ECE 536 – Integrated Optics and Optoelectronics
Lecture 27 – April 26, 2022

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Tu-Th 11:00am-12:20pm

Prof. Umberto Ravaioli

ECE Department, University of Illinois

Lecture 27 Outline

- Modulators

Electro-optic modulation

The Kerr effect was discovered in 1875. Optically isotropic materials become birefringent in a strong electric field, with the optic axis parallel to the applied field.

A similar but much weaker effect was discovered about 20 years later in certain crystals by Pockel. Isotropic crystals become uniaxial in the presence of electric fields. Uniaxial crystals become biaxial.

In the Kerr effect, polarization depends quadratically on the field. Instead, the Pockel effect is linear and is better suited for modulation applications.

Anisotropic crystal

The displacement vector and the electric field are not necessarily parallel and are related by the following expression, where *i* and *j* refer to Cartesian coordinates

$$\mathbf{D}_i = \epsilon_{ij}\mathbf{E}_j$$

Through energy conservation arguments, it can be shown (e.g., Yariv, *Quantum Electronics* [1989]) that

$$\epsilon_{ij} = \epsilon_{ji}$$

meaning that there are only six possible values of ϵ_{ij} **.**

Propagation in anisotropic crystal

The Poynting vector travels perpendicular to $\mathbf{E}$ and $\mathbf{H}$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Maxwell's equations for a single frequency plane wave

$$\begin{aligned} \mathbf{k} \times \mathbf{E} &= \omega\mu_0\mathbf{H} \\ \mathbf{k} \times \mathbf{H} &= -\omega\mathbf{D} \end{aligned}$$

Not necessarily = 0

Eliminating $\mathbf{H}$ one obtains

$$\mu_0\omega^2\mathbf{D} = k^2\mathbf{E} - (\mathbf{k} \cdot \mathbf{E})\mathbf{k}$$

Since power flows perpendicular to $\mathbf{E}$ then power may not travel in the same direction as $\mathbf{k}$. This is the cause of distortions seen in double-refracting crystals.

Index Ellipsoid 1

The stored electric energy in the medium is

$$w = \frac{1}{2}\mathbf{E}\cdot\mathbf{D} = \frac{1}{2}E_i\epsilon_{ij}E_j$$

Dielectric tensor

In Cartesian coordinates, expansion of the stored energy terms yields

$$2w = \epsilon_{xx}E_x^2 + \epsilon_{yy}E_y^2 + \epsilon_{zz}E_z^2 + 2\epsilon_{yz}E_yE_z + 2\epsilon_{xz}E_xE_z + 2\epsilon_{xy}E_xE_y$$

Simplifications are possible by using the principal dielectric axes, which depend on the crystal structure.

Index Ellipsoid 2

Many crystals lie along the familiar x, y, z axes, but other lie in non-orthogonal directions. The principal axes are the orientations where an applied electric field E produces a parallel displacement vector D, and are found by diagonalizing the dielectric tensor.

In terms of principal axes x', y', z' the energy is defined as

$$2w = \epsilon_{x'} E_{x'}^2 + \epsilon_{y'} E_{y'}^2 + \epsilon_{z'} E_{z'}^2$$

and

$$2w\epsilon_0 = \frac{D_{x'}^2}{\epsilon_{x'}/\epsilon_0} + \frac{D_{y'}^2}{\epsilon_{y'}/\epsilon_0} + \frac{D_{z'}}{\epsilon_{z'}/\epsilon_0}$$

Using $\boxed{\mathbf{D}/\sqrt{2\omega\epsilon_0} = \mathbf{r}}$

$\boxed{\textbf{Index Ellipsoid}}$

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

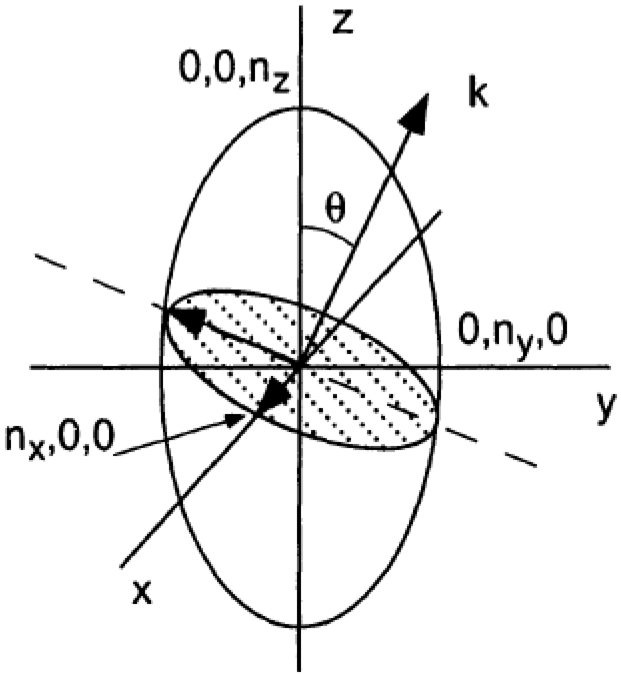
Electro-optic modulation

The index of refraction is defined by an ellipsoid in terms of principal dielectric axes (which are orthogonal for many crystals but not always)

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

index ellipsoid

The distance from the origin to the surface of the ellipse is equal to the index of refraction for an electric field polarized in that direction.



Electro-optic modulation

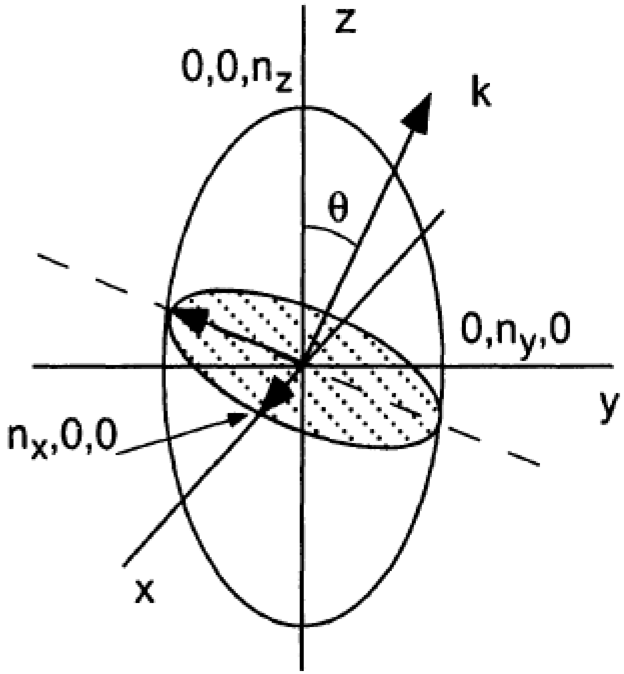
The index of refraction experienced by the wave depends on the orientation of the polarization. A special case occurs if the field is polarized along one of the principal axes.

Consider the case of a wave polarized in the *x*-direction, travelling in the *yz* plane.

As the angle θ varies, the width in the *x*-direction remains constant, indicating that the index n_x is independent of angle θ. This is called the ordinary wave.

If the polarization lies in the *yz* plane, then the index depends on the angle θ, ranging from n_y when $\theta = 0$, to n_z when $\theta = 90°$. This is the extraordinary wave with index expressed by

$$\frac{1}{n_{ext}^2} = \frac{\cos^2\theta}{n_y^2} + \frac{\sin^2\theta}{n_z^2}$$



Electro-optic modulation

A plane wave in a crystal has two polarization eigenstates, corresponding to polarization that does not change as it propagates. **If the field is linearly polarized along either axis of the corresponding ellipse, it will remain linearly polarized.**

Fields with polarizations that do not lie along the major or minor axis will not remain linearly polarized. The electric field will be decomposed into two linearly polarized components oriented along each axis. These components will travel separately accumulating phase according to the index of the axis. **The general field will be elliptically polarized as it propagates through the crystal.**

Uniaxial crystal = index of refraction is identical along two axes.
Examples: quartz, sapphire

Biaxial crystal = has three unique indices of refraction
Examples: calcite, tourmaline, forsterite

Pockel's Effect

The linear electro-optic effect, or Pockel's effect, is the change in the index of refraction that occurs when an external electric field is applied to a crystal. The Pockel effect makes it possible for an electric field to alter the index of refraction of a material and directly modulate the phase, the intensity, or the polarization of the light.

We are interested in how Pockel's effect alters propagation in a crystal and it is convenient to use the general modified index ellipsoid

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 2yz + \left(\frac{1}{n^2}\right)_5 2xz + \left(\frac{1}{n^2}\right)_6 2xy = 1$$

$(1/n^2)_i$ **represents the dielectric tensor along the regular Cartesian coordinates**

Pockel's Effect

If one choses the principal axes x', y', z' at E=0 the terms reduce to

$$\left(\frac{1}{n^2}\right)_1 = \frac{1}{n_x'^2}, \qquad \left(\frac{1}{n^2}\right)_2 = \frac{1}{n_y'^2}, \qquad \left(\frac{1}{n^2}\right)_3 = \frac{1}{n_z'^2}$$

$$\left(\frac{1}{n^2}\right)_4 = \left(\frac{1}{n^2}\right)_5 = \left(\frac{1}{n^2}\right)_6 = 0$$

An applied field modifies the index of refraction. The change of the index ellipsoid is defined in terms of electro-optic coefficients r

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Electro-optic tensor

Electro-optic tensor

The matrix r_{ij} is called the electro-optic tensor and, unlike the dielectric tensor, even if the axes are aligned along the principal axes, the cross-terms 4, 5, and 6 are not necessarily zero.

Coefficients are zero in directions of inversion symmetry. In manycrystals of interest, most of the r coefficients are zero.

Example: Electro-optic tensor of GaAs

$$r_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

Electro-optic tensor

The form of the electro-optic tensor can be determined from a knowledge of the crystal symmetry.

The magnitudes of the coefficients are determined through molecular polarizability calculations or experimental measurement and examples are given in the table on the next slide.

More extensive tables can be found in:

R. J. Pressey, ed. *CRC Handbook of Lasers*, Chemical Rubber Co., Cleveland, Ohio, (1971)

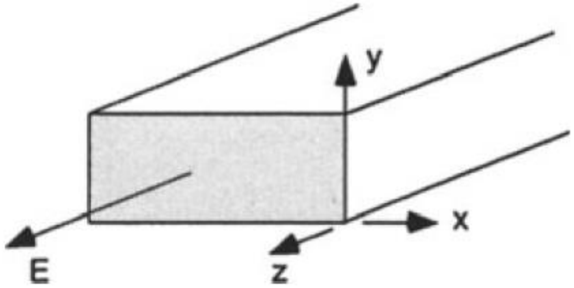
A. Yariv, *Optical Electronics, 4th ed.* Ch. 9, Holt, Rinehart, and Winston, New York (1991)

Linear Electro-optic Coefficients for Some Relevant Crystals

Material	Symmetry	Wavelength (μm)	Electro-optic coefficient (10^{-12} m/V)	Index of Refraction
$LiNbO_3$	3m	0.632	$r_{13} = 9.6$	$n_0 = 1.8830$
			$r_{22} = 6.8$	$n_e = 1.7367$
			$r_{33} = 30.9$	
			$r_{51} = 32.6$	
$LiIO_3$	6	0.633	$r_{13} = 4.1$	$n_0 = 1.8830$
			$r_{41} = 1.4$	$n_e = 1.7376$
GaAs	$\bar{4}3m$	0.9	$r_{41} = 1.1$	$n = 3.60$
		1.15	$r_{41} = 1.43$	
KDP	$\bar{4}2m$	0.633	$r_{63} = 11$	$n_o = 1.5074$
			$r_{41} = 8$	$n_e = 1.4669$
ADP	$\bar{4}2m$	0.633	$r_{63} = 8.5$	$n_o = 1.52$
			$r_{41} = 28$	$n_e = 1.48$
Quartz	32	≈ 0.632	$r_{41} = 0.2$	$n_0 = 1.54$
			$r_{63} = 0.93$	$n_e = 1.55$
$BaTiO_3$	$4mm$	≈ 0.632	$r_{33} = 23$	$n_0 = 2.437$
			$r_{13} = 8$	$n_e = 2.180$
			$r_{42} = 820$	
$LiTaO_3$	$3m$	≈ 0.632	$r_{33} = 30.3$	$n_0 = 2.175$
			$r_{13} = 5.7$	$n_e = 2.365$

Example – Electro-optic effect in GaAs

Consider the effect of an electric field along the propagation direction of a waveguide oriented along the [001] axis of the crystal

