ECE 536 – Integrated Optics and Optoelectronics
Lecture 25 – April 19, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

Prof. Umberto Ravaioli

ECE Department, University of Illinois

Lecture 25 Outline

- Integrated Optics components
- Directional coupler
- Modulators

The final exam will consist of the final project presentation (10 minutes each). Have a Powerpoint slide show ready on data stick.

Date: Thursday May 12, 8:00-11:00am.

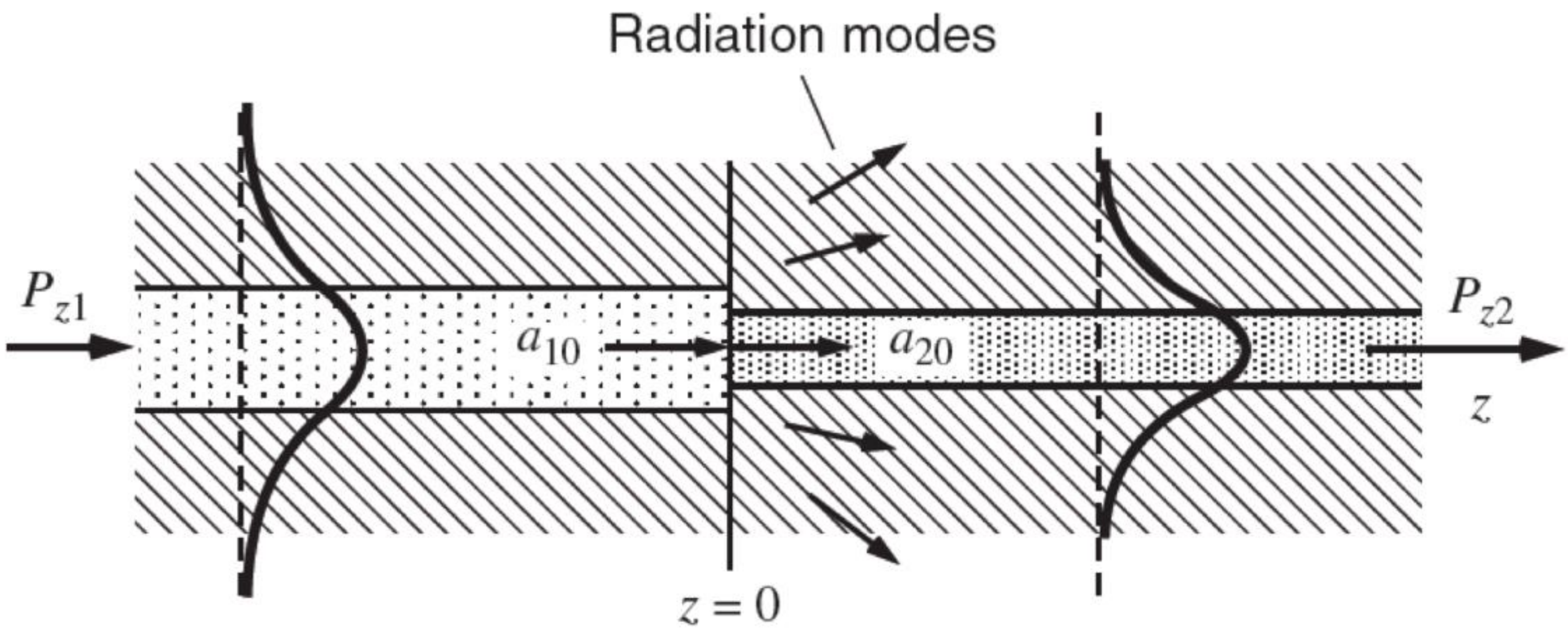
The **Take-Home Exam** has been posted. Take-Home Exam and final paper are also due on May 12.

Integrated Optics Structures

Coupled Waveguides

Coupling Methods

- Edge (Butt) Coupling

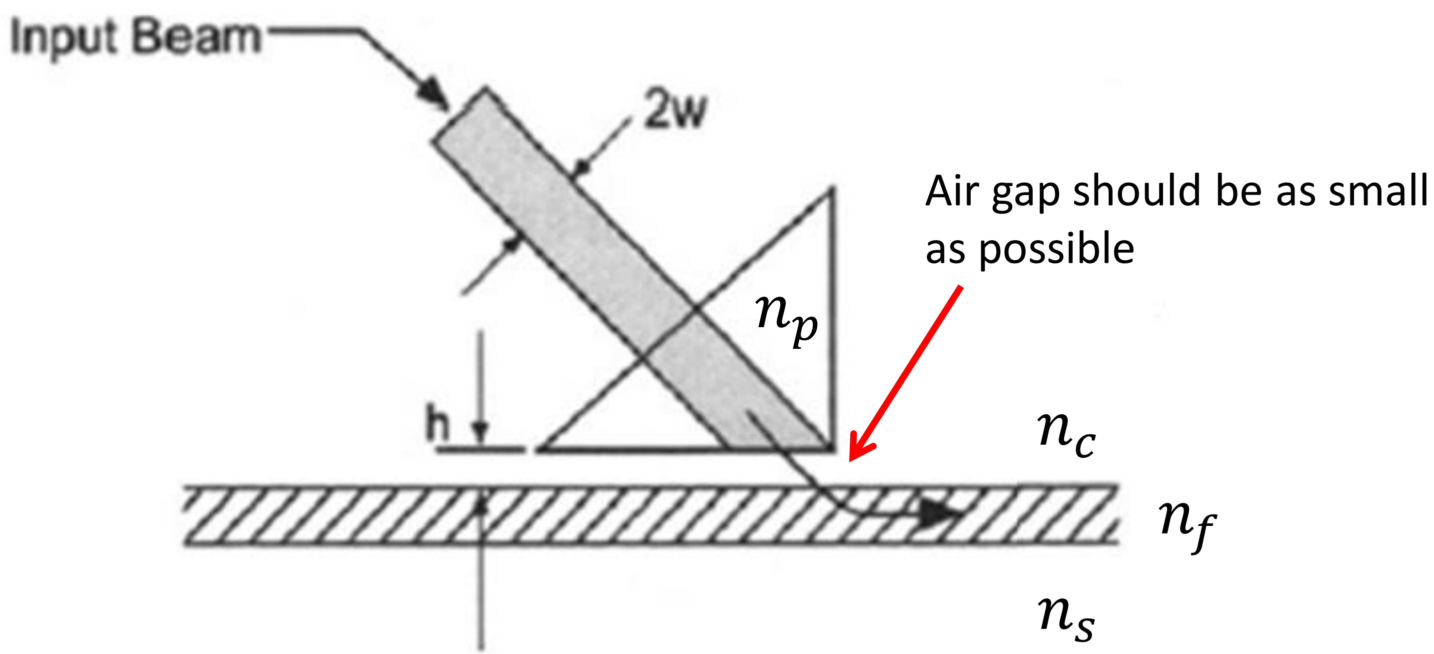


$$\frac{P_{20}\left(0^{+}\right)}{P_{10}\left(0^{-}\right)} \approx \left| \sqrt{\frac{4\bar{n}_1\bar{n}_2}{\left(\bar{n}_1+\bar{n}_2\right)^2}} \int \mathbf{U}_{20}^{*} \bullet \mathbf{U}_{10}\, dA \right|^2$$

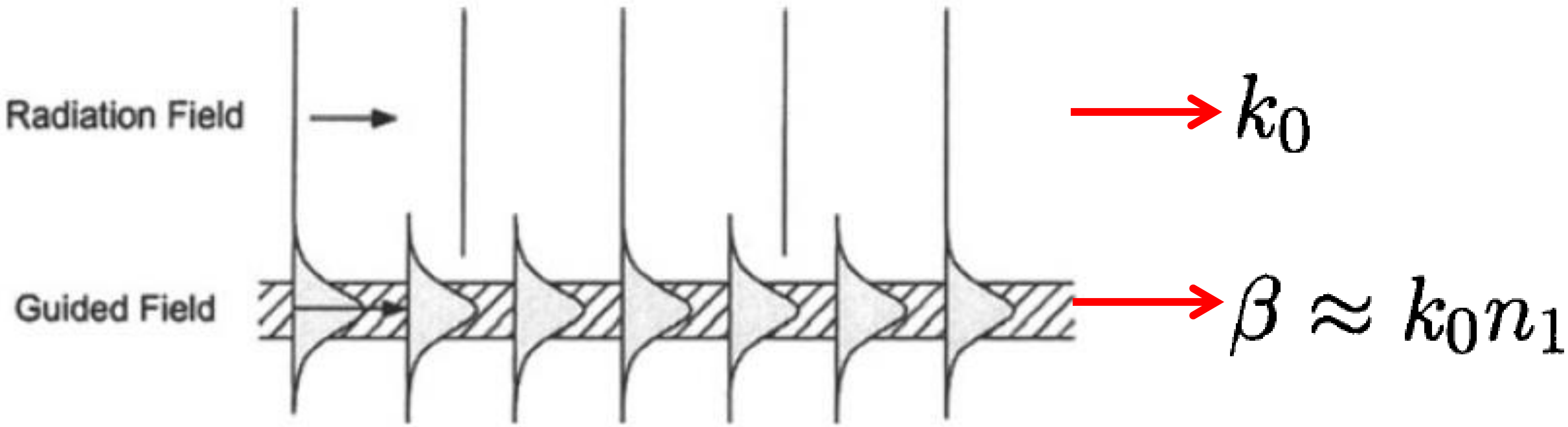
Coupling Methods

Effective coupling between two fields requires that they be phase matched, i.e. the two waves travel at the same phase velocity in the waveguide.

- Prism Coupler



Why a radiation mode does not excite modes in a waveguide?



- In a distance L, such that $(\beta - k_0)L = \pi$

  the field induced at $z = 0$ is exactly out of phase with the field induced at $z = L$.

  The polarization perturbation created by the radiation field within the waveguide excites the guided field but it cannot transfer energy.

