ECE 536 – Integrated Optics and Optoelectronics
Lecture 24 – April 14, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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Lecture 24 Outline

- More on VCSELs
- Examples

VCSEL – Relative Confinement factor or Gain Enhancement factor

$$\Gamma_{\mathrm{r}} = \frac{L}{d_{\mathrm{a}}} \frac{\int_{d_{\mathrm{a}}} |E(z)|^2 \, \mathrm{d}z}{\int_L |E(z)|^2 \, \mathrm{d}z}$$

Approximate field in the central part of the cavity ($z = 0$ at center of cavity)

$$E(z) = E_0 \cos\left(2\pi \langle \bar{n} \rangle z / \lambda\right)$$

for an optimally positioned active region one can show that

$$\Gamma_{\mathrm{r}} = 1 + \frac{\sin(2\pi \langle \bar{n} \rangle d_{\mathrm{a}} / \lambda)}{2\pi \langle \bar{n} \rangle d_{\mathrm{a}} / \lambda} \quad \geq \mathbf{1}$$

For the 3 QW system shown earlier, $\Gamma_{\mathrm{r}} = 1.8$

For a single thin QW, $\Gamma_{\mathrm{r}} \rightarrow 2$

For $d_{\mathrm{a}} = m\lambda / (2\langle \bar{n} \rangle)$ we have $\Gamma_{\mathrm{r}} = 1$

VCSEL – Laser Threshold

Adapting the earlier model for a simple estimate:

$d_a = 24$ nm (equivalent active thickness of MQW)

$L = 2\ \mu\text{m}$

$D = 8\mu\text{m}$

$\Gamma_r = 1.8$

$\alpha_i = 20\ \text{cm}^{-1}$

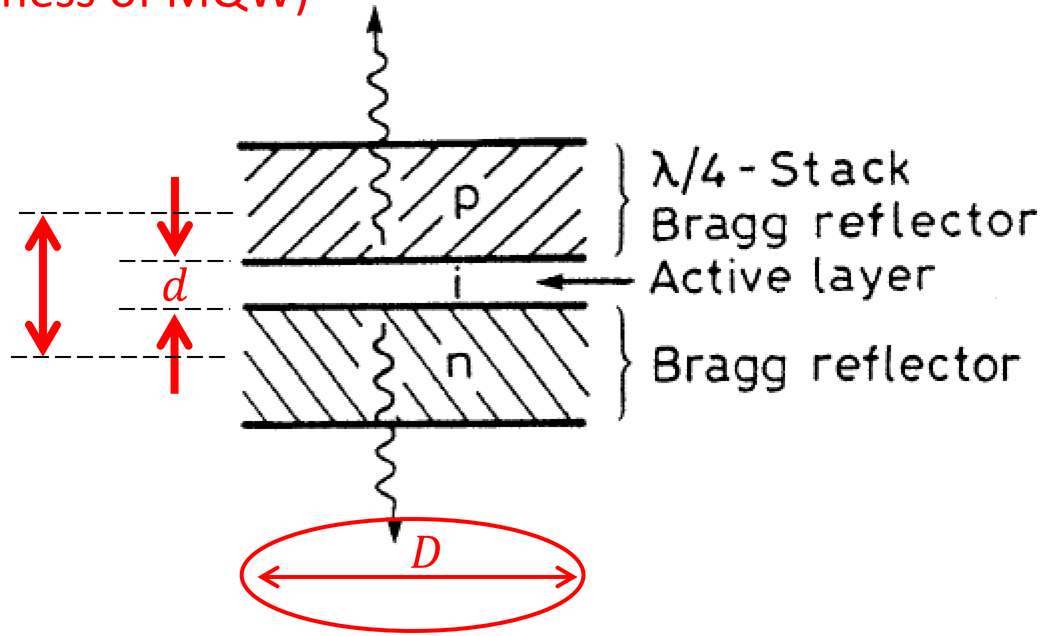
$R = 0.995$

$g' = 6 \times 10^{-16}\ \text{cm}^2$

$N_{tr} = 2 \times 10^{18} \text{cm}^{-3}$

$\eta_i \approx 1$

$\tau_e = 2$ ns



$$\overline{\gamma} = \alpha_i L + \ln(1/R) = 1.4 \times 10^{-2}$$

$$N_{th} = \frac{\overline{\gamma}}{\Gamma_r g' d_a} + N_{tr} = 5.4 \times 10^{18} + 2 \times 10^{18}\ \text{cm}^{-3} = 7.4 \times 10^{18}\ \text{cm}^{-3}$$

$$J_{th} = \left(\frac{q\ d_a}{\eta_i\ \tau_e}\right) N_{th} = 9.6 \times 10^{-17} \times 7.4 \times 10^{18}\ \text{cm}^{-3} = 2.26 \times 10^{3}\ \text{A/cm}^2$$

$$I_{th} = \left(\frac{\pi D^2}{4}\right) J_{th} = 1.14\ \text{mA}$$

Gratings Calculation

For the design of high performance VCSELs it is important to know the electric field distribution in the resonator. Assuming linearly polarized waves in a 1-D scalar approach, we have to solve the Helmholtz equation

$$\frac{\mathrm{d}^2 E(z)}{\mathrm{d}z^2} + \gamma^2 E(z) = 0$$

for the phasor of the transverse electric field component $E = E_x$

In each homogeneous n^{th} layer

$$\gamma_n = \beta_n - \mathrm{i}\alpha_n/2$$

$$\beta_n = 2\pi\bar{n}_n/\lambda$$

The absorption coefficient fulfills $\alpha_n \geq 0$ except from the QW layers, where gain leads to $\alpha_n < 0$.

Gratings Calculation

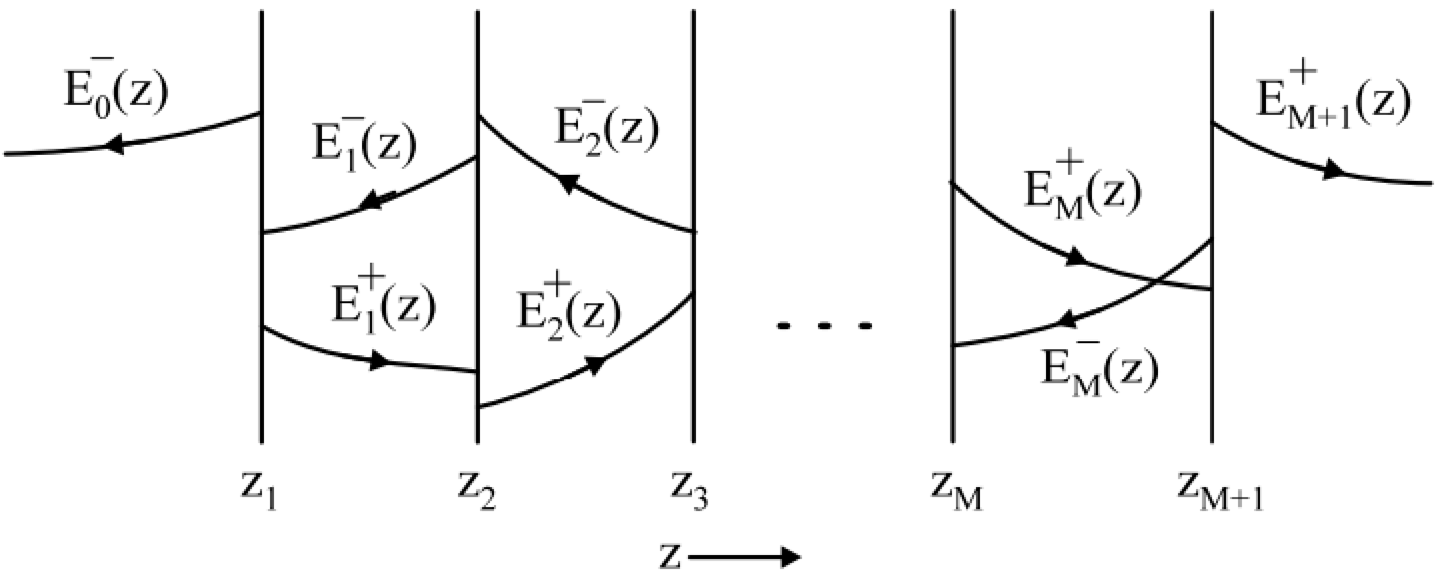
In each layer, the electric field is the superposition of two monochromatic plane waves counterpropagating in the *z*-direction

$$E_n(z) = E_n^+ \exp\{-\mathrm{i}\gamma_n (z - z_n)\} + E_n^- \exp\{+\mathrm{i}\gamma_n (z - z_n)\}$$

E_n^+ and E_n^- denote the complex field amplitudes of the waves at the interface

$z = z_n$ with $z_n \leq z \leq z_{n+1}$

$$n = 1, \ldots, M$$



Gratings Calculation

The continuity conditions for the transverse components of electric and magnetic fields lead to relations, between amplitudes in subsequent layers

$$E_n^+ = (\gamma_n^+ E_{n+1}^+ + \gamma_n^- E_{n+1}^-) \exp\{i\gamma_n(z_{n+1} - z_n)\} ,$$
$$E_n^- = (\gamma_n^- E_{n+1}^+ + \gamma_n^+ E_{n+1}^-) \exp\{-i\gamma_n(z_{n+1} - z_n)\}$$

