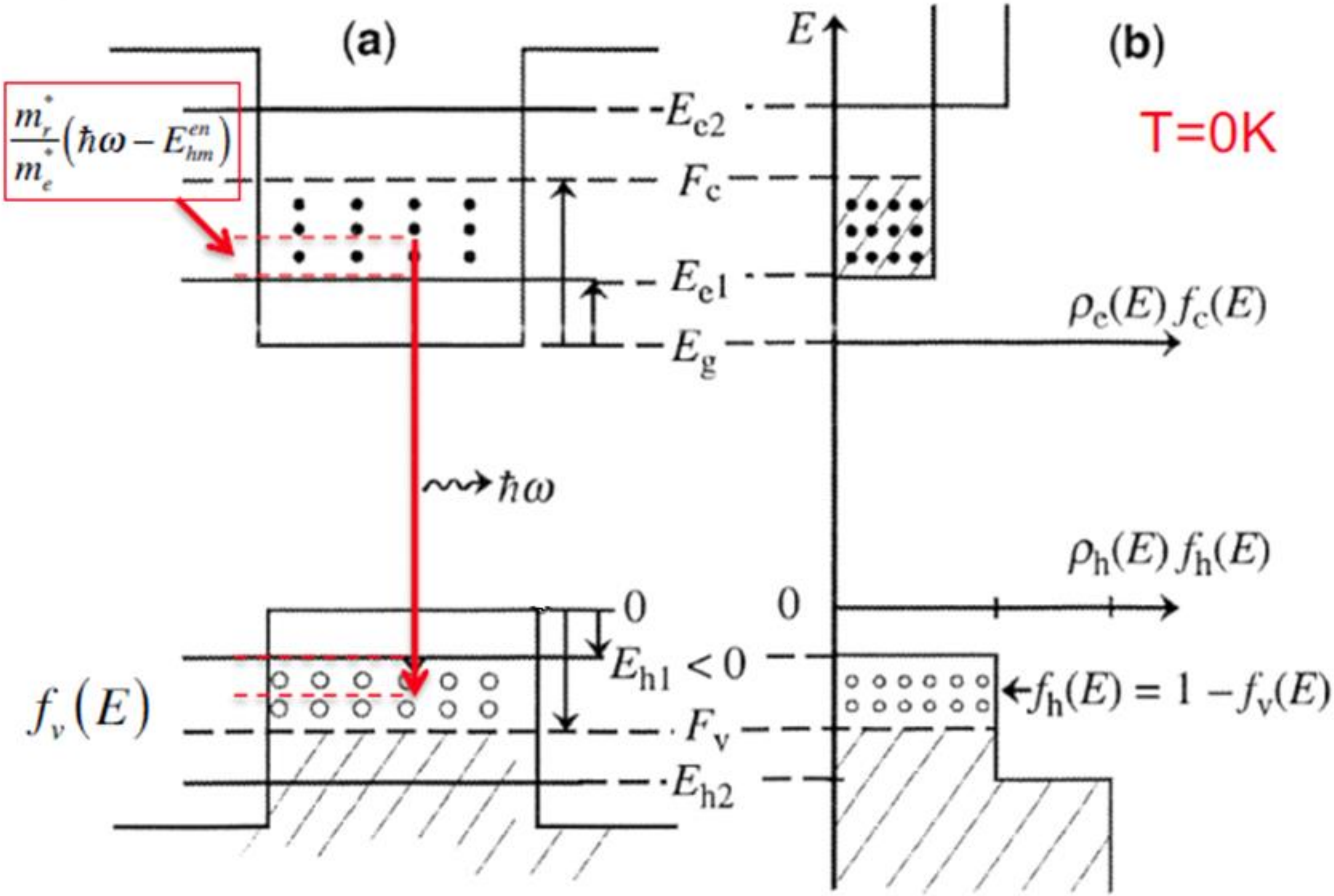
L_w=6nm InGaAs/InGaAsP QW lattice matched to InP

$$\frac{\left\langle \left| \hat{e} \cdot \mathbf{M} \right|^2 \right\rangle}{M_b^2}$$





Electron and Hole Occupancy



Electron and Hole Occupancy

Occupation Factor for electrons in nth subband of conduction band

$$f_c^n\left(\hbar\omega - E_{hm}^{en}\right) = \frac{1}{1 + e^{\left[E_{en} + \left(m_r^*/m_e^*\right)\left(\hbar\omega - E_{hm}^{en}\right) - F_c\right]/k_B T}}$$

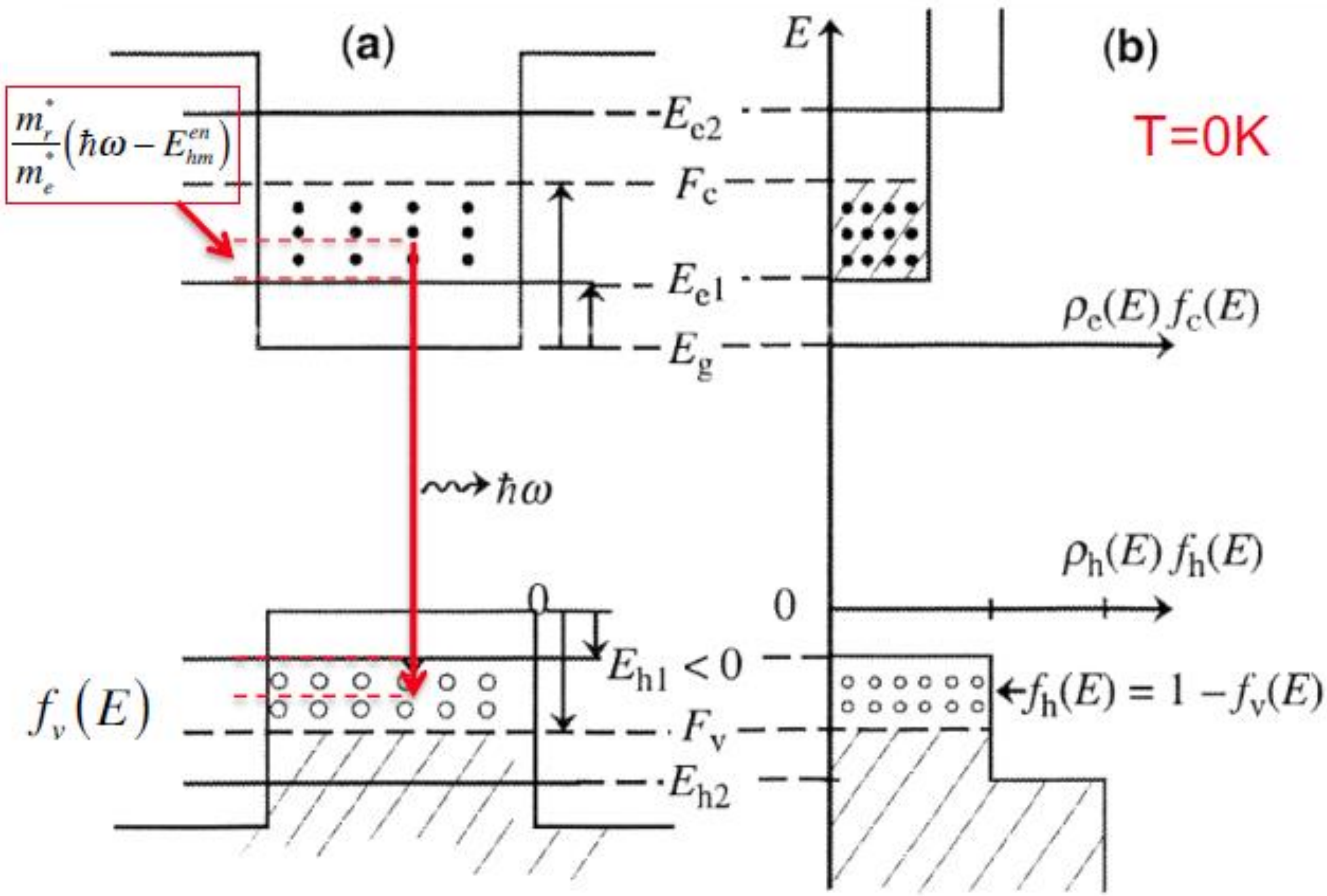
Occupation Factor for electrons in mth subband of the valence band

$$f_v^m\left(\hbar\omega - E_{hm}^{en}\right) = \frac{1}{1 + e^{\left[E_{hm} + \left(m_r^*/m_h^*\right)\left(\hbar\omega - E_{hm}^{en}\right) - F_v\right]/k_B T}}$$

For holes: $f_h(E) = 1 - f_v(E)$

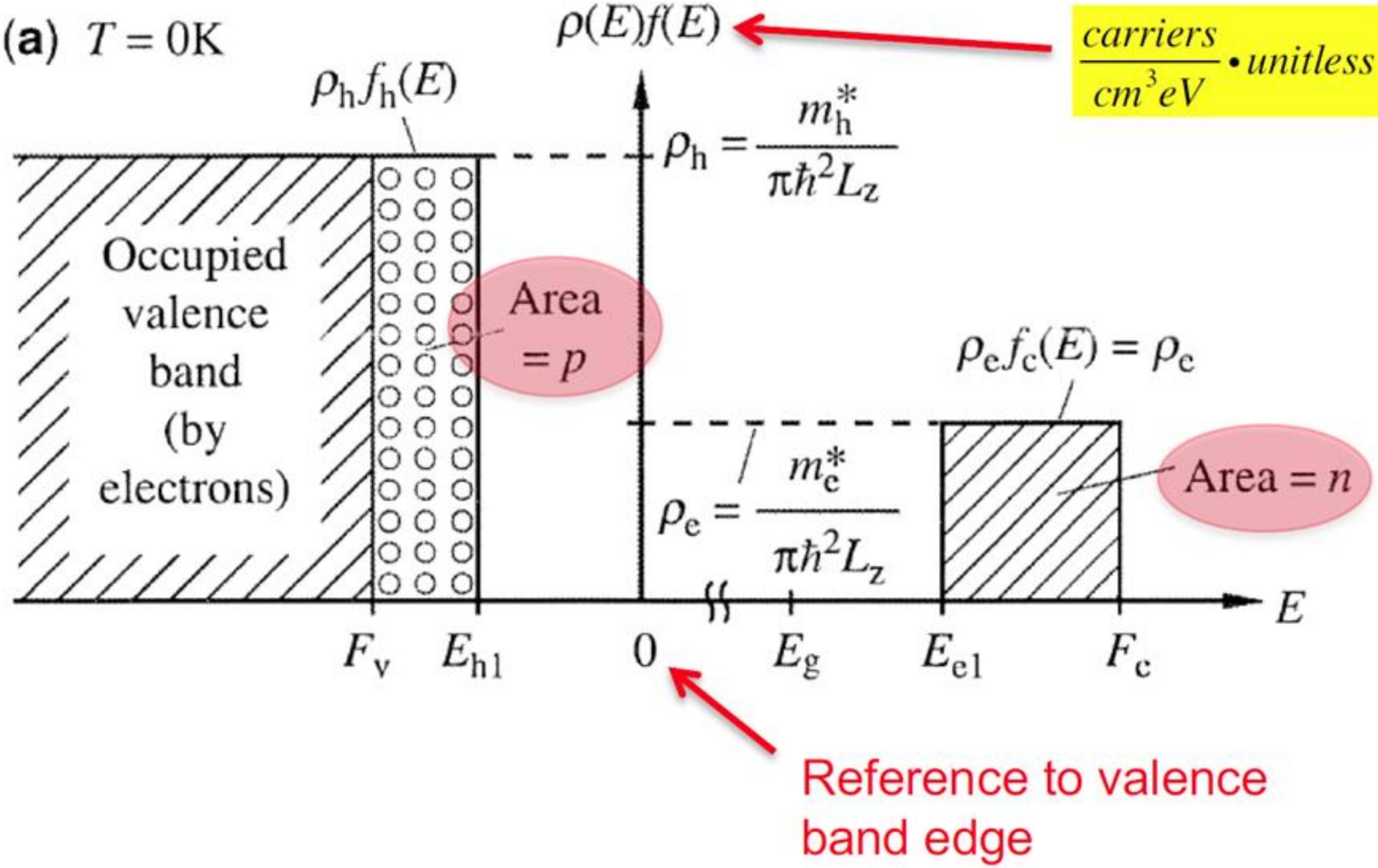
Condition for Population Inversion → Gain

$$f_c^n > f_v^m$$

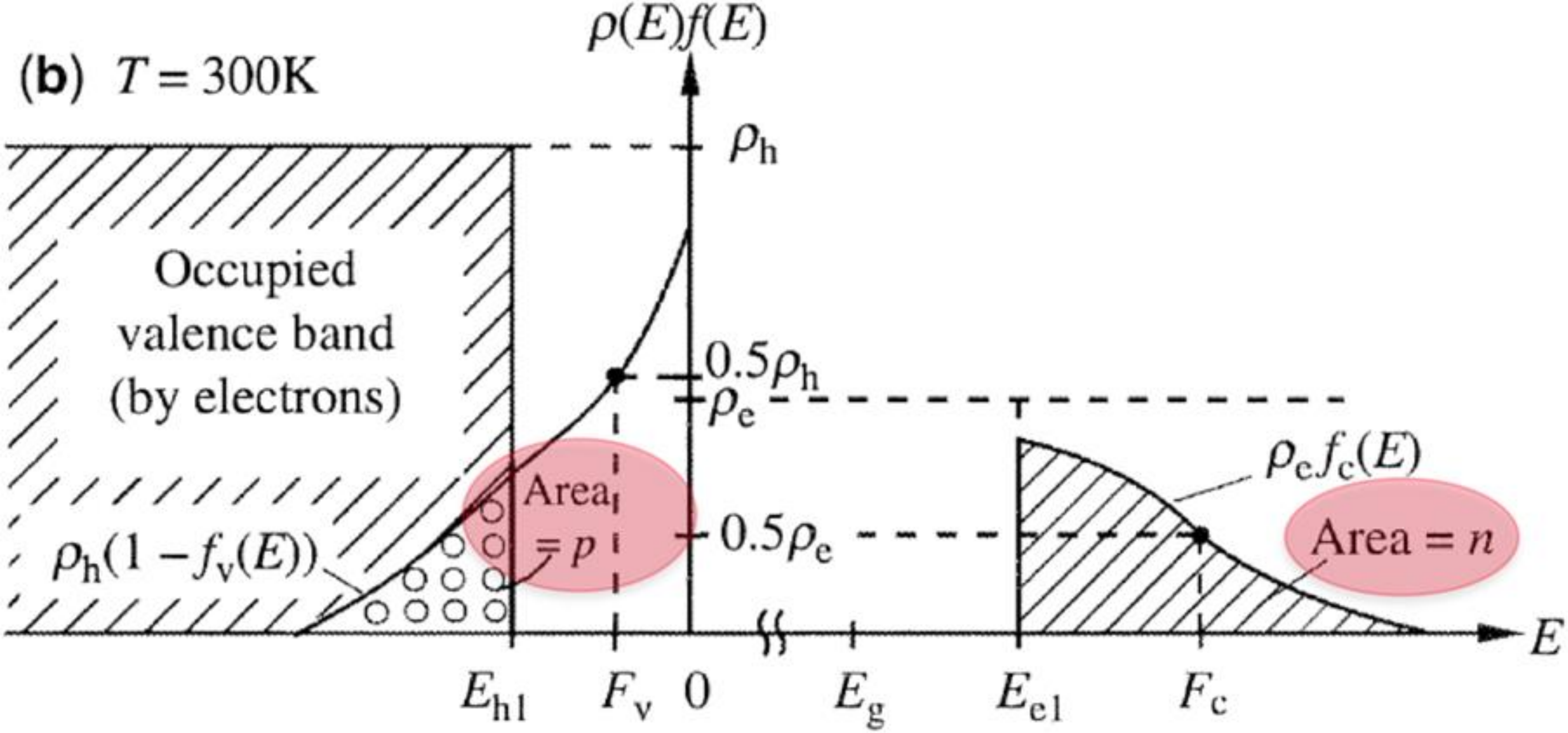


$$E_g + E_{e1} - E_{h1} < \hbar\omega < E_g + F_c - F_V$$

Quasi-Fermi Levels at $T = 0K$



Quasi-Fermi Levels at Room Temperature



Carrier densities

Charge neutrality

$$n + N_A^- = p + N_D^+$$

Electrons

$$n = \int_0^\infty dE \rho_e(E) f_c(E)$$

$$\rho_e(E) = \frac{m_e^*}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} H(E - E_{en})$$

Holes

correction to (10.3.3)

$$p = \int_{-\infty}^{0} dE \rho_h(E) \left[1 - f_v(E)\right]$$

$$\rho_h(E) = \frac{m_h^*}{\pi \hbar^2 L_z} \sum_{m=1}^{\infty} H(E_{hm} - E)$$

Gain Spectrum at $T = 0K$

$$f_c(E) = \begin{cases} 1 & E < F_c \\ 0 & E > F_c \end{cases}$$

In general the electron concentration is

$$n = \frac{m_e^*}{\pi\hbar^2 L_z} \sum_{\substack{n \text{ occupied} \\ \text{subbands}}} \left(F_c - E_{en}\right)$$

Consider a single occupied state

$$\begin{cases} n = \dfrac{m_e^*}{\pi\hbar^2 L_z}\left(F_c - E_{e1}\right) \\ p = \dfrac{m_h^*}{\pi\hbar^2 L_z}\left(E_{h1} - F_v\right) \end{cases}$$