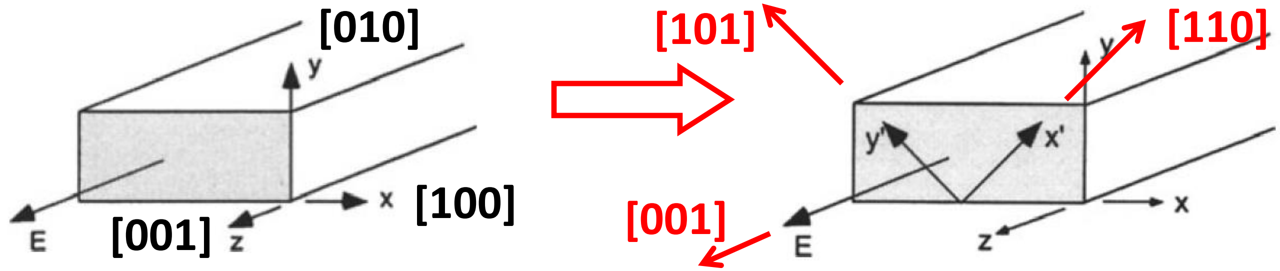


expressed in terms of the general modified index ellipsoid

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 2yz + \left(\frac{1}{n^2}\right)_5 2xz + \left(\frac{1}{n^2}\right)_6 2xy = 1$$

The electro-optic interaction for GaAs can be written as

$$\frac{x^2}{n^2} + \frac{y^2}{n^2} + \frac{z^2}{n^2} + 2r_{41}E_x yz + 2r_{41}E_y xz + 2r_{41}E_z xy = 1$$



For a z-directed field, the expression reduces to

$$\frac{x^2 + y^2 + z^2}{n^2} + 2r_{41}E_z xy = 1$$

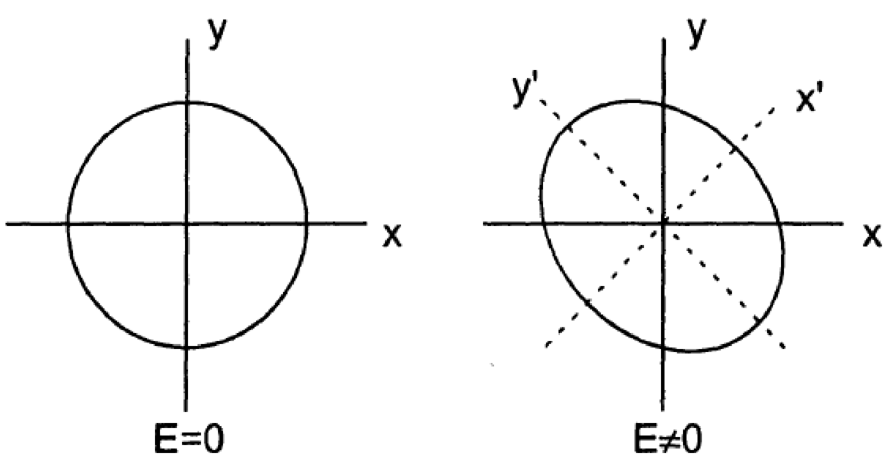
$$\begin{aligned} x &= x' \cos 45^\circ + y' \sin 45^\circ \\ y &= -x' \sin 45^\circ + y' \cos 45^\circ \end{aligned}$$

transformed equation

$$\underbrace{\left(\frac{1}{n^2} - r_{14}E_z\right)}_{\text{give effective indices of refraction}} x'^2 + \underbrace{\left(\frac{1}{n^2} + r_{14}E_z\right)}_{\text{give effective indices of refraction}} y'^2 + \frac{z^2}{n^2} = 1$$

give effective indices of refraction

$$\boxed{\begin{aligned} \frac{1}{n_{x'}^2} &= \frac{1}{n^2} - r_{14}E_z \\ \frac{1}{n_{y'}^2} &= \frac{1}{n^2} + r_{14}E_z \end{aligned}}$$



The ellipsoid changes from a circle when no field is present, to an ellipse rotated 45° from the *x* and *y* axes when the z-directed field is applied. The **coupled term can be removed** (diagonalized) by finding a new coordinate system that lies parallel to the principal axes, *x'* and *y'* .

If the magnitude of $r_{14}E_z$ **is small compared to** n^2

$$\frac{1}{n_{x'}^2} = \frac{1}{n^2} - r_{14}E_z$$

$$\frac{1}{n_{y'}^2} = \frac{1}{n^2} + r_{14}E_z$$

$$n_{x'} = n\left(1 - n^2 r_{14}E_z\right)^{-1/2}$$

$$n_{y'} = n\left(1 + n^2 r_{14}E_z\right)^{-1/2}$$

$$n_{x'} \approx n\left(1 + \frac{n^2 r_{14}E_z}{2}\right)$$

$$n_{y'} \approx n\left(1 - \frac{n^2 r_{14}E_z}{2}\right)$$

$$\boxed{n_{x'} \approx n + \frac{n^3 r_{14}E_z}{2}}$$

$$\boxed{n_z = n}$$

unchanged

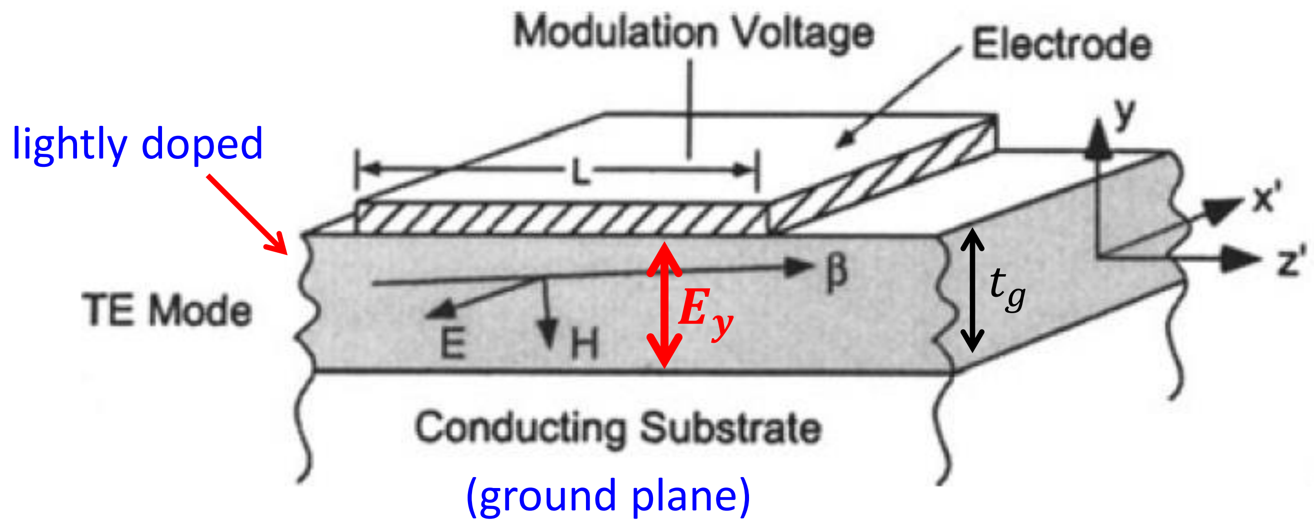
$$\boxed{n_{y'} \approx n - \frac{n^3 r_{14}E_z}{2}}$$

$$\Delta n_{x'} \approx \frac{n^3 r_{14}E_z}{2}$$

$$\Delta n_{y'} \approx -\frac{n^3 r_{14}E_z}{2}$$

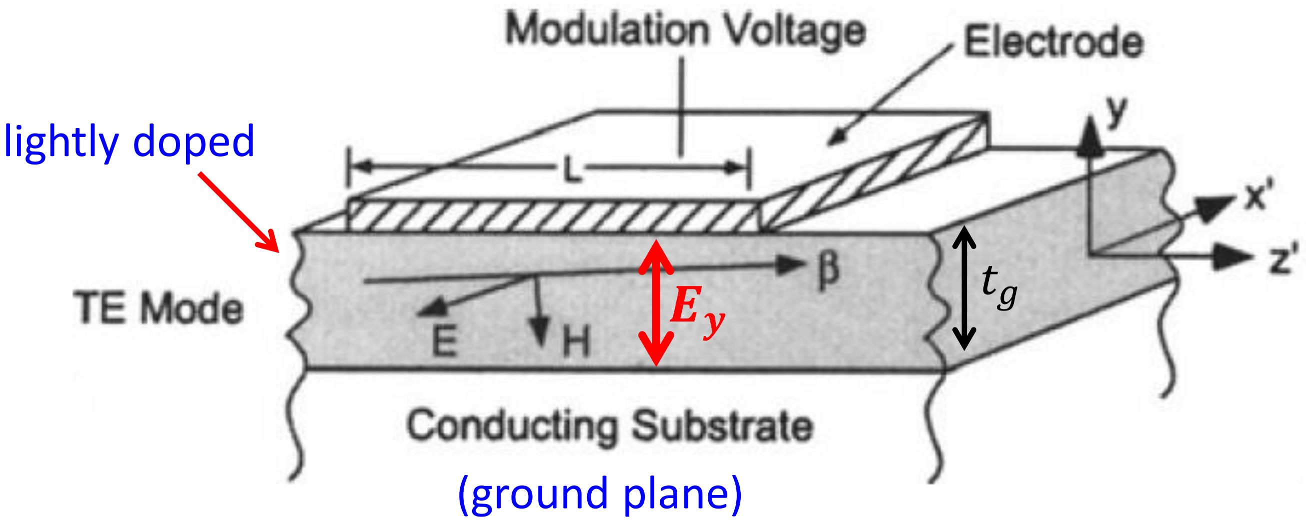
Pockel effect – Phase modulation in GaAs waveguide

Applying an electric field along any single axis will alter the indices along the remaining axes. We apply the electric field along y direction and the waveguide axis along z' oriented in the [101] direction.



An applied field causes the index to change in the z' and x' directions. The change in the z' direction has no effect. The TE mode is polarized along the x' directions so the electric field feels changes in the index n'_x. The TM mode has mainly electric field component in the y direction which is unaffected, and a component along z' which is very small in the weakly guided mode approximation with negligible modulation effect.

Pockel effect – Phase modulation in GaAs waveguide



$$E_y = \frac{V_0}{t_g}$$

The TE wave experience approximately a total phase shift over distance L

$$\Delta\phi = \Delta\beta L = k_0\,\Delta n'_x \times L = \frac{2\pi}{\lambda}\frac{Ln^3 r_{14} E_y}{2}$$

To achieve a phase modulation $\pi/2$ (ignoring a mode confinement factor)

$$E_{\pi/2} = \frac{\lambda}{2}\frac{1}{Ln^3 r_{14}} \qquad V_{\pi/2} = \frac{\lambda}{2}\frac{t_g}{L\,n^3 r_{14}}$$

The longer L, the lower the required voltage to achieve the modulation.

Limitations of this phase modulator

Bandwidth is limited by the capacitance of the electrode (GaAs has a fairly large DC dielectric constant)

Modulation only works for the TE mode. If connected to a circular fiber where polarization may not be controllable, both TE and TM would be excited in the modulator with reduction of the total modulation efficiency.

Whenever the index of refraction of a material is modified, the imaginary component of the index also changes (remember the Kramers-Kronig relations) causing a change of the intrinsic attenuation of the waveguide. An applied electric field will in practice modulate both the phase and intensity of the transmitted light, which is like an additional source of noise.

Fortunately the magnitude of imaginary component change is small and decreases as the wavelength gets further from the absorption edge of the material. Proper selection of material and operating wavelength can reduce the importance of this problem.