where we have the abbreviations

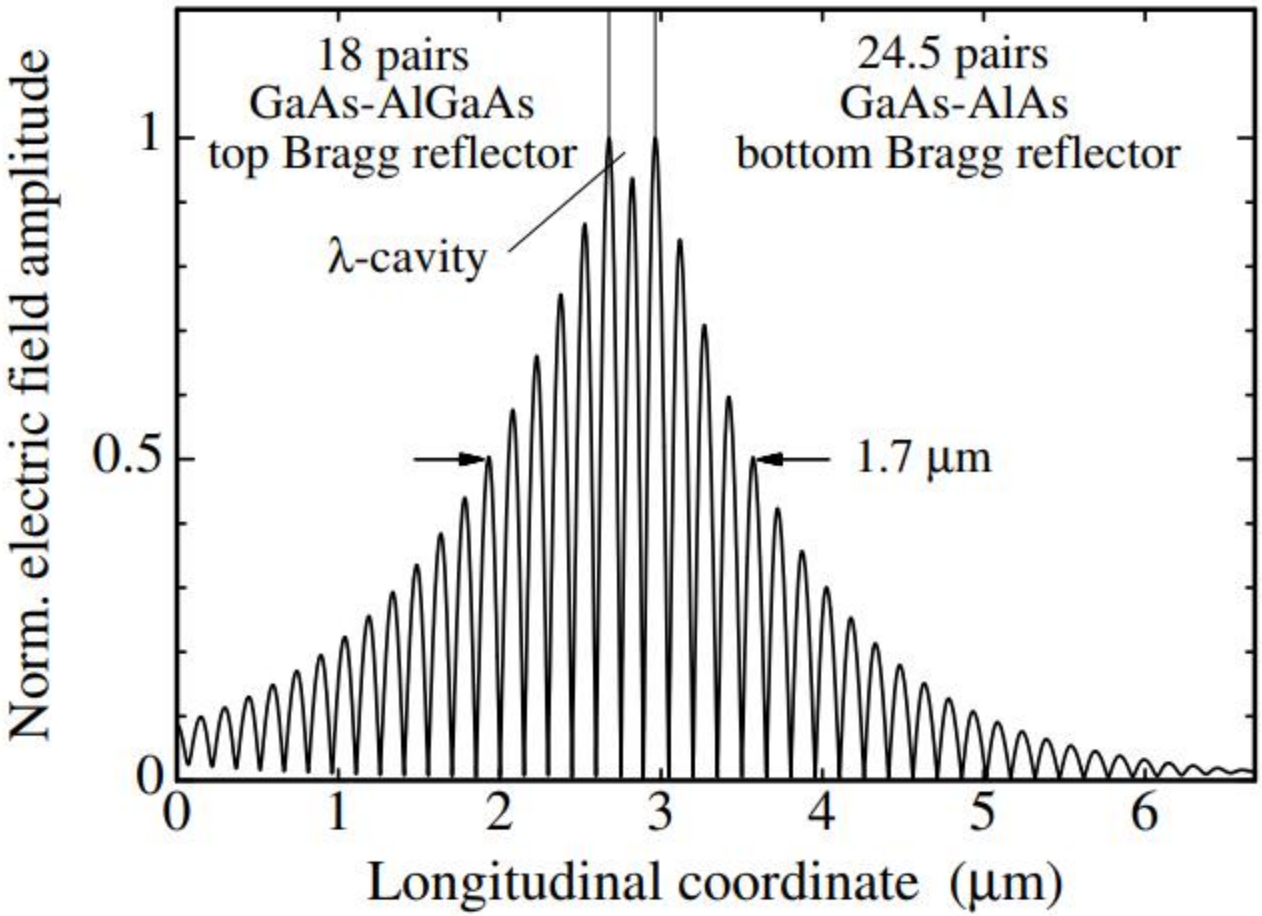
$$\gamma_n^+ = \frac{\gamma_n + \gamma_{n+1}}{2\gamma_n} \quad \text{and} \quad \gamma_n^- = \frac{\gamma_n - \gamma_{n+1}}{2\gamma_n}$$

Self-oscillation of the layer structure exclusively allows **outgoing** waves in the terminating sections

$$n = 0 \qquad \boxed{E_0^+ = 0}$$

$$n = M + 1 \qquad \boxed{E_{M+1}^- = 0}$$

Gratings Calculations



due to the high reflectivities of the mirrors, a pronounced resonant enhancement of the field amplitude is built up

Power

Poynting vector

$$\boldsymbol{S} = \mathrm{Re}\{\boldsymbol{E} \times \boldsymbol{H}^*\}$$

In the 1-D scalar approach the magnetic field is

$$H = H_y = \frac{\mathrm{i}}{\omega\mu_0}\frac{\mathrm{d}E}{\mathrm{d}z}$$

The energy flux in section m of the multilayer structure can be expressed as

$$S_n(z) = \frac{\beta_n}{\omega\mu_0}|E_n^+|^2 \exp\{-\alpha_n(z - z_n)\} - \frac{\beta_n}{\omega\mu_0}|E_n^-|^2 \exp\{\alpha_n(z - z_n)\} + \frac{\alpha_n}{\omega\mu_0}\mathrm{Im}\left\{E_n^+(E_n^-)^* \exp\{-\mathrm{i}2\beta_n(z - z_n)\}\right\} .$$

Power

Above threshold current I_{th}, top and bottom light output powers P_t and P_{b} linearly increase with driving current I. We can write

$$P_{\mathrm{t,b}} = \tilde{\eta}_{\mathrm{dt,b}} \frac{\hbar\omega}{q} \left(I - I_{\mathrm{th}}\right)$$

$$\tilde{\eta}_{\mathrm{dt,b}} = \eta_{\mathrm{dt,b}}\, \eta_{\mathrm{i}}$$

where the differential quantum efficiency

$$\eta_{\mathrm{dt,b}}$$

characterizes the percentage of generated coherent light that is available as top or bottom emission. In well designed VCSELs with high quality active QWs

$$\eta_{\mathrm{i}} > 90\,\%$$

Due to absorption in the mirrors we always find $\eta_{\mathrm{dt}} + \eta_{\mathrm{db}} < 100\,\%$

Gratings Calculation

The differential quantum efficiency is identified as the fraction of the generated flux that is emitted through top or bottom mirrors

$$\eta_{\mathrm{d}} = \eta_{\mathrm{dt}} + \eta_{\mathrm{db}}$$

$$\eta_{\mathrm{d}} = \frac{g_{\mathrm{th}}}{g_{\mathrm{th}} + \alpha_{\mathrm{a}}} \left(1 + \frac{\sum_{i,\mathrm{pass.}} \Delta S_i}{\sum_{i,\mathrm{act.}} \Delta S_i} \right) \quad \text{with} \quad \Delta S_i = S(z_{i+1}) - S(z_i)$$

Absorption leads to flux increments

$$\Delta S_i < 0 \qquad\qquad \eta_{\mathrm{d}} < 1$$

Denoting top and bottom emitted energy fluxes

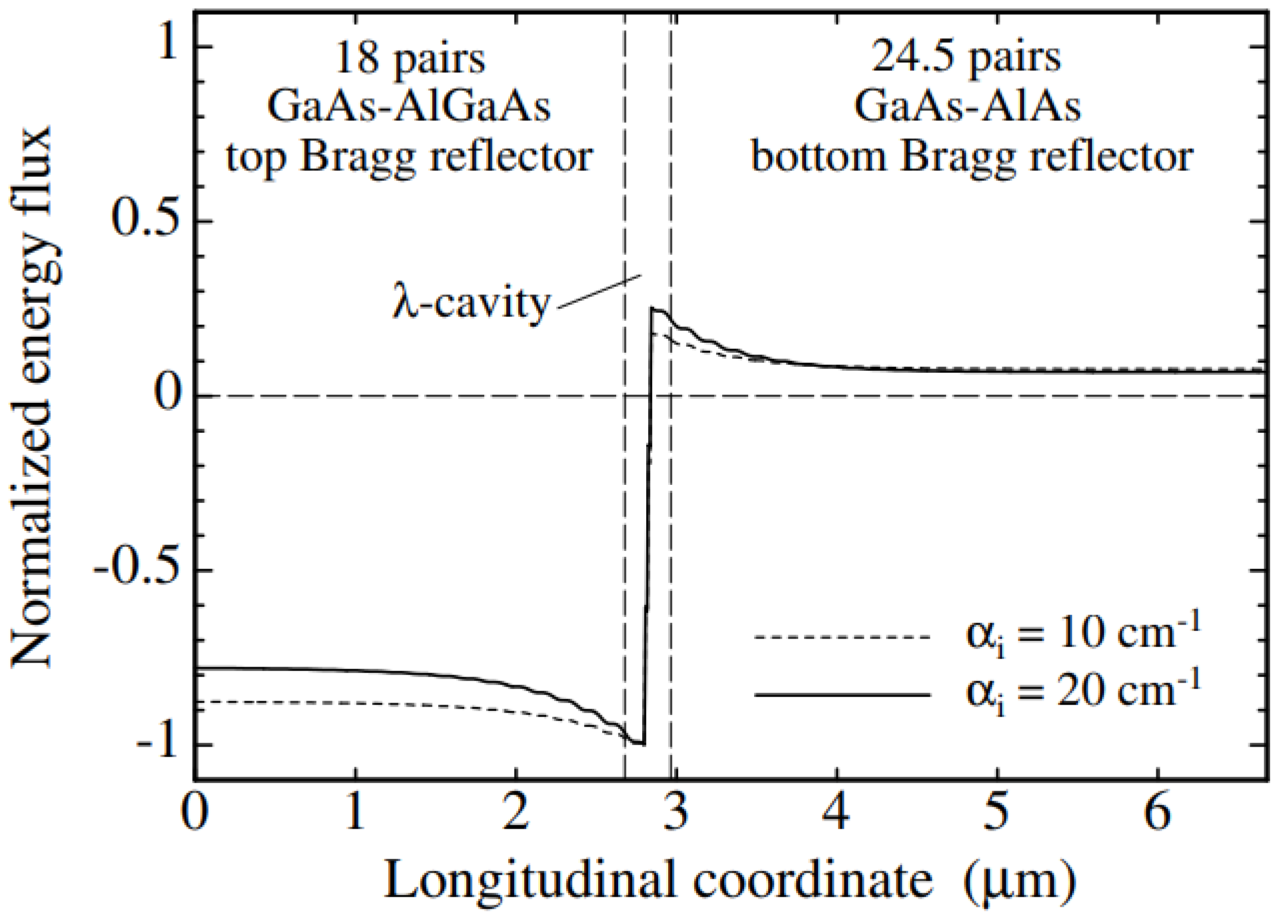
$$S_{\mathrm{t}} = S(z_1) < 0 \text{ and } S_{\mathrm{b}} = S(z_{M+1}) > 0$$

the corresponding differential quantum efficiencies are

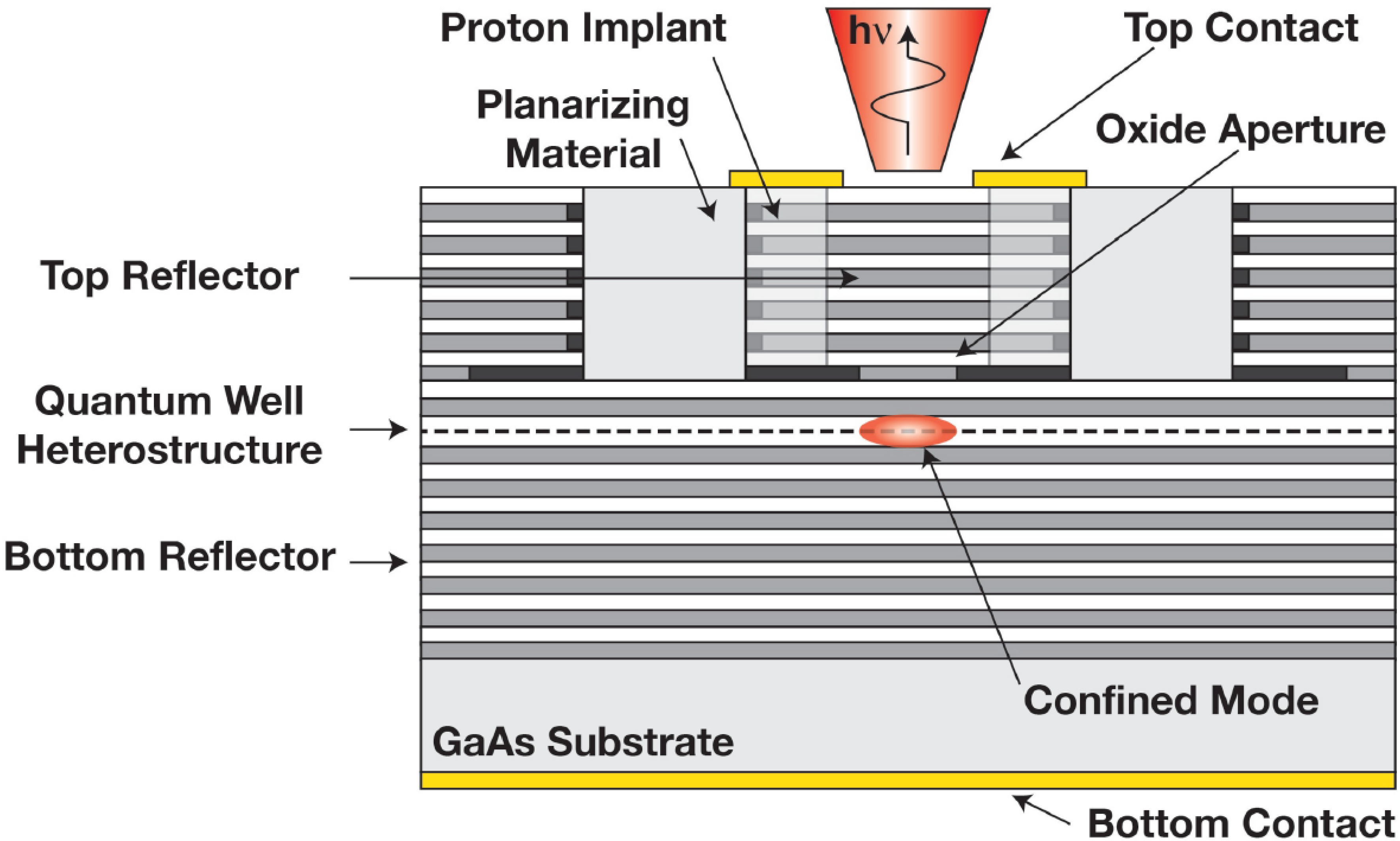
$$\eta_{\mathrm{dt}} = \eta_{\mathrm{d}} \frac{|S_{\mathrm{t}}|}{|S_{\mathrm{t}}| + |S_{\mathrm{b}}|} \qquad \text{and} \qquad \eta_{\mathrm{db}} = \eta_{\mathrm{d}} \frac{|S_{\mathrm{b}}|}{|S_{\mathrm{t}}| + |S_{\mathrm{b}}|}$$

