ECE 536 – Integrated Optics and Optoelectronics
Lecture 26 – April 21, 2022

Spring 2022

Tu-Th 11:00am-12:20pm

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Lecture 26 Outline

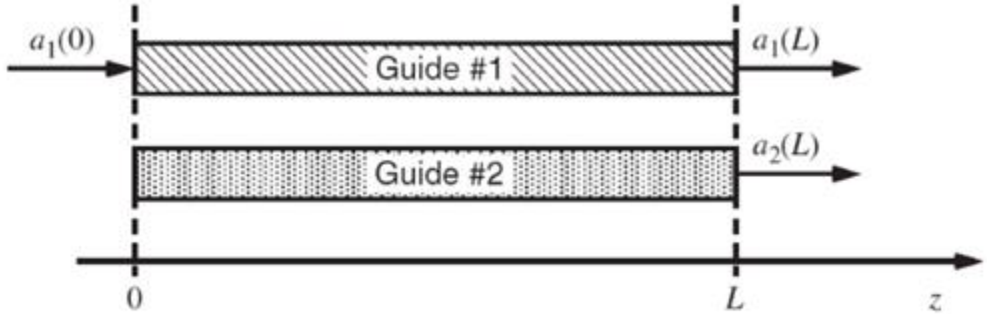
- Integrated Optics components
- Directional couplers
- Modulators

Integrated Optics Structures

Directional Couplers

Coldren, Corzine, Mašanović: Chapter 6 & Appendix 15

For different waveguides there are additional terms



Assuming we write the electric field as: $\mathbf{E} = \sqrt{2\eta_1}a_1\mathbf{U}_1 + \sqrt{2\eta_2}a_2\mathbf{U}_2$

For the 4-port coupler with input at z=0:

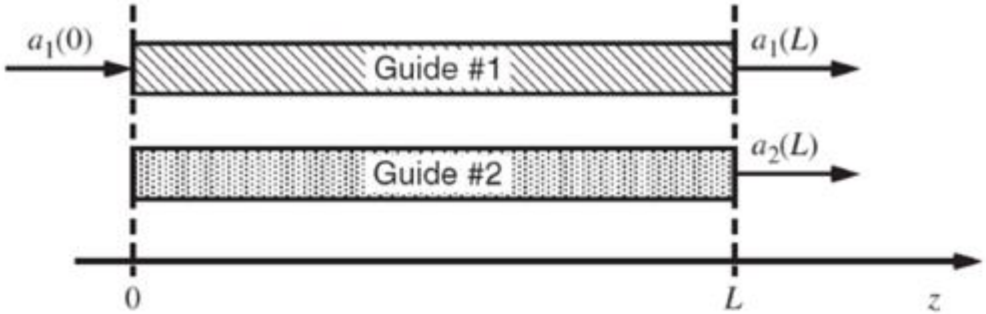
$$a_1(z) = \left[a_1(0)\left(\cos sz + j\frac{\beta_2 - \beta_1}{2s}\sin sz \right) - j\frac{K_{12}}{s}a_2(0)\sin sz \right] e^{-j\tilde{\beta}z}$$

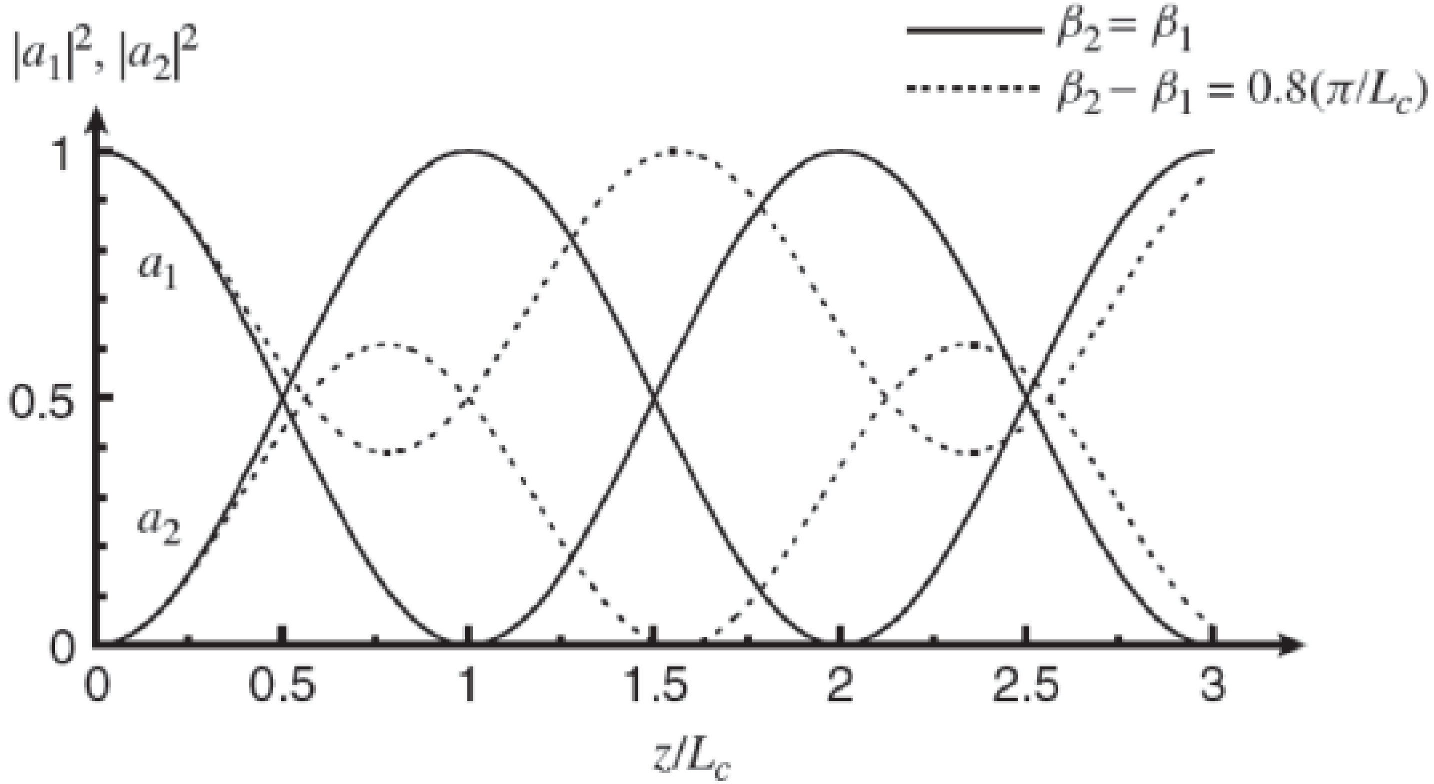
$$a_2(z) = \left[-j\frac{K_{21}}{s}a_1(0)\sin sz + a_2(0)\left(\cos sz - j\frac{\beta_2 - \beta_1}{2s}\sin sz \right) \right] e^{-j\tilde{\beta}z}$$

For identical guides: $\beta_1 = \beta_2$ and $s = \sqrt{K_{12}K_{21}} \equiv K$ and $\left| \dfrac{a_2(L)}{a_1(0)} \right|^2 = \sin^2 KL$

The critical length for full coupling from 1 to 2 is: $L_c = \dfrac{\pi}{2K}$

Coupling Coefficients (Weak Coupling)

