ECE 536 – Integrated Optics and Optoelectronics
Lecture 18 – March 25, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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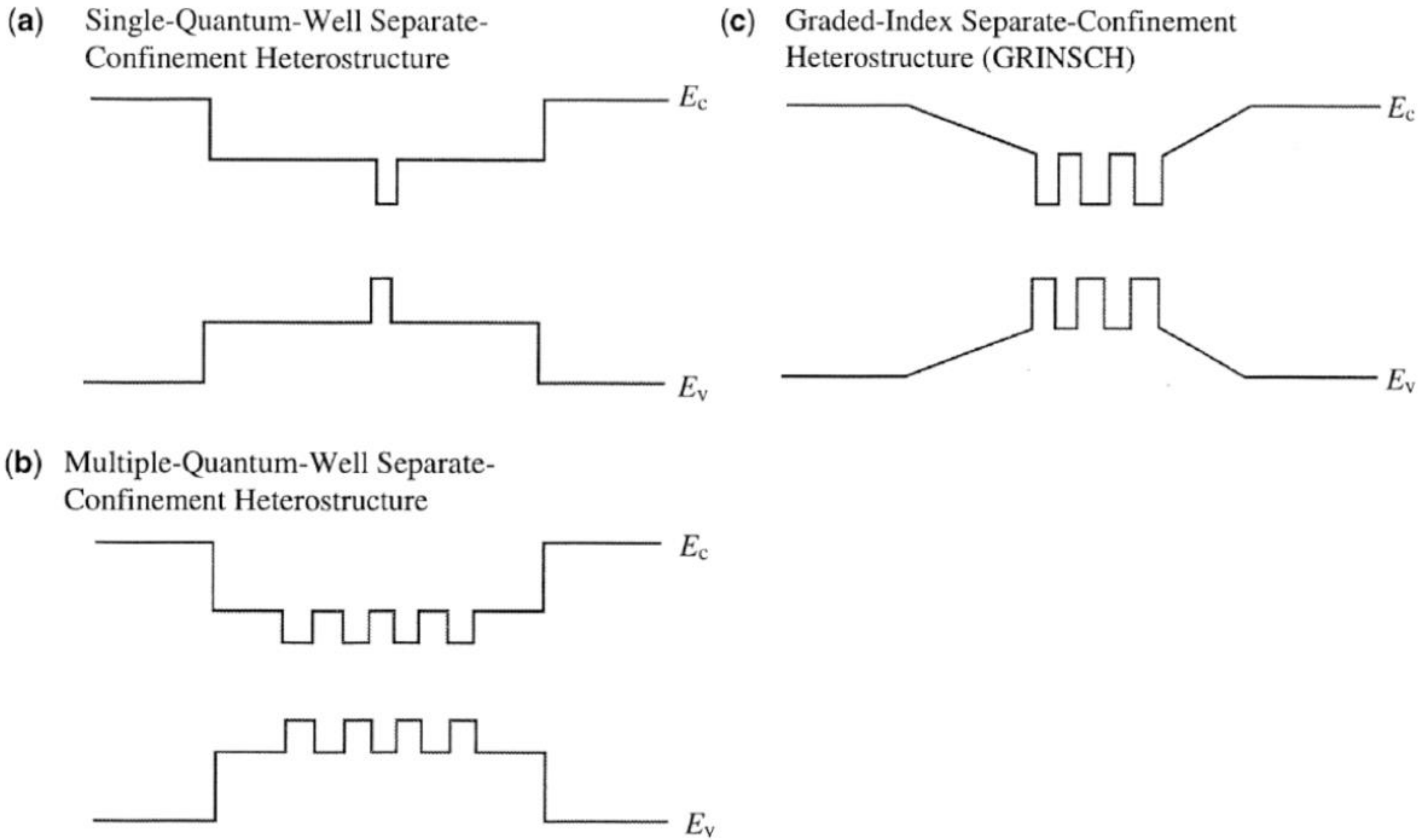
Lecture 18 Outline

- Scaling Law for Multiple Quantum Wells
- Strain Effects
- Quantum Dot Lasers

Quantum Well (QW) Lasers

Types of QW Lasers



Peak Gain versus Current Density

For a given carrier concentration n

$$J = J_{rad} + J_{Aug} + J_{leak}$$

$$J_{rad} = qL_z \underbrace{R_{sp}(n)}_{Bn^2} \qquad J_{Aug} = qL_z \underbrace{R_{Aug}(n)}_{Cn^3}$$

Commonly used empirical formula

$$g_p(J) = g_0\left[1 + \ln\frac{J}{J_0}\right] = g_0 \ln\frac{J}{J_{tr}}$$

Scaling Laws for Multiple Quantum Well (MQW) Lasers

Let's define

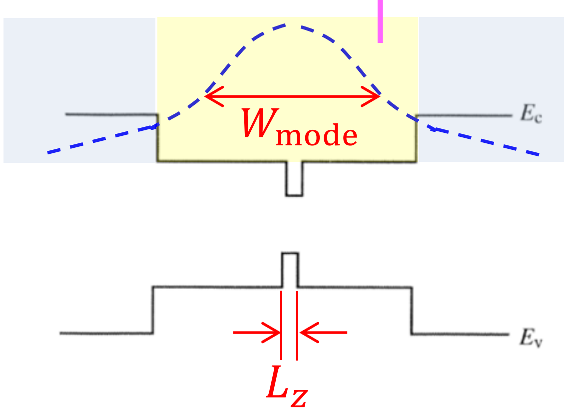
$$J_w = \eta J_{applied} = \text{ injected current density for SQW}$$

$$g_w = g_0\left[\ln\left(\frac{J_w}{J_0}\right)+1\right] = \text{peak gain coefficient for SQW}$$

where usually $g_w \propto L_z^{-1}$

$$J_{tr} = J_0 e^{-1} = \text{transparency current density}$$

$$\Gamma_w = \text{ optical confinement factor per well}$$



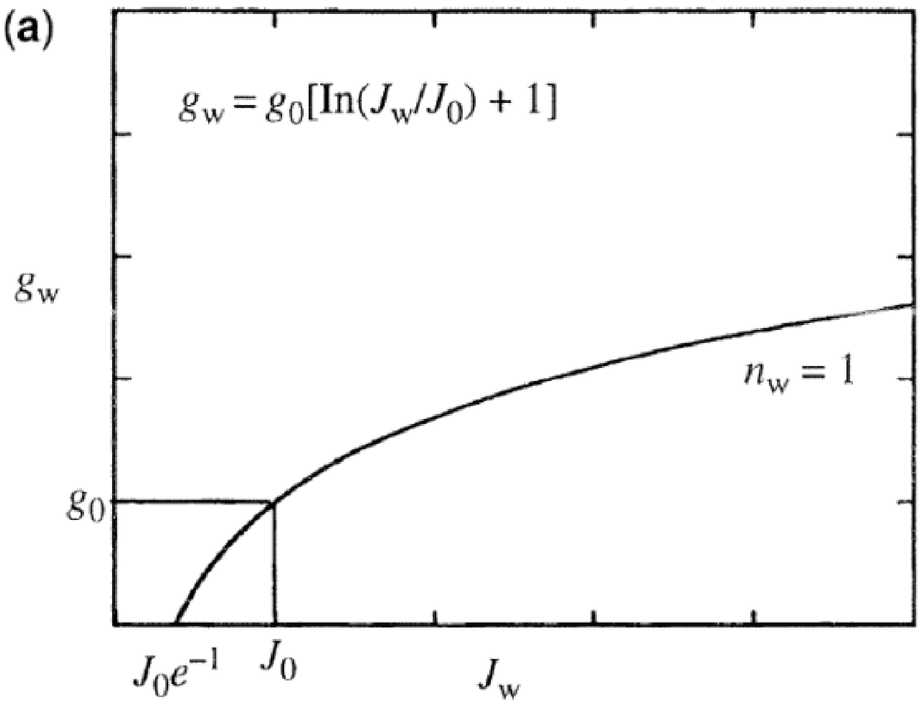
$$\Gamma_w = \Gamma_{op}\frac{L_z}{W_{\text{mode}}}$$

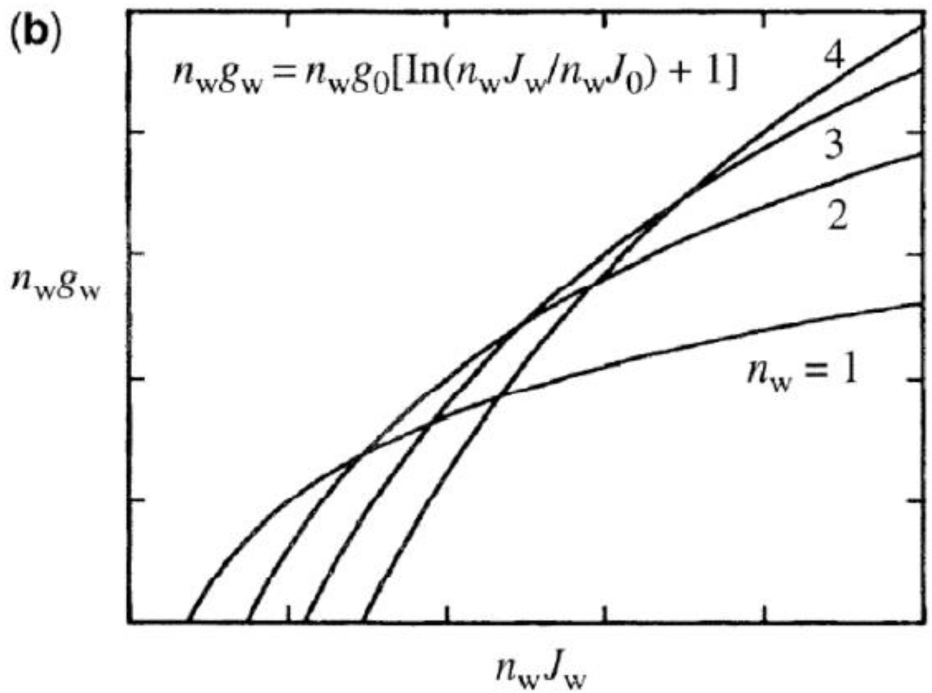
full-width at half maximum of optical mode

Scaling Laws for Multiple Quantum Well (MQW) Lasers

Modal gain for a MSQW at threshold

$$G_{th} = n_w \Gamma_w g_w = \alpha_{tot} = \alpha_i + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$





Scaling Laws for Multiple Quantum Well (MQW) Lasers

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General Expression of Modal gain for a SQW

$$\boxed{G = \Gamma_w g_w = \Gamma_w g_{\max} \left[f_c(\hbar\omega) - f_v(\hbar\omega) \right] = \frac{\Gamma_{op} L_z}{W_{\text{mode}}} g_{\max}}$$

$$\boxed{\hbar\omega = E_{h1}^{e1}(0)}$$

$$g_{\max} = C_0 \left| \hat{e} \bullet \mathbf{M}_{\mathbf{ch}} \right|^2 \frac{m_r^*}{\pi \hbar^2 L_z} \delta_{nm}$$

Modal gain for a MQW

$$\boxed{G = n_w \Gamma_w g_w}$$

Threshold Current Density

- **Injected Current Density per QW at Threshold**

$$J_{w,th} = \frac{\eta J_{th}}{n_w}$$

- **Peak gain**

SQW)

$$g_w = g_0 \left[\ln\left(\frac{J_w}{J_0} \right) + 1 \right]$$

MQW)

$$n_w g_w = n_w g_0 \left[\ln\left(\frac{J_w}{J_0} \right) + 1 \right] = n_w g_0 \left[\ln\left(\frac{n_w J_w}{n_w J_0} \right) + 1 \right]$$

$n_w J_w =$ total injected current density

$n_w J_0 e^{-1} =$ total injected current density for transparency

Threshold Current Density for MQW

$$n_w g_w = n_w g_0 \left[\ln\left(\frac{J_w}{J_0}\right) + 1 \right] = n_w g_0 \left[\ln\left(\frac{n_w J_w}{n_w J_0}\right) + 1 \right]$$

$$\frac{n_w J_w}{n_w J_0} = e^{\left(\frac{n_w g_w}{n_w g_0}\right) - 1} \quad \Rightarrow \quad \eta J_{th} = n_w J_w = n_w J_0 \exp\left[\left(\frac{g_w}{g_0}\right) - 1 \right]$$

$$J_{th} = \frac{n_w J_0}{\eta} \exp\left[\left(\frac{g_w}{g_0}\right) - 1 \right]$$

At threshold Gain = Loss

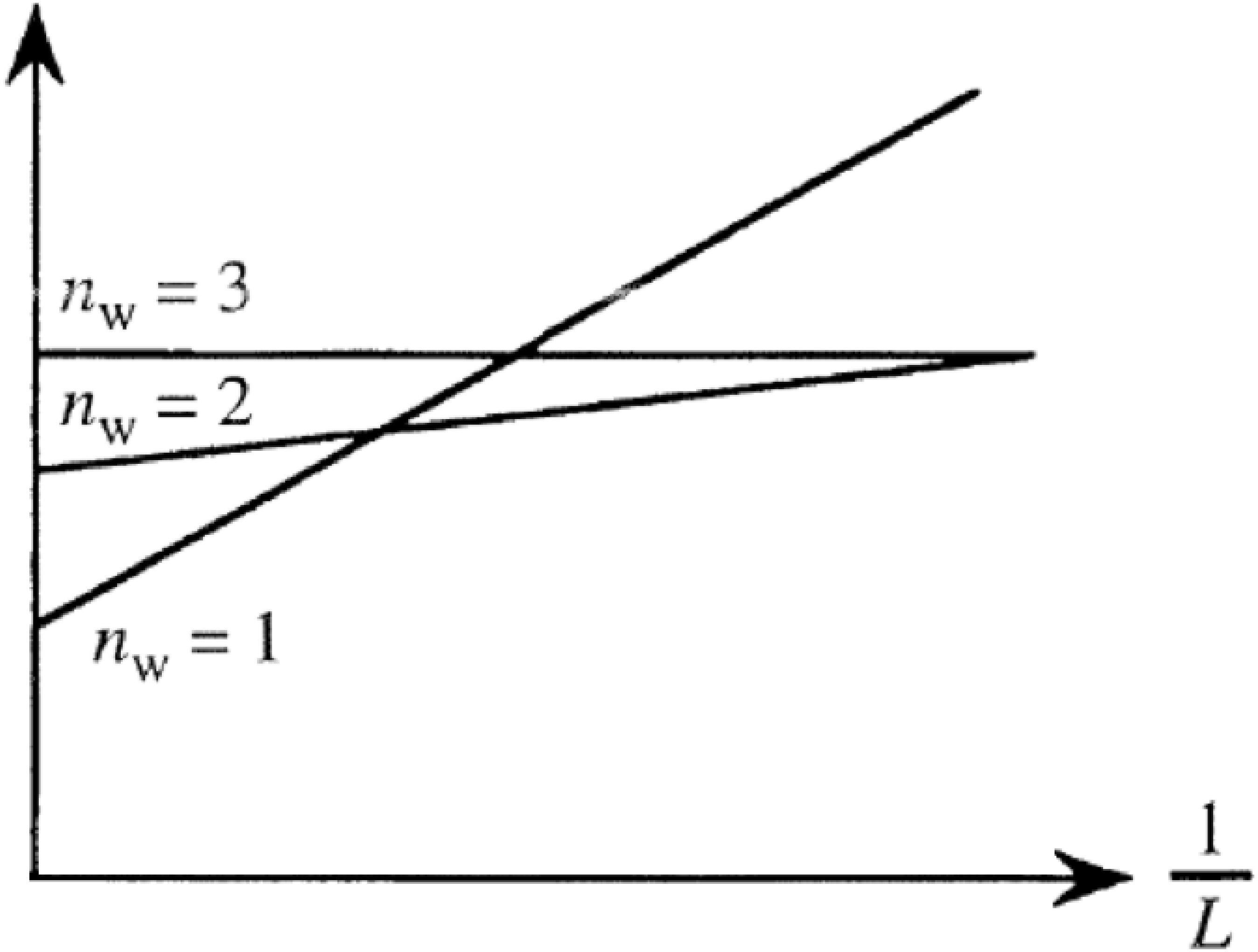
$$n_w \Gamma_w g_w = \alpha_{tot}$$

$$J_{th} = \frac{n_w J_0}{\eta} \exp\left[\left(\frac{\alpha_{tot}}{n_w \Gamma_w g_0}\right) - 1 \right]$$