Gratings Calculation

Normalized energy flux density



The Modern VCSEL



VCSEL emission characteristics

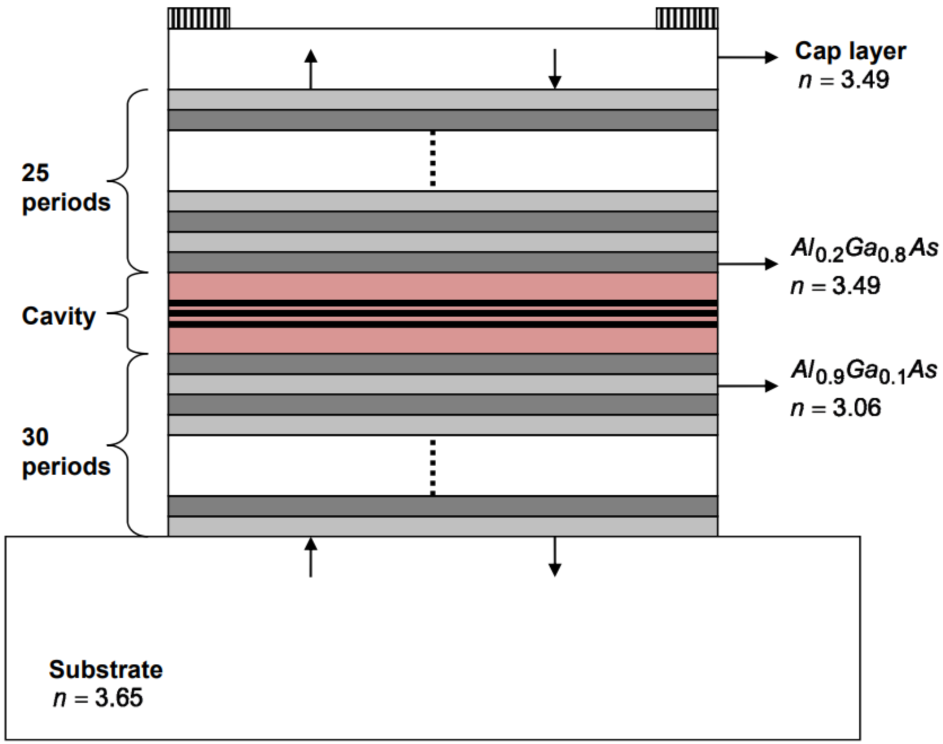
The active diameter of the VCSEL can be made quite small (few micron size) to reach the lowest possible threshold current (down to sub-100 μA) but it can also exceed 100 μm to get power beyond 100mW.

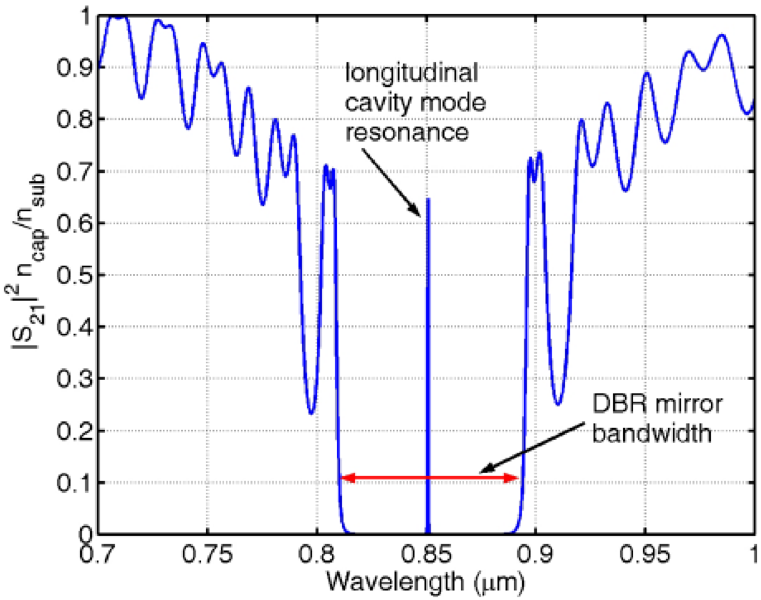
Rule of Thumb: Up to active region diameters of about 4 μm the VCSEL emits in the single fundamental transverse mode. Larger devices start lasing on several higher order radial and azimuthal modes.

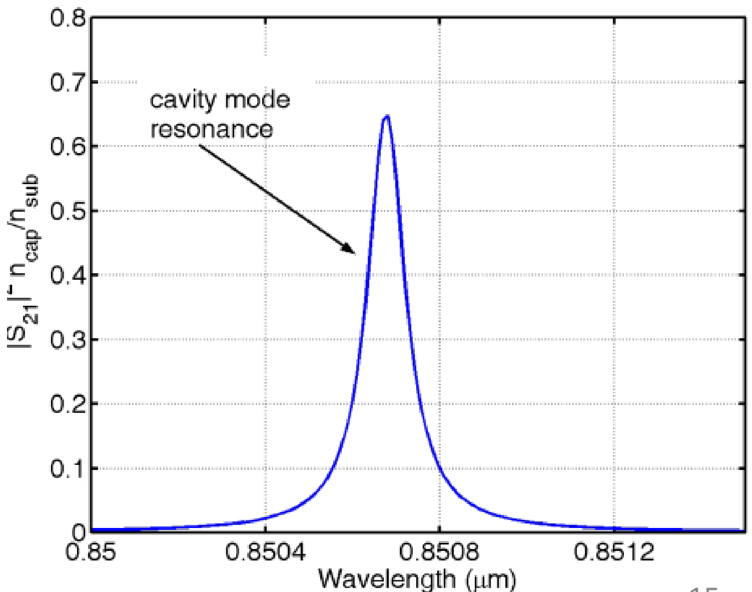
Due to the small length of the laser cavity (1 to 2 μm) consecutive longitudinal modes are widely spaced in wavelength ($\Delta\lambda \approx 100 \text{ nm}$) .

If one mode is made to coincide with the peak reflectivity of the mirrors, the two adjacent modes fall outside the high-reflectivity band and single longitudinal mode oscillation can also be established.

VCSEL emission characteristics







Lasing Condition

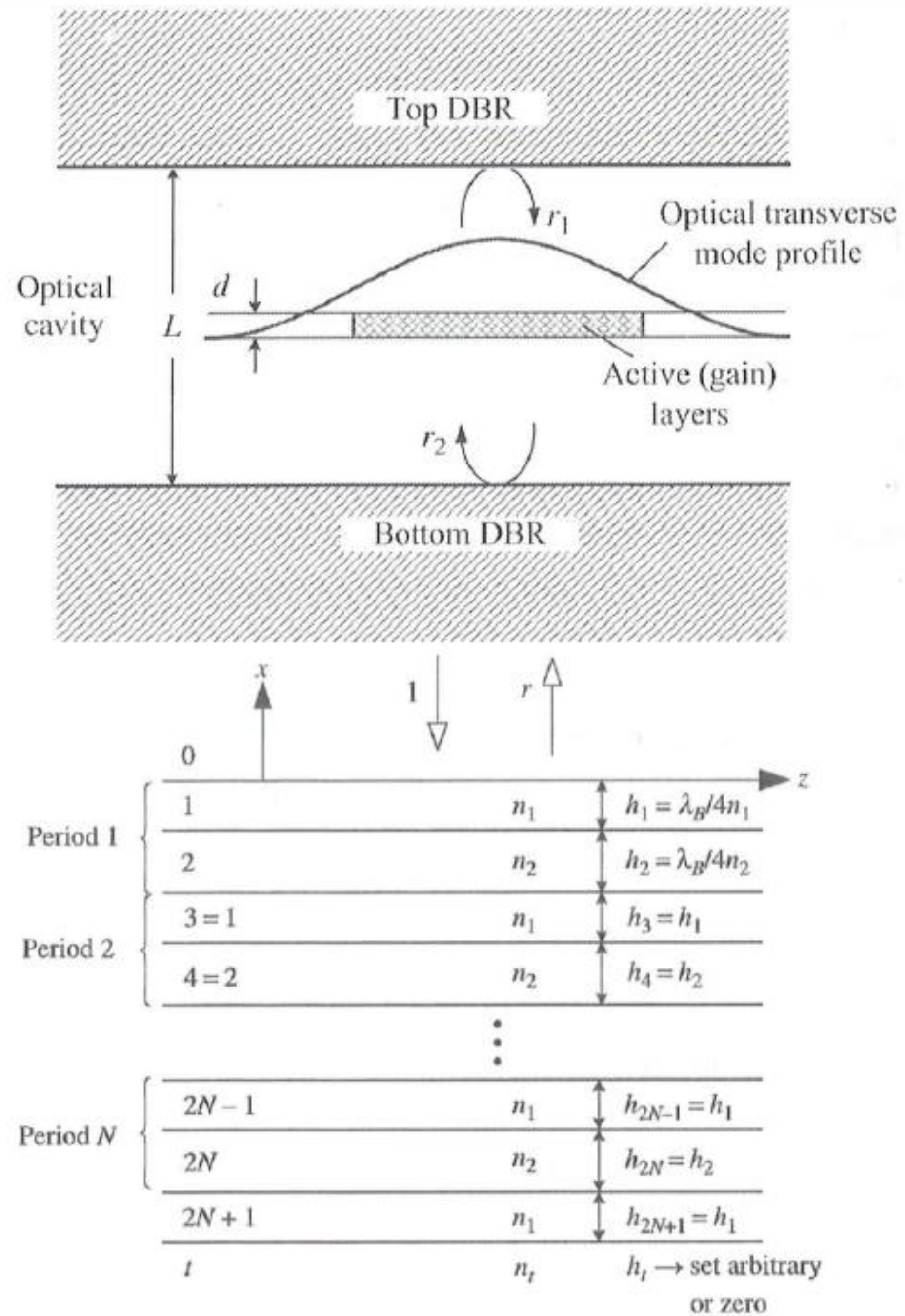
Round - Trip Amplitude and Phase :

$$r_1 r_2 e^{i2\beta L} = 1$$

where $\beta = \beta_c + i\dfrac{\alpha - \Gamma g}{2}$

The reflection coefficients are complex:

$$r_1 = |r_1|\, e^{i\varphi_1} \quad \text{and} \quad r_2 = |r_2|\, e^{i\varphi_2}$$