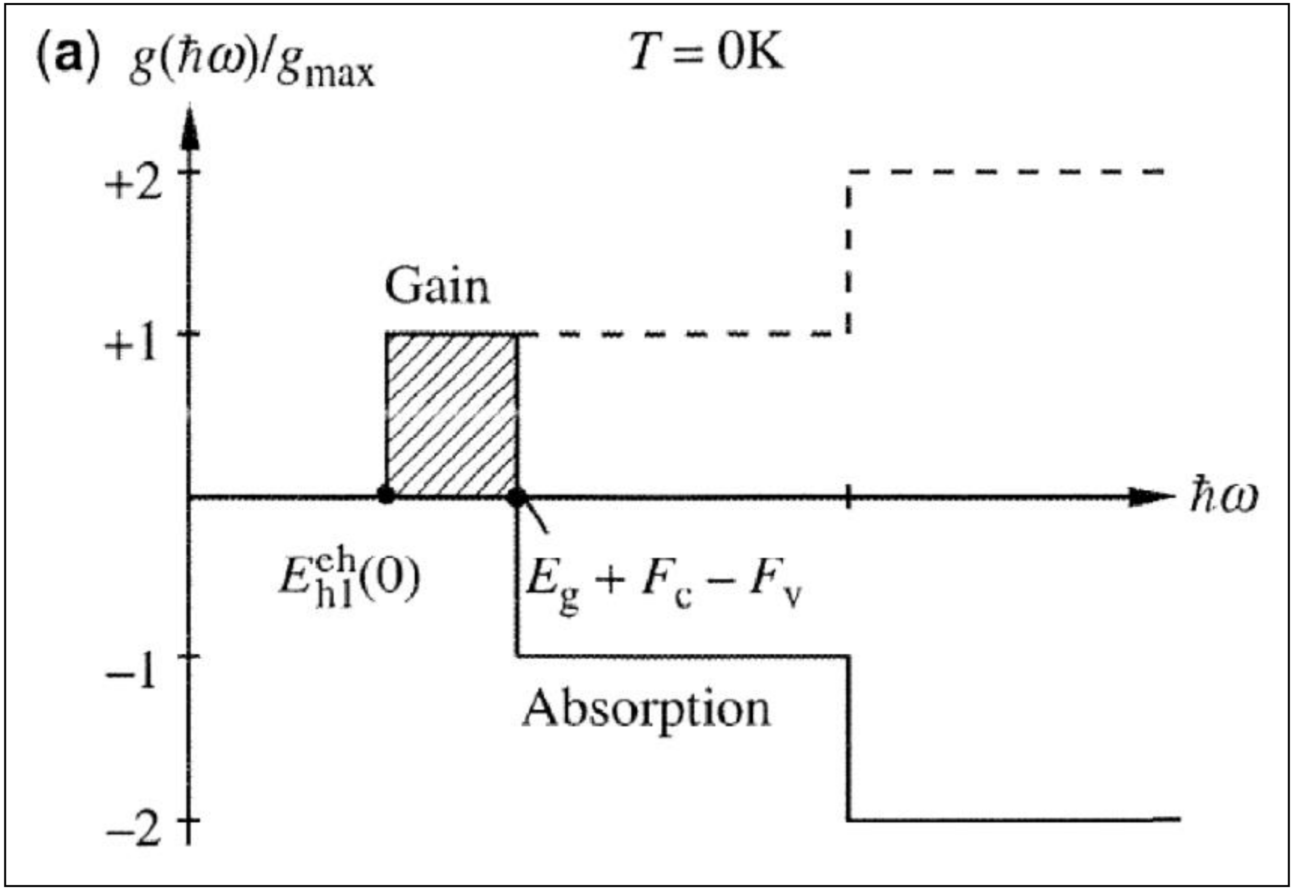
$$\text{if } \begin{cases} n \approx p \\ m_h^* > m_e^* \end{cases} \Rightarrow \left(F_c - E_{e1}\right) > \left(E_{h1} - F_v\right)$$

(quasi-Fermi level is deeper into conduction band than in valence band)

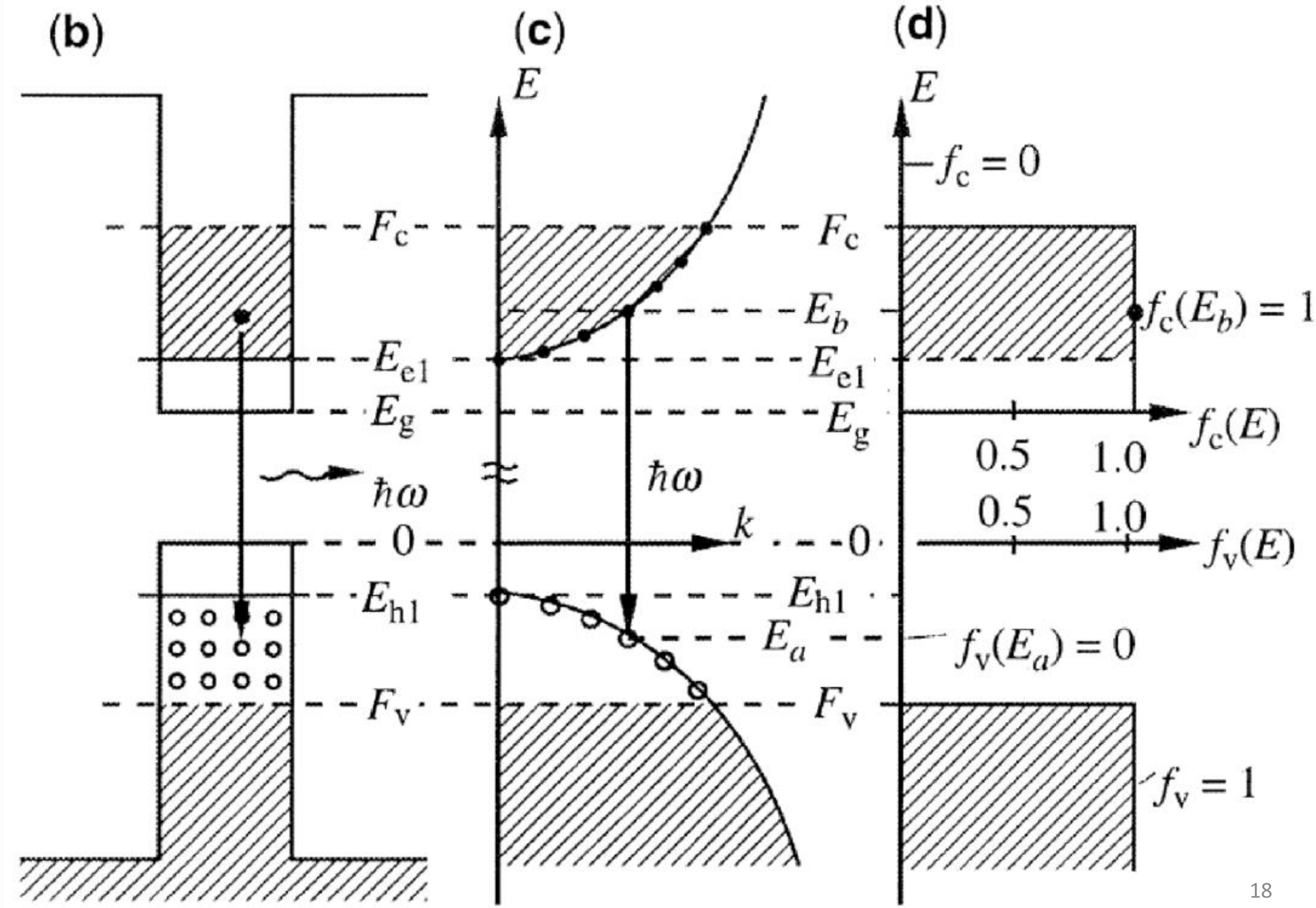
Gain Spectrum at $T = 0K$

Gain

$$g(\hbar\omega) = \begin{cases} g_{max} & E_{h1}^{e1} < \hbar\omega < E_g + F_c - F_v \\ -g_{max} \sum_{n=m} H\left(\hbar\omega - E_{hm}^{en}\right) & \text{otherwise} \end{cases}$$



State occupation at $T = 0K$



Gain Spectrum at Finite Temperature

Define

$$n_c = \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \qquad n_v = \frac{m_v^* k_B T}{\pi \hbar^2 L_z}$$

In general the electron concentration is

$$n = \sum_{n=1}^{\infty} \int_0^{\infty} \rho_e^{2D}(E)\ f_c^n(E)\, dE = \sum_{n=1}^{\infty} n_c \ln\left[1 + e^{(F_c - E_{en})/k_B T}\right]$$

Similarly, the hole concentration is

$$p = \sum_{n=1}^{\infty} n_v \ln\left[1 + e^{(E_{hm} - F_v)/k_B T}\right]$$

(Sum over both heavy-hole and light-hole bands)

Gain Spectrum at Finite Temperature

Assuming a single subband

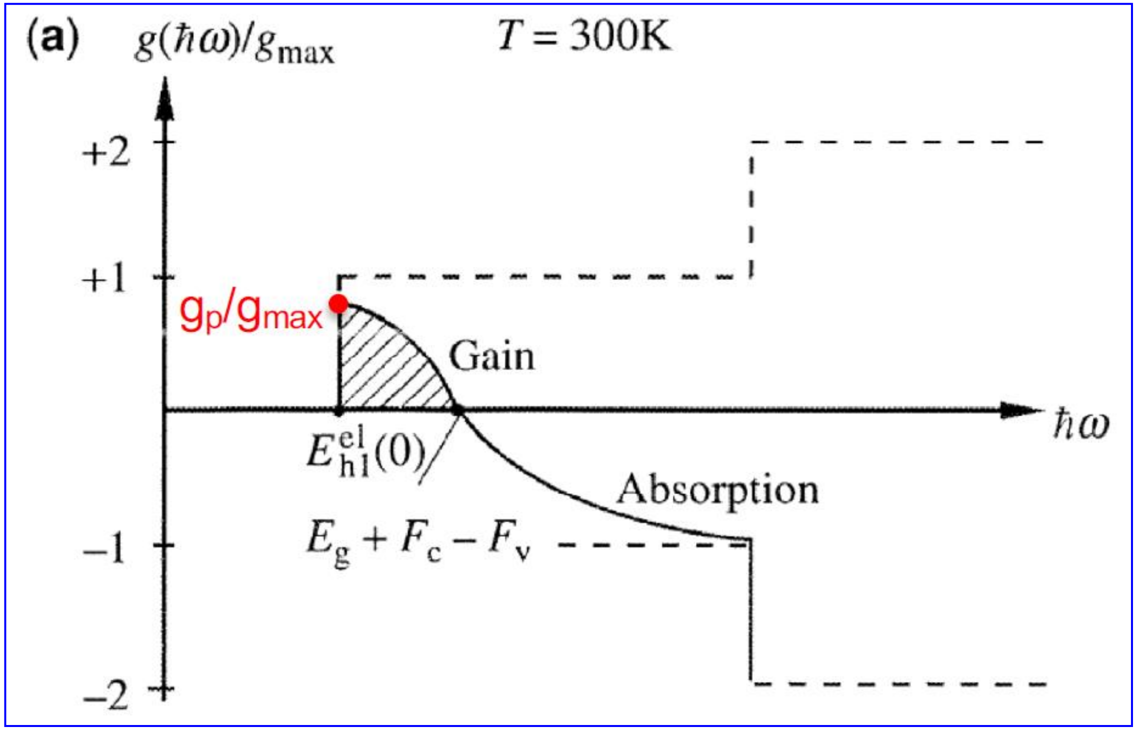
$$g(\hbar\omega) = g_{\max}\left[f_c\left(\hbar\omega - E_{h1}^{el}(0)\right) - f_v\left(\hbar\omega - E_{h1}^{el}(0)\right)\right]$$

Peak Gain

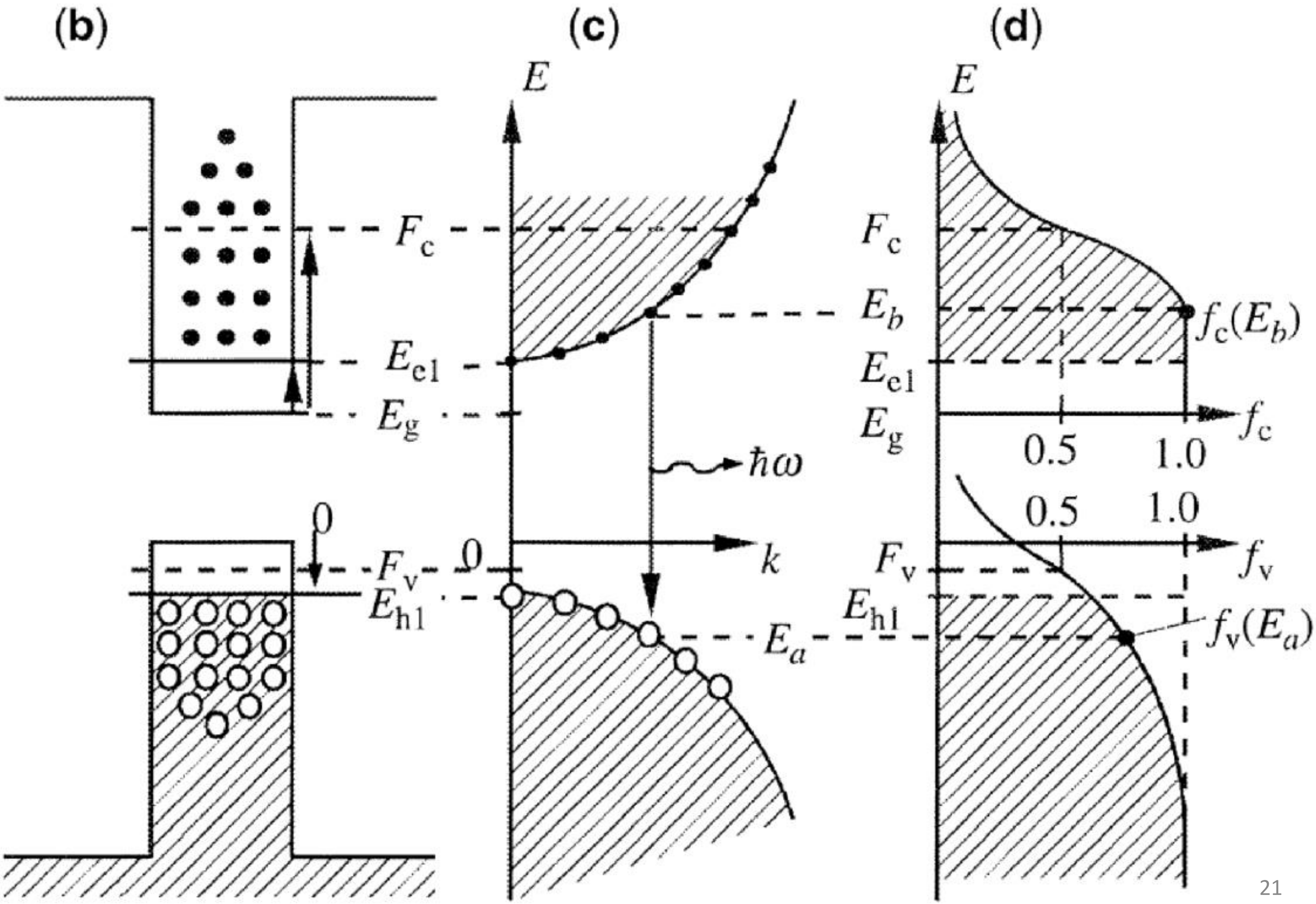
$$g_p = g_{\max}\left[f_c\left(\hbar\omega = E_{h1}^{el}\right) - f_v\left(\hbar\omega = E_{h1}^{el}\right)\right]$$

with

$$f_c\left(\hbar\omega = E_{h1}^{el}\right) = \frac{1}{1+e^{\left(E_{e1}-F_c\right)/k_B T}} \qquad f_v\left(\hbar\omega = E_{h1}^{el}\right) = \frac{1}{1+e^{\left(E_{h1}-F_v\right)/k_B T}}$$



State occupation at Finite Temperature



Peak Gain versus Carrier Density

Redefine as:

$$n_c \simeq \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} e^{(E_{e1}-E_{en})/k_B T}$$

$$n_v \simeq \frac{m_v^* k_B T}{\pi \hbar^2 L_z} \sum_{m=1}^{\infty} e^{(E_{hm}-E_{h1})/k_B T}$$

The occupation probabilities can be expressed with the approximate inverted forms

$$f_c\left(\hbar\omega = E_{h1}^{e1}\right) \approx 1 - e^{-n/n_c}$$

$$f_v\left(\hbar\omega = E_{h1}^{e1}\right) \approx e^{-p/n_v}$$

Vahala and Zah, *Appl. Phys. Lett.* vol. 52, p. 1945, 1988.

Expressions are essentially exact for single subband occupation in conduction and valence bands

and we can write the peak gain as a function of n

$$g_p = g_{\max}\left(f_c - f_v\right) = g_{\max}\left[1 - e^{-n/n_c} - e^{-p/n_v}\right]$$

Peak Gain versus Carrier Density

Defining the ratio of effective masses $\quad \mathrm{R} \equiv \frac{m_h^*}{m_e^*} \approx \frac{n_v}{n_c}$

$$g_p = g_{\max}\left(f_c - f_v\right) = g_{\max}\left[1 - e^{-n/n_c} - e^{-p/n_v}\right]$$