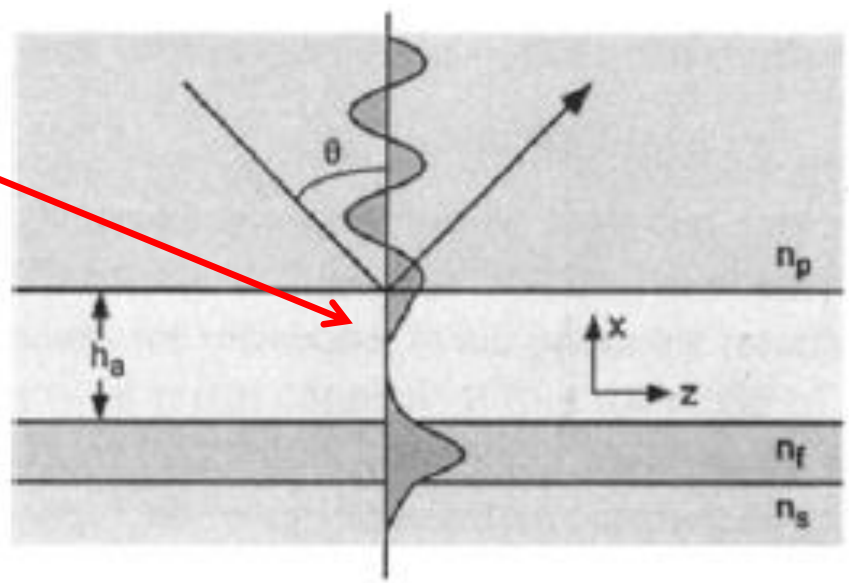
Angle of incidence must be greater than the critical angle

$$\theta > \theta_c = \sin^{-1}(n_a/n_p)$$

The k-vector for the field in the prism

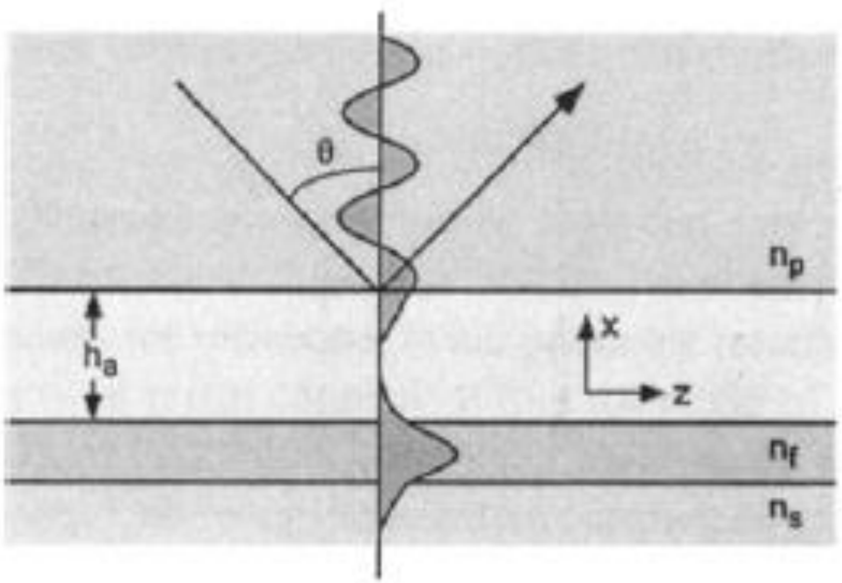
$$\begin{aligned} \hat{k}_o n_p &= n_p(\pm k_x \hat{x} + k_z \hat{z}) \\ &= n_p(\pm k_0 \cos\theta \hat{x} + k_0 \sin\theta \hat{z}) \end{aligned}$$

Evanescent overlap coupling



Below the prism-air interface, field decays exponentially with increasing distance. The x-component of the propagation coefficient is imaginary in this region, but the z-component remains the same as inside the prism.

Since the z-component of *k* depends on the angle of incidence, it is possible to adjust the angle so the waves travel at the same velocity as in the waveguide. When this happens, strong coupling occurs.

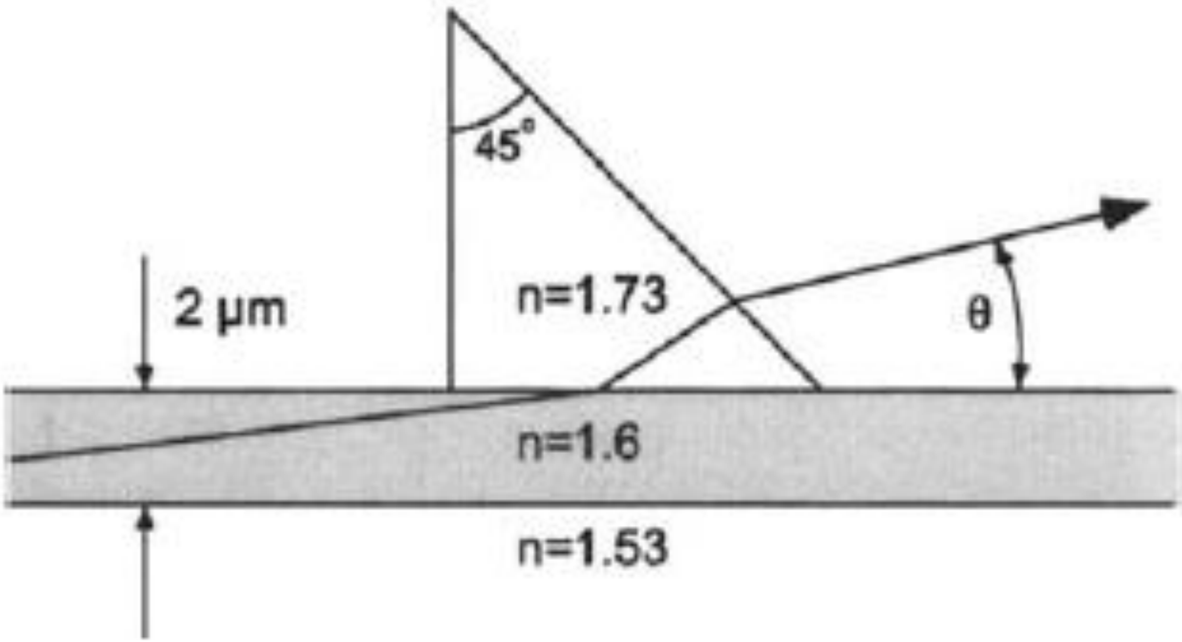


Phase Matching Condition

$$k_0 n_p \sin\theta = \beta_f$$

Mode analysis using a prism coupler

A step index thin film waveguide is constructed on a glass substrate. The guiding film has an index of 1.53, and the guiding film has an index of 1.6, with thickness 2 μm. The waveguide is excited by a HeNe laser operating at $\lambda = 0.6328\mu$m. We can assume that all possible spatial modes in the waveguide have been excited by the source. If a 45-45-90° prism made from SF-14 glass with an index 1.73 is placed on the surface of the waveguide, at what angle will each mode couple out of the prism?



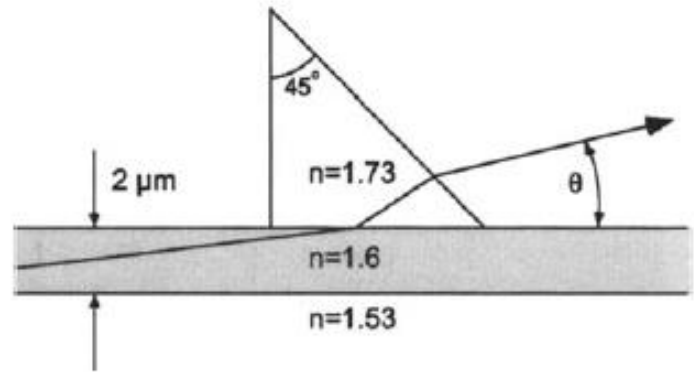
Mode analysis using a prism coupler

- We assume TE polarization. There are 3 allowed TE modes with $\beta_0 = 15.826\mu\text{m}^{-1}$, $\beta_1 = 15.651\mu\text{m}^{-1}$, and $\beta_2 = 15.369\mu\text{m}^{-1}$

$$k_0 = 9.93\mu\text{m}^{-1}$$

- From the phase matching condition

$$\begin{aligned}\theta_0 &= \sin^{-1}\frac{\beta_0}{k_0 n_p} = \frac{15.826}{1.73\cdot 9.93} = 67.13^\circ \\ \theta_1 &= \sin^{-1}\frac{\beta_1}{k_0 n_p} = \frac{15.651}{1.73\cdot 9.93} = 65.65^\circ \\ \theta_2 &= \sin^{-1}\frac{\beta_2}{k_0 n_p} = \frac{15.369}{1.73\cdot 9.93} = 63.46^\circ\end{aligned}$$

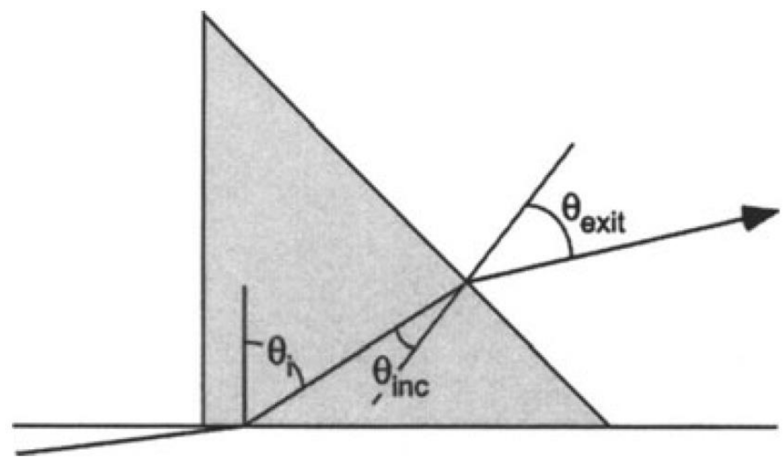


- Incident angle on interface

$$\theta_{inc} = \theta_i - 45^\circ$$

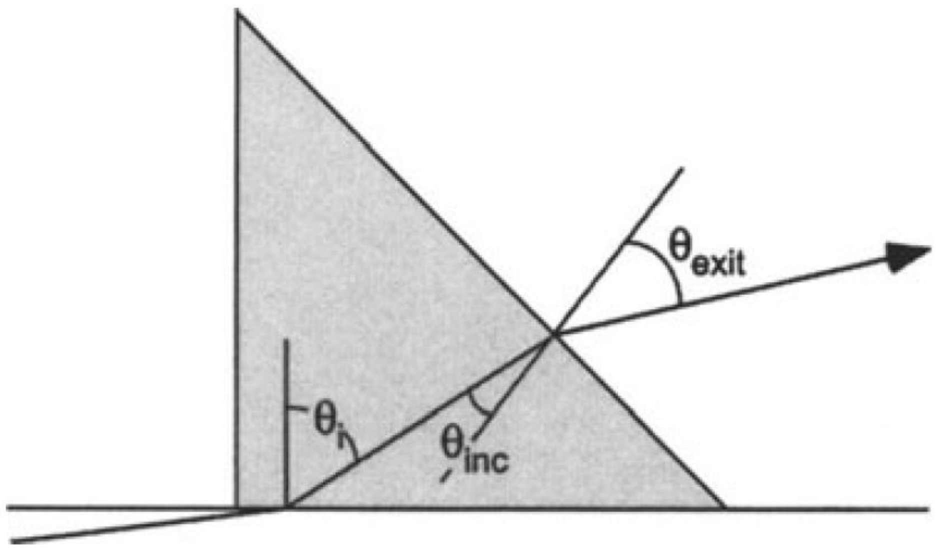
- From Snell's Law

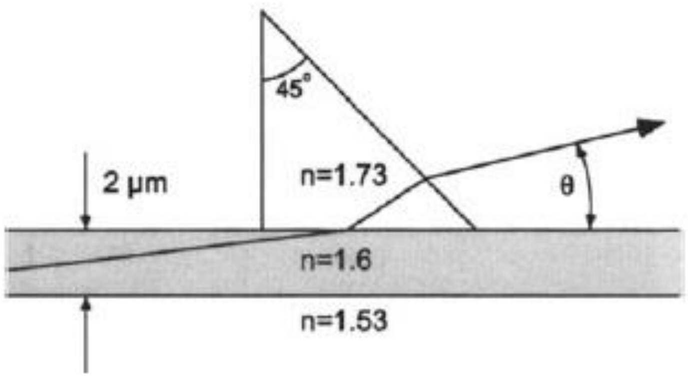
$$\theta_{exit} = \sin^{-1}(1.73\sin\theta_i)$$