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{1}{n_w \Gamma_w g_0}\left(\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) - 1$$

Threshold Current Density for MQW

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{1}{n_w \Gamma_w g_0}\left(\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) - 1$$

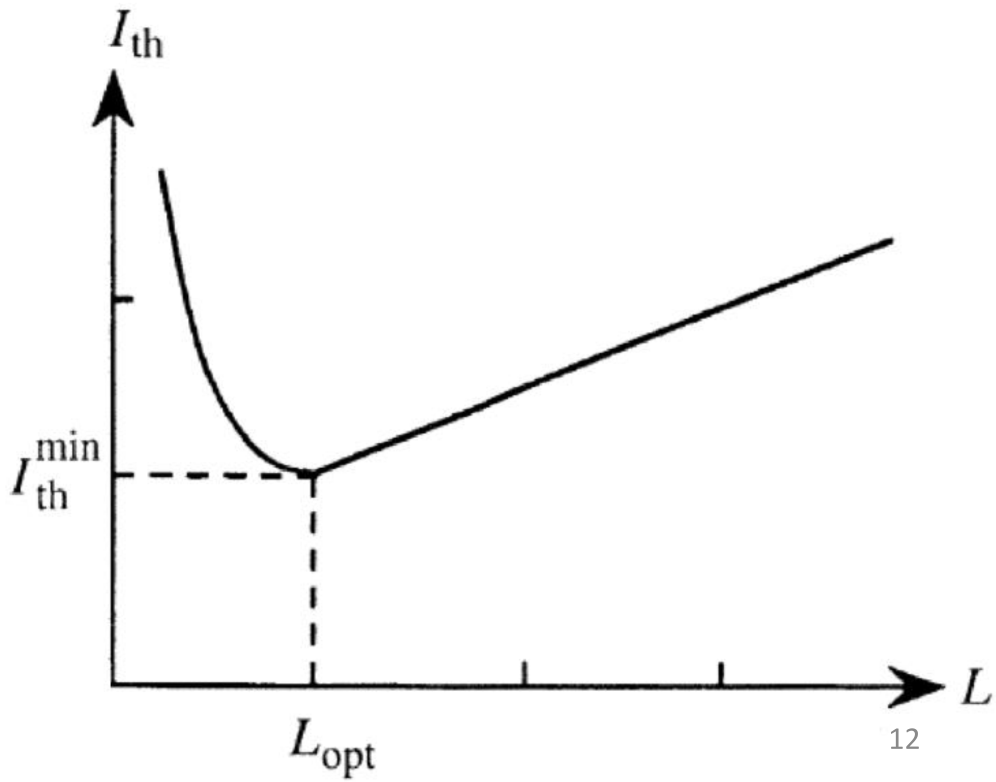


Optimal Cavity Length to Minimize Threshold

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{1}{n_w \Gamma_w g_0}\left(\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) - 1$$

$$= \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1$$

$$L_{opt} = \frac{1}{2}\ \frac{1}{n_w \Gamma_w g_0}\ \ln\frac{1}{R_1 R_2}$$



Minimum Threshold Current for L_{opt}

$$\ln J_{th} = \ln\left(\frac{n_w J_0}{\eta}\right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1$$

$$\boxed{I_{th} = \frac{w\, L\, n_w J_0}{\eta} \exp\left[\frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1\right] = \mathrm{const} \cdot L \exp\left(L_{opt}/L\right)}$$

Minimum Threshold Current at Optimal Cavity Length

$$\frac{\partial}{\partial L} I_{th} = I_{th}\left(\frac{1}{L} - \frac{L_{opt}}{L^2}\right) = 0 \quad \Rightarrow \quad \boxed{\frac{1}{L} = \frac{L_{opt}}{L^2} \quad \Rightarrow \quad L = L_{opt}}$$

$$\boxed{I_{th}^{\min} = \frac{w\, L_{opt}\, n_w J_0}{\eta} \exp\left[\frac{\alpha_i}{n_w \Gamma_w g_0}\right]}$$

Optimal Number of Wells for Fixed Cavity Length

$$I_{th} = \frac{wLn_w J_0}{\eta} \exp\left[\left(\frac{\alpha_{tot}}{n_w \Gamma_w g_0}\right) - 1\right]$$

$$\boxed{n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0}}$$

$$= \text{const} \cdot n_w \exp\left(\frac{\alpha_{tot}}{n_w \Gamma_w g_0}\right) = \text{const} \cdot n_w \exp\left(\frac{n_{opt}}{n_w}\right)$$

$$\boxed{\frac{\partial}{\partial n_w} I_{th} = I_{th}\left(\frac{1}{n_w} - \frac{n_{opt}}{n_w^2}\right) = 0}$$

(implicit assumption of no coupling between wells)

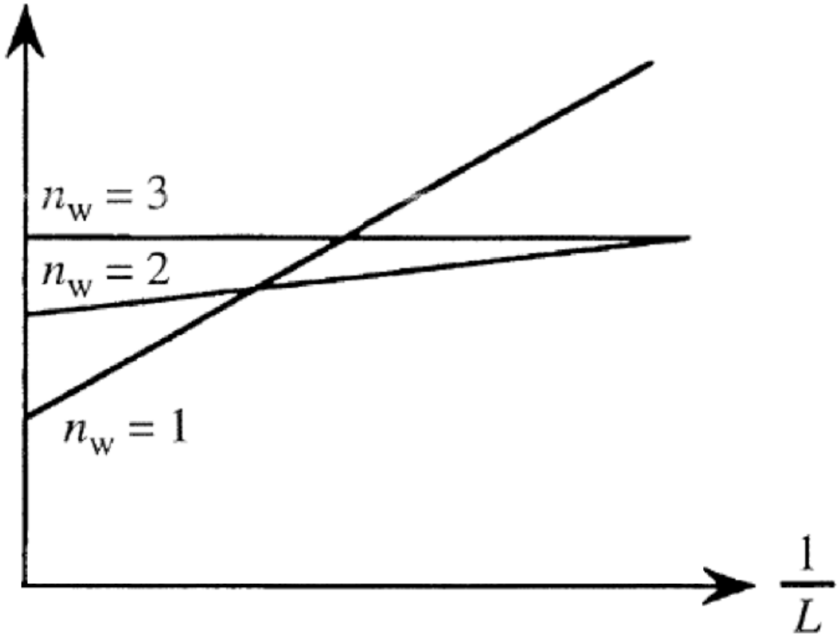
$$n_w = n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0} = \frac{1}{\Gamma_w g_0}\left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right)$$

Take the closest integer!

$$\boxed{I_{th}^{\min} = \frac{wLn_{opt} J_0}{\eta}}$$

Threshold Current Optimization

It is not possible to optimize simultaneously for cavity length and number of quantum wells.

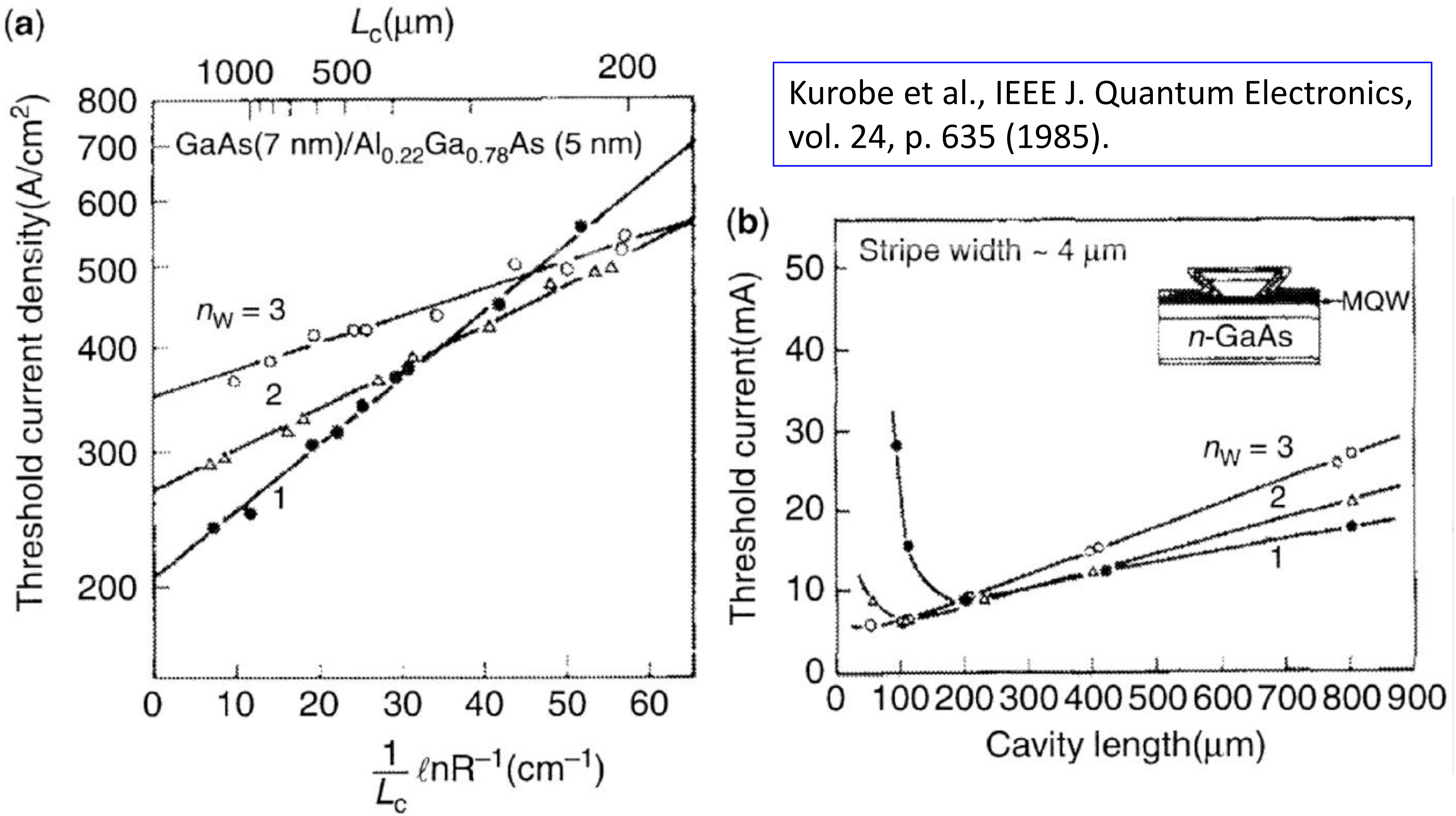


If $\boxed{L = L_{opt}}$

$$\Rightarrow \quad \boxed{n_w = \frac{\alpha_m}{\Gamma_w g_0} \neq n_{opt}}$$

(unless $\alpha_i = 0$ which is unphysical)

Some representative results



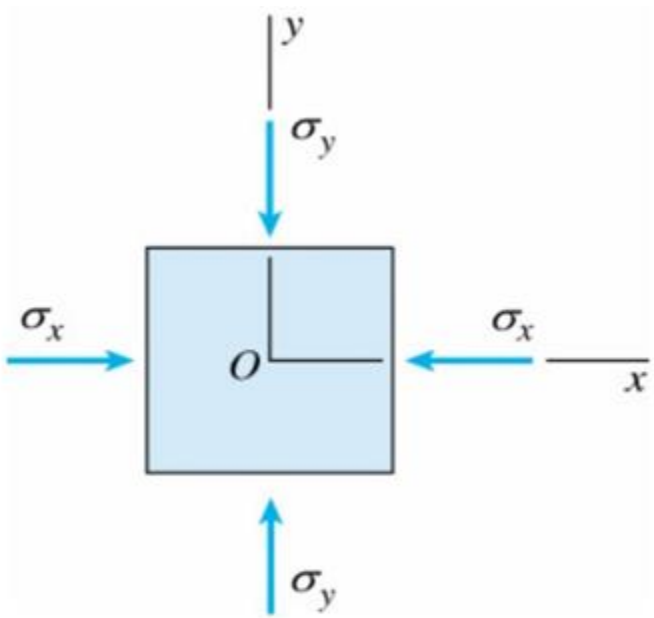
Kurobe et al., IEEE J. Quantum Electronics, vol. 24, p. 635 (1985).

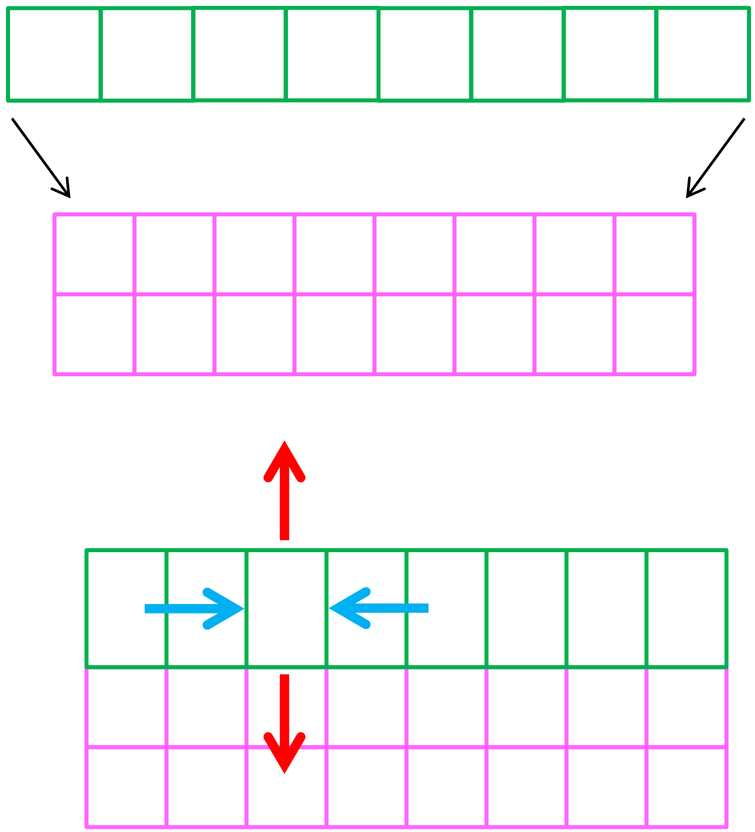
Strain Effects

Definitions

Biaxial Compression

The strained material ha a larger lattice constant resulting in **compressive strain** in the **plane of the wafer** and **tension** in the direction **perpendicular to the surface.**