Power required by the phase modulator

Assume the modulation of the signal to send one bit requires energy W stored in the capacitor formed by electrode and substrate (neglect losses)

$$W = \frac{\epsilon}{2} \int_V E_0^2 \, dV$$

The power required to send a signal is approximately

$$P = W \cdot \Delta f$$

where Δf is the bit rate or the bandwidth. For a constant field in the volume

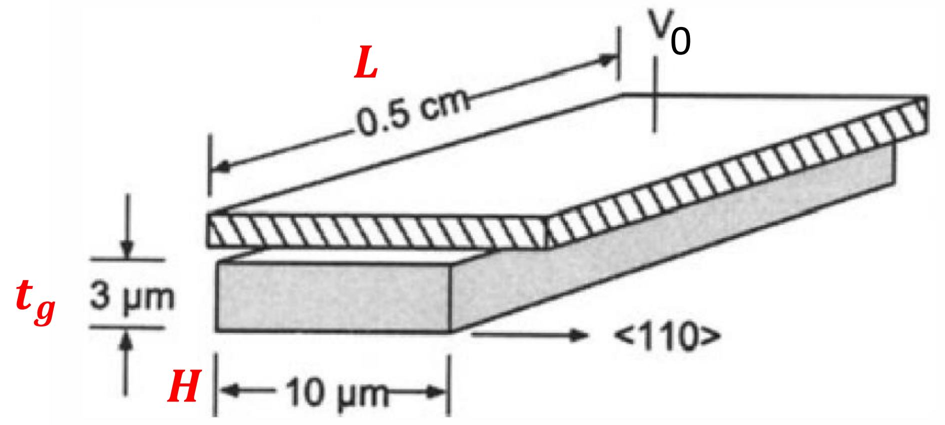
$$W = \frac{\epsilon}{2} H L t_g \, E_0^2 \quad \Rightarrow \quad P = \frac{\epsilon}{2} H L t_g E_0^2 \Delta f$$

For the GaAs modulator, if we assume that to effect a change from 0 to 1 or 1 to 0 requires changing the phase by $\pi/2$, with the previous result

$$E_{\pi/2} = \frac{\lambda}{2} \frac{1}{L n^3 r_{14}} \quad \Rightarrow \quad \frac{P}{\Delta f} = \frac{\epsilon}{8} \frac{H \, t_g}{L} \frac{\lambda^2}{n^6 r_{14}^2}$$

Example

Consider the GaAs modulator structure illustrated earlier



$n(\text{GaAs}) \approx 3.6$

$r_{41} \approx 1.1 \times 10^{-12}\ \text{m/V}$

$\lambda = 0.9\ \mu\text{m}$

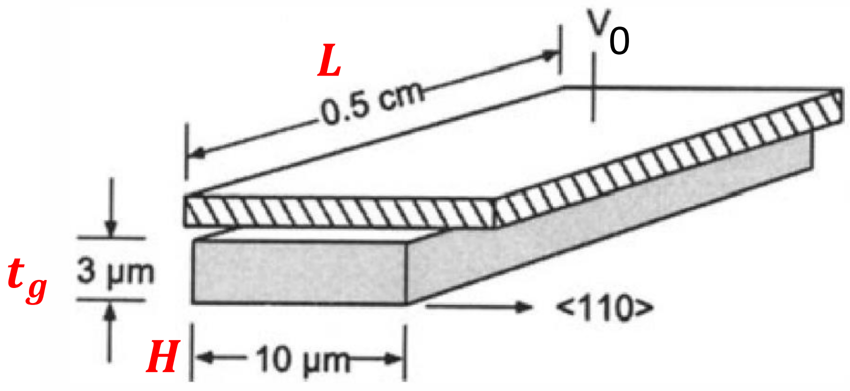
The peak voltage required to achieve a $\pi/2$ phase shift is

$$V_{\pi/2} = \frac{\lambda}{2} \frac{t_g}{L\, n^3 r_{14}}$$

$$= \frac{0.9 \times 10^{-6} m}{2} \frac{3 \times 10^{-6} m}{(5 \times 10^{-3} m)\,(3.6)^3\,(1.1 \times 10^{-12} m/V)} = 5.26V$$

Across the $3\mu m$ film this corresponds to a field $E = 1.73 \times 10^6\ \text{V/m}$

Example



$n(\text{GaAs}) \approx 3.6$ $\varepsilon_r \approx 12.9$

$r_{41} \approx 1.1 \times 10^{-12}$ m/V

$\lambda = 0.9\ \mu\text{m}$

$$\begin{aligned}
\frac{P}{\Delta f} &= \frac{\epsilon}{8}\frac{H\, t_g}{L}\frac{\lambda^2}{n^6 r_{14}^2} \\
&= \frac{12 \cdot 8.85 \times 10^{-12}}{8}\,\frac{10 \times 10^{-6} \cdot (3 \times 10^{-6})}{5 \times 10^{-3}}\,\frac{(0.9 \times 10^{-6})^2}{3.6^6 (1.1 \times 10^{-12})^2} \\
&= 2.63 \times 10^{-11}\ \text{W/Hz} = 24.5\ \mu\text{W/MHz}
\end{aligned}$$

Compare to a bulk (non-waveguide) modulator

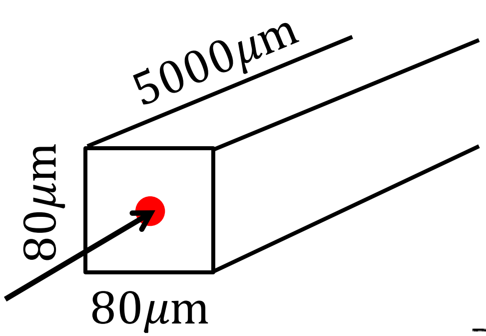
We consider the same length $L = 0.5\text{cm}$. To minimize the size of the bulk modulator, the optical beam should be focused through the crystal. The smallest that the beam can be made and get through the crystal without excessive losses is determined by the confocal parameter $2z_0$

$$2z_0 = \frac{2\pi n\omega_0^2}{\lambda}$$

ω_0 = beam radius at the focus

The *confocal parameter* is the distance over which an optical beam doubles its area due to diffraction from a focal spot. It represents the region where the optical beam is the most collimated.

A value 0.5cm yields $\omega_0 = 14\mu\text{m}$. At the face of the crystal the beam has largest radius $\omega = \sqrt{2}\omega_0 = 20\mu\text{m}$. To limit losses the crystal should have a lateral dimension at least twice the input beam diameter ($80\mu\text{m}$)



$$V_{\pi/2} = \frac{0.9\times10^{-6}}{2}\,\frac{80\times10^{-6}}{(5\times10^{-3})\,(3.6)^3\,(1.1\times10^{-12})} = 140V$$

Electric field = $140/80\times10^{-6} \approx 1.73\times10^{6}\,\text{V/m}$ **(same as before)**

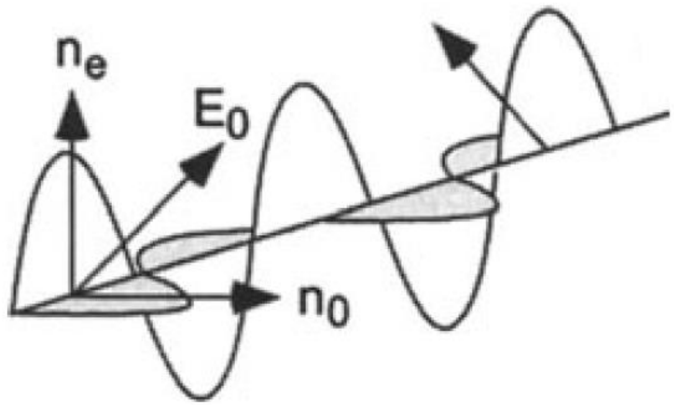
$$\frac{P}{\Delta f} = \frac{12\cdot 8.85\times10^{-12}}{8}\,\frac{80\times10^{-6}\cdot(80\times10^{-6})}{5\times10^{-3}}\,\frac{(0.9\times10^{-6})^2}{3.6^6(1.1\times10^{-12})^2}$$

$$= 5.22\times10^{-9}\ \text{W/Hz} = 5.2\ \text{mW/MHz}$$ **(200 times greater)**

Electro-optic Intensity Modulators

The phase modulation introduced by the electro-optic effect can be used to create an intensity modulation via changes in polarization, or through interferometric effects.

Polarization Modulation



A linearly polarized wave at 45° to the polarization axes of a birefringent crystal will travel as two waves, an ***ordinary wave*** (index n_{o}), and an ***extraordinary wave*** (index n_{e}). They travel with different phase velocities accumulating relative phase difference over a distance L

$$\Delta\phi = k_0 L(n_x - n_y)$$

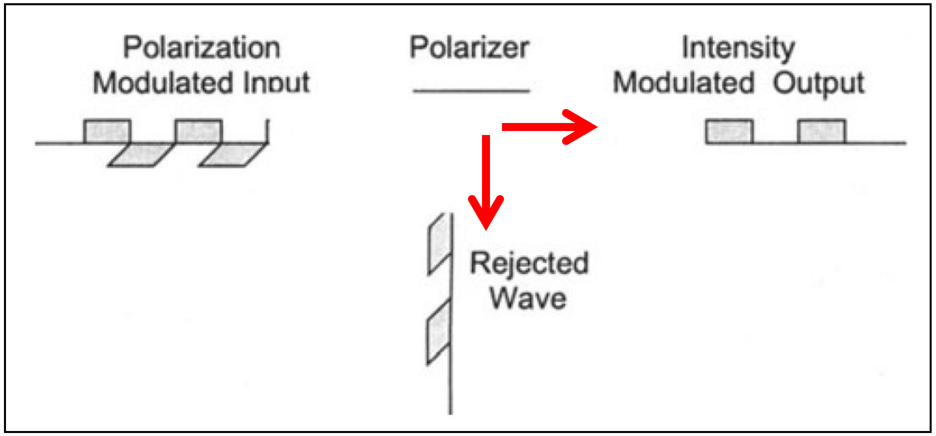
When the phase difference is $\Delta\phi = \pi, 3\pi, 5\pi$ ... the superposition will result in a linear polarization that is rotated by 90° relative to the input polarization.

For $\Delta\phi = 2\pi, 4\pi, 6\pi$ ... the original polarization state is restored.

Polarization Modulation

This effect can be used to control (modulate) the polarization through Pockel's effect.

To convert this polarization rotation into an intensity modulation, it is necessary to run the output through a linear polarizer, also known as an *analyzer* which transmits only one polarization component, either ejecting the other component into another direction or attenuating it.



As the polarization is electro-optically rotated over 90° the transmitted intensity will vary from continuously from a maximum to zero.

Polarization Modulation

Polarization modulation is not widely adopted because of various reasons:

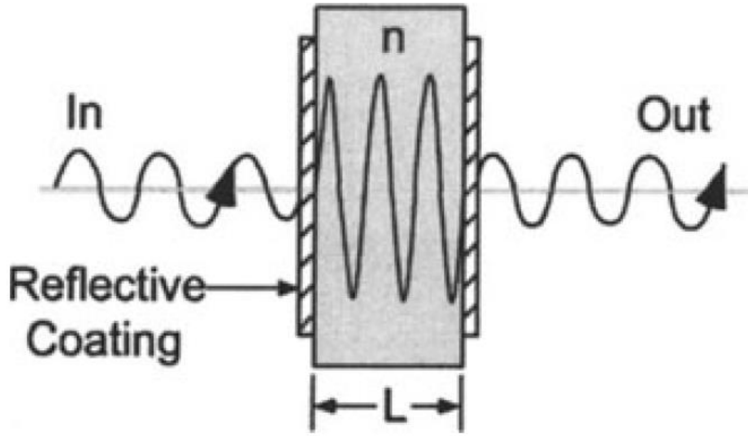
- **Most suitable electro-optic materials may have also natural birefringence ($n_x \neq n_y$) which ends up overwhelming the Pockel effect. Natural birefringence may have $\Delta n \approx 10^{-2}$ while the Pockel effect $\Delta n \approx 10^{-5}$.**
- **Theses crystals have temperature dependent index properties which caused polarization drifts**
- **Integration of polarizers remains difficult. In fiber optics systems, the problem may be solved with the use of certain polarization maintaining fibers, but for purely integrated optics applications technical difficulties remain.**

Interferometric Modulation

Phase modulation can be converted into intensity modulation through constructive interference between two waves. Two important examples are:

