ECE 536 – Integrated Optics and Optoelectronics
Lecture 23 – April 13, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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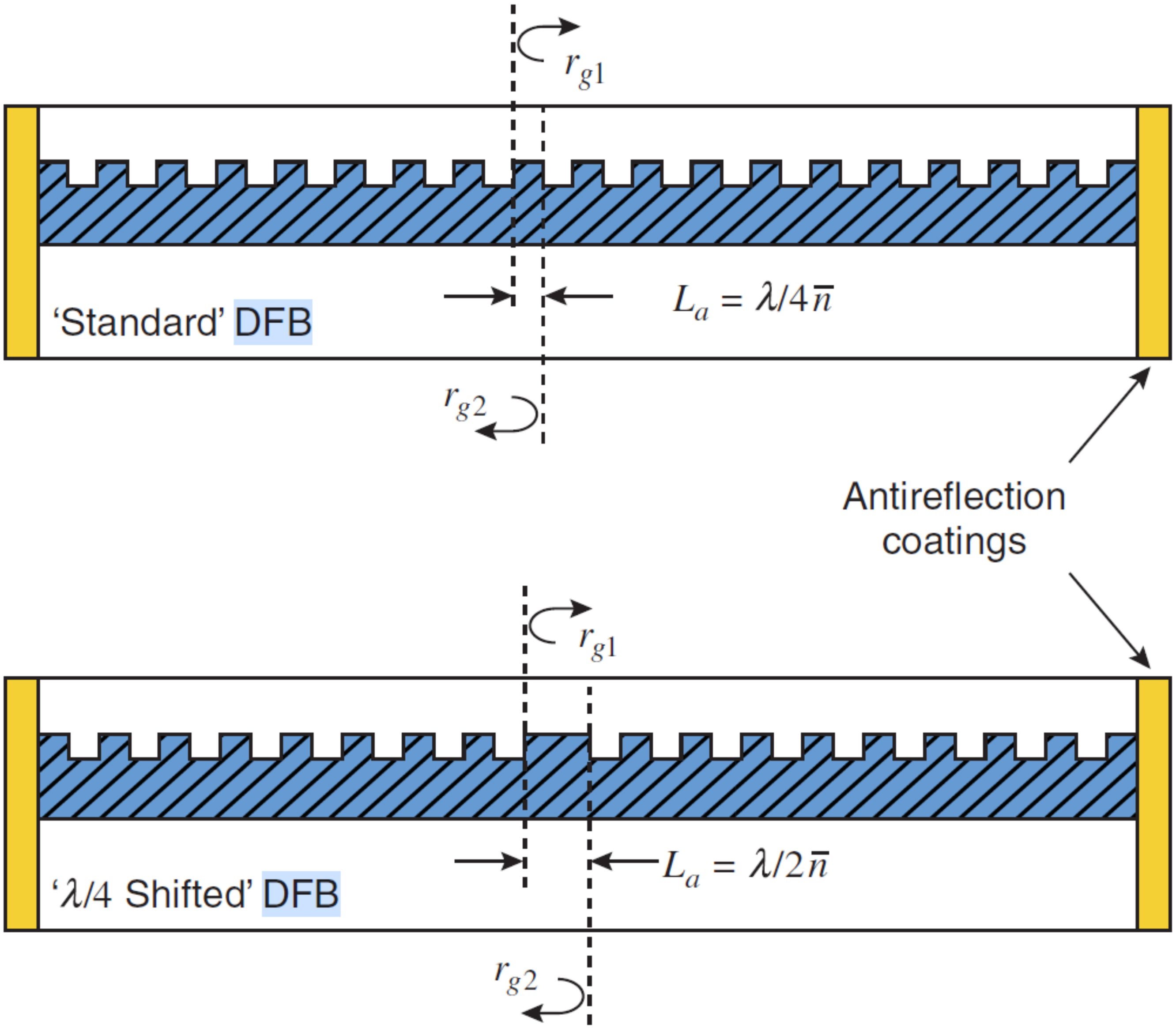
Lecture 23 Outline

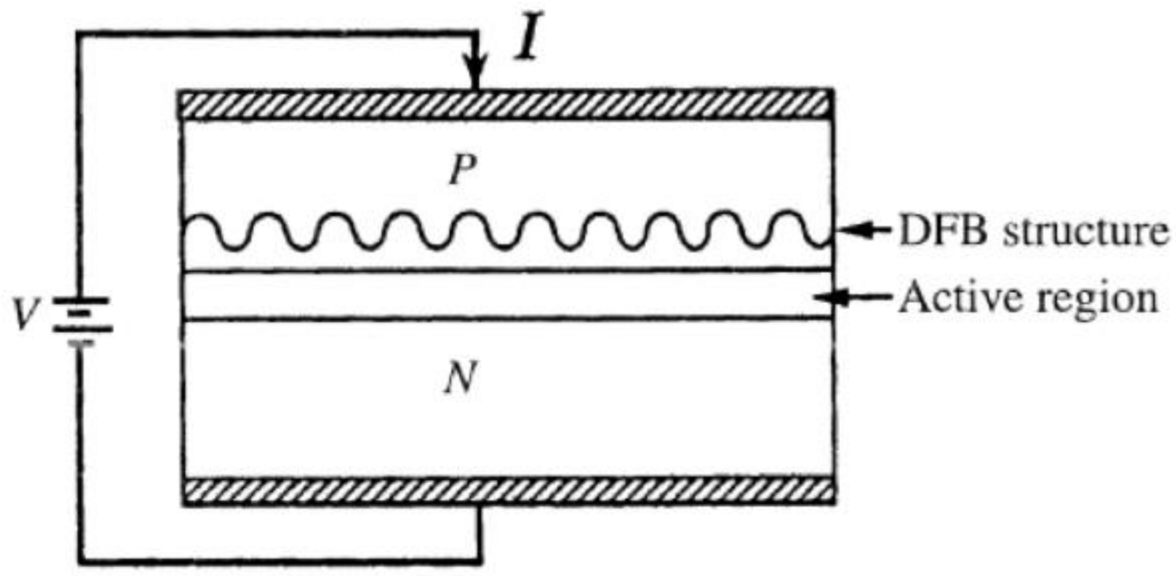
- Distributed Bragg Reflector Lasers
  - Vertical Cavity Surface Emitting (VCSEL) Laser
  - In plane edge emitting DBR Laser

Applications of gratings in lasers – Distributed Feedback (DFB) Laser



from Coldren, Corzine and Mašanović

Distributed Feedback (DFB) Laser



DFB Lasers provide tight control of channel wavelength and minimize chromatic dispersion (important in systems for long distance optical communications)

The reflection coefficient depends on

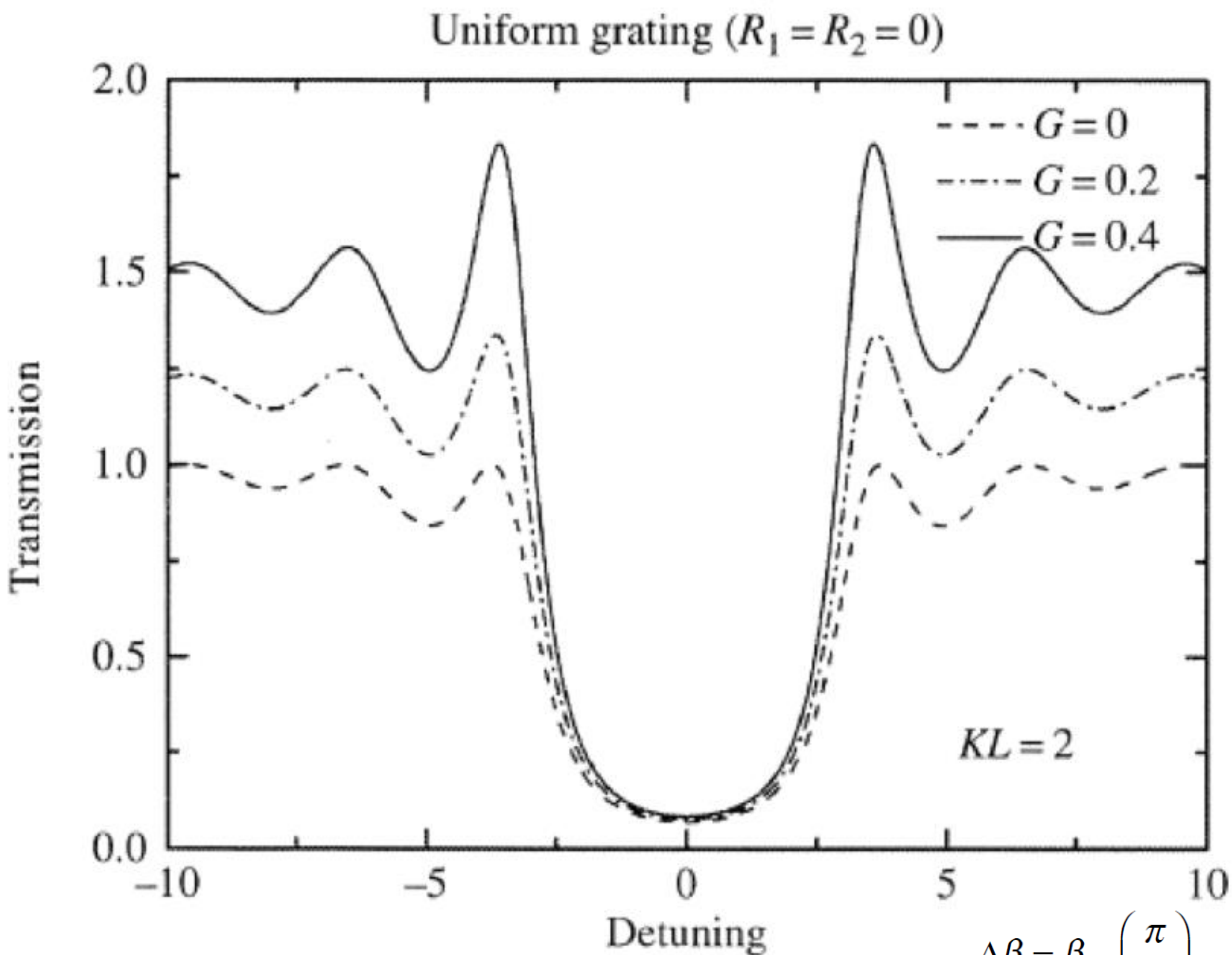
K Coupling coefficient

δ Detuning parameter

$\beta_B = \ell \pi / \Lambda$ Bragg wave number

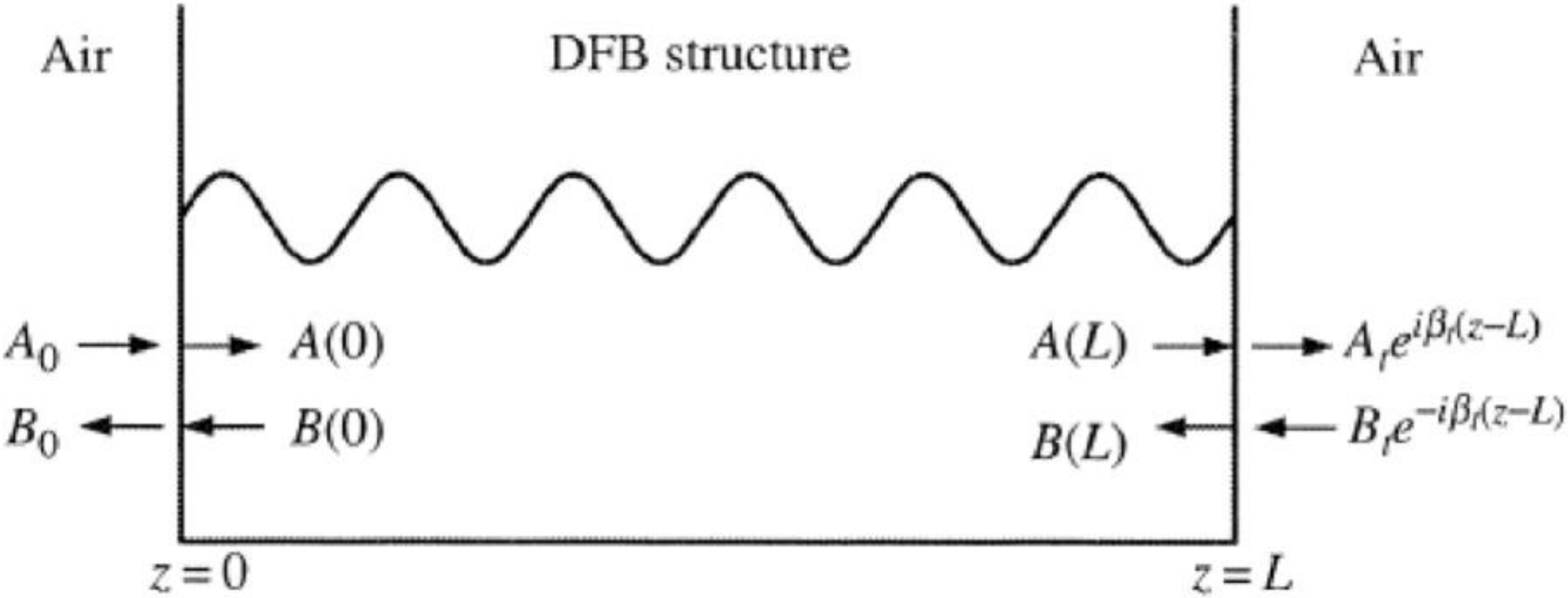
ℓ Order of grating (integer)