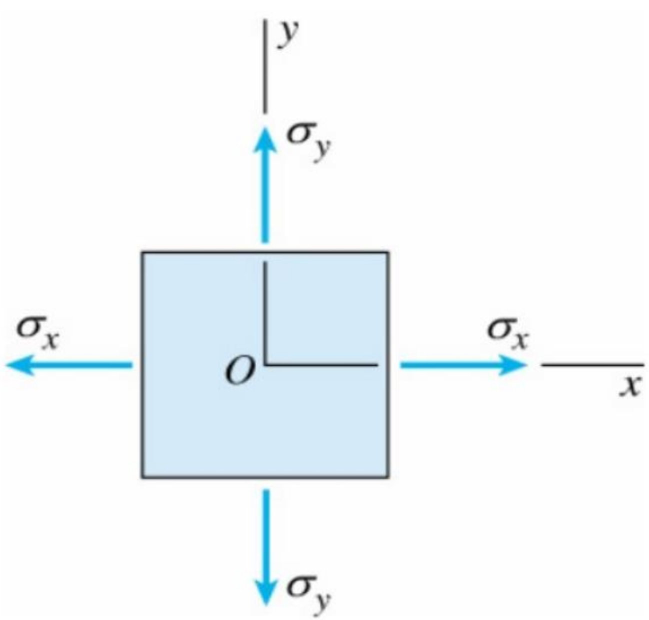


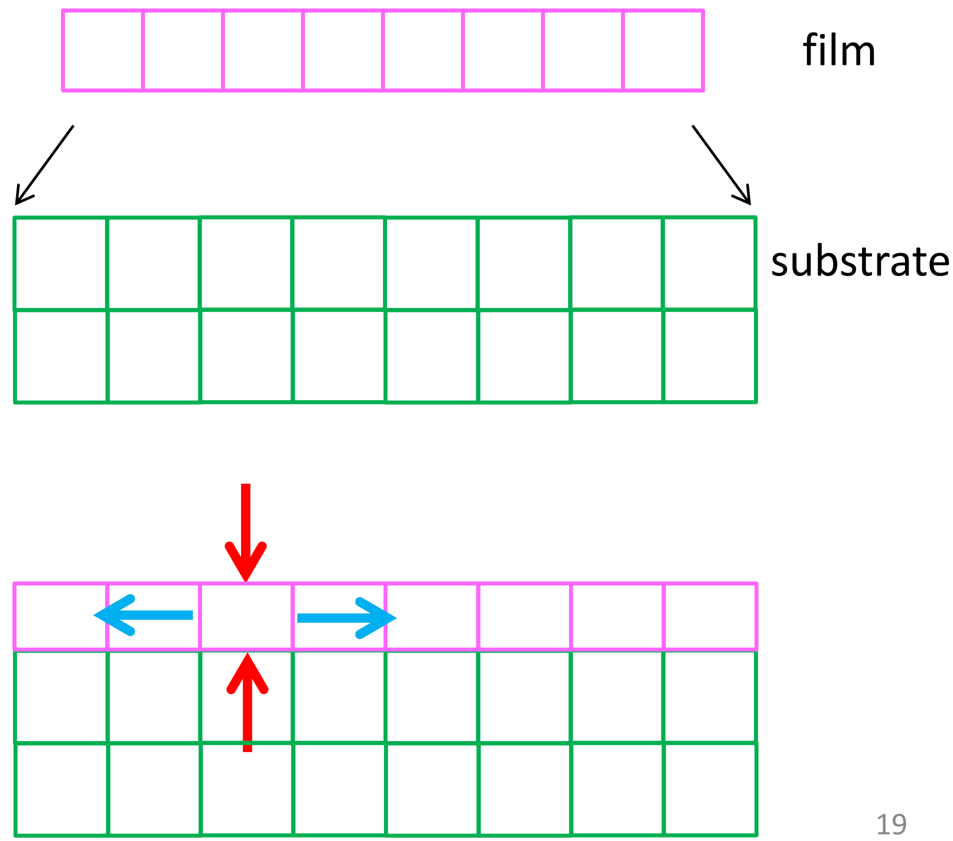


Definitions

Biaxial Tension

The strained material ha a smaller lattice constant resulting in **tensile strain** in the **plane of the wafer** and **compression** in the direction **perpendicular to the surface.**



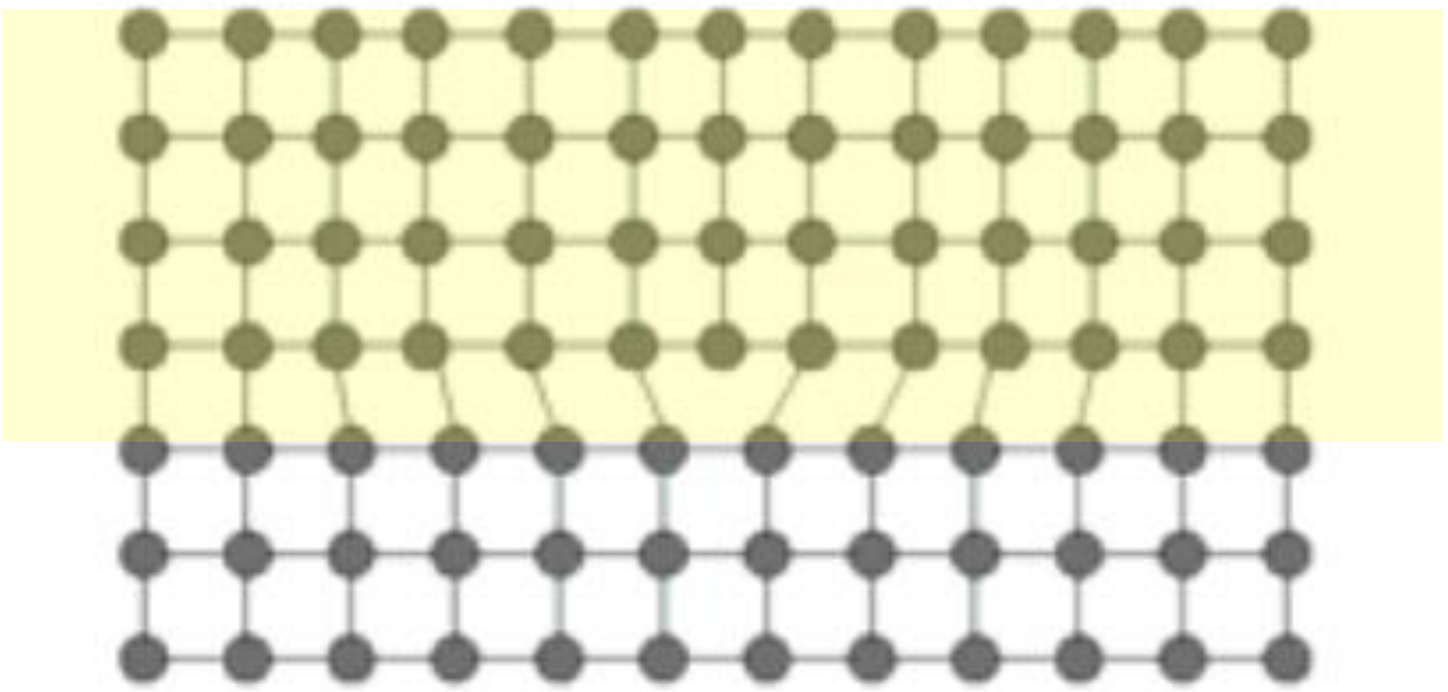


Definitions

Critical Layer Thickness

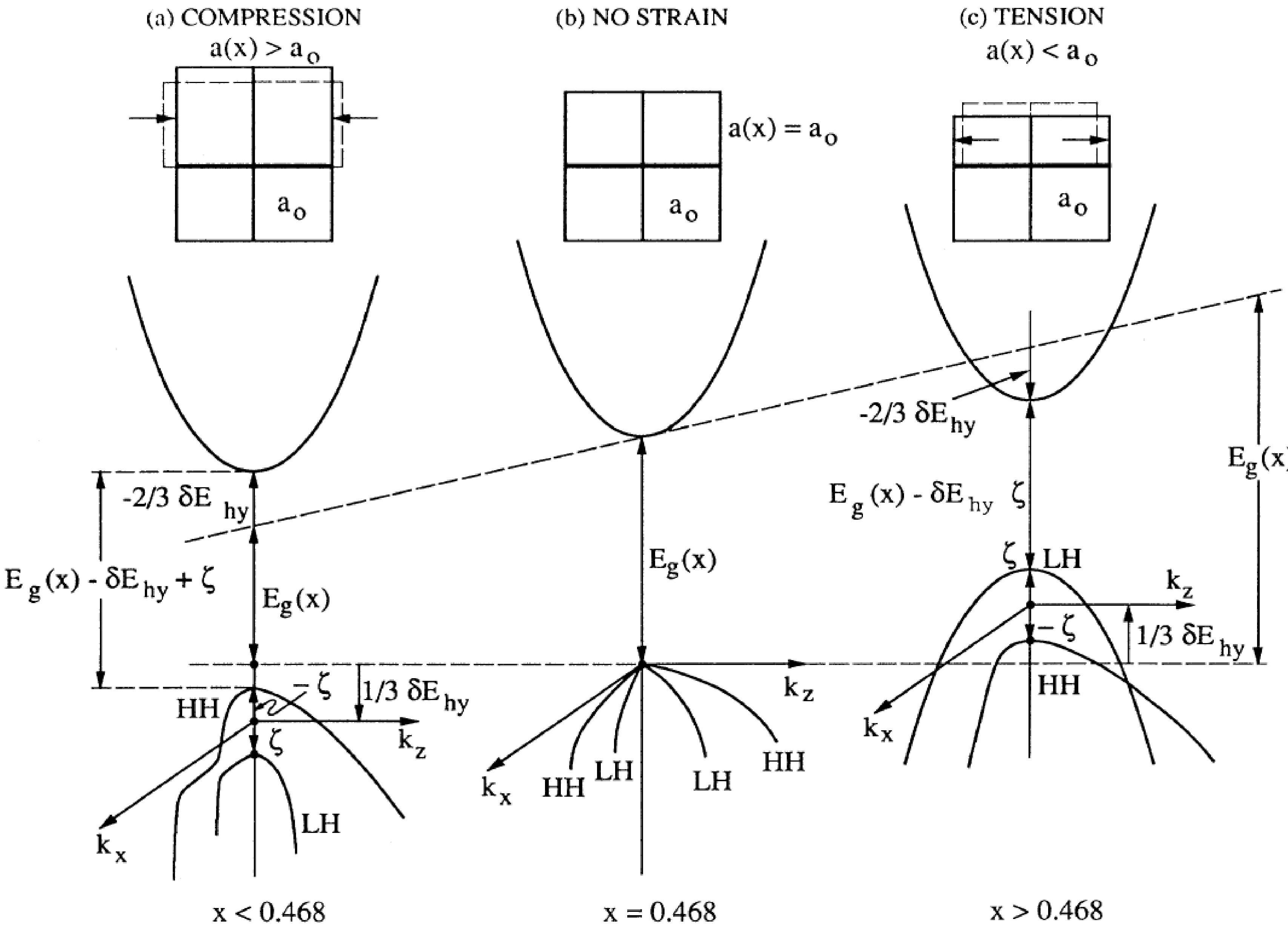
Thickness beyond which dislocations form to accommodate mismatch.

edge dislocation



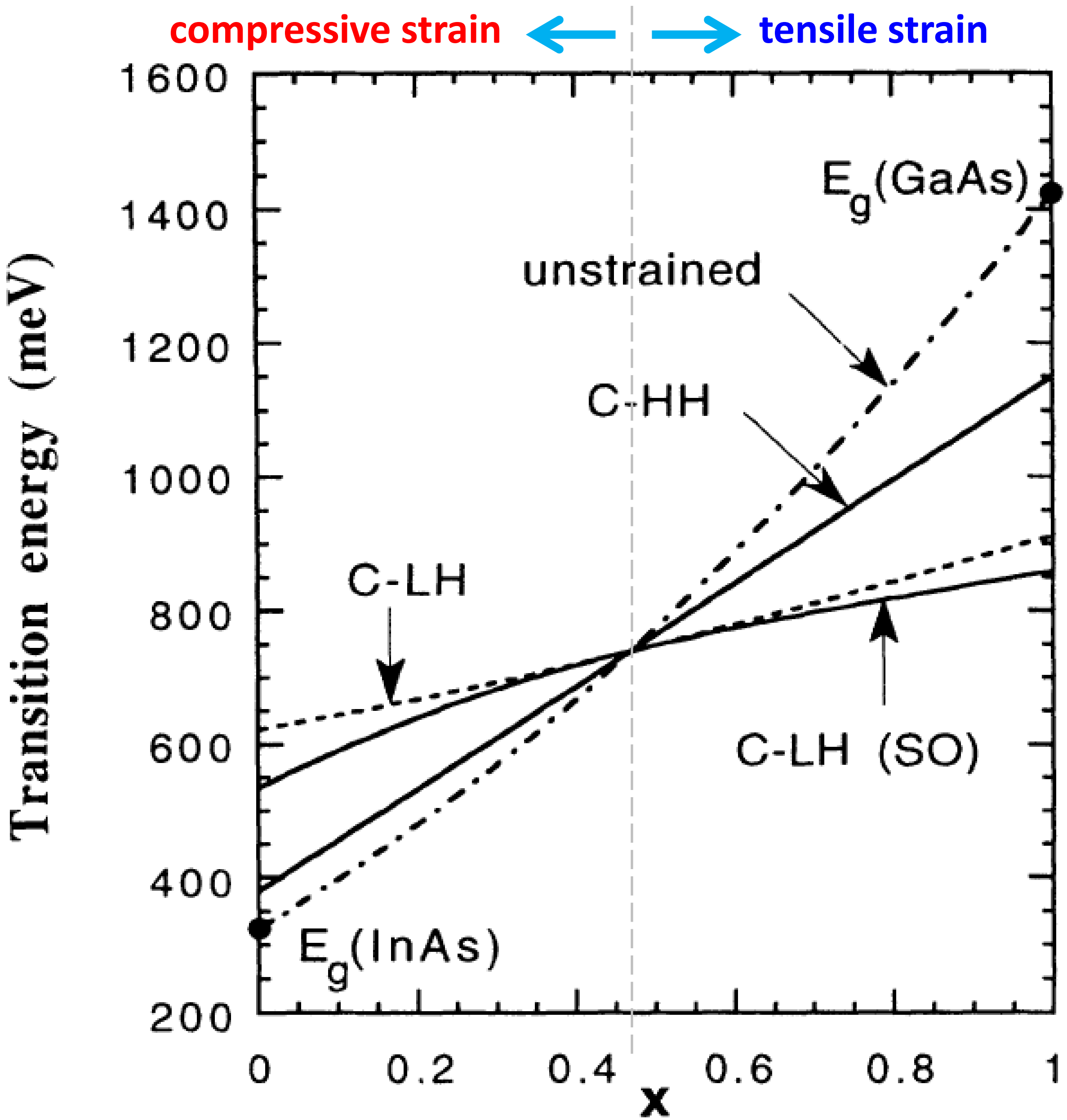
Some Key Points

- Strain may modify significantly the band structure of the valence band.
- Both bandgap energy and carrier effective mass may change (e.g., heavy hole effective mass becomes lighter and light hole effective mass becomes heavier in the direction of strain.
- Degeneracy of hh/lh bands is broken.
- Strain may change the threshold current of lasers
- Strain may change polarization of emitted light.
- Reduction in threshold current density may reduce the importance of non-radiative processes such as Auger recombination.