- **Fabry-Pérot modulator**
- **Mach-Zehnder modulator**

Fabry-Pérot Modulator



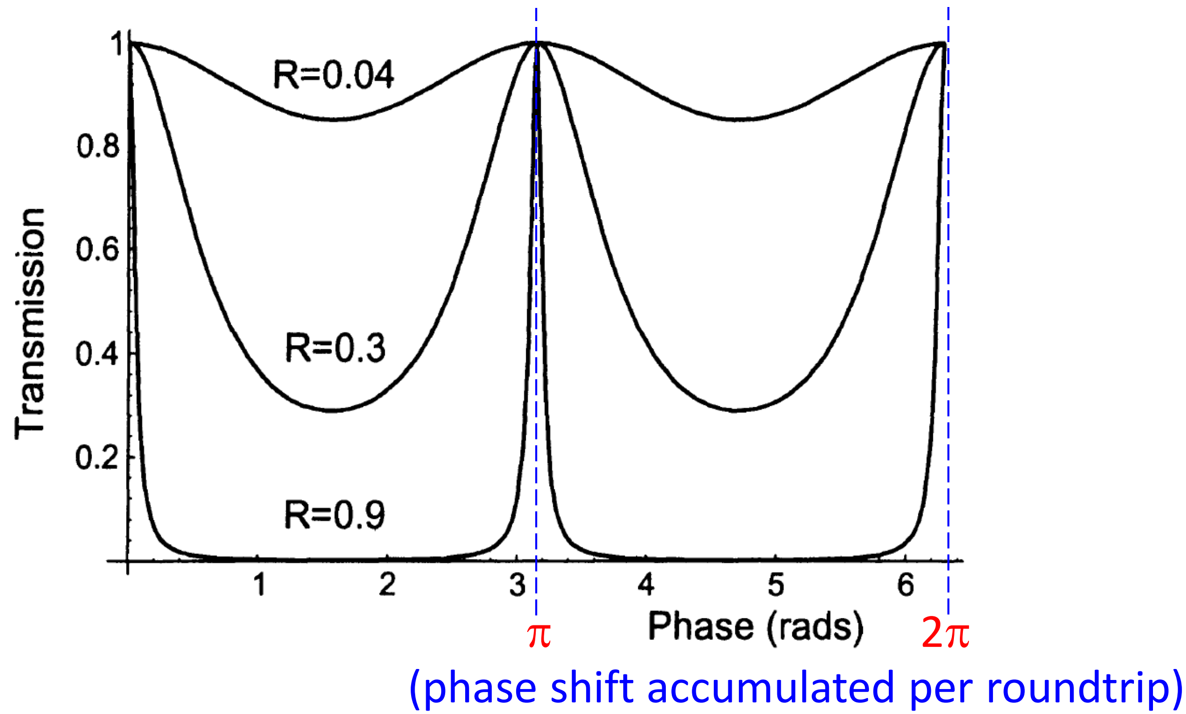
Transmission through the Fabry-Perot etalon is maximum when

$$L = \frac{m\lambda}{2n}$$

The transmission is given by

$$T = \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2\left(\frac{4\pi}{\lambda} nL\right)}$$

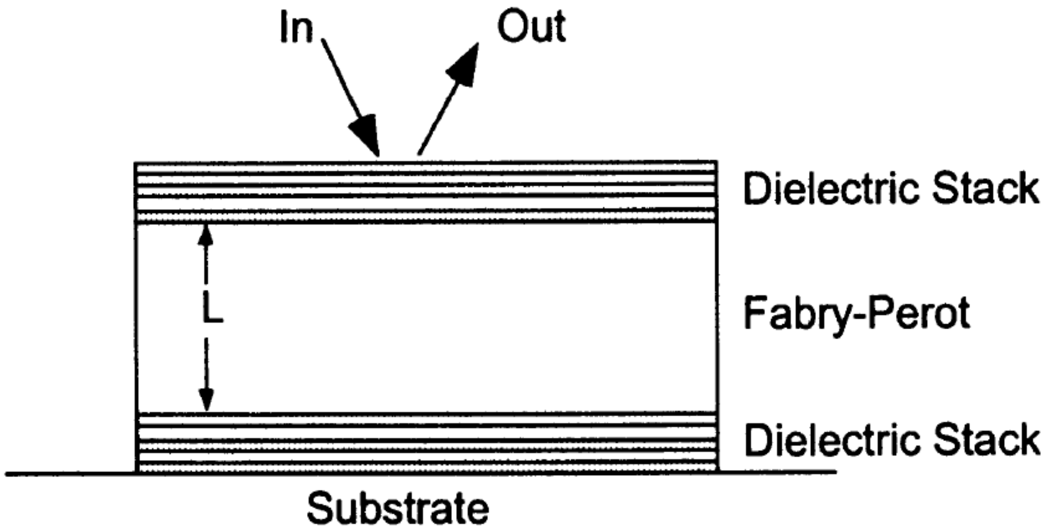
Fabry-Pérot Modulator



Selectivity increases with reflectance *R*. For a reflection equivalent to the Fresnel reflection from fused silica (≈4% per surface) the modulation depth is ≈ 8% between the minimum and maximum transmission points. For reflectance ≈ 30% (GaAs) the modulation depth is ≈ 80%. In general, reflectivity is adjusted using dielectric coatings.

Fabry-Pérot Modulator

This interferometer can operate as intensity modulator with electro-optic material between the mirrors. The goal is to switch the device between T_{max} and T_{min} by applying an electric field across the bulk layer, which changes the material index and shifts the transmission peak to a different wavelength.



To reduce the insertion loss for this type of modulator, the output from this device is usually taken from the reflection off the front surface. In anti-resonance, the reflection is nearly 100%, while at resonance the reflection is minimum.

Fabry-Pérot Modulator

A well designed modulator has a Free Spectral Range (FSR – the spectral distance between two adjacent transmission maxima) which is larger than the bandwidth of the modulated signal.

$$\mathrm{FSR} = \frac{c}{2nL}$$

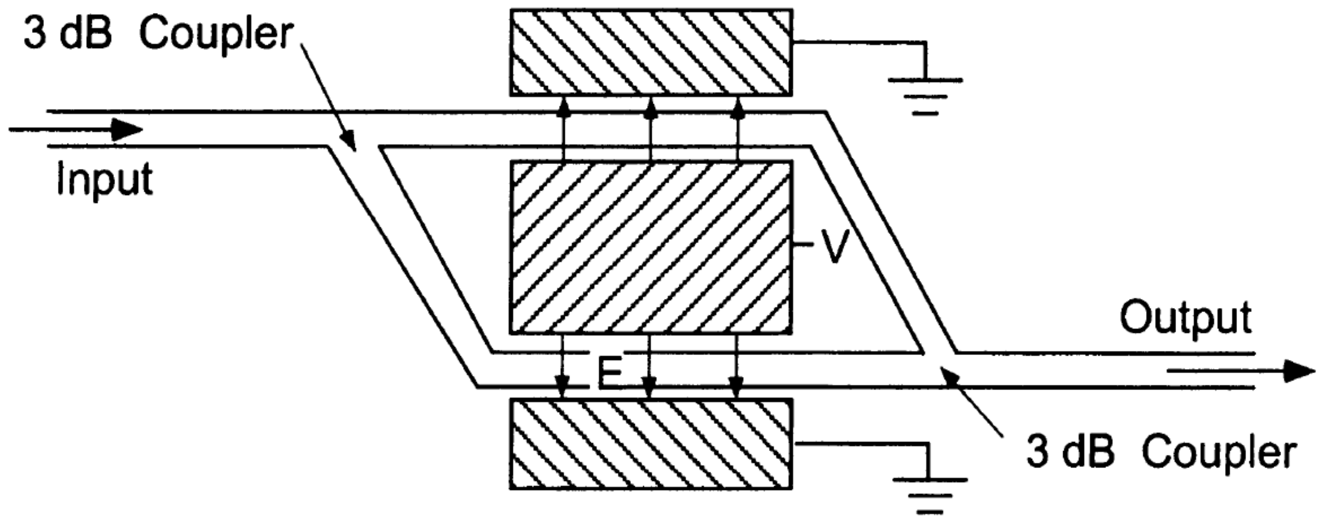
Also the spectral transmission at resonance must be broad enough to transmit the entire signal.

The resolution is often defined in terms of ***finesse F***

$$F = \frac{\mathrm{FSR}}{\text{Full width at Half Maximum}} = \frac{\pi (R_1 R_2)^{(1/4)}}{1 - (R_1 R_2)^{1/2}}$$

Mach-Zehnder Modulator

In a waveguide structure, the Mach-Zehnder interferometer can use interference between two waves to convert phase modulation into intensity variation.



The split beams travel different paths of length l_1 and l_2, then recombine at another Y-junction. Combining two arms with different phase modulation converts phase modulation into intensity modulation.

Mach-Zehnder Modulator

If the optical path lengths of the two arms have an integer number of optical wavelengths, the two waves arrive at the Y-junction in-phase and constructively interfere propagating down the output waveguide.

If the optical path lengths are unequal and the relative phase difference between the two combining beams is $\pi/2$, the two beams destructively interfere and no output wave propagates.

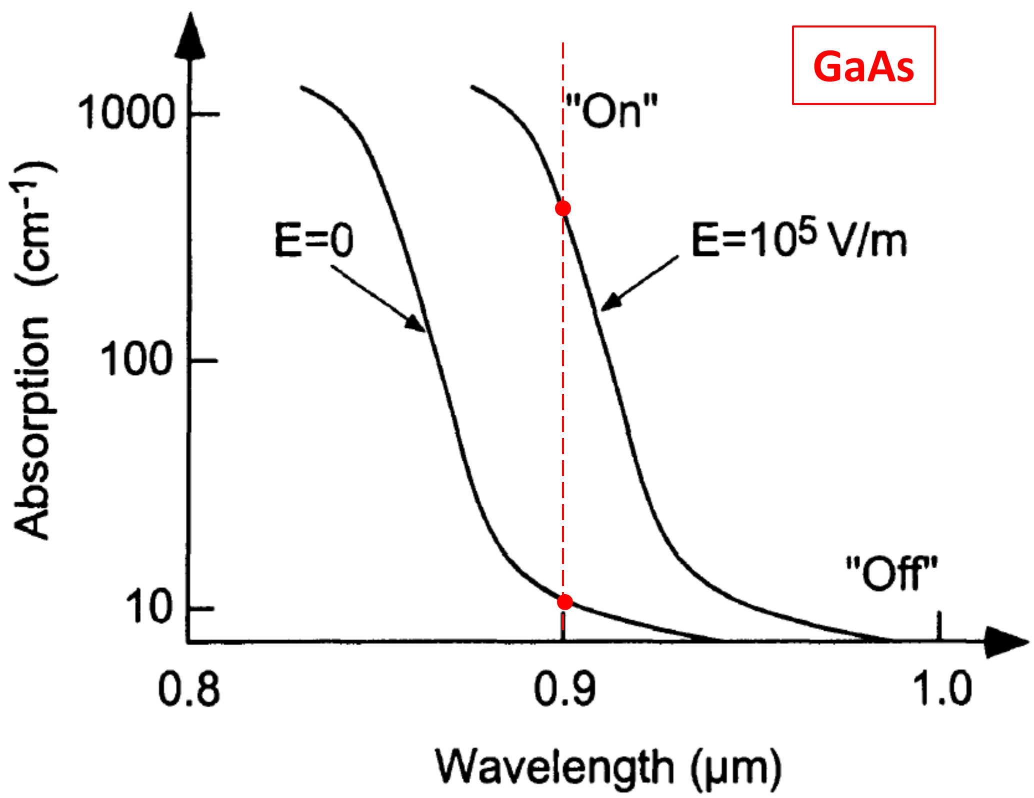
The relative phase difference of the beams can be controlled electro-optically by applying a voltage to the center electrode. The change in index Δn depends on the orientation of the crystal and the direction of the applied electric field. Appropriate choice of the crystal axes causes the applied field to increase the index in one arm and decrease it in the other arm, to alter the relative phase of the recombining fields.

Electro-Absorption Modulator

(Franz-Keldysh effect)

Franz-Keldysh Effect

The absorption edge of a semiconductor shifts in the presence of an electric field. Application of a large field shifts the absorption profile toward longer wavelengths



Electro-Absorption Modulators

The total change in intensity depends on the path length through the modulator. The transmitted signal behaves as

$$I(z) = I(0)e^{-\alpha z}$$

The contrast ratio is

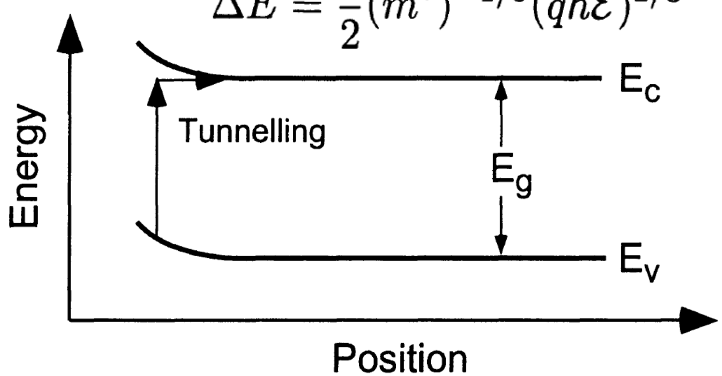
$$\frac{I_{max}}{I_{min}} = \frac{e^{-\alpha_1 z}}{e^{-\alpha_2 z}}$$

The length of the electrode is chosen to maximize the contrast ratio

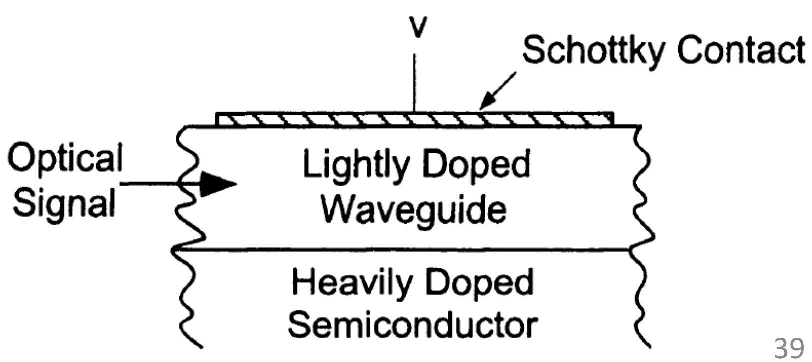
The Franz-Keldysh effect arises from band bending near the surface of the semiconductor where photon absorption can occur at energy lower than the bandgap.