Transmission Characteristic for DFB



$$\Delta\beta = \beta - \left(\frac{\pi}{\Lambda}\right)$$

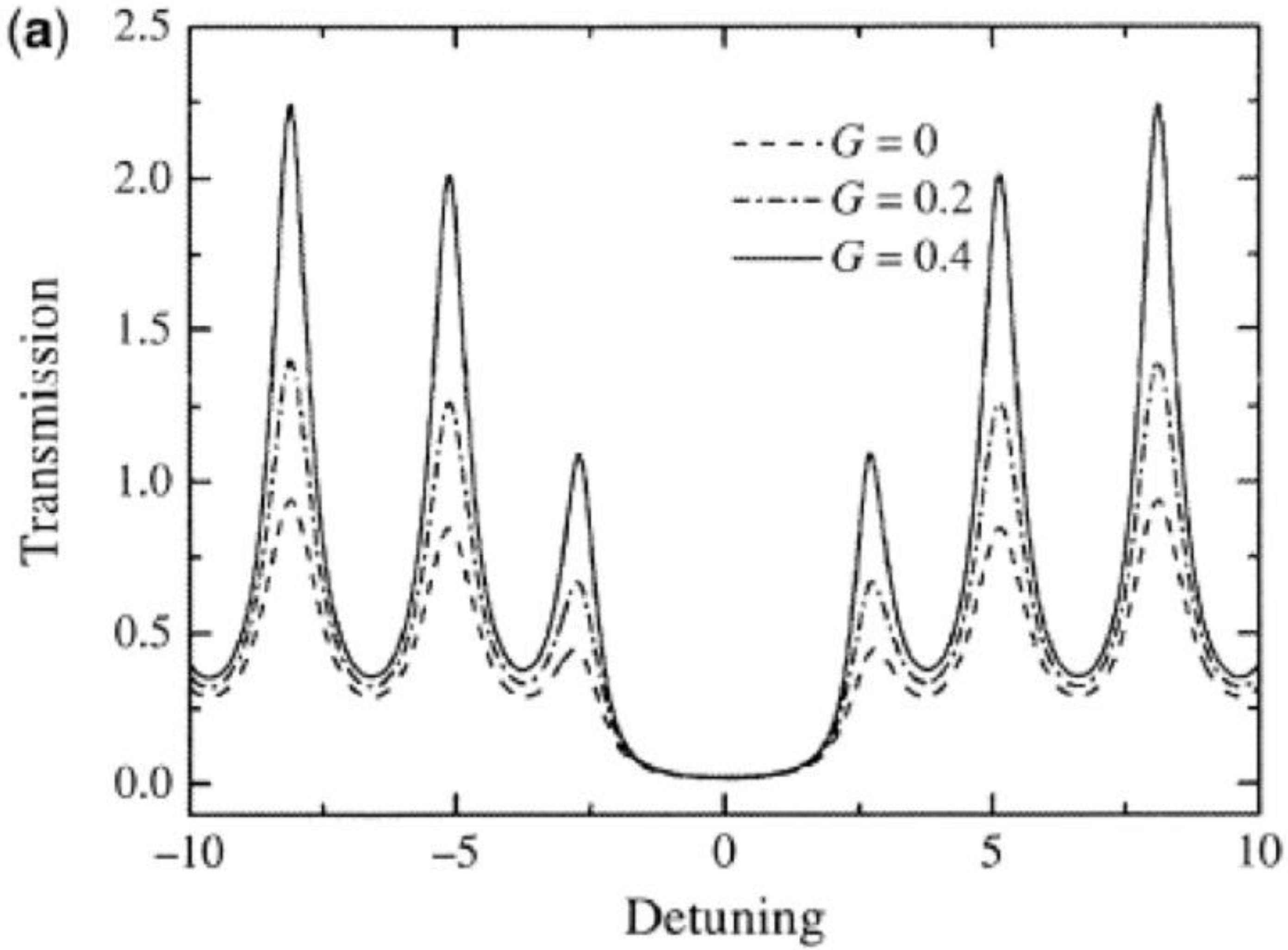
DBF Structure with End Facets



The transmission matrix method can be applied to relate A_t with $A(O)$

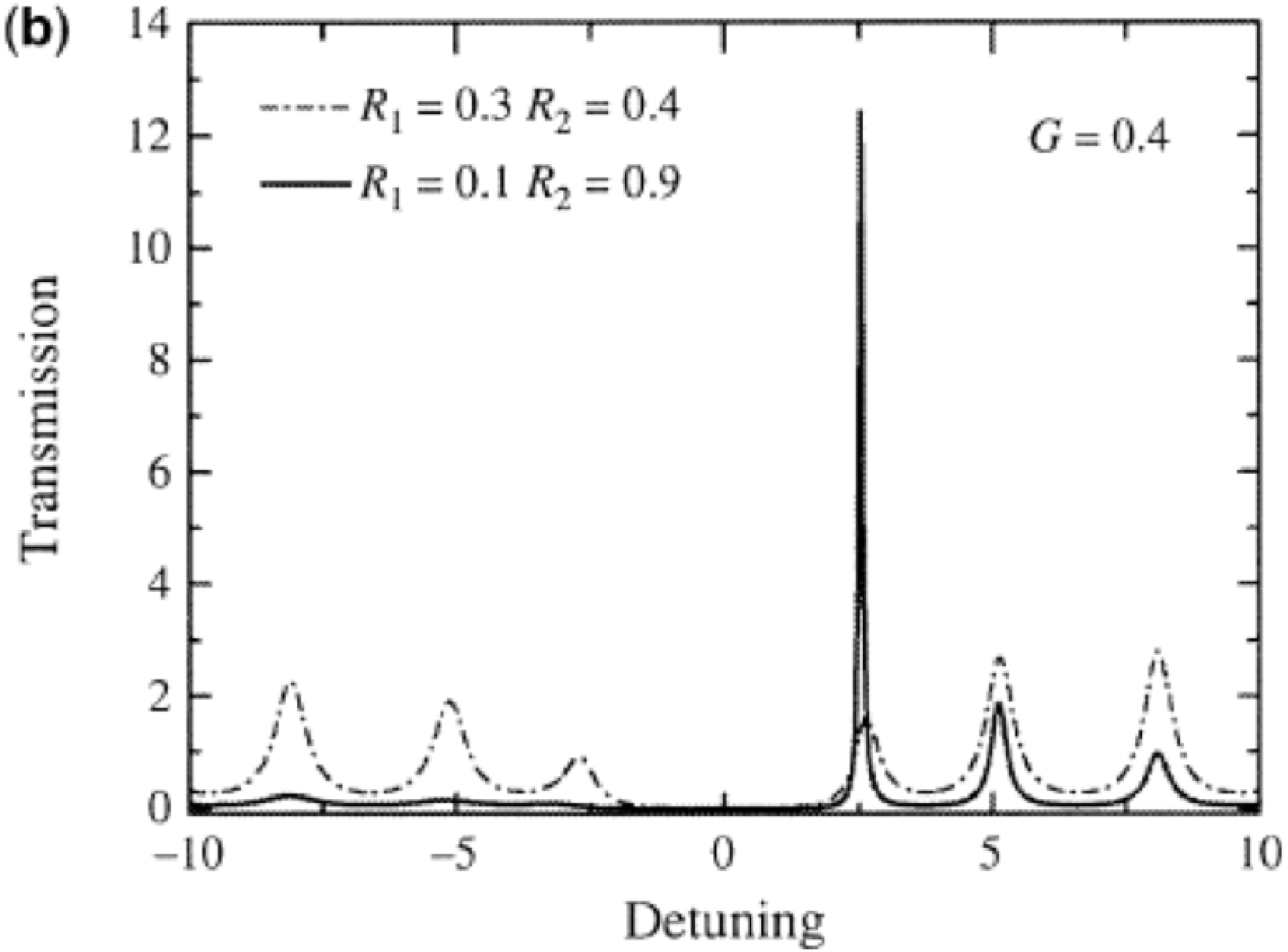
Effect of Cavity End Conditions: Facet Coating