$$g_p = g_{\max}\left[1 - e^{-n/n_c} - e^{-p/(Rn_c)}\right]$$

only function of n_c

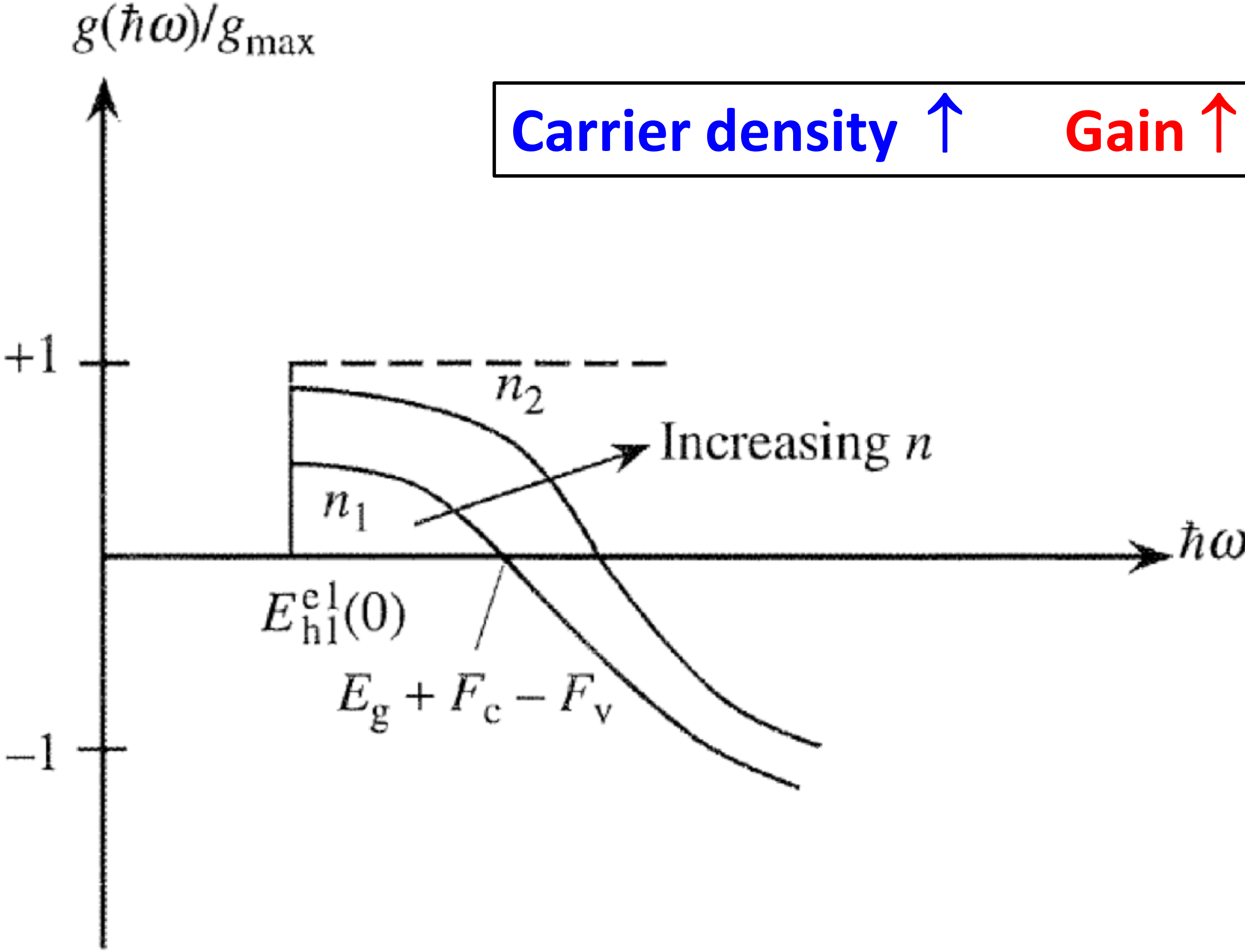
Transparency condition occurs for

$$e^{-n/n_c} + e^{-p/(Rn_c)} = 1$$

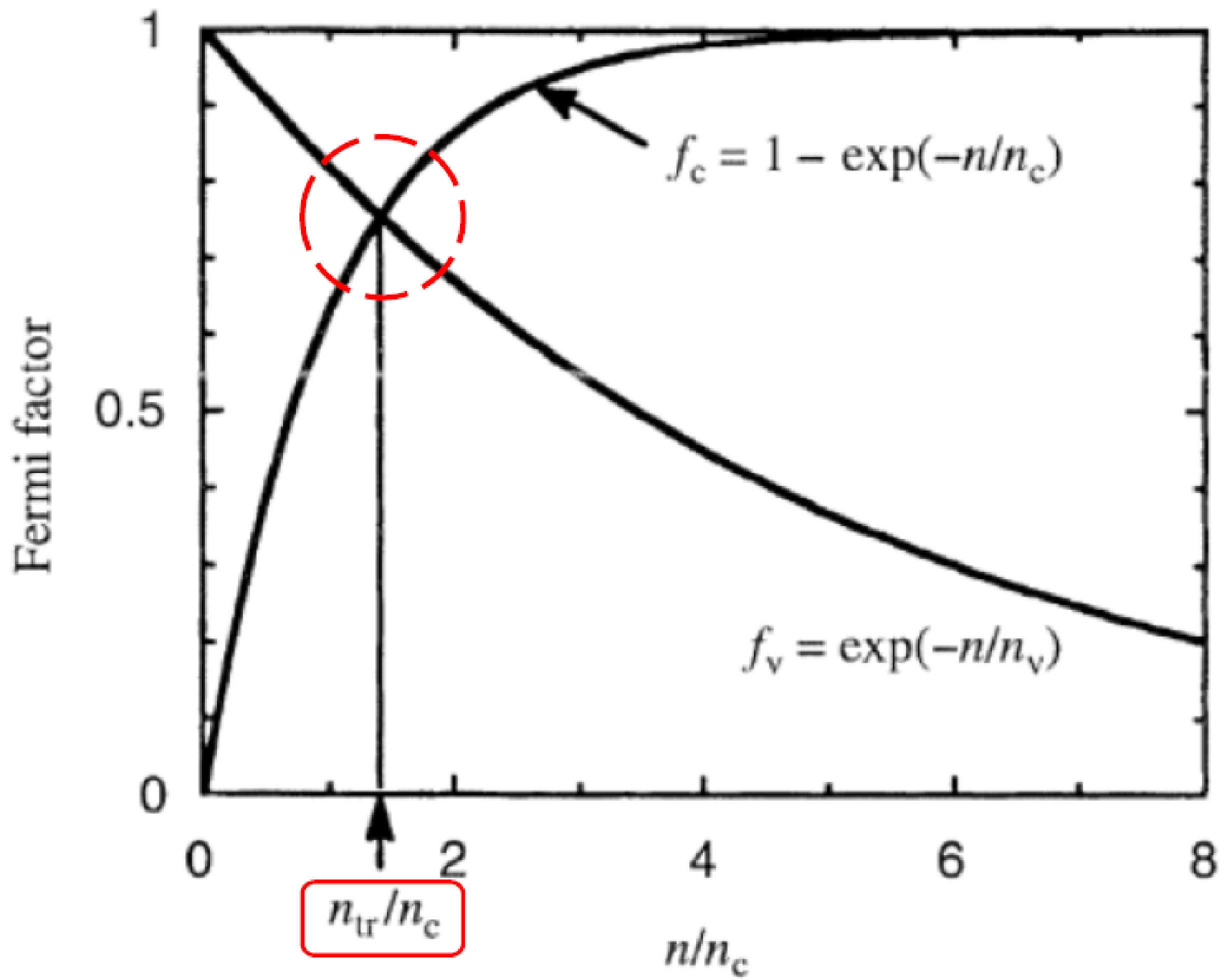
In the case $m_e^* = m_h^*$ **then** $R = 1 \rightarrow n_{tr} = n_c \ln 2$

Peak Gain versus Carrier Density

Carrier density ↑ Gain ↑



Fermi Levels versus Carrier Density

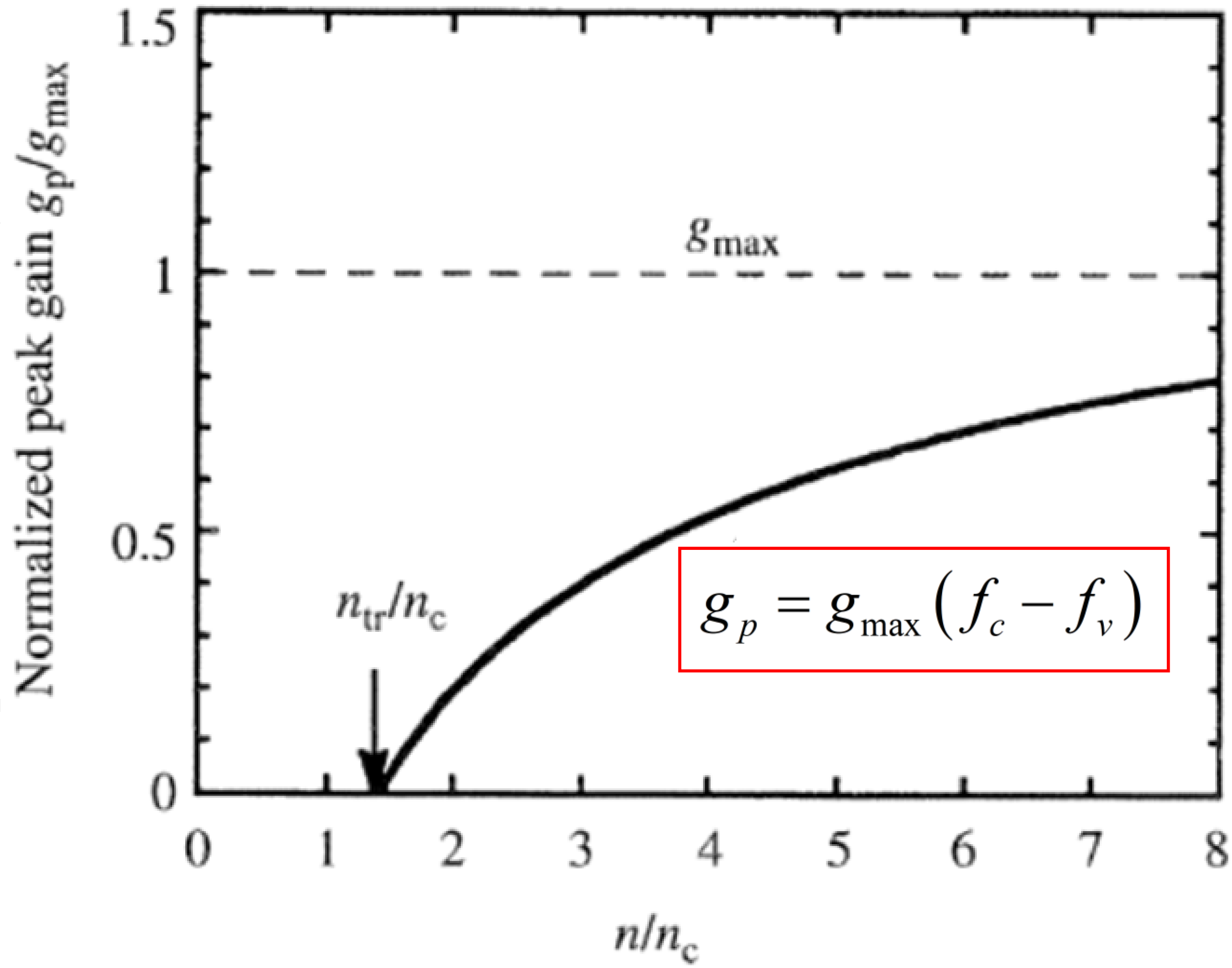


$$g_p = g_{max}\left(f_c - f_v\right)$$

$$\text{if} \quad f_c = f_v \Rightarrow g_p = 0$$

transparency condition

Peak Gain versus Carrier Density



$$g_p = g_{max}\left(f_c - f_v\right)$$

$$\text{if} \quad f_c = f_v \Rightarrow g_p = 0$$

transparency condition

Differential Gain

From the analytical expression we can derive also the differential gain

$$g_p = g_{\max}\left[1 - e^{-n/n_c} - e^{-p/(Rn_c)}\right]$$

$$\frac{\partial}{\partial n} g_p(n) = \frac{g_{\max}}{n_c}\left(e^{-n/n_c} + \frac{1}{R} e^{-n/Rn_c}\right)$$

Empirically, the peak gain curve is often fitted with a logarithmic function as

$$g_p(n) \simeq g_0\left(1 + \ln\frac{n}{n_0}\right)$$

$$g_p(n = n_0) = g_0 \qquad g_p = 0 \quad \text{at} \quad n = n_0 e^{-1} = n_{tr}$$

Peak Gain versus Current Density

For a given carrier concentration n

$$J = J_{rad} + J_{Aug} + J_{leak}$$

$$J_{rad} = qL_z \underbrace{R_{sp}(n)}_{Bn^2} \qquad J_{Aug} = qL_z \underbrace{R_{Aug}(n)}_{Cn^3}$$

Commonly used empirical formula

$$g_p(J) = g_0\left[1 + \ln\frac{J}{J_0}\right] = g_0 \ln\frac{J}{J_{tr}}$$

transparency at $J = J_{tr} = J_0 e^{-1}$

Scaling Laws for Multiple Quantum Well (MQW) Lasers

Let's define

injection efficiency or fraction of applied current captured by QW

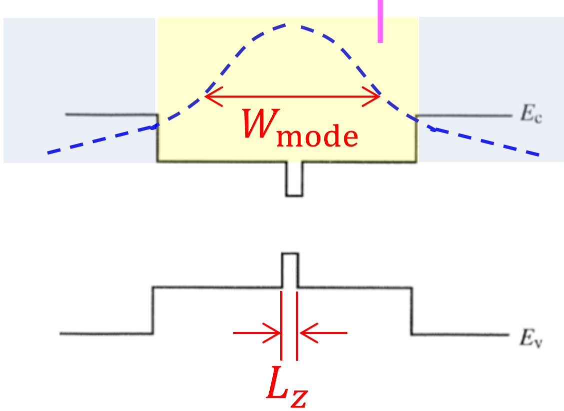
$$J_w = \eta J_{applied} = \text{ injected current density for SQW}$$

$$g_w = g_0\left[\ln\left(\frac{J_w}{J_0}\right)+1\right] = \text{peak gain coefficient for SQW}$$

where usually $g_w \propto L_z^{-1}$

$$J_{tr} = J_0 e^{-1} = \text{transparency current density}$$

$$\Gamma_w = \text{ optical confinement factor per well}$$



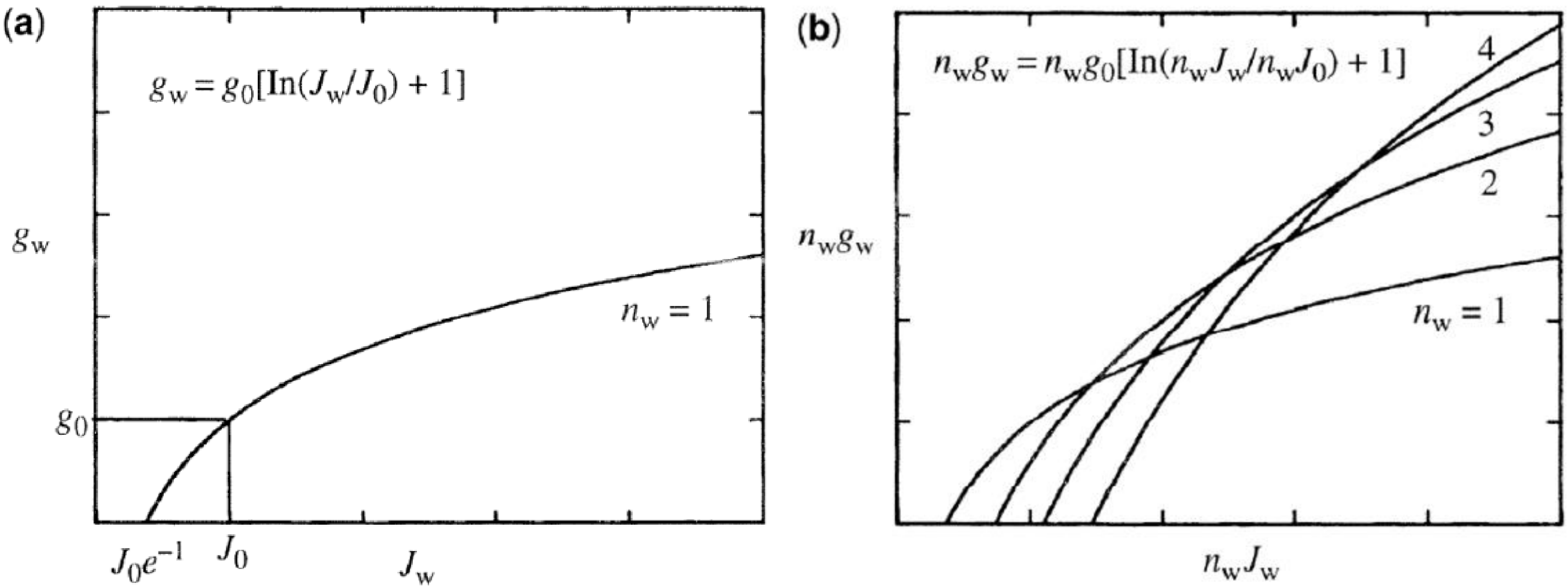
$$\Gamma_w = \Gamma_{op}\frac{L_z}{W_{\text{mode}}}$$

full-width at half maximum of optical mode

Scaling Laws for Multiple Quantum Well (MQW) Lasers

Modal gain for a MSQW at threshold

$$G_{th} = n_w \Gamma_w g_w = \alpha_{tot} = \alpha_i + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$



Scaling Laws for Multiple Quantum Well (MQW) Lasers

Modal gain for a MSQW at threshold

$$G_{th} = n_w \Gamma_w g_w = \alpha_{tot} = \alpha_i + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2} \right)$$

General Expression of Modal gain for a SQW

$$G = \Gamma_w g_w = \Gamma_w g_{\max} \left[f_c(\hbar\omega) - f_v(\hbar\omega) \right] = \frac{\Gamma_{op} L_z}{W_{\text{mode}}} g_{\max}$$

$$\hbar\omega = E_{h1}^{e1}(0)$$

$$g_{\max} = C_0 \left| \hat{e} \bullet \mathbf{M}_{\mathbf{ch}} \right|^2 \frac{m_r^*}{\pi \hbar^2 L_z} \delta_{nm}$$

Modal gain for a MQW

$$G = n_w \Gamma_w g_w$$

Threshold Current Density

- **Injected Current Density per QW at Threshold**

$$J_{w,th} = \frac{\eta J_{th}}{n_w}$$

- **Peak gain**

SQW)

$$g_w = g_0 \left[\ln\left(\frac{J_w}{J_0} \right) + 1 \right]$$

MQW)

$$n_w g_w = n_w g_0 \left[\ln\left(\frac{J_w}{J_0} \right) + 1 \right] = n_w g_0 \left[\ln\left(\frac{n_w J_w}{n_w J_0} \right) + 1 \right]$$

$n_w J_w =$ total injected current density

$n_w J_0 e^{-1} =$ total injected current density for transparency

Threshold Current Density for MQW

$$n_w g_w = n_w g_0 \left[\ln\left(\frac{J_w}{J_0} \right) + 1 \right] = n_w g_0 \left[\ln\left(\frac{n_w J_w}{n_w J_0} \right) + 1 \right]$$

$$\frac{n_w J_w}{n_w J_0} = e^{\left(\frac{n_w g_w}{n_w g_0} \right) - 1} \quad \Rightarrow \quad \eta J_{th} = n_w J_w = n_w J_0 \exp\left[\left(\frac{g_w}{g_0} \right) - 1 \right]$$

$$J_{th} = \frac{n_w J_0}{\eta} \exp\left[\left(\frac{g_w}{g_0} \right) - 1 \right]$$