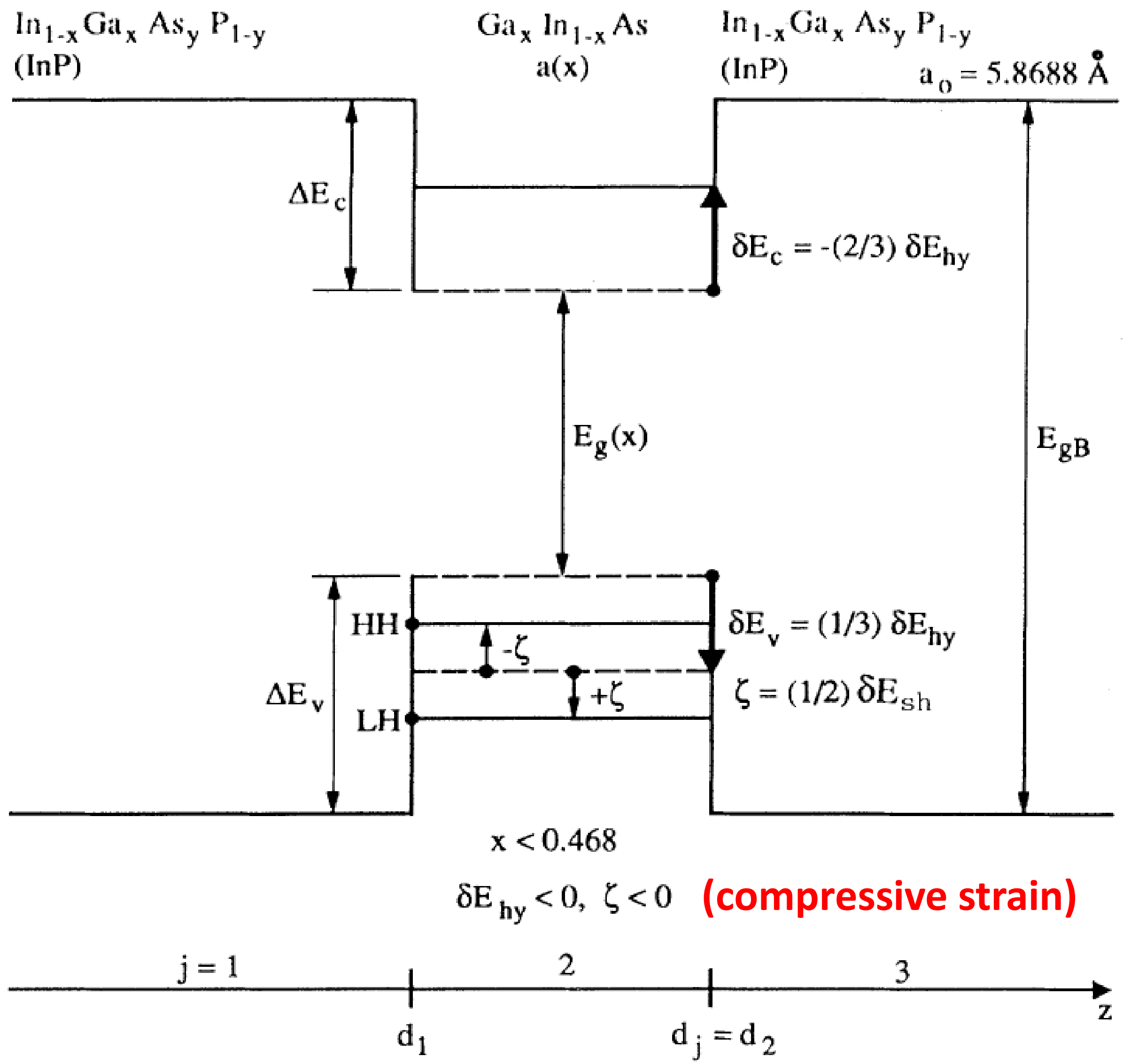
Effect of strain on the band structure of $In_{1-x}Ga_xAs$

(S.L. Chuang, Phys Rev B, vol. 43, p. 9649 (1991)



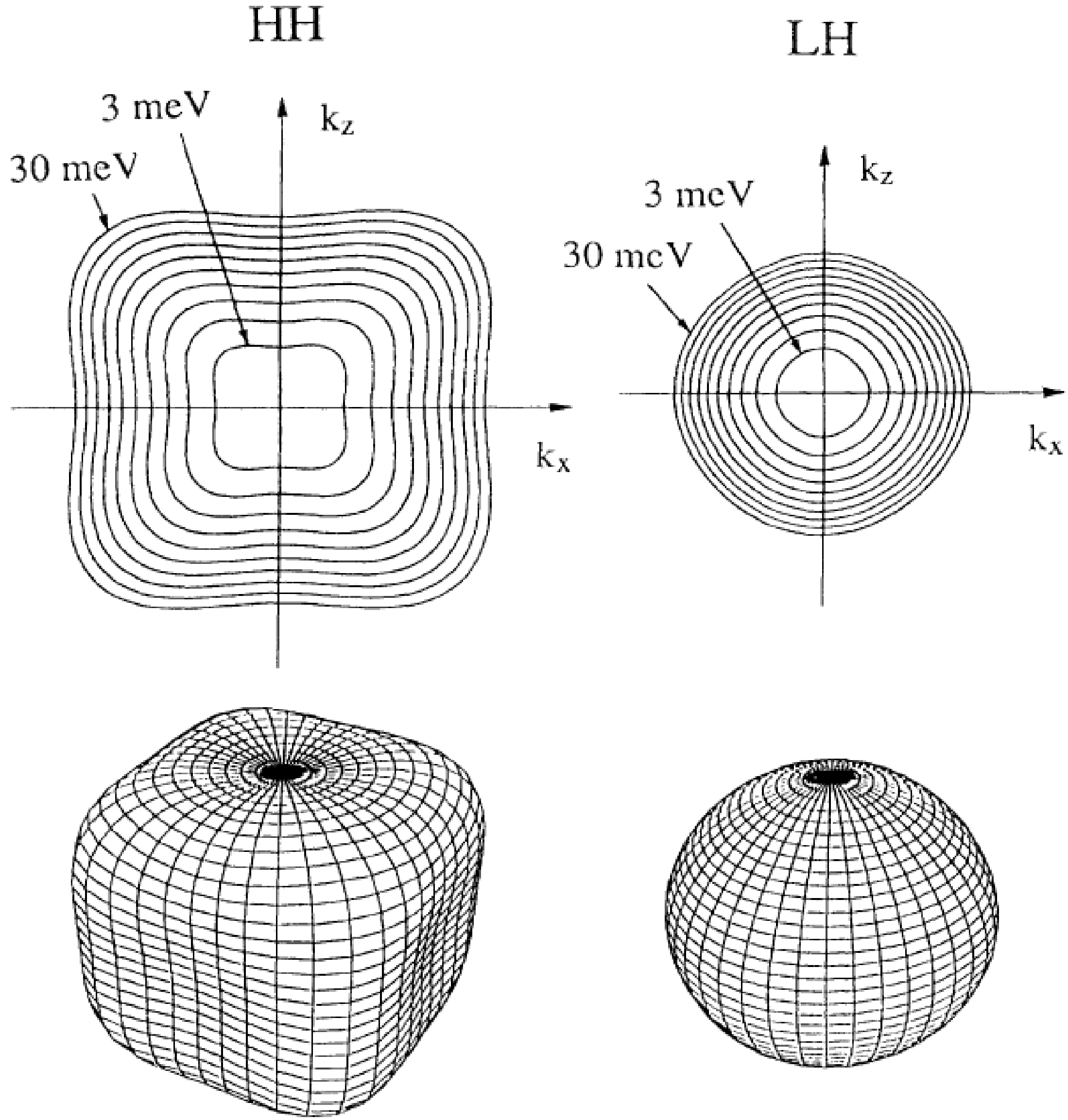
Energy band gap of $In_{1-x}Ga_xAs$ bulk and as grown pseudomorphically on InP

(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).



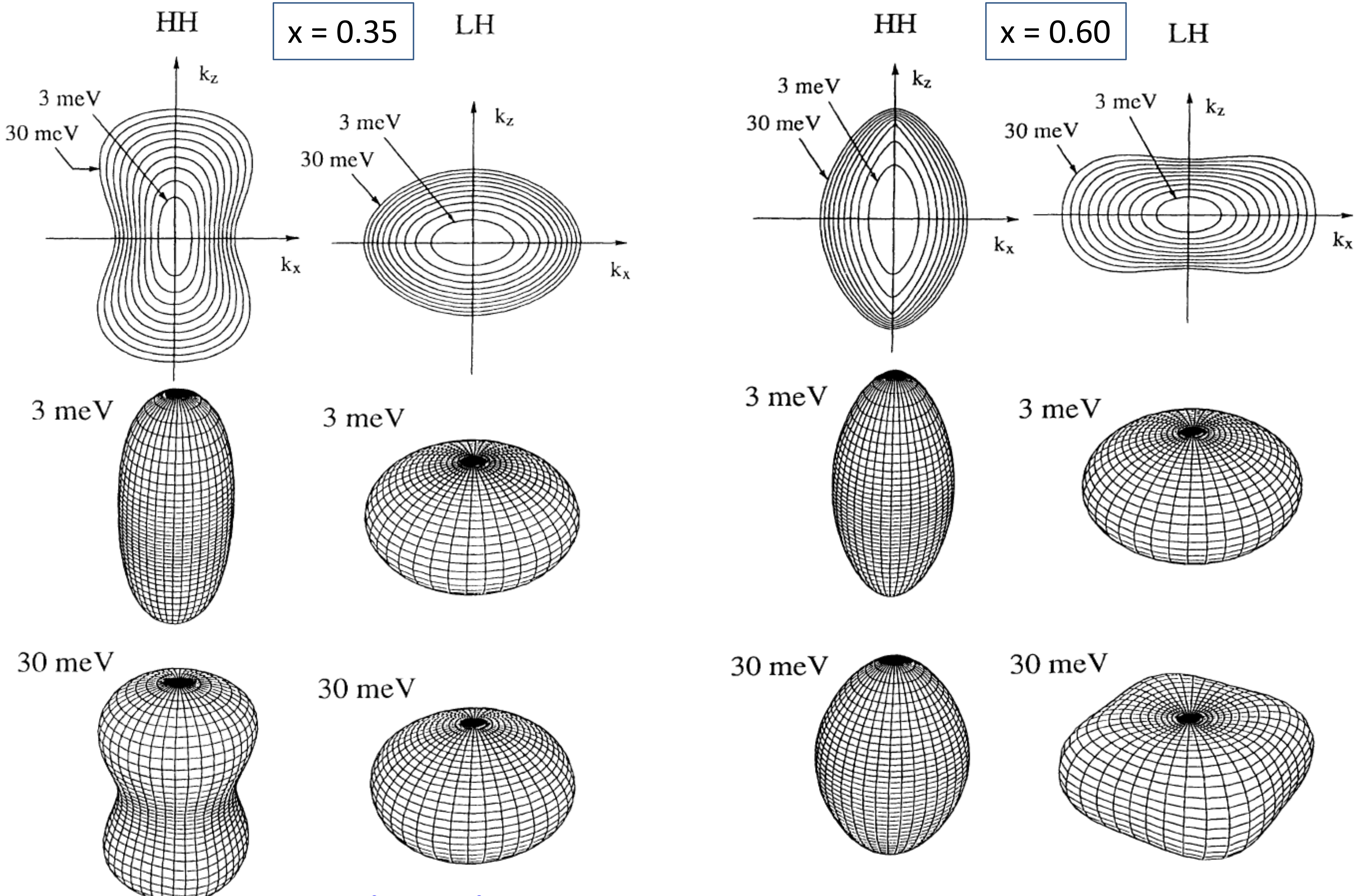
Energy band diagram of $In_{1\text{-}x}Ga_xAs$ quantum well grown on $In_{1\text{-}x}Ga_xAs_yP_{1\text{-}y}$
(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).

x = 0.468



Hole bands energy isosurfaces for unstrained $In_{1-x}Ga_xAs$ lattice matched to InP

(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).



compressive strain

tensile strain

Hole bands energy isosurfaces for $In_{1-x}Ga_xAs$ under strain

(C. Y.-P. Chao and S.L. Chuang, Phys Rev B, vol. 46, p. 4110 (1992).

Effective mass effect on Quasi-Fermi levels

- **Reducing the effective mass of holes reduces the density of states in the valence band**

- **Quasi-Fermi levels become more symmetrical with respect to the band edges, when effective masses are similar**

- **With increased symmetry, the quasi-Fermi level has to penetrates less into the conduction band to reach density for population inversion $\rightarrow$ Less degenerate**

- **Also, the carrier density necessary to reach the transparency condition is reduced**