Symmetrical Cavity

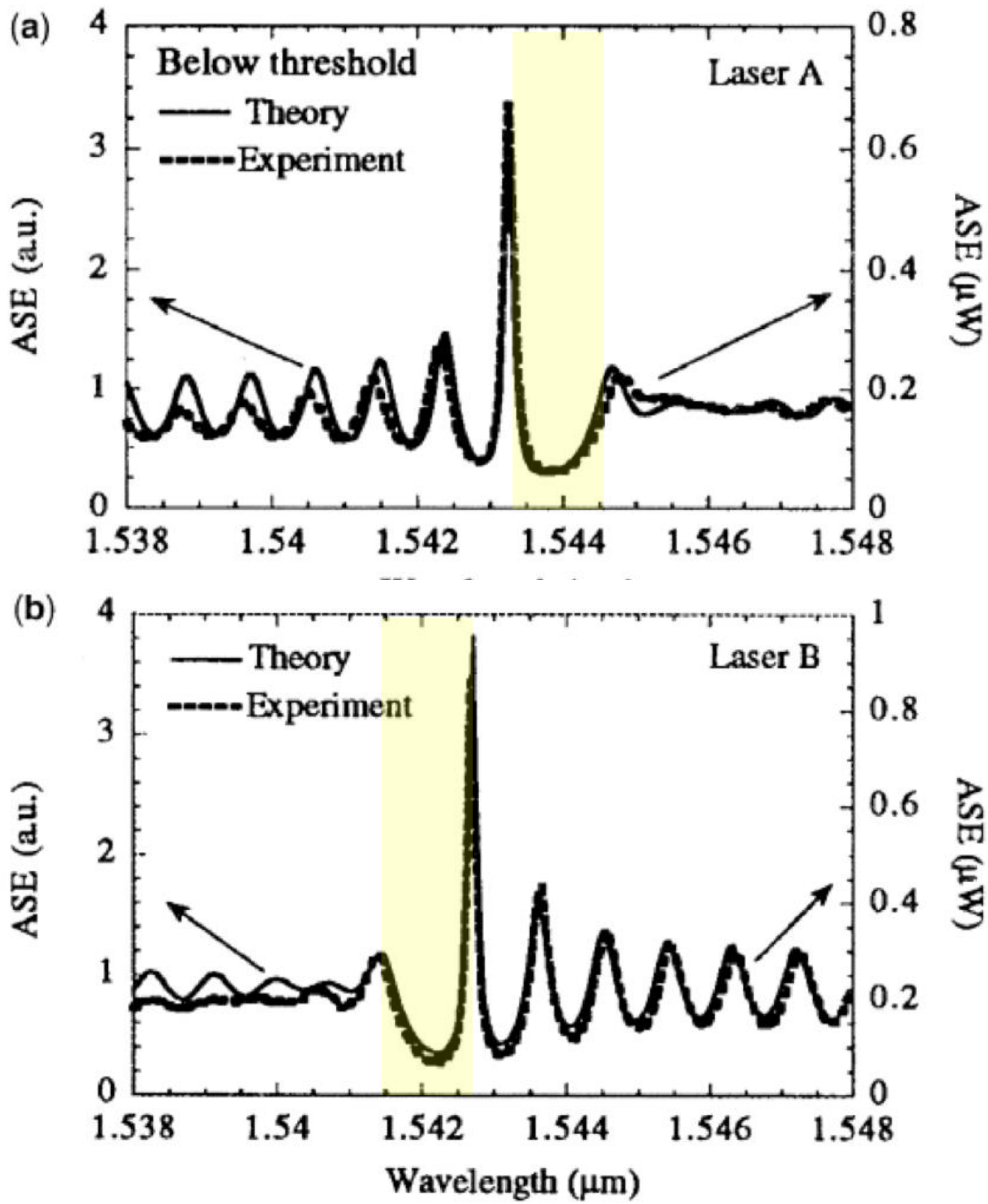


Effect of Cavity End Conditions: Facet Coating

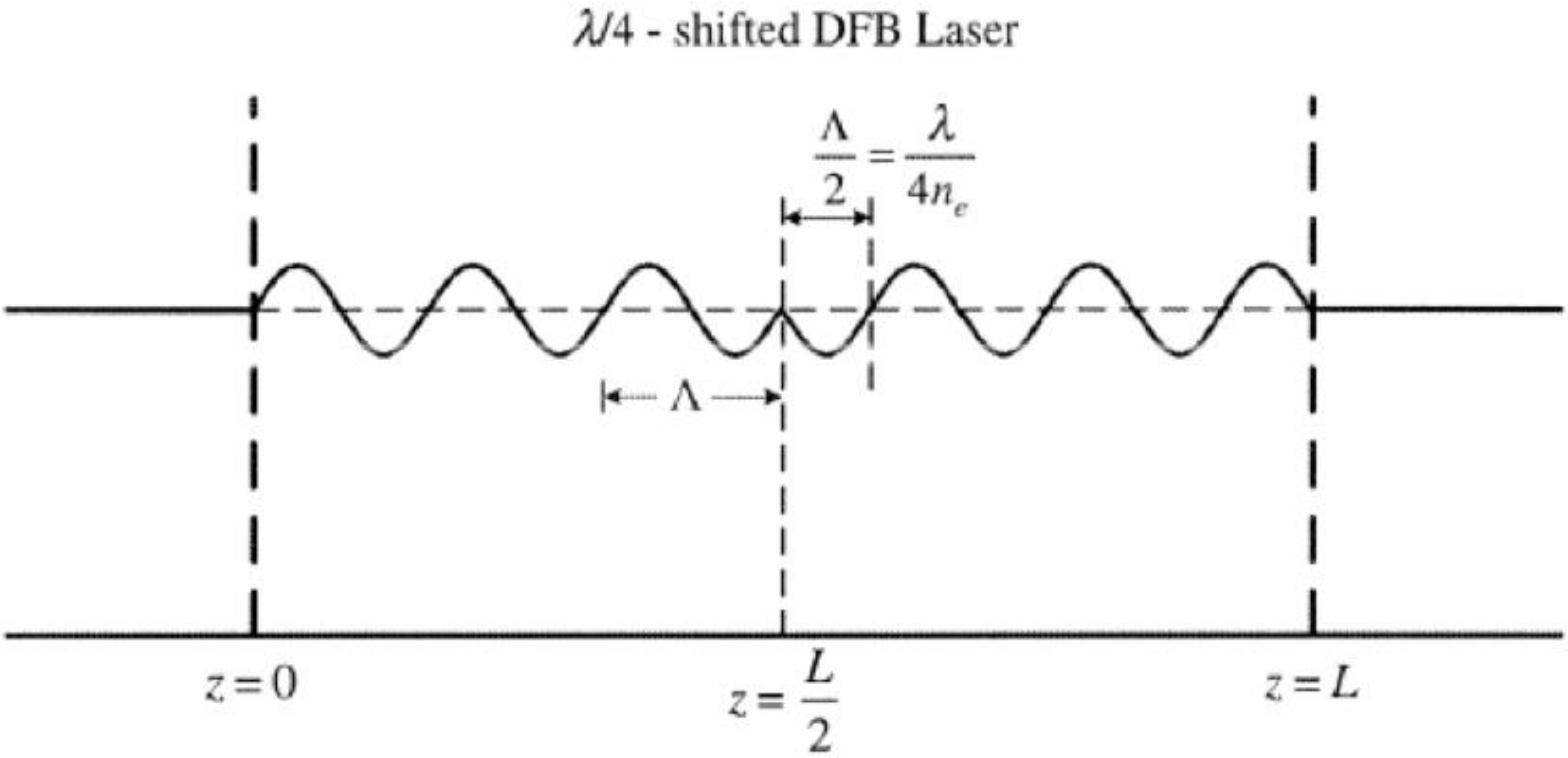
Asymmetrical Cavity



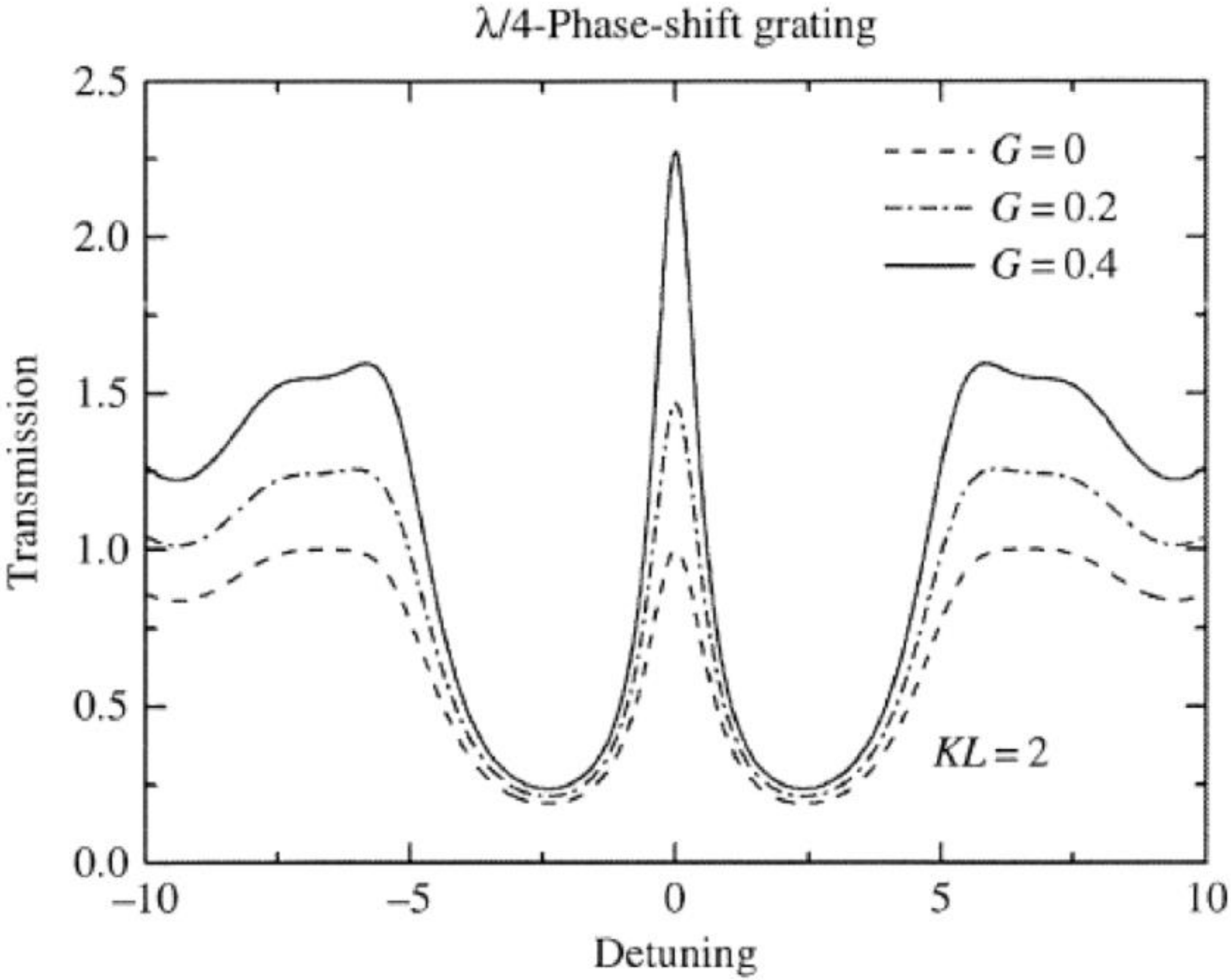
Lasing Peak can be obtained to operate on either side of stopband



Phase Shifted DFB breaks the degeneracy



Phase Shifted DFB breaks the degeneracy



Intuitive Picture

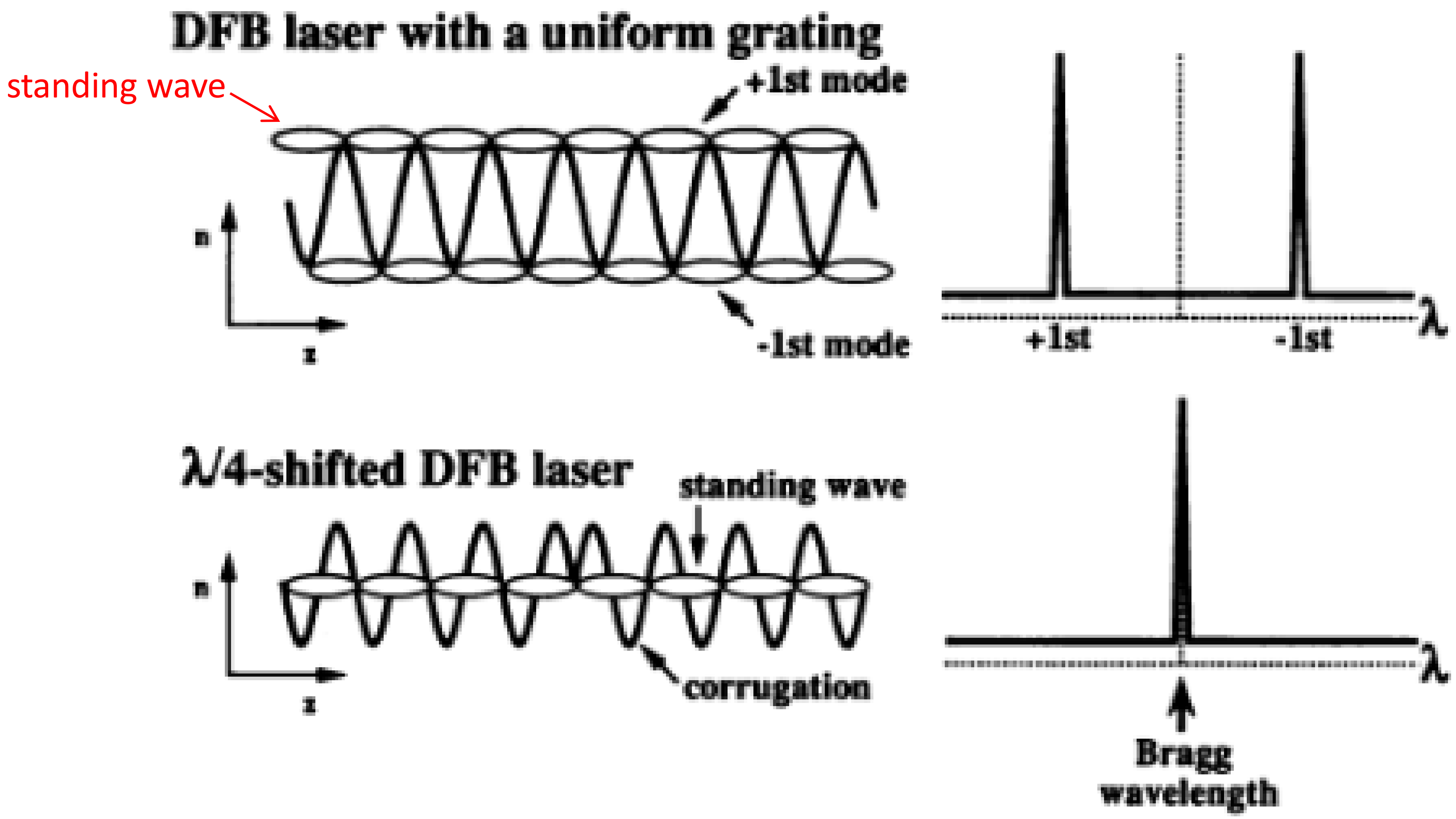
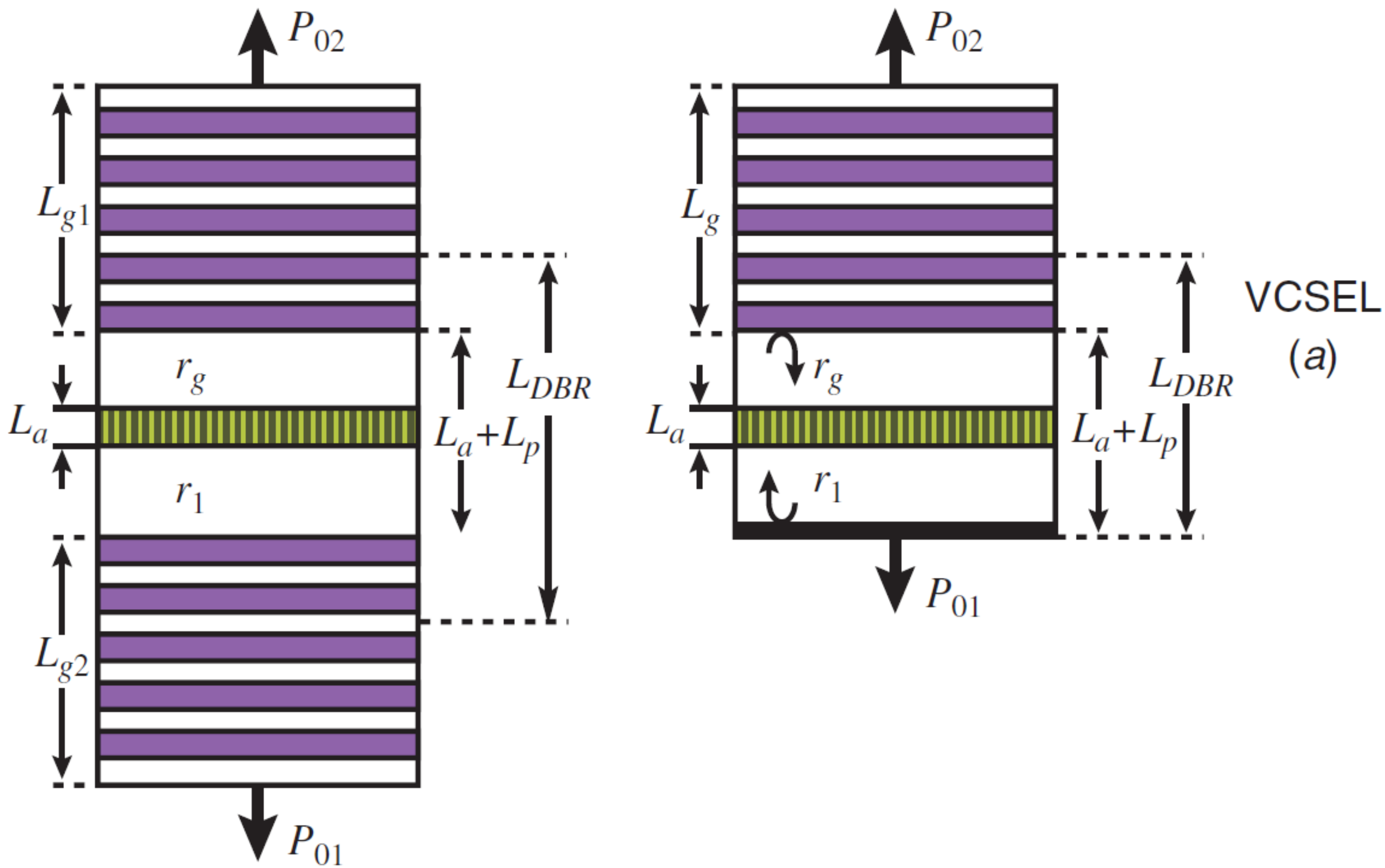


FIGURE 2.6. Schematic representation of longitudinal modes in DFB lasers with a uniform grating and λ/4-shifted DFB lasers. Corrugations and standing waves around the center of the laser axis are shown at the left-hand side of the figure.