Effective change of the bandgap

$$\Delta E = \frac{3}{2}(m^*)^{-1/3}(q\hbar\mathcal{E})^{2/3}$$



Basic electro-absorption modulator



Acousto-optic Modulator

Acousto-optic Modulators

Also acousto-optic modulators control transmission of light by local changes of index of refraction in the medium.

- the modulation occurs by means of a travelling sound wave which induces a stress-related modification of the local index
- acoustic interactions travel at the speed of sound in the material, while electro-optic interactions can occur at nearly the speed of light
- while electro-optic interactions can be established with DC fields, acousto-optic modulation based on sound waves always involves interaction with travelling or standing waves in the solid

Acousto-optic Modulators

The photoelastic effect involves reflecting light due to the change of the index of refraction in a dielectric caused by strain. This effect is nonlinear and characterized by the fourth rank ***photoelastic tensor*** p_{ijkl}

term of the index ellipsoid $$\Delta\left(\frac{1}{n^2}\right)_{ij} = p_{ijkl}S_{kl}$$

strain $$S_{kl}(r) = \frac{1}{2}\left[\frac{\partial u_k(r)}{\partial x_l} + \frac{\partial u_l(r)}{\partial x_k}\right]$$

The acousto-optic strain interacts with an electric field component $\boldsymbol{E_j}$ to generate a polarization $\Delta\boldsymbol{P_i}$. The acoustic power $\boldsymbol{P_a}$ is related to the change in index of refraction though the relation

$$\Delta n = \sqrt{n^6 p^2 P_a / 2\rho v_a^3 A}$$

area cross-section

photoelastic tensor element

mass density

acoustic velocity

Acousto-optic Modulators

Rewriting in terms of a figure of merit $M = n^6 p^2 / \rho v_a^3$

$$\Delta n = \sqrt{M P_a / 2A}$$

Materials commonly used in Acousto-optic Modulators

Materials	$\lambda(\mu m)$	n	$\rho(g/cm^3)$	$v_s(10^3$ m/s	M
Fused Quartz	0.63	1.46	2.2	5.95	1.51×10^{-15}
GaAs	1.15	3.37	5.34	5.15	104×10^{-15}
$LiNbO_3$	0.63	2.20	4.7	6.57	6.99×10^{-15}
YAG	0.63	1.83	4.2	8.53	0.012×10^{-15}
As_2S_3	1.15	2.46	3.20	2.6	433×10^{-15}
$PbMO_4$	0.63	2.4		3.75	73×10^{-15}

A good (old) reference on photoelastic tensor:

J. F. Nye, *Physical Properties of Crystals,* Oxford Clarendon Press, London, pp. 241 (1957)

Acousto-optic Modulators

Acousto-optic modulators used in integrated optics generally use travelling wave acoustic fields. The acoustic field creates a grating structure which can diffract the incident optical field.

Optical wave interaction can be produced by either bulk acoustic waves travelling in the volume of the material, or by surface acoustic waves (SAW) which propagate on the surface within approximately one acoustic wavelength of the surface.

SAW devices are well suited to integrated optics applications, because the energy of the acoustic field is concentrated in the region of the optical waveguide.

Diffraction in Acoustic Grating

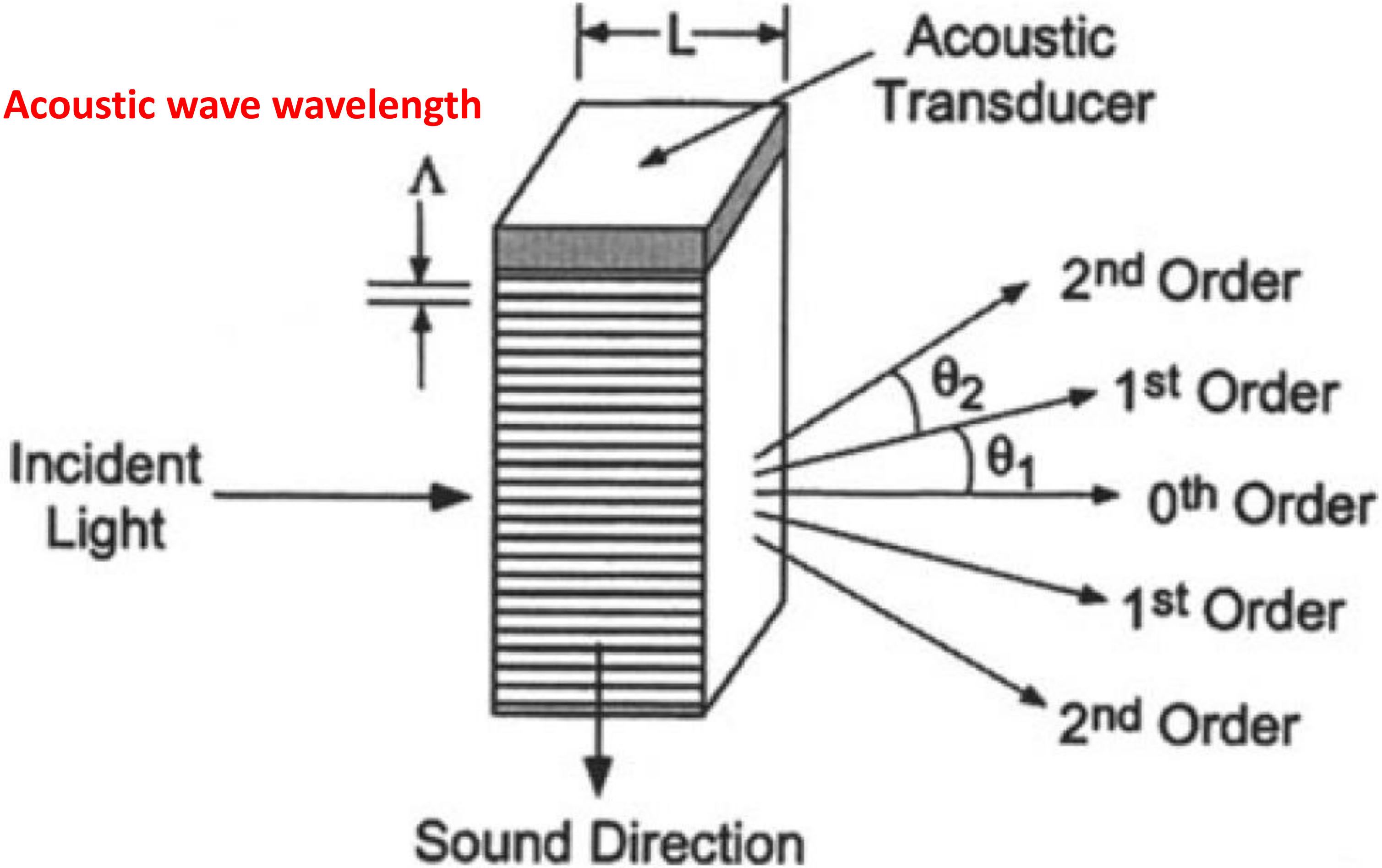
Acusto-optic interaction can be described as a diffraction of an optical wave by a travelling phase grating induced by an acoustic wave.

As the phase grating Doppler-shifts the optical frequency, it can be used to deflect, modulate, or filter the optical beam.

Isotropic acusto-optic interactions which do not change the polarization of the optical beam can result in either multiple or single diffracted optical orders.

Acousto-optic Modulators

Acoustic wave wavelength



Acousto-optic Modulators

There are two basic configurations:

If the optical field propagates transverse to the acoustic beam, and the interaction length of the two beams is relatively short we have ***Raman-Nath*** diffraction

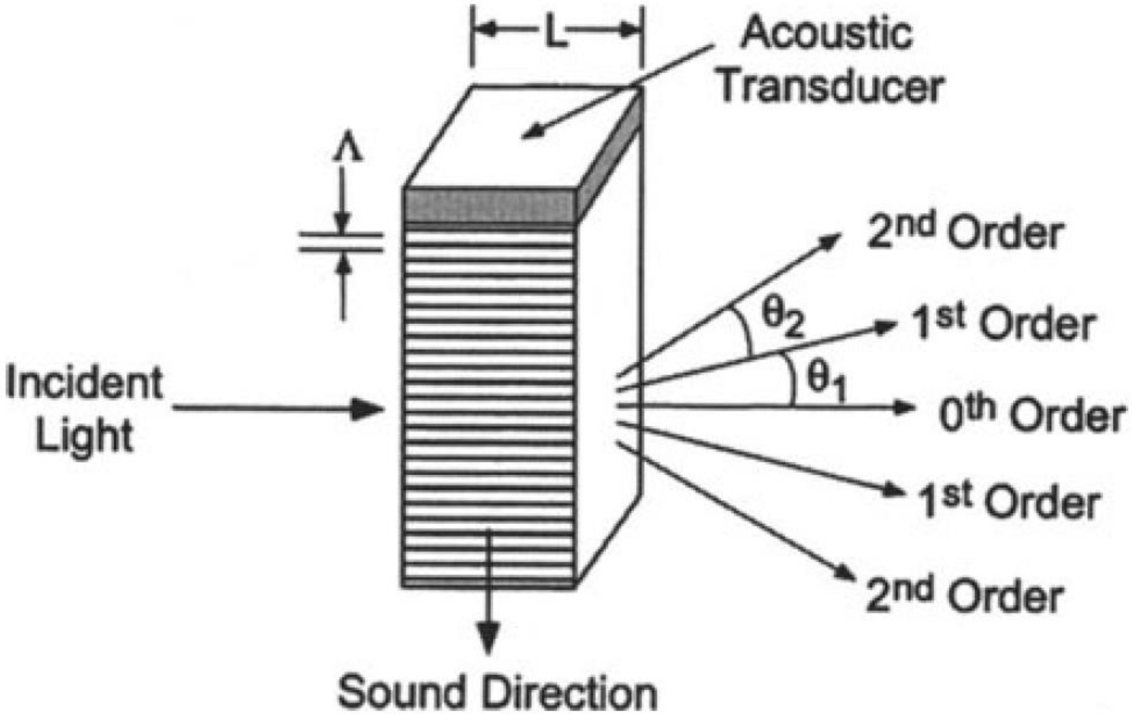
$$L \ll \frac{\Lambda^2}{(\lambda/n)}$$

If the acoustic field is large so that **multiple refraction** can occur, the interaction is called ***Bragg*** modulation.

$$L \gg \frac{\Lambda^2}{(\lambda/n)}$$

Raman-Nath Diffraction Regime

The Raman-Nath regime is observed at relatively low acoustic frequencies at small acusto-optic interaction lengths. This type of diffraction takes place at an arbitrary incident angle of light roughly normal to the acoustic beam and the diffraction pattern contains many diffraction orders of symmetrically distributed light intensity (which can be studied with Bessel functions).