At threshold Gain = Loss

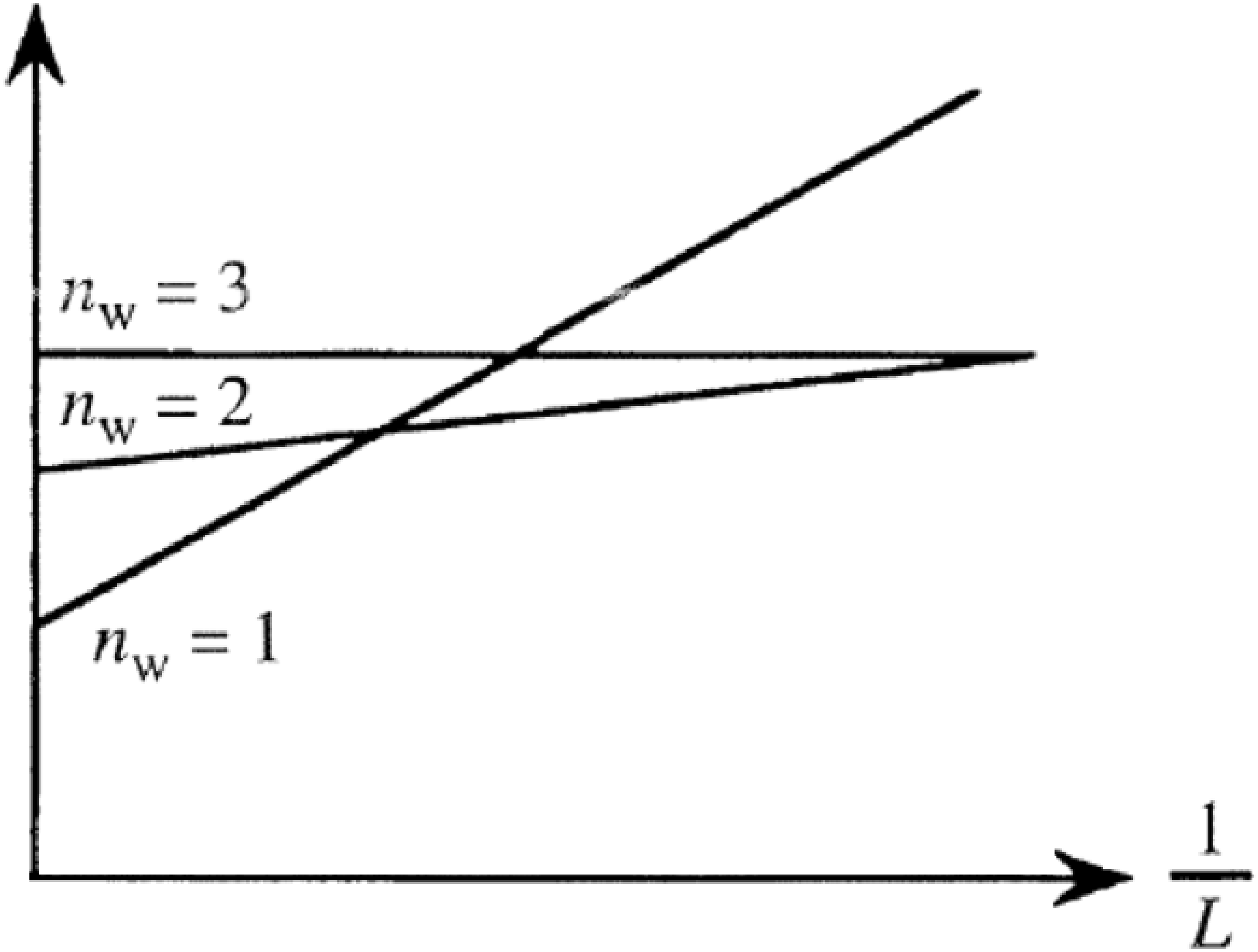
$$n_w \Gamma_w g_w = \alpha_{tot}$$

$$J_{th} = \frac{n_w J_0}{\eta} \exp\left[\left(\frac{\alpha_{tot}}{n_w \Gamma_w g_0} \right) - 1 \right]$$

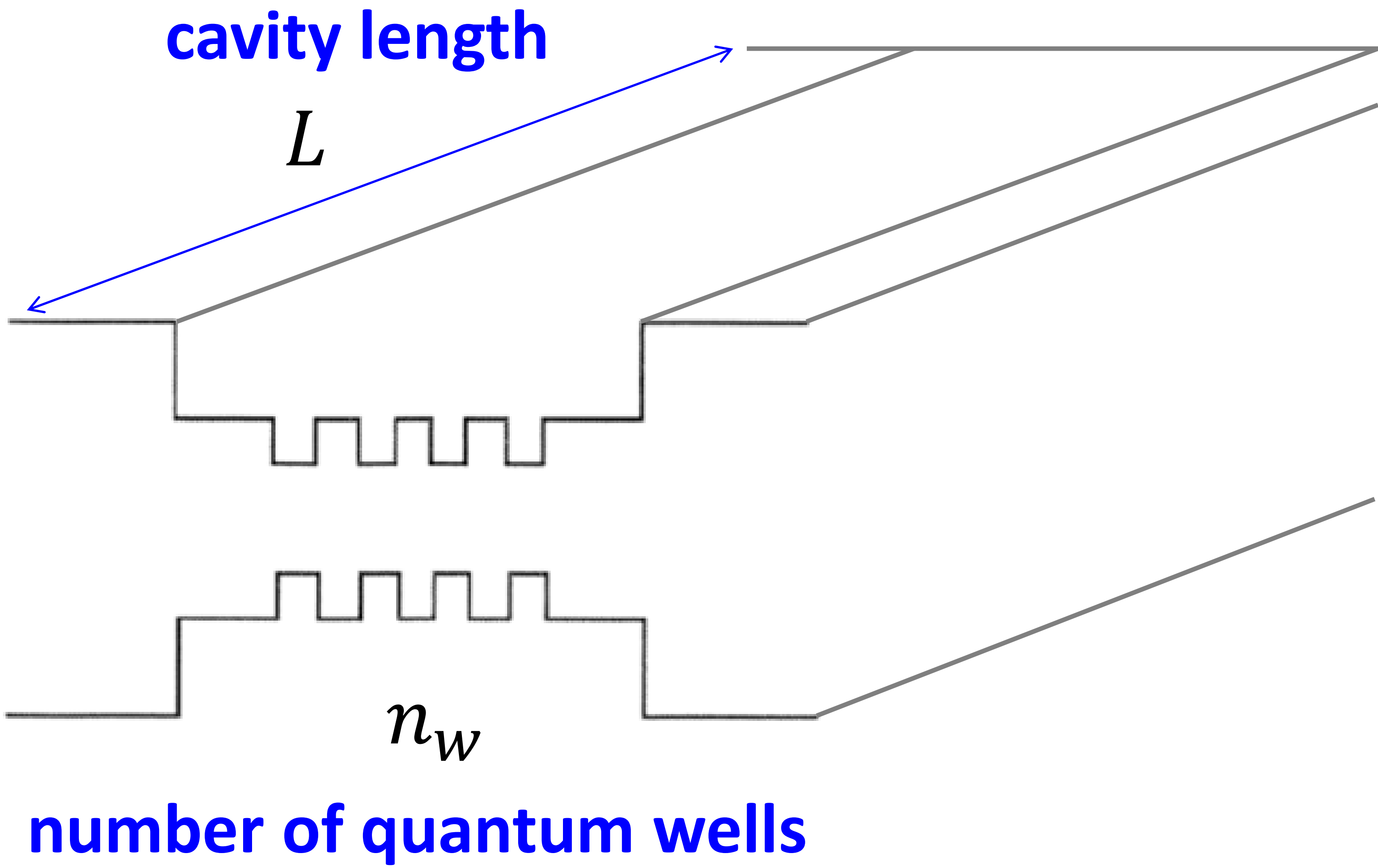
$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0} \left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) - 1$$

Threshold Current Density for MQW

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{1}{n_w \Gamma_w g_0}\left(\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) - 1$$



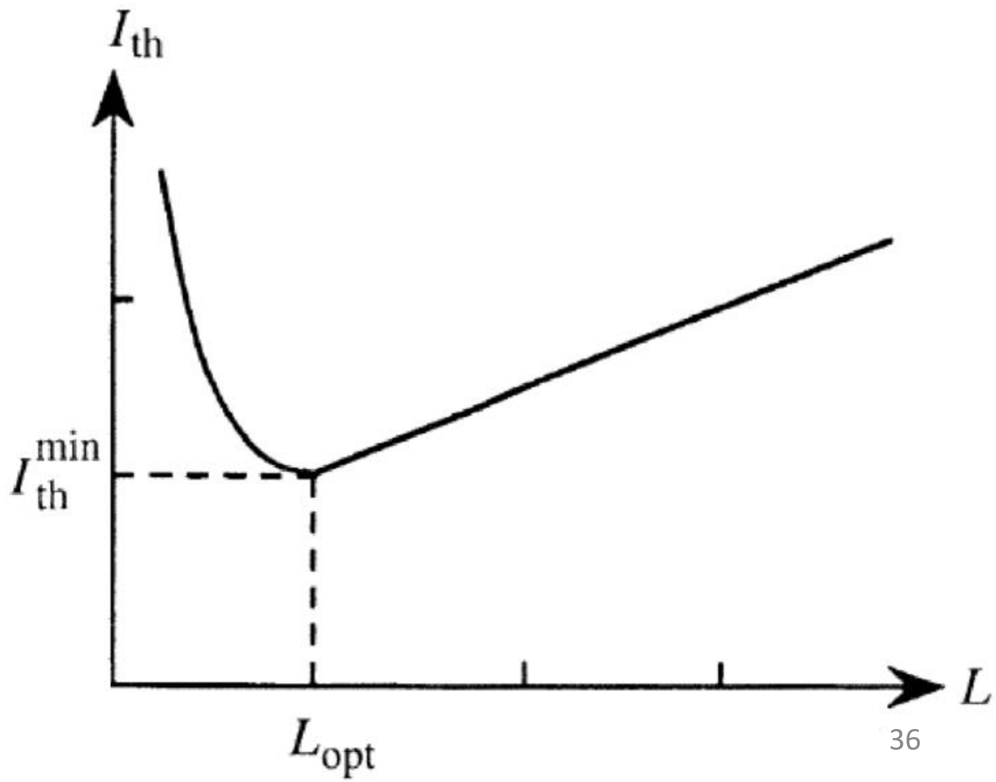
We have two main parameters for device optimization



Optimal Cavity Length to Minimize Threshold

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{1}{n_w \Gamma_w g_0}\left(\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) - 1$$
$$= \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1$$

$$L_{opt} = \frac{1}{2}\,\frac{1}{n_w \Gamma_w g_0}\,\ln\frac{1}{R_1 R_2}$$



Minimum Threshold Current for L_{opt}

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1$$

$$I_{th} = \frac{w\, L\, n_w J_0}{\eta} \exp\left[\frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1\right] = \text{const} \cdot L \exp\left(L_{opt}/L\right)$$

Minimum Threshold Current at Optimal Cavity Length

$$\frac{\partial}{\partial L} I_{th} = I_{th}\left(\frac{1}{L} - \frac{L_{opt}}{L^2}\right) = 0 \quad \Rightarrow \quad \frac{1}{L} = \frac{L_{opt}}{L^2} \quad \Rightarrow \quad L = L_{opt}$$

$$I_{th}^{\min} = \frac{w\, L_{opt}\, n_w J_0}{\eta} \exp\left[\frac{\alpha_i}{n_w \Gamma_w g_0}\right]$$

Optimal Number of Wells for Fixed Cavity Length

$$I_{th} = \frac{w L n_w J_0}{\eta} \exp\left[\left(\frac{\alpha_{tot}}{n_w \Gamma_w g_0}\right) - 1\right]$$

$$n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0}$$

$$= \text{const} \cdot n_w \exp\left(\frac{\alpha_{tot}}{n_w \Gamma_w g_0}\right) = \text{const} \cdot n_w \exp\left(\frac{n_{opt}}{n_w}\right)$$

$$\frac{\partial}{\partial n_w} I_{th} = I_{th}\left(\frac{1}{n_w} - \frac{n_{opt}}{n_w^2}\right) = 0$$

(implicit assumption of no coupling between wells)

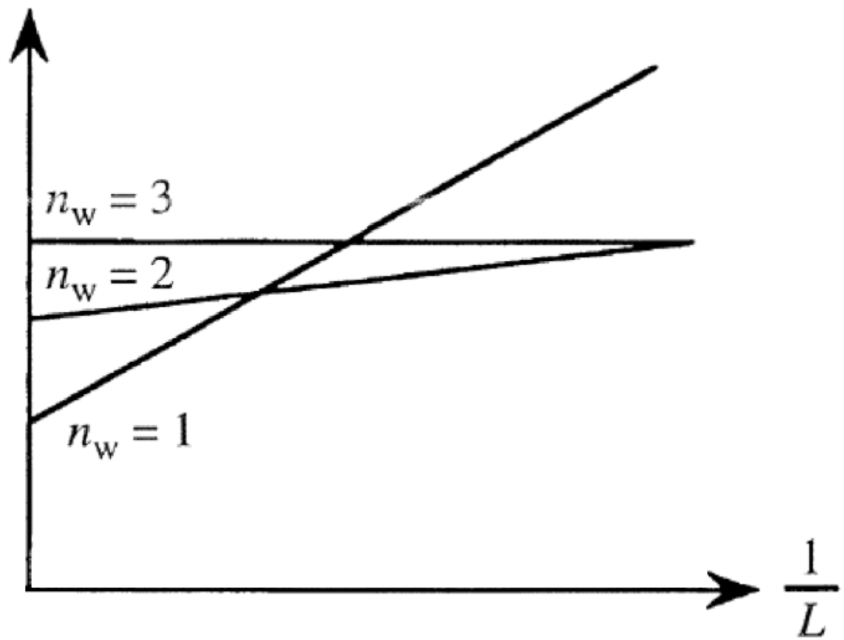
$$n_w = n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0} = \frac{1}{\Gamma_w g_0}\left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right)$$

Take the closest integer!

$$I_{th}^{\min} = \frac{w L n_{opt} J_0}{\eta}$$

Threshold Current Optimization

It is not possible to optimize simultaneously for cavity length and number of quantum wells.



If $$L = L_{opt}$$

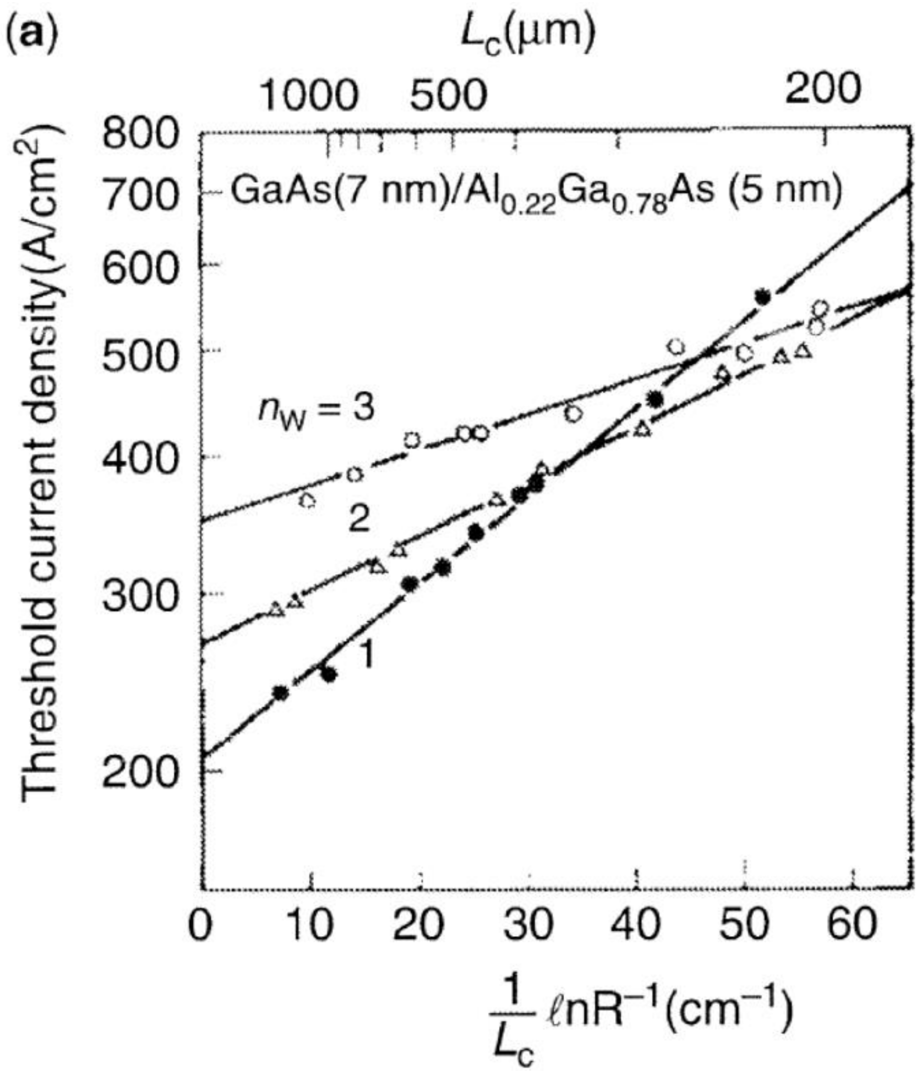
$$\Rightarrow \quad n_w = \frac{\alpha_m}{\Gamma_w g_0} \neq n_{opt}$$

using:

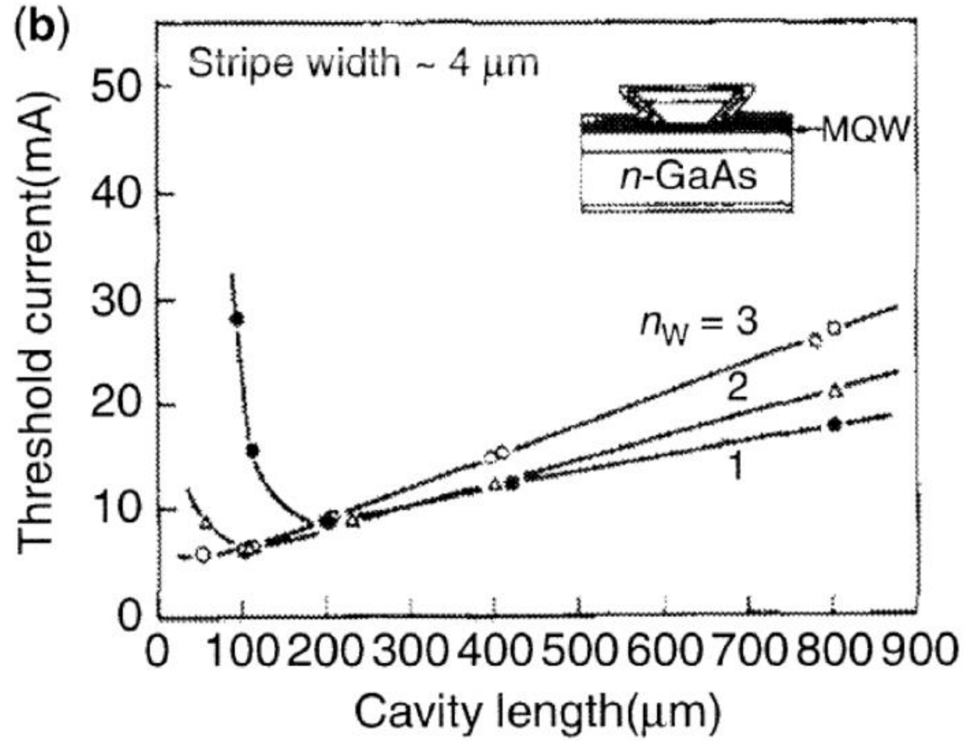
$$L_{opt} = \frac{1}{2} \frac{1}{n_w \Gamma_w g_0} \ln \frac{1}{R_1 R_2}$$

(unless $\alpha_i = 0$ which is unphysical)

Some representative results



Kurobe et al., IEEE J. Quantum Electronics, vol. 24, p. 635 (1985).