[N. Chinone and M. Okai, "Distributed Feedback Semiconductor Lasers" Chapter 2 in *Semiconductor Lasers: Past, Present, and Future*, G.P. Agrawal, editor (1995)]

Applications of gratings in lasers – Distributed Bragg Reflector (DBR) laser

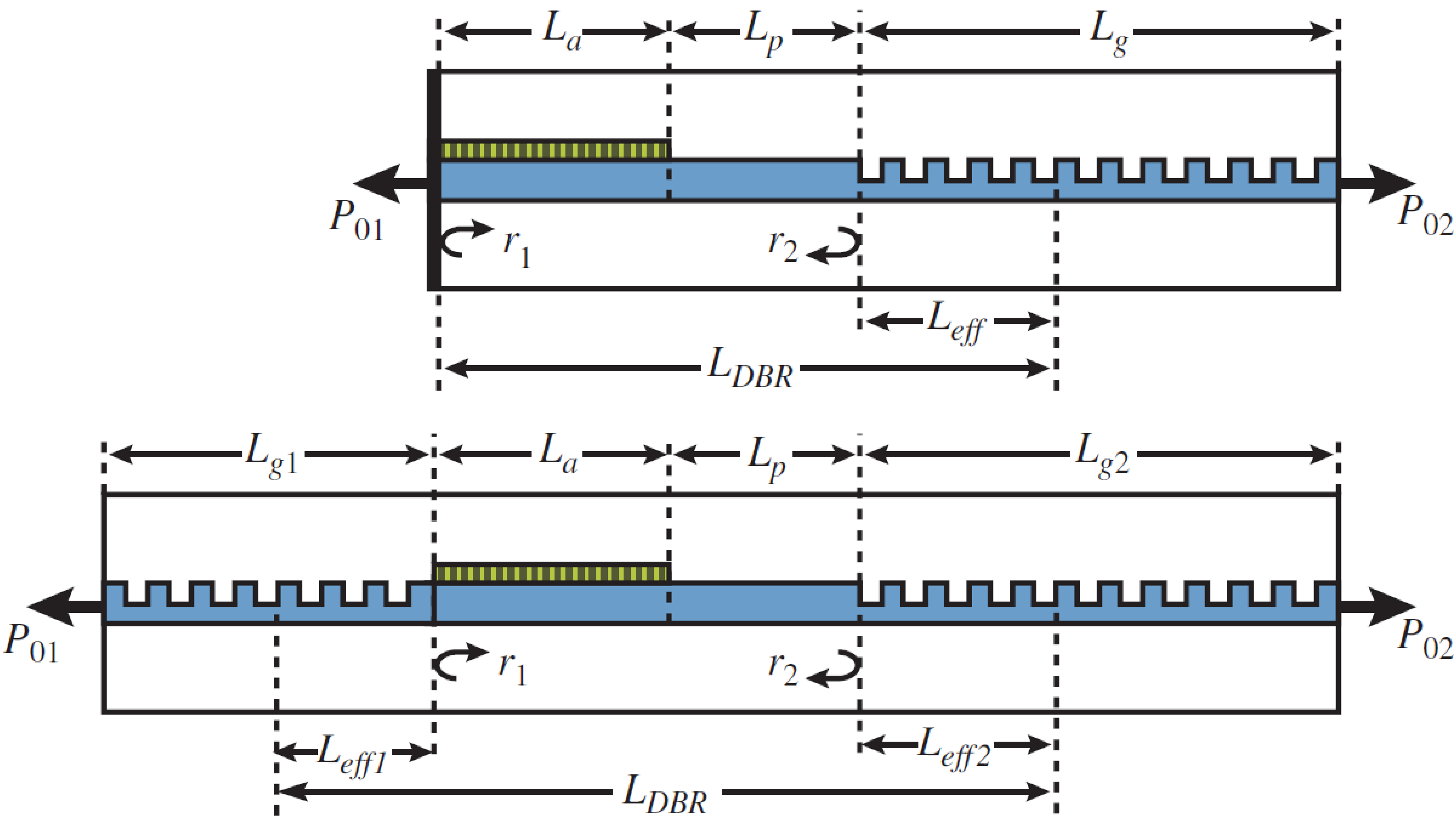
Vertical Cavity Surface Emitting (VCSEL) lasers



from Coldren, Corzine and Mašanović

Applications of gratings in lasers – Distributed Bragg Reflector (DBR) laser

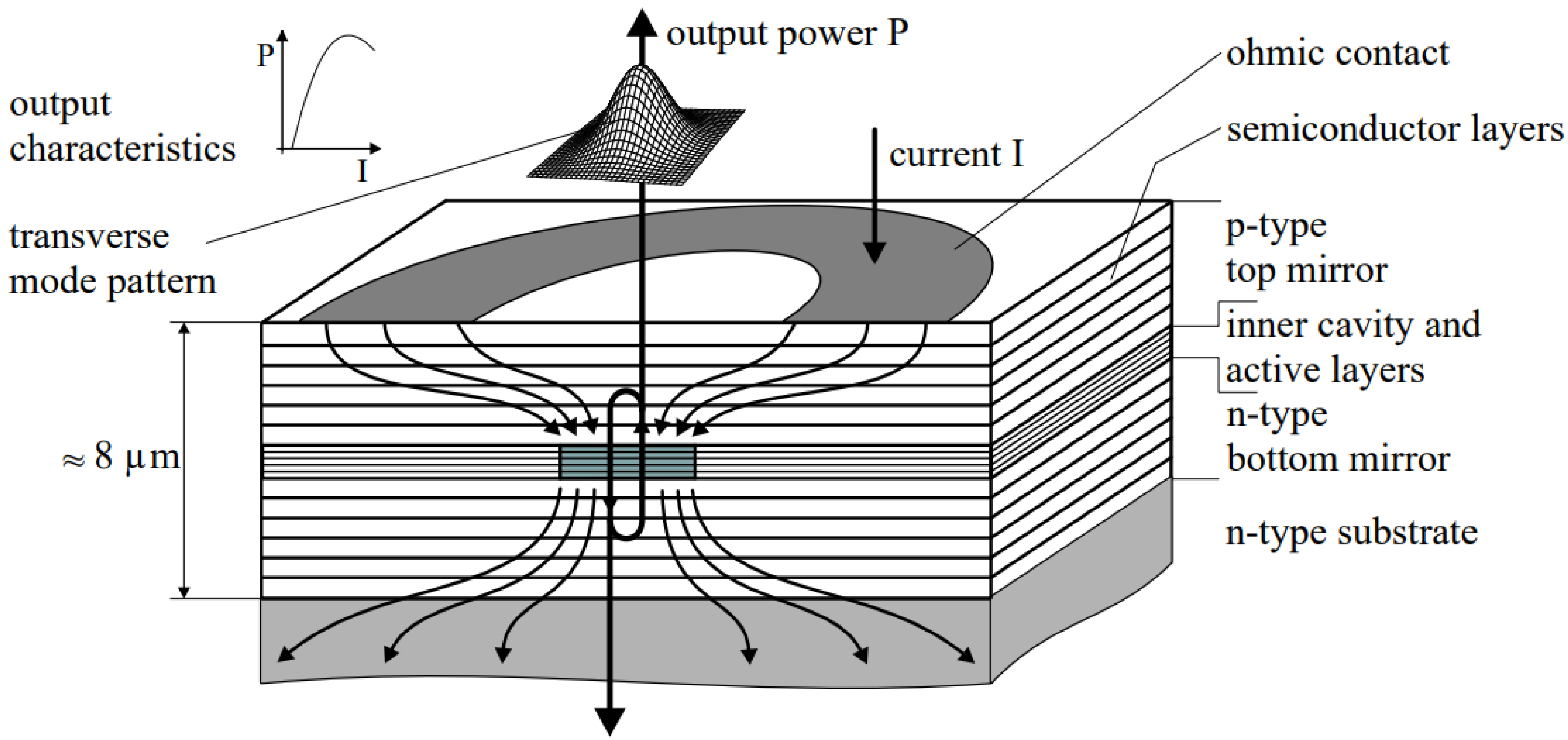
Edge emitting lasers



from Coldren, Corzine and Mašanović

VCSEL

Basic VCSEL Structure



Review: Laser Threshold

$$\alpha_i = \alpha_0(1-\Gamma) + \alpha_g\Gamma$$

$$\alpha_m = \frac{1}{2L}\ln\frac{1}{R_1R_2}$$

$$\Gamma g_{th} = \alpha_i + \alpha_m$$

$$\alpha_i L + \frac{1}{2}\ln\frac{1}{R_1R_2} = \gamma$$

total loss per pass

$$g(n) = g'(n - n_{tr})$$

$$\begin{cases} g_{th} = \dfrac{(\alpha_i + \alpha_m)}{\Gamma} \\ n_{th} = n_{tr} + \dfrac{(\alpha_i + \alpha_m)}{\Gamma g'} \end{cases}$$

If $R_1 = R_2 = R$

$$\alpha_m = \frac{1}{L}\ln\frac{1}{R}$$

$$J_{th} = \frac{qd}{\eta_i}\frac{n_{th}}{\tau_e(n_{th})}$$

Review: Laser Threshold

confinement factor

$$\Gamma = \frac{\int_{-d/2}^{+d/2} |U|^2 dx}{\int_{-\infty}^{+\infty} |U|^2 dx}$$

good approximation

$$\Gamma \cong D^2/(2 + D^2)$$

$$D = 2\pi (n_1^2 - n_2^2)^{1/2} d/\lambda$$

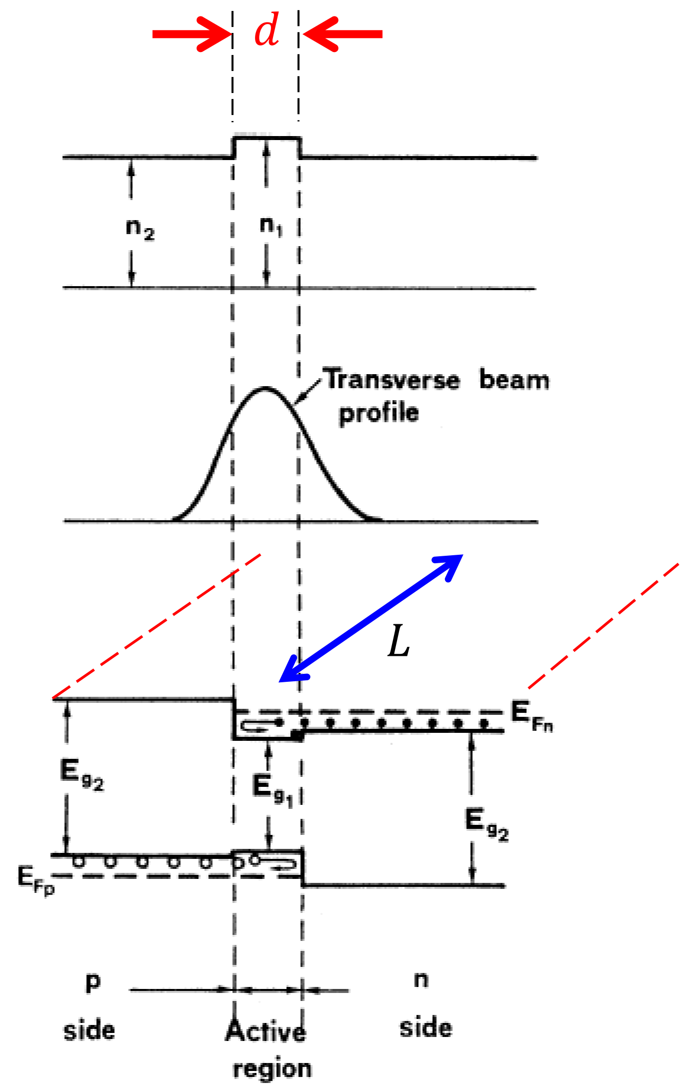
normalized thickness
of active layer

DH Laser – Example

$d = 0.1\ \mu\text{m}$
$\lambda = 850\ \text{nm}$
$n_1 = 3.6$
$n_2 = 3.4$

$D \approx 0.875$

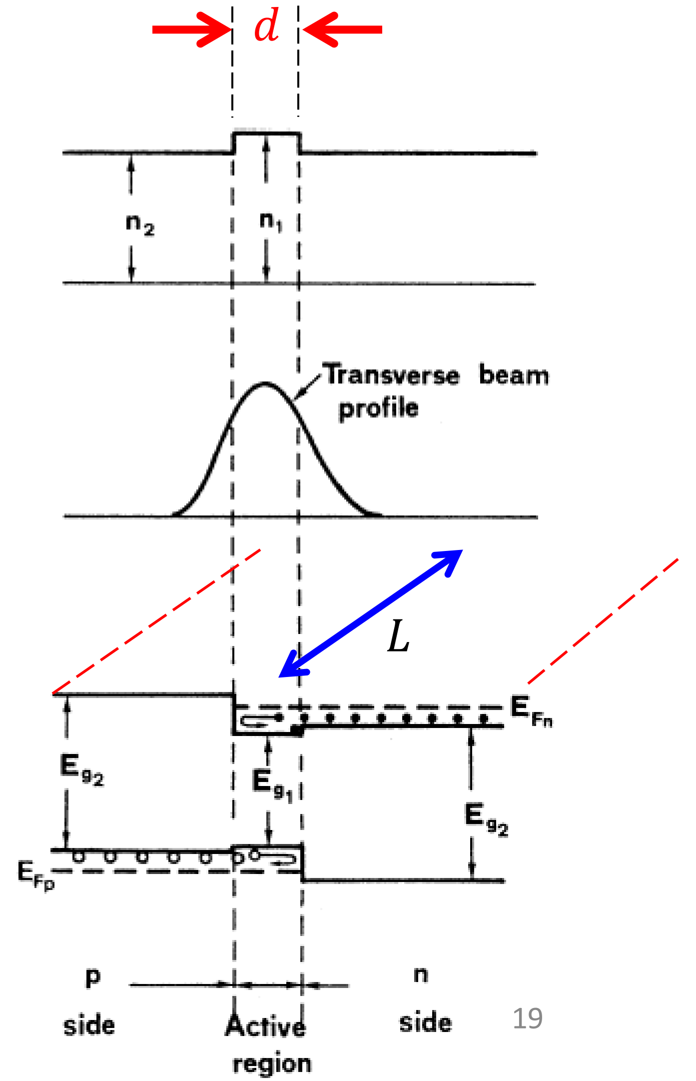
$\Gamma \approx 0.28$



Review: Laser Threshold

DH Laser – Example

$d = 0.1\ \mu\text{m}$
$\Gamma = 0.28$
$L = 300\ \mu\text{m}$
$\alpha_i = 10\ \text{cm}^{-1}$
$R = 0.32$
$g' = 1.5 \times 10^{-16}\ \text{cm}^2$
$N_{tr} = 2 \times 10^{18}\text{cm}^{-3}$
$\eta_i \approx 1.0$
$\tau_e = 4\ \text{ns}$