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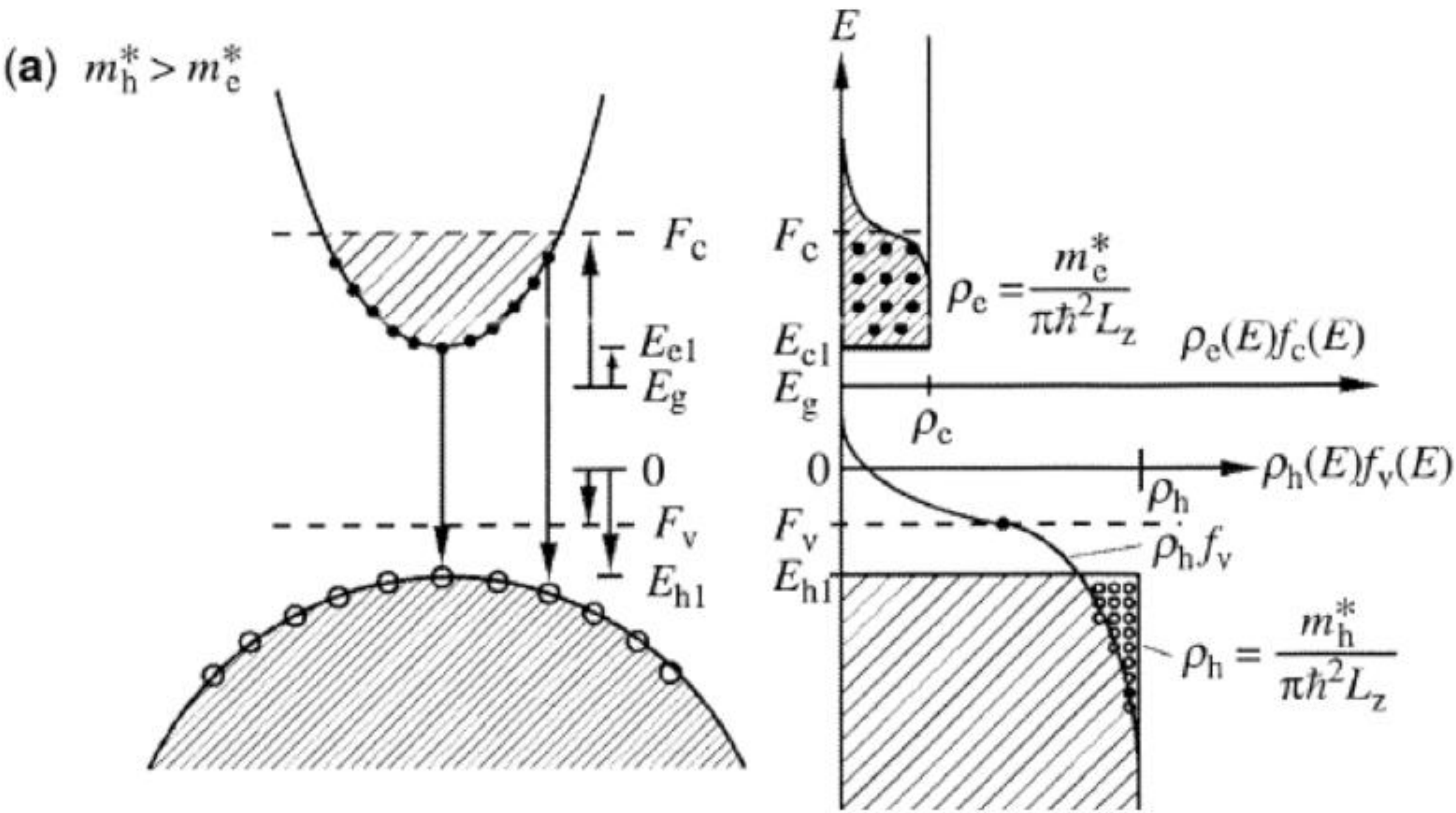
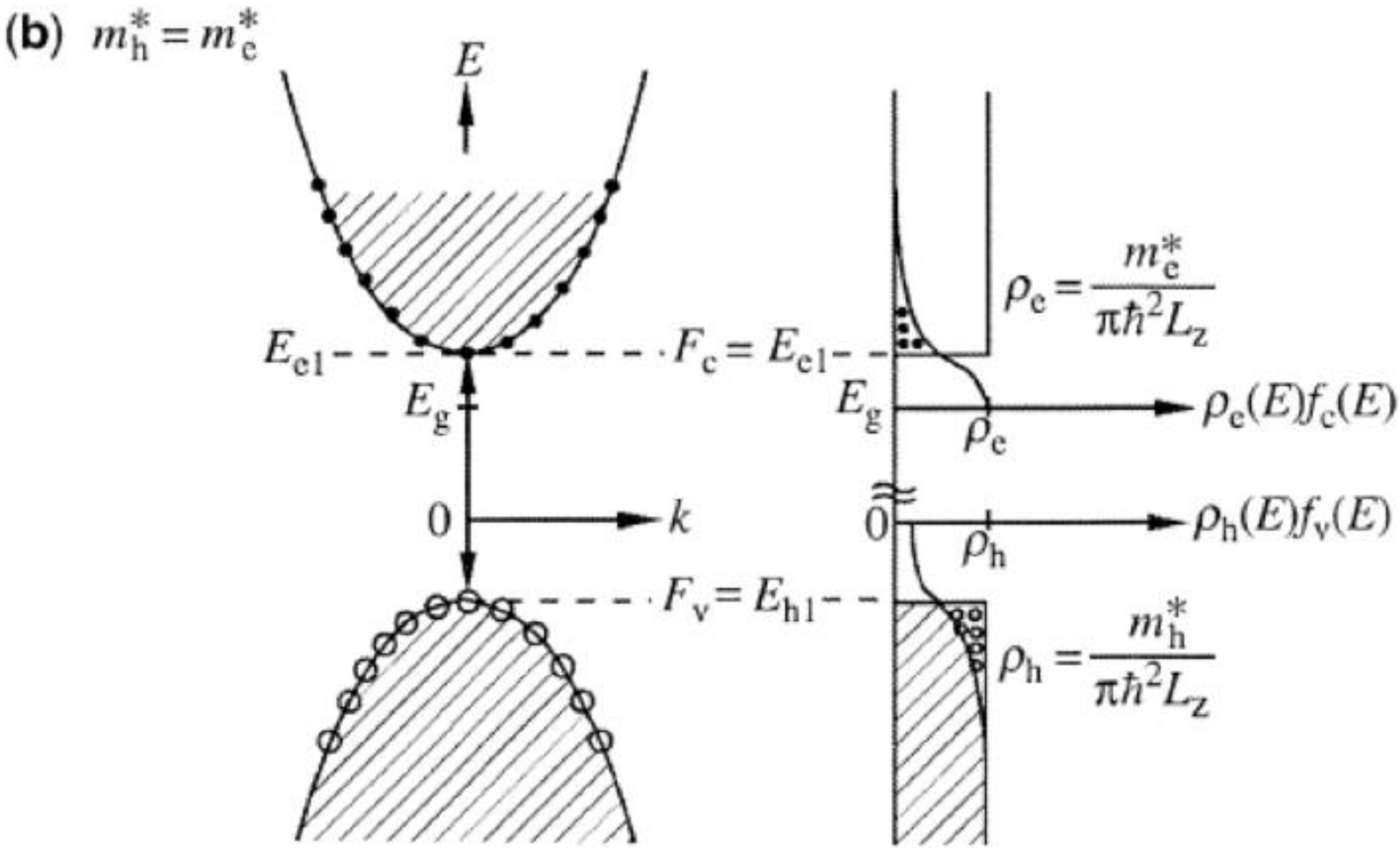


population inversion condition

$$F_c - F_v + E_g > \hbar\omega > E_g + E_{e1} - E_{h1}$$



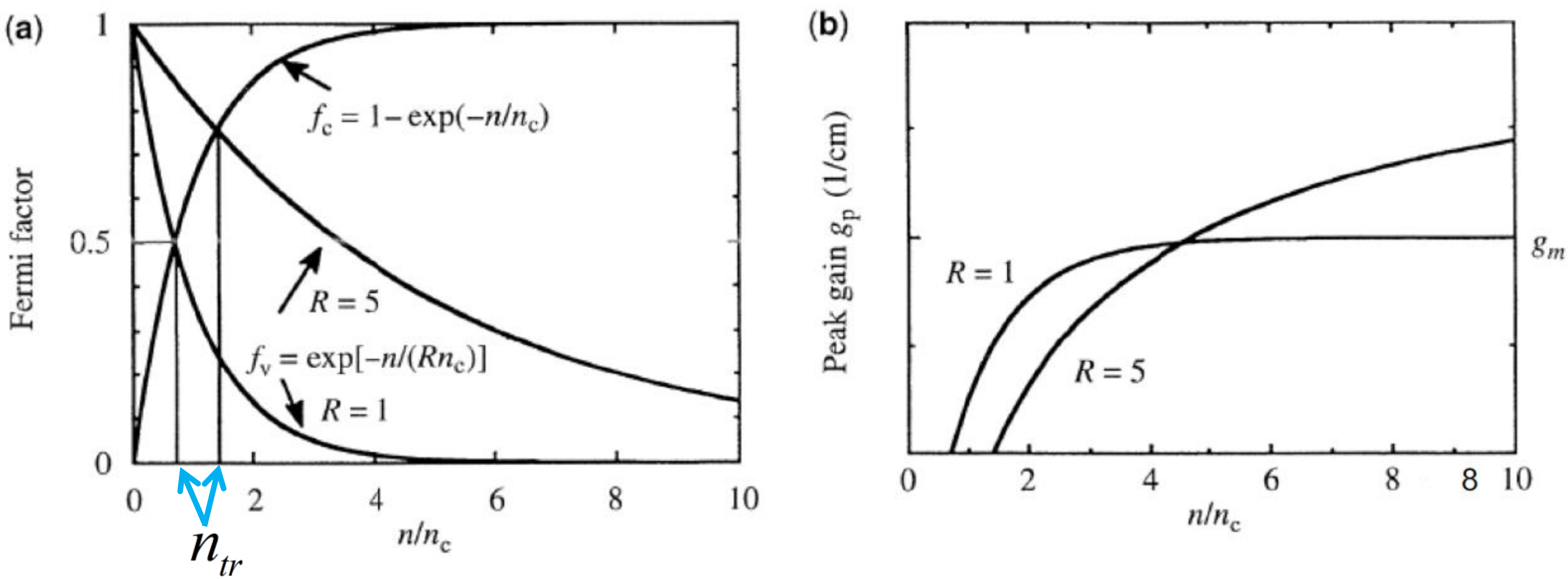
population inversion condition

$$F_c - F_v + E_g > \hbar\omega > E_g + E_{e1} - E_{h1}$$

Transparency Density and Peak Gain

- Remember our simple model for Peak Gain

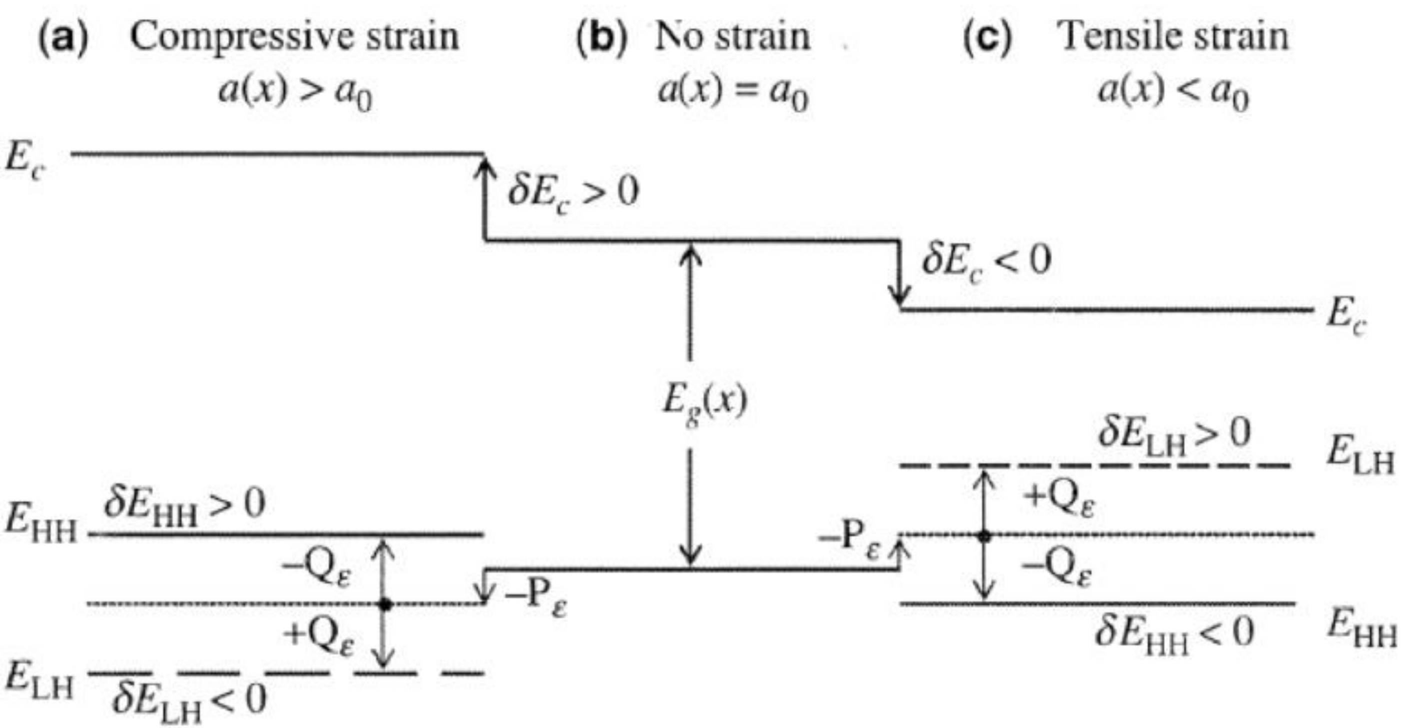
$$g_p(n) = g_{\max}\left(f_c - f_v\right) \simeq g_{\max}\left(1 - e^{-n/n_c} - e^{-n/Rn_c}\right) \qquad \text{with } R = \frac{m_h^*}{m_e^*}$$



Transparency density decreases with R but maximum achievable gain decreases $\rightarrow$ **Design trade-off**

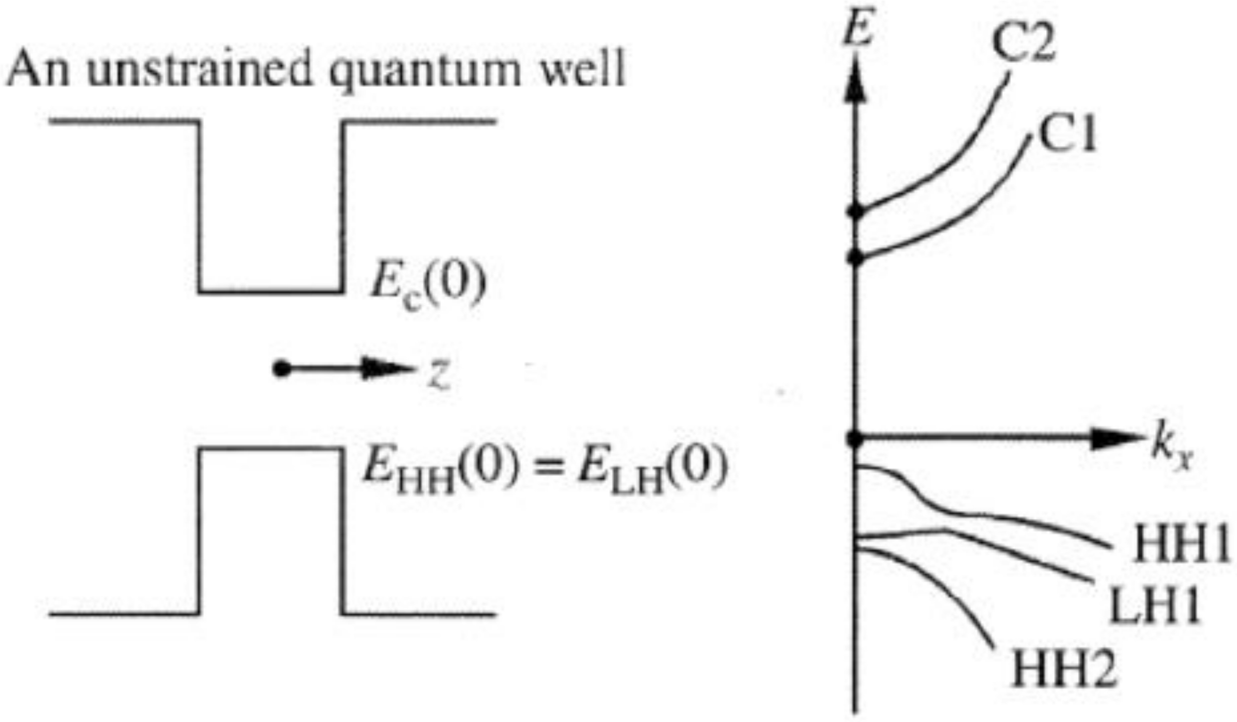
Strain Effects on Band-Edge Energies

- **Compressive strain generally increases the bandgap.**
- **Tensile strain generally decreases the bandgap.**
- **The LH band follows these trends but the HH band goes against.**
- **Strain affects also the conduction band structure of the conduction band. Mainly, the Γ, L, and X valleys shift in energy at different rates.**