Mode analysis using a prism coupler

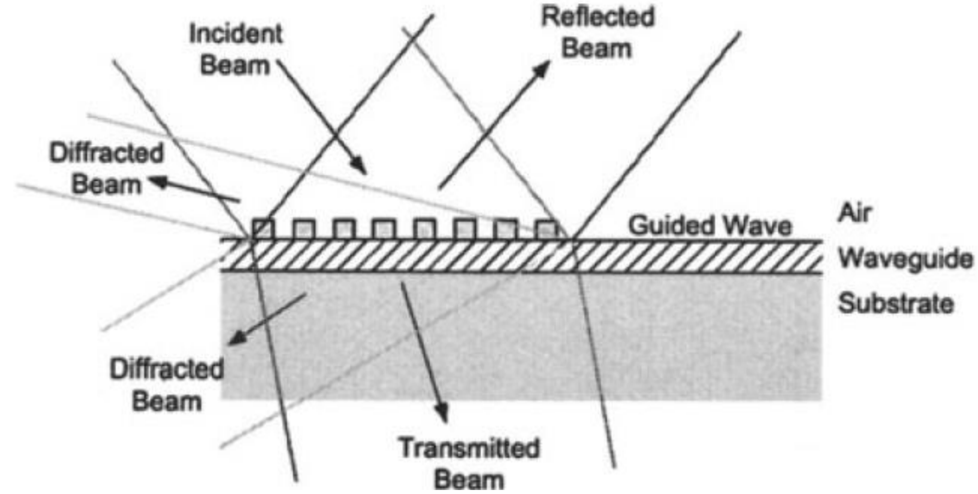
For the three angles determined above, the exit angles with respect to the prism hypotenuse are $40.67°$, $37.64°$, and $33.25°$ for β_0, β_1, and β_2, respectively. Since the hypotenuse makes a 45° angle with respect to the substrate, we should subtract these angles from 45° to find the angle, θ as indicated in the figure for each of the modes. In this case we find that the lowest order modes travels at an angle of $4.33°$ from the substrate, while the other modes travel at $7.36°$ and $11.75°$ from the substrate. These modes can be easily distinguished from each other on a card placed a small distance from the prism coupler.