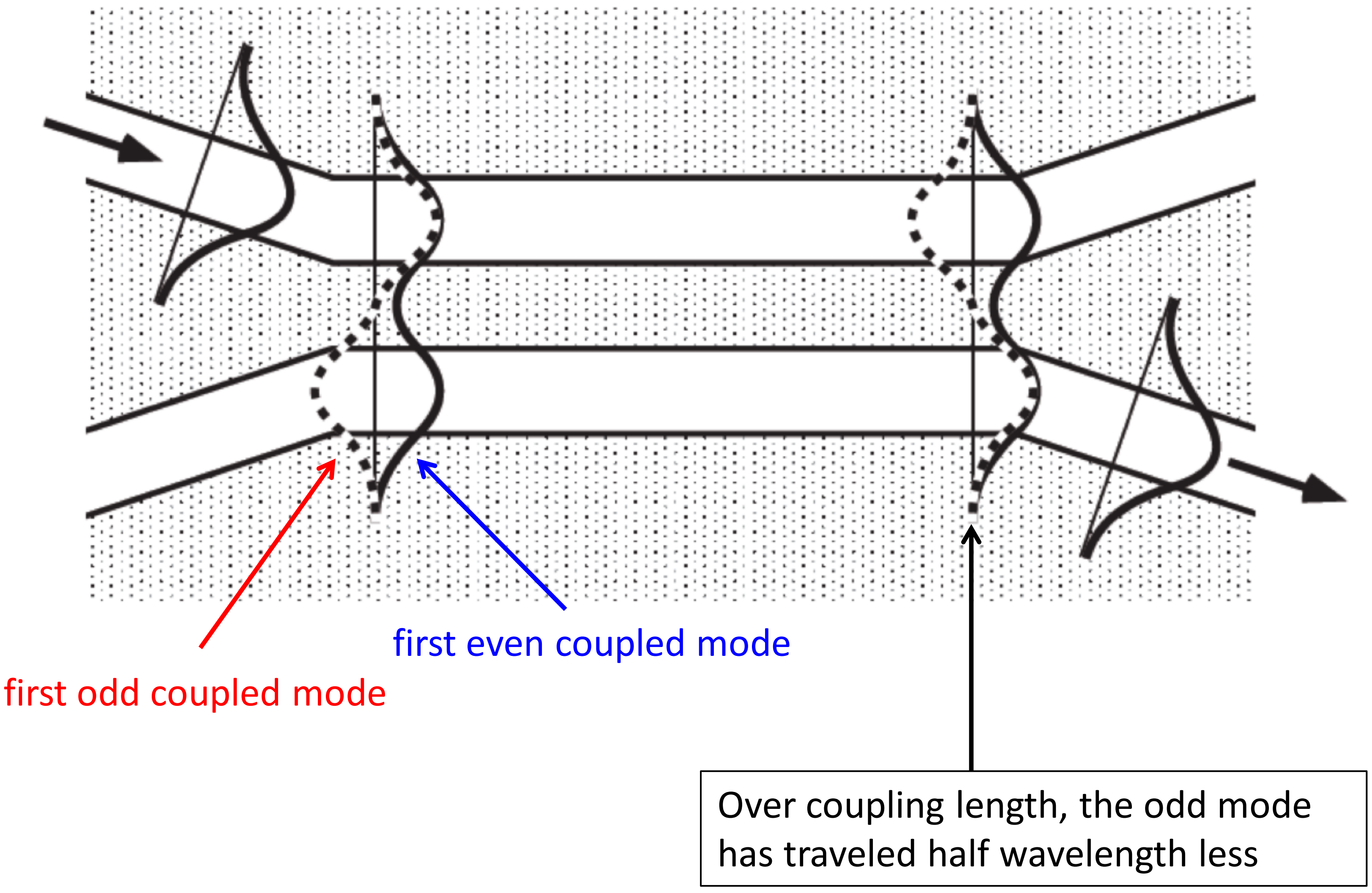




100% power transfer when $KL = \pi/2$, i.e. $L = \pi/2K$

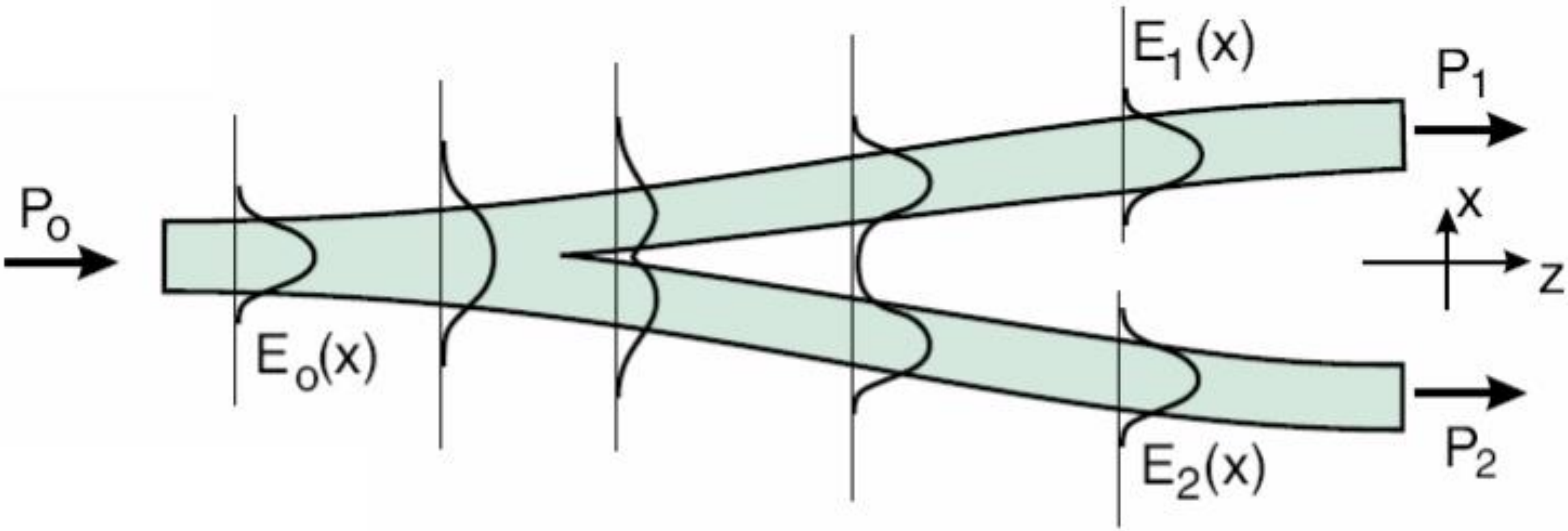
50:50 power split when $KL = \pi/4$, i.e. $L = \pi/4K$

Directional Coupler – Superposition of Modes



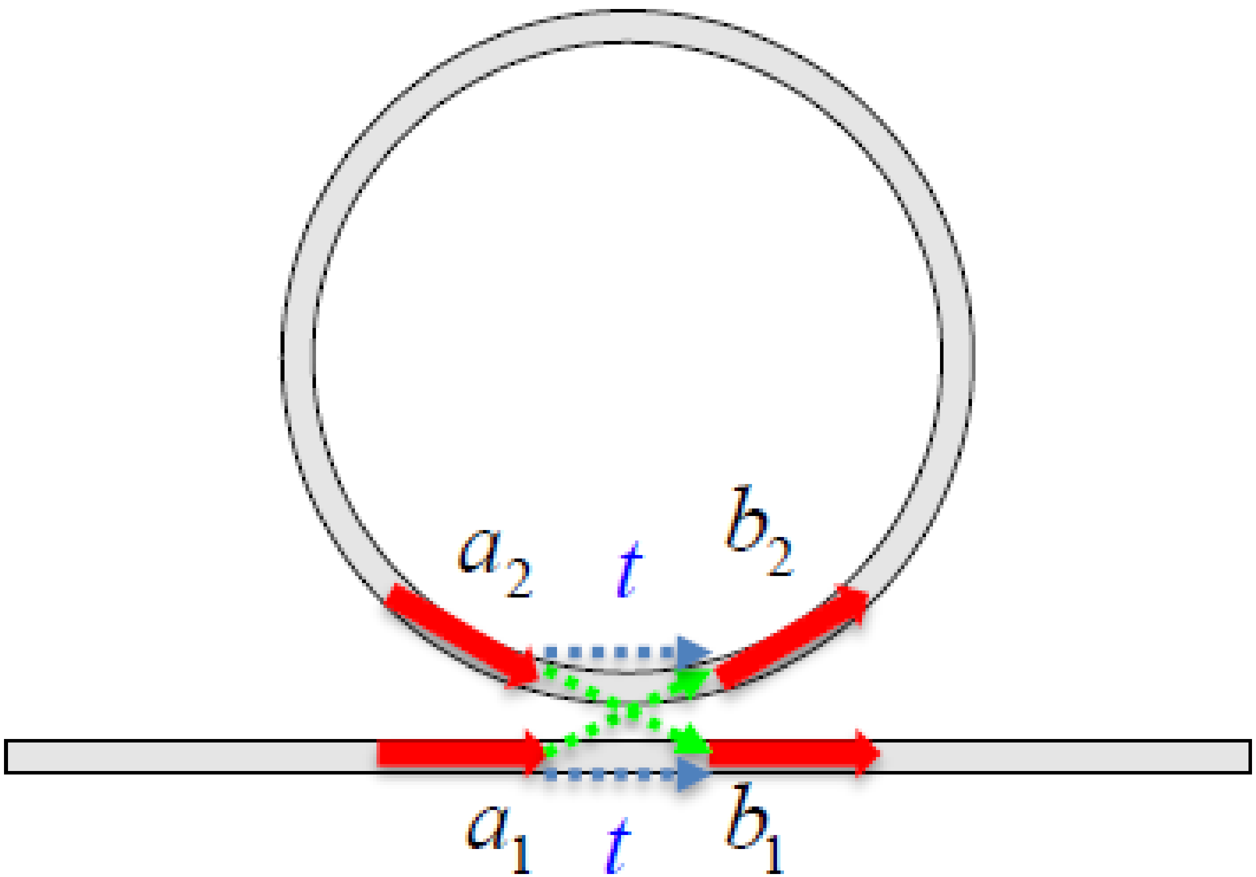
Coupling Methods

- Y-Coupler



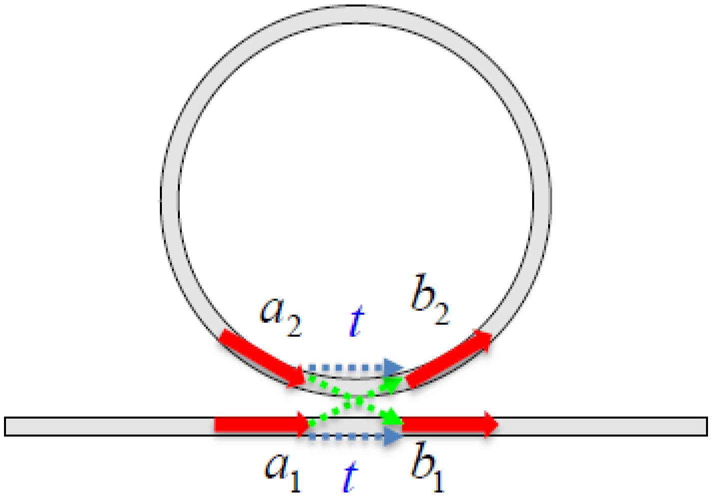
Coupling Methods

- Microring Resonator



Microring Resonator

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} t & i\kappa \\ i\kappa & t \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



For a lossless coupler, $t^2 + \kappa^2 = 1$.

Roundtrip condition: $a_2 = ae^{i\theta} b_2$

$$a = e^{-\alpha L/2}$$

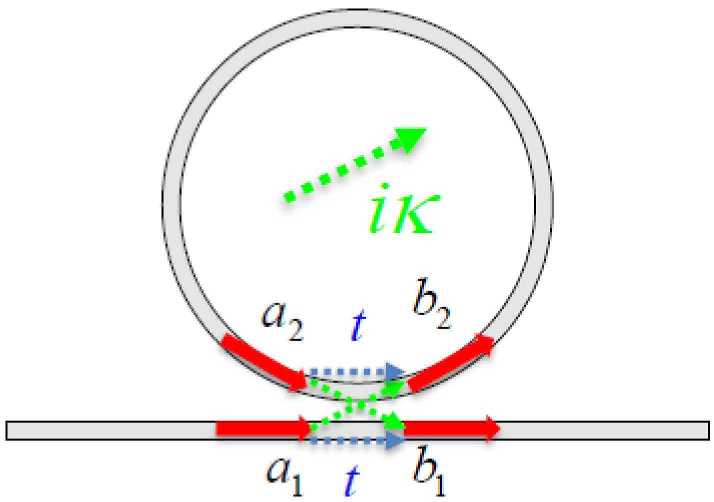
$$\theta = \frac{\omega}{c} n_{eff} L = \frac{2\pi n_{eff}}{\lambda} L$$

$$L = 2\pi r$$

Microring Resonator

General Solution

$$b_1 = \frac{t - ae^{i\theta}}{1 - ate^{i\theta}} a_1, \qquad b_2 = \frac{i\kappa}{1 - ate^{i\theta}} a_1.$$



Assuming t real

$$T = \left|\frac{b_1}{a_1}\right|^2 = \frac{t^2 + a^2 - 2at\cos(\theta)}{1 + (at)^2 - 2at\cos(\theta)}$$

At resonance, an integer number of wavelengths fit along the circumference ($L = m\frac{\lambda_m}{n_{eff}}$) and so: $\theta = 2\pi m$.

