ECE 536 – Integrated Optics and Optoelectronics
Lecture 19 – March 29, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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Lecture 19 Outline

- Semiconductor Optical Amplifiers
- Integrated Optics Simulation Approaches
- Transmission Matrix Method

Semiconductor Optical Amplifier (SOA)

Basic Characteristics of SOA

- **SOAs are typically optical active regions in a semiconductor that are used without any optical feedback.**

- **An optical signal input experiences gain through stimulated emission.**

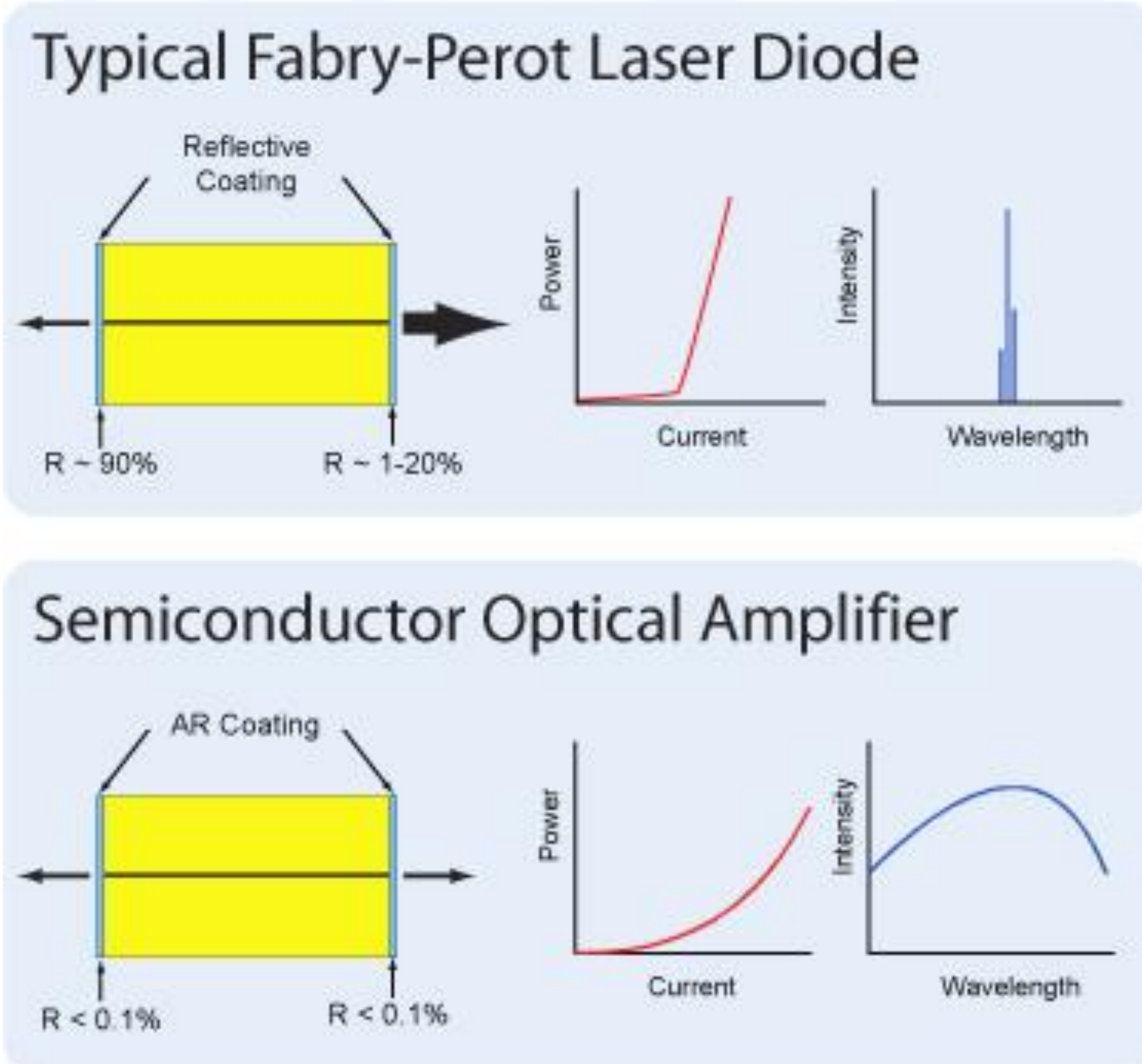
- **Spontaneous emission is added to the signal and then amplified through stimulated amplification while propagating in the structure.**

Basic Characteristics of SOA

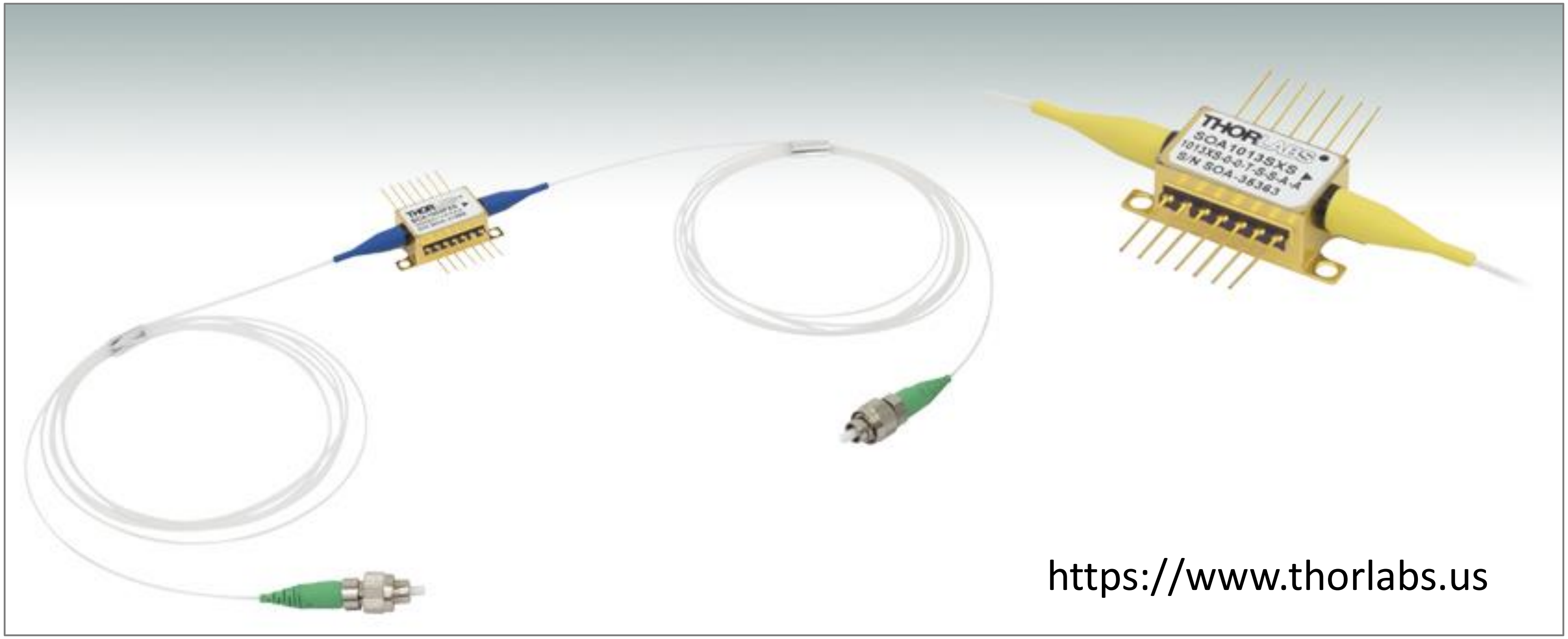
- **Noise is added by spontaneous emission and amplified spontaneous emission**

- **Carrier density and gain are nor clamped if there is no feedback in the cavity. Larger fraction of carriers recombine through spontaneous emission, compared to a laser.**

- **SOAs are use mainly for photonic integration, but discrete SOAs also exist.**



https://www.thorlabs.us/images/TabImages/SOA-LD-Comparison.jpg



Specifications[a]		
Item #	**SOA1013SXS**	**BOA1004PXS**
Operating Wavelength	1528 - 1562 nm	1500 - 1600 nm
Optical Isolation (P_{IN} / P_{OUT})[b]	≥42 dB	≥40 dB
Extinction Ratio[c]	60 dB	70 dB
Switching Speed	1 ns	1 ns
Max Output Power for CW Input Signal	17 dBm	18 dBm
Max Output Power for Modulated Input Signal	9 dBm	10 dBm

a. Typical values. For complete specifications, please see *Specs* tab.
b. At 0 mA and 1550 nm
c. At P_{IN} = -20 dBm and 1550 nm

SOA

Four important parameters characterize the performance of SOA:

- Signal gain
- Frequency bandwidth
- Saturation output power
- Noise figure

The measured signal gain of an SOA, in decibels, is given by

$$G = 10 \log[P_{\text{out}}/P_{\text{in}}]$$

If it refers to a single light path from input to output (Travelling Wave Amplifier, TWA) the resulting gain is known as “single pass gain” $G = G_S$. If positive feedback is provided by reflections from end-facets (Fabry-Pérot amplifier)

$$G = \frac{G_{\text{s}}}{1 + F_{\text{B}} G_{\text{s}}}$$

F_B = proportion of output signal fed back to the input

SOA

The signal gain of an optical amplifier is limited by a finite range of input and output power. Experimentally, once the input power is increased to a certain level, the gain starts to drop.

material gain coefficient

unsaturated value of g_m

$$G_s = \exp(g_m L) = \exp[g_0 L/(1 + I_{out}/I_{sat})]$$

output light intensity

saturation output intensity

The pumping source creates a fixed amount of population inversion. As we increase the input power, as some point the rate of draining due to amplification is greater than the rate of pumping and the population inversion level starts to fall.

Gain saturation simply arises because of conservation of energy.

SOA Gain (Section 8.2.5 in Coldren, Corzine and Mašanović)

Gain of an SOA of Length L :

$$G_0 = e^{\Gamma g(N)L}$$

Rate Equation :

$$\frac{dN}{dt} = \frac{\eta_i I}{qV} - \frac{N}{\tau} - \frac{\Gamma_{xy}\, g}{wd}\frac{P}{h\nu}$$

(V is volume of active region)

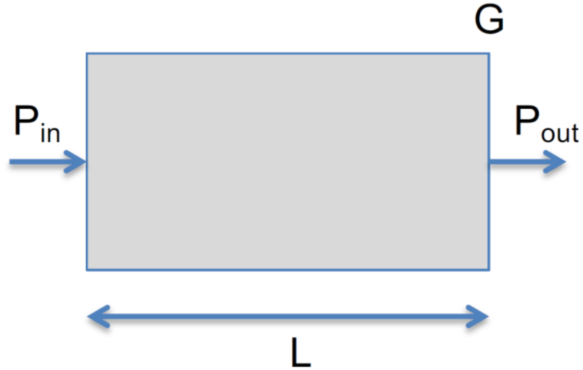
Steady State:

$$\frac{dN}{dt} = 0$$

$$\frac{dN}{dt} \approx \frac{\eta_i I}{qV} - \frac{N}{\tau} = 0$$

$$N = N_0$$

$$\frac{\eta_i I}{qV} = \frac{N_0}{\tau}$$



SOA Gain

At low optical powers :

$$N = N_0 = \frac{\eta_i I \tau}{qV}$$

$$g_0 = a\left(N - N_{tr}\right) = a\left[\frac{\eta_i I \tau}{\left(qV\right)} - N_{tr}\right]$$

differential gain

$$\frac{dN}{dt} \approx \frac{\eta_i I}{qV} - \frac{N}{\tau} = 0$$

$$N = N_0$$

$$\frac{\eta_i I}{qV} = \frac{N_0}{\tau}$$

At large input / output powers, stimulated emission is included :

$$N = \frac{\eta_i I \tau}{qV} - \frac{\Gamma_{xy} g \tau}{wd} \frac{P}{h\nu} \quad \rightarrow \quad g = \frac{wdh\nu}{\Gamma_{xy} \tau P}\left(\frac{\eta_i I \tau}{qV} - N\right)$$

$$g = \frac{g_0}{1 + P / P_s} \quad \text{and} \quad P_S = \frac{wdh\nu}{a\Gamma_{xy} \tau} \quad \left(\mathrm{P_s} \text{ typically 1-20 mW, depends upon } \Gamma\right)$$