$$\gamma = \alpha_i L + \ln(1/R) = 1.4394$$

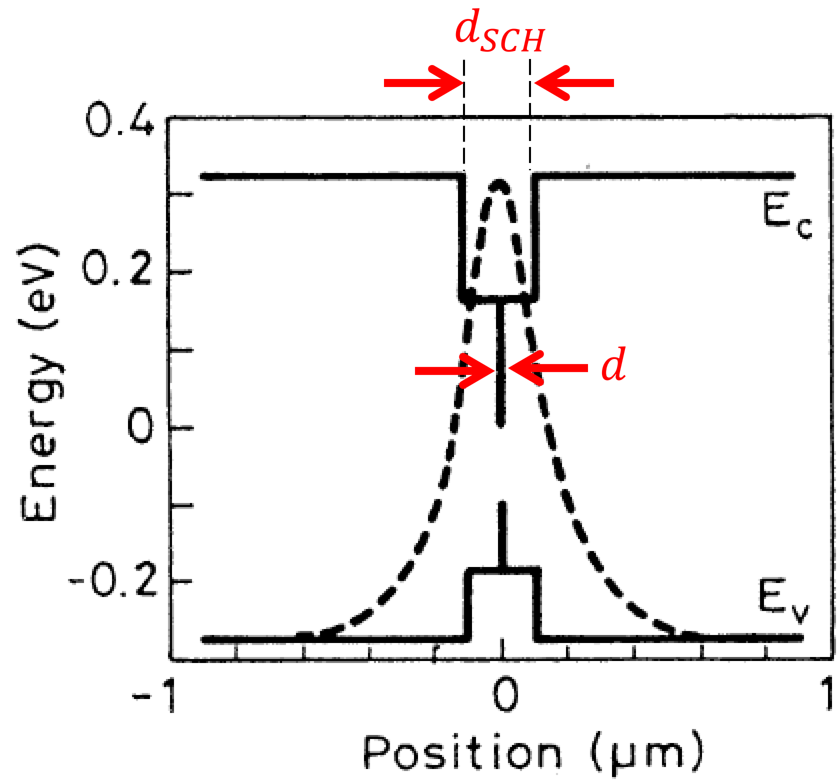
$$N_{th} = \frac{\gamma}{\Gamma g' L} + N_{tr} = 1.14 \times 10^{18} + 2 \times 10^{18}\ \text{cm}^{-3}$$

$$J_{th} = \left(\frac{q\ d}{\eta_i\ \tau_e}\right) N_{th} = 4.0 \times 10^{-16} \times 3.14 \times 10^{18}\ \text{cm}^{-3}$$
$$= 1.256 \times 10^{3}\ \ \text{A/cm}^2$$

Review: Laser Threshold

confinement factor

$$\Gamma = \frac{\int_{-d/2}^{+d/2} |U|^2 dx}{\int_{-\infty}^{+\infty} |U|^2 dx}$$



QW – Approximate with

$$U \propto \exp\left[-x^2/w_{spot}^2\right]$$

$$d_{SCH} = 2w_{spot}$$

$$\Gamma \approx \frac{d}{0.62\, d_{SCH}}$$

QW Laser – Example

$d_{SCH} = 1.0\ \mu\text{m}$

$d = 10\ \text{nm}$

$\Gamma \approx 1.6 \times 10^{-2}$

Review: Laser Threshold

QW Laser – Example

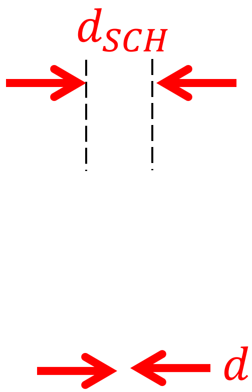
$d_{SCH} = 1.0\ \mu\text{m}$
$d = 10\ \text{nm}$
$\Gamma = 1.6 \times 10^{-2}$
$L = 300\ \mu\text{m}$
$\alpha_i = 10\ \text{cm}^{-1}$
$R = 0.32$
$g' = 6 \times 10^{-16}\ \text{cm}^2$
$N_{tr} = 2 \times 10^{18}\text{cm}^{-3}$
$\eta_i \approx 1$
$\tau_e = 4\ \text{ns}$

$$\gamma = \alpha_i L + \ln(1/R) = 1.4394$$

$$N_{th} = \frac{\gamma}{\Gamma g' L} + N_{tr} = 5 \times 10^{18} + 2 \times 10^{18}\ \text{cm}^{-3}$$

$$J_{th} = \left(\frac{q\ d}{\eta_i\ \tau_e}\right) N_{th} = 4 \times 10^{-17} \times 7 \times 10^{18}\ \text{cm}^{-3}$$
$$= 280\ \ \text{A/cm}^2$$

QW Laser – Example
$d_{SCH} = 1.0\ \mu\text{m}$
$d = 10\ \text{nm}$
$\Gamma = 1.6 \times 10^{-2}$
$L = 300\ \mu\text{m}$
$\alpha_i = 10\ \text{cm}^{-1}$
$R = 0.32$
$g' = 6 \times 10^{-16}\ \text{cm}^2$
$N_{tr} = 2 \times 10^{18}\text{cm}^{-3}$
$\eta_i \approx 1$
$\tau_e = 4\ \text{ns}$
$\gamma = \alpha_i L + \ln(1/R) = 1.4394$
$N_{th} = \frac{\gamma}{\Gamma g' L} + N_{tr} = 5 \times 10^{18} + 2 \times 10^{18}\ \text{cm}^{-3}$
$J_{th} = \left(\frac{q\ d}{\eta_i\ \tau_e}\right) N_{th} = 4 \times 10^{-17} \times 7 \times 10^{18}\ \text{cm}^{-3} = 280\ \ \text{A/cm}^2$



Review: Laser Threshold

QW Laser

How to reduce J_{th} further?

→ **Reduce cavity loss**

Example:

$$\alpha_i = 3\text{cm}^{-1}, R = 80\% :$$

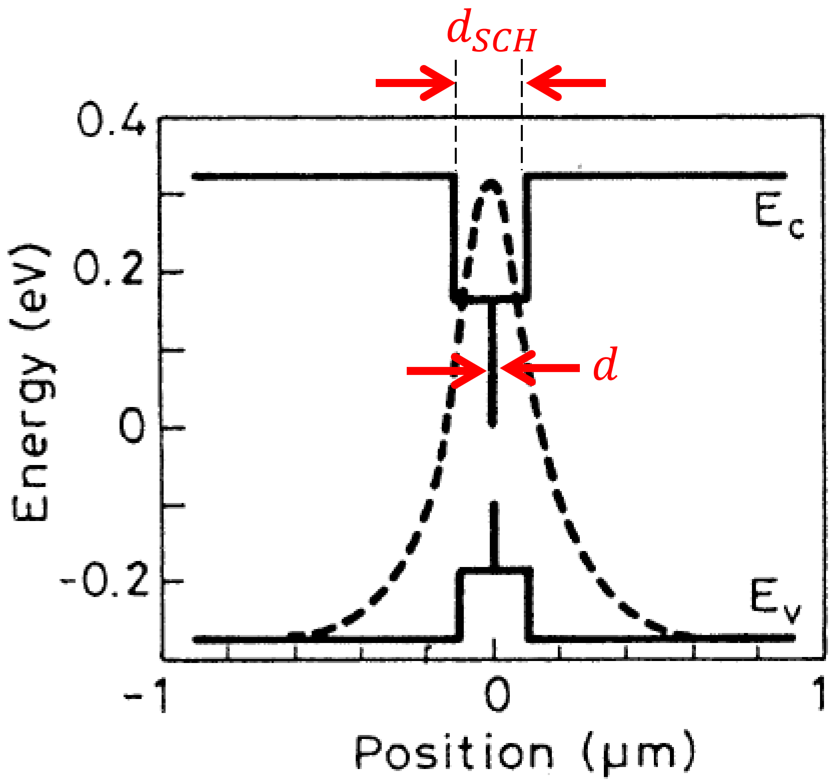
$$\gamma = 0.28$$

$$N_{th} = (2.3 + 2) \times 10^{18} = 4.43 \times 10^{18}\text{cm}^{-3}$$

$$\rightarrow \quad J_{th} \approx 170 \ \text{A/cm}^2$$

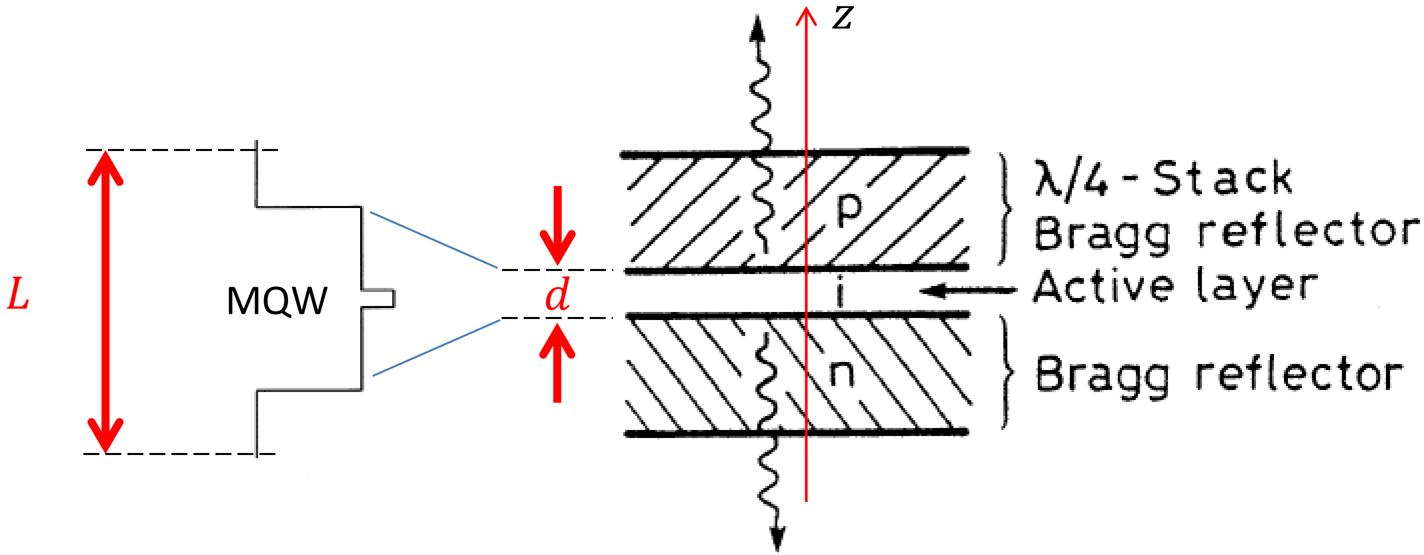
→ **Introduce strain**

- Reduces transparency density
- Increase differential gain



VCSEL

It is not straightforward to formulate a simple model for threshold

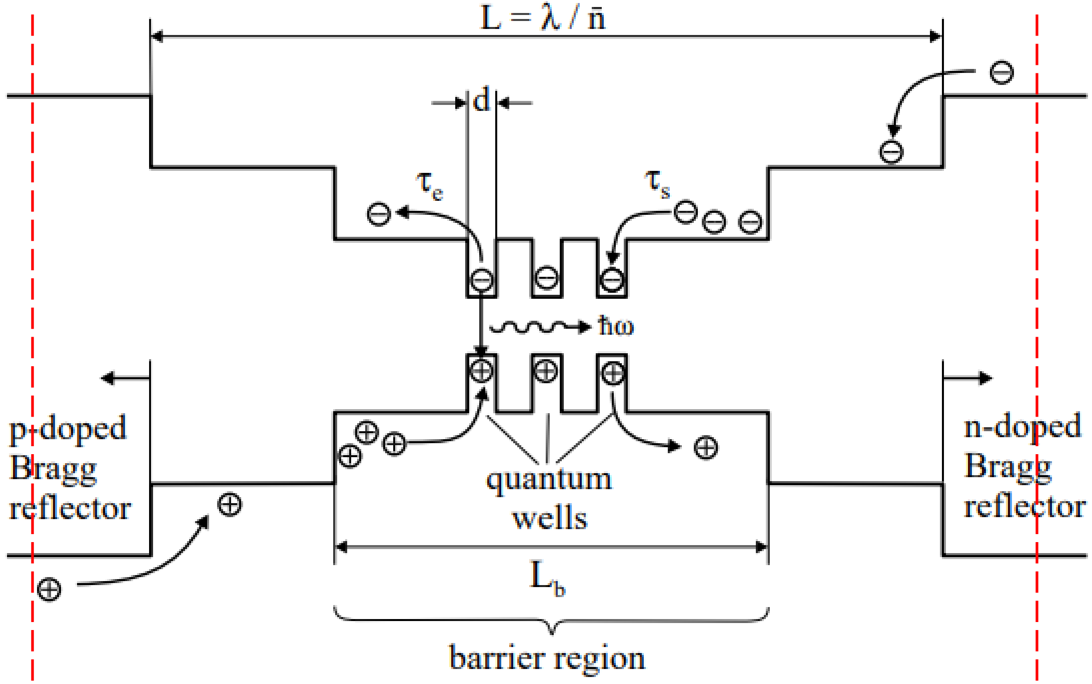


confinement factor in the transverse plane $\Gamma_{x,y} = \Gamma_t$

confinement factor along the axis *z* Γ_z

Very short cavity L = **Small gain**
somewhat compensated by **high reflectivity mirrors**