Grating reflection coefficient

$$r = \frac{\left(\frac{n_2}{n_1}\right)^{2N} - \frac{n_1^2}{n_0 n_t}}{\left(\frac{n_2}{n_1}\right)^{2N} + \frac{n_1^2}{n_0 n_t}}$$

$N \to \infty$

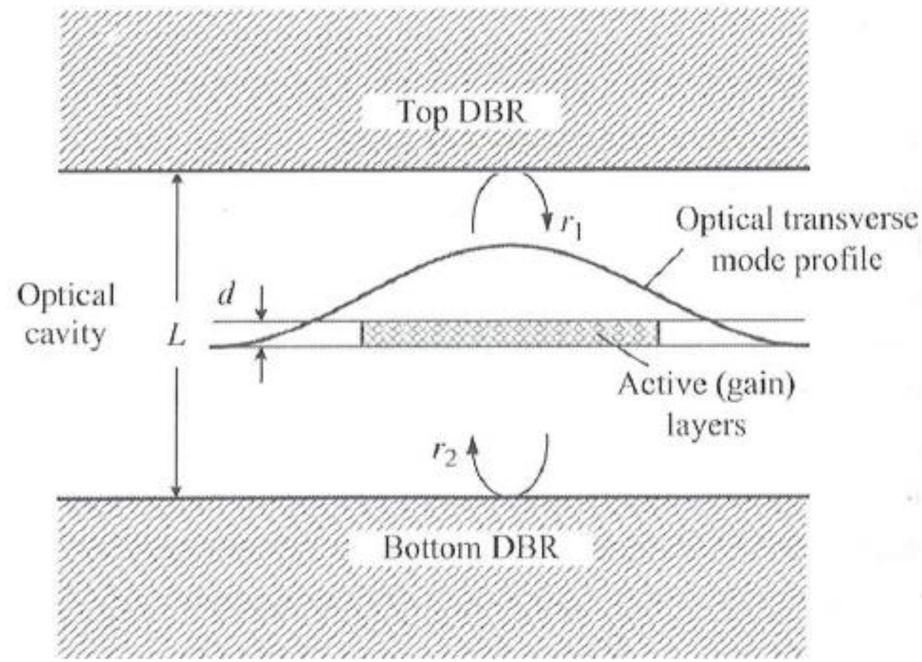
$r \to 1 \;\; [n_1 < n_2]$

$r \to -1 \;\; [n_2 < n_1]$

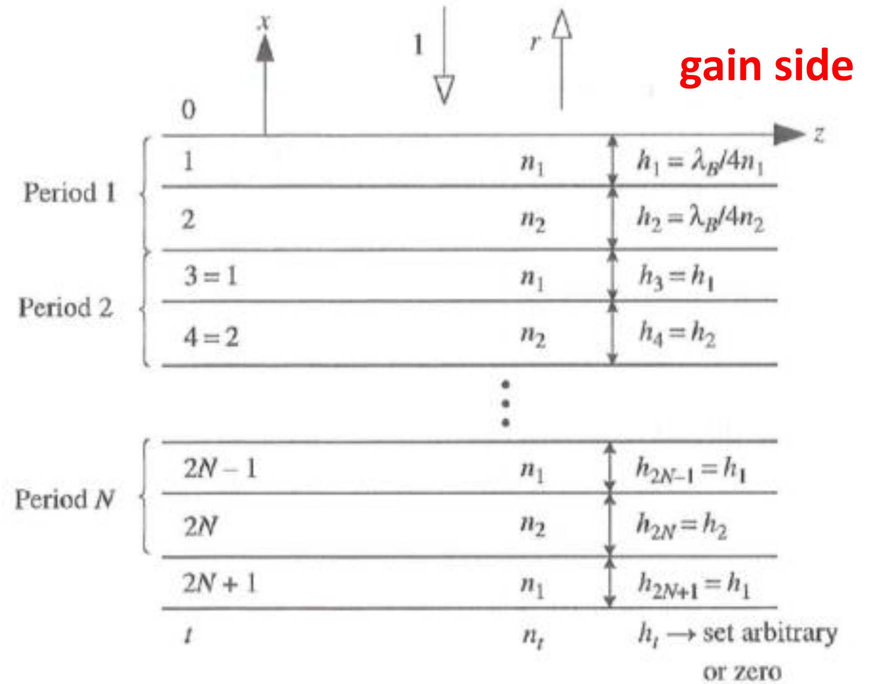
$$r = \frac{1 - \left(\frac{n_1}{n_2}\right)^{2N} \frac{n_1^2}{n_0 n_t}}{1 + \left(\frac{n_1}{n_2}\right)^{2N} \frac{n_1^2}{n_0 n_t}}$$

Section 5.9 in Chuang

Appendix 7 in Coldren, Corzine and Mašanović



gain side



transmitted side

Lasing Condition

Phase Condition : $\varphi_1 + \varphi_2 + 2\beta_c L = 2m\pi$

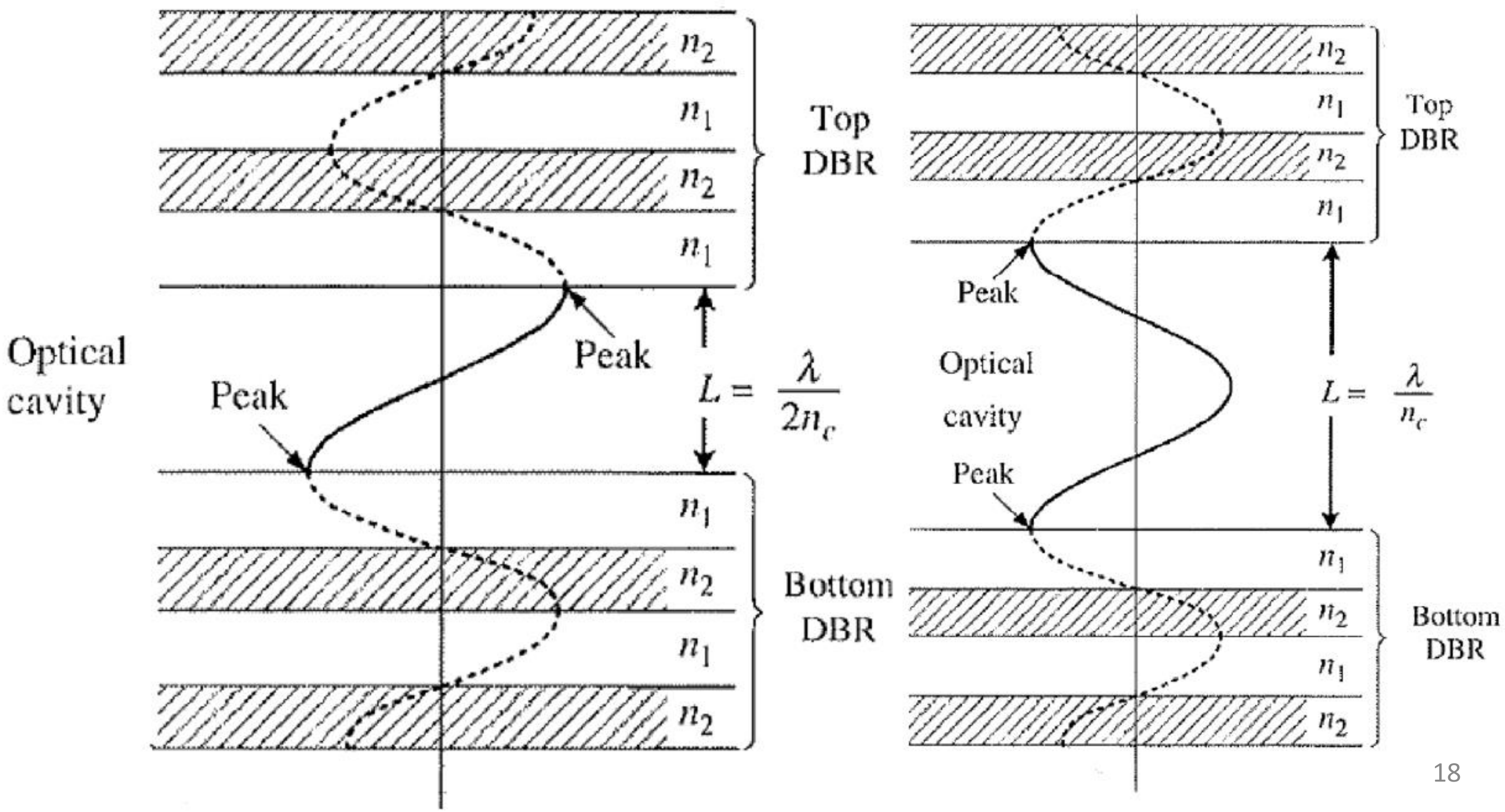
Threshold Condition : $\Gamma g = \alpha + \frac{1}{2L}\ln\left(R_1 R_2\right)$

Case 1: $\varphi_1 = \varphi_2 = 0\ (n_1 < n_2)$,

then $L = m\frac{\lambda}{2n}$ (peak at DBR interface)