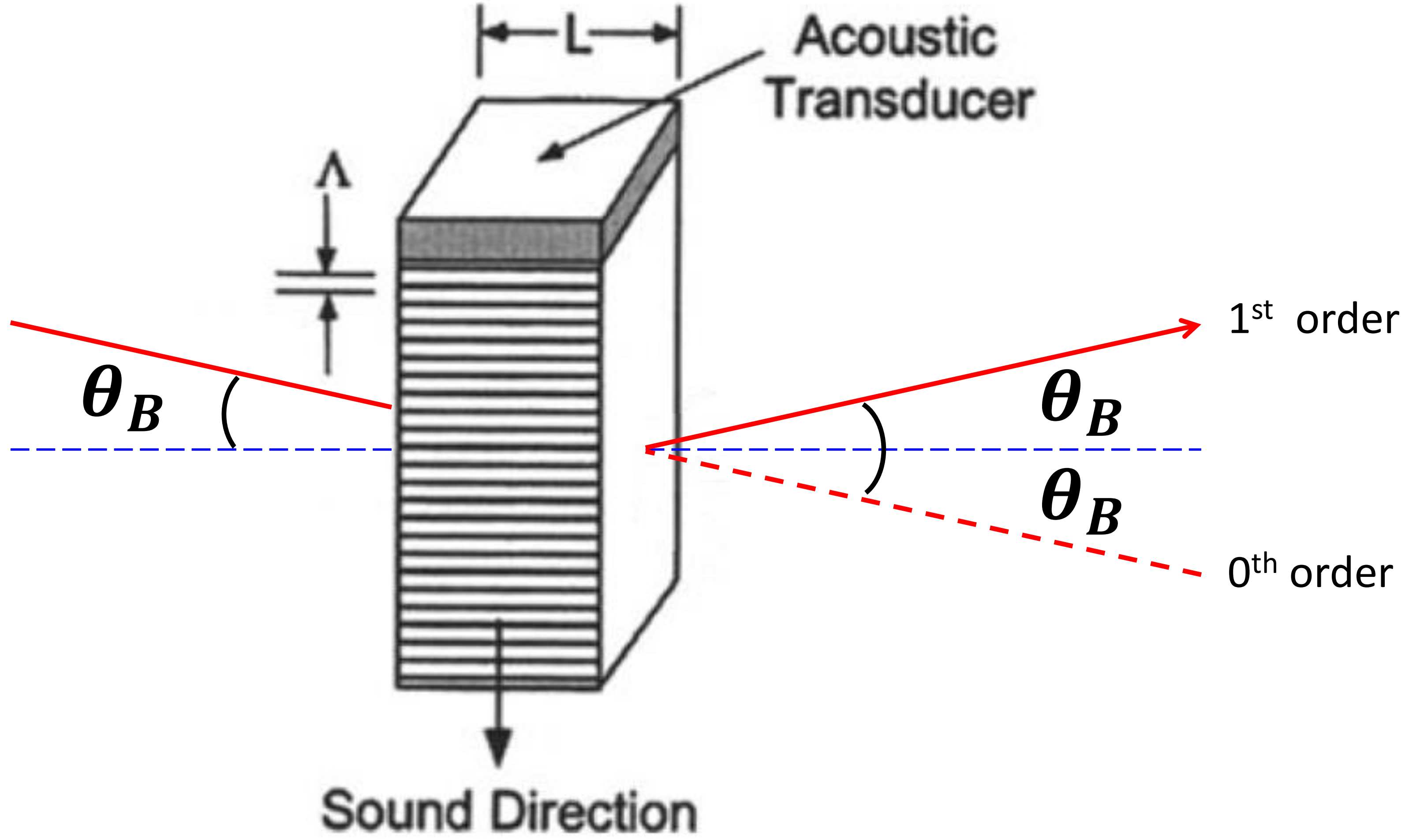
Bragg Diffraction Regime

By contrast, the Bragg regime is observed at high acoustic frequencies usually exceeding 100MHz. The diffraction pattern of this regime even at large acoustic power consists of two diffraction maxima of zero and first orders.

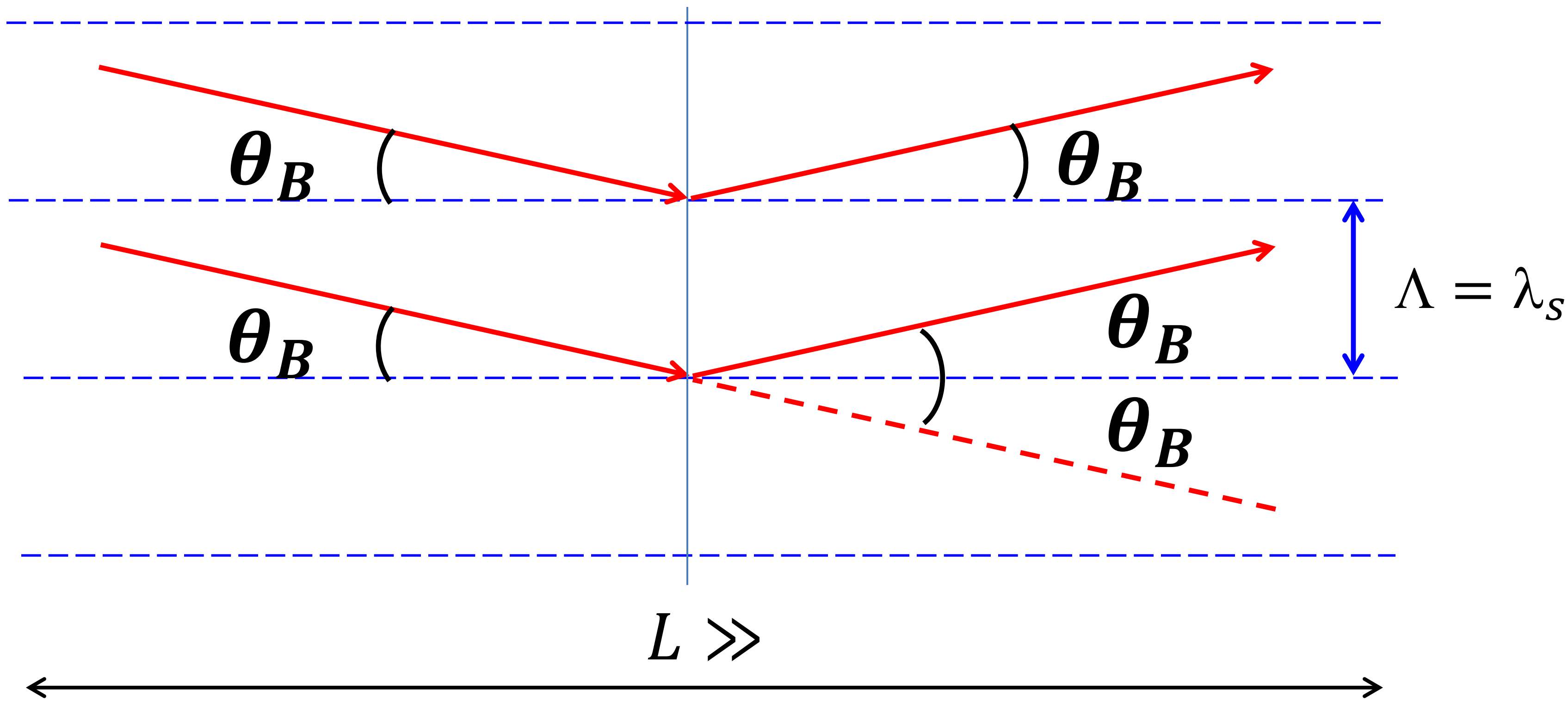
At the Bragg angle of incidence θ_B only one diffraction order is produced while others are annihilated by destructive interference.

This single-order isotropic Bragg diffraction is much more efficient and therefore is widely used in practical devices.

Bragg Diffraction Regime



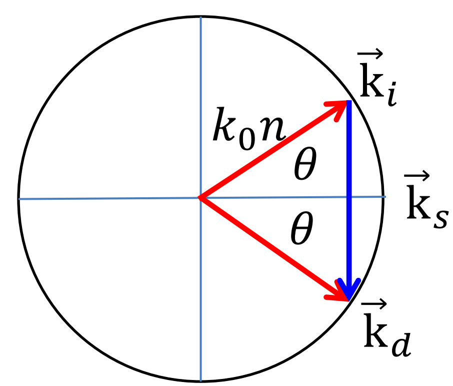
Bragg Diffraction Regime



$$2k_0 n \sin\theta_B = k_s$$

$$2\frac{2\pi}{\lambda_0} n \sin\theta_B = \frac{2\pi}{\Lambda}$$

$$2\Lambda \sin\theta_B = \lambda_0 / n$$

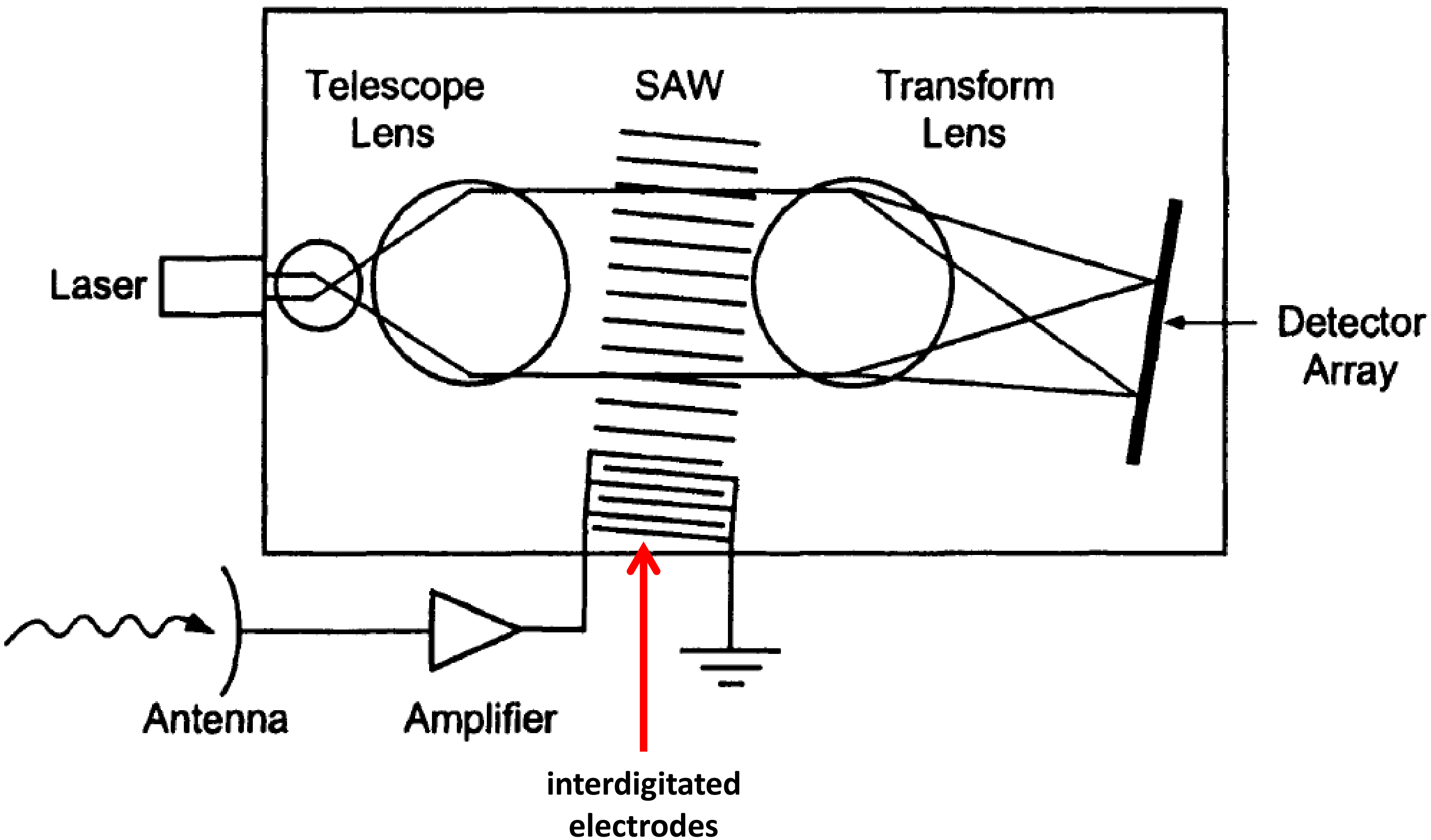


Spectral detection of radio signals

One of the most significant examples of acousto-optic modulation applications in integrated optics is in spectral analysis of radio frequency signals.

A direct application of this device is to allow a pilot to obtain an instantaneous spectrum analysis of a radar signal, in order to determine if the plane is being tracked by a ground-based station, air-to-air missile, or other vehicle. The signature of the radar signal can be deciphered to extract this information.

Spectral detection of radio signals



The Surface Acoustic Wave (SAW) is generated by the incoming electrical signal. An antenna collects the RF signal, and sends it to an amplifier. The amplified signal is applied to an interdigitated array of electrodes on the surface of the planar waveguide.