VCSEL



$L_{eff} = L +$ penetration depths into Bragg reflectors

Simple picture

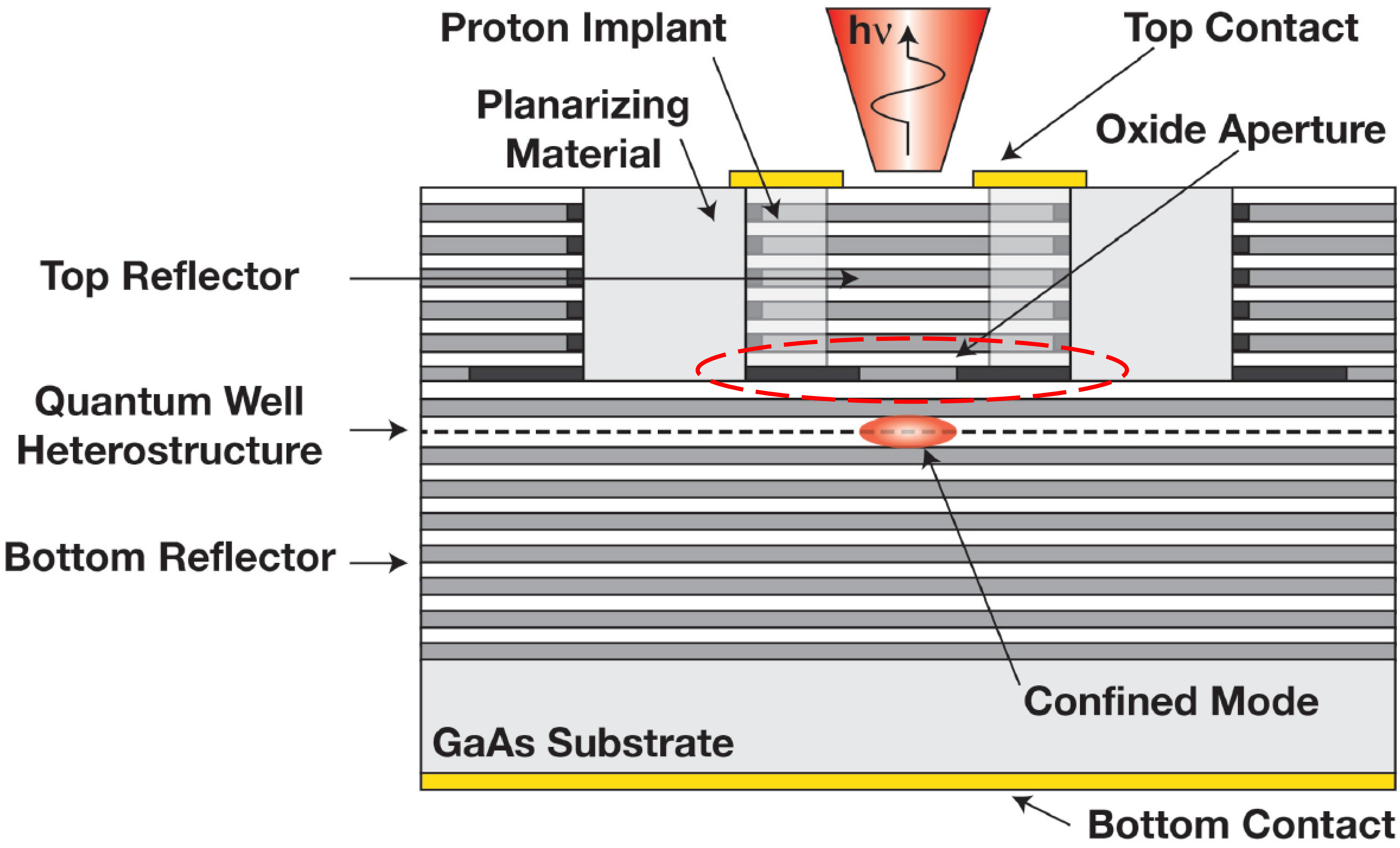
$$\frac{V_w}{V_p} = \Gamma = \Gamma_z \cdot \Gamma_t = \frac{d_a}{L_{eff}} \cdot \Gamma_t$$

volume occupied by active carriers (V_w)

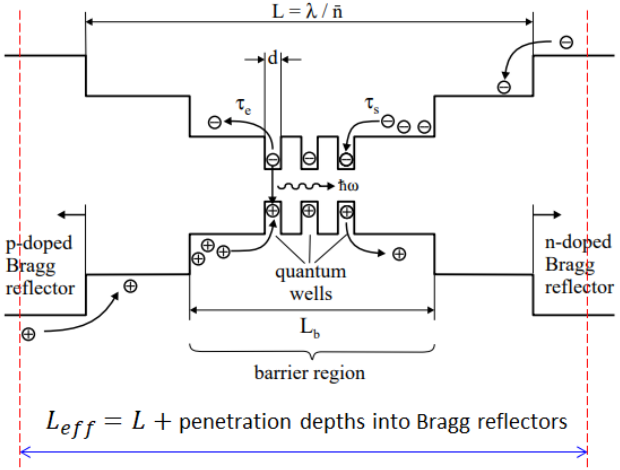
cavity volume occupied by photons (V_p)

where $0 < \Gamma_t \leq 1$ and often $\Gamma_t \approx 1$

The Modern VCSEL



VCSEL



Some authors use L_{eff} for the total length of the cavity as above.

Coldren, Corzine and Mašanović use the same to indicate the effective length at which to place a mirror that gives the same reflection coefficient as the grating as in the diagram below

$$L_{eff} = \frac{1}{2\kappa}\tanh(\kappa L_g), \qquad (|\delta L_g| \ll \pi)$$

where

$$\kappa L_g \equiv 2mr$$

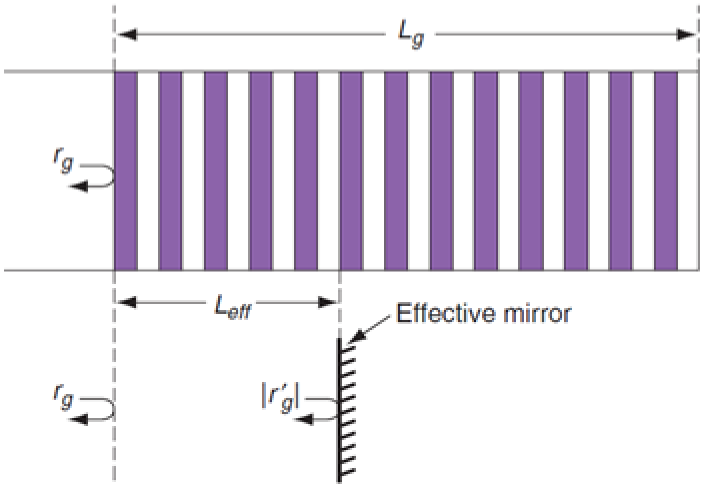
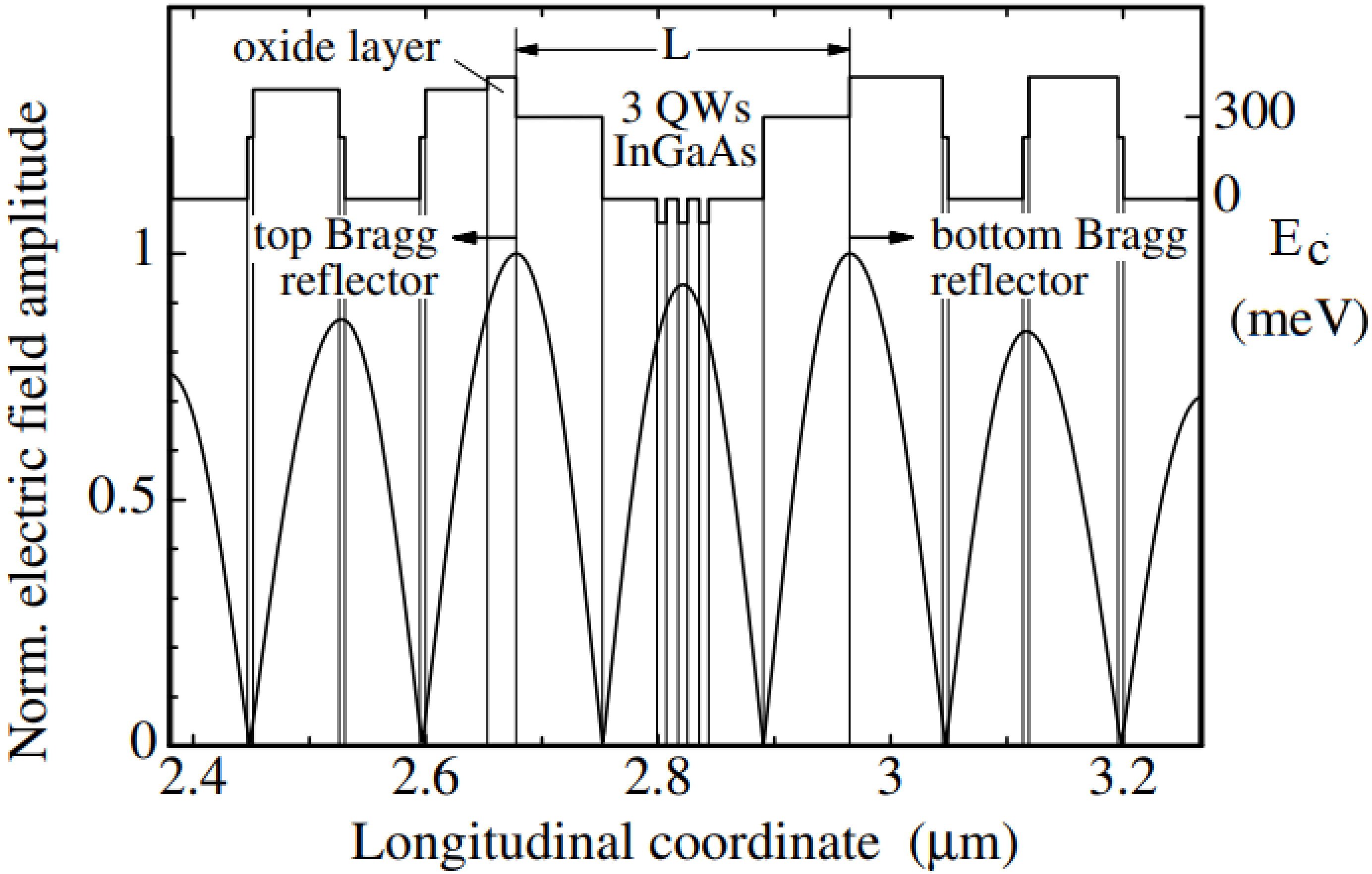


FIGURE 3.15: Definition of an effective mirror for a grating reflector.

VCSEL - Resonance condition



Resonance condition for this design

$$\langle \bar{n} \rangle L = m\lambda/2$$

spatially averaged refractive index ($\langle \bar{n} \rangle$)

integer (m)

place active layers at antinode of the standing wave pattern for good electron-photon coupling

VCSEL – Relative Confinement factor or Gain Enhancement factor

$$\Gamma_{\mathrm{r}} = \frac{L}{d_{\mathrm{a}}} \frac{\int_{d_{\mathrm{a}}} |E(z)|^2 \, \mathrm{d}z}{\int_L |E(z)|^2 \, \mathrm{d}z}$$

Approximate field in the central part of the cavity (*z* = 0 at center of cavity)

$$E(z) = E_0 \cos\left(2\pi \langle \bar{n} \rangle z / \lambda\right)$$

for an optimally positioned active region one can show that

$$\Gamma_{\mathrm{r}} = 1 + \frac{\sin(2\pi \langle \bar{n} \rangle d_{\mathrm{a}} / \lambda)}{2\pi \langle \bar{n} \rangle d_{\mathrm{a}} / \lambda} \quad \geq \mathbf{1}$$

For the 3 QW system shown earlier, $\Gamma_{\mathrm{r}} = 1.8$

For a single thin QW, $\Gamma_{\mathrm{r}} \rightarrow 2$

For $d_{\mathrm{a}} = m\lambda / (2\langle \bar{n} \rangle)$ we have $\Gamma_{\mathrm{r}} = 1$

VCSEL – Threshold Gain and Photon Lifetime

$$g_{\mathrm{th}} = \alpha_{\mathrm{a}} + \frac{1}{\Gamma_{\mathrm{r}} d_{\mathrm{a}}} \left[\alpha_{\mathrm{i}} (L_{\mathrm{eff}} - d_{\mathrm{a}}) + \ln \frac{1}{\sqrt{R_{\mathrm{t}} R_{\mathrm{b}}}} \right]$$

↑ losses in active region

OK for lossless mirrors (but gratings are usually lossy)

$$\frac{1}{\tau_{\mathrm{p}}} = \frac{d_{\mathrm{a}}}{L_{\mathrm{eff}}} \langle v_{\mathrm{gr}} \rangle \Gamma_{\mathrm{r}} g_{\mathrm{th}} \approx \langle v_{\mathrm{gr}} \rangle \left[\alpha_{\mathrm{i}} + \frac{1}{L_{\mathrm{eff}}} \ln \frac{1}{\sqrt{R_{\mathrm{t}} R_{\mathrm{b}}}} \right]$$

$$\boxed{\langle v_{\mathrm{gr}} \rangle = c / \langle \bar{n}_{\mathrm{gr}} \rangle}$$

Example

$d_a = 24$ nm

$\Gamma_r = 1.8$

$L_{eff} = 1.3$ μm

$\alpha_a \ll g_{th}$

$R_t = R_b = 0.995$

$\langle \bar{n}_{gr} \rangle = 3.6$

$\alpha_i = 10 \text{ cm}^{-1}$

$$\boxed{g_{th} = 1{,}460 \text{ cm}^{-1}}$$

$$\boxed{\tau_p = 2.5 \text{ ps}}$$

VCSEL – Laser Threshold

Adapting the earlier model for a simple estimate:

$d_a = 24$ nm (equivalent active thickness of MQW)