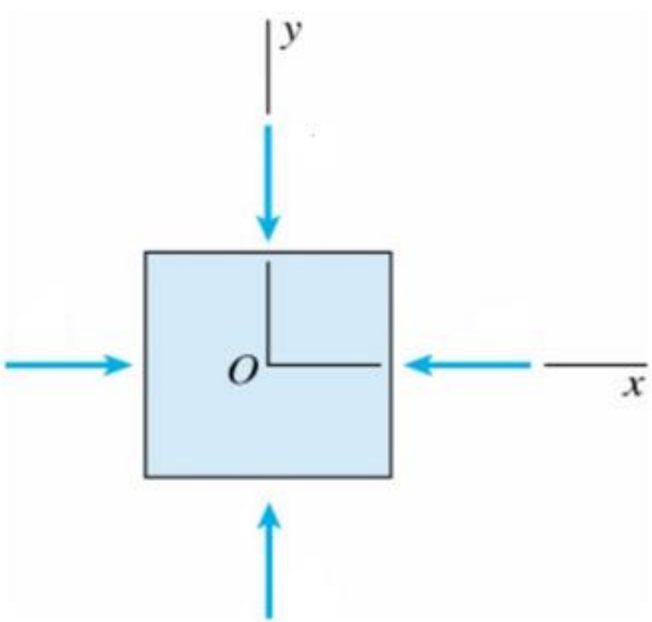


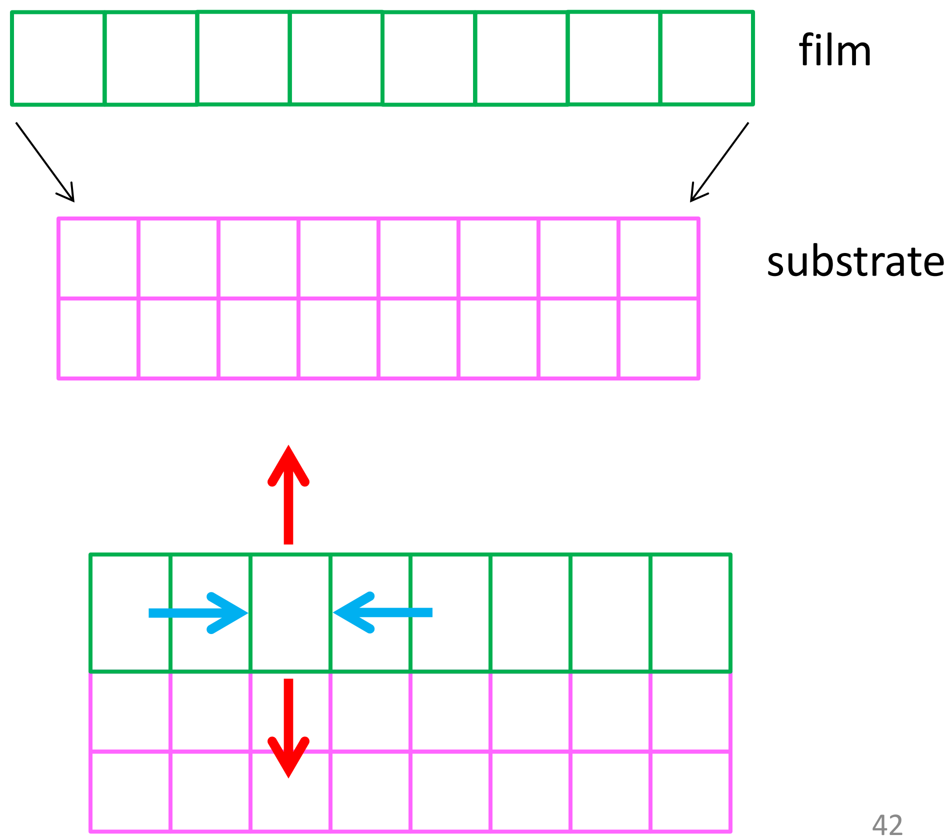
Strain Effects

Definitions

Biaxial Compression

The strained material has a larger lattice constant resulting in **compressive strain** in the **plane of the wafer** and **tension** in the direction **perpendicular to the surface.**

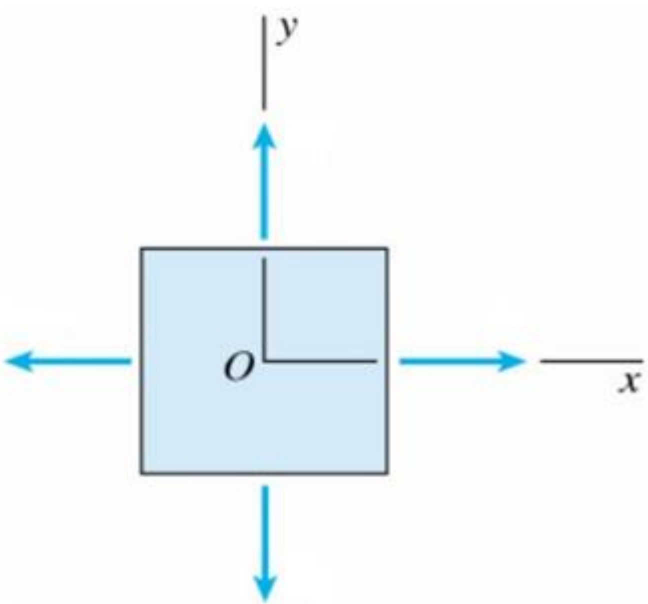


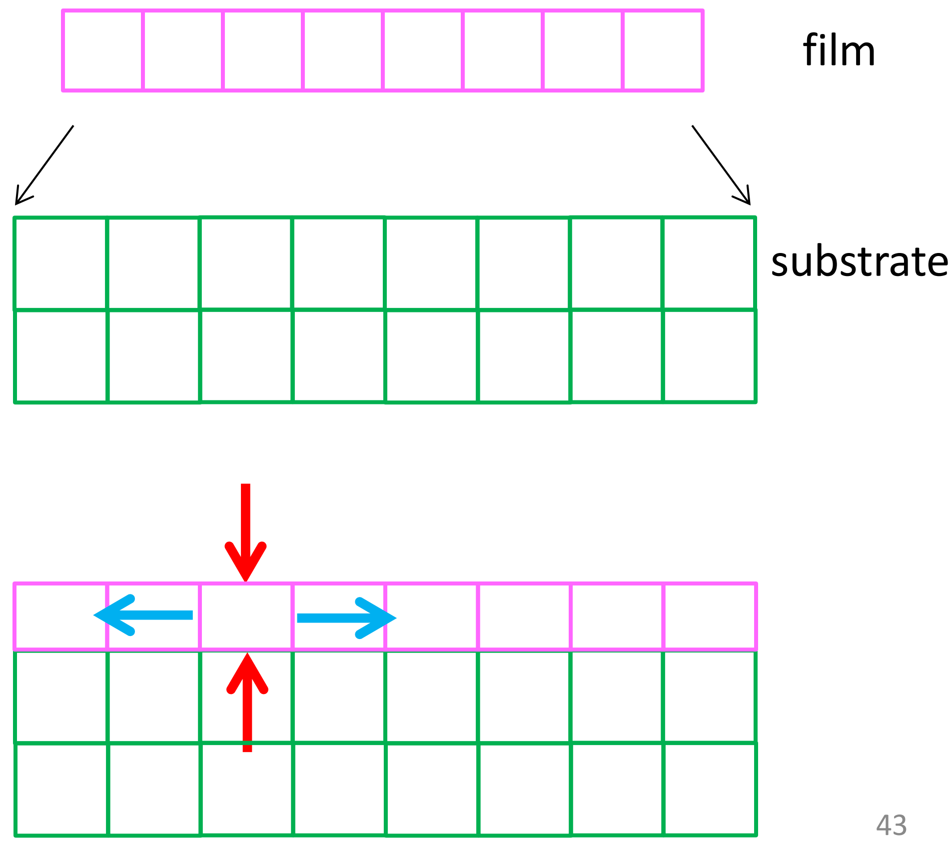


Definitions

Biaxial Tension

The strained material ha a smaller lattice constant resulting in **tensile strain** in the **plane of the wafer** and **compression** in the direction **perpendicular to the surface.**



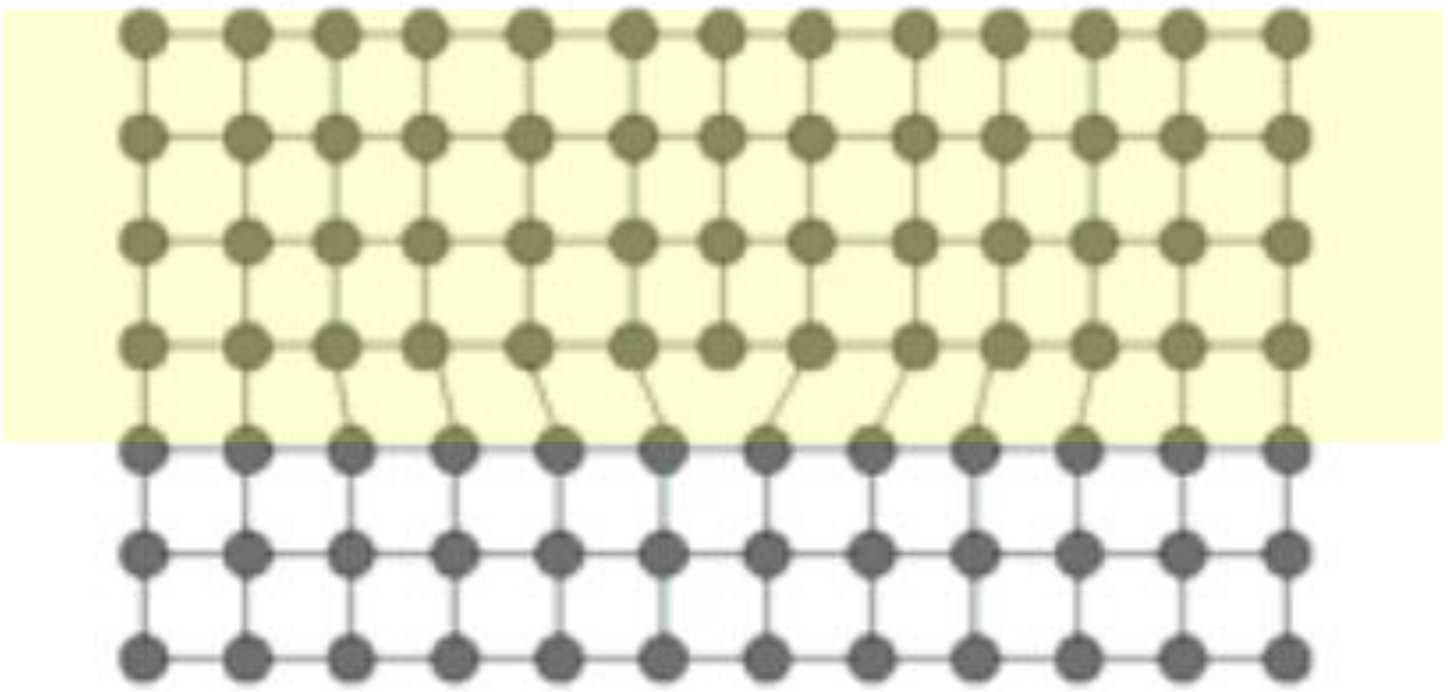


Definitions

Critical Layer Thickness

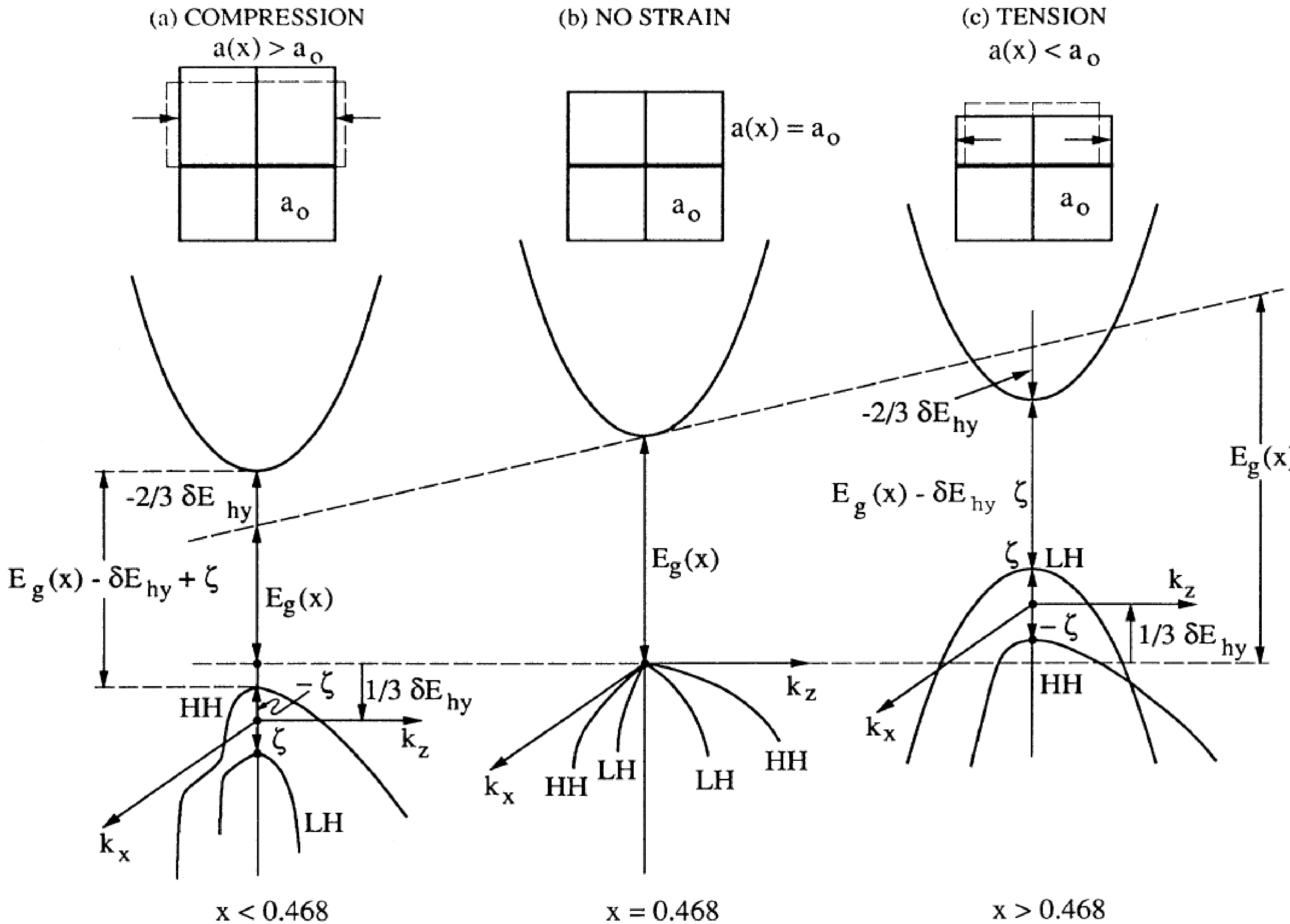
Thickness beyond which dislocations form to accommodate mismatch.

edge dislocation



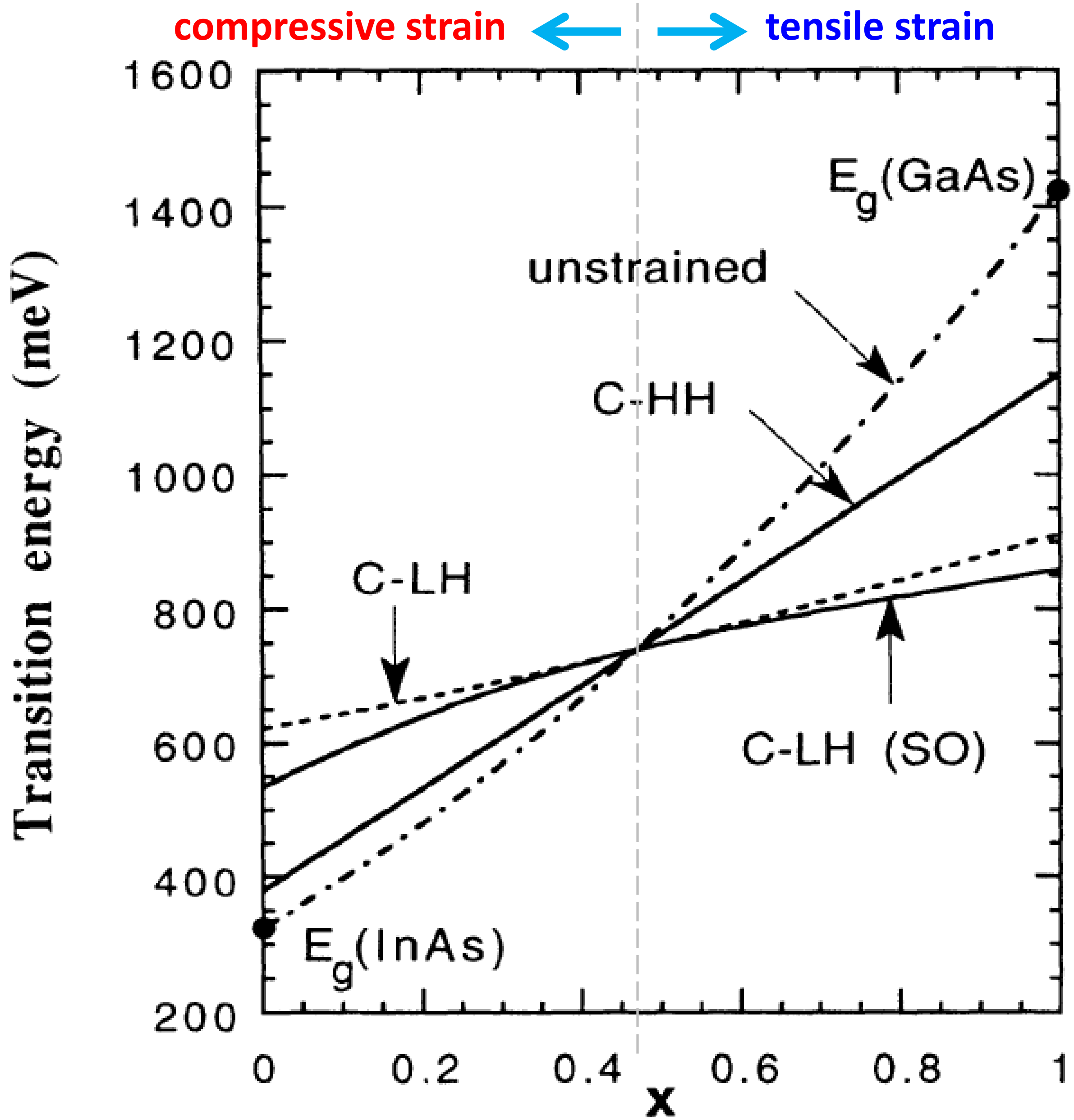
Some Key Points

- **Strain may modify significantly the band structure of the valence band.**
- **Both bandgap energy and carrier effective mass may change (e.g., heavy hole effective mass becomes lighter and light hole effective mass becomes heavier in the direction of strain.**
- **Degeneracy of hh/lh bands is broken.**
- **Strain may change the threshold current of lasers**
- **Strain may change polarization of emitted light.**
- **Reduction in threshold current density may reduce the importance of non-radiative processes such as Auger recombination.**