[QW SCH]

SOA Amplifier Response

Net Amplifier Response :

Integrate over gain length for amplifier response:

$$\frac{dP}{dz} = gP = \frac{g_0}{1 + P / P_s} P = g_0 \left(\frac{1}{P} + \frac{1}{P_s} \right)^{-1}$$

$$G = \frac{P(L)}{P(0)} = \frac{P_{out}}{P_{in}} = G_0 \exp\left[-\frac{G-1}{G} \frac{P_o}{P_s} \right]$$

large signal gain

$G_0 = e^{gL} = e^{g_0 L}$ is the unsaturated gain when $P_o \ll P_s$

SOA Amplifier Response

Proof :

$$\int \left(\frac{1}{P} + \frac{1}{P_s} \right) dP = \int g_0 dz$$

$$\ln\left(\frac{P(L)}{P(0)} \right) + \frac{P(L) - P(0)}{P_s} = g_0 L$$

$$\ln(G) + \frac{P(0)}{P_s}(G-1) = g_0 L$$

$$\ln(G) + \frac{P(L)}{P_s}\left(\frac{G-1}{G} \right) = g_0 L$$

$$\ln(G) = g_0 L - \left(\frac{G-1}{G} \right) \frac{P(L)}{P_s}$$

$$G = G_0 \exp\left[-\left(\frac{G-1}{G} \right) \frac{P(L)}{P_s} \right] \text{where } \mathrm{G}_0 = e^{g_0 L}$$

SOA Amplifier Response

Output Saturation Power :

The output saturation power $P_{o,sat}$ is defined as the power that causes the gain G to drop to half of G_0 :

$$G = G_0 \exp\left[-\frac{G-1}{G}\frac{P_{o,sat}}{P_s}\right] = \frac{1}{2}G_0$$

$$P_{o,sat} = \frac{G_0 \ln 2}{G_0 - 2} P_s$$

Proof :

$$G = G_0 \exp\left[-\left(\frac{G-1}{G}\right)\frac{P(L)}{P_s}\right]$$

$$P(L) = -P_s\left(\frac{G}{G-1}\right)\ln\left(\frac{G}{G_0}\right)$$

$$P_{o,sat} = -P_s\left(\frac{G_0/2}{G_0/2-1}\right)\ln\left(\frac{1}{2}\right)$$

$$\ln 2 = -\ln\frac{1}{2} \approx 0.693$$

SOA Noise Figure :

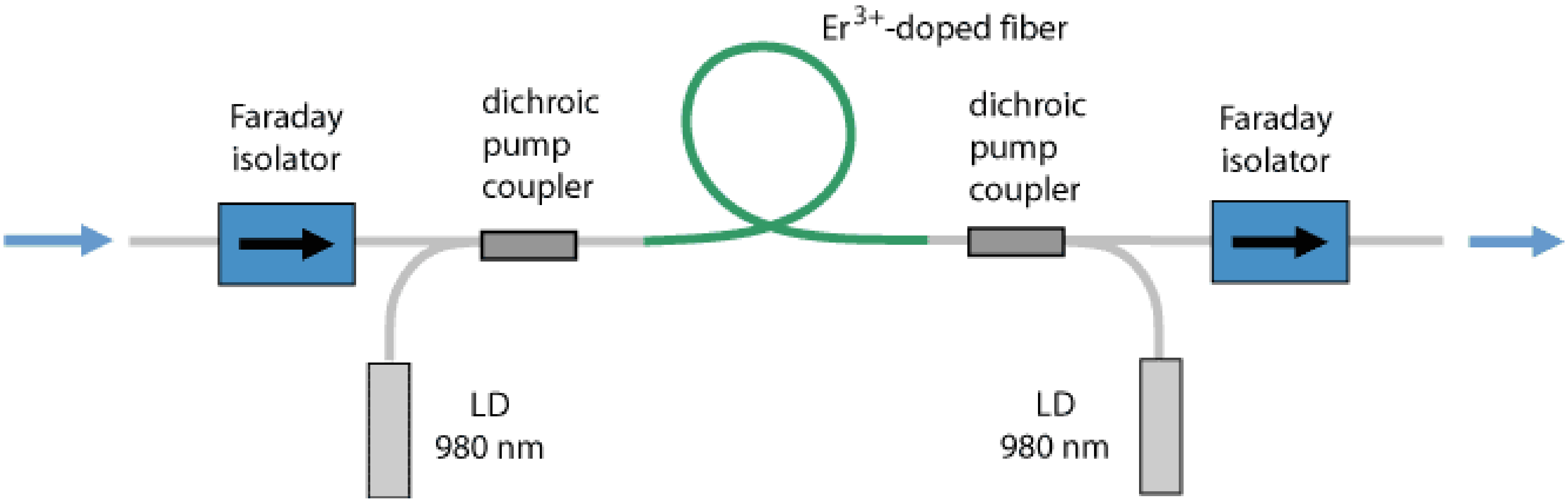
$$F_A = 2n_{sp}\left(\frac{g}{g-\alpha_i}\right) = 2\frac{f_2(1-f_1)}{f_2-f_1}\left(\frac{g}{g-\alpha_i}\right) \quad \text{(typically ~5dB)}$$

(Neglecting facets feedback)

Saitoh T, Mukai T. Traveling-wave semiconductor laser amplifiers. In: Yamamoto Y, editor. *Coherence, amplification and quantum effects in semiconductor lasers*. New York: Wiley; 1991. Chapter 7.

- For photonic integrated circuits including optical receivers, SOAs should not have polarization dependence. Structures needs to be designed for gain and optical confinement factors are similar for TE and TM modes.
- Inclusion of **strain** in the active region is an approach commonly used to achieve this.

Famous application of optical amplifiers: Erbium-doped fiber



Schematic setup of a simple erbium-doped fiber amplifier. Two laser diodes (LDs) provide the pump power for the erbium-doped fiber. The pump light is injected via dichroic fiber couplers. Pig-tailed optical isolators reduce the sensitivity of the device to back-reflections.

https://www.rp-photonics.com/erbium_doped_fiber_amplifiers.html

Erbium-doped fiber example

400 mW pump power at 980 nm from each side

1 mW input signal power. Amplified Spontaneous Emission is included

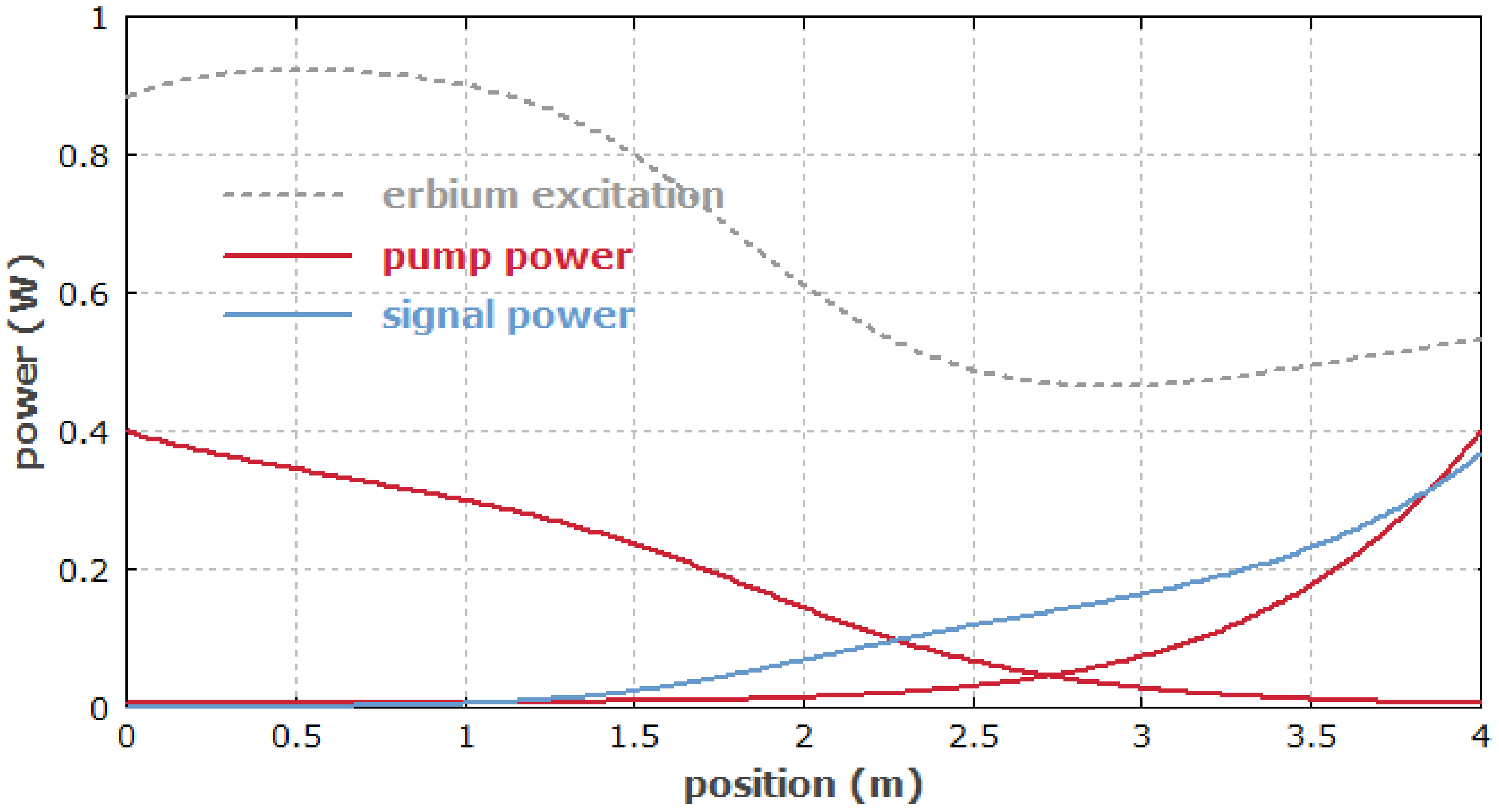


Figure 1: Power distribution in the fiber.

https://www.rp-photonics.com/fiberpower_edfa.html

Erbium-doped fiber example

Backward ASE is significantly stronger at short wavelengths. This is the usual case for erbium doped fiber amplifiers (quasi-three-level gain medium).