$L = 2\ \mu\text{m}$

$D = 8\mu\text{m}$

$\Gamma_r = 1.8$

$\alpha_i = 20\ \text{cm}^{-1}$

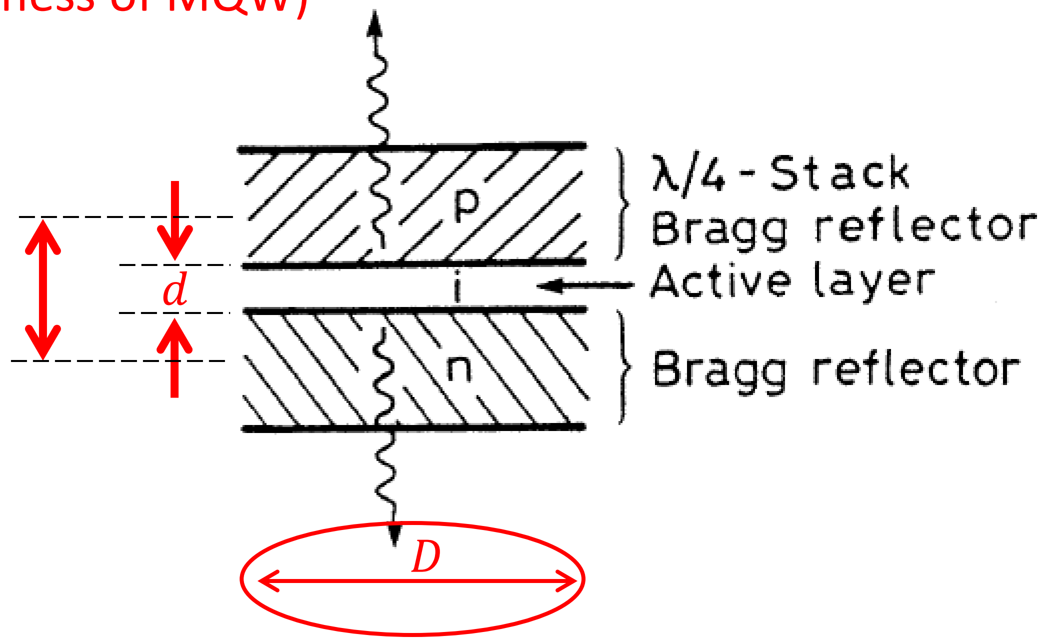
$R = 0.995$

$g' = 6 \times 10^{-16}\ \text{cm}^2$

$N_{tr} = 2 \times 10^{18}\text{cm}^{-3}$

$\eta_i \approx 1$

$\tau_e = 2$ ns



$$\gamma = \alpha_i L + \ln(1/R) = 1.4 \times 10^{-2}$$

$$N_{th} = \frac{\gamma}{\Gamma_r g' d_a} + N_{tr} = 5.4 \times 10^{18} + 2 \times 10^{18}\ \text{cm}^{-3} = 7.4 \times 10^{18}\ \text{cm}^{-3}$$

$$J_{th} = \left(\frac{q\ d_a}{\eta_i\ \tau_e}\right) N_{th} = 9.6 \times 10^{-17} \times 7.4 \times 10^{18}\ \text{cm}^{-3} = 2.26 \times 10^3\ \text{A/cm}^2$$

$$I_{th} = \left(\frac{\pi D^2}{4}\right) J_{th} = 1.14\ \text{mA}$$

Gratings Calculation

For the design of high performance VCSELs it is important to know the electric field distribution in the resonator. Assuming linearly polarized waves in a 1-D scalar approach, we have to solve the Helmholtz equation

$$\frac{\mathrm{d}^2 E(z)}{\mathrm{d}z^2} + \gamma^2 E(z) = 0$$

for the phasor of the transverse electric field component $E = E_x$

In each homogeneous *m*th layer

$$\gamma_m = \beta_m - \mathrm{i}\alpha_m/2$$

$$\beta_m = 2\pi\bar{n}_m/\lambda$$

The absorption coefficient fulfills $\alpha_m \geq 0$ except from the QW layers, where gain leads to $\alpha_m < 0$.

Gratings Calculation

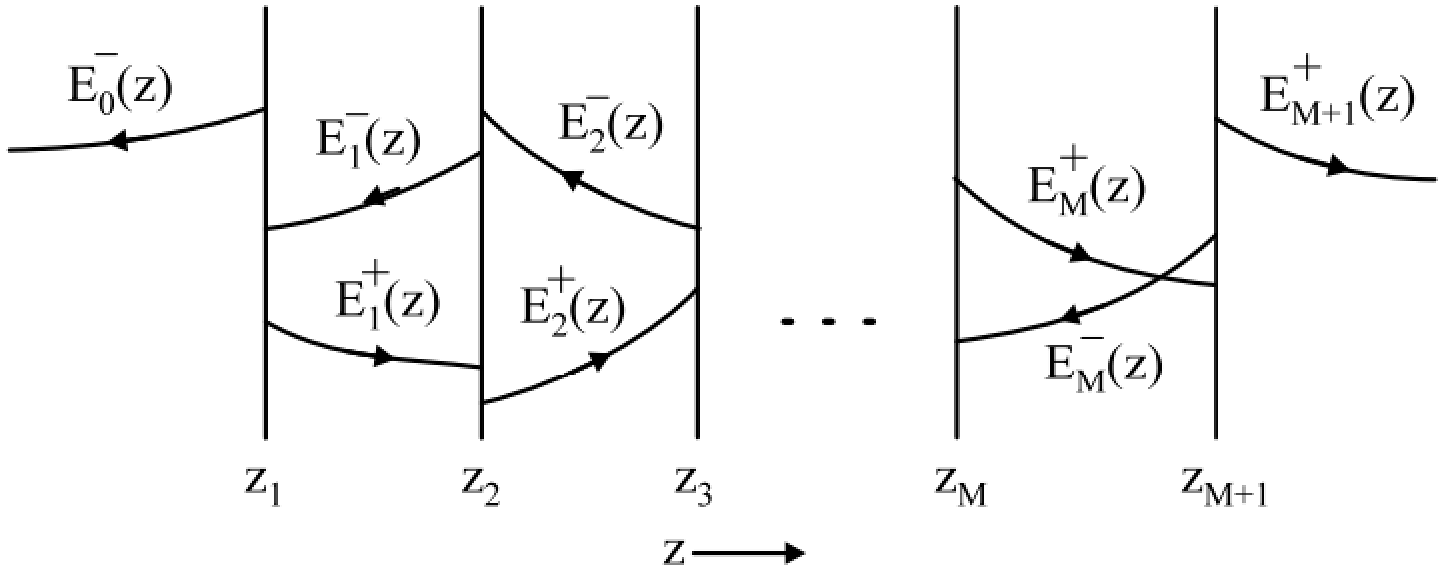
In each layer, the electric field is the superposition of two monochromatic plane waves counterpropagating in the *z*-direction

$$E_m(z) = E_m^+ \exp\{-\mathrm{i}\gamma_m(z - z_m)\} + E_m^- \exp\{+\mathrm{i}\gamma_m(z - z_m)\}$$

E_m^+ and E_m^- denote the complex field amplitudes of the waves at the interface

$z = z_m$ with $z_m \leq z \leq z_{m+1}$

$$m = 1, \ldots, M$$



Gratings Calculation

The continuity conditions for the transverse components of electric and magnetic fields lead to relations, between amplitudes in subsequent layers

$$E_m^+ = (\gamma_m^+ E_{m+1}^+ + \gamma_m^- E_{m+1}^-) \exp\{i\gamma_m(z_{m+1} - z_m)\} ,$$
$$E_m^- = (\gamma_m^- E_{m+1}^+ + \gamma_m^+ E_{m+1}^-) \exp\{-i\gamma_m(z_{m+1} - z_m)\}$$

where we have the abbreviations

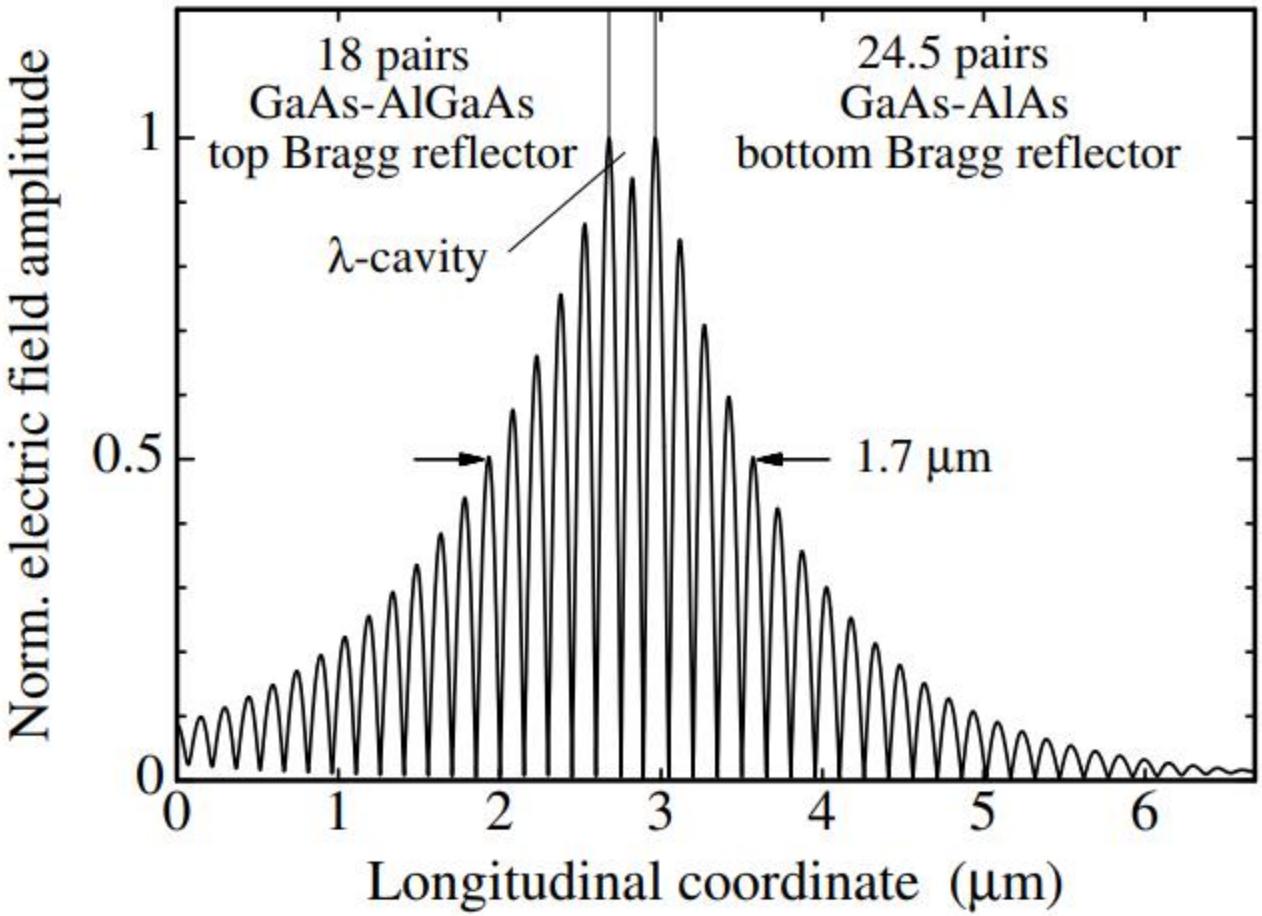
$$\gamma_m^+ = \frac{\gamma_m + \gamma_{m+1}}{2\gamma_m} \quad \text{and} \quad \gamma_m^- = \frac{\gamma_m - \gamma_{m+1}}{2\gamma_m}$$

Self-oscillation of the layer structure exclusively allows **outgoing** waves in the terminating sections

$$m = 0 \qquad \boxed{E_0^+ = 0}$$

$$m = M + 1 \qquad \boxed{E_{M+1}^- = 0}$$

Gratings Calculations



due to the high reflectivities of the mirrors, a pronounced resonant enhancement of the field amplitude is built up

Gratings Calculation

Poynting vector

$$\boldsymbol{S} = \mathrm{Re}\{\boldsymbol{E} \times \boldsymbol{H}^*\}$$

In the 1-D scalar approach the magnetic field is

$$H = H_y = \frac{\mathrm{i}}{\omega\mu_0}\frac{\mathrm{d}E}{\mathrm{d}z}$$

The energy flux in section m of the multilayer structure can be expressed as

$$S_m(z) = \frac{\beta_m}{\omega\mu_0}|E_m^+|^2 \exp\{-\alpha_m(z - z_m)\} - \frac{\beta_m}{\omega\mu_0}|E_m^-|^2 \exp\{\alpha_m(z - z_m)\} + \frac{\alpha_m}{\omega\mu_0} \mathrm{Im}\left\{E_m^+(E_m^-)^* \exp\{-\mathrm{i}2\beta_m(z - z_m)\}\right\} .$$

Gratings Calculation

Above threshold current I_{th}, top and bottom light output powers P_t and P_t linearly increase with driving current I. We can write

$$P_{\mathrm{t,b}} = \tilde{\eta}_{\mathrm{dt,b}} \frac{\hbar\omega}{q} (I - I_{\mathrm{th}})$$

$$\tilde{\eta}_{\mathrm{dt,b}} = \eta_{\mathrm{dt,b}}\, \eta_{\mathrm{i}}$$

where the differential quantum efficiency

$$\eta_{\mathrm{dt,b}}$$

characterizes the percentage of generated coherent light that is available as top or bottom emission. In well designed VCSELs with high quality active QWs

$$\eta_{\mathrm{i}} > 90\,\%$$

Due to absorption in the mirrors we always find $\eta_{\mathrm{dt}} + \eta_{\mathrm{db}} < 100\,\%$

Gratings Calculation

The differential quantum efficiency is identified as the fraction of the generated flux that is emitted through top or bottom mirrors

$$\eta_{\mathrm{d}} = \eta_{\mathrm{dt}} + \eta_{\mathrm{db}}$$

$$\eta_{\mathrm{d}} = \frac{g_{\mathrm{th}}}{g_{\mathrm{th}} + \alpha_{\mathrm{a}}} \left(1 + \frac{\sum_{i,\mathrm{pass.}} \Delta S_i}{\sum_{i,\mathrm{act.}} \Delta S_i} \right) \quad \text{with} \quad \Delta S_i = S(z_{i+1}) - S(z_i)$$