Free Spectral Range

$$FSR = \frac{c}{n_{eff}L}$$

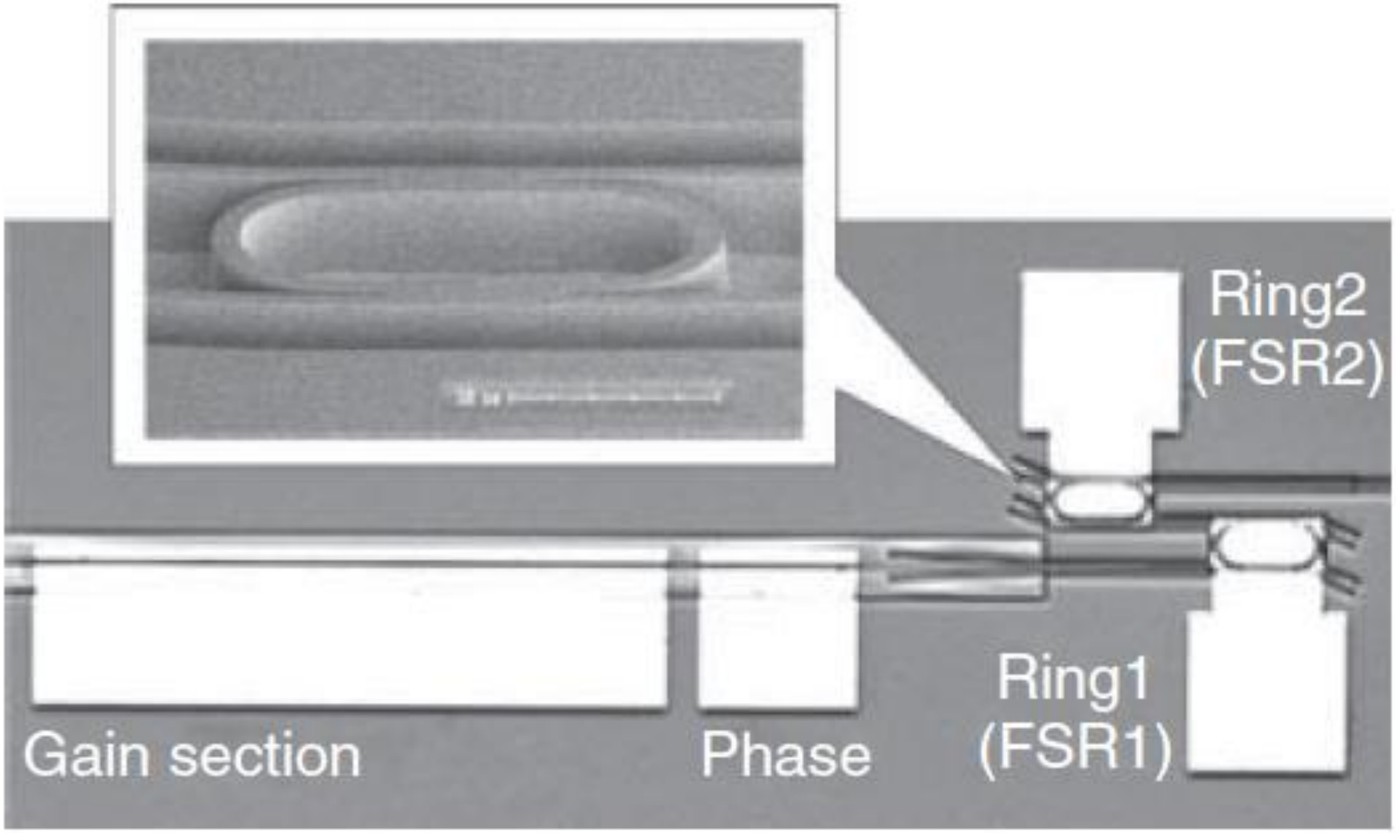
In Fabry-Pérot cavity

$$FSR = \frac{c}{n_{eff}2L}$$

$$T_{resonance} = \left|\frac{b_1}{a_1}\right|^2 = \frac{t^2 + a^2 - 2at}{1 + (at)^2 - 2at} = \frac{(t-a)^2}{(1-at)^2}$$

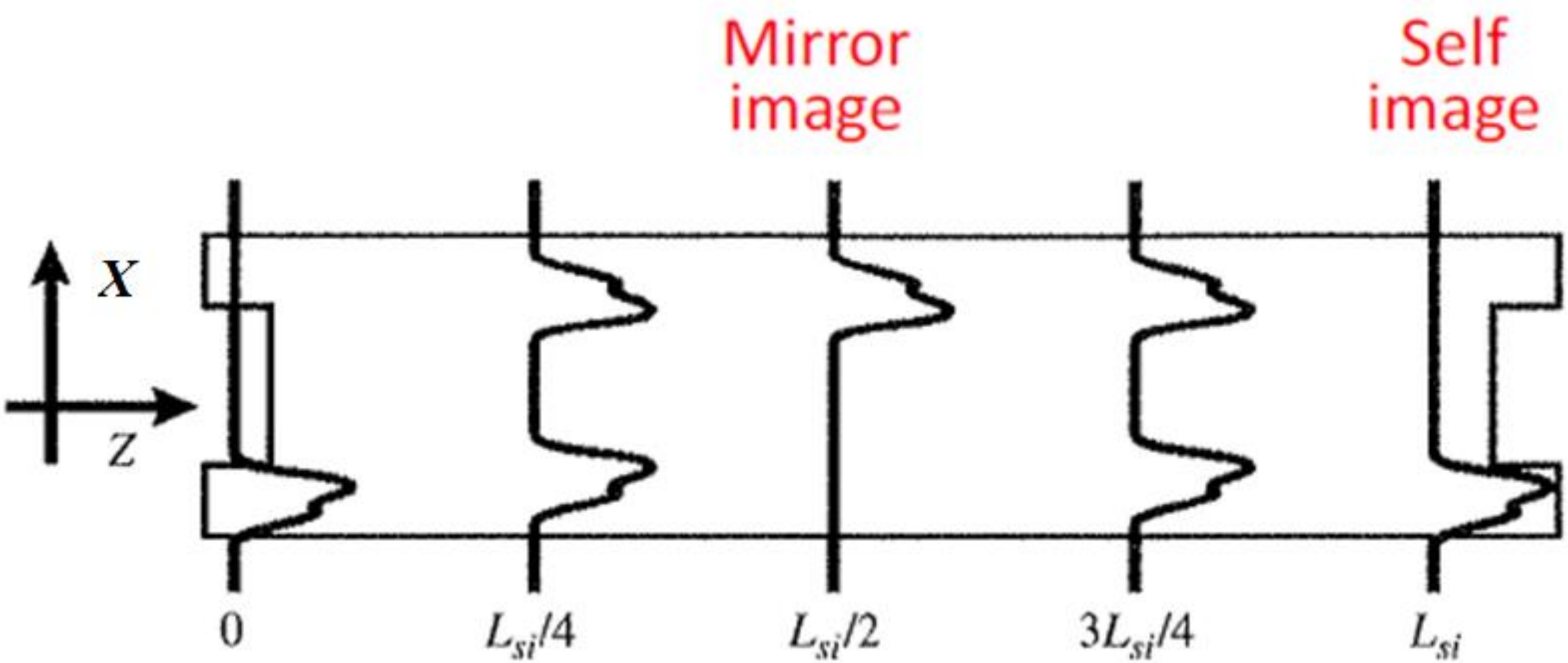
Coupling Methods

- Double Ring Resonator (DRR) Tunable Laser



Coupling Methods

- Multimode Interference (MMI) Coupler



Self-imaging distance:

$$L_{si} \approx \frac{8 n_l d_{eff}^2}{\lambda_0}$$

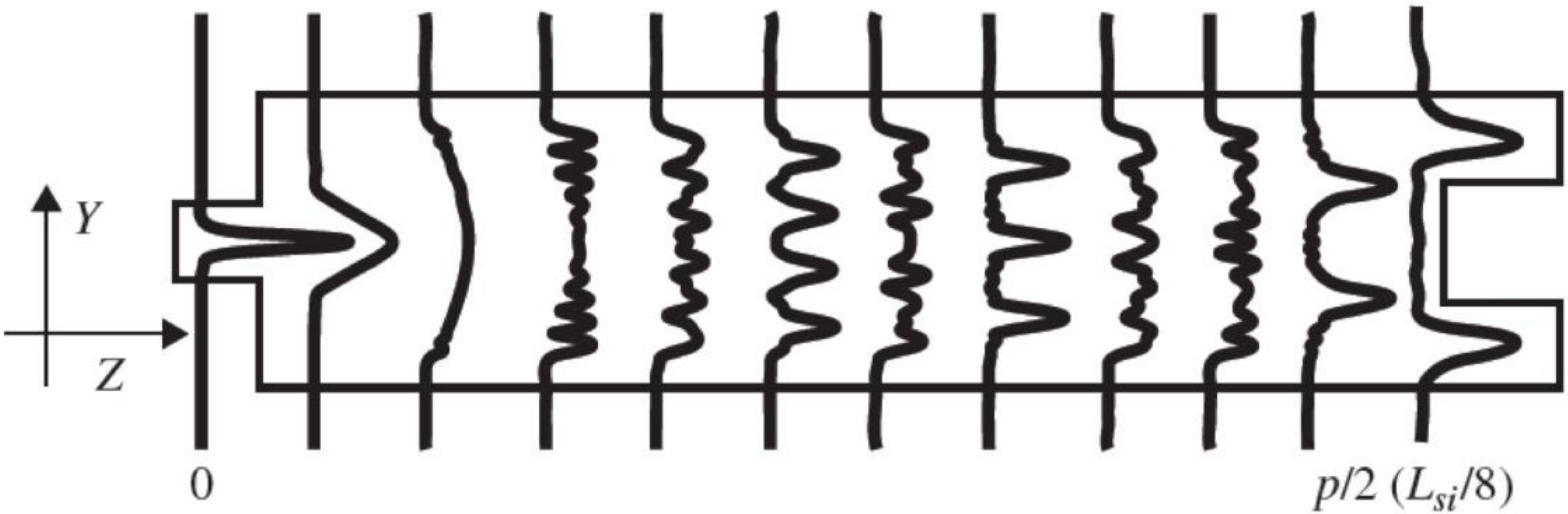
n_l = waveguide transverse effective index