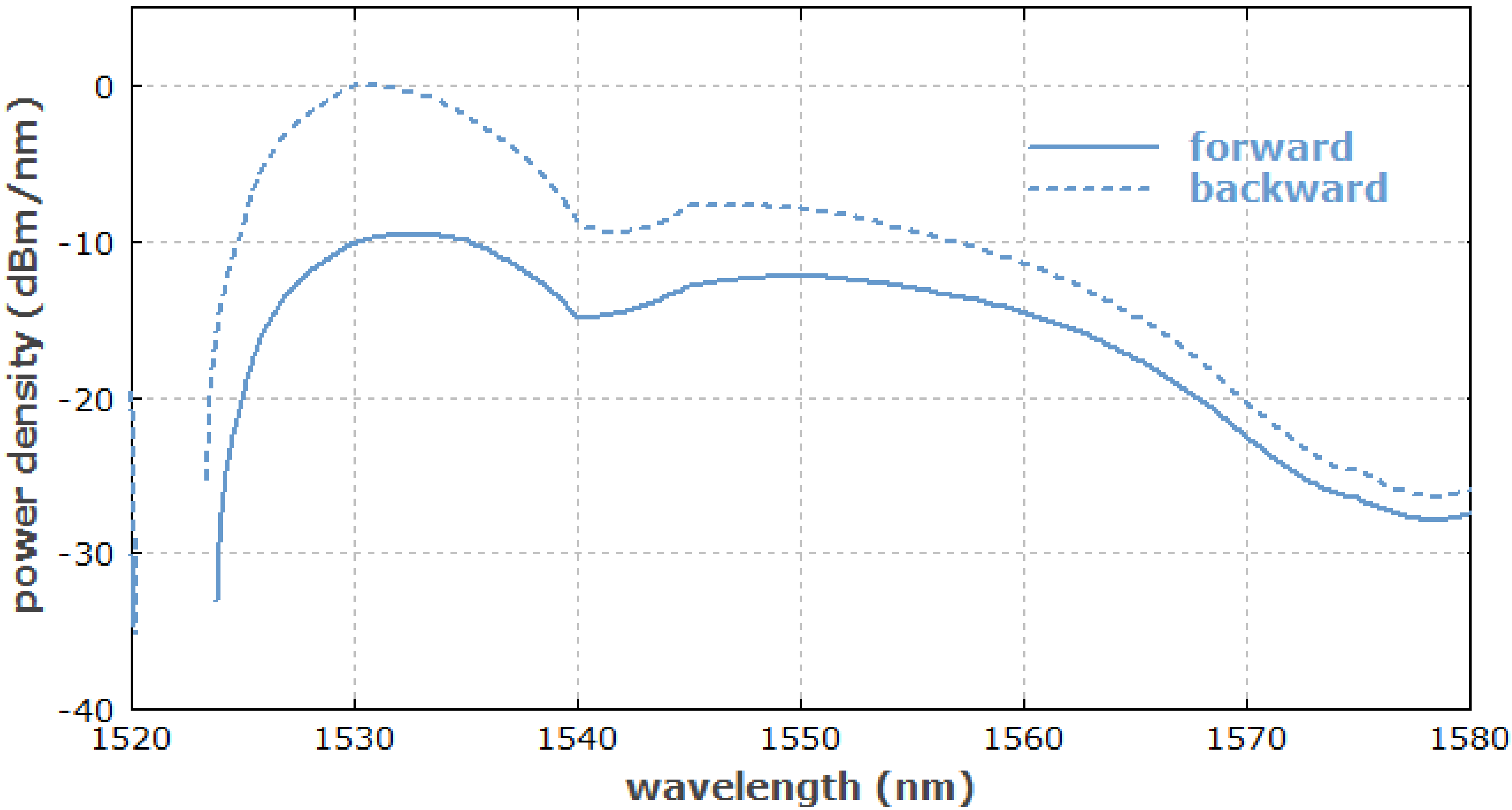
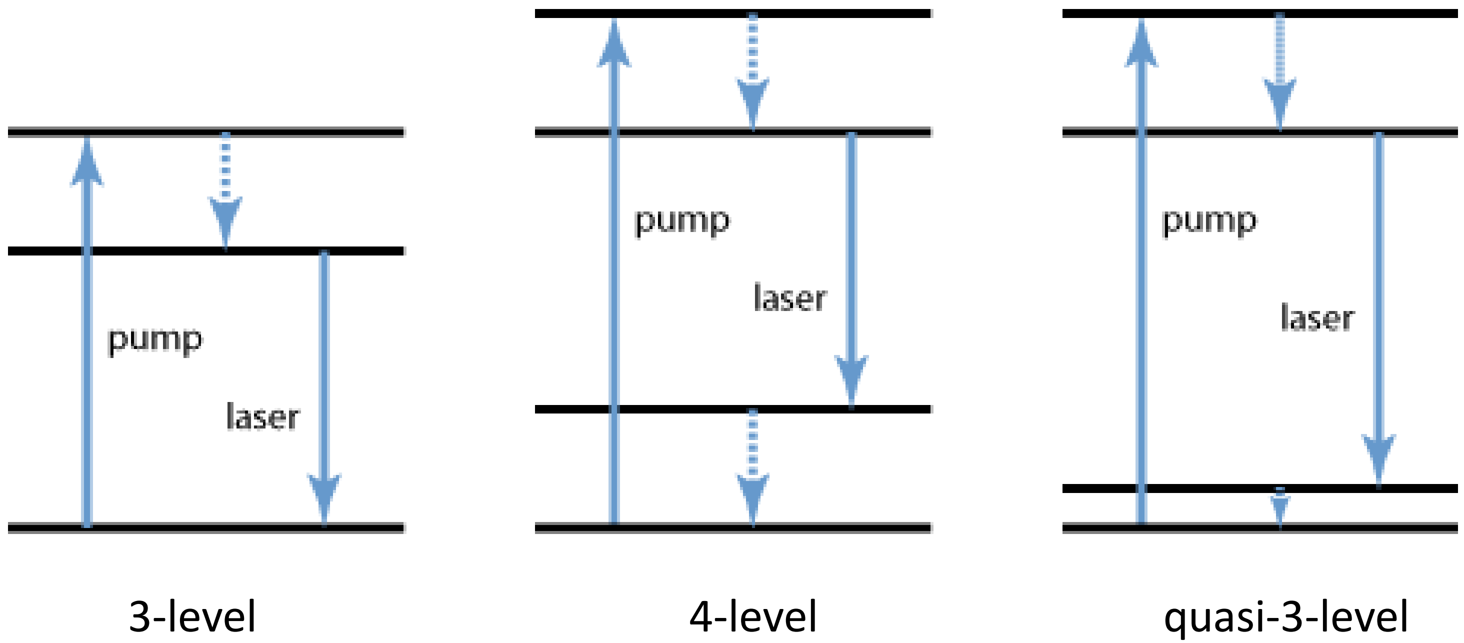


Figure 2: Spectrum of the amplified spontaneous emission (ASE).

https://www.rp-photonics.com/fiberpower_edfa.html

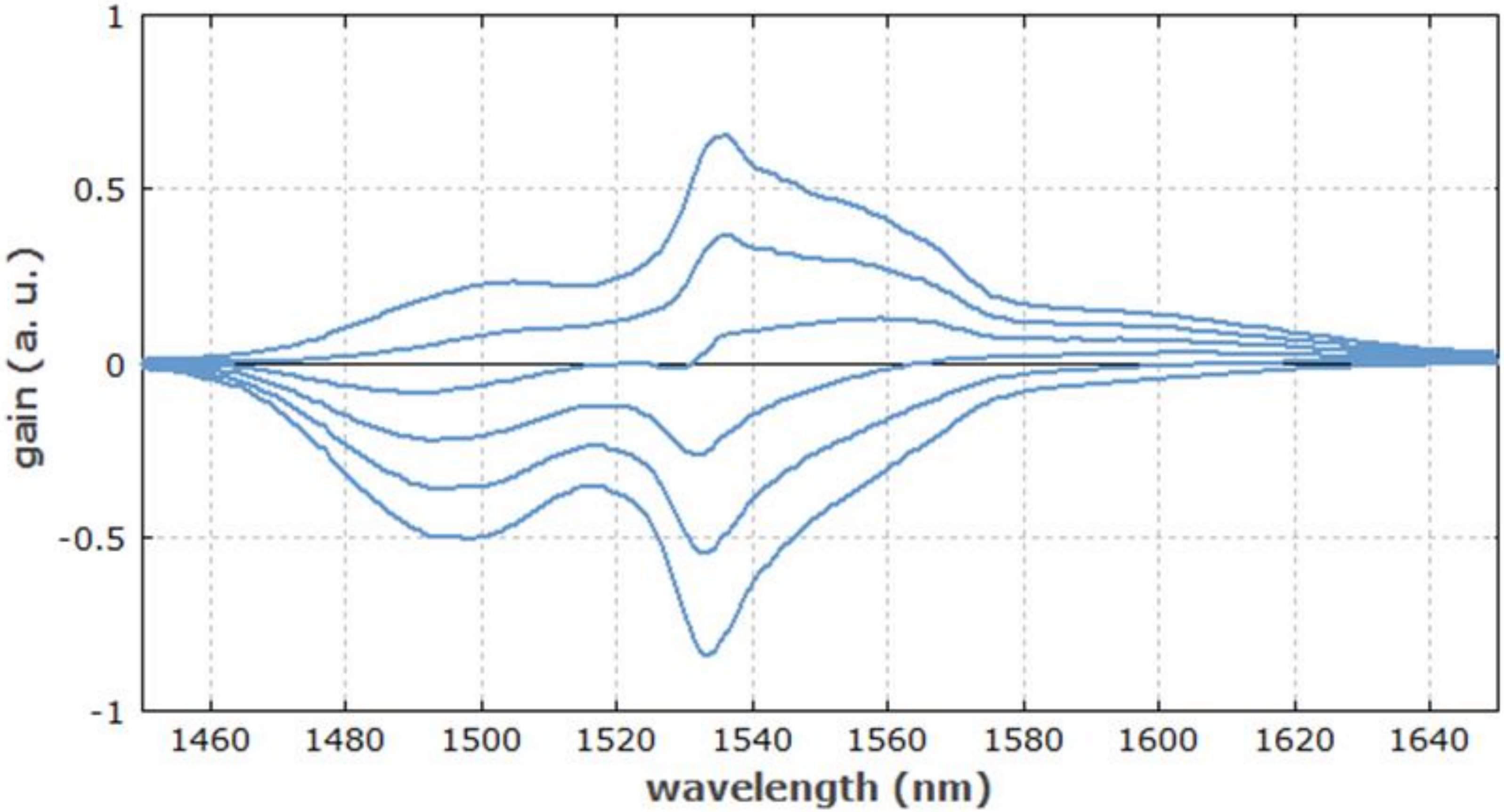
Erbium-doped fiber example

In the quasi-three-level gain medium the lower laser level is so close to the ground state that there is always a population in thermal equilibrium. The unpumped gain medium causes some reabsorption loss at the laser wavelength, so that transparency is reached only for some finite pump intensity



https://www.rp-photonics.com/four_level_and_three_level_gain_media.html

Erbium-doped fiber example



Gain and absorption (negative gain) of erbium (Er^{3+}) ions in germano-alumino-silicate glass for excitation levels from 0 to 100% in steps of 20%. Strong three-level behavior (with transparency reached only for > 50% excitation) occurs at 1530 nm. At longer wavelengths (e.g. 1580 nm), a lower excitation level is required for obtaining gain, but the maximum gain is smaller.

https://www.rp-photonics.com/four_level_and_three_level_gain_media.html

Erbium-doped fiber example

The amplifier gain is reduced for high input power levels. Curves are shown for pump powers of 100 mW to 500mW from each side.

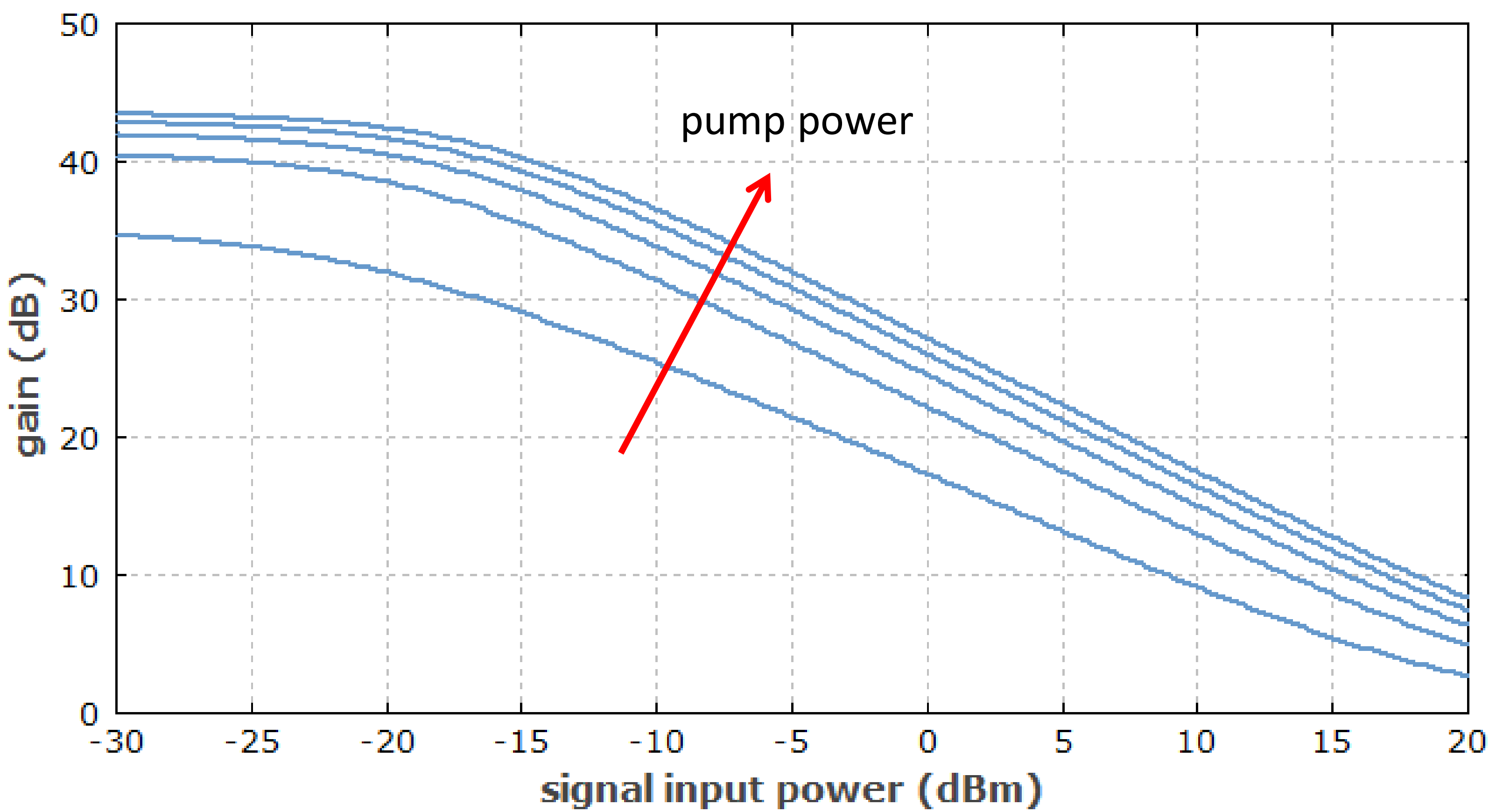


Figure 3: Saturation characteristics.

https://www.rp-photonics.com/erbium_doped_fiber_amplifiers.html

Erbium-doped fiber example

Dependence of amplifier output signal power on fiber length, which appears not to be a critical parameter for the power efficiency.