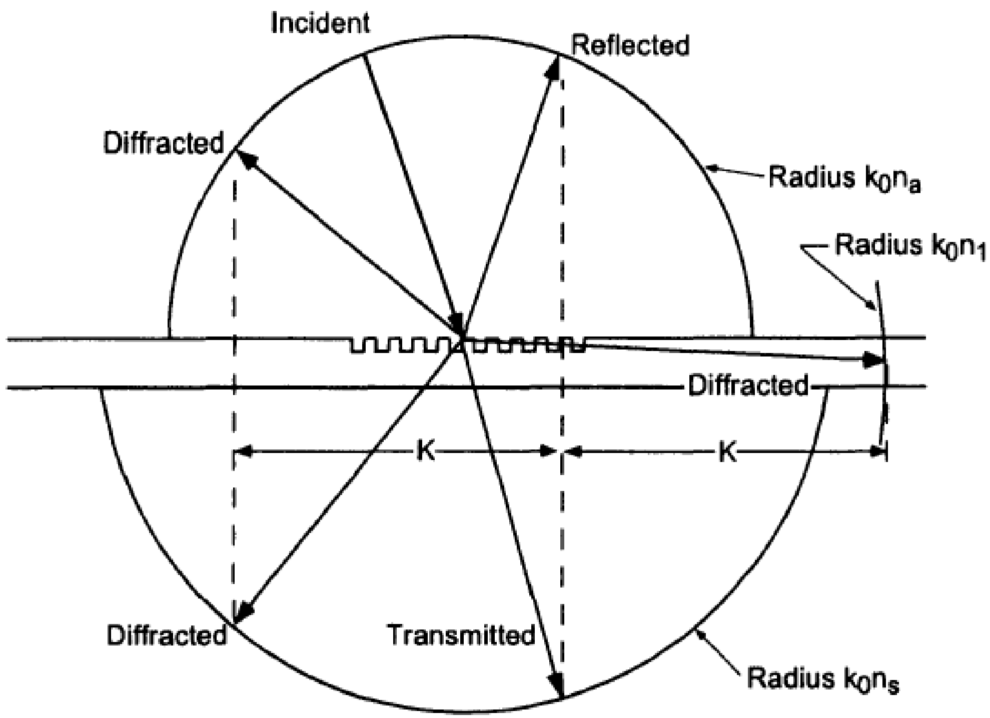




Coupling Methods

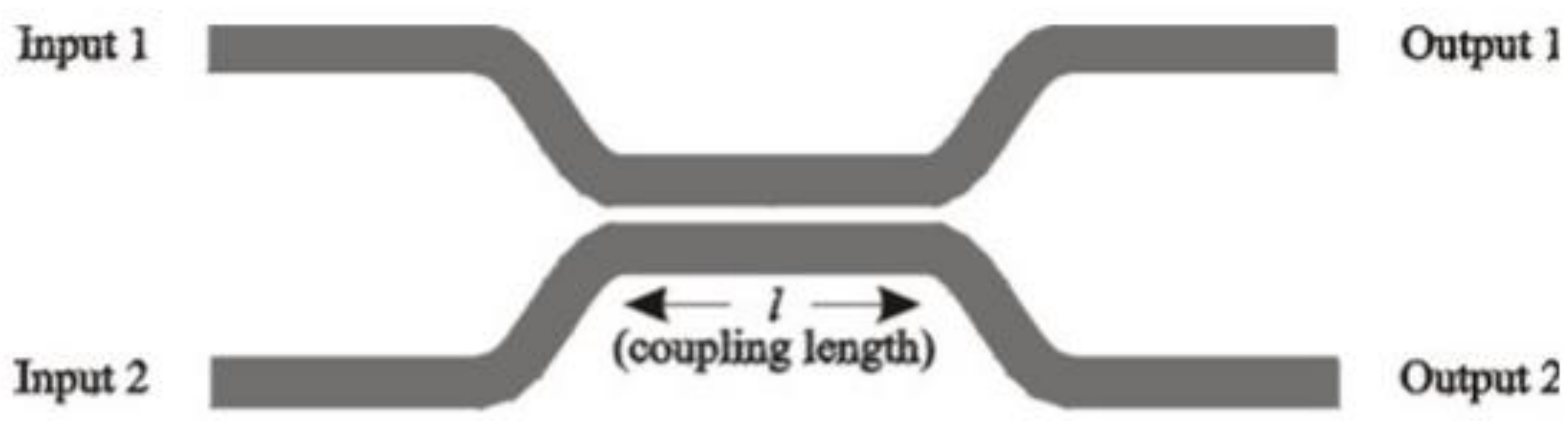
- Grating Coupler



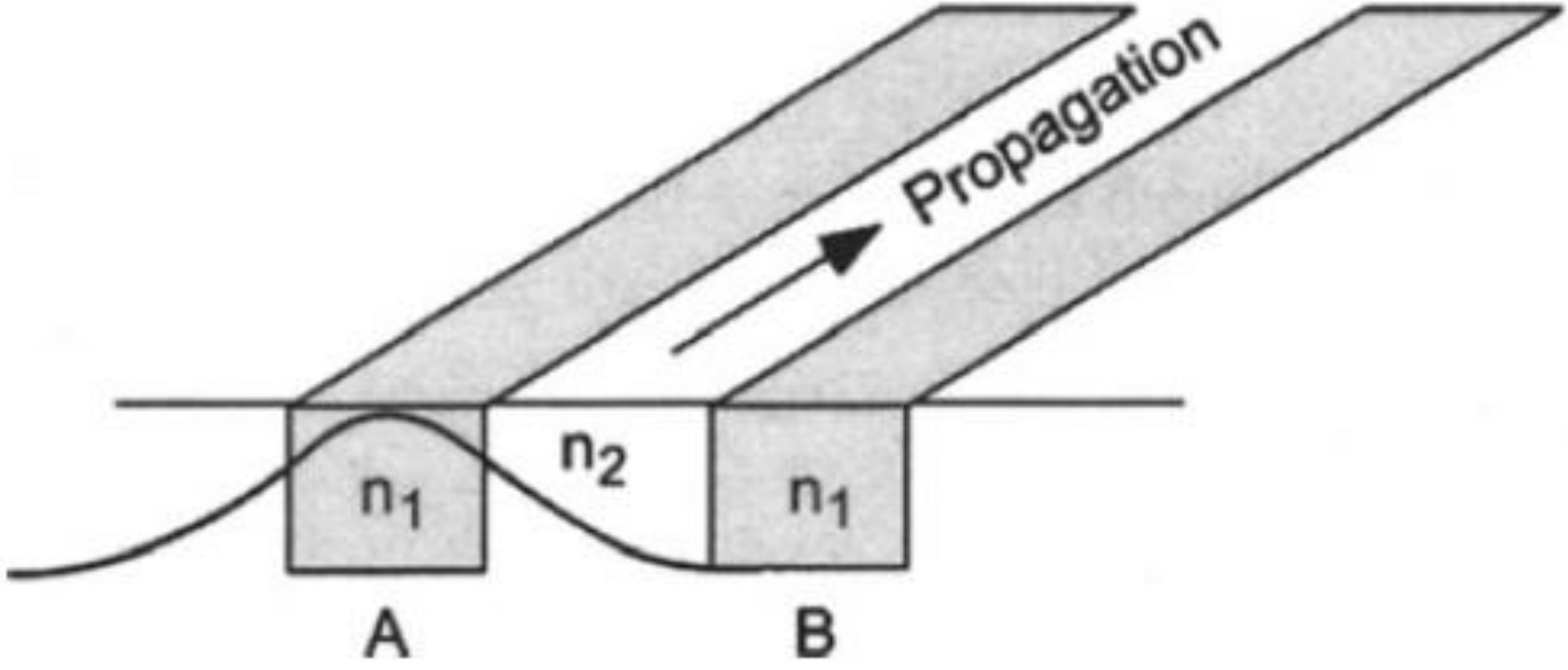


Coupling Methods

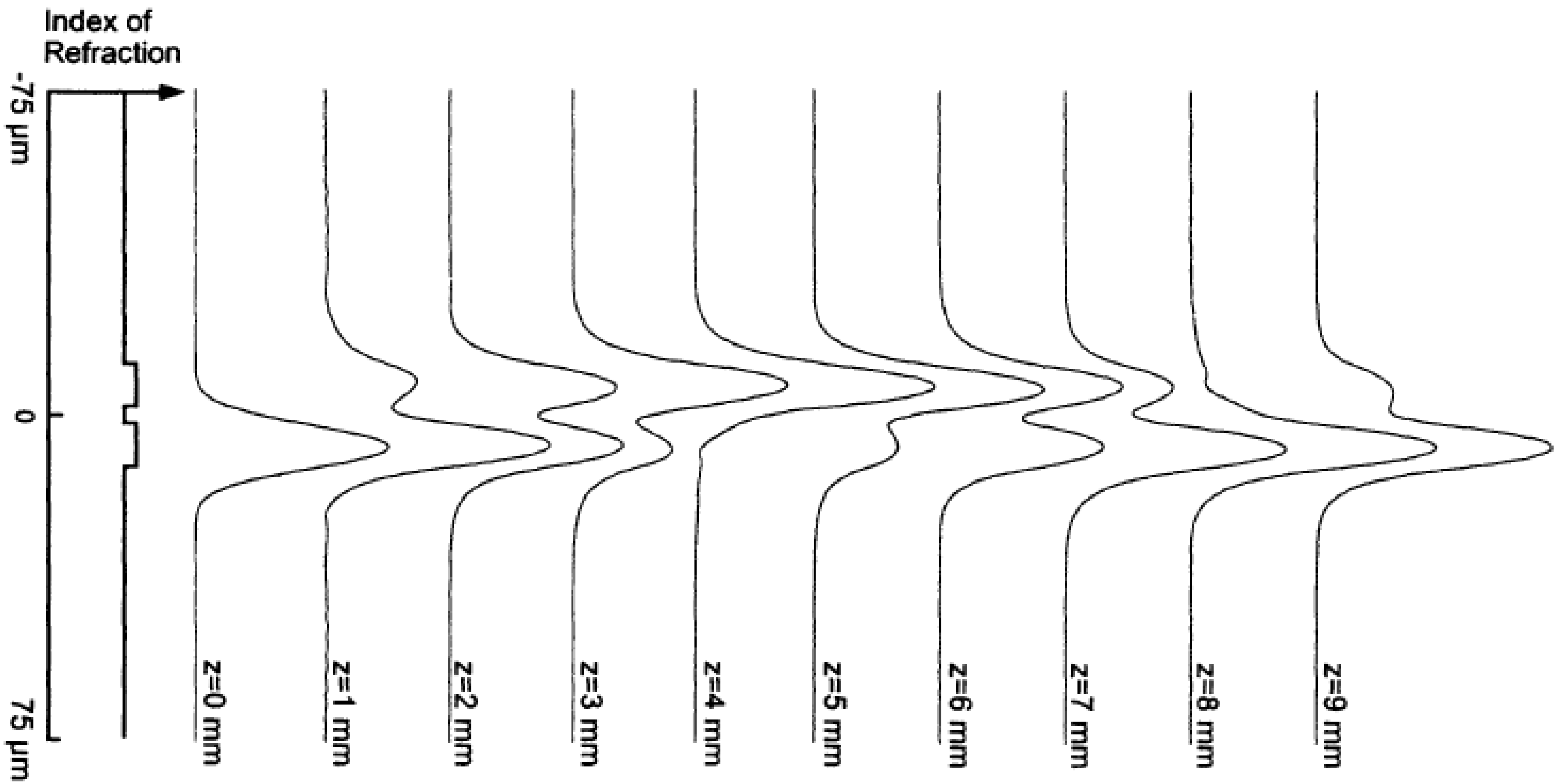
- Parallel Waveguide Coupling (Directional Coupler)



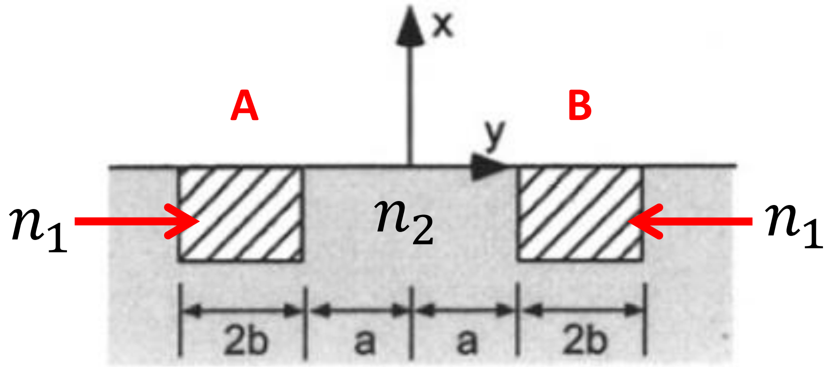
Coupled optical waveguides



Coupled optical waveguides



Coupled optical waveguides



UNCOUPLED

$$
\begin{aligned}
E_A(x,y,z) &= A\cos(\kappa_x x + \phi_x)\cos(\kappa_y(y+(a+b))e^{-j\beta z} \\
E_B(x,y,z) &= B\cos(\kappa_x x + \phi_x)\cos(\kappa_y(y-(a+b))e^{-j\beta z}
\end{aligned}
$$

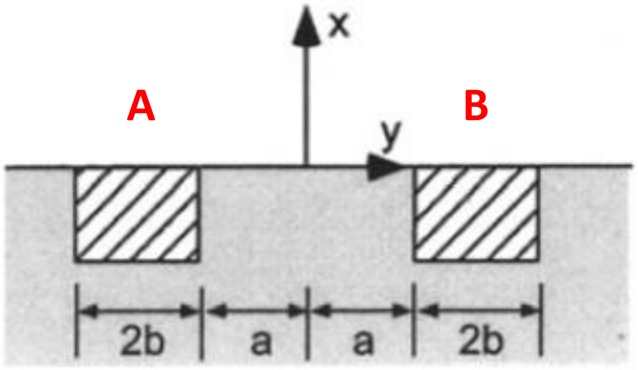
Same transverse and longitudinal wavevectors

The polarization perturbation in waveguide B arises from the presence of the evanescent tail of mode A. The perturbation is actually due to the difference in index the evanescent field sees when in the core of waveguide B, compared to the normal cladding index. The perturbation induced by mode A acts as a source to excite mode B.

Coupled optical waveguides

The perturbation seen by B is of the form

$$P_{pert}(x,y,z) = \overbrace{\epsilon_0(n_1^2(x,y) - n_2^2)}^{\Delta\varepsilon} E_A(x,y)$$
$$= \epsilon_0(n_1^2(x,y) - n_2^2)\left[\frac{A}{2}\mathcal{E}_A(y)e^{-j(\beta z - \omega t)} + c.c.\right]$$



The device operates with codirectional coupling, so no backward wave is included (due to phase matching, it is impossible to couple energy into the backward wave).

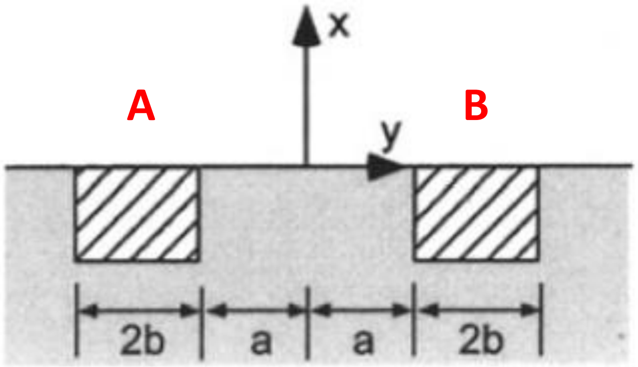
$\mathcal{E}$ Indicates the field of a specific mode

Equation of motion

$$-\frac{\partial B}{\partial z}e^{-j(\beta z-\omega t)} + c.c. = -\frac{j}{2\omega}\frac{\partial^2}{\partial t^2}\int_S \mathcal{E}_B(x,y)P_{pert}dS$$
$$= \frac{j\omega}{2}\int_{-\infty}^{\infty}\epsilon_0(n^2(x,y) - n_2^2)\mathcal{E}_B(x,y)\left[\frac{A}{2}\mathcal{E}_A(y)e^{-j(\beta z-\omega t)} + c.c.\right]dx\,dy$$
$$= j\mathcal{K}Ae^{-j(\beta z-\omega t)}$$

Coupled optical waveguides

Because of symmetry there is identical coupling from B to A



$$-\frac{\partial A}{\partial z} e^{-j(\beta z - \omega t)} = j\mathcal{K} B e^{-j(\beta z - \omega t)}$$

The system reduces to

$$\frac{\partial A}{\partial z} = -j\mathcal{K}B$$
$$\frac{\partial B}{\partial z} = -j\mathcal{K}A$$

$$\Rightarrow \quad \frac{\partial^2 A}{\partial z^2} = -\mathcal{K}^2 A$$

The uncoupled equation can be solved directly. With B.C. $A(0) = 1$ and $B(0) = 0$

$$\begin{aligned} A(z) &= \cos(\mathcal{K}z) \\ B(z) &= -j\sin(\mathcal{K}z) \end{aligned}$$

$$\frac{\partial A}{\partial z} = -j\kappa B$$
$$\frac{\partial B}{\partial z} = -j\kappa A$$

$$\frac{\partial}{\partial z}\left(\frac{\partial A}{\partial z}\right) = -j\kappa\frac{\partial B}{\partial z}$$

$$\frac{\partial^2 A}{\partial z^2} = -j\kappa(-j\kappa A) \Rightarrow \frac{\partial^2 A}{\partial z^2} = -\kappa^2 A$$

General solution $A(z) = C_1\cos(\kappa z) + C_2\sin(\kappa z)$

B.C. $A(0) = 1$ $\quad C_1 = 1$

B.C. $B(0) = 0$ $\quad \frac{\partial A(0)}{\partial z} = -j\kappa B(0) = 0 \Rightarrow C_2 = 0$