$$k_{xm}^2 + \beta_m^2 = k_0^2 n_l^2$$

d_{eff} = effective width of fundamental mode

Coupling Methods

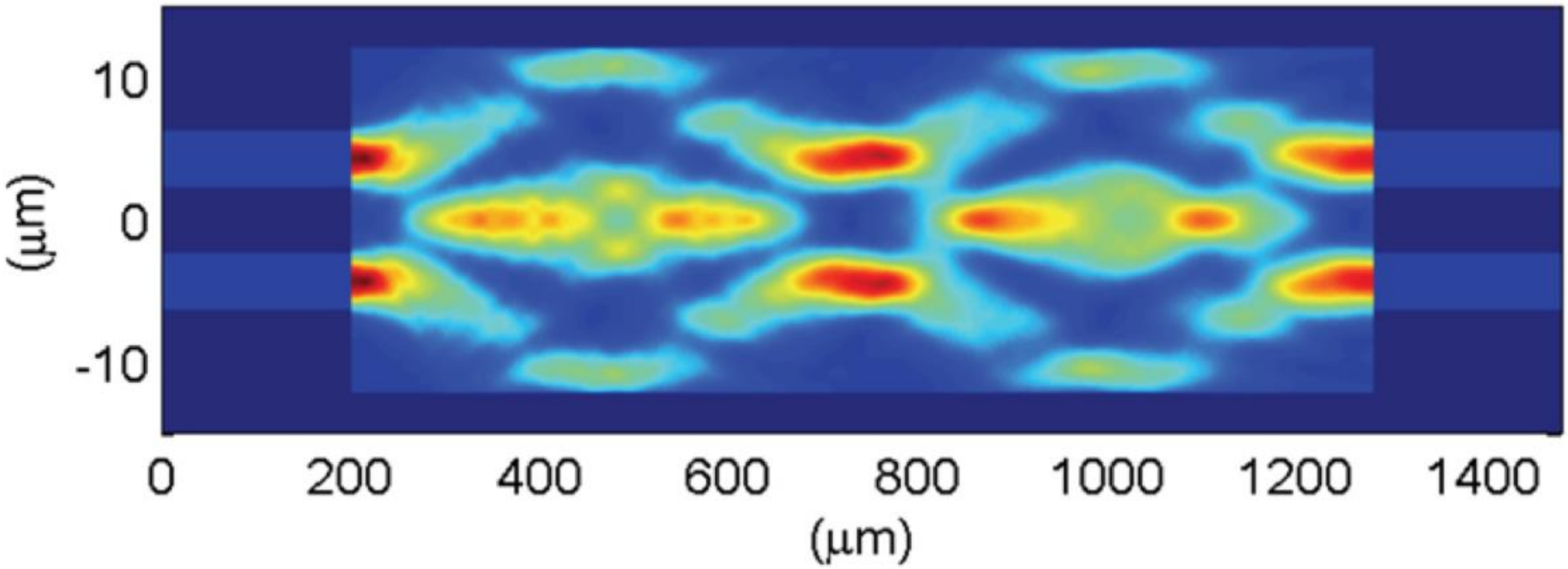
- MMI Splitter (1×2)



$p =$ integer

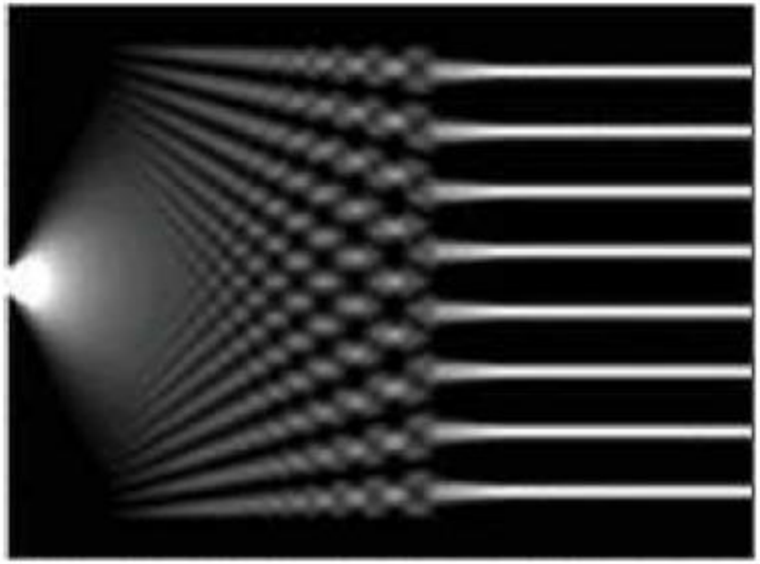
Coupling Methods

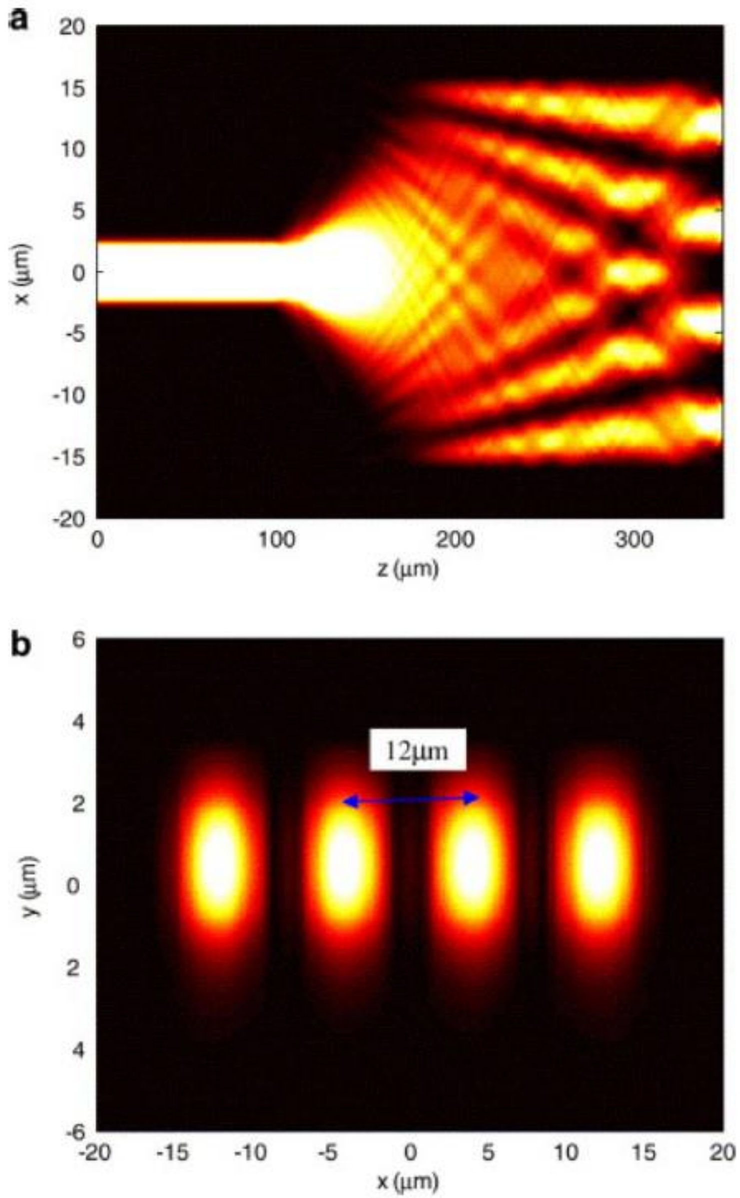
- MMI Splitter (2×2)



Coupling Methods

- MMI Splitter (1 × N)