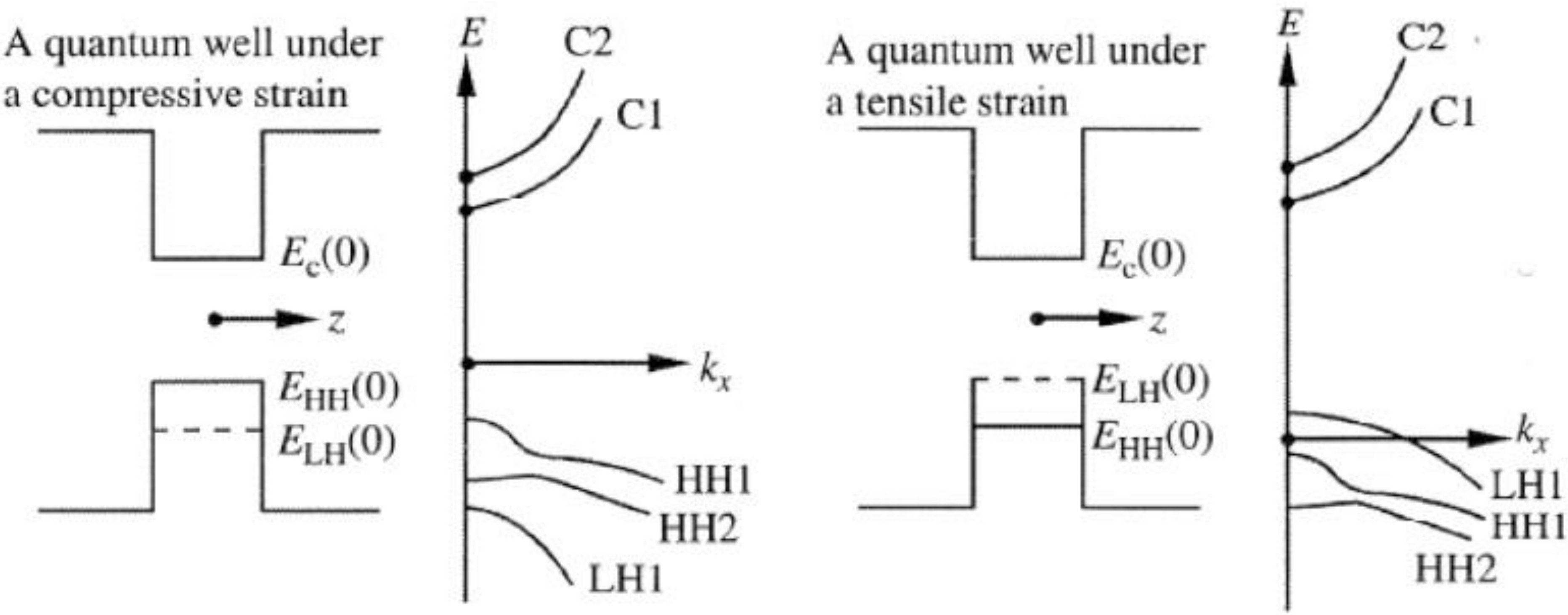
Strain Effects in Quantum Wells

- **For unstrained material, bandgap is the same for HH and LH. The energy levels in a quantum well, corresponding to HH and LH, differ because of unequal effective masses**



Strain Effects in Quantum Wells

- **For strained materials HH and LH bandgaps and the energy offsets in CB and VB are different. HH and LH are in different potential wells.**

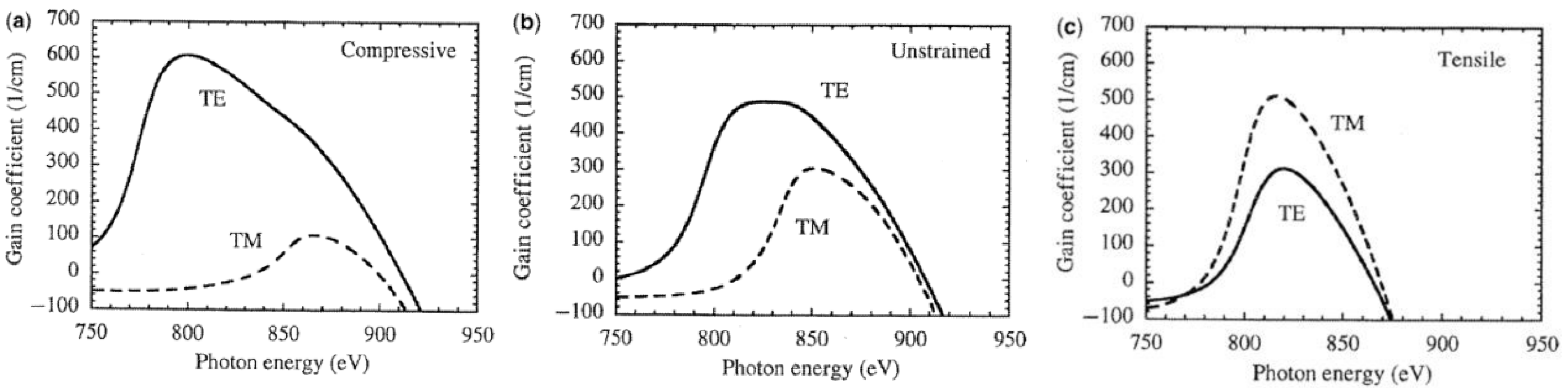


- **QW under tensile strain brings HH and LH quantum level closer to each other (LH has deeper well but higher energy levels)**

Gain Spectrum of Strained Quantum Wells

Recall that

- **C1-HH1 transition is mostly favored by TE**
- **C1-LH1 transition is mostly favored by TM**



- **Tensile strain can improve balance between TE and TM gain**
- **Trade-off is maximum gain linked to joint density of states through the ratio of effective masses**

Momentum Matrix Elements

Recall the matrix element depends on transverse wave vector k_t

For $\mathrm{k}_t = 0$,

TE Polarization :

$$\left|\hat{x} \bullet \mathbf{M}_{\text{c-hh}}\right|^2 = \left|\hat{y} \bullet \mathbf{M}_{\text{c-hh}}\right|^2 = \frac{3}{2} M_b^2$$

$$\left|\hat{x} \bullet \mathbf{M}_{\text{c-}l\text{h}}\right|^2 = \left|\hat{y} \bullet \mathbf{M}_{\text{c-}l\text{h}}\right|^2 = \frac{1}{2} M_b^2$$

TM Polarization :

$$\left|\hat{z} \bullet \mathbf{M}_{\text{c-hh}}\right|^2 = 0$$

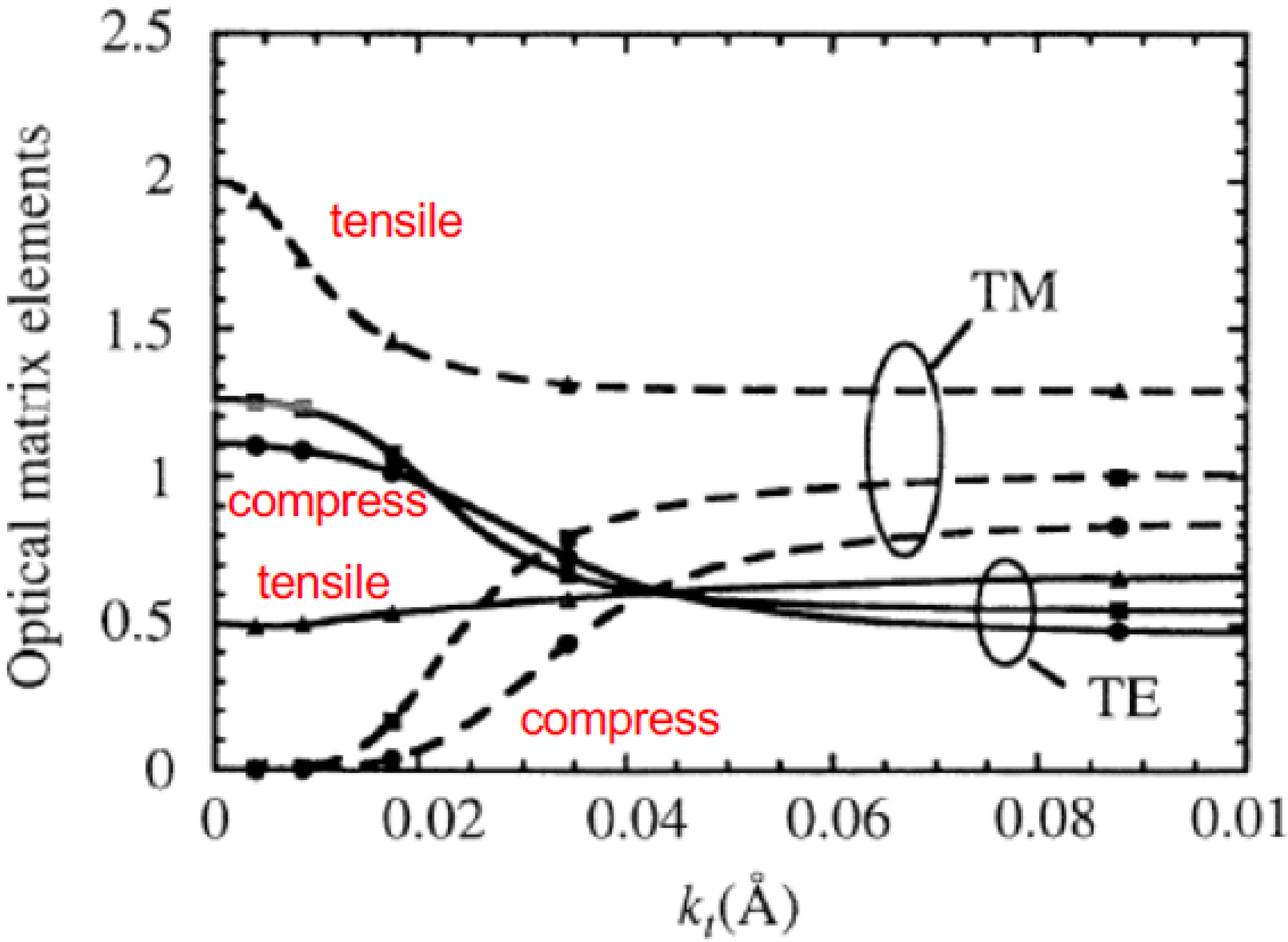
$$\left|\hat{z} \bullet \mathbf{M}_{\text{c-}l\text{h}}\right|^2 = 2 M_b^2$$

Gain depends on surface (sheet) carrier concentration

$$\boxed{n_s = n \times L_z}$$

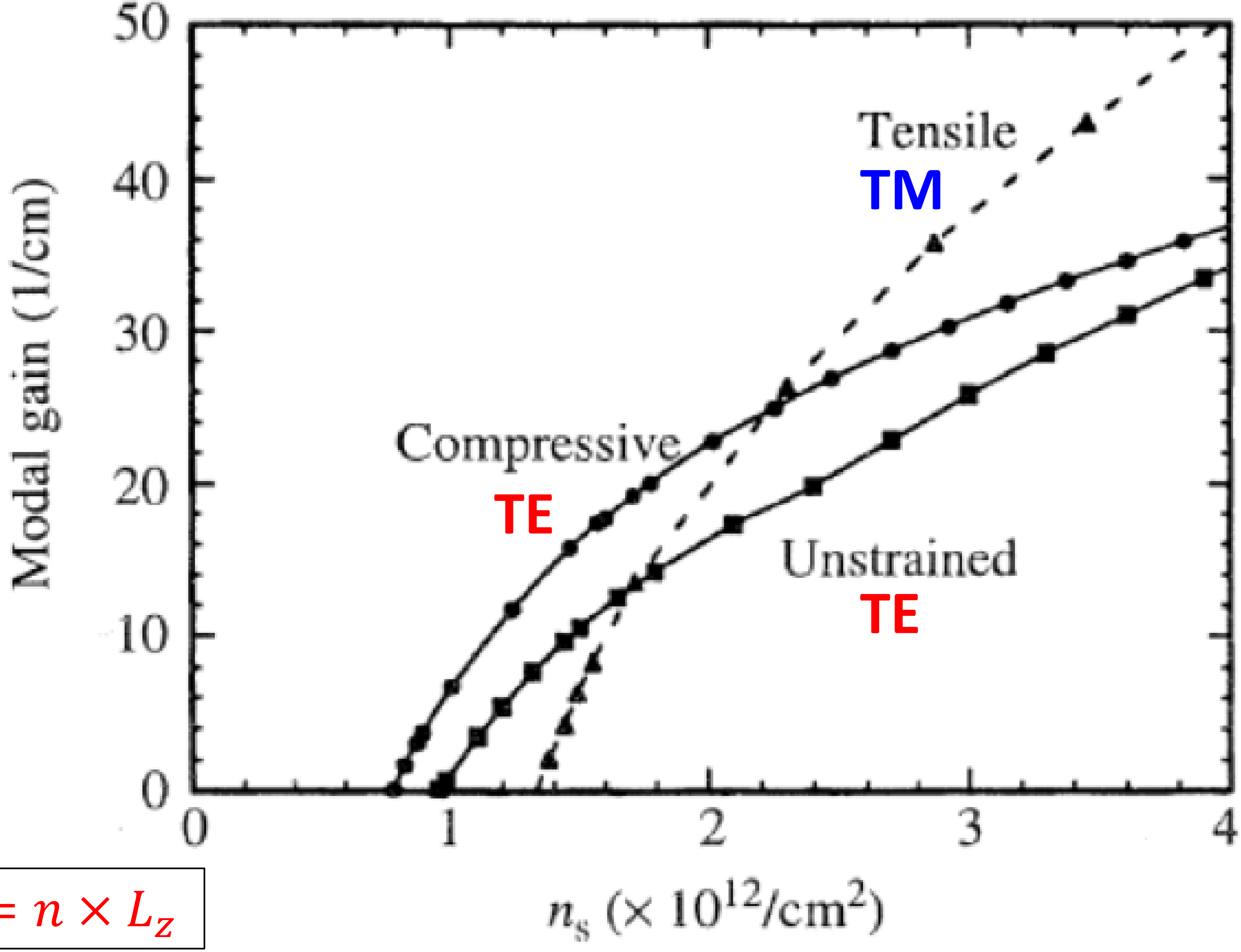
Momentum Matrix Element

- Normalized as $2|M_{nm}(k_t)|^2/M_b^2$ with $n = C1, m = HH1$ for compressive strain and $m = LH1$ for tensile strain]