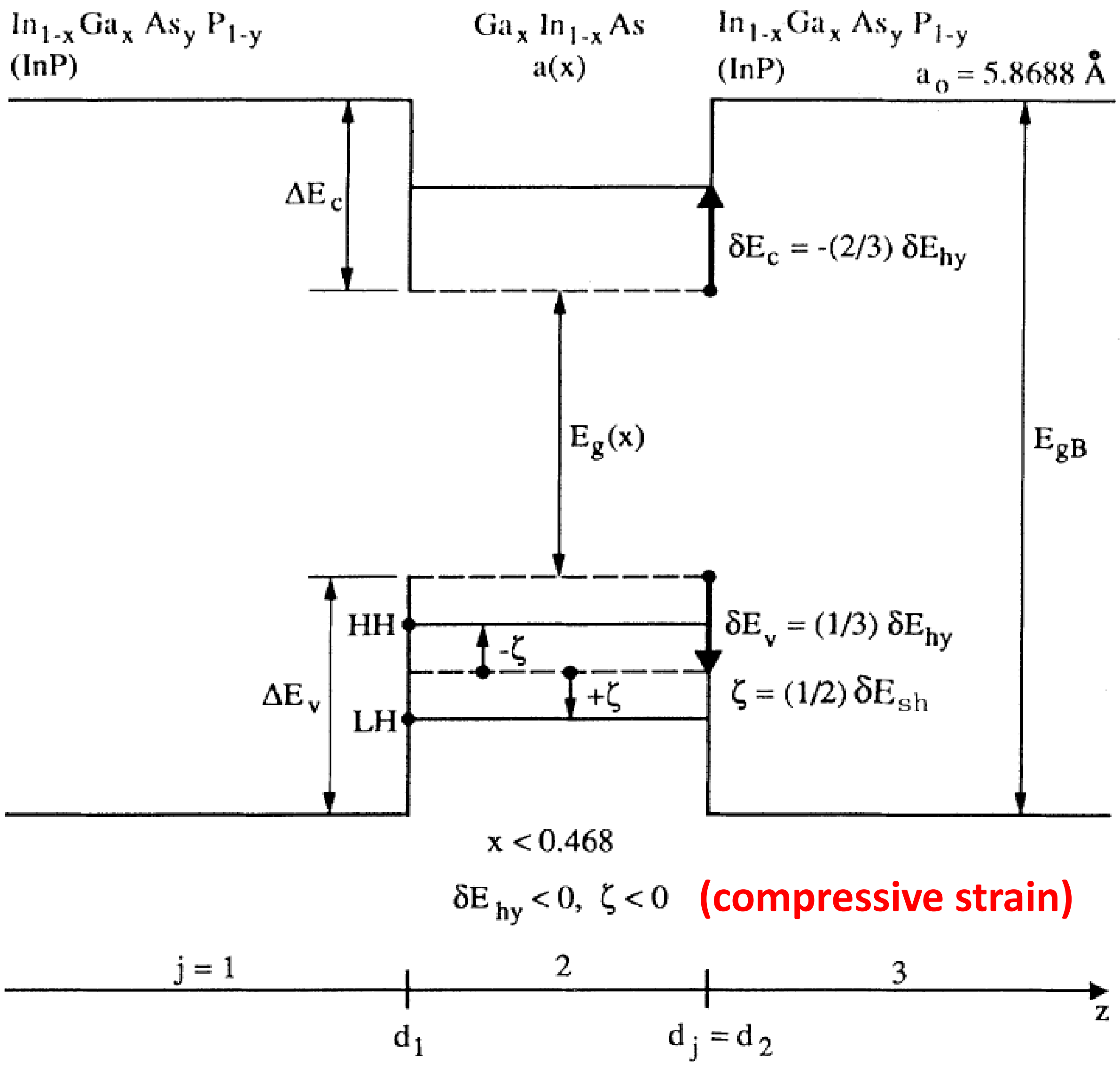
Effect of strain on the band structure of $In_{1-x}Ga_xAs$

(S.L. Chuang, Phys Rev B, vol. 43, p. 9649 (1991)



Energy band gap of $In_{1-x}Ga_xAs$ bulk and as grown pseudomorphically on InP

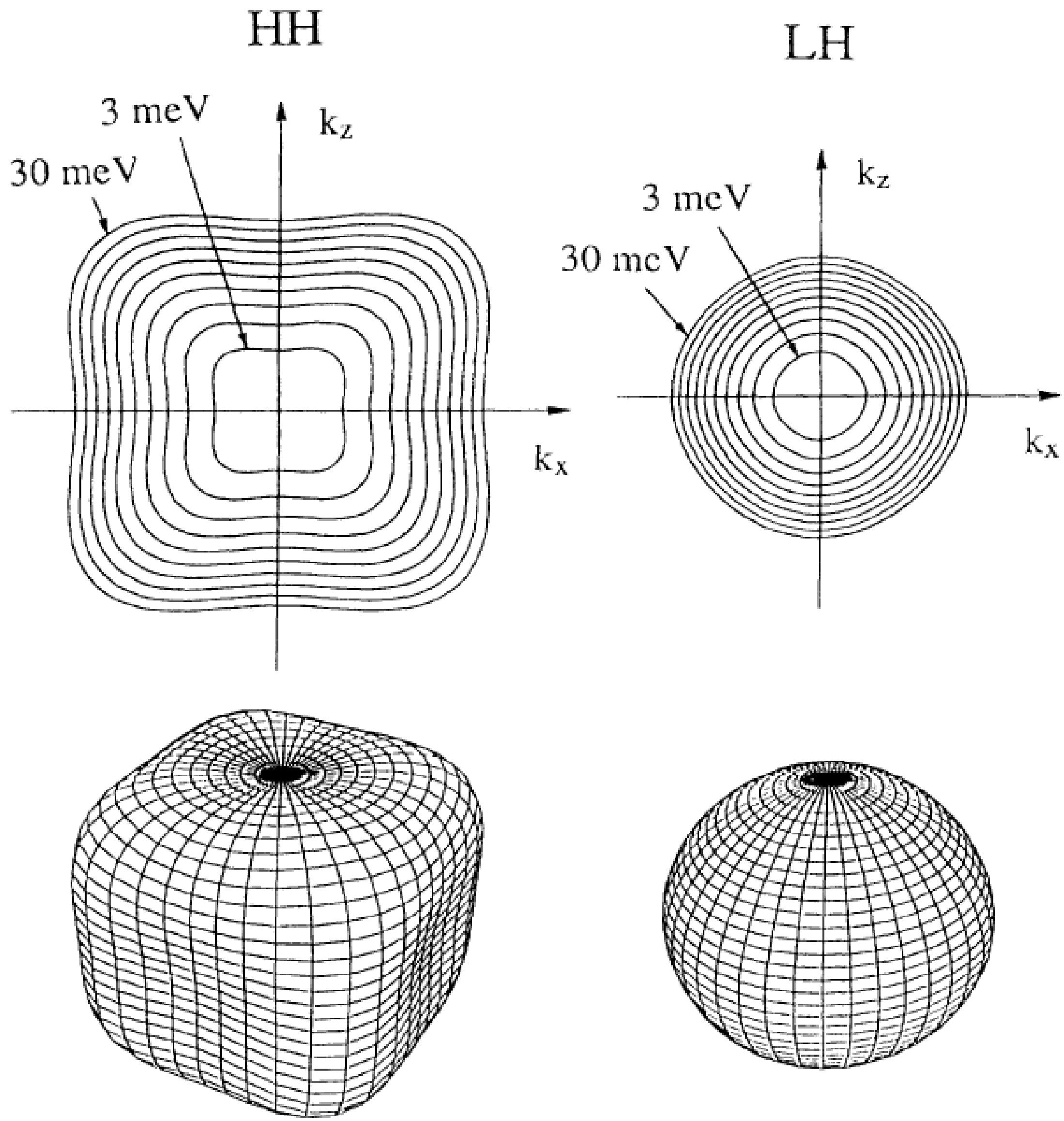
(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).



Energy band diagram of $In_{1-x}Ga_xAs$ quantum well grown on $In_{1-x}Ga_xAs_yP_{1-y}$

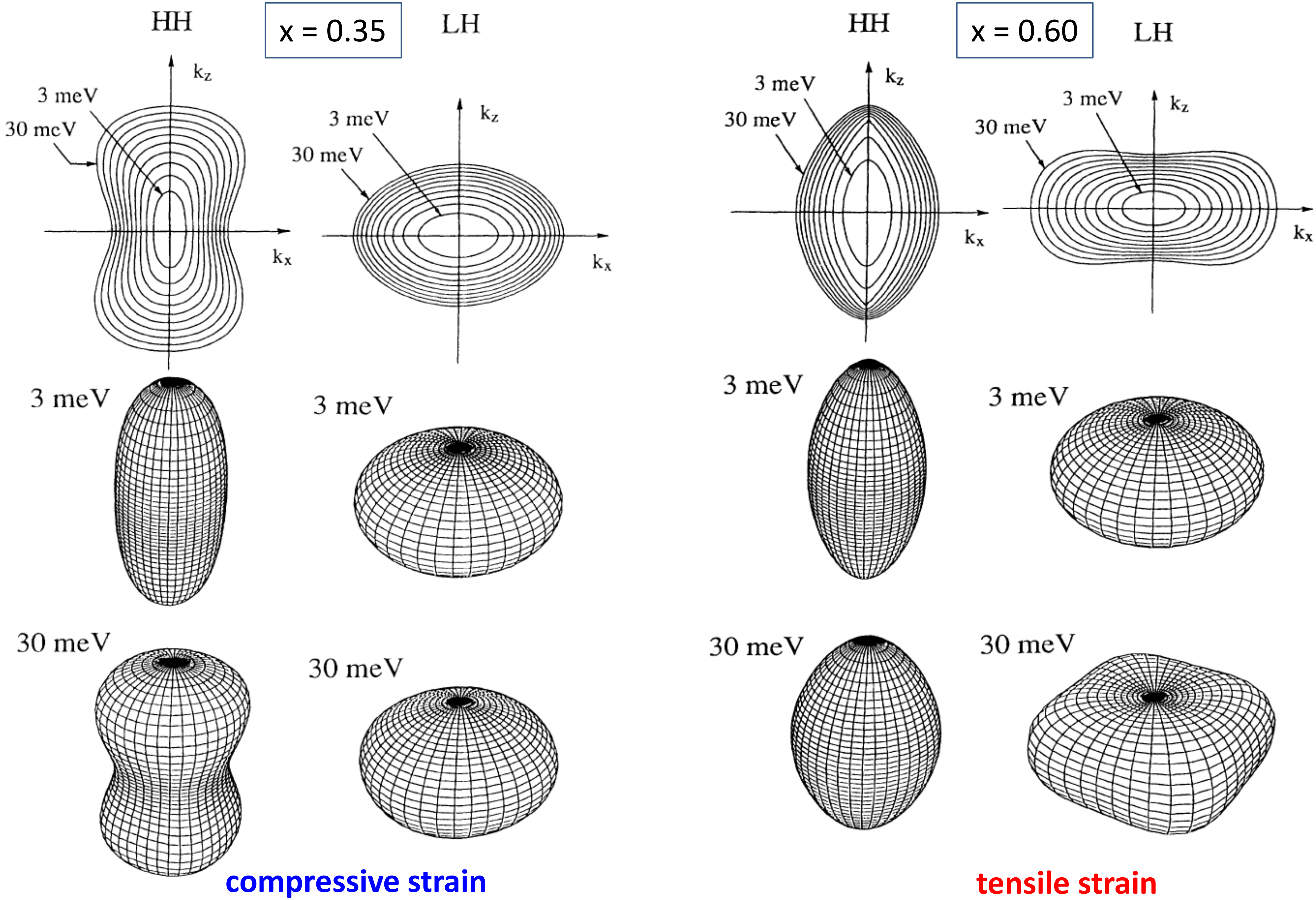
(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).

x = 0.468



Hole bands energy isosurfaces for unstrained $In_{1-x}Ga_xAs$ lattice matched to InP

(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).



Hole bands energy isosurfaces for $In_{1-x}Ga_xAs$ under strain

(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).

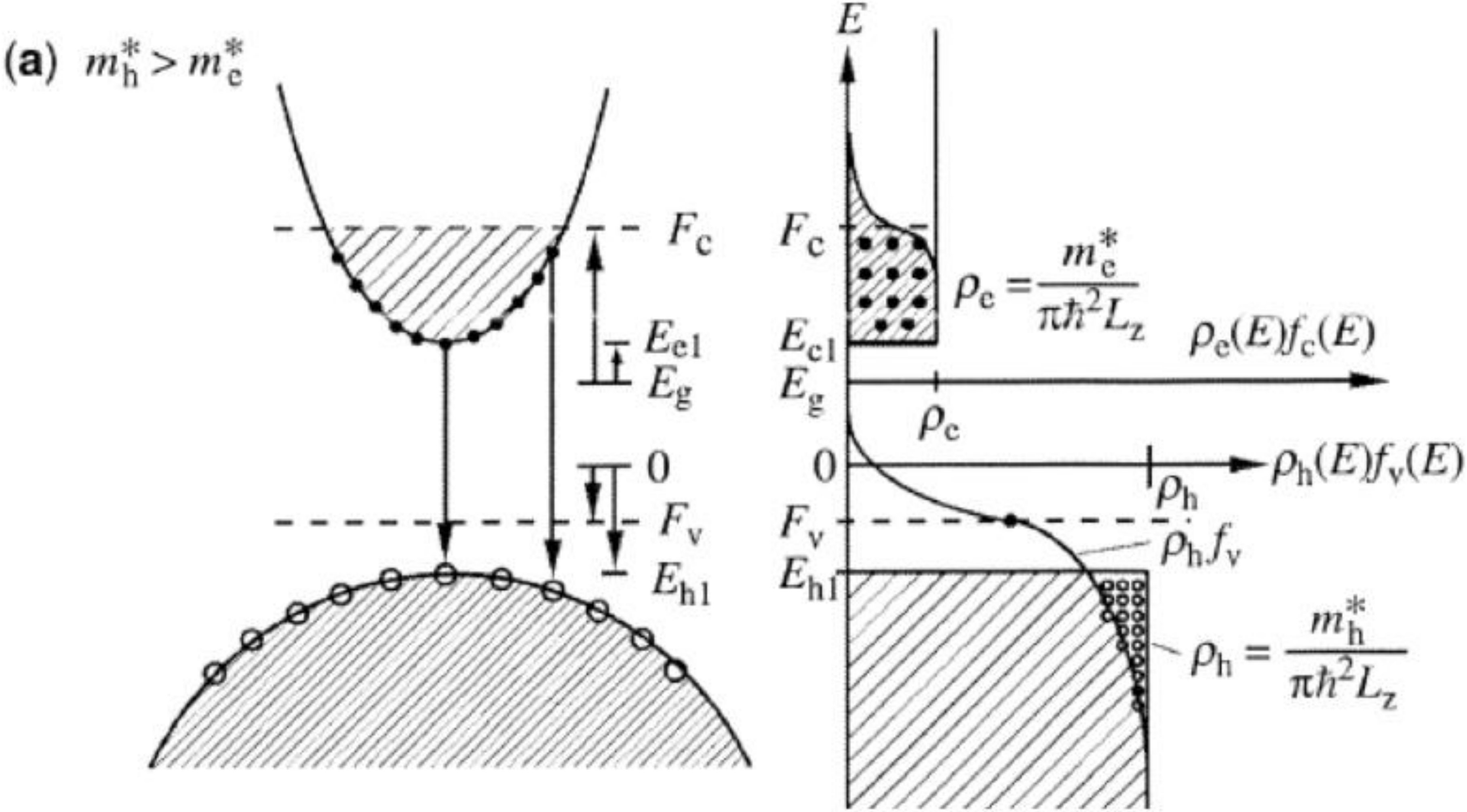
Effective mass effect on Quasi-Fermi levels

- **Reducing the effective mass of holes reduces the density of states in the valence band**

- **Quasi-Fermi levels become more symmetrical with respect to the band edges, when effective masses are similar**