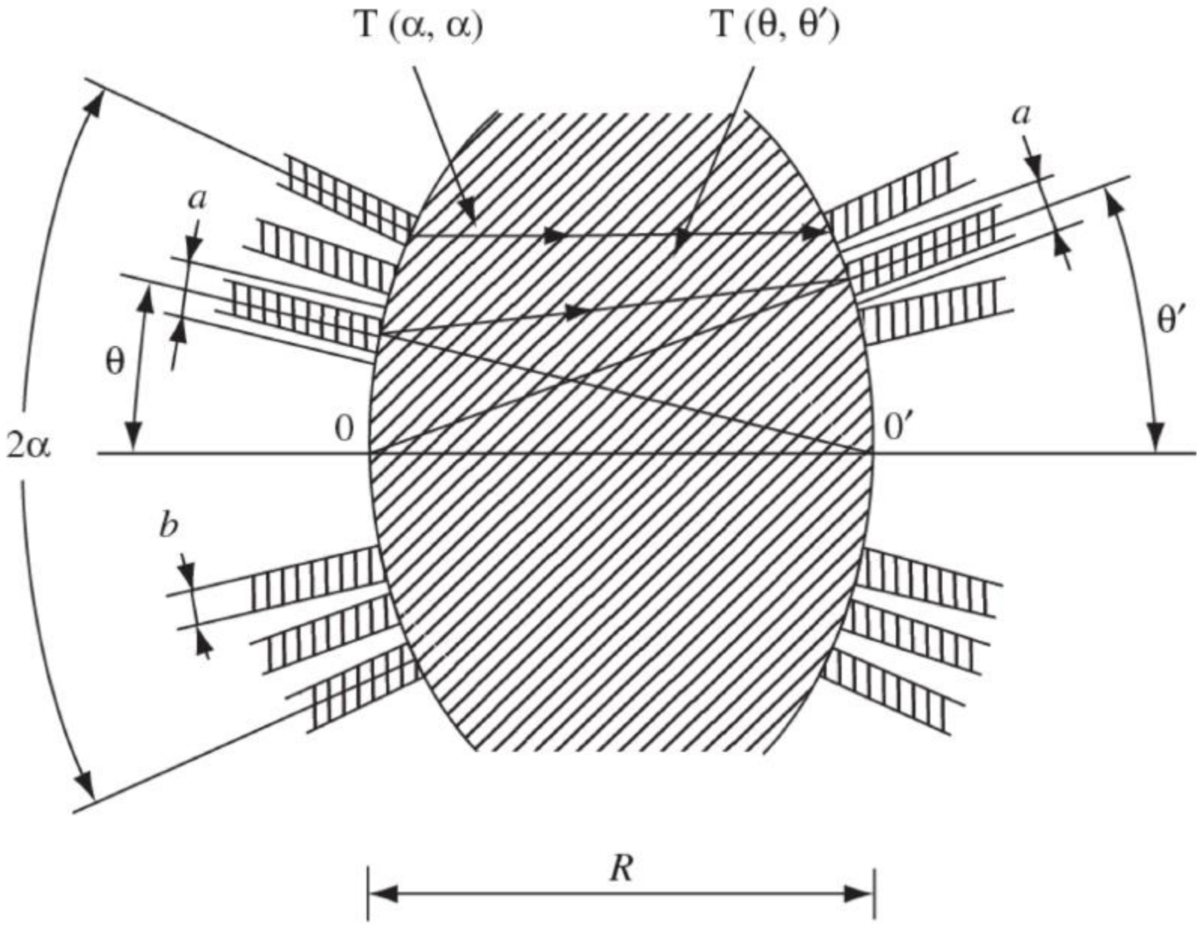




Coupling Methods

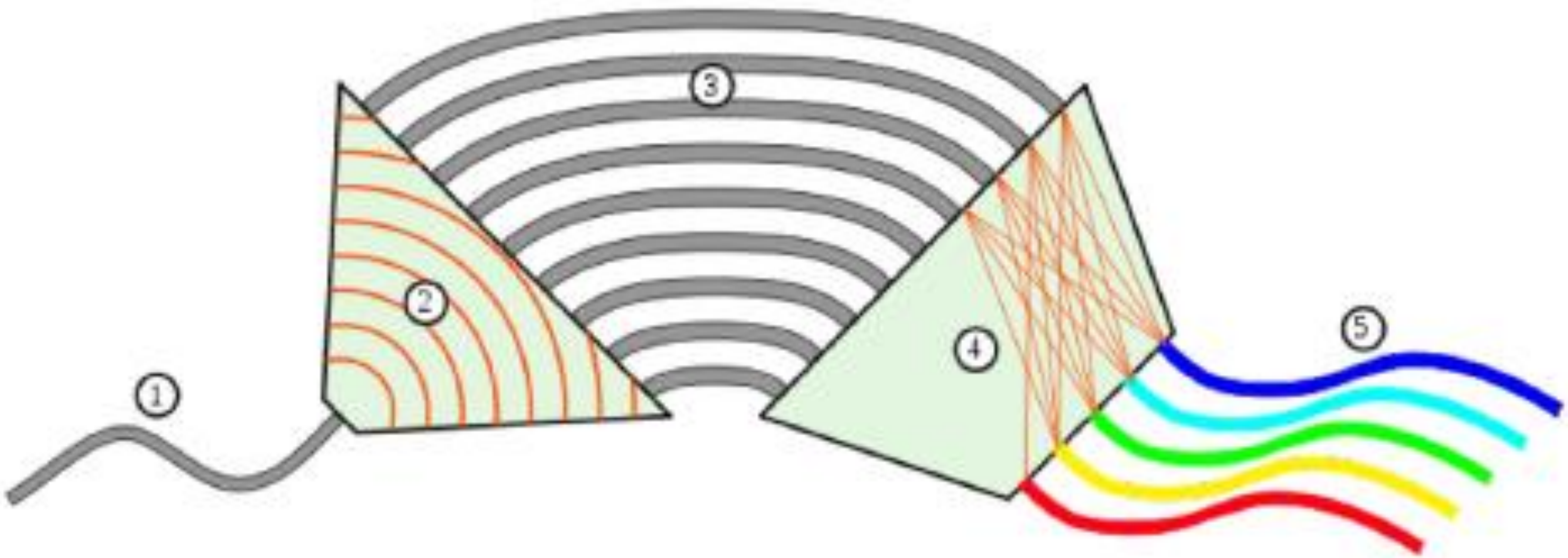
- Star Coupler (N × M)

  Uses diffraction instead of modal interference



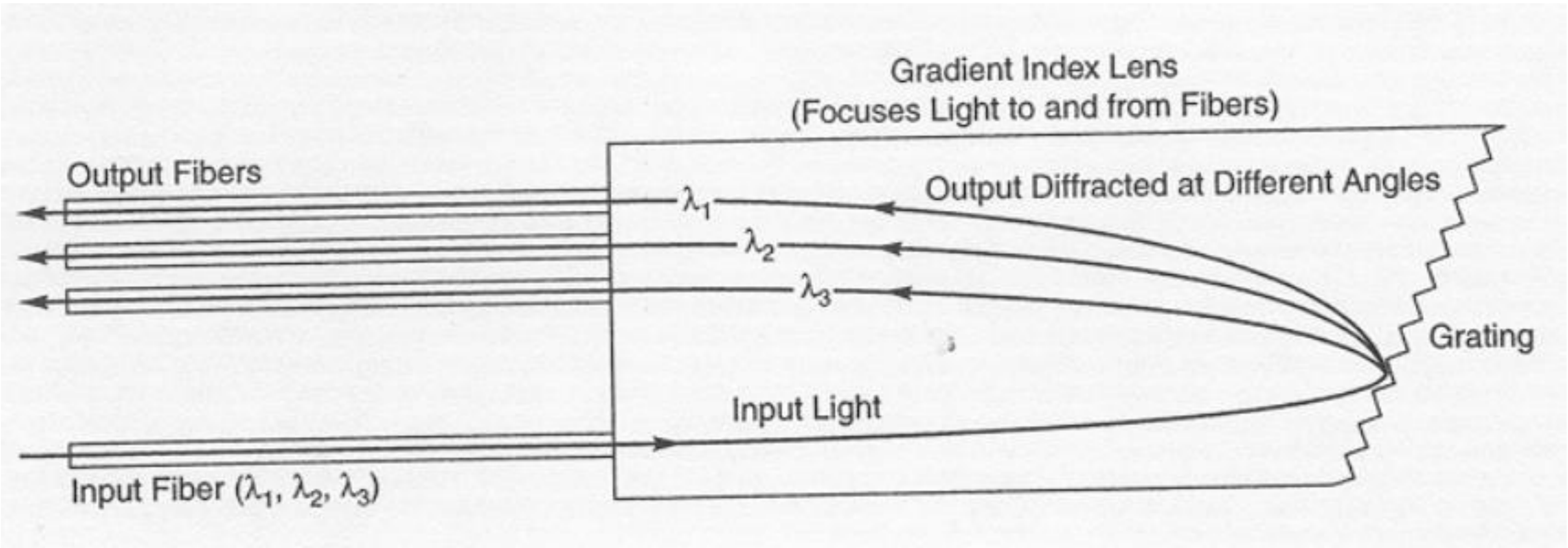
Coupling Methods

- Arrayed Waveguide Grating Devices



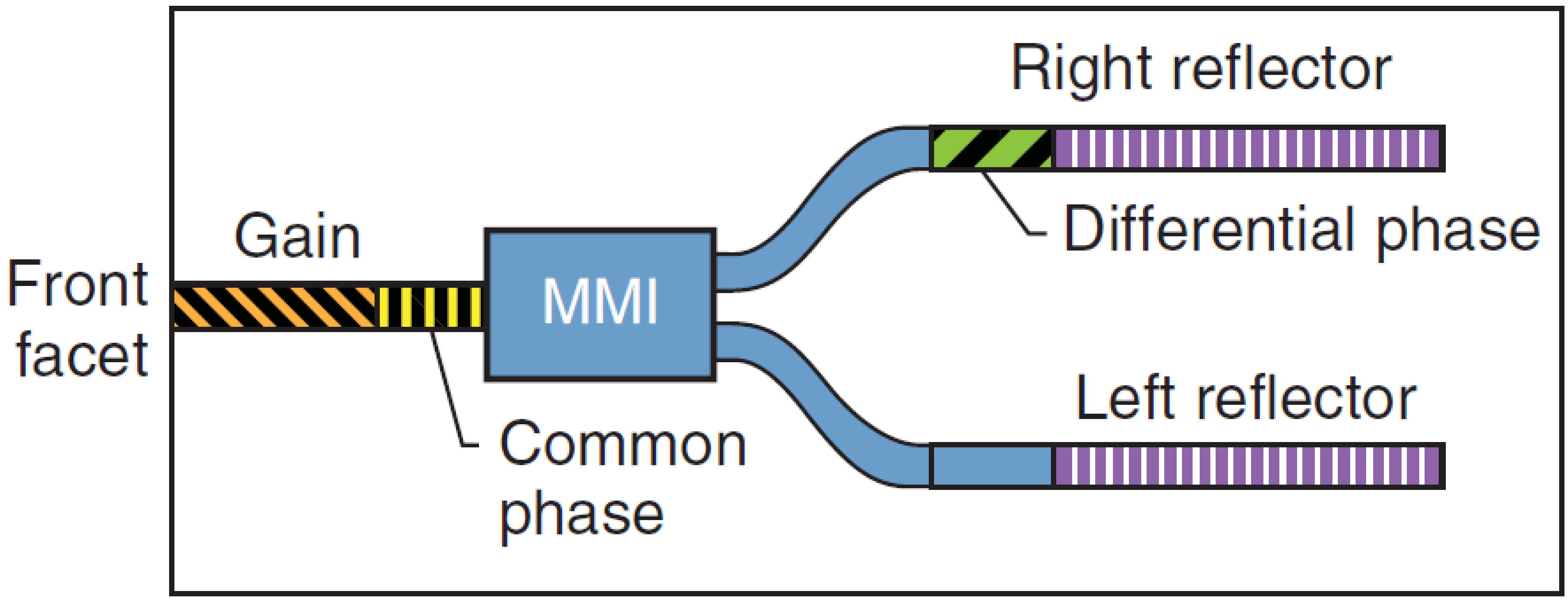
Coupling Methods

- Arrayed Waveguide Grating Devices



An advanced application example

- Modulated grating Y-branch tunable laser



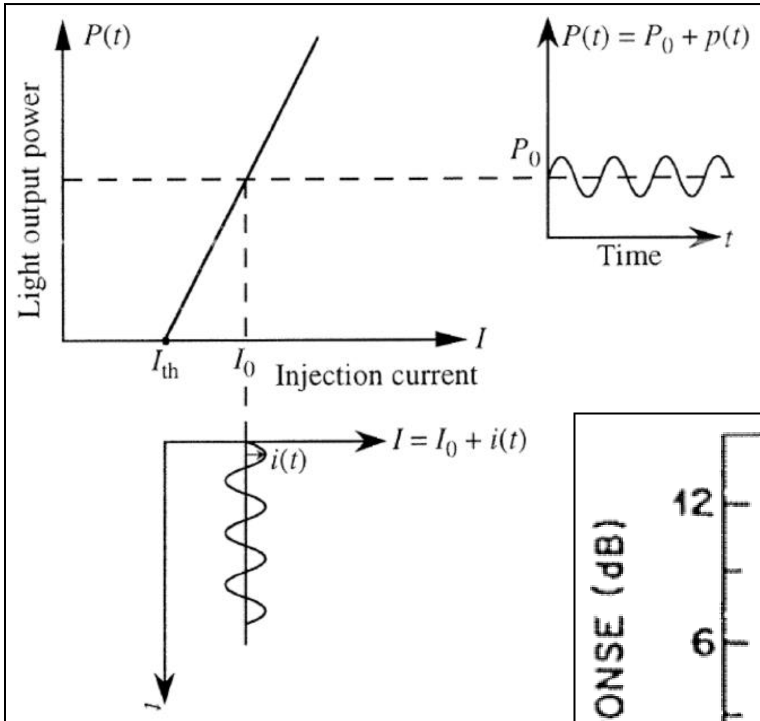
Mach-Zehnder interferometer based cavity