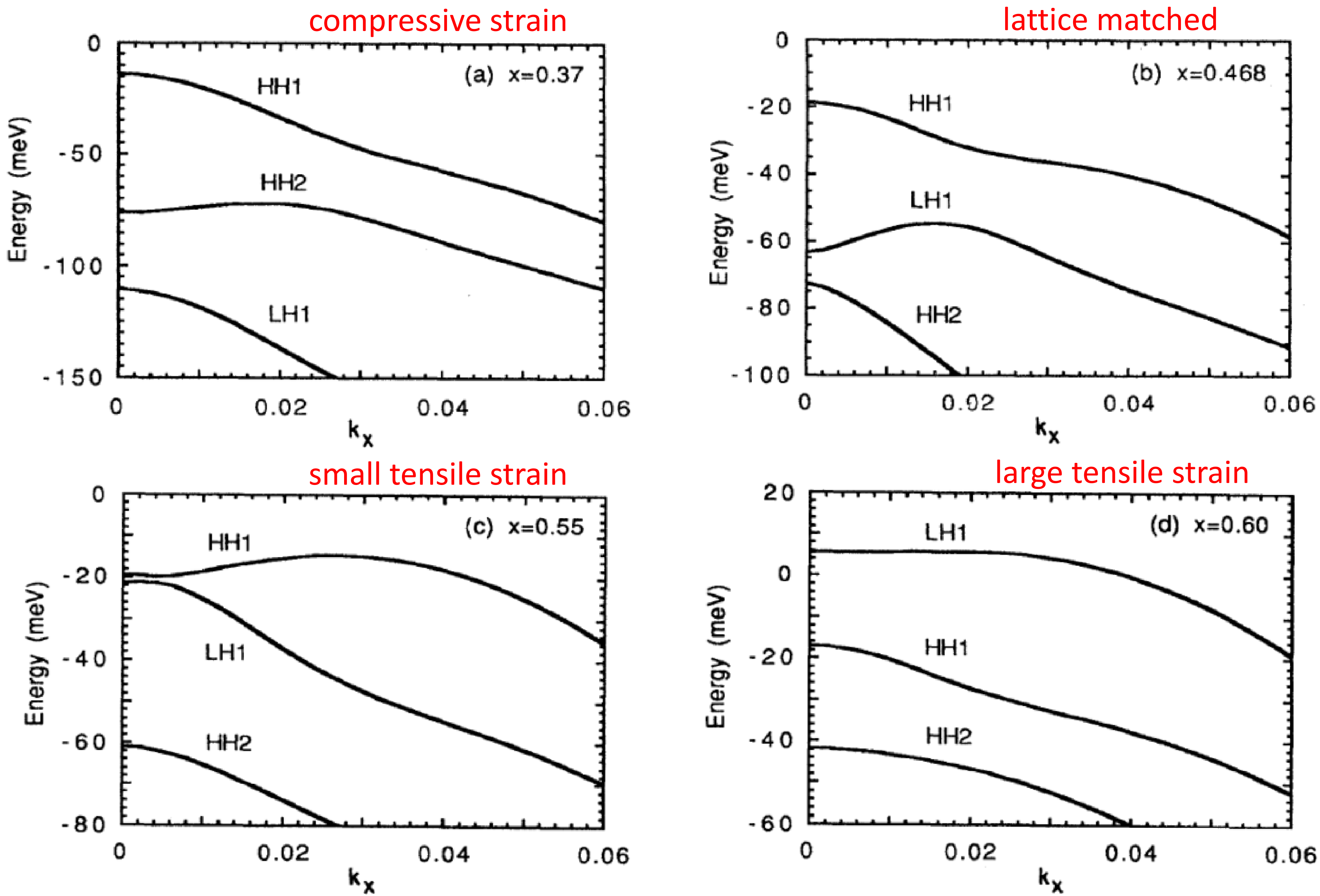
Modal gain versus sheet concentration

For $In_{1-x}Ga_xAs$ / $In_{1-x}Ga_xAs_yP_{1-y}$ quantum well laser working near 1.55μm



$$n_s = n \times L_z$$

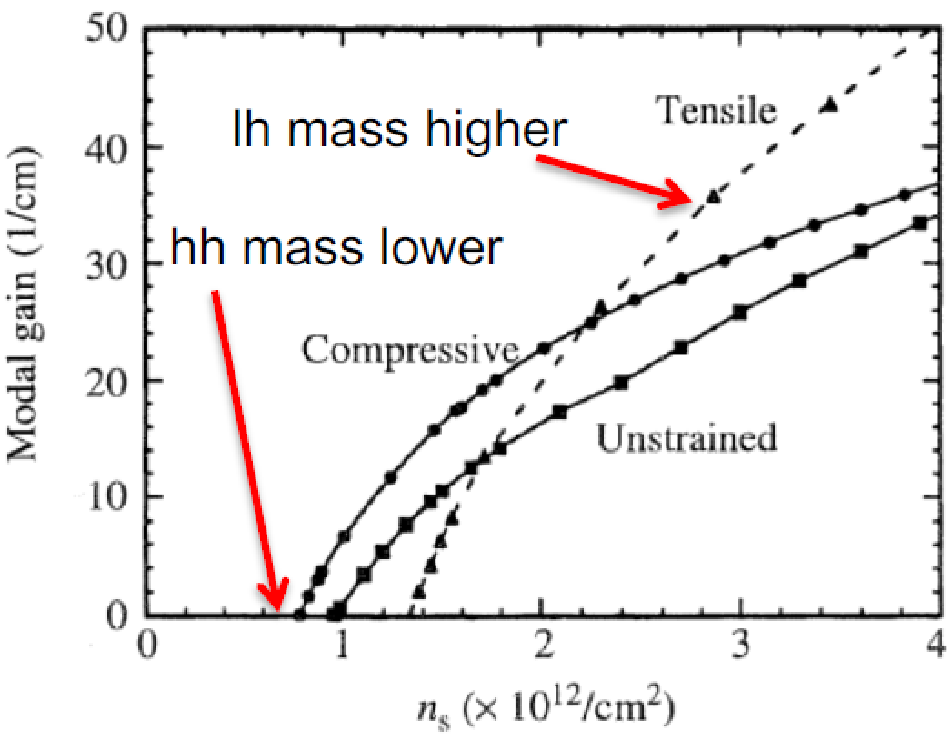
Band Distortion in Strained Quantum Wells



(S.L. Chuang, Phys Rev B, vol. 43, p. 9649 (1991)

Some comments

- **LH can become heavier, HH can become lighter**
- **Compressively-strained materials can have lower valence band density of states (because HH at the top of the VB can have effective mass lower than LH at the top of the VB in the case of tensile strain)**



$$\boxed{n_s = n \times L_z}$$

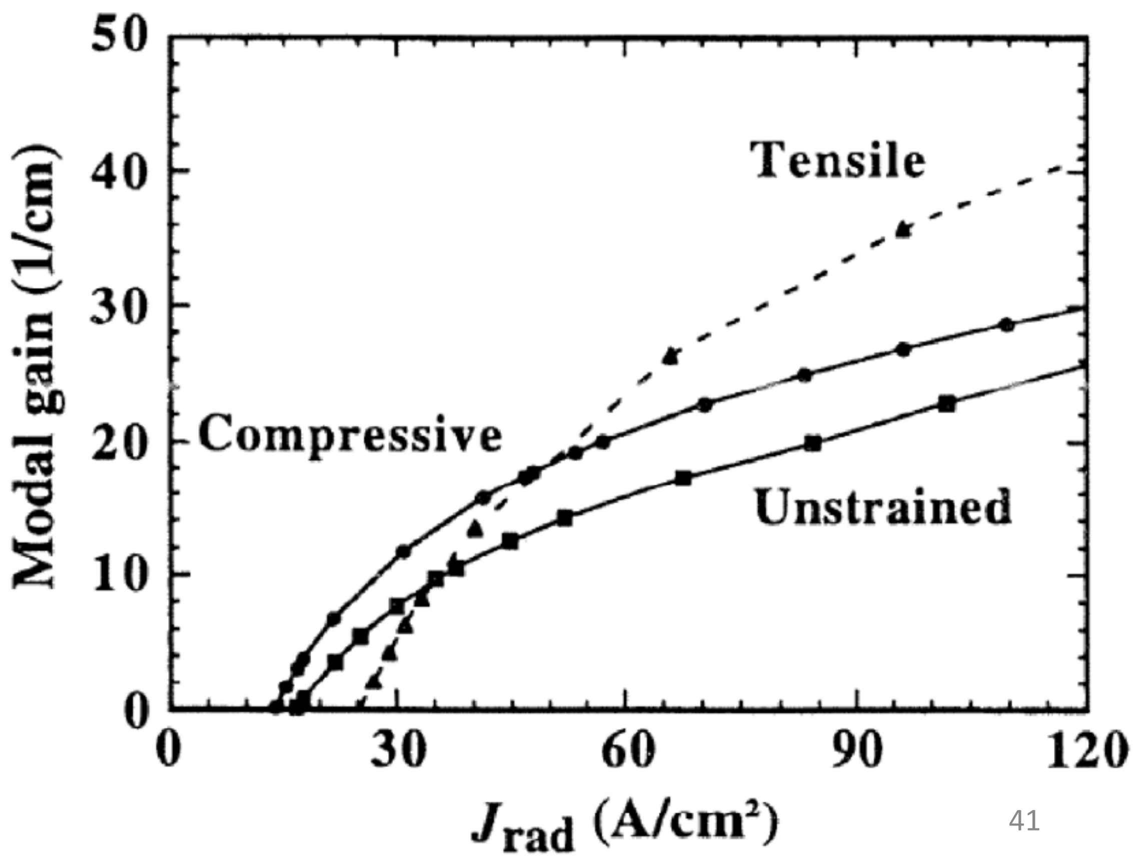
Modal Gain versus Current Density

- **Modal Gain** $$\Gamma g \propto \frac{L_z}{W_{\text{mode}}} g$$
- **Empirical relationship** $$G = n_w \Gamma_w g_w = n_w \Gamma_w g_0 \left[\ln\left(\frac{n_w J_w}{n_w J_0} \right) + 1 \right]$$

Compressive strain
Smaller transparency carrier density but saturates faster.

Tensile strain
Larger transparency carrier density but increases faster (higher differential gain).

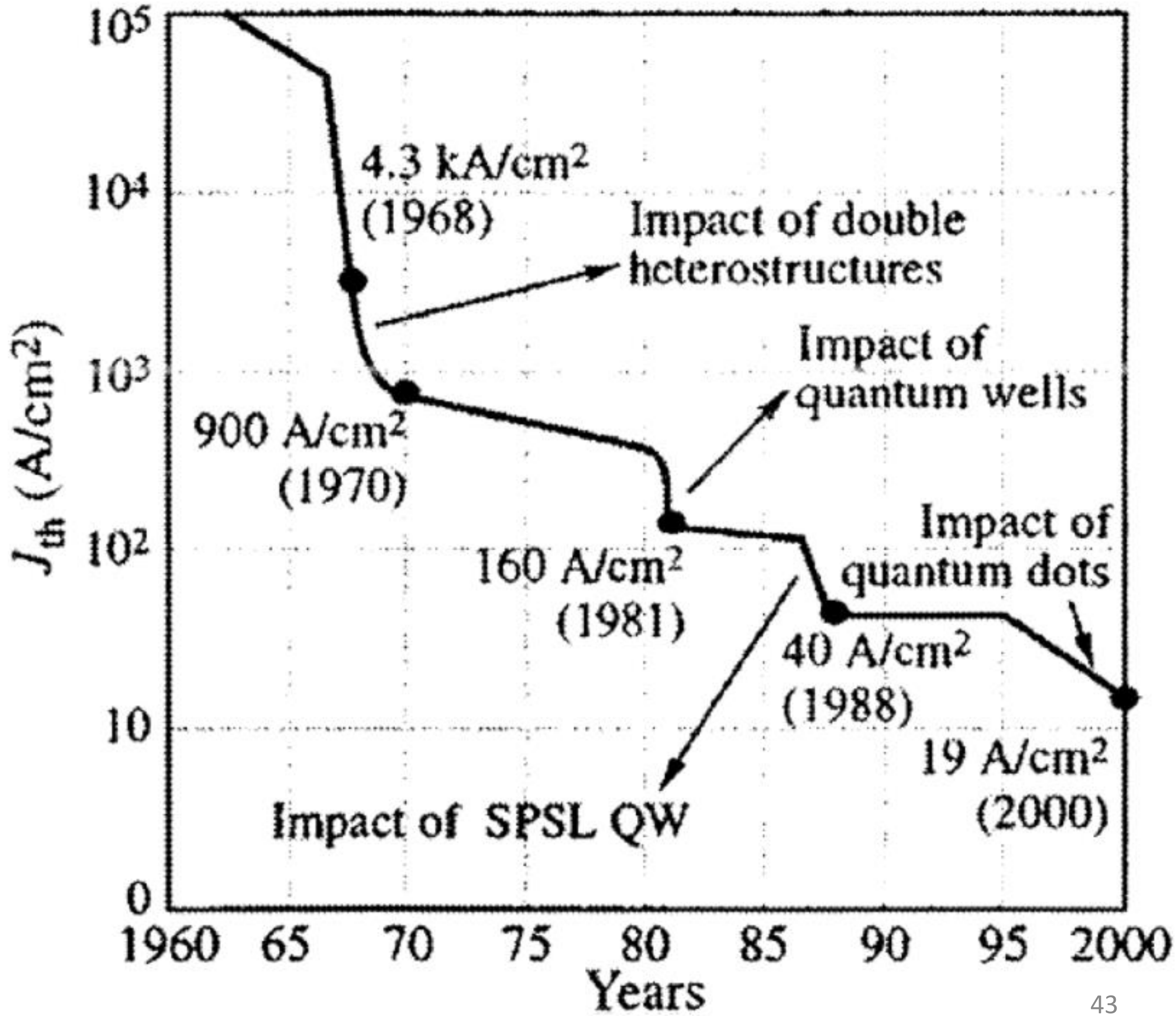
Loss mechanisms (Auger, recombination, intervalence band absorption) need to be factored in when considering current density.



Quantum Dot Lasers

(brief considerations)

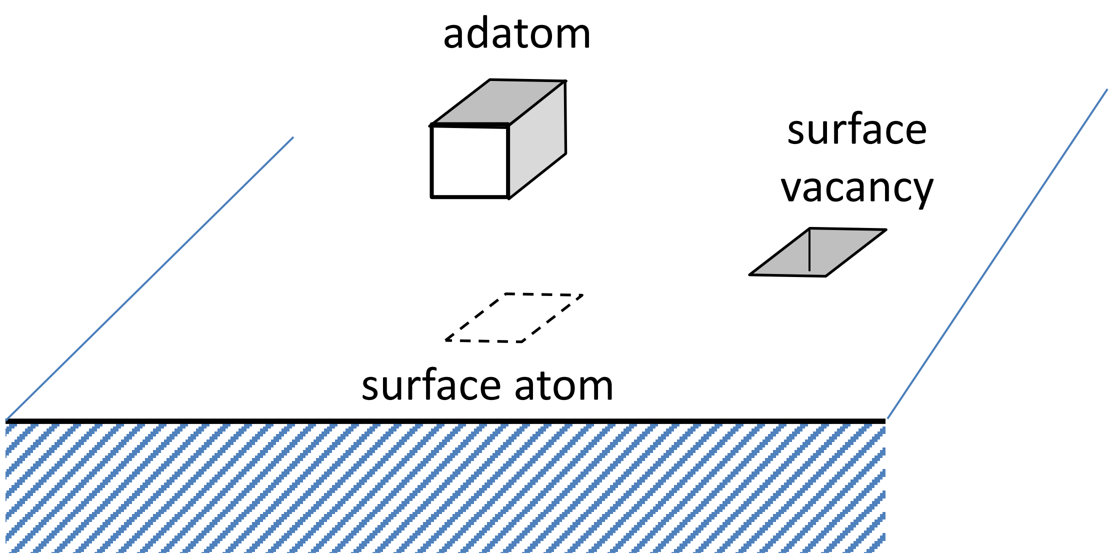
Technology Advanced in Laser Diodes



SPSL = Short Period Superlattice

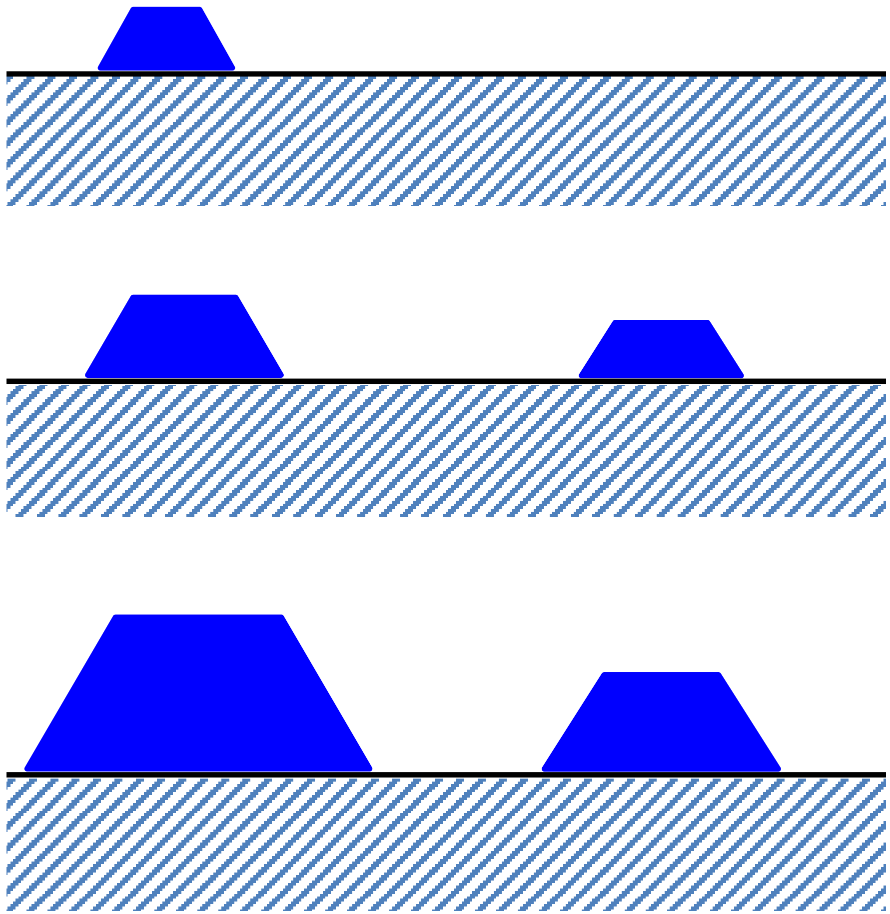
(Alternate monolayers of GaAs & AlAs instead of growing AlGaAs gives less fluctuation in QW width)