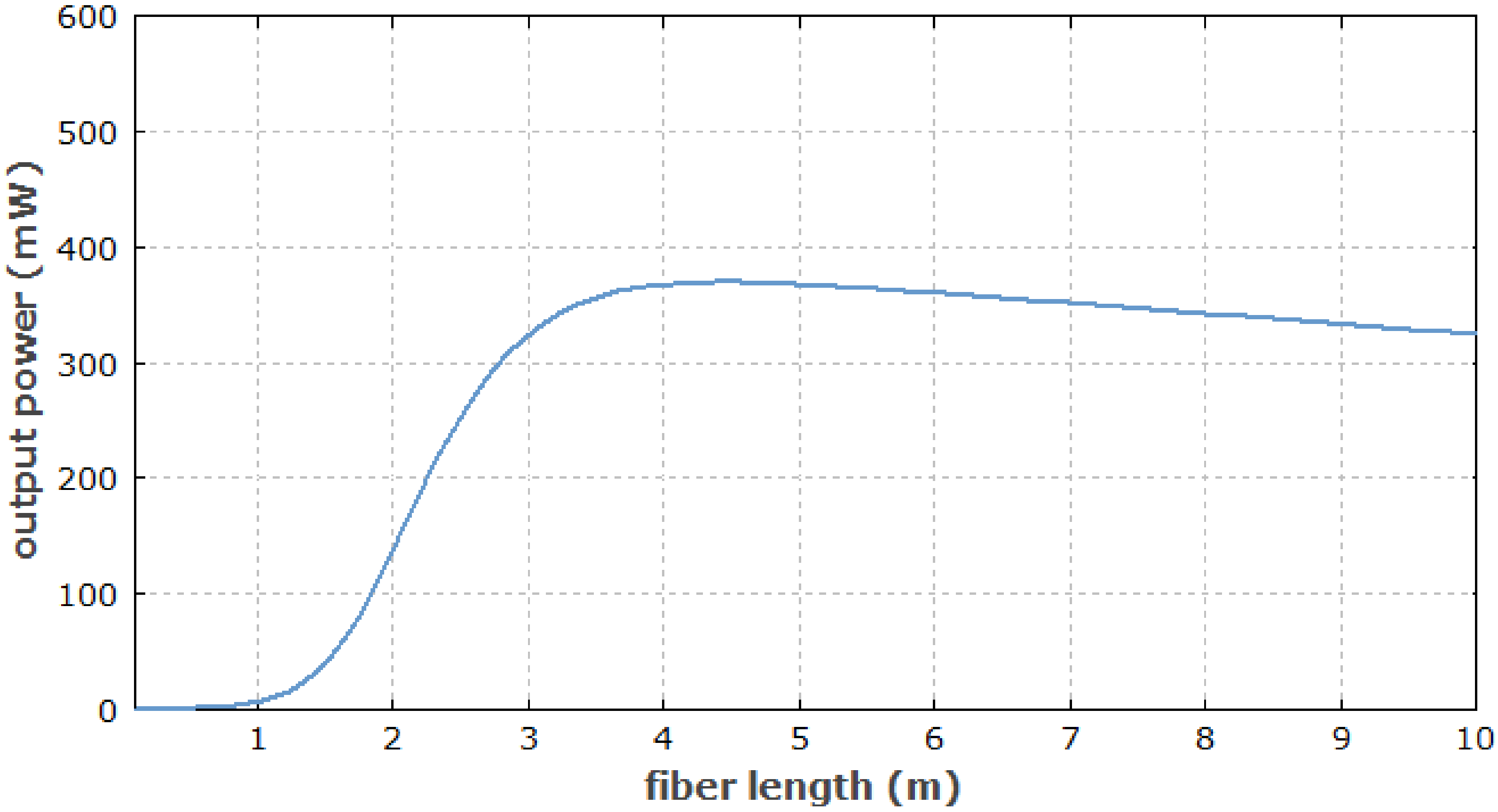


Figure 4: Variation of the fiber length.

https://www.rp-photonics.com/erbium_doped_fiber_amplifiers.html

Erbium-doped fiber example

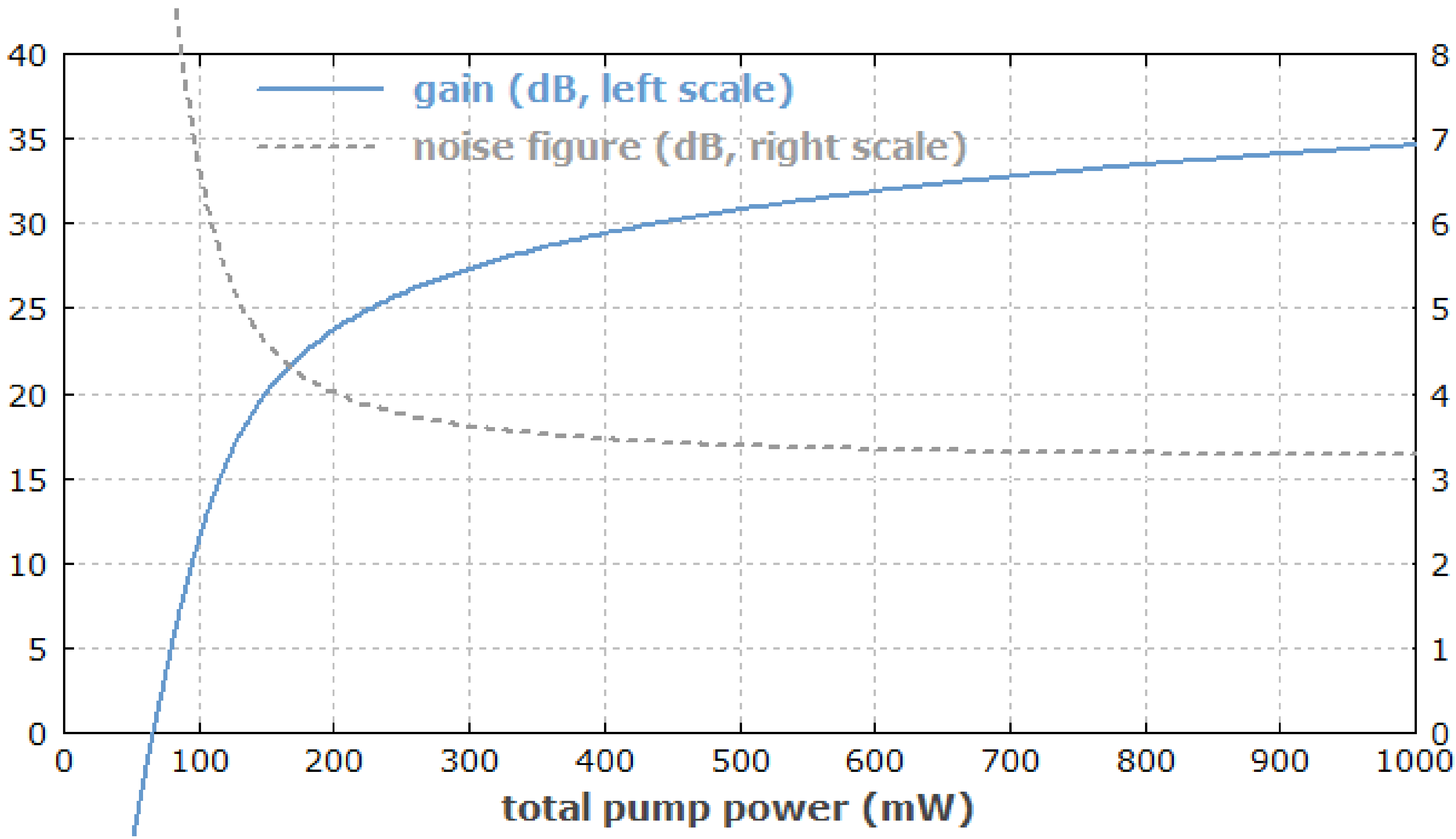


Figure 5: Gain and noise figure as functions of pump power for a signal input power of 1 mW.

https://www.rp-photonics.com/erbium_doped_fiber_amplifiers.html

Most popular numerical simulation approaches for optical waveguide devices

- **Full solution of Maxwell's equations:**

  Finite-Differences Time-Domain (FDTD) Method

- **Approximate EM solution techniques for light propagation in slowly varying waveguides**

  Beam Propagation Method

- **Scattering formalism (Transmission Line)**

  Transmission Matrix Method

Maxwell's Equation

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\partial_t \mathbf{B}(\mathbf{r}, t),$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \partial_t \mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t),$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t),$$

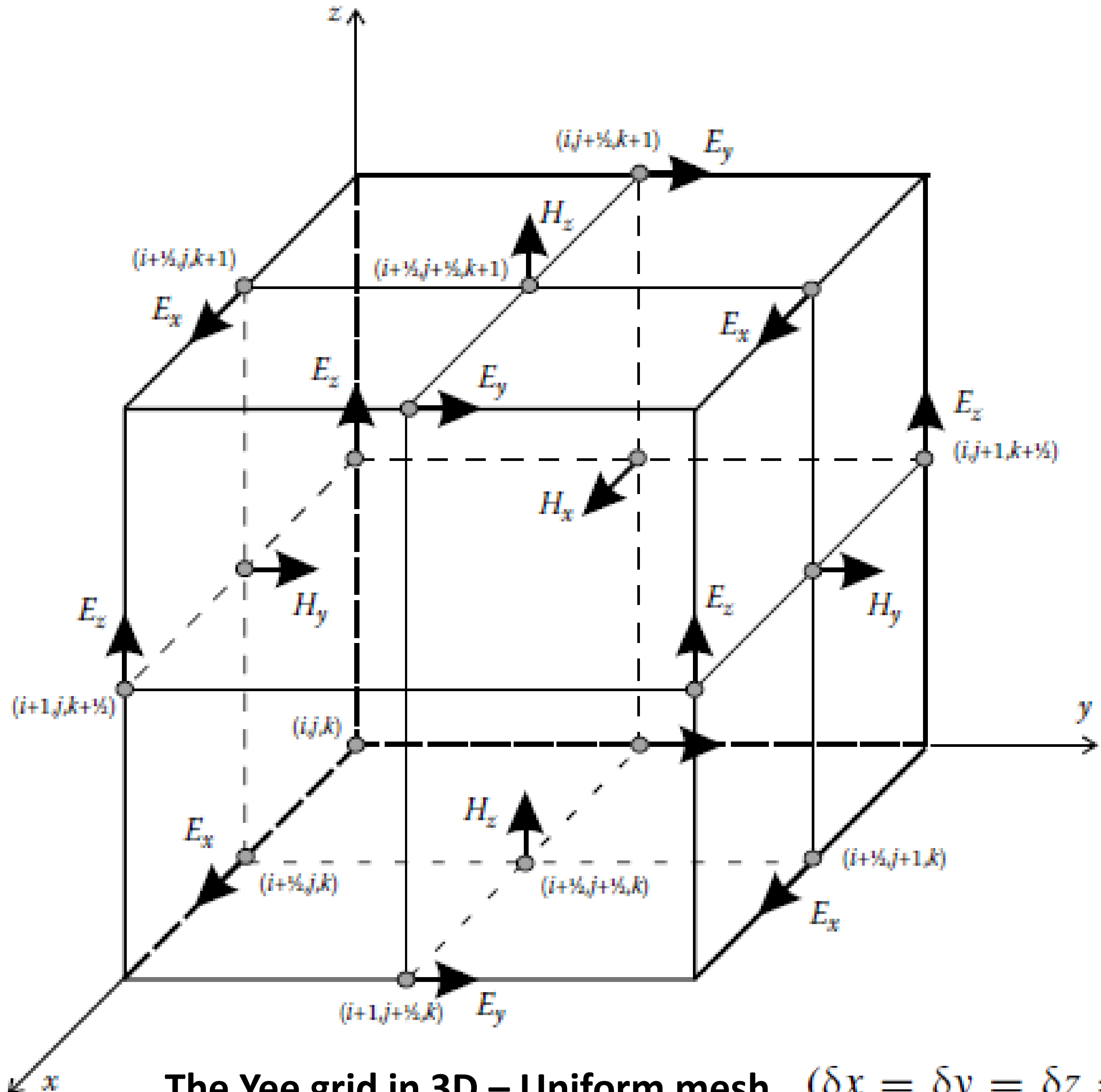
$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0,$$