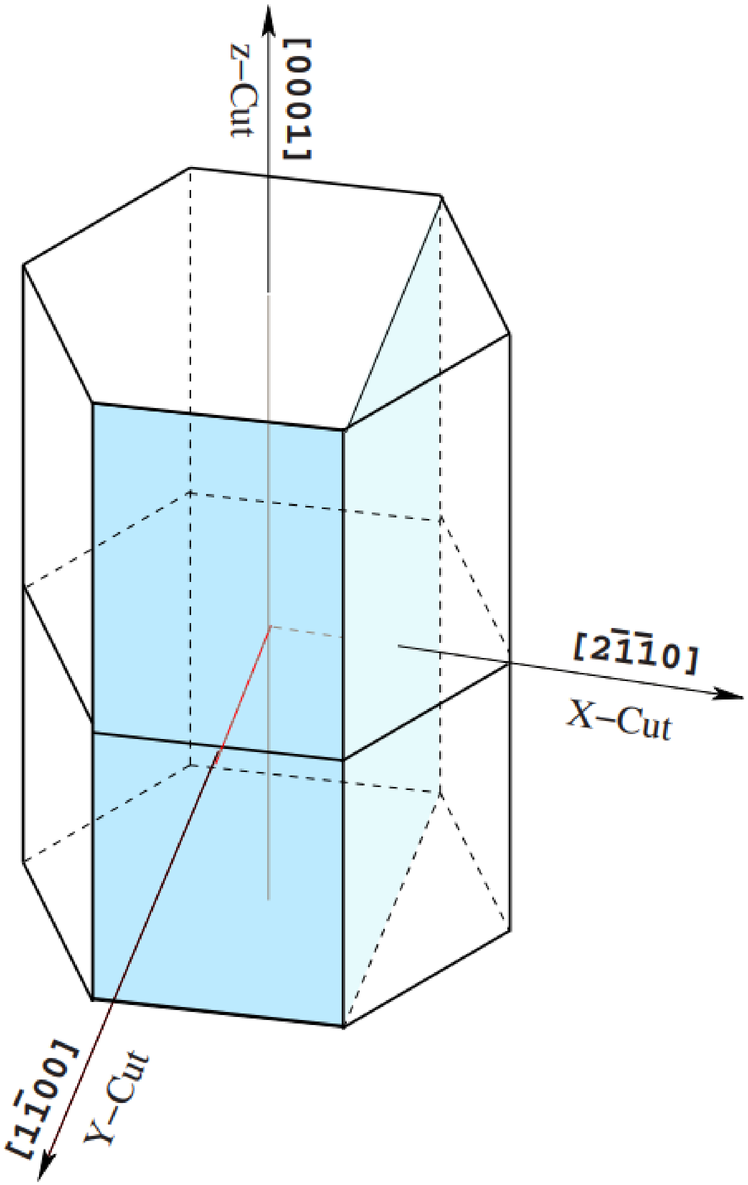
Acousto-optic Modulators

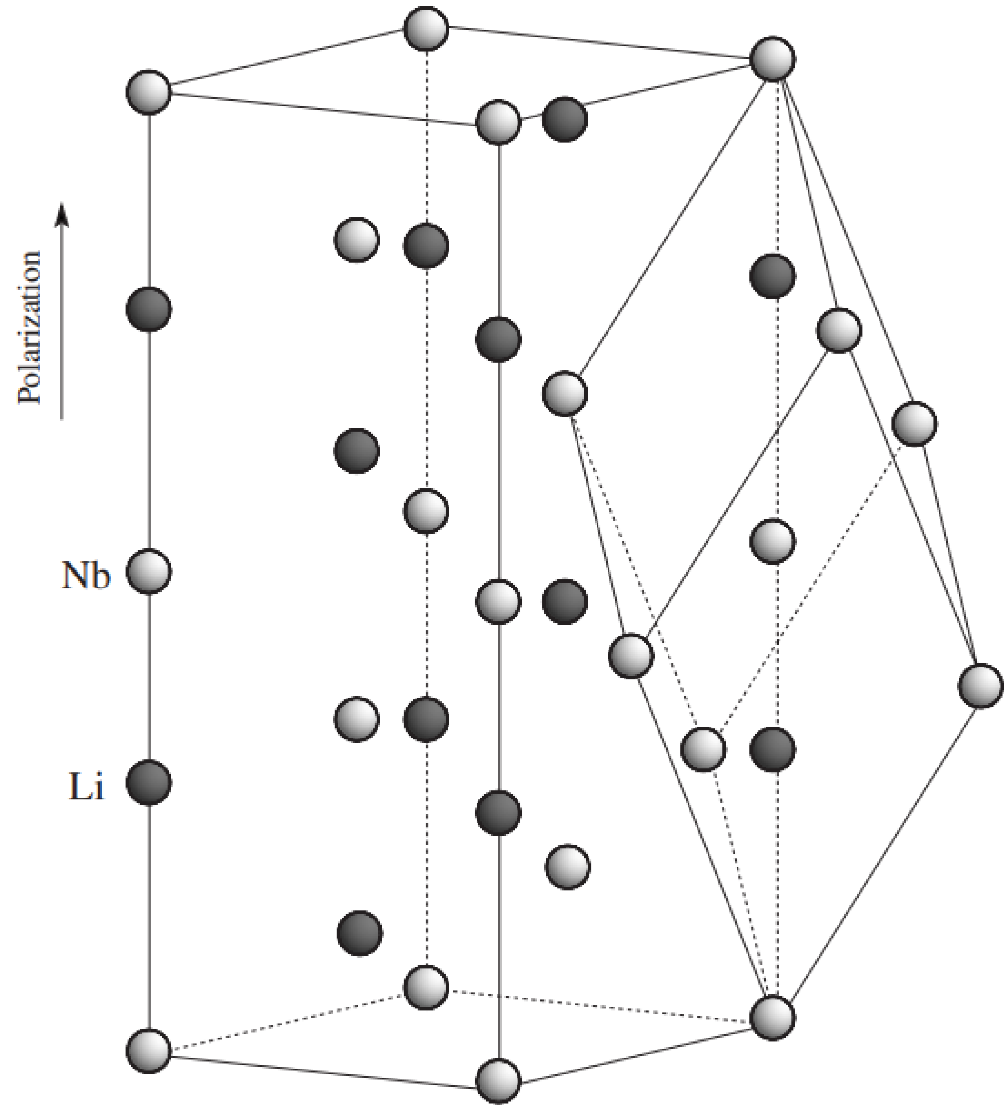
With a piezoelectric material, such as X-cut LiNb03, the electric field between the fingers of the electrode periodically constrict and expand the surface material, establishing an acoustic wave that propagates across the waveguide.

The spatial period of the acoustic wave depends on the frequency of the applied RF signal.

$LiNbO_3$

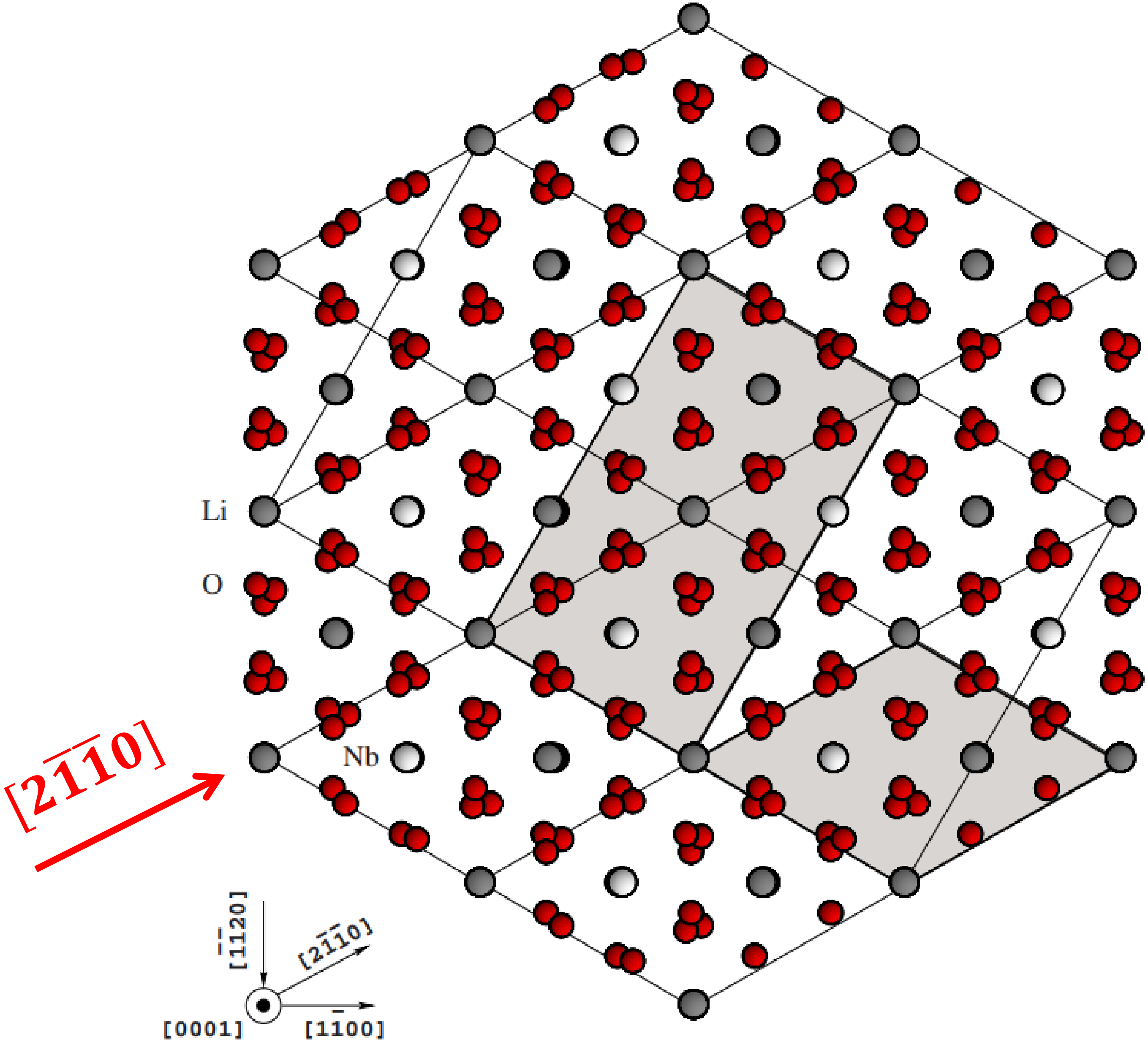
(Oxygen omitted)





Sanna and Schmidt, PHYSICAL REVIEW B **81**, 214116 (2010)

$LiNbO_3$ X-cut



Sanna and Schmidt, PHYSICAL REVIEW B **81**, 214116 (2010)

Spectral detection of radio signals

The optical beam leaving the SAW region contains several distinct beams travelling in different directions, depending on the spectral content of the applied electrical signal. But the beams are essentially fully spatially overlapped.

To separate the beams, a lens converts the angular variation of the incoming rays (k-space description) into a spatial variation (x-space) at the focus of the output beam (equivalent to a Fourier Transform). The focused output of this lens is directed onto an array of detectors, each one corresponding to a specific frequency of the incoming electrical signal.

All frequencies present in the incoming signal are simultaneously detected at the array, so a *signature* of the signal can be readily determined.

Plasmonic modulation

Plasmonics

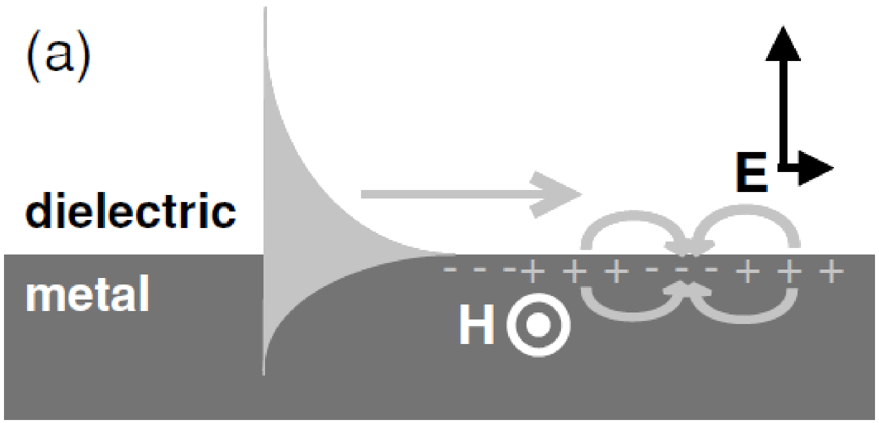
Plasmonics is an extension of ***Photonics*** and it refers to the generation, detection, and manipulation of optical signals at nanoscale metal-dielectric interfaces.

Plasmonics utilizes ***surface plasmon polaritons*** (SPPs) which are **coherent electron oscillations** travelling together with an electromagnetic wave at the interface between a dielectric (glass, air, etc.) and a metal (silver, gold, etc.)