Light Modulation

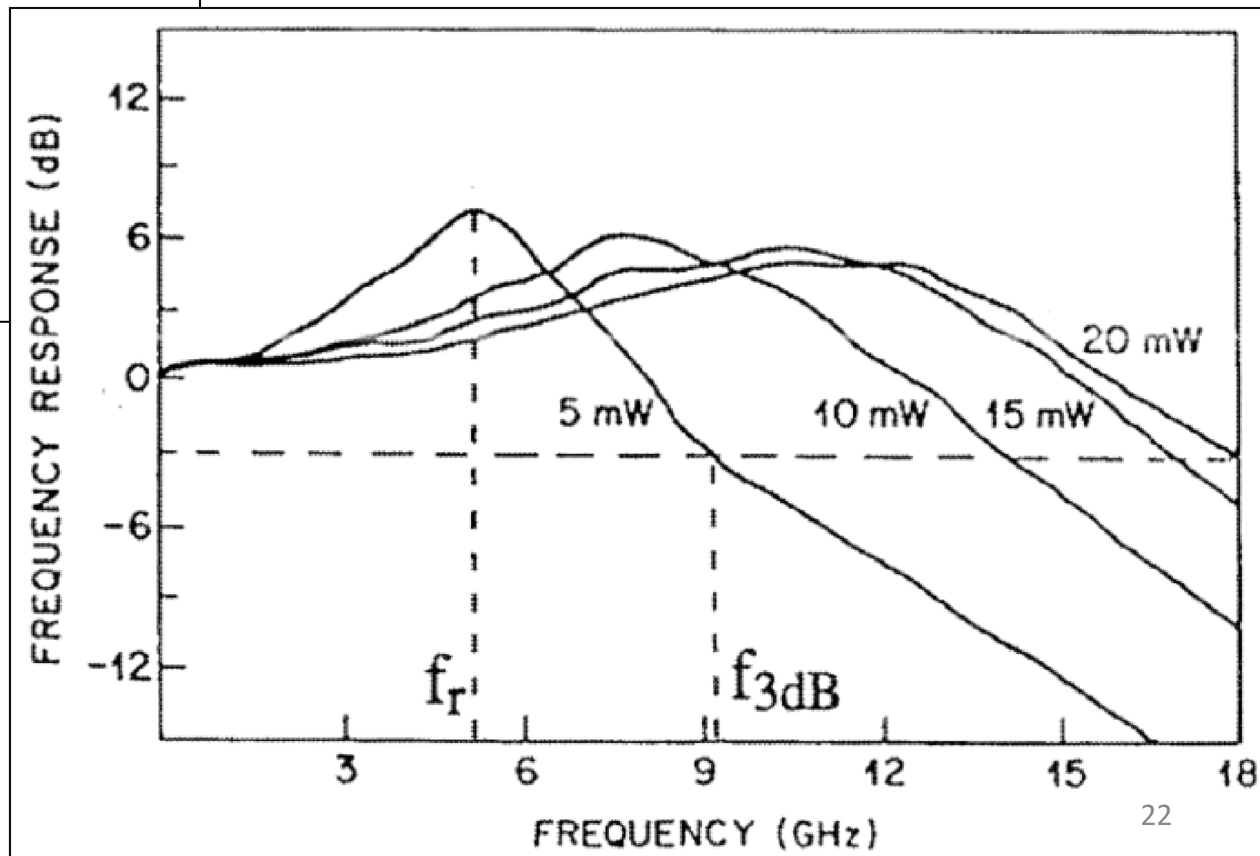
Modulators

- Laser diodes can be modulated **directly** via the applied bias.
- However, the continuous wave of the laser can also be modulated **externally**.
- With external modulation a relative less expensive laser can be used. The burden of encoding the information is placed on the modulator alone.
- Separation of the generation and modulation functions may make the system work better at the cost of added system complexity.

Direct Laser Modulation – Modulation Response



Although direct modulation has been for a long time the most common method, with the rapid increase of data rates in all applications, external modulators are becoming more important to overcome the limitations of practical devices, particularly in integrated optics applications.



It is difficult to modulate typical lasers above a few GHz and maintain single mode operation.

Direct phase modulation is also not practical.

External Modulators

Electro-optical modulators exploit the electro-optic effect in a **Pockel cell**. They can be used for modifying the polarization, **phase** or power of a beam, or for **pulse** picking in the context of **ultrashort pulse** **amplifiers**.

Acousto-optical modulators are based on the acousto-optic effect. They are used for switching or continuously adjusting the amplitude of a laser beam, for shifting its optical frequency, or its spatial direction.

External Modulators

Electroabsorption modulators are intensity modulators, used for instance to transmit data in **optical fiber communications**.

Interferometric *modulators*, e.g. Mach–Zehnder modulators, are often realized in **photonic integrated circuits** for **optical data transmission**.

Plasmonic modulators exploit the formation of plasmons at metal surfaces, which lead to surface plasmon polaritons (SPPs). They can be extremely fast with low energy consumption.

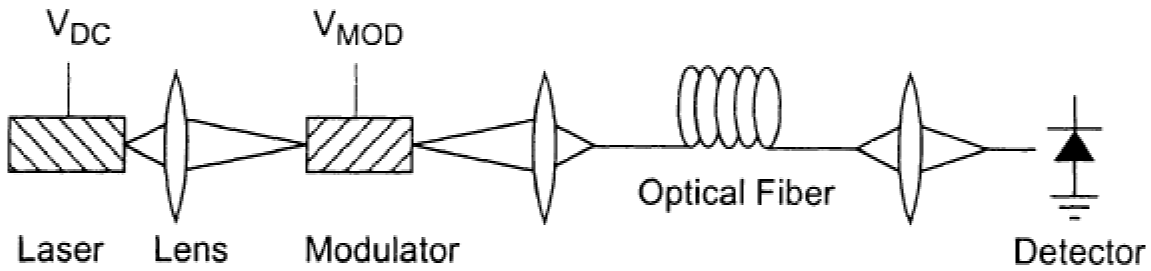
External Modulators

Liquid crystal modulators are suitable for optical displays and ultrafast **pulse shapers**.

Chopper wheels can switch periodically the **optical power** of a light beam, as required for certain optical measurements using a lock-in amplifier.

MEMS modulators are based on 2D arrays of micro-mirror, and are widely used for projection displays.

Figures of merit for an External Modulator



Modulation efficiency – The definition may depend on the form of modulation. For intensity modulation:

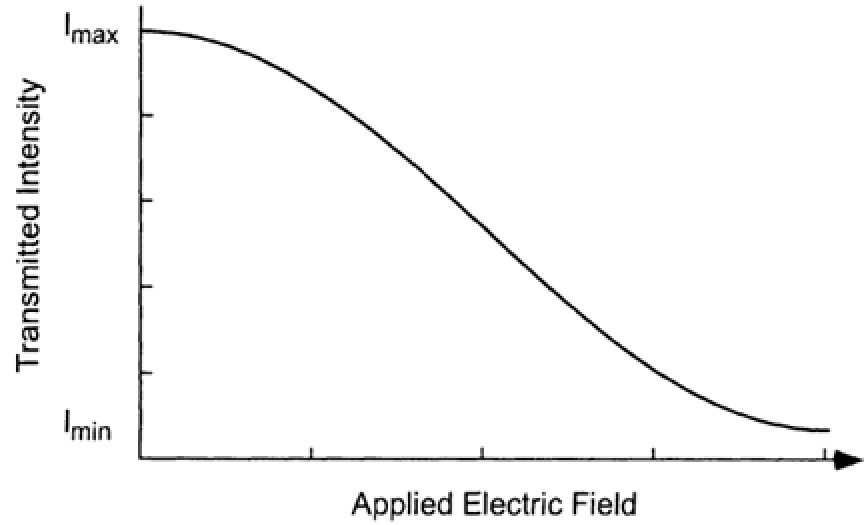
$$\eta = \frac{I_{max} - I_{min}}{I_{max}} \ (\times \ 100\%)$$

Contrast Ratio – It defines with decibels the modulation depth

$$\text{Contrast Ratio} = 10 \log \frac{I_{max}}{I_{min}}$$

For phase modulation, we use $\eta = \sin^2(\Delta\phi/2)$ **where** $\Delta\phi$ **is the extreme value of the modulation. This form describes the intensity contrast derived from an interferometric measurement of the phase shift.**

Figures of merit for a Modulator



Transmissions intensity of an electro-optic modulator is function of applied field.

Modulation bandwidth – It is defined by the frequencies where the the modulation index is reduced to 50% of its maximum value (3 dB). Bandwidth establishes the maximum information transfer rate for a modulator. If the switching time τ is defined instead (as the 10-to-90% rise time), the equivalent bandwidth is

$$\Delta\nu = \frac{0.35}{\tau} \quad \text{Hz}$$

Figures of merit for a Modulator

Insertion Loss – It describes the fraction of power lost when the modulator is placed in the system (it does not include additional losses induced by the modulator)

$$L = 10 \log \frac{P_{out}}{P_{in}}$$

P_{out} is the transmitted power of the system when the modulator is not in the beam path, P_{in} is the transmitted power when the modulator is inserted in the beam and is adjusted to provide maximum transmission.

Insertion loss is a passive loss, arising from reflection, absorption, and imperfect mode coupling between the modulator and the source. It must be compensated with either a higher power optical source, a more sensitive detector, or an optical amplifier.

Insertion loss provides isolation between the optical sources and reflection from the light destination, but it is not necessarily an efficient way to accomplish this isolation.

Figures of merit for a Modulator

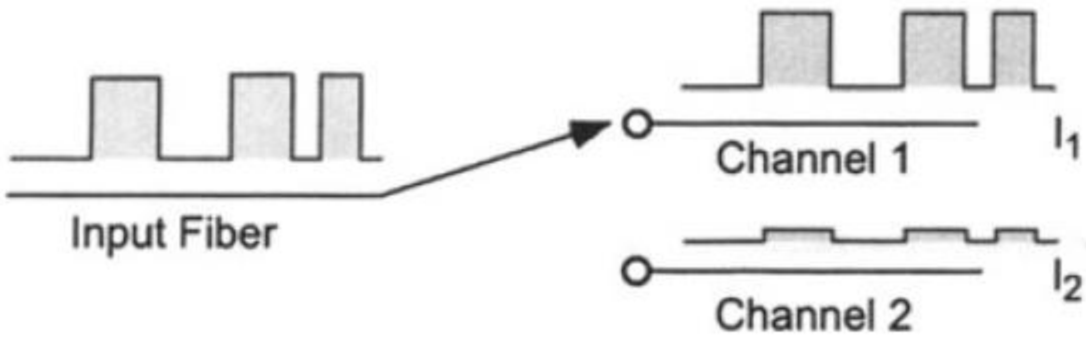
Power consumption – It is determined by the power per unit bandwidth required for intensity modulation. In the case of phase modulation it is the power per unit bandwidth per unit radian of modulation.