There is a null at a DBR interface going from low index to high index

$\varphi_1 = \varphi_2 = 0\ (n_1 < n_2)$



Lasing Condition

Case 2: $\varphi_1 = \varphi_2 = \pi \ (n_1 > n_2)$,

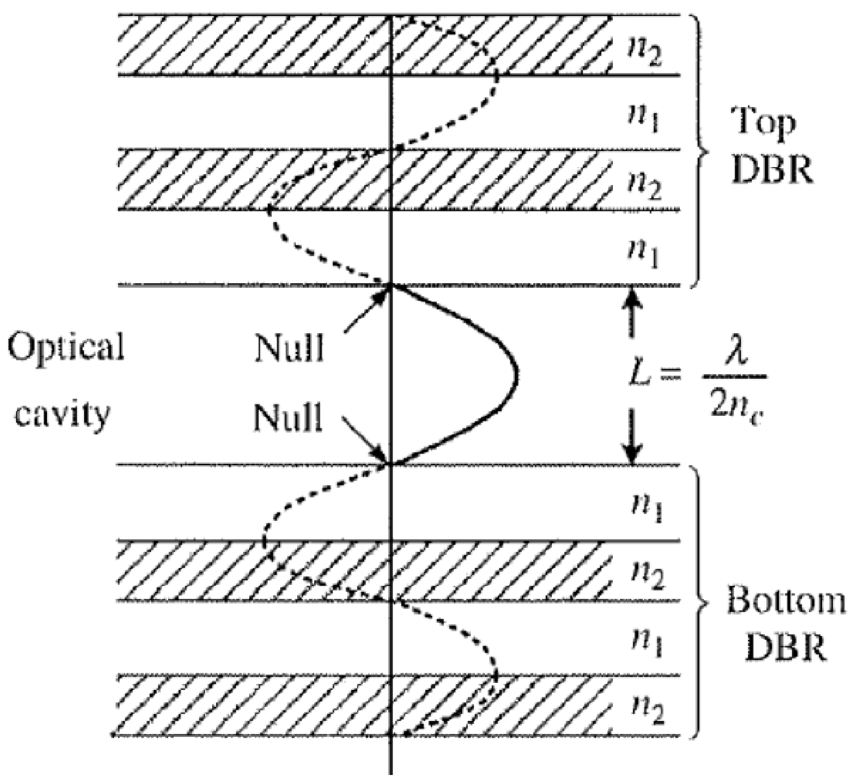
then $L = (m-1)\dfrac{\lambda}{2n}$ (zero at DBR interface)

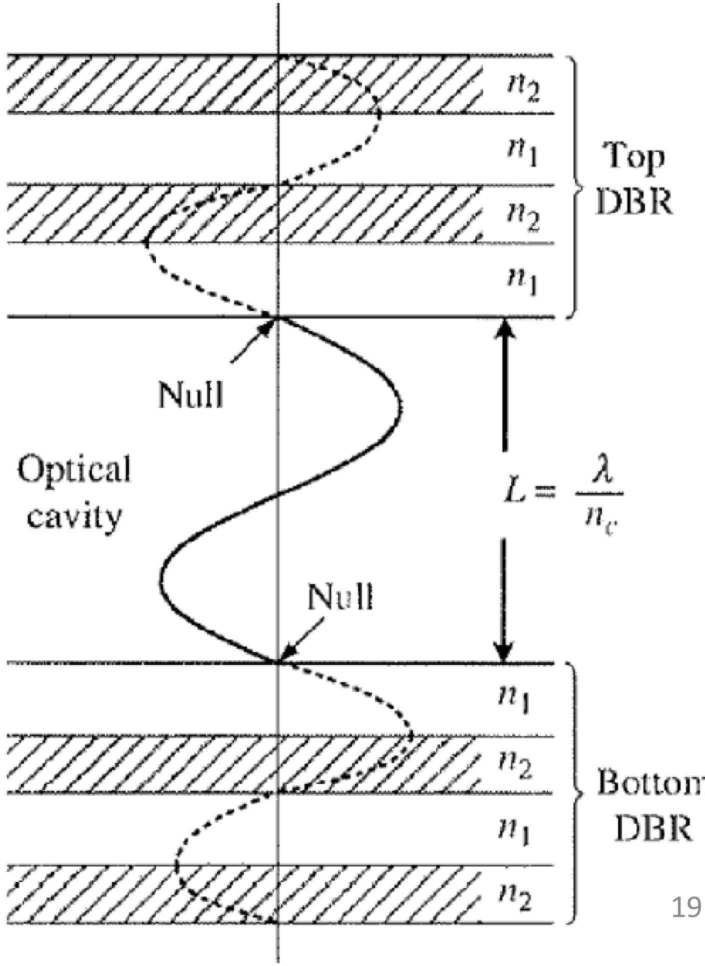
Phase Condition : $\varphi_1 + \varphi_2 + 2\beta_c L = 2m\pi$

Threshold Condition : $\Gamma g = \alpha + \dfrac{1}{2L}\ln(R_1 R_2)$

There is a null at a DBR interface going from low index to high index

$\varphi_1 = \varphi_2 = \pi \ (n_1 > n_2)$





Simplified Optical Confinement Factor

In the book by Chuang

$$\Gamma = \gamma \frac{d}{L} \Gamma_t$$

d= active layer thickness
L = cavity length

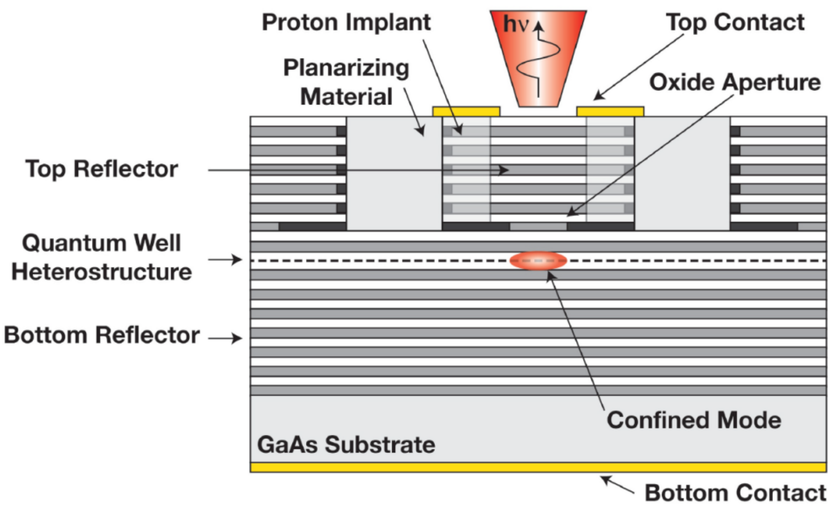
$$\gamma = \begin{cases} 2 \;\; for\ thin\ active\ layer\ at\ antinode \\ 1 \; for\ a\ thick\ active\ layer \\ 0 \;\; for\ a\ thin\ active\ layer\ at\ a\ node \end{cases}$$

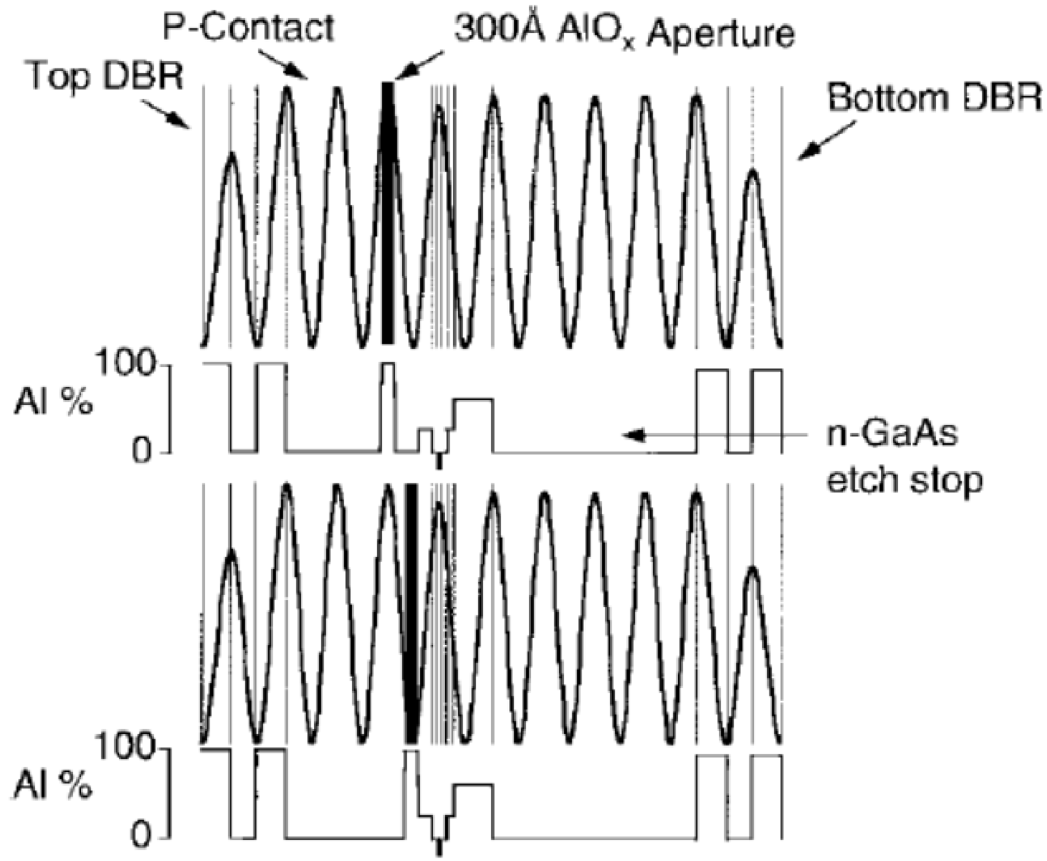
γ is the same as the Gain Enhancement factor Γ_r introduced in the previous lecture. Γ_t is the transverse confinement factor (often $\Gamma_t \approx 1$)

As we saw earlier, the value of the gain enhancement factor depends on the number of quantum wells forming the active layer

Often, for MQW the choice is made of γ (or Γ_r) = 2 but remember that this is an approximation. For instance, in the case of 3 QW's we have Γ_r = 1.8

Placement of the oxide aperture





Oxide aperture at the peak of the Electric Field standing wave causes high optical loss.

Oxide aperture at the null of the Electric Field standing wave causes low optical loss.

Example: Estimation of Number of DBR pairs needed

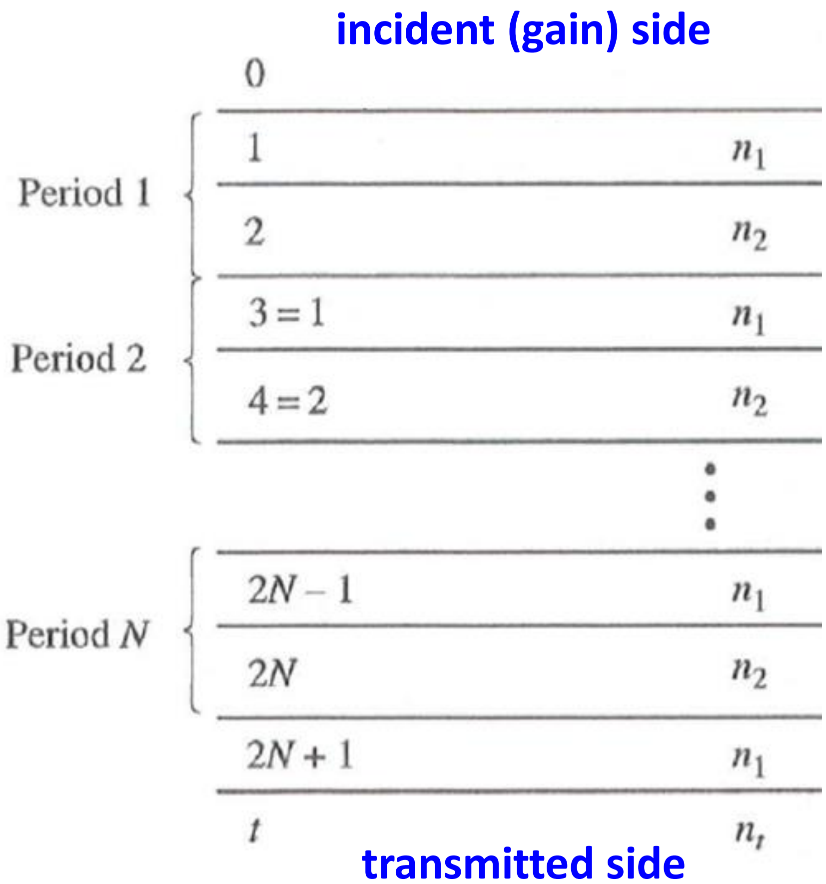
Consider only mirror loss (cavity is small) and identical mirror reflectivities. Neglect intrinsic losses and diffraction losses in cavity, and use $\Gamma_t \approx 1$.