Simple case of linear, isotropic, and non-dispersive medium

$$\mathbf{D}(\mathbf{r}, t) = \varepsilon_0 \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}, t),$$

$$\mathbf{B}(\mathbf{r}, t) = \mu_0 \mu(\mathbf{r}) \mathbf{H}(\mathbf{r}, t).$$

Finite Differences Time Dependent (FDTD)



The Yee grid in 3D – Uniform mesh $(\delta x = \delta y = \delta z = h)$

Finite Differences Time Dependent (FDTD)

- **Components of the electric fields are defined in the middle of the edges of the cube**
- **Components of the magnetic field are defined in the centers of the cube faces**
- **Time discretization: leapfrog scheme with time step δt**

Electric Field

$$\frac{E_i^{n+1/2} - E_i^{n-1/2}}{\delta t} = -\frac{1}{\varepsilon_0 \varepsilon_i} \frac{H_{i+1/2}^n - H_{i-1/2}^n}{h}.$$

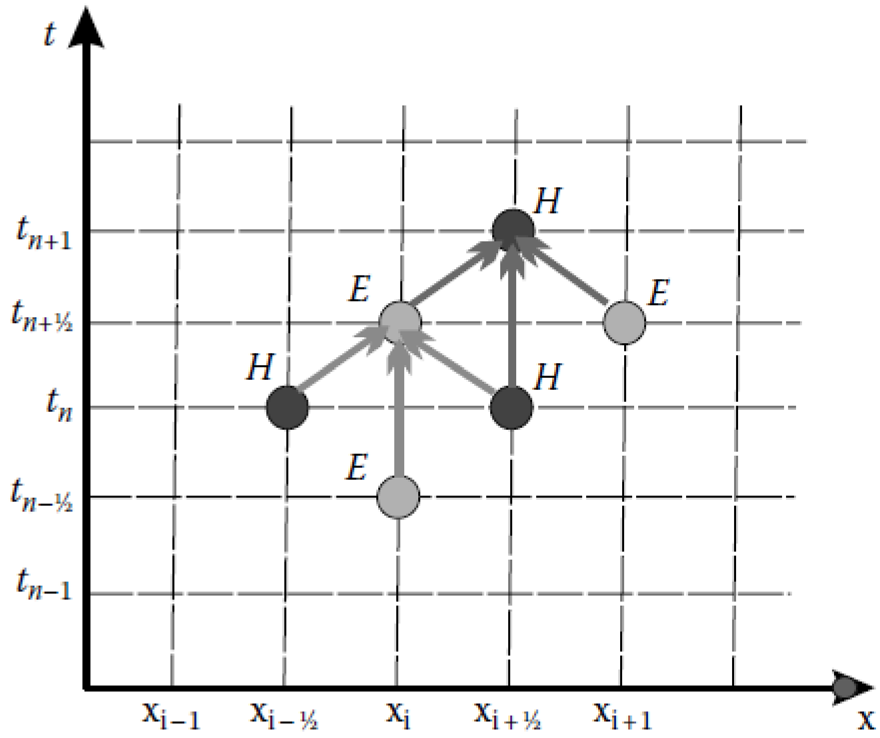
$$E_i^{n+1/2} = E_i^{n-1/2} - \frac{\delta t}{h \varepsilon_0 \varepsilon_i} (H_{i+1/2}^n - H_{i-1/2}^n).$$

Finite Differences Time Dependent (FDTD)

Magnetic Field

$$\frac{H_{i+1/2}^{n+1} - H_{i+1/2}^{n}}{\delta t} = -\frac{1}{\mu_0 \mu_{i+1/2}} \frac{E_{i+1}^{n+1/2} - E_{i}^{n+1/2}}{h},$$

$$H_{i+1/2}^{n+1} = H_{i+1/2}^{n} - \frac{\delta t}{h \mu_0 \mu_{i+1/2}} \left(E_{i+1}^{n+1/2} - E_{i}^{n+1/2}\right).$$



Finite Differences Time Dependent (FDTD)

$$E_x\big|_{i+1/2,j,k}^{n+1/2} = E_x\big|_{i+1/2,j,k}^{n-1/2} + \frac{\delta t}{\varepsilon_0 \varepsilon_{i+1/2,j,k}} \times \left(\frac{H_z\big|_{i+1/2,j+1/2,k}^{n} - H_z\big|_{i+1/2,j-1/2,k}^{n}}{\delta y} - \frac{H_y\big|_{i+1/2,j,k+1/2}^{n} - H_y\big|_{i+1/2,j,k-1/2}^{n}}{\delta z} \right),$$

Similar discretization for y and z

$$H_x\big|_{i,j+1/2,k+1/2}^{n+1} = H_x\big|_{i,j+1/2,k+1/2}^{n} + \frac{\delta t}{\mu_0 \mu_{i,j+1/2,k+1/2}} \times \left(\frac{E_y\big|_{i,j+1/2,k+1}^{n+1/2} - E_y\big|_{i,j+1/2,k}^{n+1/2}}{\delta z} - \frac{E_z\big|_{i,j+1,k+1/2}^{n+1/2} - E_z\big|_{i,j,k+1/2}^{n+1/2}}{\delta y} \right),$$

Similar discretization for y and z

Example of discretization

Finite Differences Time Dependent (FDTD)

$$E^{n+1}_{x;i,j,k} = E^{n}_{x;i,j,k} + \frac{Q}{\varepsilon_{i,j,k}}\left(H^{n}_{z;i,j,k} - H^{n}_{z;i,j-1,k} - H^{n}_{y;i,j,k} + H^{n}_{y;i,j,k-1}\right),$$

$$E^{n+1}_{y;i,j,k} = E^{n}_{y;i,j,k} + \frac{Q}{\varepsilon_{i,j,k}}\left(H^{n}_{x;i,j,k} - H^{n}_{x;i,j,k-1} - H^{n}_{z;i,j,k} + H^{n}_{z;i-1,j,k}\right),$$

$$E^{n+1}_{z;i,j,k} = E^{n}_{z;i,j,k} + \frac{Q}{\varepsilon_{i,j,k}}\left(H^{n}_{y;i,j,k} - H^{n}_{y;i-1,j,k} - H^{n}_{x;i,j,k} + H^{n}_{x;i,j-1,k}\right),$$

$$H^{n+1}_{x;i,j,k} = H^{n}_{x;i,j,k} + \frac{Q}{\mu_{i,j,k}}\left(E^{n+1}_{y;i,j,k+1} - E^{n+1}_{y;i,j,k} - E^{n+1}_{z;i,j+1,k} + E^{n+1}_{z;i,j,k}\right),$$

$$H^{n+1}_{y;i,j,k} = H^{n}_{y;i,j,k} + \frac{Q}{\mu_{i,j,k}}\left(E^{n+1}_{z;i+1,j,k} - E^{n+1}_{z;i,j,k} - E^{n+1}_{x;i,j,k+1} + E^{n+1}_{x;i,j,k}\right),$$

$$H^{n+1}_{z;i,j,k} = H^{n}_{z;i,j,k} + \frac{Q}{\mu_{i,j,k}}\left(E^{n+1}_{x;i,j+1,k} - E^{n+1}_{x;i,j,k} - E^{n+1}_{y;i+1,j,k} + E^{n+1}_{y;i,j,k}\right),$$

Scaling $\hat{E} = \dfrac{E}{\eta_0}$ **Uniform mesh** $(\delta x = \delta y = \delta z = h)$ $Q = \dfrac{c_0 \delta t}{h}$

Beam Propagation Method (BPM)

Approximate simulation technique for light propagation in slowly varying optical waveguides

Helmholtz equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + k^2 n^2(x, y, z) E = 0.$$

Electric Field

$$E(x, y, z) = \underbrace{\phi(x, y, z)}_{\text{slowly varying term}} \, \underbrace{\exp(-jk\underbrace{n_0}_{\text{cladding index}} z)}_{\text{rapidly varying term}}.$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\boxed{\nabla^2 \phi - j2kn_0 \frac{\partial \phi}{\partial z} + k^2(n^2 - n_0^2)\phi = 0,}$$

Beam Propagation Method (BPM)

Assuming weakly guiding condition

$$(n^2 - n_0^2) \cong 2\, n_0\, (n - n_0)$$

$$\boxed{\nabla^2\phi - j2kn_0\frac{\partial\phi}{\partial z} + k^2(n^2 - n_0^2)\phi = 0,}$$

$$\downarrow$$

$$\frac{\partial\phi}{\partial z} = -j\frac{1}{2kn_0}\nabla^2\phi - jk(n - n_0)\phi.$$

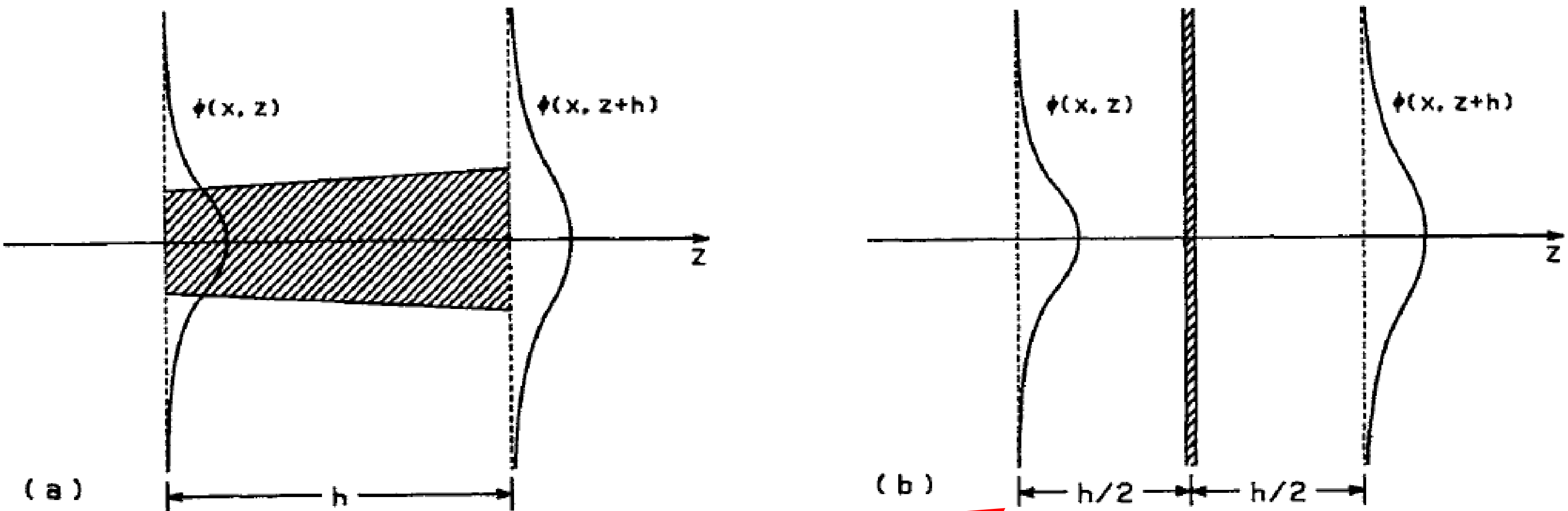
free-light propagation term

guidance term

Beam Propagation Method (BPM)