Absorption leads to flux increments

$$\Delta S_i < 0 \qquad\qquad \eta_{\mathrm{d}} < 1$$

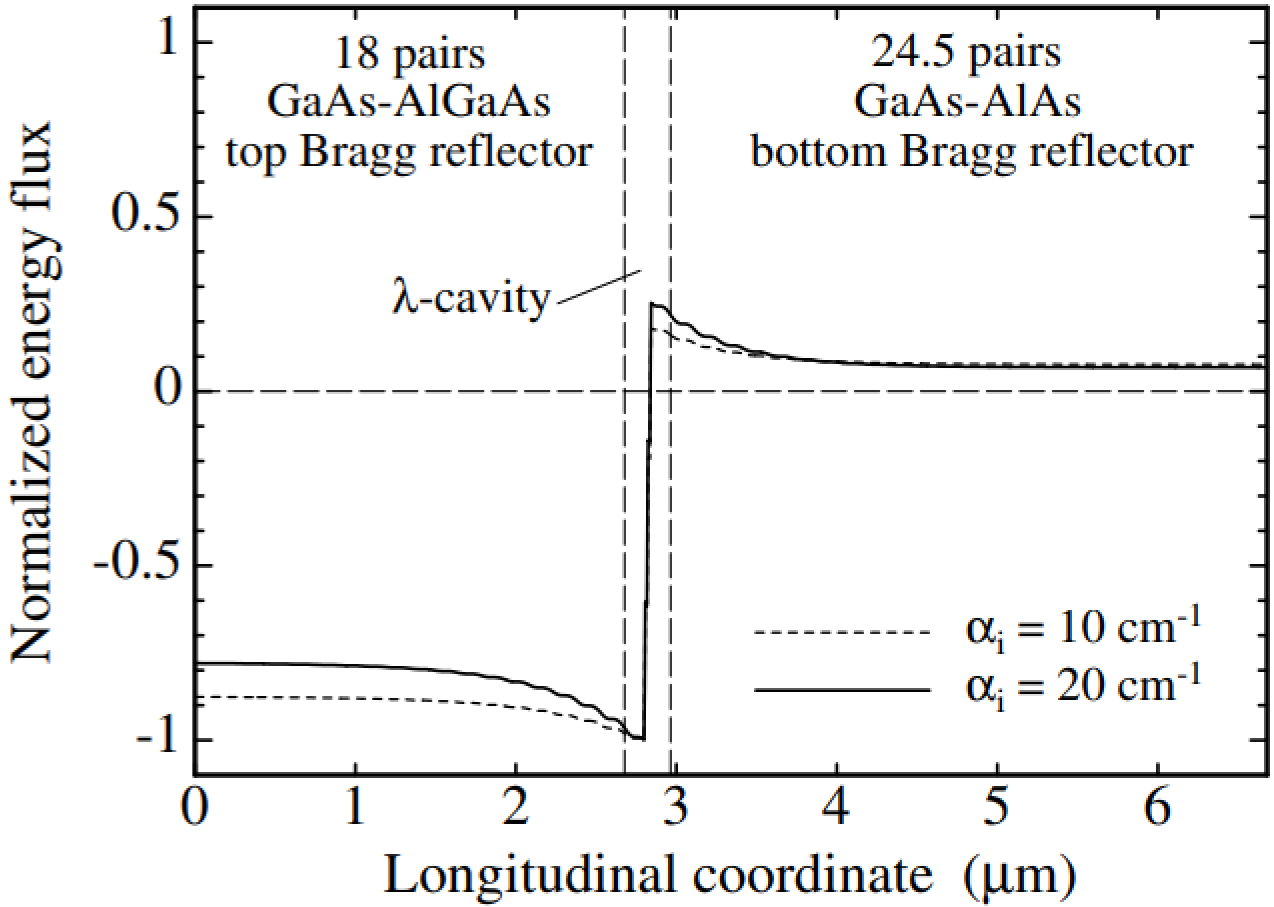
Denoting top and bottom emitted energy fluxes

$$S_{\mathrm{t}} = S(z_1) < 0 \text{ and } S_{\mathrm{b}} = S(z_{M+1}) > 0$$

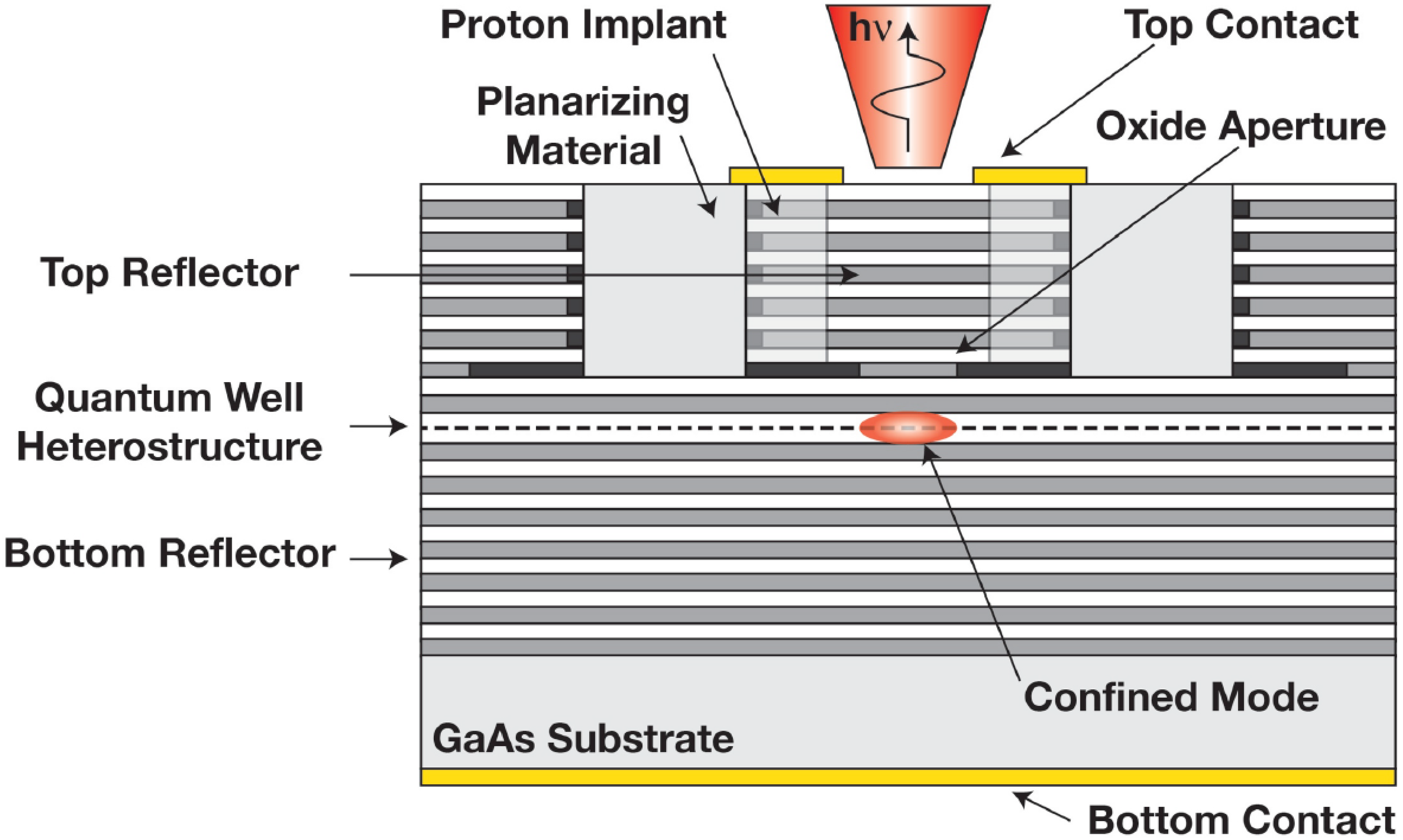
the corresponding differential quantum efficiencies are

$$\eta_{\mathrm{dt}} = \eta_{\mathrm{d}} \frac{|S_{\mathrm{t}}|}{|S_{\mathrm{t}}| + |S_{\mathrm{b}}|} \qquad \text{and} \qquad \eta_{\mathrm{db}} = \eta_{\mathrm{d}} \frac{|S_{\mathrm{b}}|}{|S_{\mathrm{t}}| + |S_{\mathrm{b}}|}$$

Normalized energy flux density



The Modern VCSEL



VCSEL emission characteristics

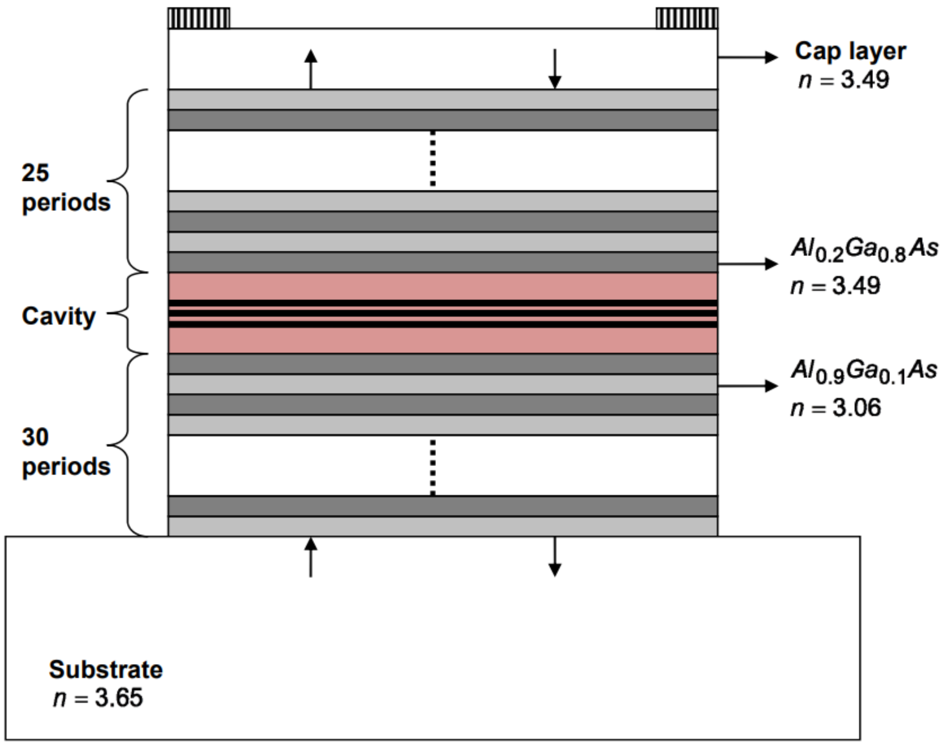
The active diameter of the VCSEL can be made quite small (few micron size) to reach the lowest possible threshold current (down to sub-100 μA) but it can also exceed 100 μm to get power beyond 100mW.

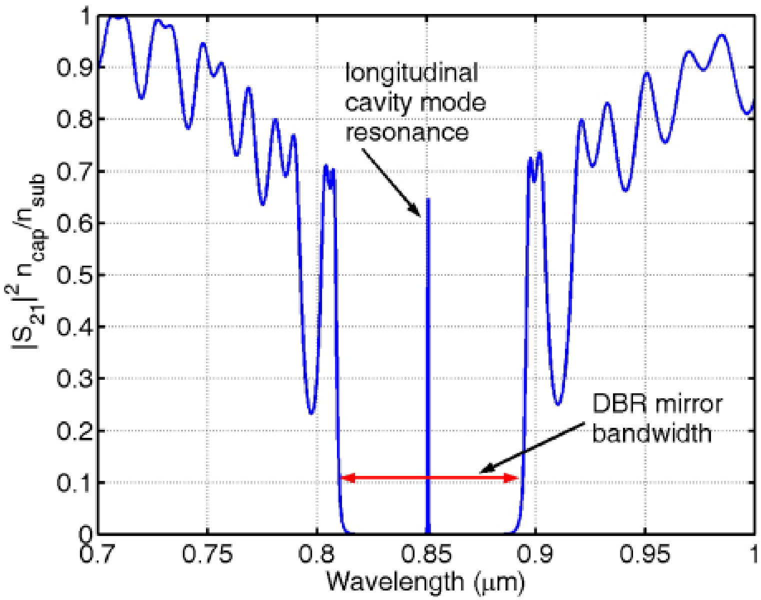
Rule of Thumb: Up to active region diameters of about 4 μm the VCSEL emits in the single fundamental transverse mode. Larger devices start lasing on several higher order radial and azimuthal modes.

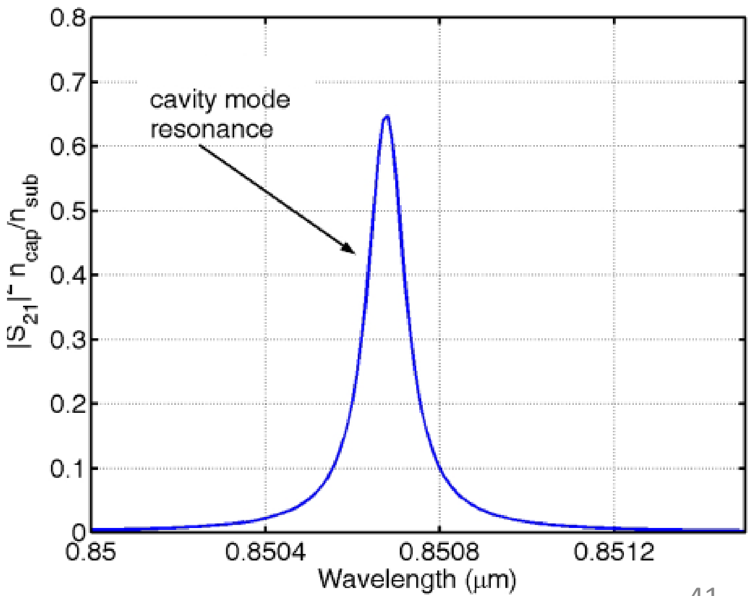
Due to the small length of the laser cavity (1 to 2 μm) consecutive longitudinal modes are widely spaced in wavelength ($\Delta\lambda \approx 100$ nm) .

If one mode is made to coincide with the peak reflectivity of the mirrors, the two adjacent modes fall outside the high-reflectivity band and single longitudinal mode oscillation can also be established.

VCSEL emission characteristics







Reading Assignments:

- Section 8.5 of Chuang's book
- Sections 11.1 and 11.2 of Chuang's book

- Chapter 3 and Appendix 5 in Coldren, Corzine and Mašanović
- Section 8.2 in Coldren, Corzine and Mašanović