$$\Gamma g_{th} \approx \Gamma_r \frac{d}{L} g_{th} \approx 2\frac{d_a}{L} g_{th} \approx \frac{1}{L}\ln\left(\frac{1}{R}\right)$$

$$g_{th} 2 n_w L_z = \ln\left(\frac{1}{R}\right) = -\ln(R) \qquad \text{with} \qquad d_a = n_w L_z$$

$$R = \exp(-2 n_w L_z g_{th}) \approx 1 - 2 n_w L_z g_{th}$$

Example: Estimation of Number of DBR pairs needed



Consider:

$n_1(\mathrm{GaAs}) = 3.52$
$n_2(\mathrm{AlAs}) = 2.95$
match: $n_0 = n_t = n_1$
$L_z = 10\ \mathrm{nm}$
$g_{th} = 500\ \mathrm{cm}^{-1}$

$$R \approx 1 - 2 n_w L_z g_{th} = 1 - 10^{-3} n_w$$

$n_w = 1$	$R = 0.999$
$n_w = 2$	$R = 0.998$
$n_w = 3$	$R = 0.997$

$$R \approx 1 - n_w \times 2 \times 500\mathrm{cm}^{-1} \times 10 \times 10^{-7}\mathrm{cm}$$
$$= 1 - n_w \times 10^{4} \times 10^{-7}$$

Example: Estimation of Number of DBR pairs needed

$\boxed{n_w = 1}$ $R = 0.999$

$$r = \frac{1 - \left(\frac{3.52}{2.95}\right)^{2\times 23}}{1 + \left(\frac{3.52}{2.95}\right)^{2\times 23}} = -0.999408776 \qquad R = \left|r^2\right| = 0.9988179$$

23 periods

$\boxed{n_w = 2}$ $R = 0.998$

$$r = \frac{1 - \left(\frac{3.52}{2.95}\right)^{2\times 21}}{1 + \left(\frac{3.52}{2.95}\right)^{2\times 21}} = -0.998801874 \qquad R = \left|r^2\right| = 0.997605183$$

21 periods

$\boxed{n_w = 3}$ $R = 0.997$

$$r = \frac{1 - \left(\frac{3.52}{2.95}\right)^{2\times 20}}{1 + \left(\frac{3.52}{2.95}\right)^{2\times 20}} = -0.998294571 \qquad R = \left|r^2\right| = 0.996592051$$

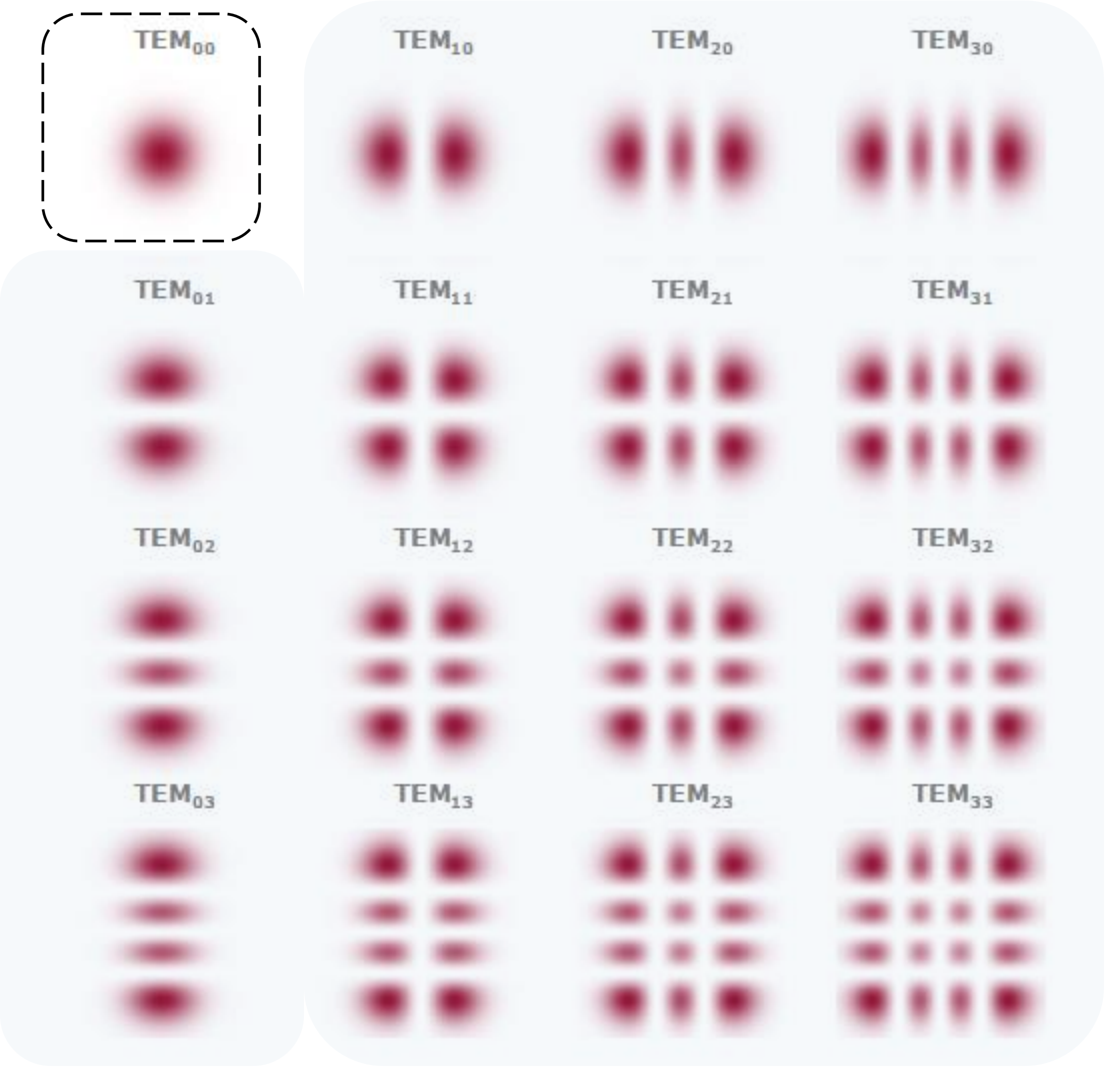
20 periods

Transverse modes in a VCSEL

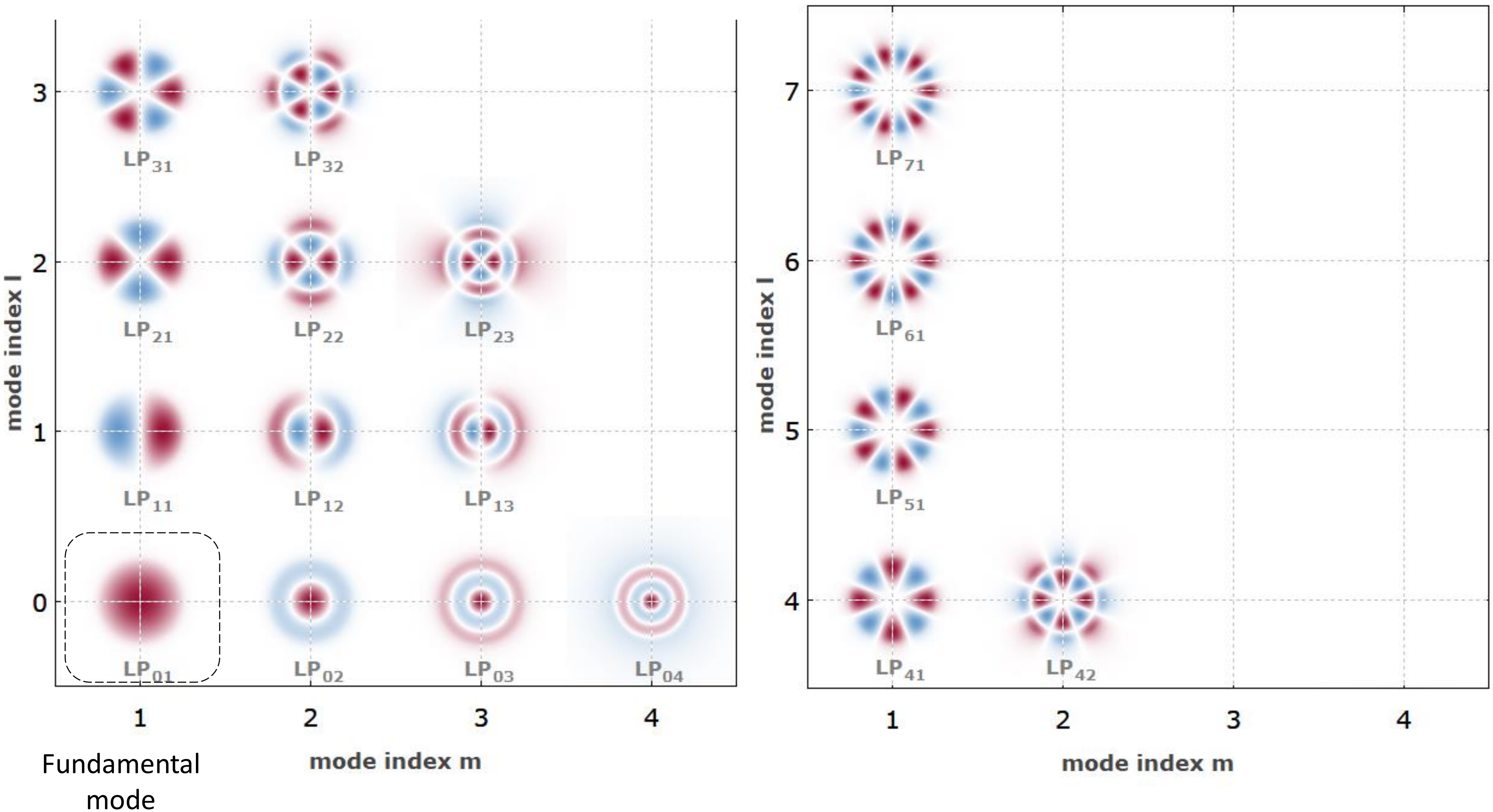
Family of modes for a beam – Hermite Gaussian modes