The two terms are applied separately

Approximation with BPM step



- The electric field $\varphi(x, z)$ is first propagated freely in index n_0 over a distance $h/2$.
- Phase retardation of the entire length h is taken into account at the center of the interval
- The resulting electric field is propagated again freely in index n_0 for another distance $h/2$ to obtain $\varphi(x, z + h)$

Beam Propagation Method (BPM)

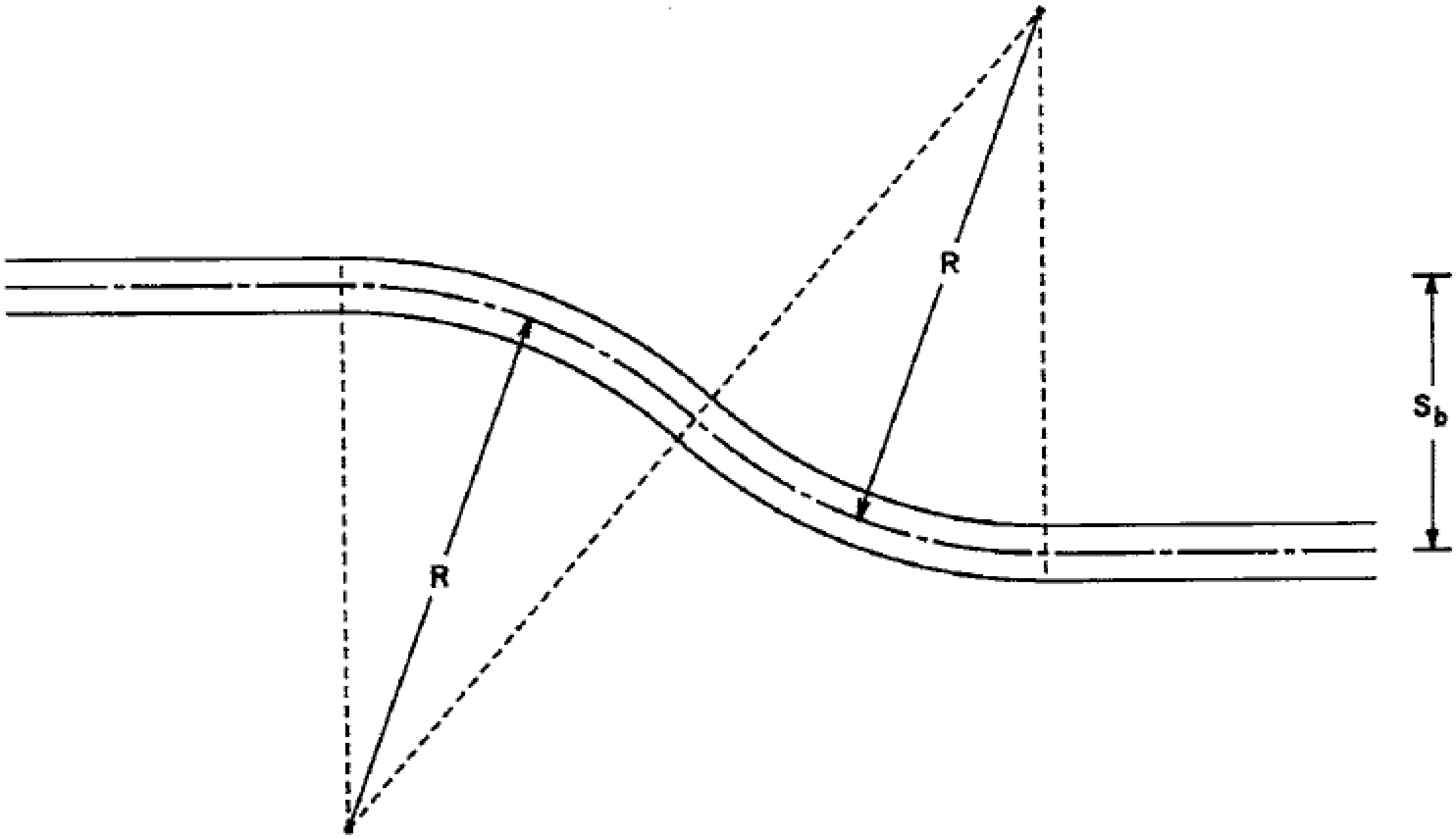
An extensive theoretical reference for BPM is Chapter 7 of

K. Okamoto, "Fundamentals of Optical Waveguides" Elsevier (2006)

(Available for download from the digital library)

Some representative examples follow

Beam Propagation Method (BPM) - Examples



S-shaped bent waveguide without offset.

Beam Propagation Method (BPM) - Examples

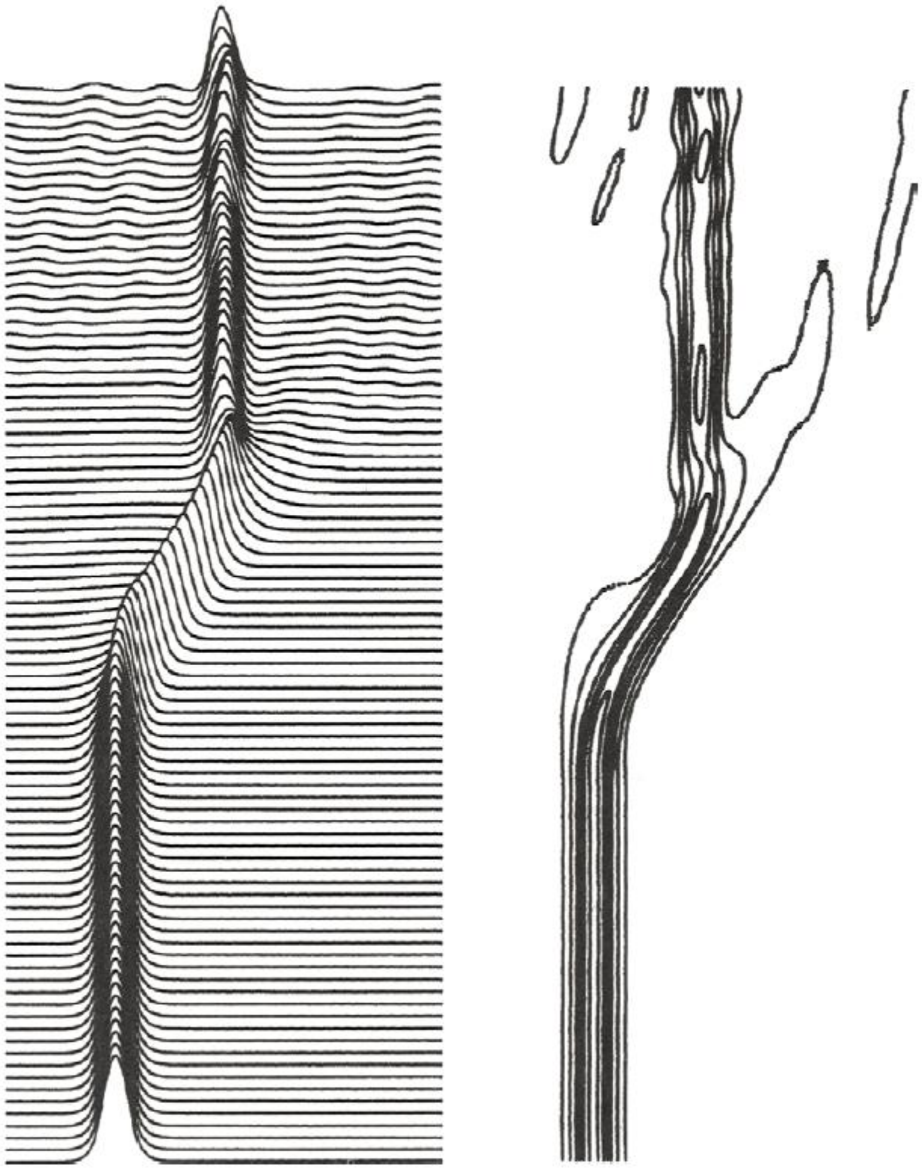


Figure 7.12 BPM simulation of the light propagation in an S-bend waveguide consisting of a fixed radius of curvature without offset.

Beam Propagation Method (BPM) - Examples

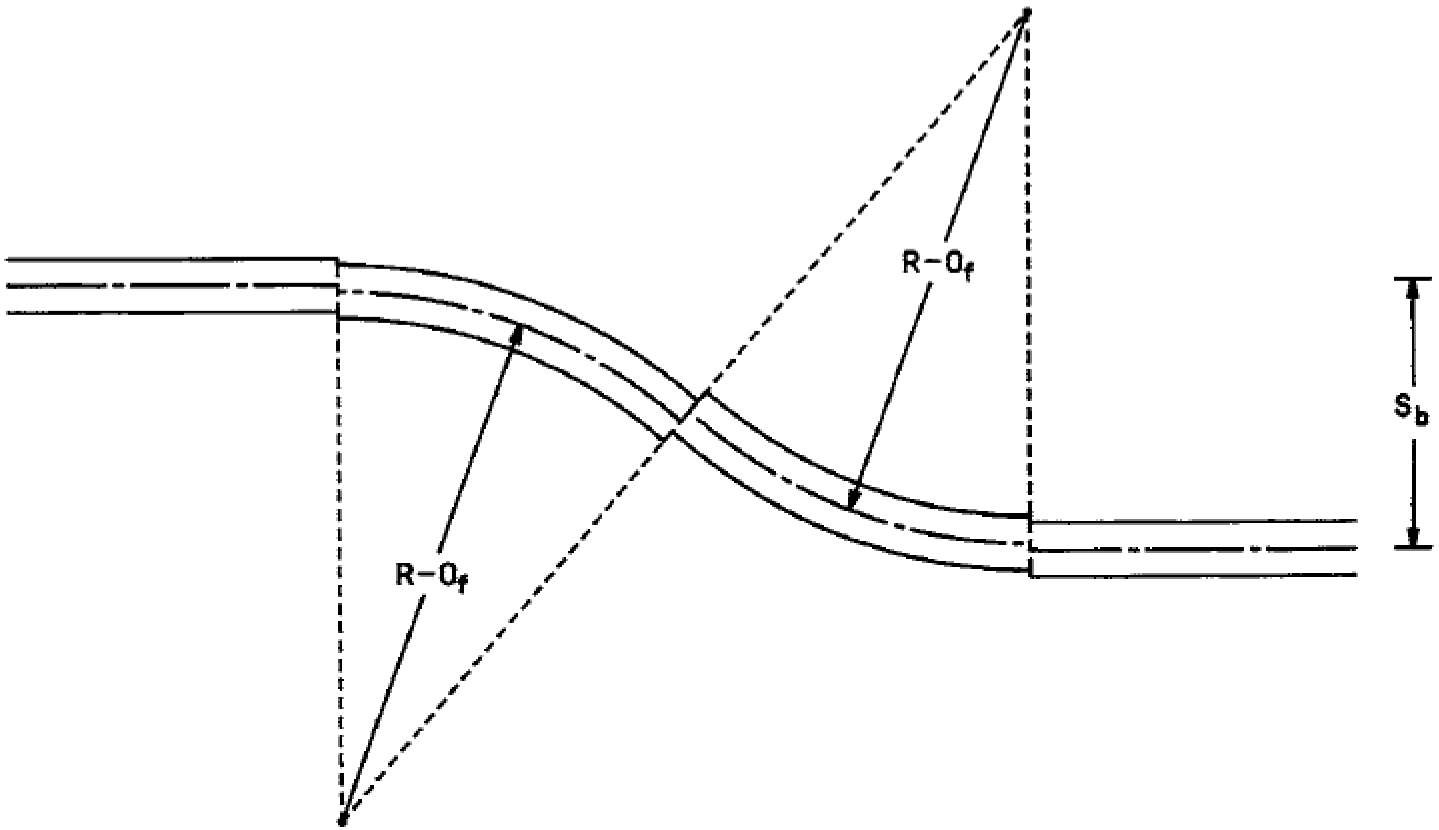


Figure 7.13 S-shaped bent waveguide with a waveguide offset.