Power consumption depends on the physical properties of the material used but typically also on the volume needed to accomplish the function. A waveguide modulator with small effective volume has usually better power consumption that a bulk modulator.

Total power consumption determines how many devices can be put on a single substrate before thermal loading or power supply loading become a serious problem.

Figures of merit for a Modulator

Isolation – It describes how effectively a signal is isolated between two unconnected channels.



Isolation quantifies how much of the input signal shows up in the unconnected Channel 2, typically because of coupling due to evanescent fields, scattering, or unwanted reflections. Isolation is specified in decibels

$$\text{Isolation [dB]} = 10 \log \frac{I_2}{I_1}$$

A switch coupling 0.1% energy to the unconnected channel has 30 dB isolation. Local Area Networks, for instance, typically specify isolation in excess of 40 dB.

Electro-optic modulation

The Kerr effect was discovered in 1875. Optically isotropic materials become birefringent in a strong electric field, with the optic axis parallel to the applied field.

A similar but much weaker effect was discovered about 20 years later in certain crystals by Pockel. Isotropic crystals become uniaxial in the presence of electric fields. Uniaxial crystals become biaxial.

In the Kerr effect, polarization depends quadratically on field. Instead, the Pockel effect is linear and is better suited for modulation applications.

We are very familiar with the fact that travel of light waves in a material depends on the index of refraction. In anisotropic crystals $\mathbf{D}_i = \epsilon_{ij}\mathbf{E}_j$ **where the subscripts refer to cartesian coordinates and we nave now a dielectric tensor. Since we have from energy conservation arguments that** $\epsilon_{ij} = \epsilon_{ji}$ **there are six possible values for tensor elements** ϵ_{ij} **.**

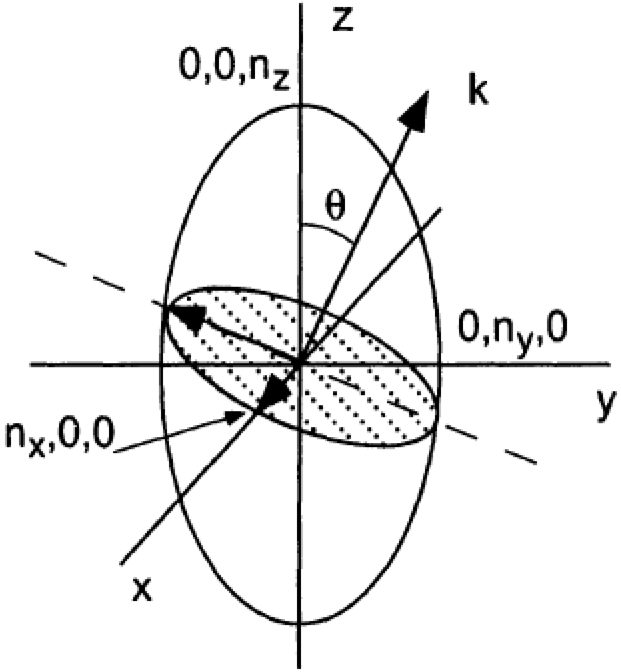
Electro-optic modulation

The index of refraction is defined by an ellipsoid in terms of principal dielectric axes (which are orthogonal for many crystals but not always)

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

index ellipsoid

The distance from the origin to the surface of the ellipse is equal to the index of refraction for an electric field polarized in that direction.



Electro-optic modulation

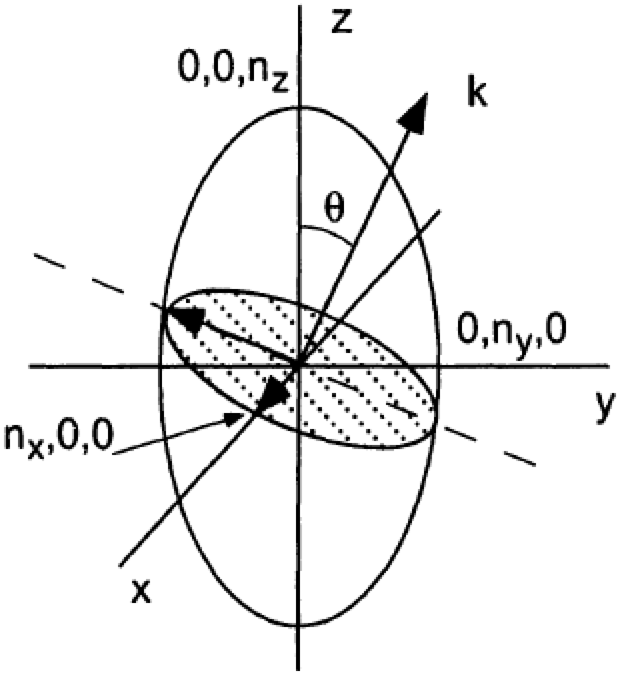
The index of refraction experienced by the wave depends on the orientation of the polarization. A special case occurs if the field is polarized along one of the principal axes.

Consider the case of a wave polarized in the *x*-direction, travelling in the *yz* plane.

As the angle θ varies, the width in the *x*-direction remains constant, indicating that the index n_x is independent of angle θ. This is called the ordinary wave.

If the polarization lies in the *yz* plane, then the index depends on the angle θ, ranging from n_y when θ = 0, to n_z when θ = 90°. This is the extraordinary wave with index expressed by

$$\frac{1}{n_{ext}^2} = \frac{\cos^2\theta}{n_y^2} + \frac{\sin^2\theta}{n_z^2}$$



Electro-optic modulation

A plane wave in a crystal has two polarization eigenstates, corresponding to polarization that does not change as it propagates. **If the field is linearly polarized along either axis of the corresponding ellipse, it will remain linearly polarized.**

Fields with polarizations that do not lie along the major or minor axis will not remain linearly polarized. The electric field will be decomposed into two linearly polarized components oriented along each axis. These components will travel separately accumulating phase according to the index of the axis. **The general field will be elliptically polarized as it propagates through the crystal.**

Uniaxial crystal = index of refraction is identical along two axes.

Examples: quartz, sapphire

Biaxial crystal = has three unique indices of refraction

Examples: calcite, tourmaline, forsterite

Pockel's Effect

The linear electro-optic effect, or Pockel's effect, is the change in the index of refraction that occurs when an external electric field is applied to a crystal. The Pockel effect makes it possible for an electric field to alter the index of refraction of a material and directly modulate the phase, the intensity, or the polarization of the light.

The magnitude of the change is critically dependent on the orientation of the electric field and crystal.

Example: Electro-optic tensor of GaAs

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Electro-optic tensor

$$r_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

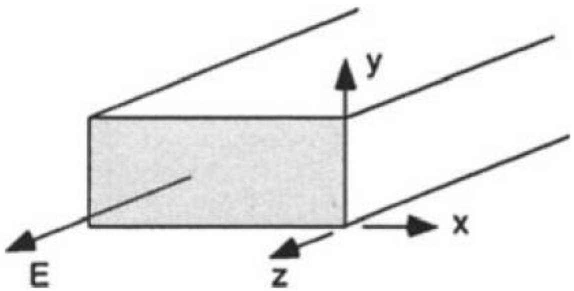
Coefficients are zero in directions of inversion symmetry

Linear Electro-optic Coefficients for Some Relevant Crystals

Material	Symmetry	Wavelength (μm)	Electro-optic coefficient (10^{-12} m/V)	Index of Refraction
$LiNbO_3$	3m	0.632	$r_{13} = 9.6$	$n_0 = 1.8830$
			$r_{22} = 6.8$	$n_e = 1.7367$
			$r_{33} = 30.9$	
			$r_{51} = 32.6$	
$LiIO_3$	6	0.633	$r_{13} = 4.1$	$n_0 = 1.8830$
			$r_{41} = 1.4$	$n_e = 1.7376$
GaAs	$\bar{4}3m$	0.9	$r_{41} = 1.1$	$n = 3.60$
		1.15	$r_{41} = 1.43$	
KDP	$\bar{4}2m$	0.633	$r_{63} = 11$	$n_o = 1.5074$
			$r_{41} = 8$	$n_e = 1.4669$
ADP	$\bar{4}2m$	0.633	$r_{63} = 8.5$	$n_o = 1.52$
			$r_{41} = 28$	$n_e = 1.48$
Quartz	32	≈ 0.632	$r_{41} = 0.2$	$n_0 = 1.54$
			$r_{63} = 0.93$	$n_e = 1.55$
$BaTiO_3$	$4mm$	≈ 0.632	$r_{33} = 23$	$n_0 = 2.437$
			$r_{13} = 8$	$n_e = 2.180$
			$r_{42} = 820$	
$LiTaO_3$	$3m$	≈ 0.632	$r_{33} = 30.3$	$n_0 = 2.175$
			$r_{13} = 5.7$	$n_e = 2.365$

Example – Electro-optic effect in GaAs

Consider the effect of an electric field along the propagation direction of a waveguide oriented along the [001] axis of the crystal

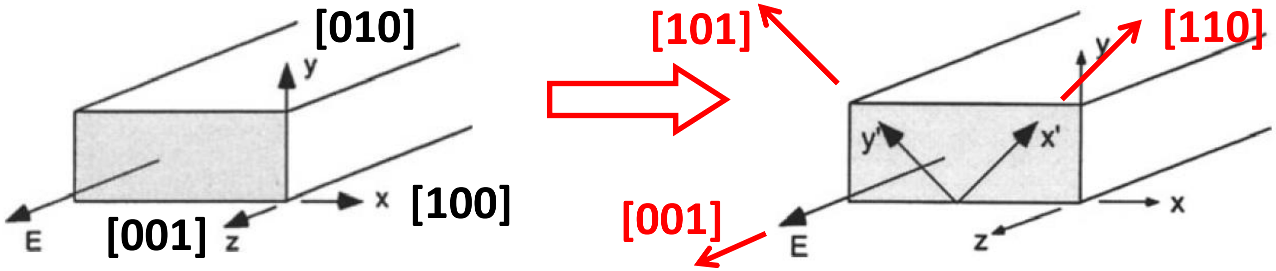


In the Pockel effect, it is convenient to use the general modified index ellipsoid