Fundamental "Gaussian" beam mode

Higher order modes



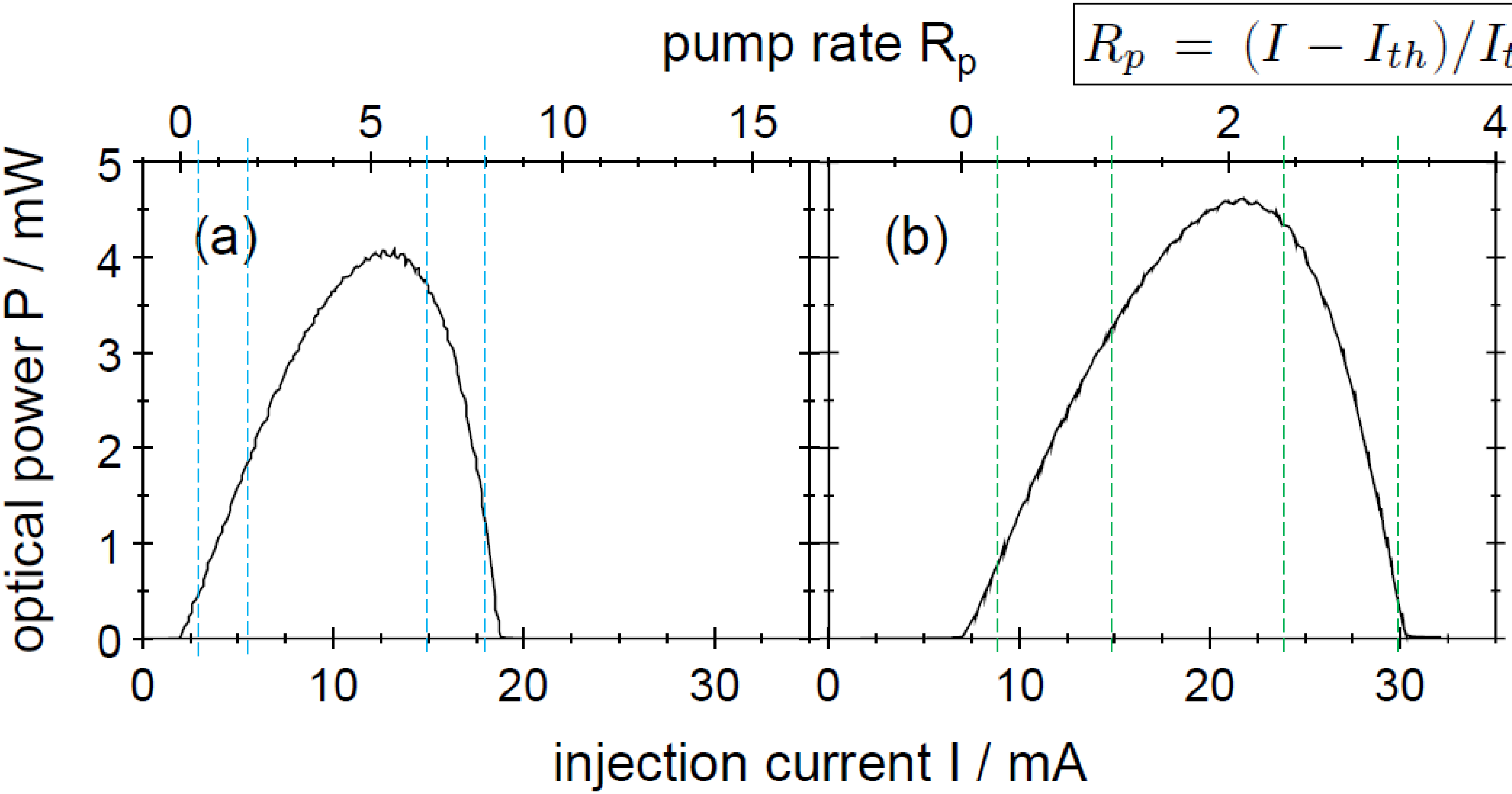
Family of Linearly Polarized (LP) core modes in a circular structure



VCSEL Transverse Modes

Oxide confined GaAs top-emitting VCSELs

$R_p = (I - I_{th})/I_{th}$



6 μm aperture diameter

11 μm aperture diameter

C. Degen, I. Fischer and W. Elsäßer, Optics Express, vol. 5, p. 38 (1999)

VCSEL Transverse Modes

6 μm aperture diameter

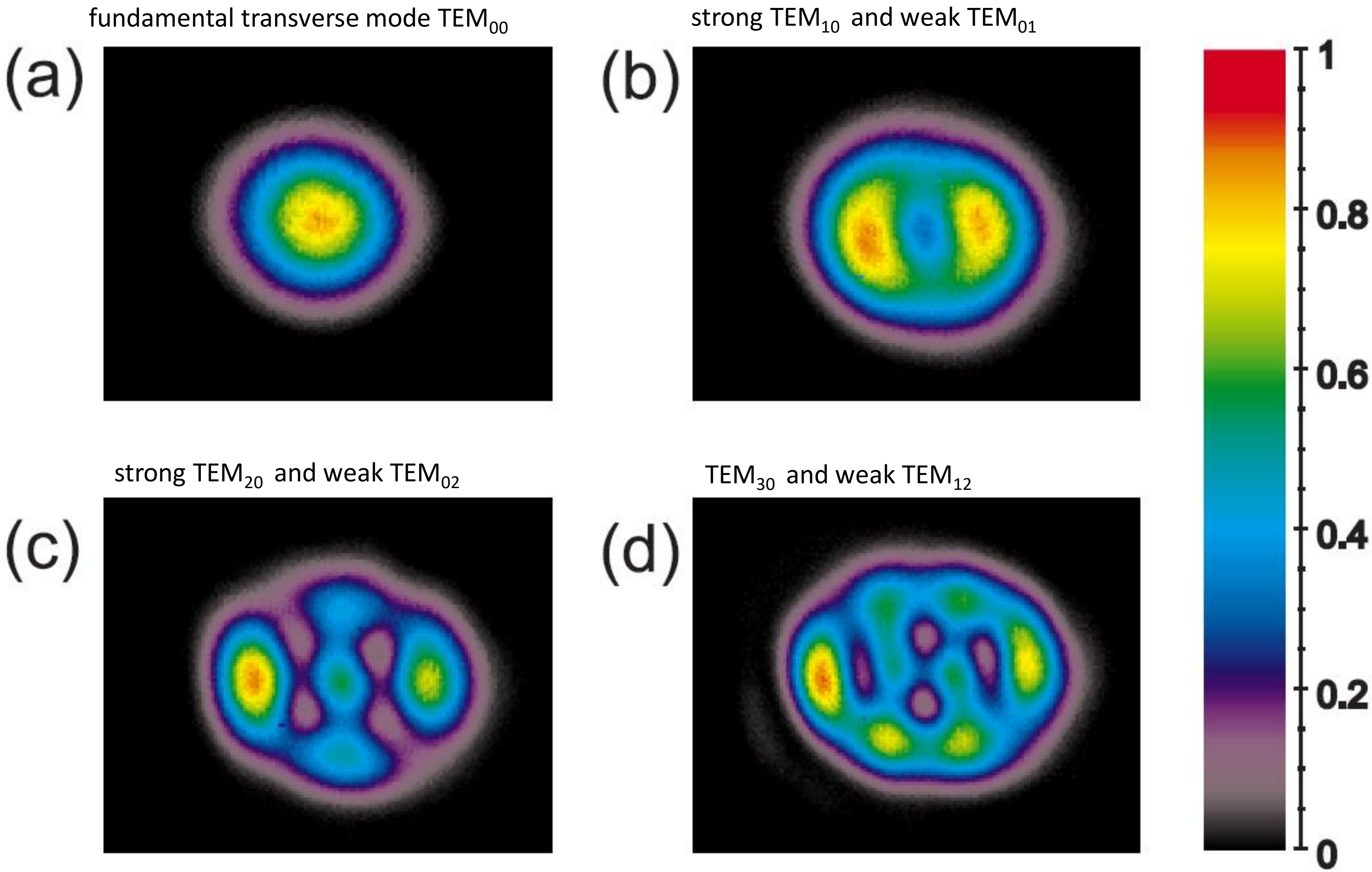


Figure 3. Nearfield images of the 6μm VCSEL at injection currents of 3.0mA (a), 6.2mA (b), 14.7mA (c), and 18mA (d)

C. Degen, I. Fischer and W. Elsäßer, Optics Express, vol. 5, p. 38 (1999)

VCSEL Transverse Modes

6 μm aperture diameter

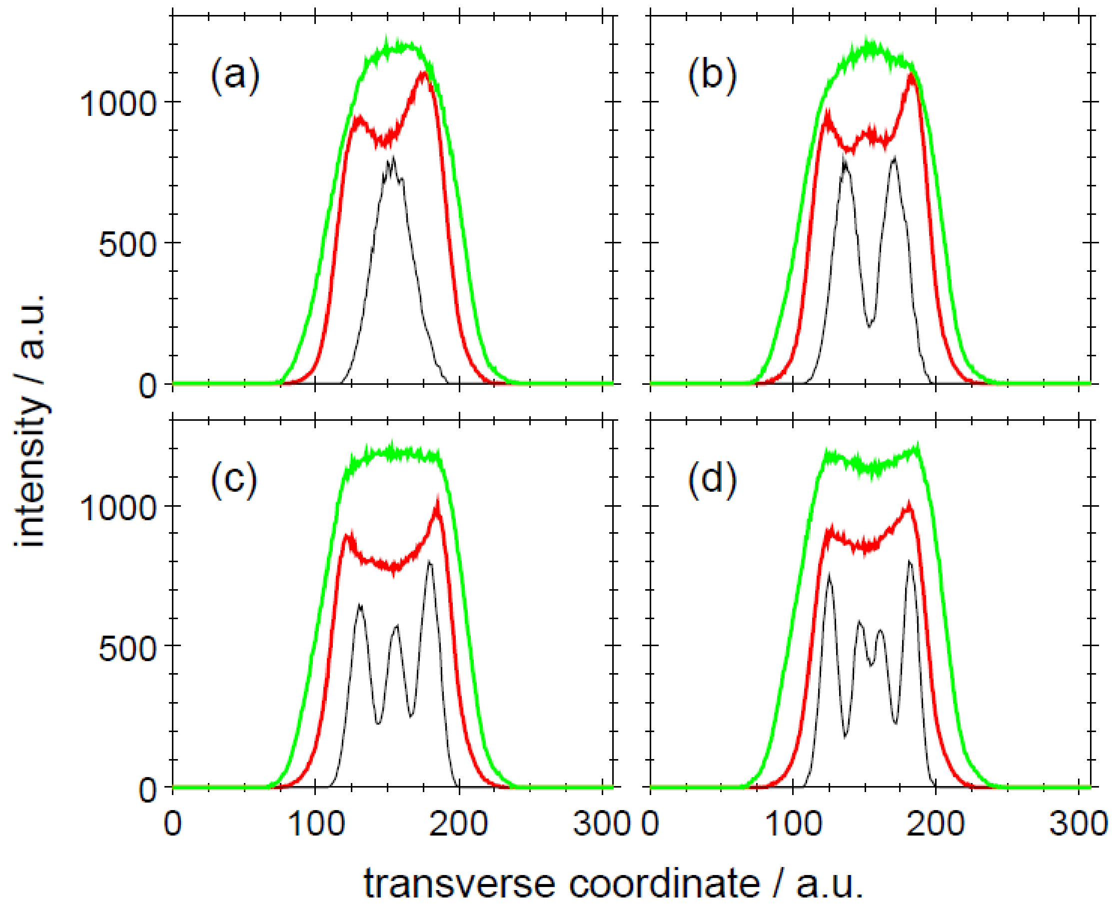


Figure 4. Transverse distribution of the laser intensity at $\lambda \approx 800nm$ (black curve), spontaneous emission at $\lambda \approx 770nm$ (red curve), and spontaneous emission at $\lambda \approx 830nm$ (green curve) of a $6\mu m$ VCSEL for injection currents I=3mA (a), I=6mA (b), I=15mA (c), and I=18mA (d)