Beam Propagation Method (BPM) - Examples

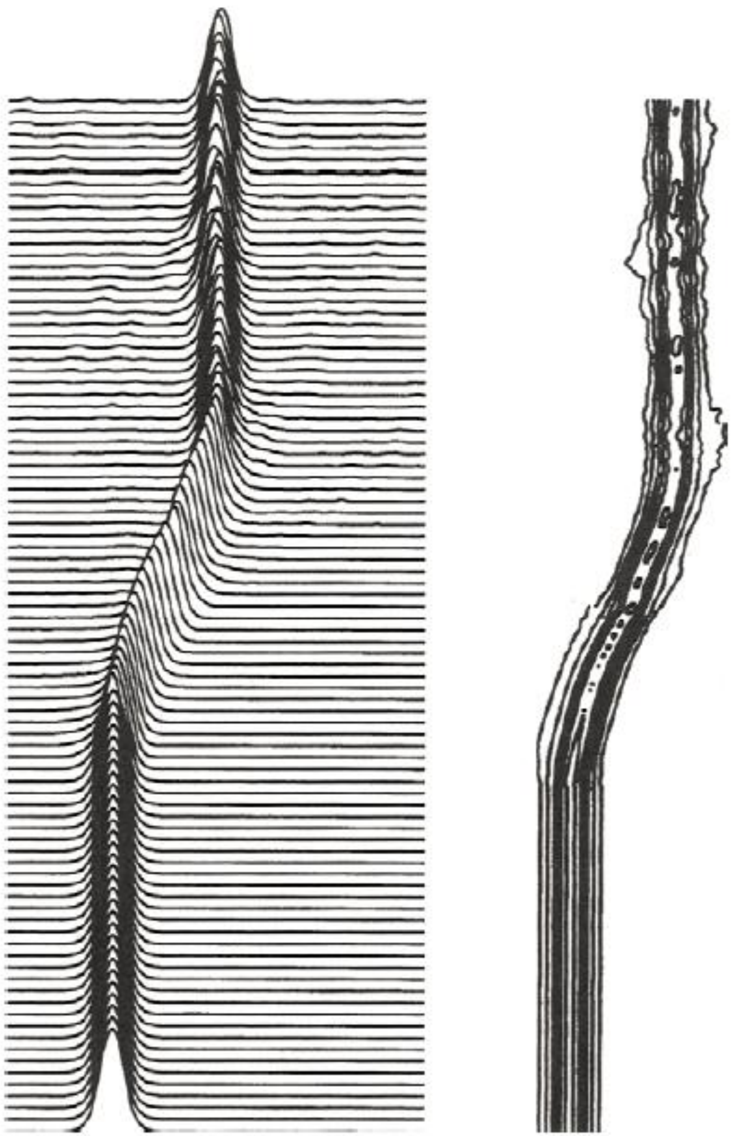


Figure 7.14 BPM simulation of the light propagation in an S-bend waveguide having an offset of $O_f = 1.4\,\mu\text{m}$.

Beam Propagation Method (BPM) - Examples

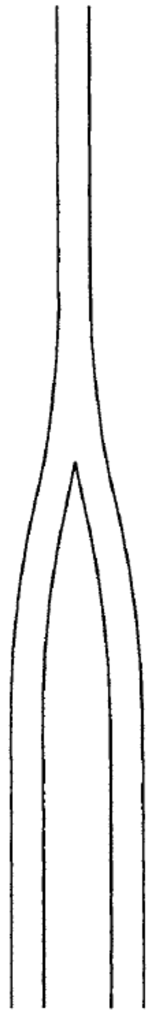


Figure 7.15 Schematic configuration of a Y-combiner consisting of single-mode waveguides.

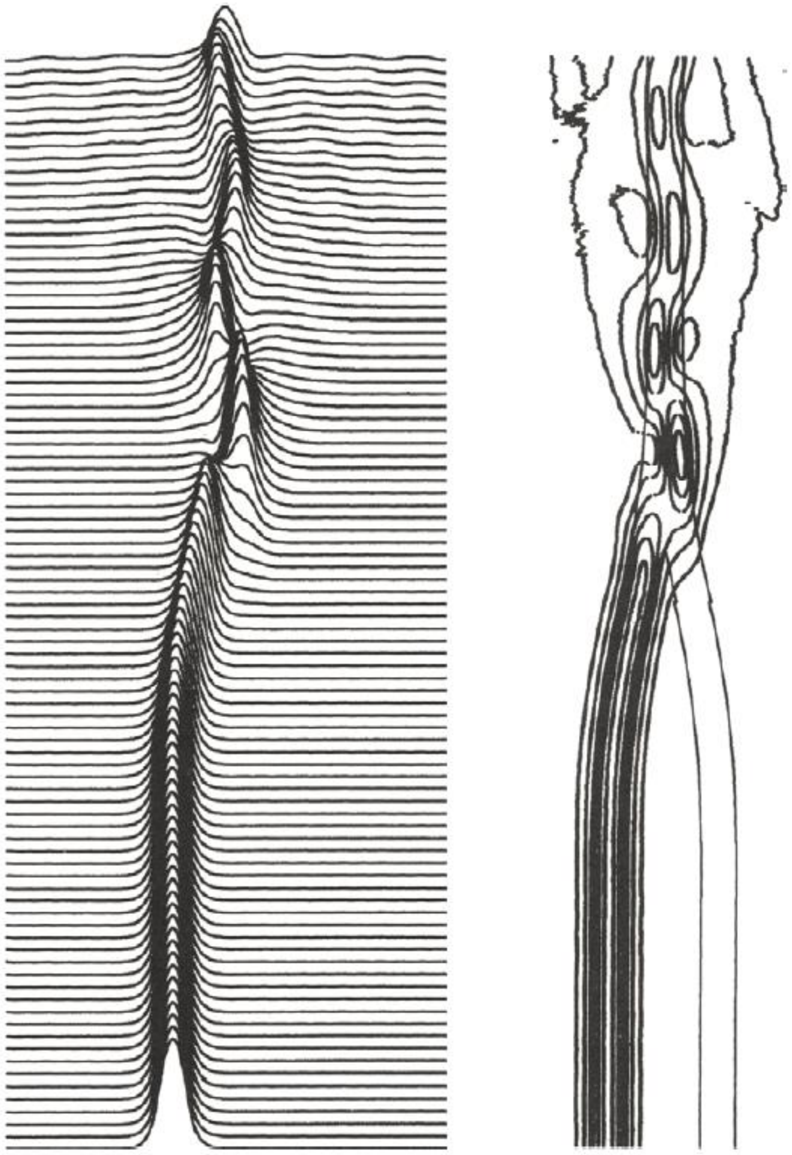


Figure 7.16 BPM analysis of single-mode Y-combiner when when light is coupled into one of the two waveguides.

Beam Propagation Method (BPM) - Examples

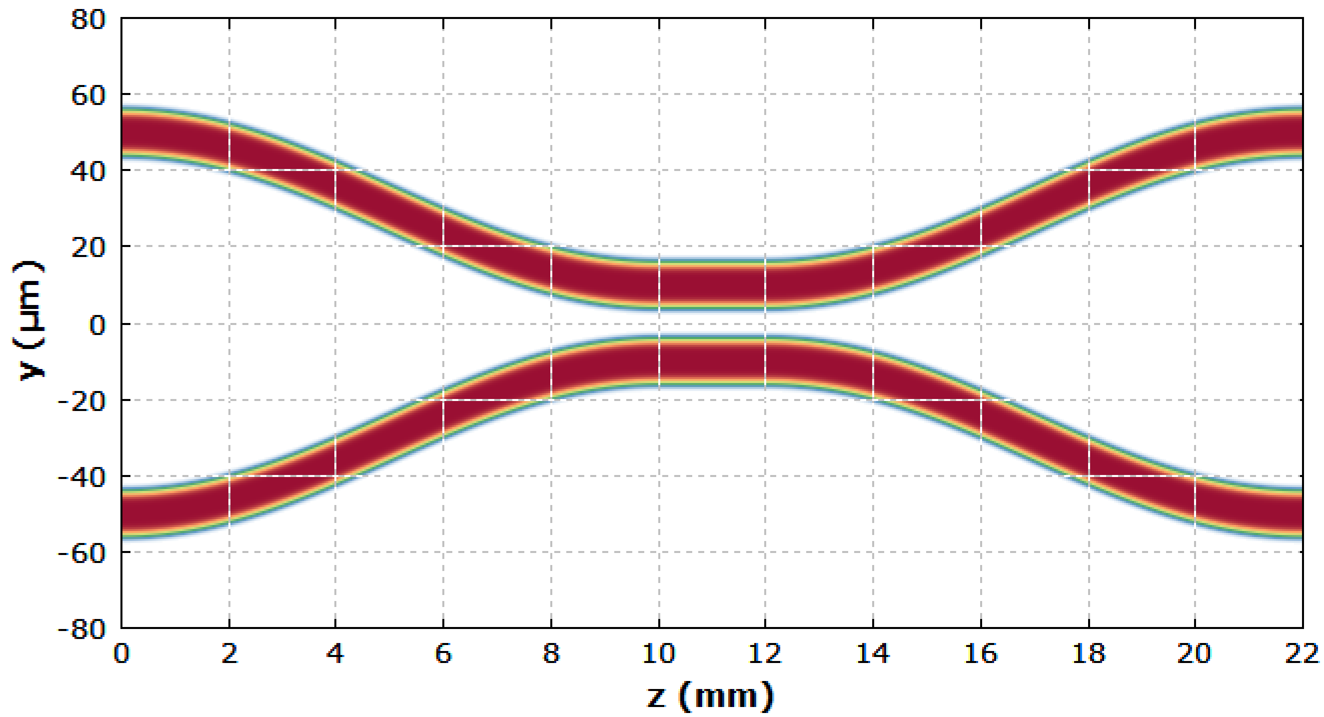


Figure 2: Refractive index profile of a fiber coupler.