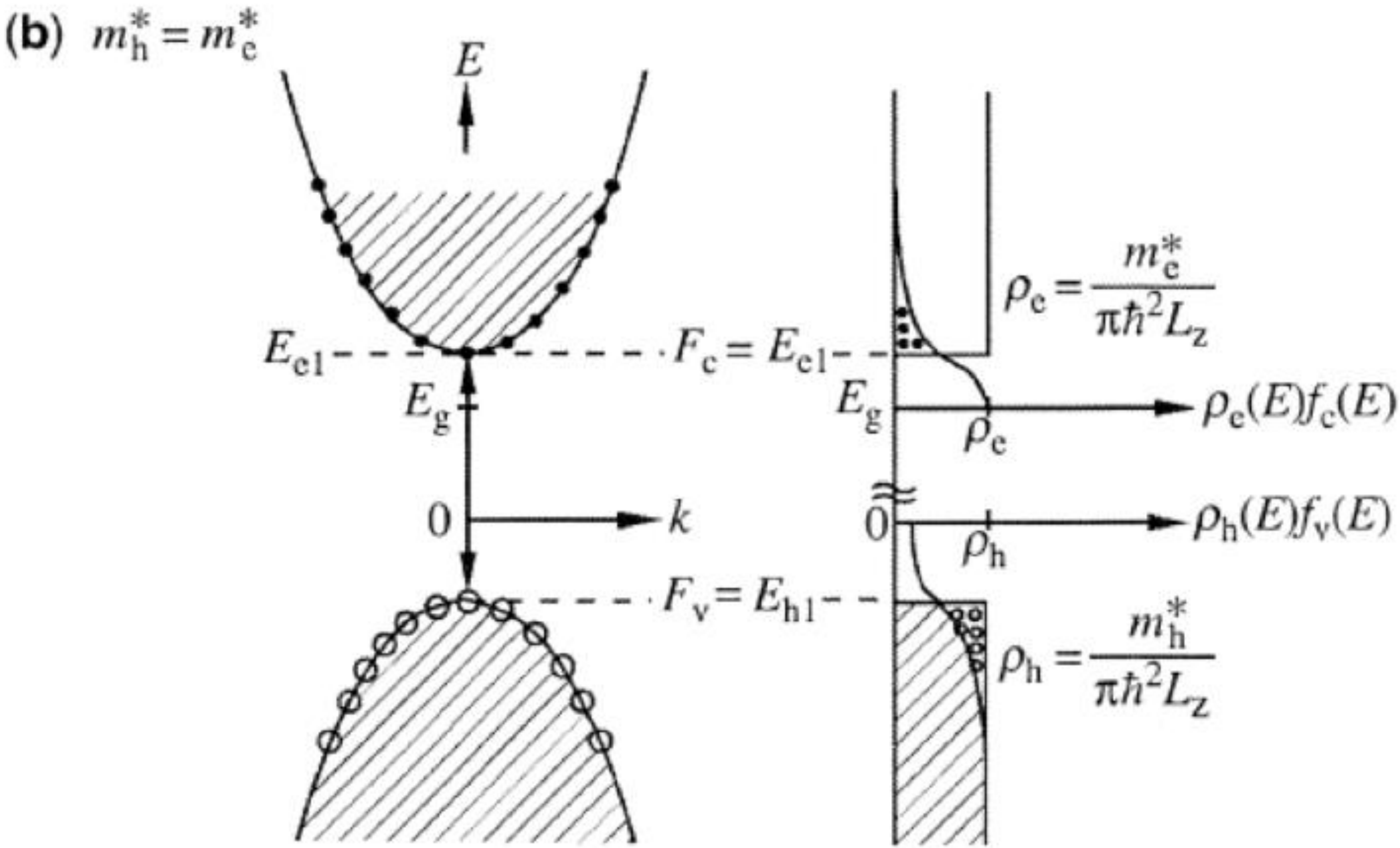
- **With increased symmetry, the quasi-Fermi level has to penetrates less into the conduction band to reach density for population inversion $\rightarrow$ Less degenerate**

- **Also, the carrier density necessary to reach the transparency condition is reduced**



population inversion condition

$$F_c - F_v + E_g > \hbar\omega > E_g + E_{e1} - E_{h1}$$



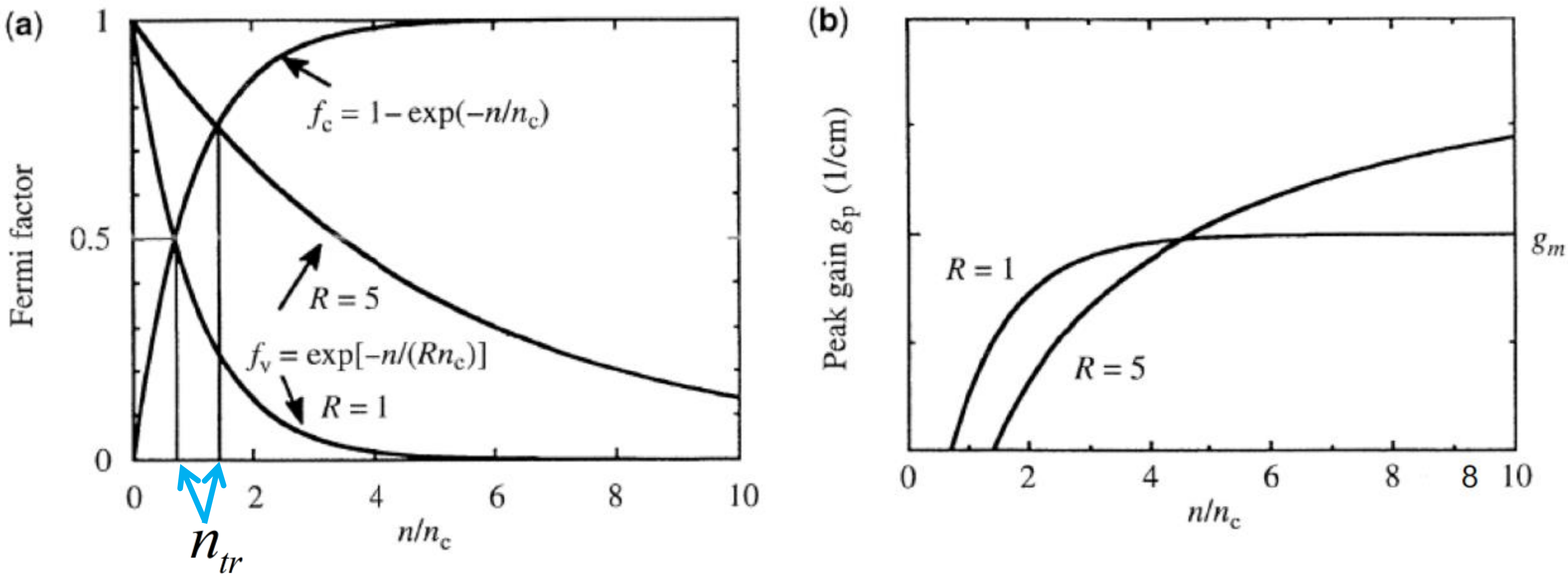
population inversion condition

$$F_c - F_v + E_g > \hbar\omega > E_g + E_{e1} - E_{h1}$$

Transparency Density and Peak Gain

- **Remember our simple model for Peak Gain**

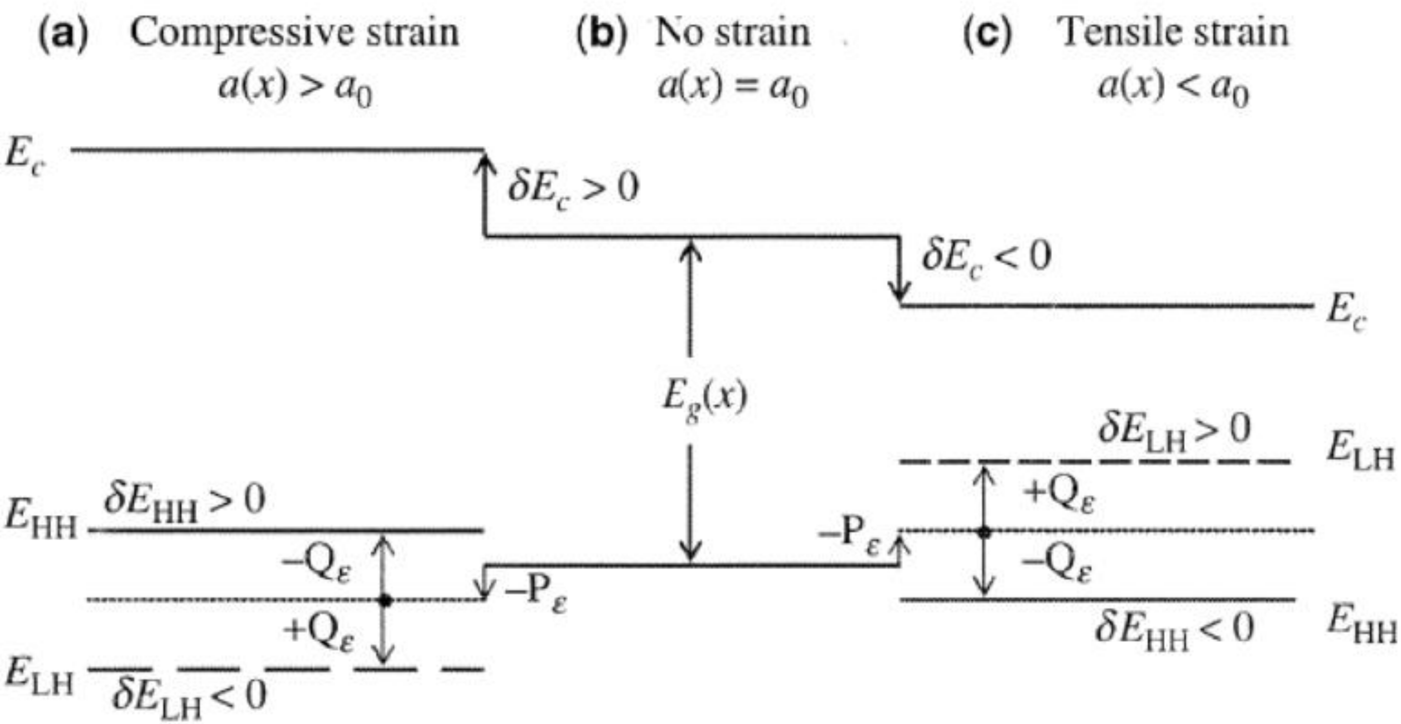
$$g_p(n) = g_{\max}\left(f_c - f_v\right) \simeq g_{\max}\left(1 - e^{-n/n_c} - e^{-n/Rn_c}\right) \qquad \textbf{with} \quad R = \frac{m_h^*}{m_e^*}$$



Transparency density decreases with R but maximum achievable gain decreases
$\rightarrow$ **Design trade-off**

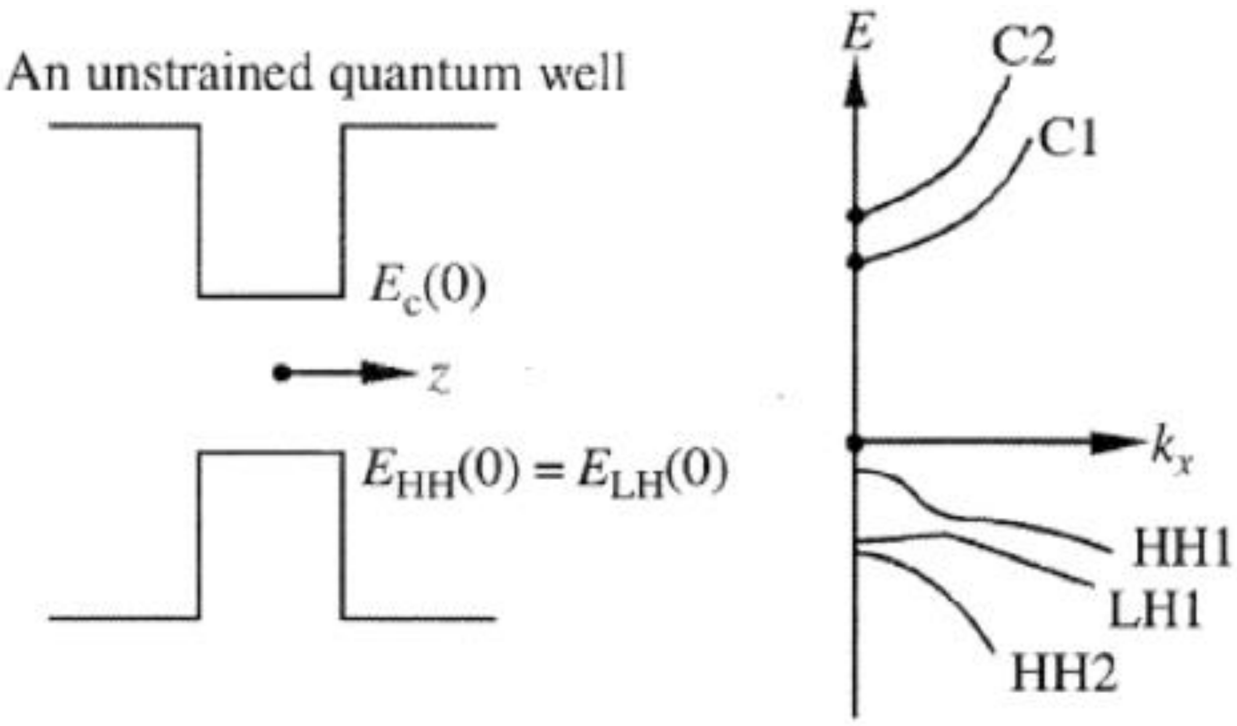
Strain Effects on Band-Edge Energies

- **Compressive strain generally increases the bandgap.**
- **Tensile strain generally decreases the bandgap.**
- **The LH band follows these trends but the HH band goes against.**
- **Strain affects also the conduction band structure of the conduction band. Mainly, the Γ, L, and X valleys shift in energy at different rates.**



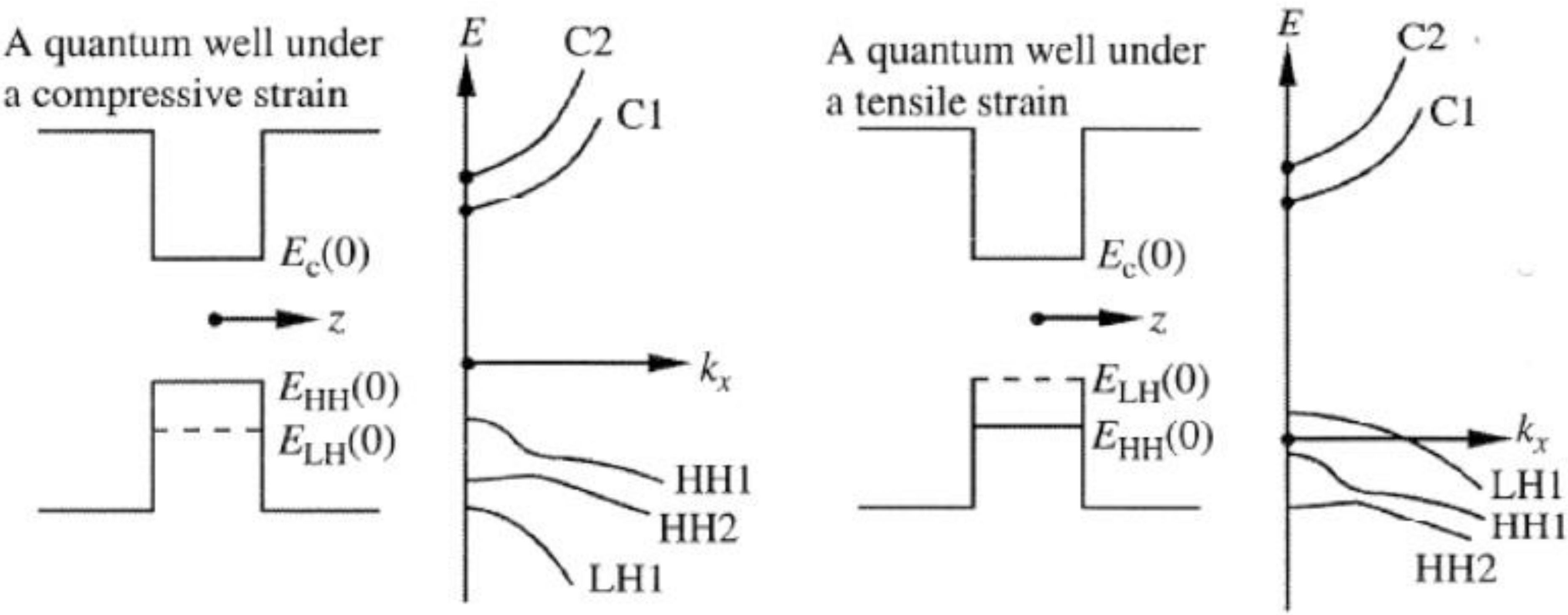
Strain Effects in Quantum Wells

- **For unstrained material, bandgap is the same for HH and LH. The energy levels in a quantum well, corresponding to HH and LH, differ because of unequal effective masses**



Strain Effects in Quantum Wells

- **For strained materials HH and LH bandgaps and the energy offsets in CB and VB are different. HH and LH are in different potential wells.**

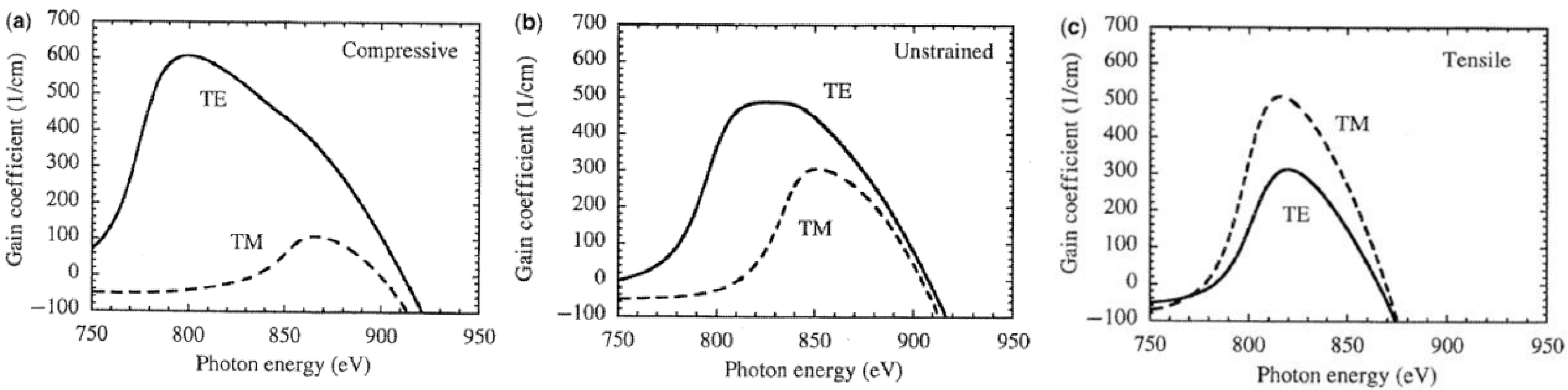


- **QW under tensile strain brings HH and LH quantum level closer to each other (LH has deeper well but higher energy levels)**

Gain Spectrum of Strained Quantum Wells

Recall that

- **C1-HH1 transition is mostly favored by TE**
- **C1-LH1 transition is mostly favored by TM**



- **Tensile strain can improve balance between TE and TM gain**
- **Trade-off is maximum gain linked to joint density of states through the ratio of effective masses**

Momentum Matrix Elements

Recall the matrix element depends on transverse wave vector k_t

For $\mathrm{k}_t = 0$,

TE Polarization:

$$|\hat{x} \cdot \mathbf{M}_{\text{c-hh}}|^2 = |\hat{y} \cdot \mathbf{M}_{\text{c-hh}}|^2 = \frac{3}{2} M_b^2$$

$$|\hat{x} \cdot \mathbf{M}_{\text{c-}l\text{h}}|^2 = |\hat{y} \cdot \mathbf{M}_{\text{c-}l\text{h}}|^2 = \frac{1}{2} M_b^2$$