C. Degen, I. Fischer and W. Elsäßer, Optics Express, vol. 5, p. 38 (1999)

VCSEL Transverse Modes

11 μm aperture diameter

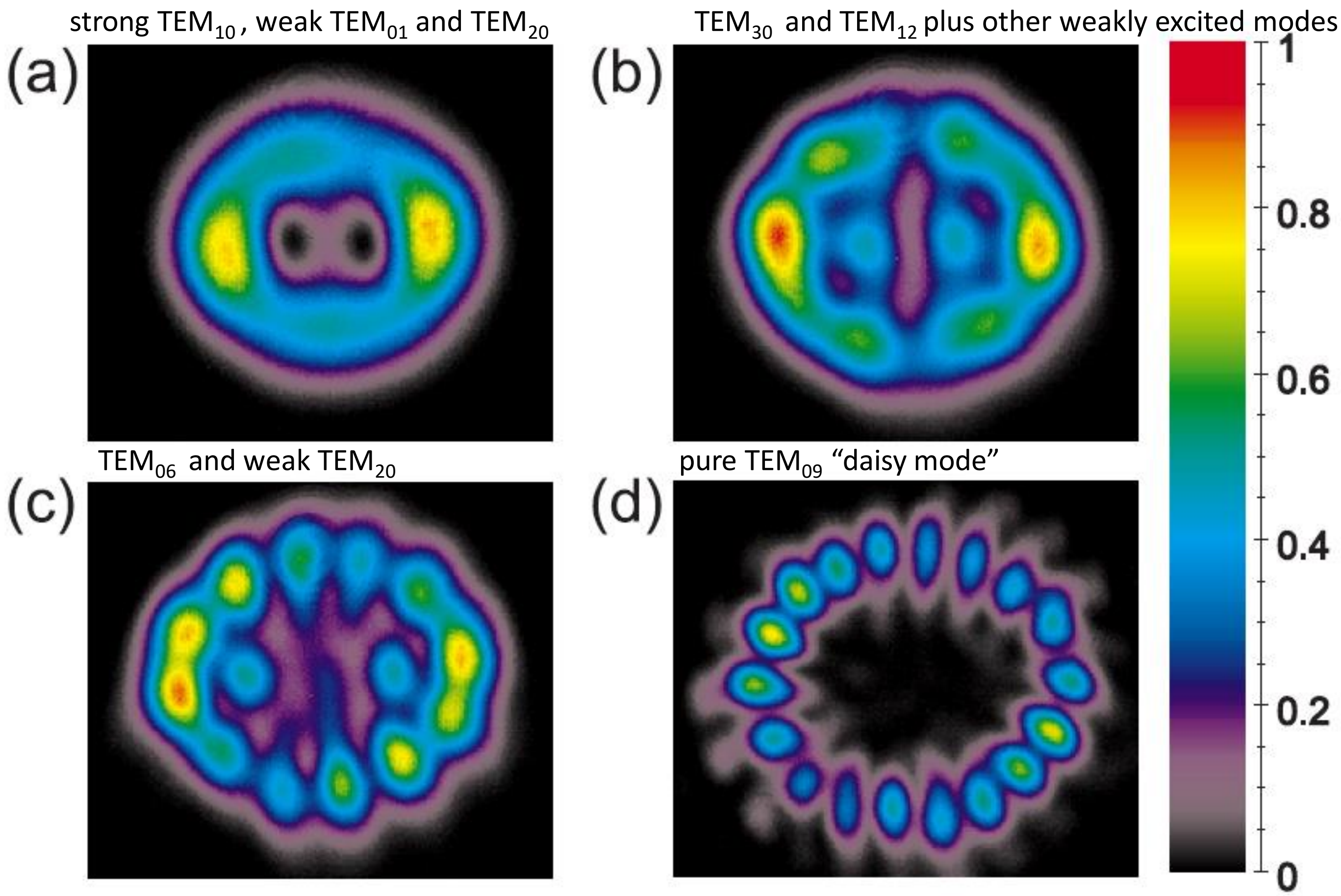


Figure 5. Nearfield images of the 11μm VCSEL at injection currents of 8.8mA (a), 15.5mA (b), 23.0mA (c) and 29.9mA (d)

C. Degen, I. Fischer and W. Elsäßer, Optics Express, vol. 5, p. 38 (1999)

VCSEL Transverse Modes

11 μm aperture diameter

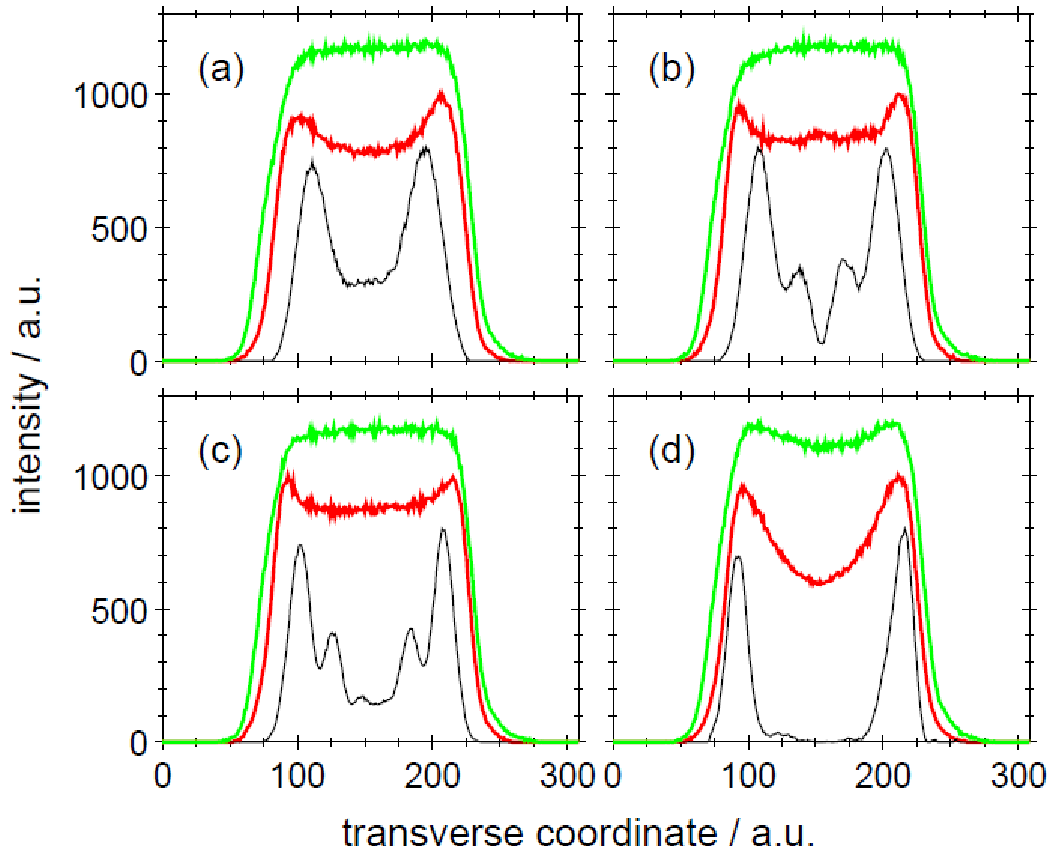


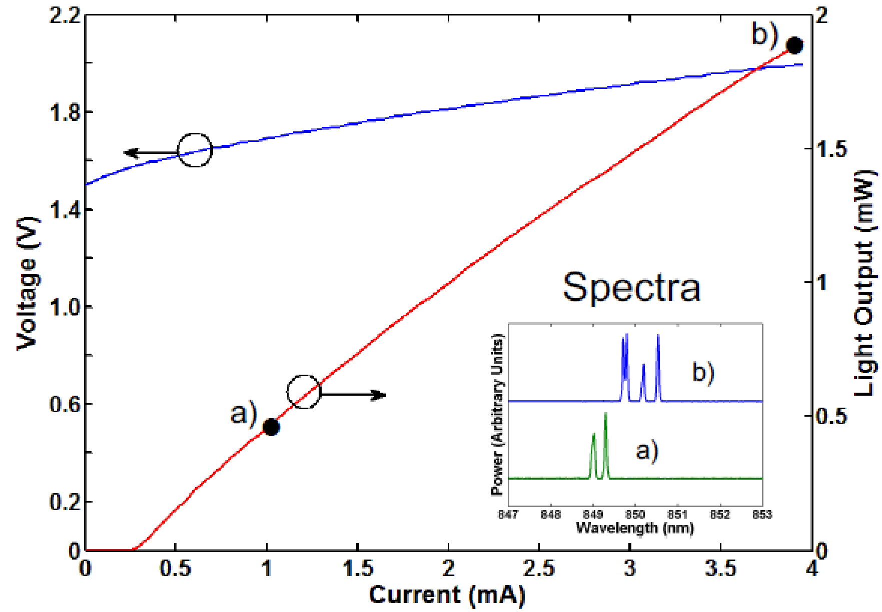
Figure 6. Transvere distribution of the optical laser field at $\lambda \approx 800$nm (black), spontaneous emission at $\lambda \approx 770$nm (red), and spontaneous emission at $\lambda \approx 830$nm (green) of a 11μm VCSEL for injection currents I=9mA (a), I=15mA (b), I=24mA (c), and I=30mA (d)

C. Degen, I. Fischer and W. Elsäßer, Optics Express, vol. 5, p. 38 (1999)

VCSEL Properties

- Lower threshold current and power dissipation than edge-emitting lasers
  - Sub-mA versus several mA laser threshold current
  - mWs versus 10's of mWs power dissipation
- Good optical beam quality
  - Circular as opposed to elliptical beam shape
  - Low divergence (<10° versus >30°)
- Better compatibility with wafer-scale manufacturing methods than edge-emitting lasers

Commercial VCSEL L-I and I-V



Comparison of Laser Parameters

	Edge Emitting Laser	VCSEL
Beam Shape	Elliptical	Circular (easy to couple to optical fiber)
Modal Behavior	Always multiple longitudinal Can be single transverse	Always single longitudinal Multiple or single transverse
Threshold Current	5-500 mA	0.1-3 mA
Drive Current	10-2000 mA	0.5-10 mA
Spectral Properties	1-3 nm linewidth with multiple modes	<0.1 nm linewidth with single mode
Temperature Dependence	0.4 nm/°C	0.08 nm/°C
Output Power	1-1000 mW	0.5-5 mW