https://www.rp-photonics.com/passive_fiber_optics8.html

Beam Propagation Method (BPM) - Examples

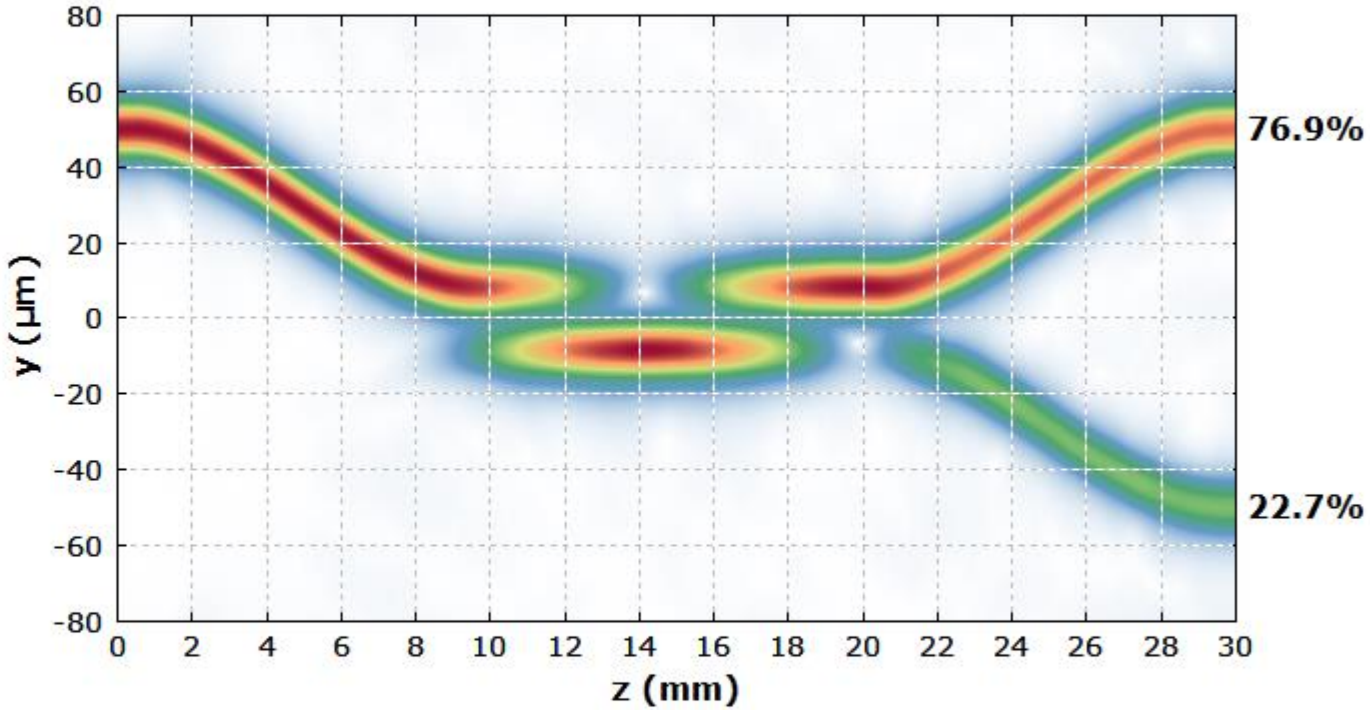


Figure 3: Amplitude distribution in a fiber coupler, obtained with a numerical simulation of beam propagation, done with the software **RP Fiber Power**.

https://www.rp-photonics.com/passive_fiber_optics8.html

Transfer Matrix Method

Sections 3.1-3.5 in Coldren, Corzine and Mašanović

Definition of the Scattering Matrix

Linear Networks **S** allows you to find outputs from inputs

Inputs: a_n

Outputs: b_n

$\mathbf{b} = \mathbf{S}\mathbf{a}$

$b_i = \sum_j S_{ij} a_j$

Can measure S_{ij} by setting $a_k = 0$ for $k \neq j$ and measuring b_i

$\mathcal{E}(x,y,z,t) = \hat{e} E_0 U(x,y) e^{j(\omega t - \tilde{\beta} z)}$

$a_j = \dfrac{E_0}{\sqrt{2\eta_j}} e^{-j\tilde{\beta} z}$ where $\eta_j = \dfrac{377\Omega}{\tilde{n}_j}$

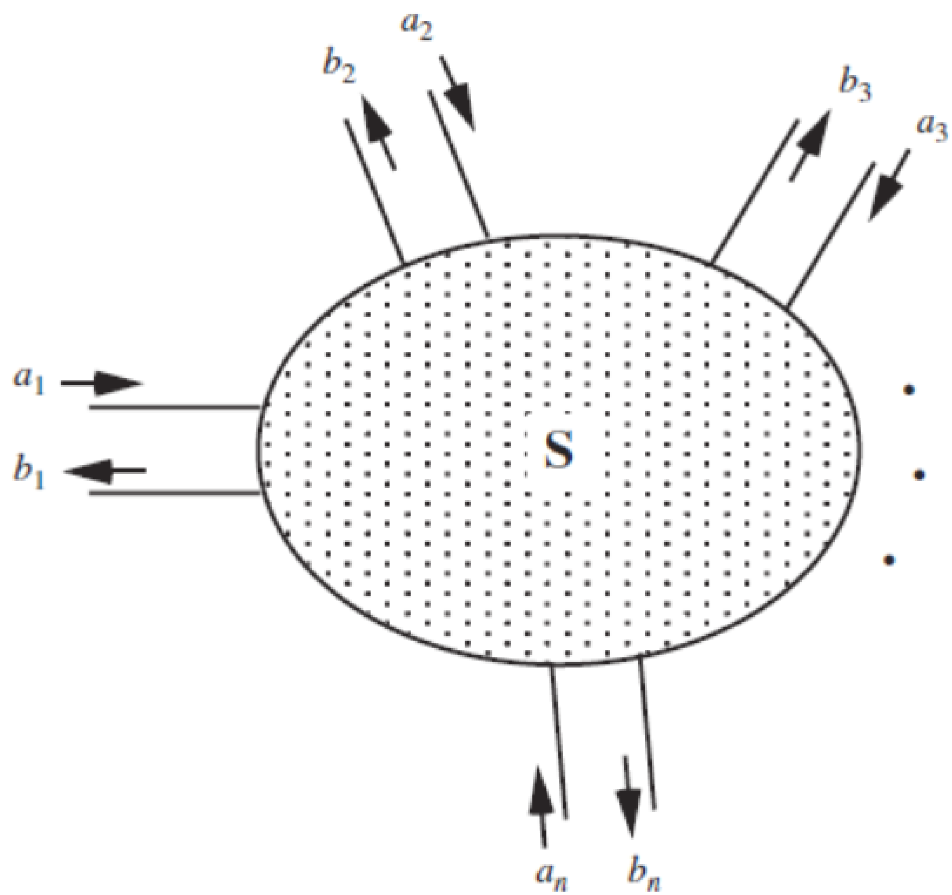
For $\int |U|^2 \, dx\, dy = 1$ we have $a_j a_j^* = P_j^+$

The net power flowing into the port is:

$P_j = a_j a_j^* - b_j b_j^*$

Important case (2-port junction):

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

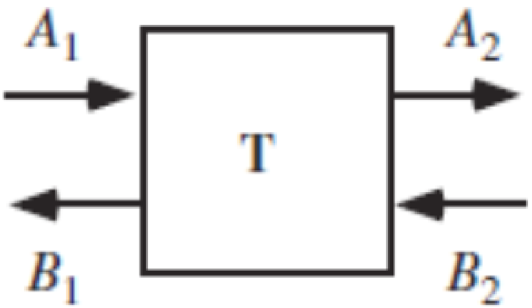


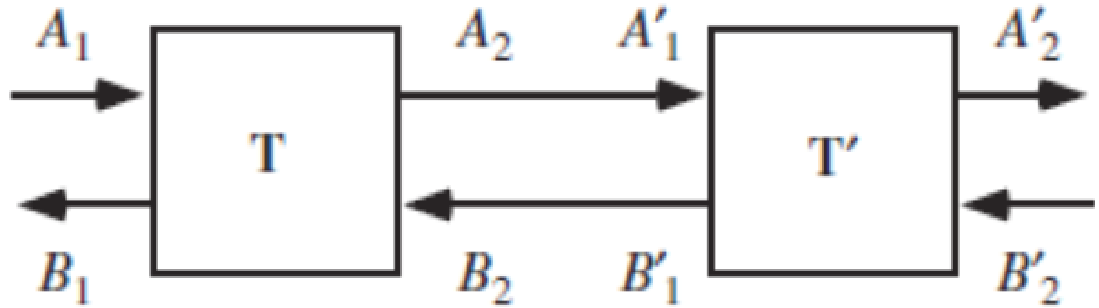
If network is reciprocal, $\mathbf{S}_t = \mathbf{S}$

If network is lossless, $\mathbf{S}$ is unitary: $\mathbf{S}_t^* \mathbf{S} = 1$

Definition of the Transmission Matrix

T allows you to cascade networks





Left side: $A_1 = a_1,\, B_1 = b_1$

Right side: $A_2 = b_2,\, B_2 = a_2$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

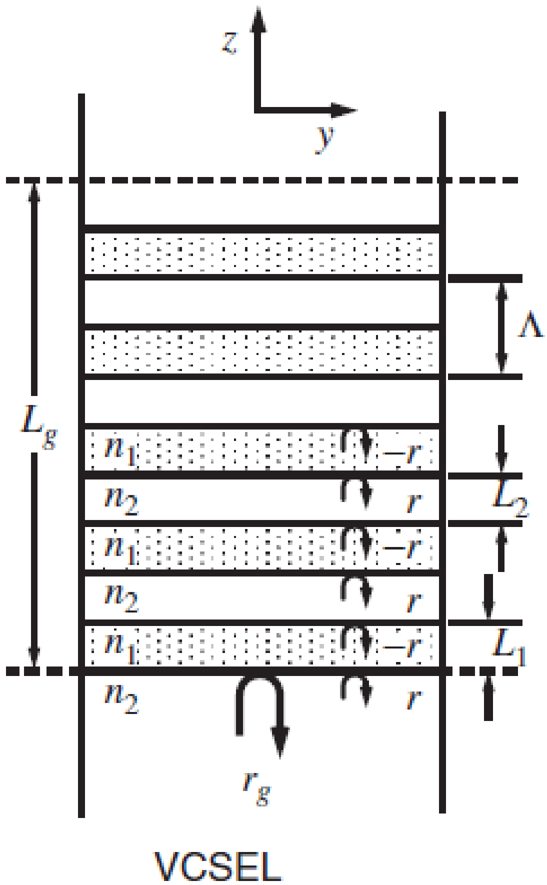
$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} A'_2 \\ B'_2 \end{bmatrix}$$

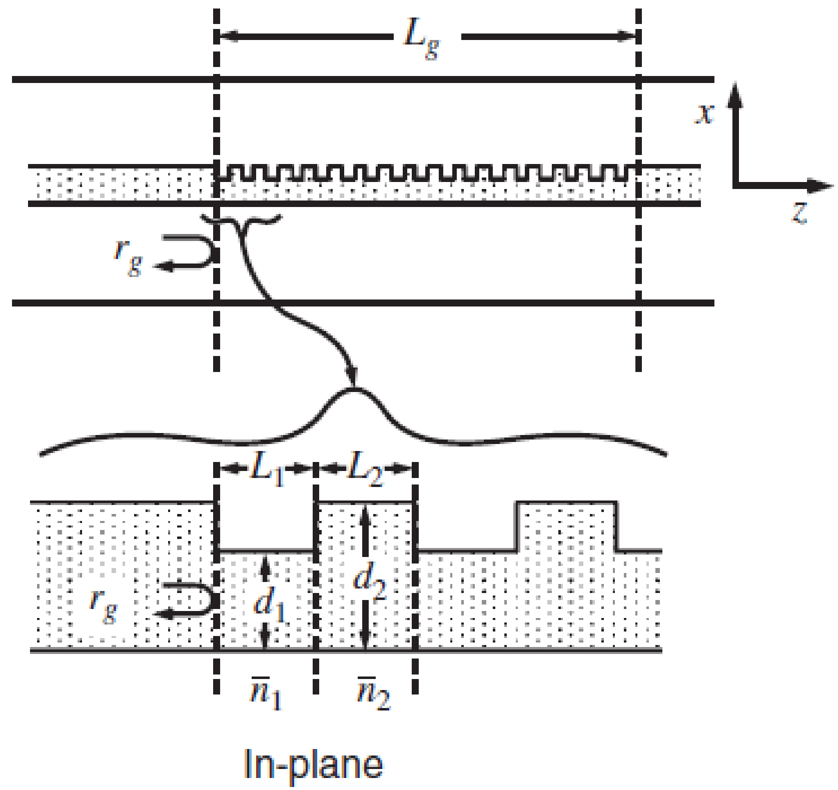
$$\begin{bmatrix} T_{11} = \dfrac{1}{S_{21}} & T_{12} = -\dfrac{S_{22}}{S_{12}} \\ T_{21} = \dfrac{S_{11}}{S_{21}} & T_{22} = -\dfrac{S_{11}S_{22} - S_{12}S_{21}}{S_{21}} \end{bmatrix}$$

If network is reciprocal, scattering matrix is symmetric and $\det \mathbf{T} = 1$

If network is lossless, $\mathbf{S}$ is unitary: $\mathbf{S}_t^* \mathbf{S} = 1$

Application to Gratings (Distributed Bragg Reflectors)





- **At the Bragg frequency, the period of the grating is half of the average optical wavelength in the medium.**
- **For each period, multiply 4 simple T-Matrices together**

Reading Assignments:

- Section 10.5 of Chuang's book

- Sections 3.1-3.5 in Coldren, Corzine and Mašanović
- Section 8.2.5 in Coldren, Corzine and Mašanović