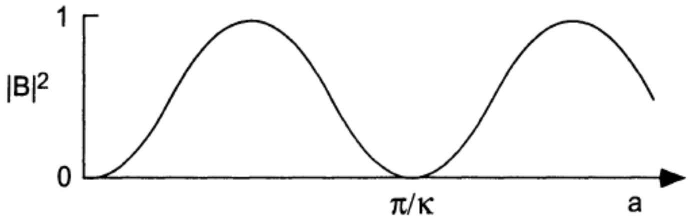
$$A(z) = \cos(\kappa z)$$

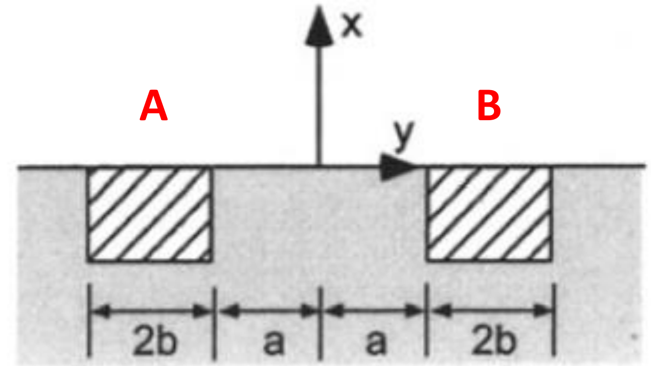
$$\frac{\partial A}{\partial z} = -\kappa\sin(\kappa z) = -j\kappa B(z)$$

$$B(z) = -j\sin(\kappa z)$$

Coupled optical waveguides

The power in waveguide B varies sinusoidally as a function of coupling length **a**





$$A(z) = \cos(\mathcal{K}z)$$
$$B(z) = -j\sin(\mathcal{K}z)$$

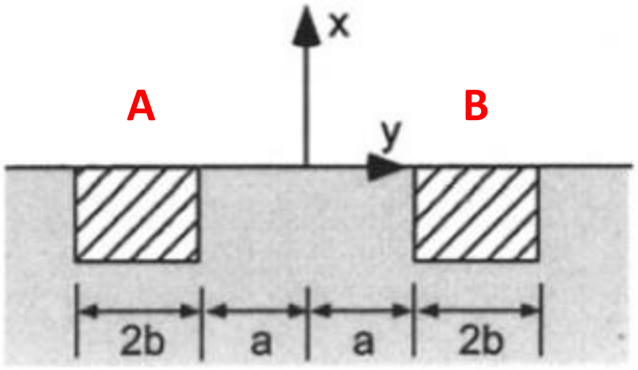
Power can travel back and forth the waveguides with 100% transfer efficiency. Also, there is always a precise phase difference between the driving and the driven field. The driven field always lags by 90°. The length of the interaction region determines the exact value of the coupling. Complete energy transfer occurs for

$$\cos(\mathcal{K}z) = 0$$

$$z_0 = \frac{\pi}{2\mathcal{K}} + \frac{q\pi}{\mathcal{K}} \quad \text{(q integer)}$$

Directional Coupler

It is possible to design couplers for different purposes by selecting the coupling length to obtain a certain percentage of transfer.



- To tap a broadcast signal it may be sufficient to couple 1% of the power (-20 dB)
- For heterodyne detection, a 50% coupler (3dB) is desirable

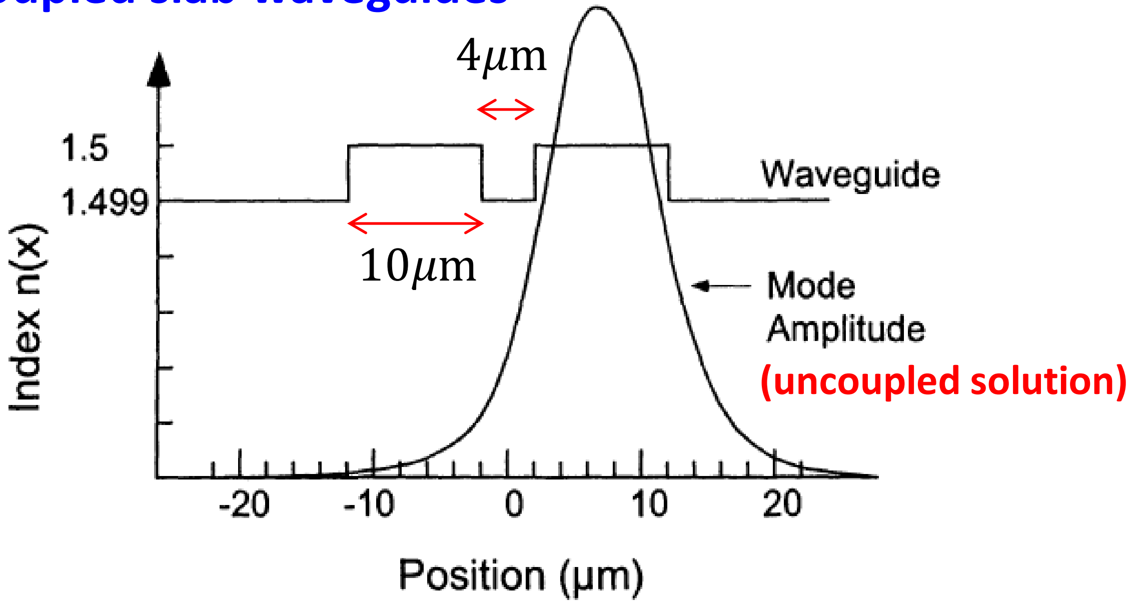
Drawbacks

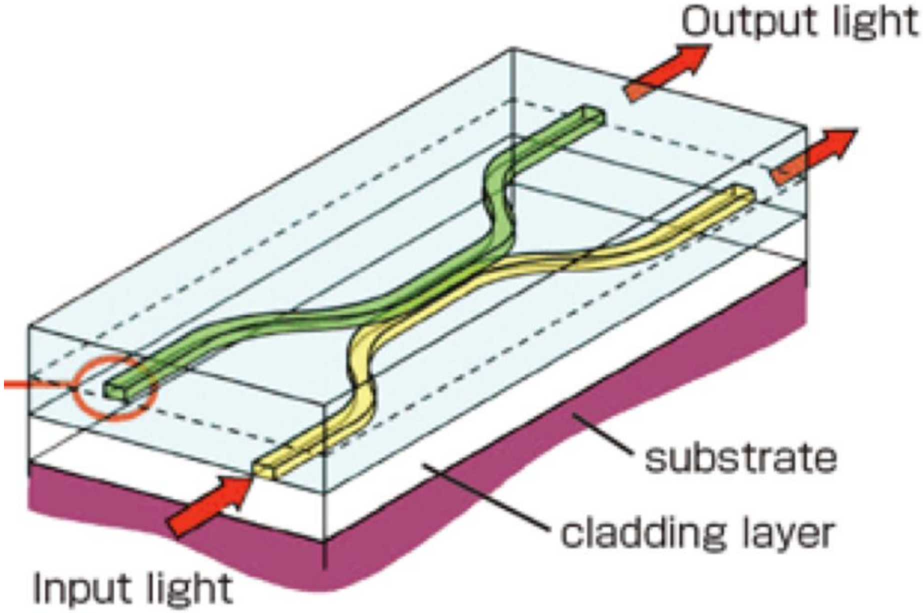
The coupling constant depends critically on β. A 3dB coupler at a certain wavelength may not yield 3dB at another wavelength

Because of the dependence on β, single mode propagation is in general necessary.