Thin film epitaxial growth



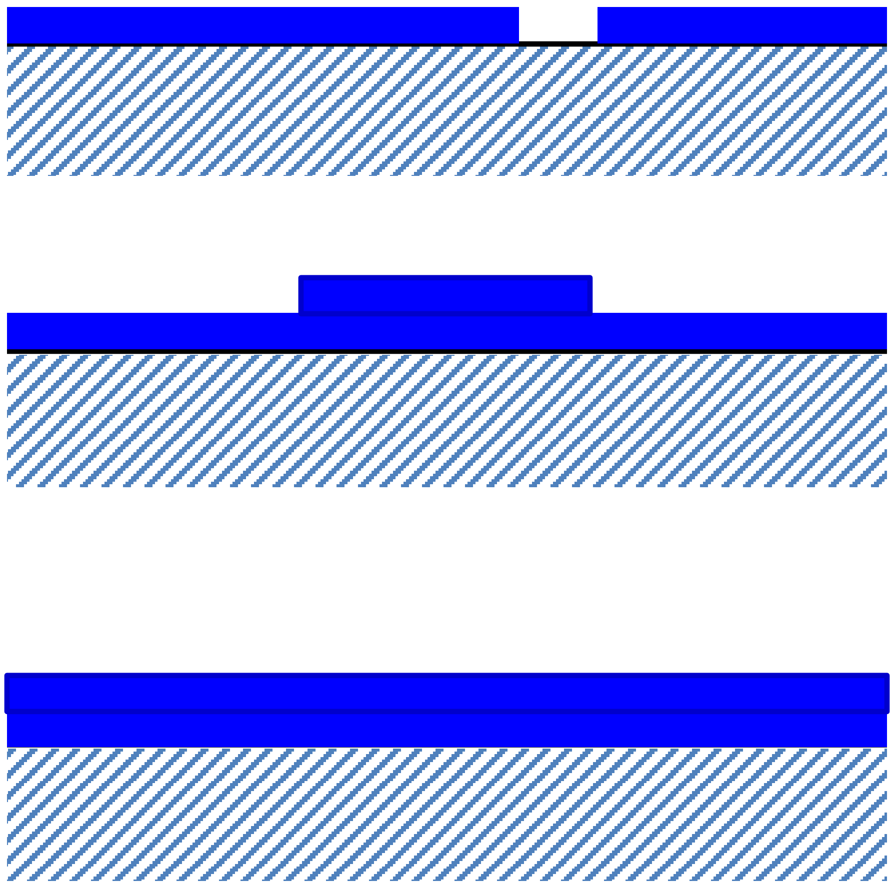
“adatom” (adsorbed atom) is an atom lying on a surface and is the opposite of a surface vacancy

Volmer-Weber growth: island formation



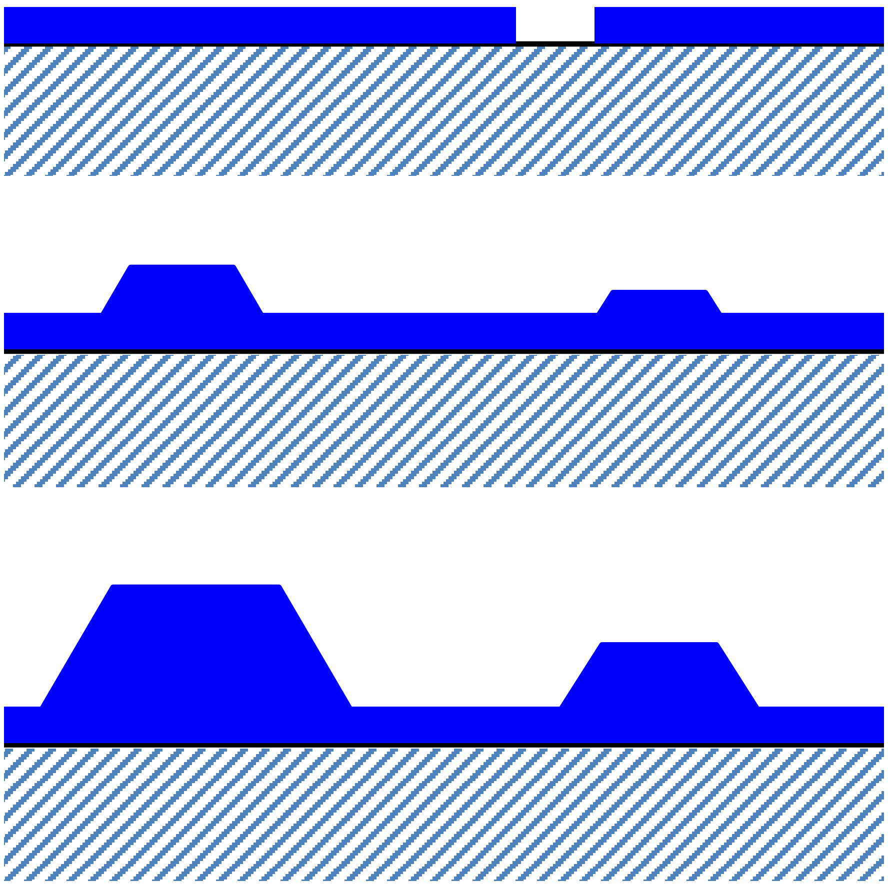
Formation of 3D clusters : *adatom-adatom* interactions stronger than adatom-surface. Formation of rough multi-layer films.

Frank-van der Merwe: layer-by-layer formation



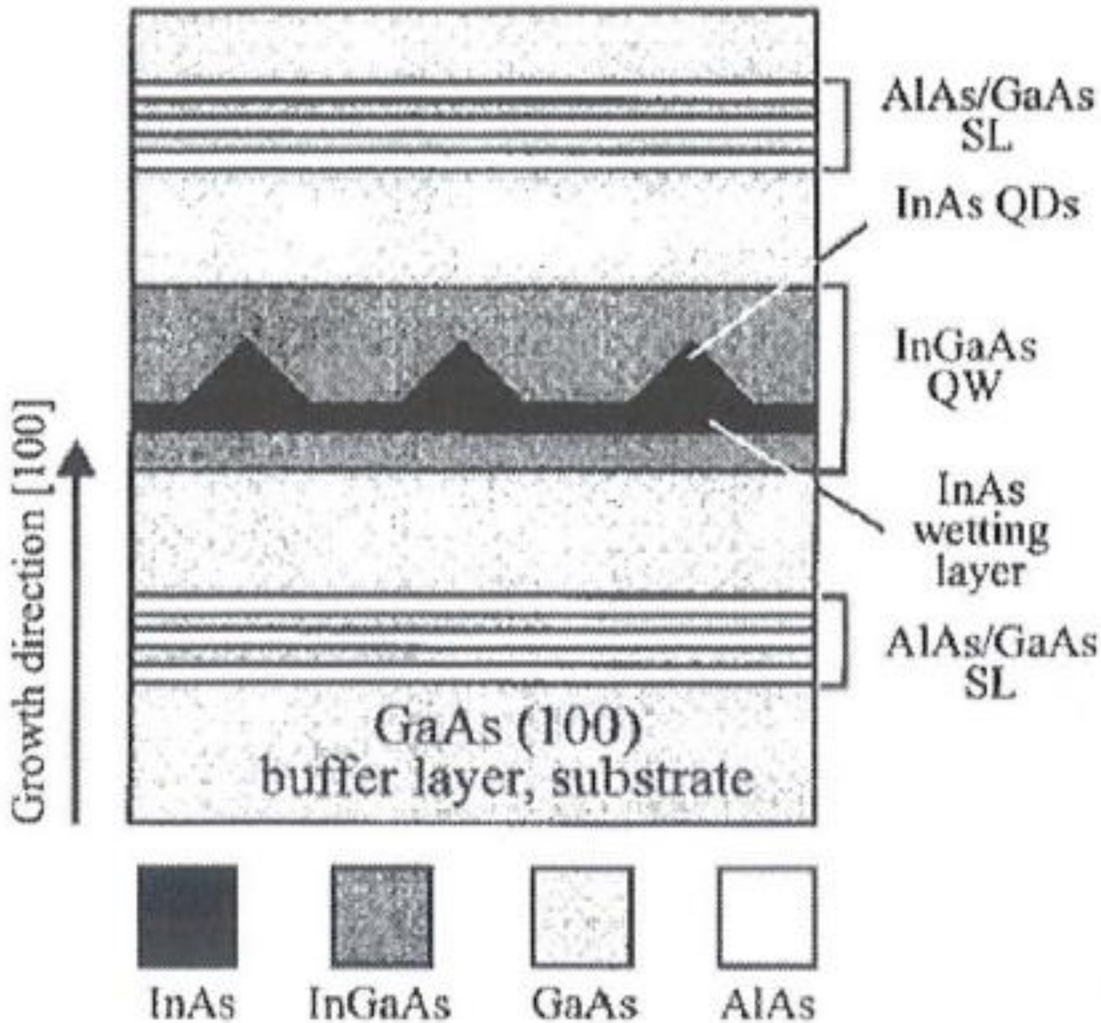
Formation of 2D layers : *adatoms* attach preferentially to surface sites. Formation of atomically smooth layers.

Stransky-Krastanov: layer-plus-island formation



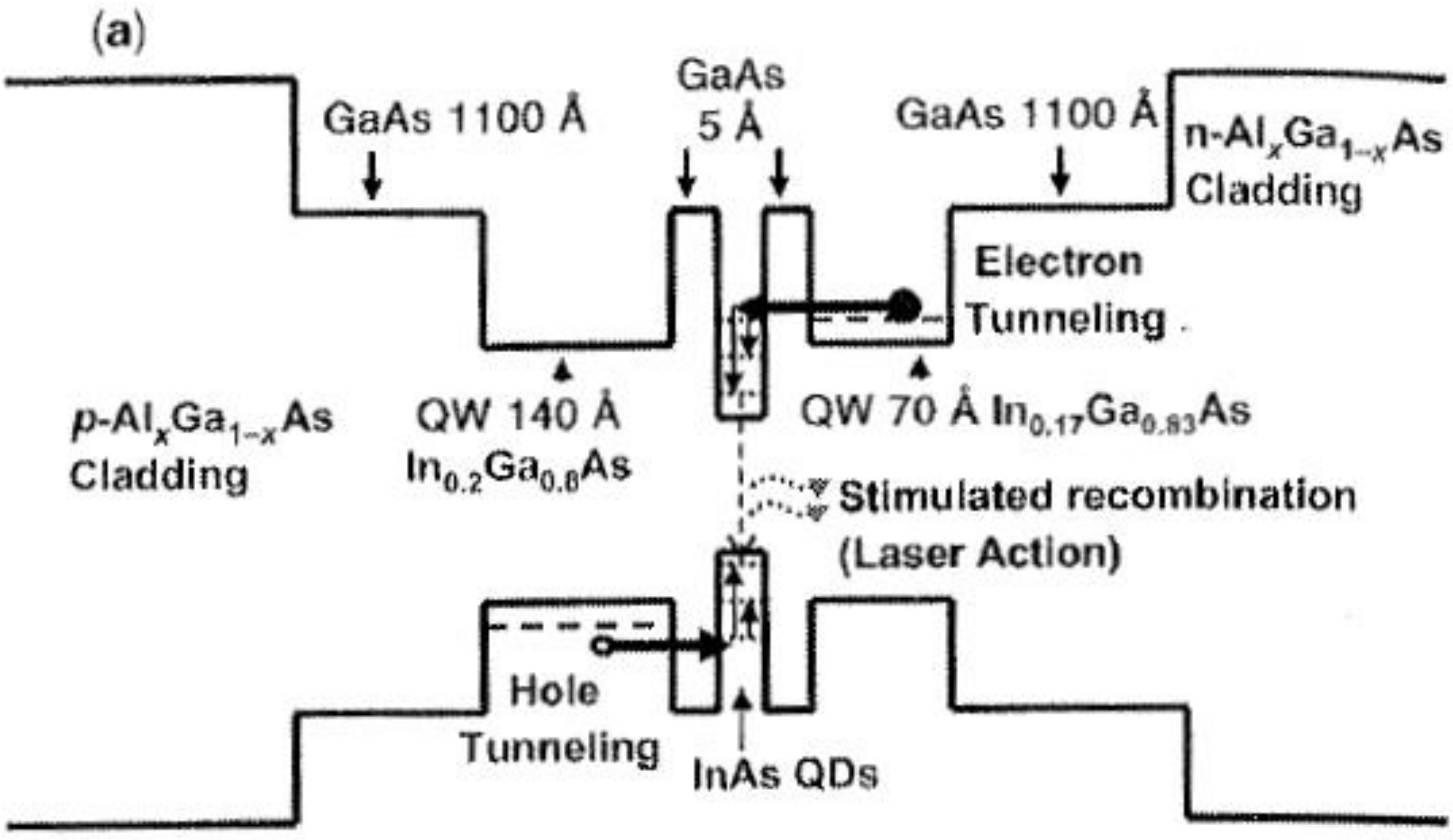
Formation of clusters : intermediate process with 2D and 3D island growth. Transition from layer-by-layer to island growth occurs at a critical layer thickness.

InAs QD array in an InGaAs QW on GaAs



Can form 3D strained islands (growth of sheets of dots on top of each other, with vertical coupling of the dots)

Tunnel Injection



Tunnel injection allows better carrier collection by the QW with reduced J_{th} , faster modulation, smaller linewidth enhancement.

Reading Assignments:

- Sections 10.3 and 10.4 of Chuang's book
- Section 8.2.5, Appendices 1,2,3,9 (supplemental) in Coldren, Corzine and Mašanović

- Section 4,5 of Chuang's book

If you are interested in a general reference source on strain you can download from the university library:

Y. Sun, S. E. Thompson, T. Nishida

Strain Effect in Semiconductors
Theory and Device Applications

Springer (2010)