TM Polarization:

$$|\hat{z} \cdot \mathbf{M}_{\text{c-hh}}|^2 = 0$$

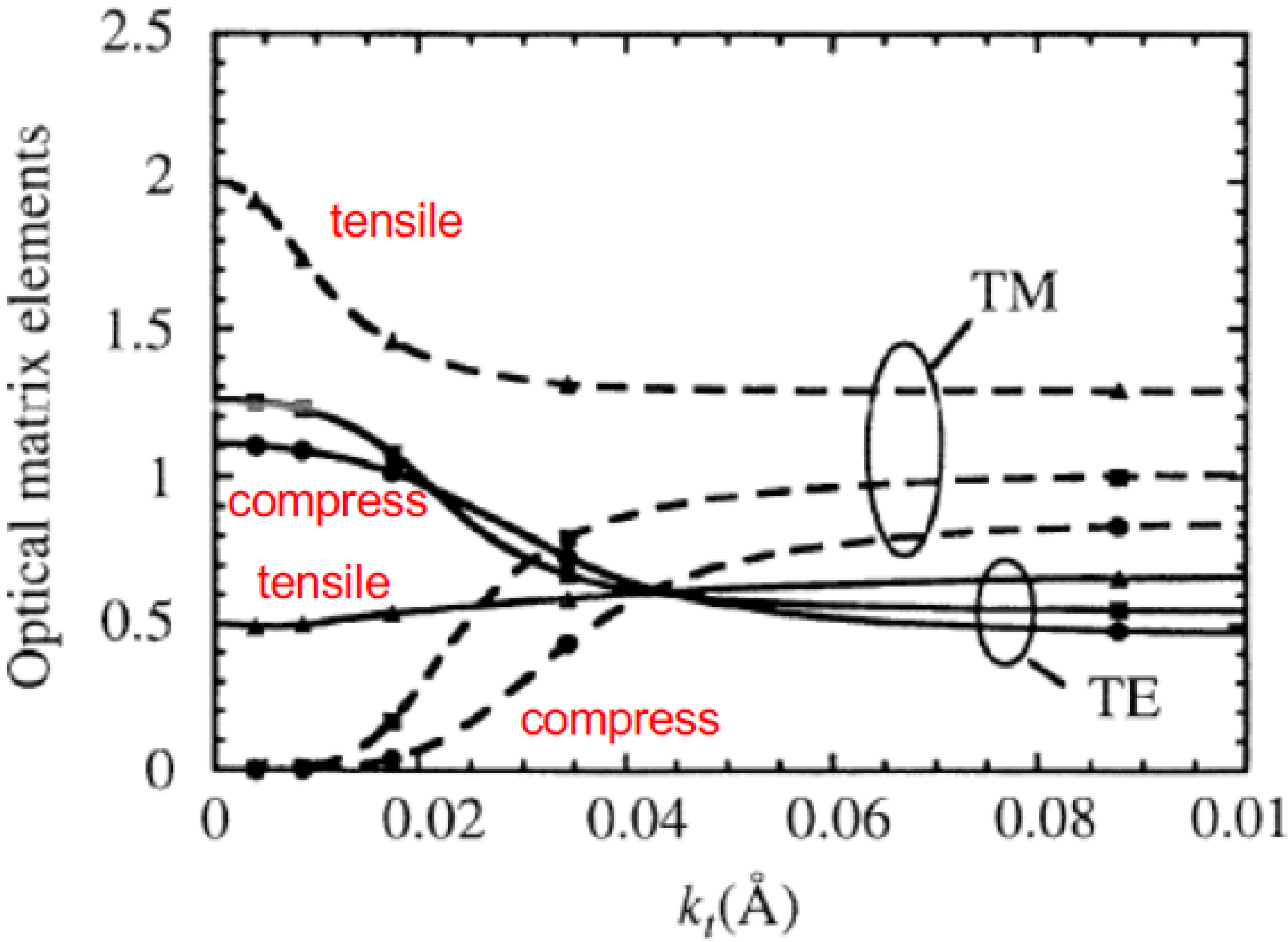
$$|\hat{z} \cdot \mathbf{M}_{\text{c-}l\text{h}}|^2 = 2 M_b^2$$

Gain depends on surface (sheet) carrier concentration

$$n_s = n \times L_z$$

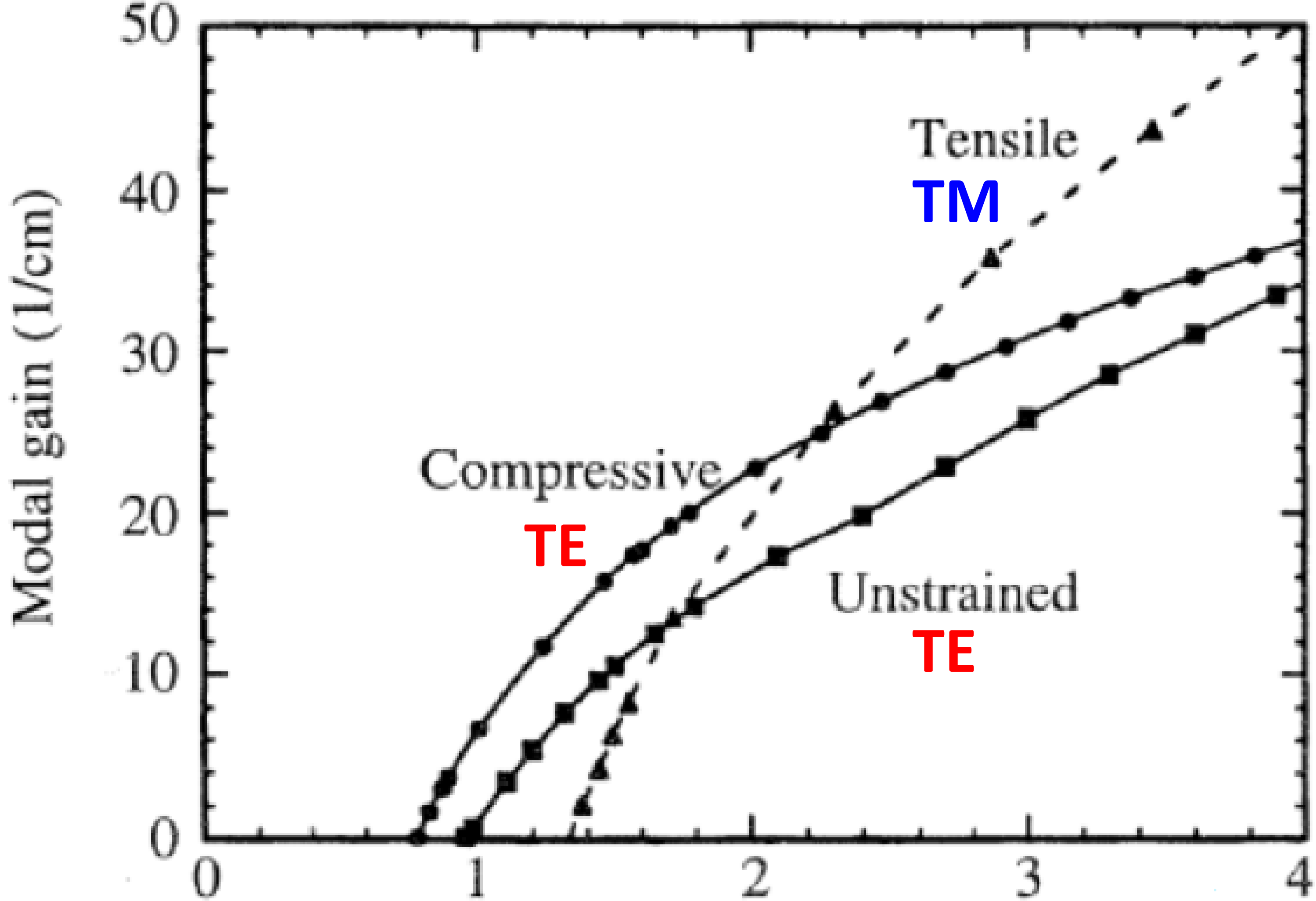
Momentum Matrix Element

- Normalized as $2|M_{nm}(k_t)|^2/M_b^2$ with $n = C1, m = HH1$ for compressive strain and $m = LH1$ for tensile strain]



Modal gain versus sheet concentration

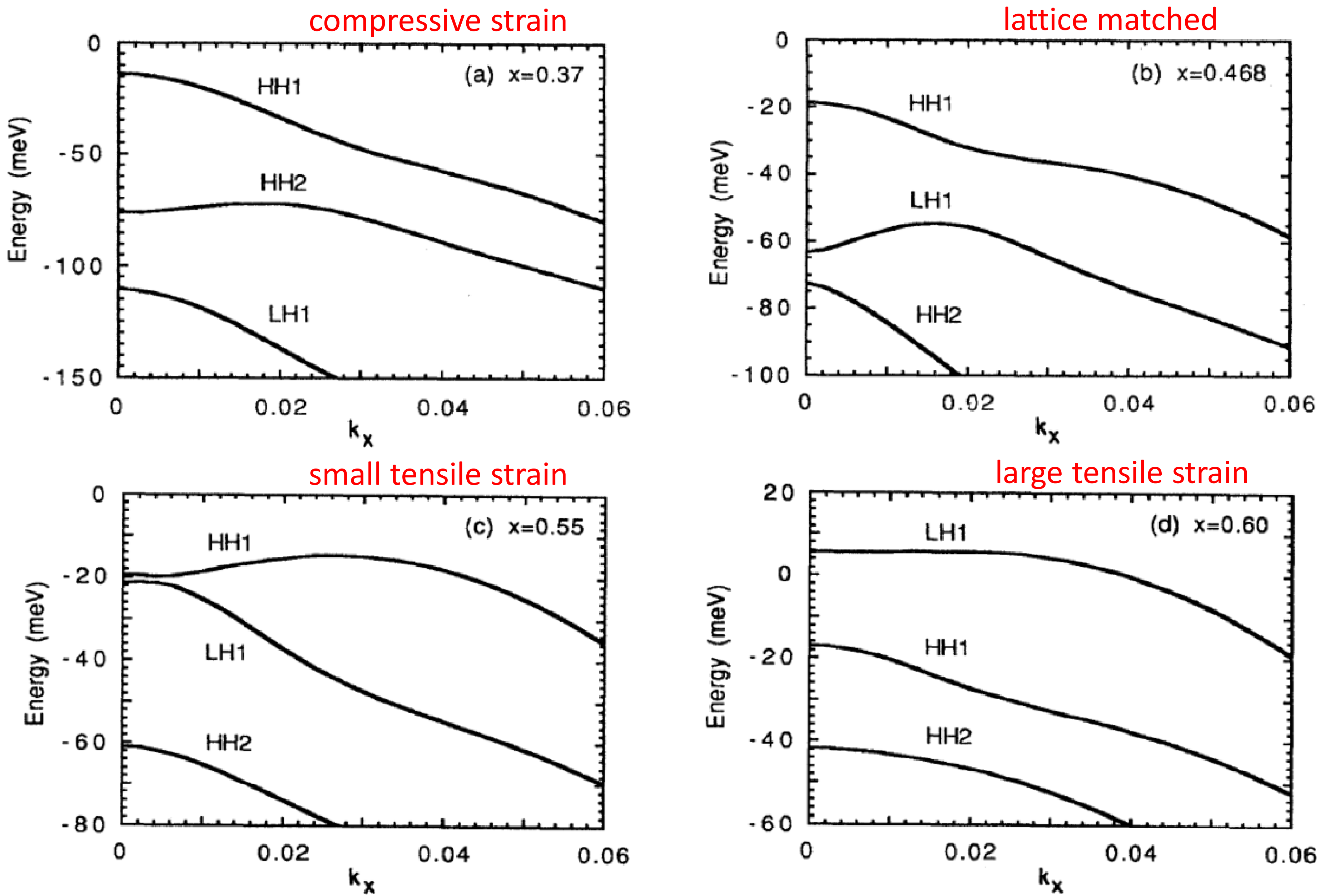
For $In_{1-x}Ga_xAs$ / $In_{1-x}Ga_xAs_yP_{1-y}$ quantum well laser working near 1.55μm



n_s (× 10^{12}/cm^2)

$$n_s = n \times L_z$$

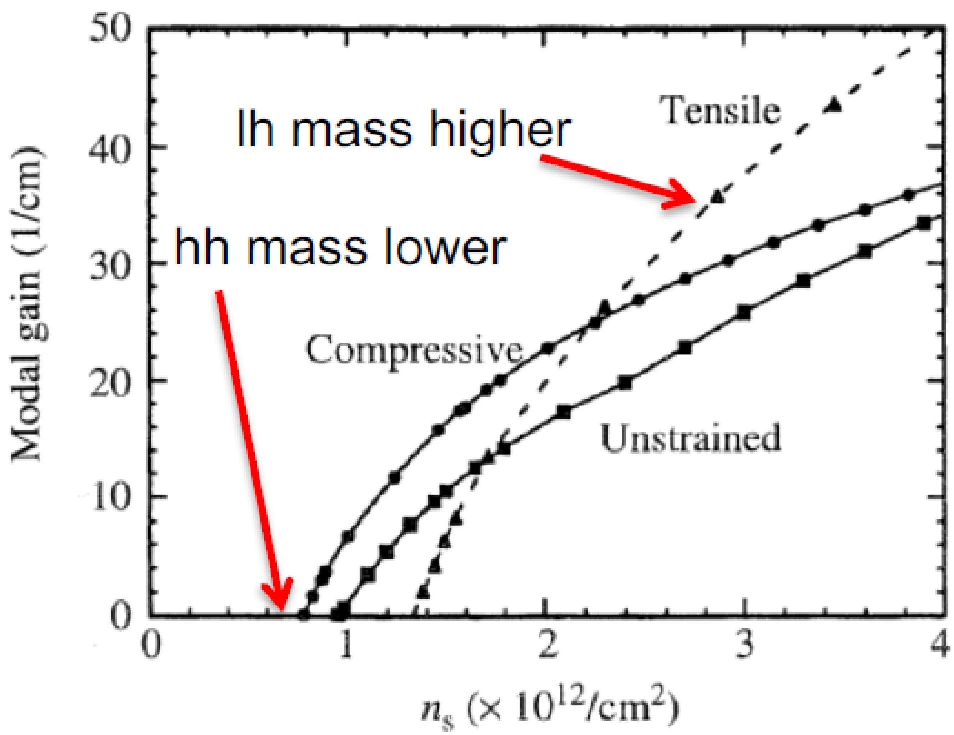
Band Distortion in Strained Quantum Wells



(S.L. Chuang, Phys Rev B, vol. 43, p. 9649 (1991)

Some comments

- **LH can become heavier, HH can become lighter**
- **Compressively-strained materials can have lower valence band density of states (because HH at the top of the VB can have effective mass lower than LH at the top of the VB in the case of tensile strain)**



$$n_s = n \times L_z$$

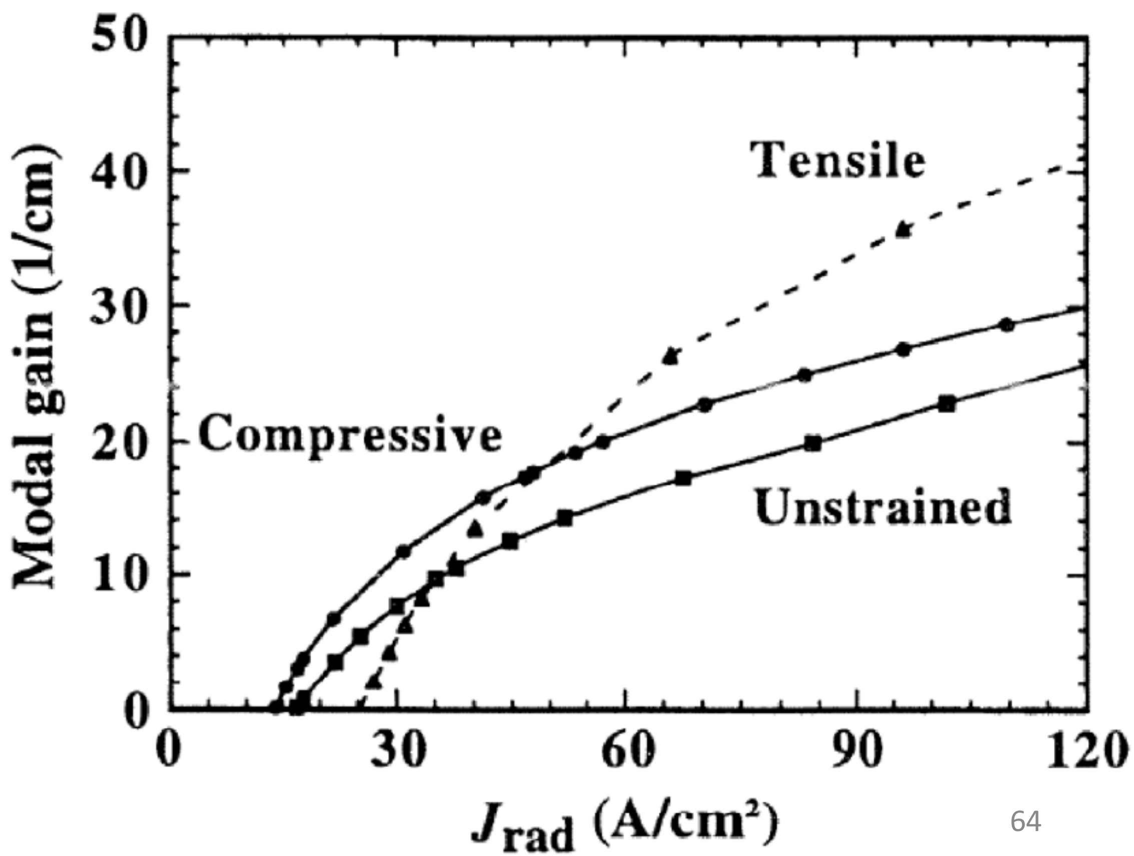
Modal Gain versus Current Density

- **Modal Gain** $$\Gamma g \propto \frac{L_z}{W_{\text{mode}}} g$$
- **Empirical relationship** $$G = n_w \Gamma_w g_w = n_w \Gamma_w g_0 \left[\ln\left(\frac{n_w J_w}{n_w J_0} \right) + 1 \right]$$

Compressive strain
Smaller transparency carrier density but saturates faster.

Tensile strain
Larger transparency carrier density but increases faster (higher differential gain).

Loss mechanisms (Auger, recombination, intervalence band absorption) need to be factored in when considering current density.



Reading Assignments:

- Sections 10.3 and 10.4 of Chuang's book
- Section 8.2.5, Appendices 1,2,3,9 (supplemental) in Coldren, Corzine and Mašanović

- Section 4.5 of Chuang's book