Directional Coupler

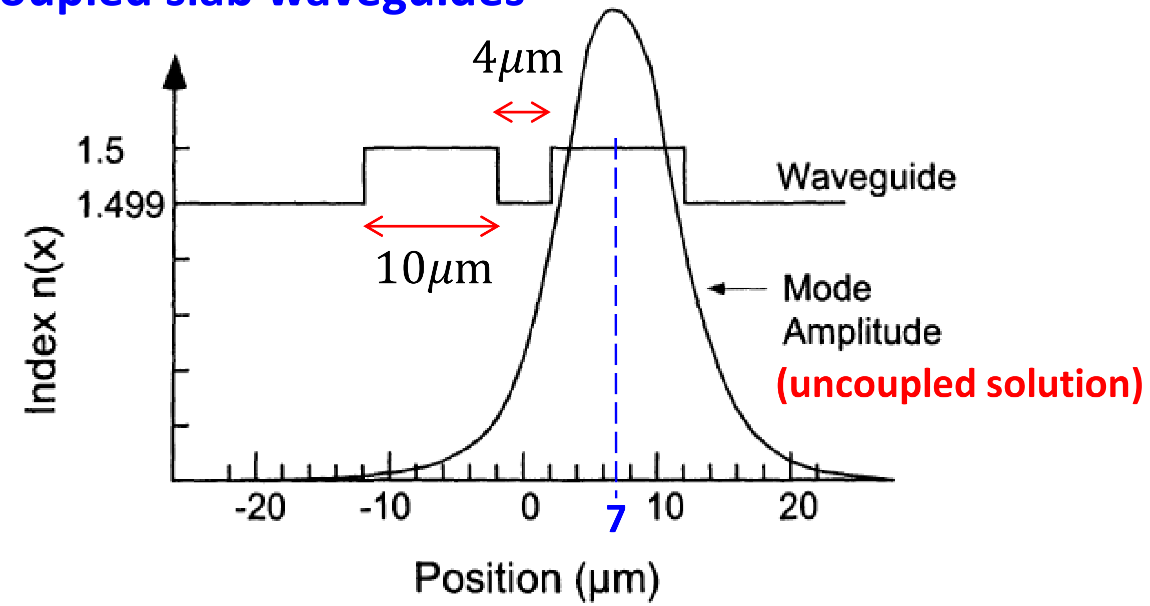
Example – two coupled slab waveguides





Directional Coupler

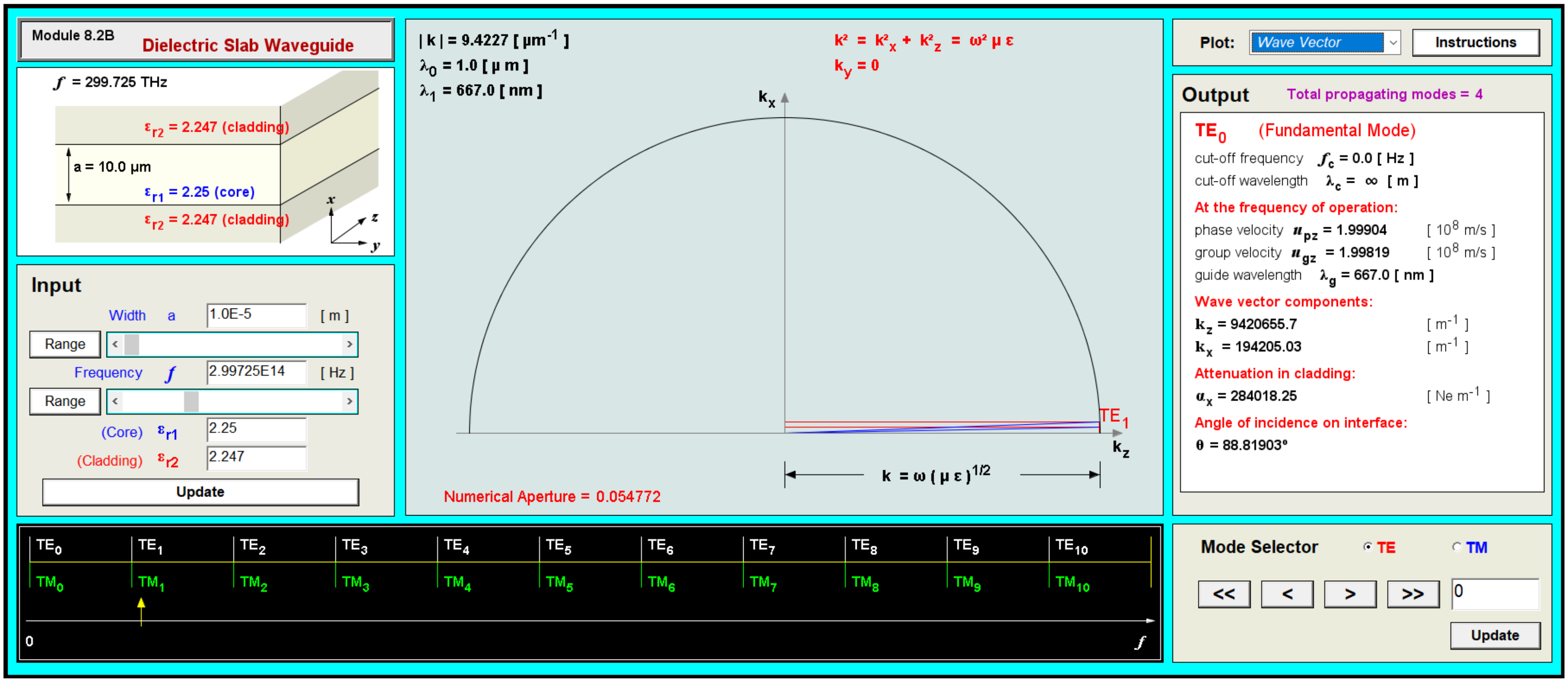
Example – two coupled slab waveguides

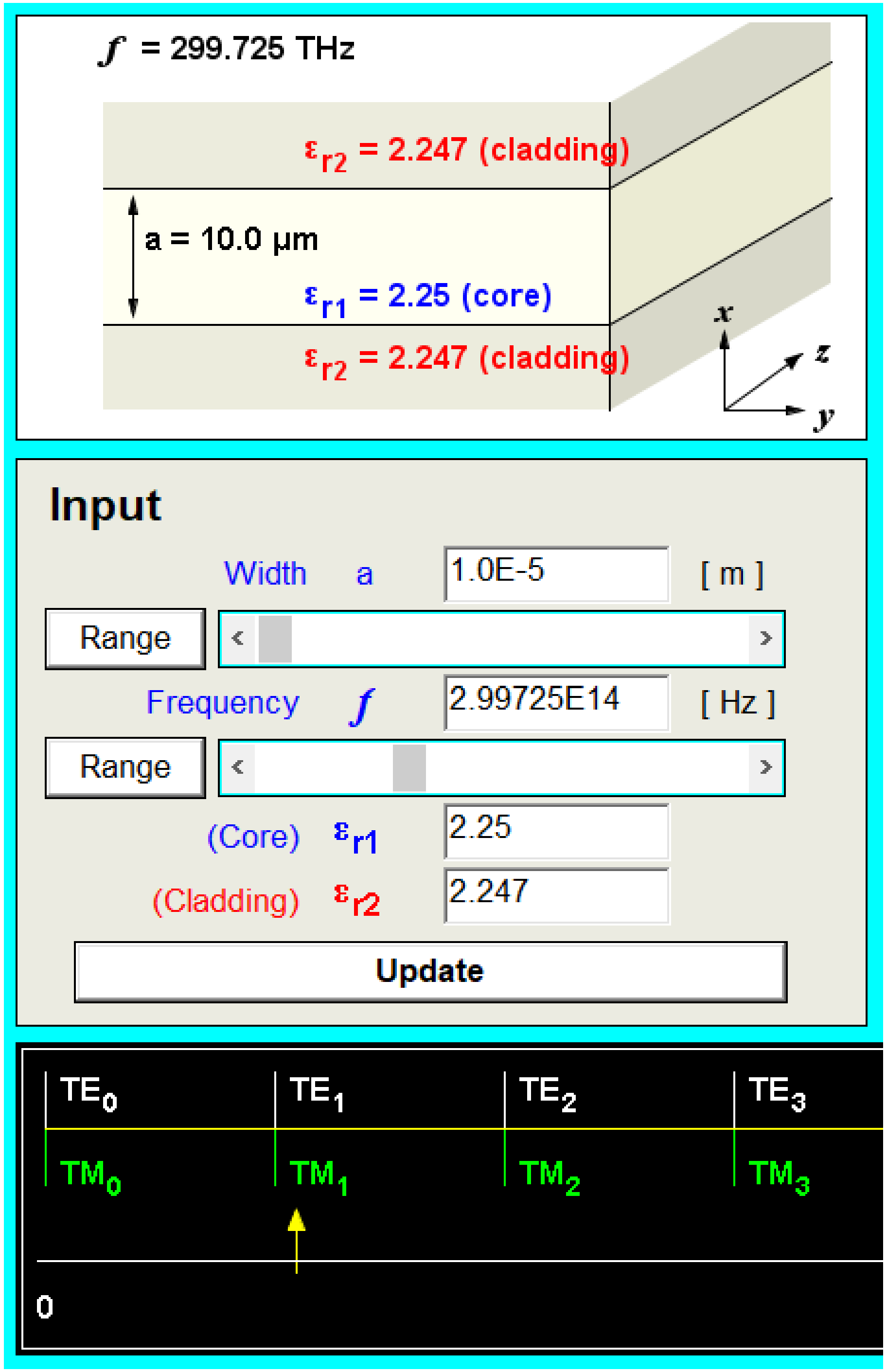


For this (uncoupled) waveguide $\beta = 94227 \text{ cm}^{-1} \; (\lambda = 1\mu\text{m})$

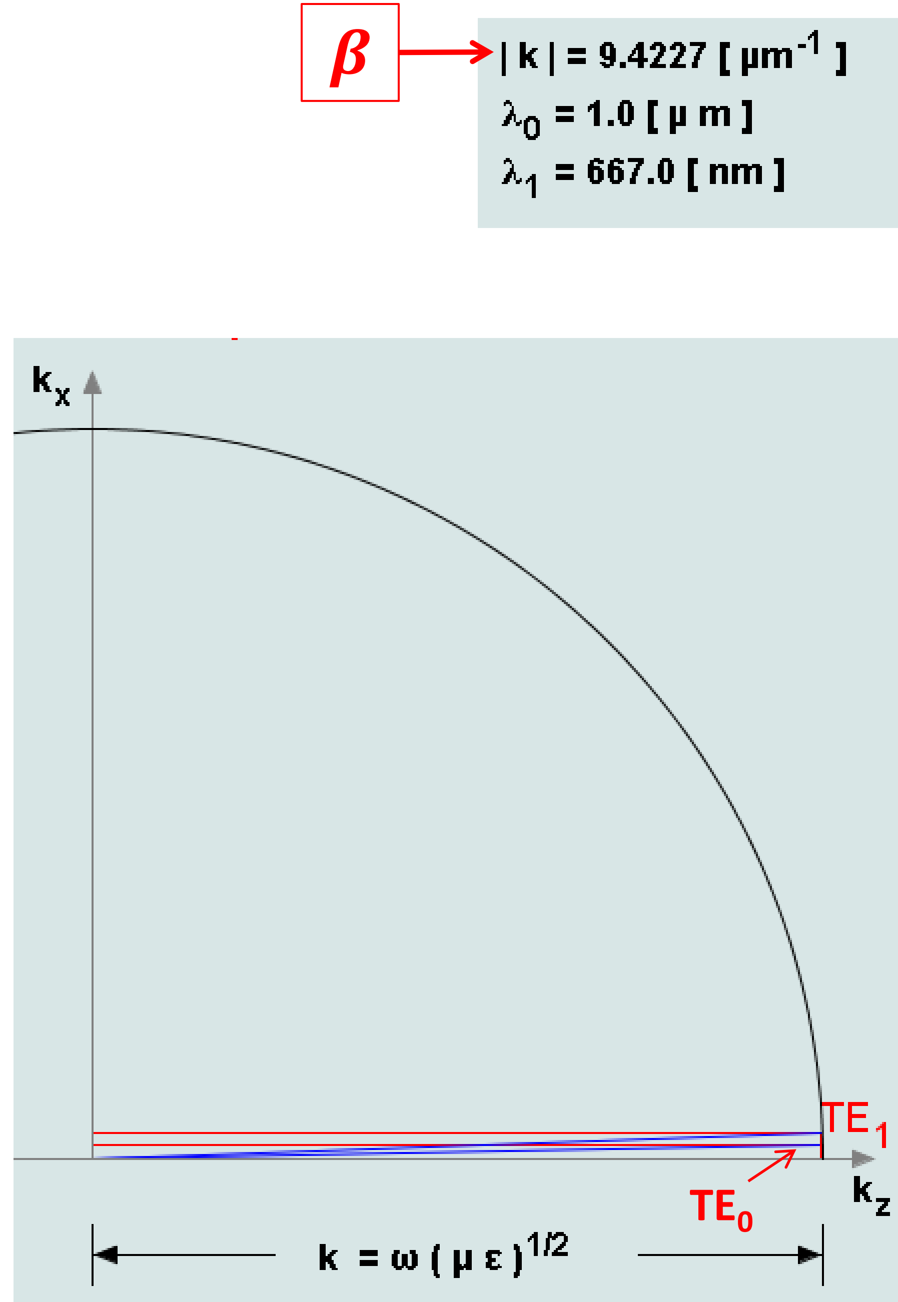
$$
\begin{aligned}
\mathcal{E}_a(x) &= C \; \exp[-\overset{\alpha}{2840}(x - 0.0012)] && (x > 0.0012\text{cm}) \\
&= C \; \frac{\cos[\overset{k_x}{1942}(x - 0.0007)]}{\cos[1942 * 0.0005]} && (0.0002 < x < 0.0012\text{cm}) \\
&= C \; \exp[+\underset{\alpha}{2840}(x - 0.0002)] && (x < 0.0002 \text{ cm})
\end{aligned}
$$

From the Dielectric Slab Waveguide Java App





f = 299.725 THz
εr2 = 2.247 (cladding)
a = 10.0 μm
εr1 = 2.25 (core)
εr2 = 2.247 (cladding)
x
z
y
Input
Width a
1.0E-5
[m]
Range
Frequency f
2.99725E14
[Hz]
Range
(Core) εr1
2.25
(Cladding) εr2
2.247
Update
TE0
TE1
TE2
TE3
TM0
TM1
TM2
TM3
0