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 2yz + \left(\frac{1}{n^2}\right)_5 2xz + \left(\frac{1}{n^2}\right)_6 2xy = 1$$

The electro-optic interaction for GaAs can be written as

$$\frac{x^2}{n^2} + \frac{y^2}{n^2} + \frac{z^2}{n^2} + 2r_{41}E_x yz + 2r_{41}E_y xz + 2r_{41}E_z xy = 1$$



For a z-directed field, the expression reduces to

$$\frac{x^2+y^2+z^2}{n^2} + 2r_{41}E_z xy = 1$$

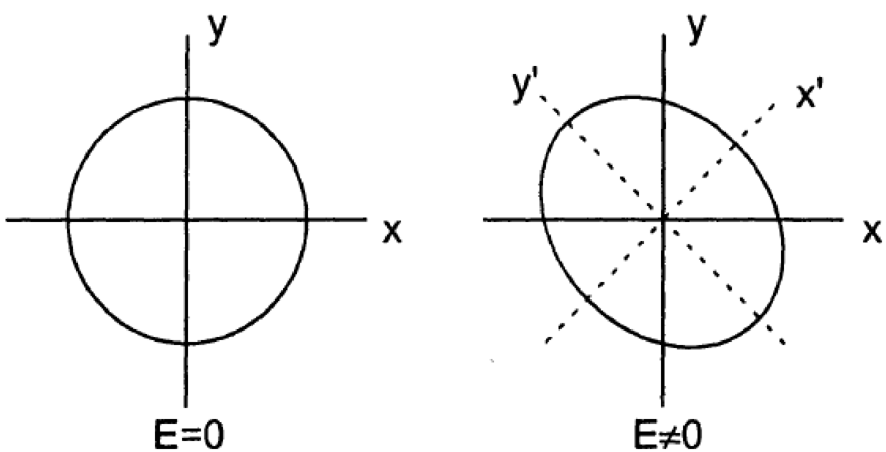
$$\begin{aligned} x &= x'\cos 45^\circ + y'\sin 45^\circ \\ y &= -x'\sin 45^\circ + y'\cos 45^\circ \end{aligned}$$

transformed equation

$$\underbrace{\left(\frac{1}{n^2} - r_{14}E_z\right)}\, x'^2 + \underbrace{\left(\frac{1}{n^2} + r_{14}E_z\right)}\, y'^2 + \frac{z^2}{n^2} = 1$$

give effective indices of refraction

$$\boxed{\begin{aligned} \frac{1}{n_{x'}^2} &= \frac{1}{n^2} - r_{14}E_z \\ \frac{1}{n_{y'}^2} &= \frac{1}{n^2} + r_{14}E_z \end{aligned}}$$



The ellipsoid changes from a circle when no field is present, to an ellipse rotated 45° from the *x* and *y* axes when the z-directed field is applied. The **coupled term can be removed** (diagonalized) by finding a new coordinate system that lies parallel to the principal axes, *x'* and *y'* .

If the magnitude of $r_{14}E_z$ **is small compared to** n^2

$$\frac{1}{n_{x'}^2} = \frac{1}{n^2} - r_{14}E_z$$

$$\frac{1}{n_{y'}^2} = \frac{1}{n^2} + r_{14}E_z$$

$$n_{x'} = n\left(1 - n^2 r_{14}E_z\right)^{-1/2}$$

$$n_{y'} = n\left(1 + n^2 r_{14}E_z\right)^{-1/2}$$

$$n_{x'} \approx n\left(1 + \frac{n^2 r_{14}E_z}{2}\right)$$

$$n_{y'} \approx n\left(1 - \frac{n^2 r_{14}E_z}{2}\right)$$

$$\boxed{n_{x'} \approx n + \frac{n^3 r_{14}E_z}{2}}$$

$$\boxed{n_z = n}$$

unchanged

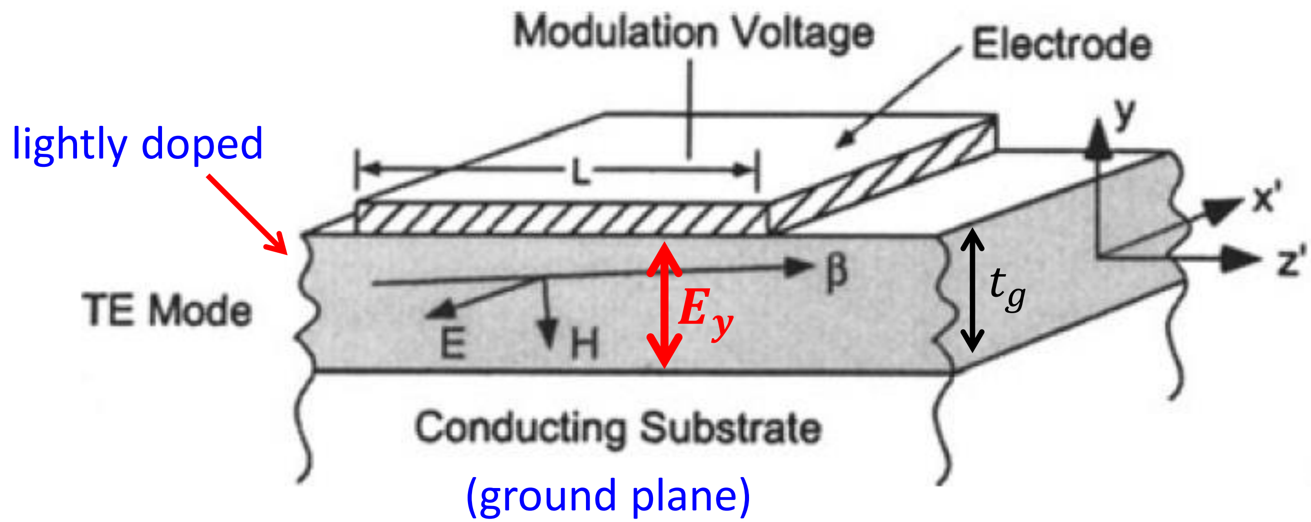
$$\boxed{n_{y'} \approx n - \frac{n^3 r_{14}E_z}{2}}$$

$$\Delta n_{x'} \approx \frac{n^3 r_{14}E_z}{2}$$

$$\Delta n_{y'} \approx -\frac{n^3 r_{14}E_z}{2}$$

Pockel effect – Phase modulation in GaAs waveguide

Applying an electric field along any single axis will alter the indices along the remaining axes. We apply the electric field along y direction and the waveguide axis along z' oriented in the [101] direction.



An applied field causes the index to change in the z' and x' directions. The change in the z' direction has no effect. The TE mode is polarized along the x' directions so the electric field feels changes in the index n'_x. The TM mode has mainly electric field component in the y direction which is unaffected, and a component along z' which is very small in the weakly guided mode approximation with negligible modulation effect.

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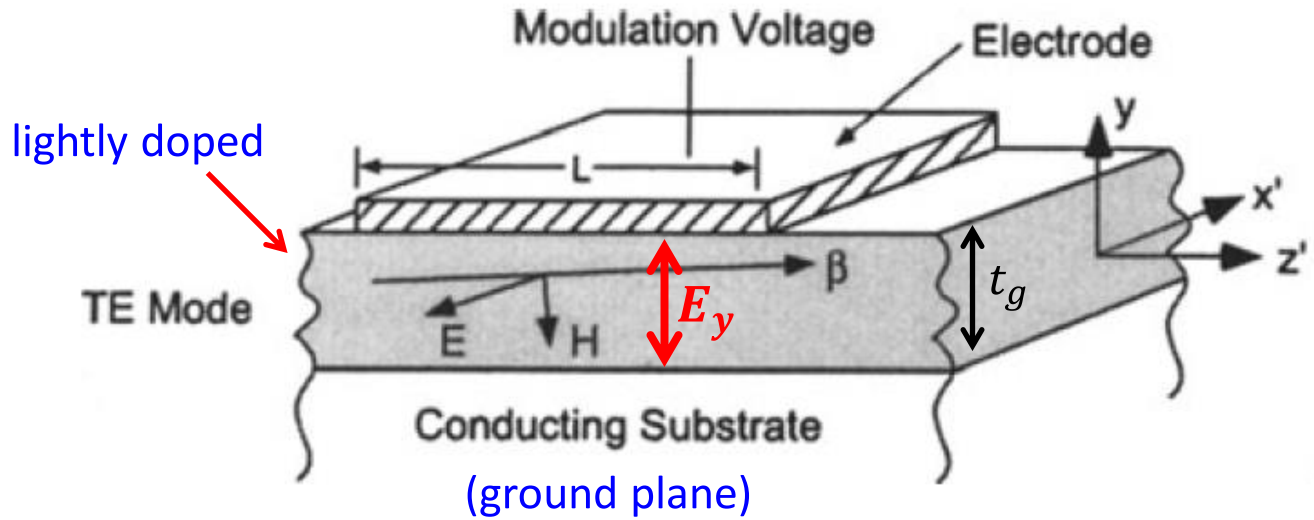

$$E_y = \frac{V_0}{t_g}$$

The TE wave experience approximately a total phase shift over distance L

$$\Delta\phi = \Delta\beta L = k_0\,\Delta n'_x \times L = \frac{2\pi}{\lambda}\frac{Ln^3 r_{14} E_y}{2}$$

To achieve a phase modulation $\pi/2$ (ignoring a mode confinement factor)

$$E_{\pi/2} = \frac{\lambda}{2}\frac{1}{Ln^3 r_{14}} \qquad V_{\pi/2} = \frac{\lambda}{2}\frac{t_g}{L\,n^3 r_{14}}$$

The longer L, the lower the required voltage to achieve the modulation.

Limitations of this phase modulator

Bandwidth is limited by the capacitance of the electrode (GaAs has a fairly large DC dielectric constant)

Modulation only works for the TE mode. If connected to a circular fiber where polarization may not be controllable, both TE and TM would be excited in the modulator with reduction of the total modulation efficiency.

Whenever the index of refraction of a material is modified, the imaginary component of the index also changes (remember the Kramers-Kronig relations) causing a change of the intrinsic attenuation of the waveguide. An applied electric field will in practice modulate both the phase and intensity of the transmitted light, which is like an additional source of noise.

Fortunately the magnitude of imaginary component change is small and decreases as the wavelength gets further from the absorption edge of the material. Proper selection of material and operating wavelength can reduce the importance of this problem.

Reading Assignments:

- Section 6.6 and Appendix 15 of Coldren, Corzine
- Section 12.1 of Chuang’s book
- Chapter 8 of Chuang’s book
- Chapter 13 of Chuang’s book