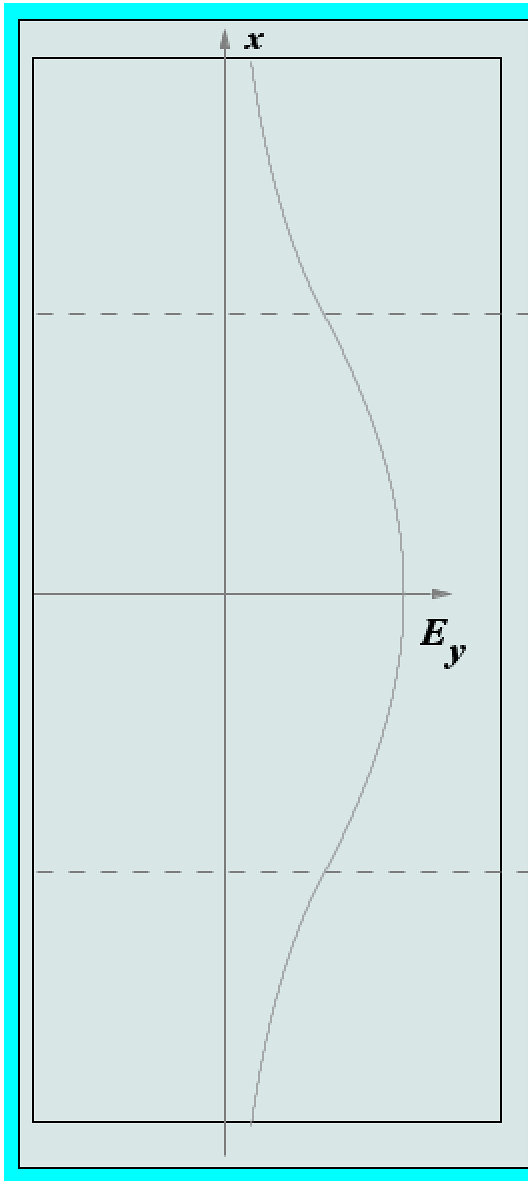


β
| k | = 9.4227 [μm-1]
λ0 = 1.0 [μ m]
λ1 = 667.0 [nm]
kx
TE1
TE0
kz
k = ω (μ ε)1/2

β → $|k| = 9.4227$ [μm^{-1}]

$\lambda_0 = 1.0$ [μ m]

$\lambda_1 = 667.0$ [nm]



TE$_0$

Output Total propagating modes = 4

TE$_0$ (Fundamental Mode)

cut-off frequency $f_c = 0.0$ [Hz]

cut-off wavelength $\lambda_c = \infty$ [m]

At the frequency of operation:

phase velocity $u_{pz} = 1.99904$ [10^8 m/s]

group velocity $u_{gz} = 1.99819$ [10^8 m/s]

guide wavelength $\lambda_g = 667.0$ [nm]

Wave vector components:

$k_z = 9420655.7$ [m^{-1}]

1942 cm^{-1} → $k_x = 194205.03$ [m^{-1}]

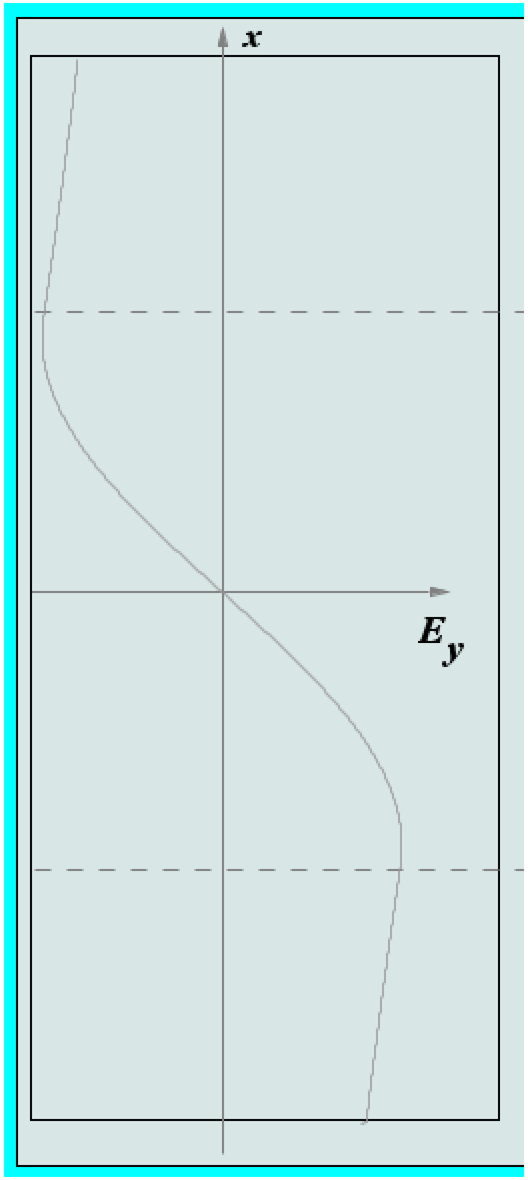
Attenuation in cladding:

2840 cm^{-1} → $\alpha_x = 284018.25$ [Ne m^{-1}]

Angle of incidence on interface:

$\theta = 88.81903°$

$|k| = 9.4227$ [μm^{-1}]
$\lambda_0 = 1.0$ [μ m]
$\lambda_1 = 667.0$ [nm]



TE $_1$

Output

Total propagating modes = 4

TE $_1$

cut-off frequency $f_c = 273.67182$ [THz]
cut-off wavelength $\lambda_c = 730.0$ [nm]

At the frequency of operation:

phase velocity $u_{pz} = 1.99993$ [10^8 m/s]
group velocity $u_{gz} = 1.99731$ [10^8 m/s]
guide wavelength $\lambda_g = 667.0$ [nm]

Wave vector components:

$k_z = 9416485.721487291$ [m^{-1}]
$k_x = 340978.1271675465$ [m^{-1}]

Attenuation in cladding:

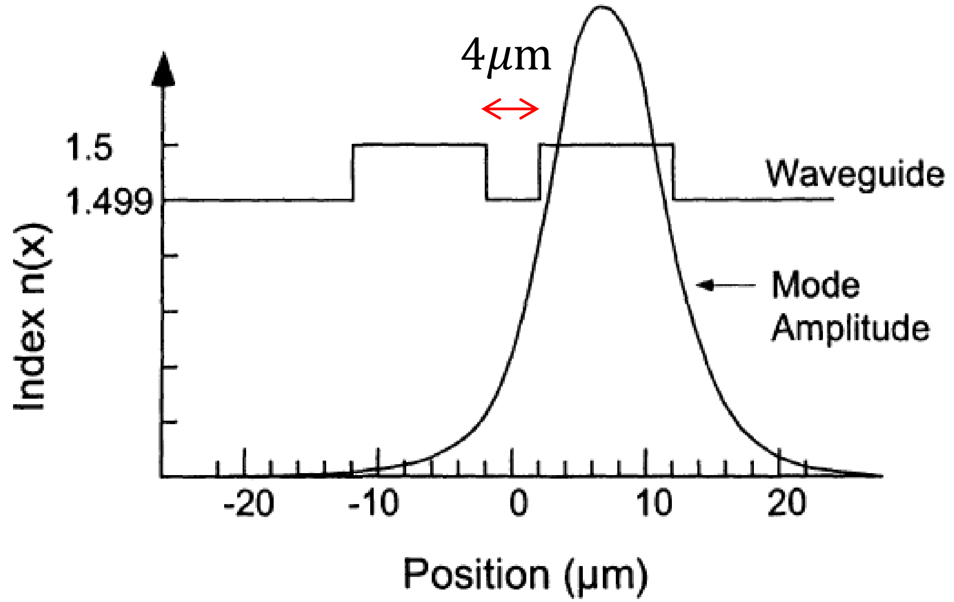
$\alpha_x = 45998.652460907346$ [Ne m^{-1}]

Angle of incidence on interface:

$\theta = 87.92618^o$

Coupled optical waveguides

Example



For normalization of eigenmode

$$\frac{1}{2}\int \mathcal{E}_i(x) \times \mathcal{H}_j(x) dA = \delta_{ij}$$

$$\int_{-\infty}^{\infty} \mathcal{E}_i(x)\mathcal{E}_j(x)dx = \frac{2\omega\mu}{\beta_i}\delta_{ij}$$

$$\beta = 94227 \text{ cm}^{-1} \ (\lambda = 1\mu\text{m})$$

$$\omega = 2\pi 300 \times 10^{12} \text{ sec}^{-1}$$

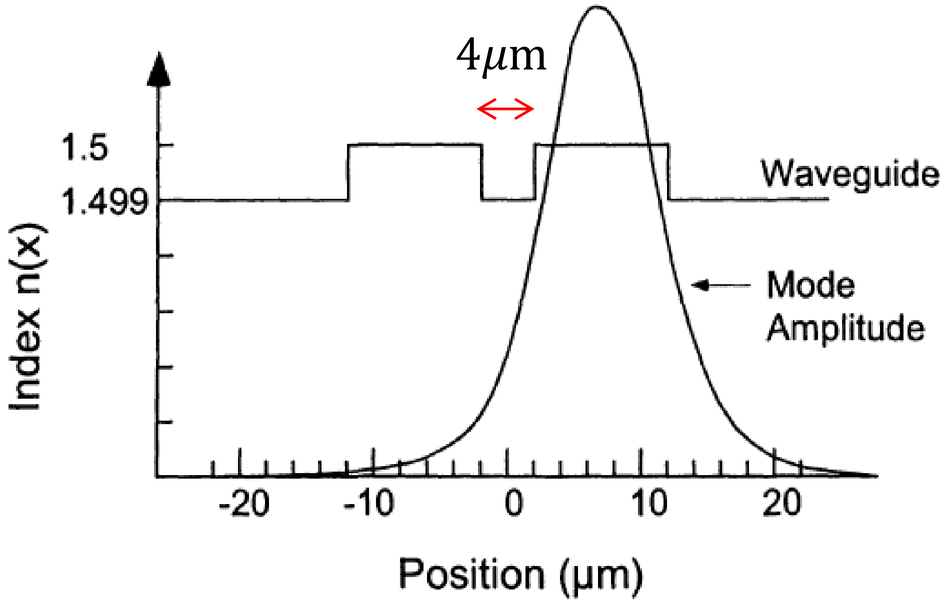
$$\mu = 4\pi \times 10^{-9} \text{ Henry/cm}$$

From numerical evaluation

$$C = 433.56.$$

Coupled optical waveguides

Example



Perturbation in the core of the left waveguide

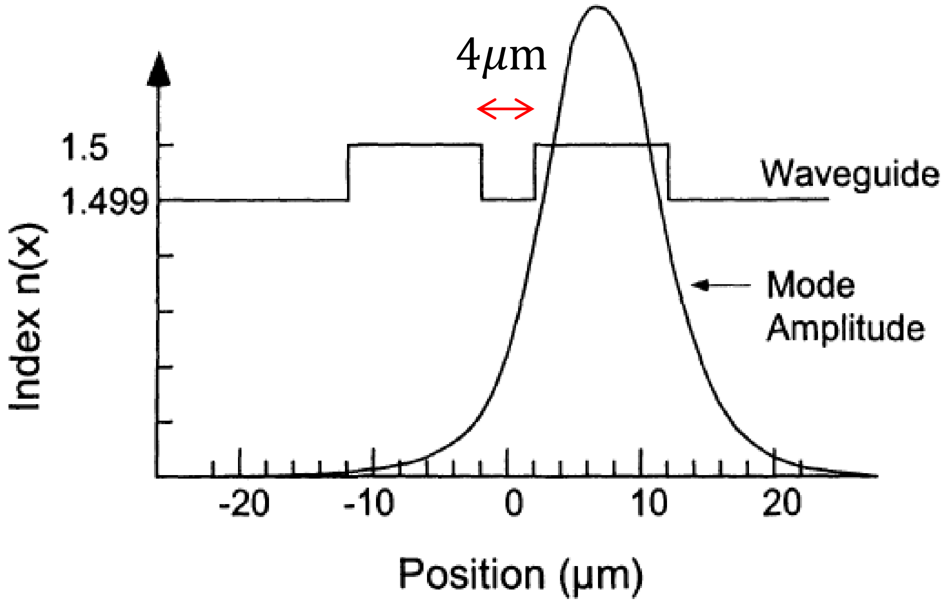
$$\begin{aligned} P_{pert} &= \mathcal{E}_a(x)\epsilon_0(n_1^2 - n_2^2)\left[\frac{A}{2}e^{-j(\beta z - \omega t)} + c.c.\right] \qquad (-0.0012 < x < -0.0002) \\ &= 0 \qquad \text{elsewhere} \end{aligned}$$

Coupling coefficient

$$\mathcal{K} = \frac{\epsilon_0 \omega}{4}\int_{-0.0012}^{-0.0002}(1.5^2 - 1.499^2)\mathcal{E}_b(x)\mathcal{E}_a(x)dx = 3.6217 \text{ cm}^{-1}$$

Coupled optical waveguides

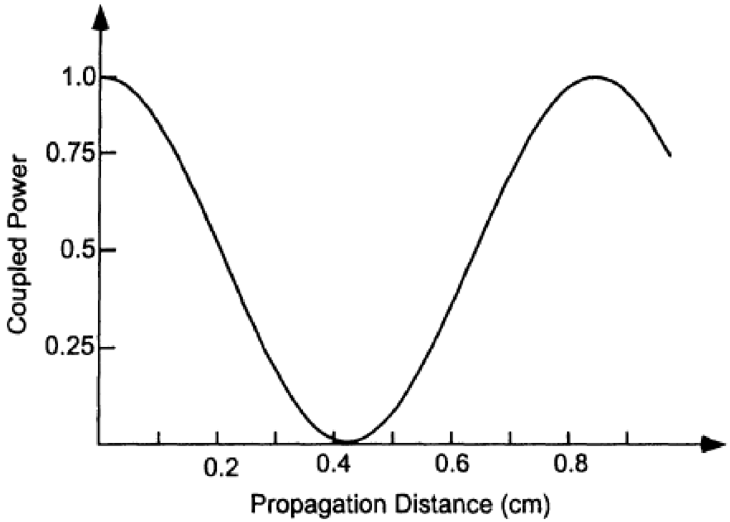
Example



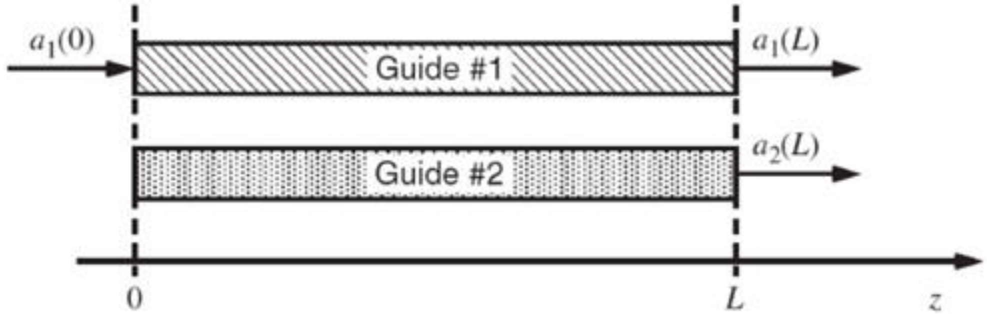
$$A(z) = A_0 \cos(\mathcal{K} z)$$

Power couples back and forth according to

$$A(z)^2 = A_0^2 \cos(\mathcal{K} z)^2$$



For different waveguides there are additional terms



Assuming we write the electric field as: $\mathbf{E} = \sqrt{2\eta_1}\, a_1 \mathbf{U}_1 + \sqrt{2\eta_2}\, a_2 \mathbf{U}_2$

For the 4-port coupler with input at z=0:

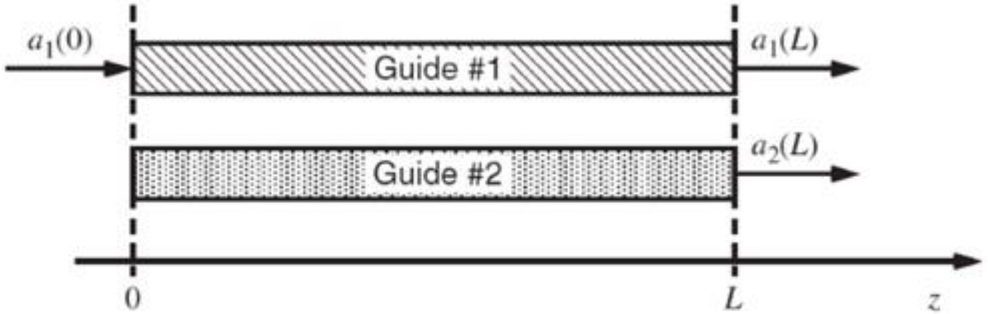
$$a_1(z) = \left[a_1(0)\left(\cos sz + j\frac{\beta_2 - \beta_1}{2s}\sin sz \right) - j\frac{K_{12}}{s} a_2(0)\sin sz \right] e^{-j\tilde{\beta}z}$$

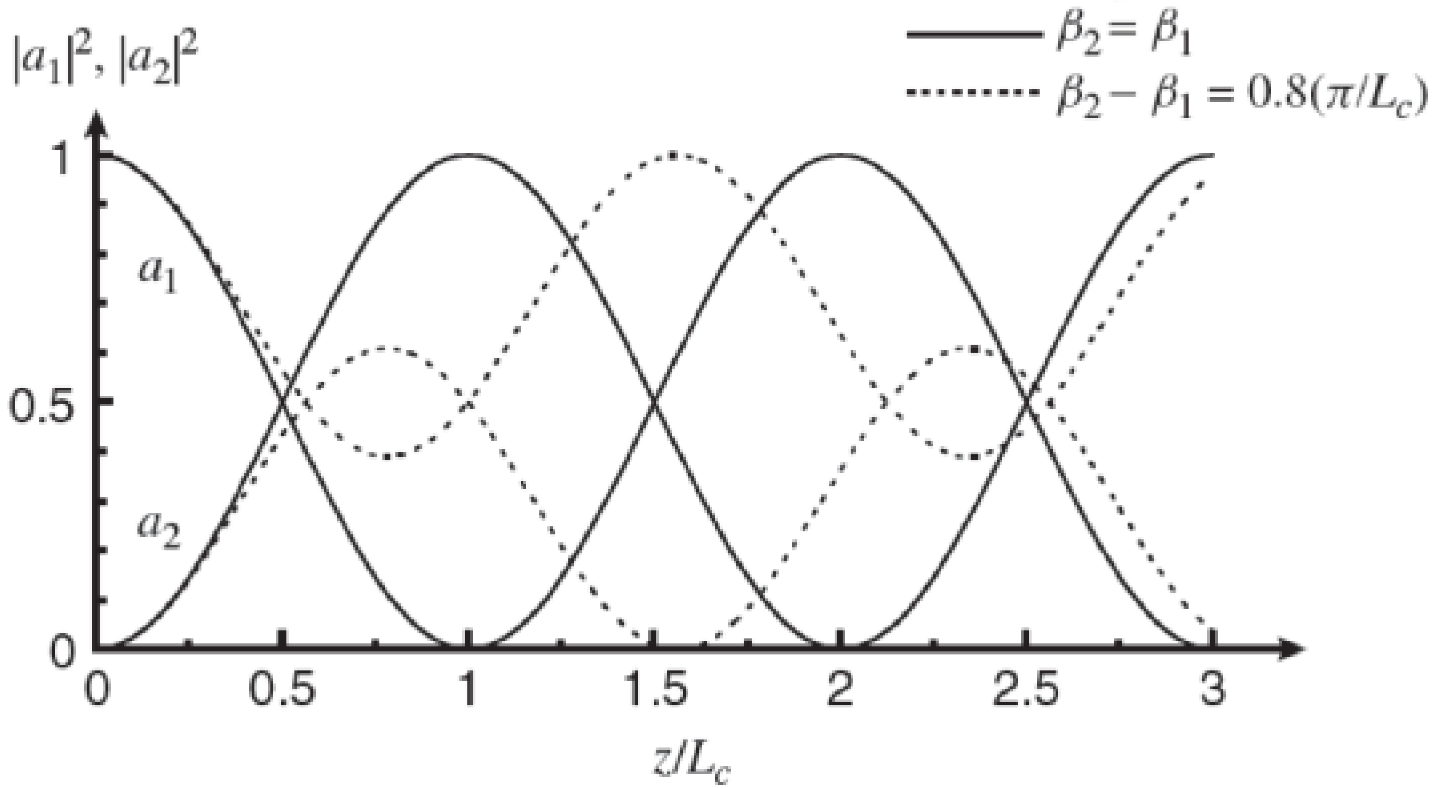
$$a_2(z) = \left[-j\frac{K_{21}}{s} a_1(0)\sin sz + a_2(0)\left(\cos sz - j\frac{\beta_2 - \beta_1}{2s}\sin sz \right) \right] e^{-j\tilde{\beta}z}$$

For identical guides: $\beta_1 = \beta_2$ and $s = \sqrt{K_{12}K_{21}} \equiv K$ and $\left| \frac{a_2(L)}{a_1(0)} \right|^2 = \sin^2 KL$

The critical length for full coupling from 1 to 2 is: $L_c = \frac{\pi}{2K}$

Coupling Coefficients (Weak Coupling)





100% power transfer when $KL = \pi/2$, i.e. $L = \pi/2K$

50:50 power split when $KL = \pi/4$, i.e. $L = \pi/4K$