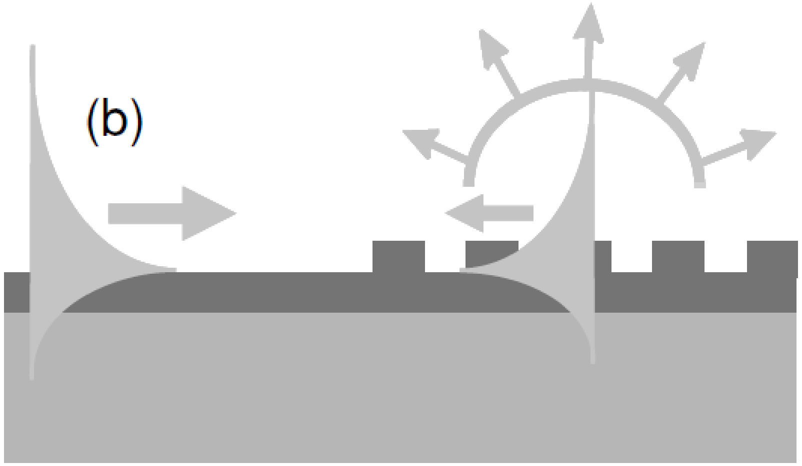
Because of ohmic losses, surface plasmons are limited to travel distances on millimeter scale. Surface roughness is also a limitation. New low-loss plasmonic materials are being investigated, including metal oxides and nitrides.

Plasmonics

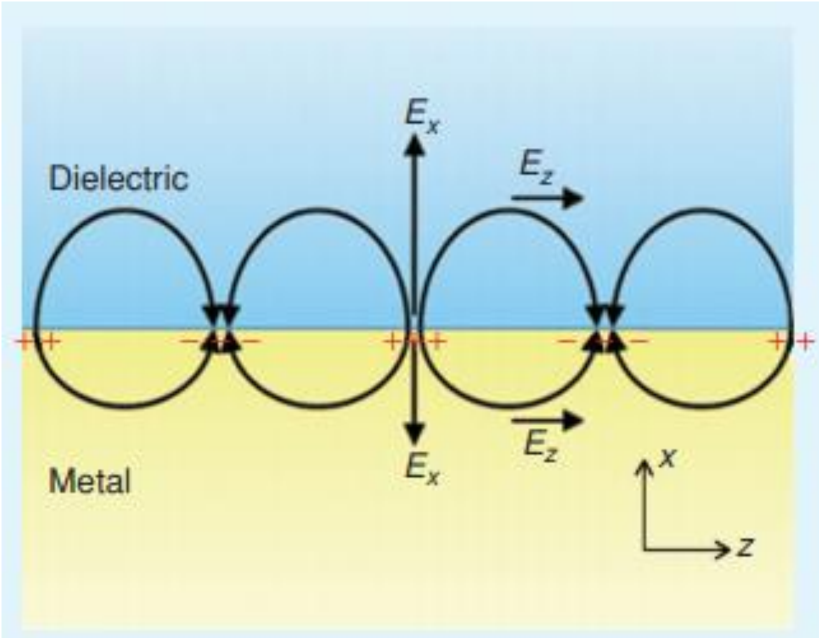
In a plasmonic structure a waveguide can be realized with a single metallic interface



A grating can reflect and scatter the wave. Scattering increases losses.

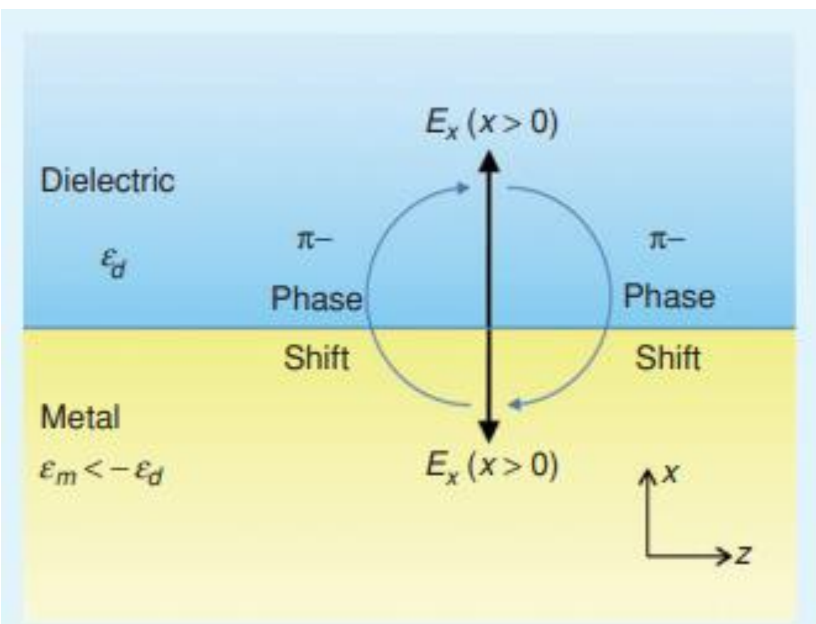


Plasmonics



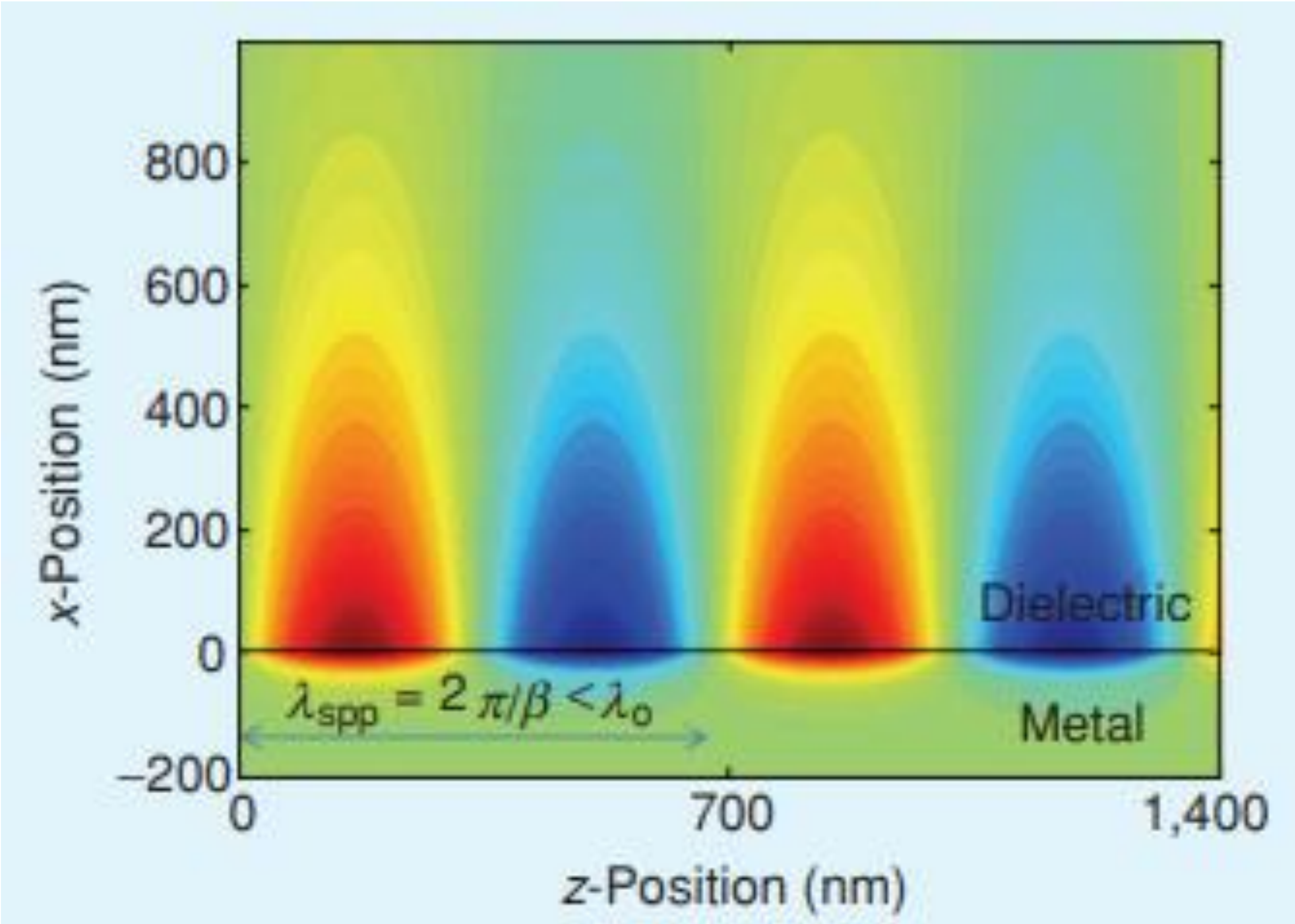
Schematic representation of SPP as charge oscillations at the interface between a metal and a dielectric.

The electric field has a longitudinal (z) component $\pi/2$ out-of-phase with the transverse (x) component.



SPP self-consistency condition allowed by a negative relative permittivity in the metal. The EM wave has phase-shift of π upon crossing the boundary, allowing self-consistency when crossing the boundary twice. This enables the SPP wave guide mode to exist at a single interface.

Plasmonics

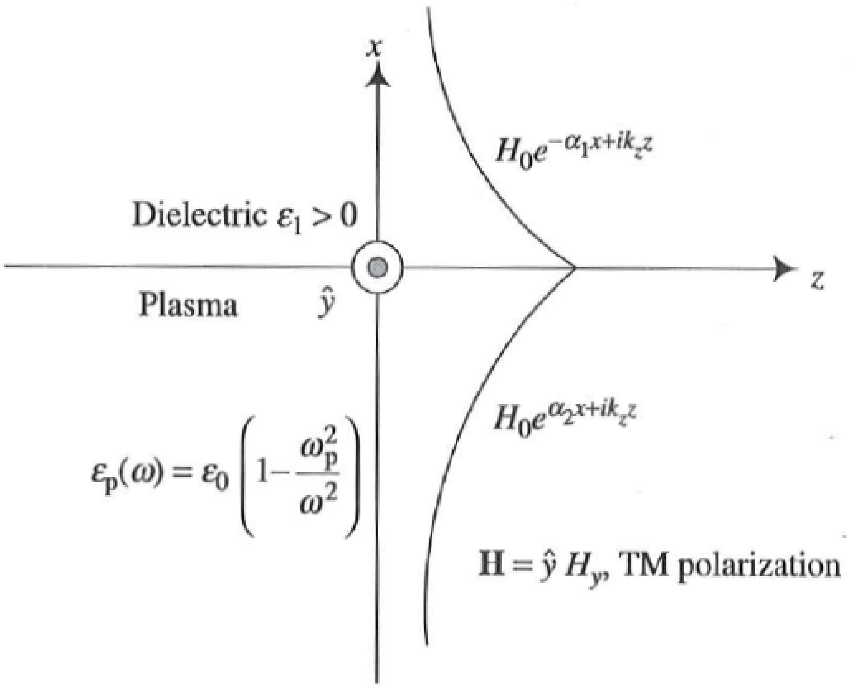


Calculated transverse magnetic field (y-direction) for an SPP above gold for $\lambda_0 = 700$ nm.

Single Plasmon Mode for a Single Interface

To form a mode, that decays in both directions away from the surface but is guided along the z-direction, we need TM polarization:

$$\mathbf{H} = H_y \hat{y} = \hat{y} H_0 e^{ik_z z} \begin{cases} e^{-\alpha_1 x} & \text{for } x \geq 0 \\ e^{\alpha_2 x} & \text{for } x \leq 0 \end{cases}$$



The wave equation in each region results in:

$$-\alpha_1^2 + k_z^2 = \omega^2 \mu_0 \varepsilon_1$$

$$-\alpha_2^2 + k_z^2 = \omega^2 \mu_0 \varepsilon_p(\omega)$$

The electric field is obtained from Maxwell's equations $\mathbf{E} = \nabla \times \mathbf{H} / (-i\omega\varepsilon)$:

$$\mathbf{E} = \begin{cases} \dfrac{1}{i\omega\varepsilon_1}\left(\alpha_1 \hat{z} + \hat{x} i k_z\right) H_0 e^{ik_z z} e^{-\alpha_1 x} & \text{for } x \geq 0 \\ \dfrac{1}{i\omega\varepsilon_p}\left(-\alpha_2 \hat{z} + \hat{x} i k_z\right) H_0 e^{ik_z z} e^{\alpha_2 x} & \text{for } x \leq 0 \end{cases}$$

Surface Plasmon Mode for a Single Interface

The tangential component for **E**, i.e. E_z must be continuous:

$$\frac{\alpha_1}{\varepsilon_1} = -\frac{\alpha_2}{\varepsilon_p}$$

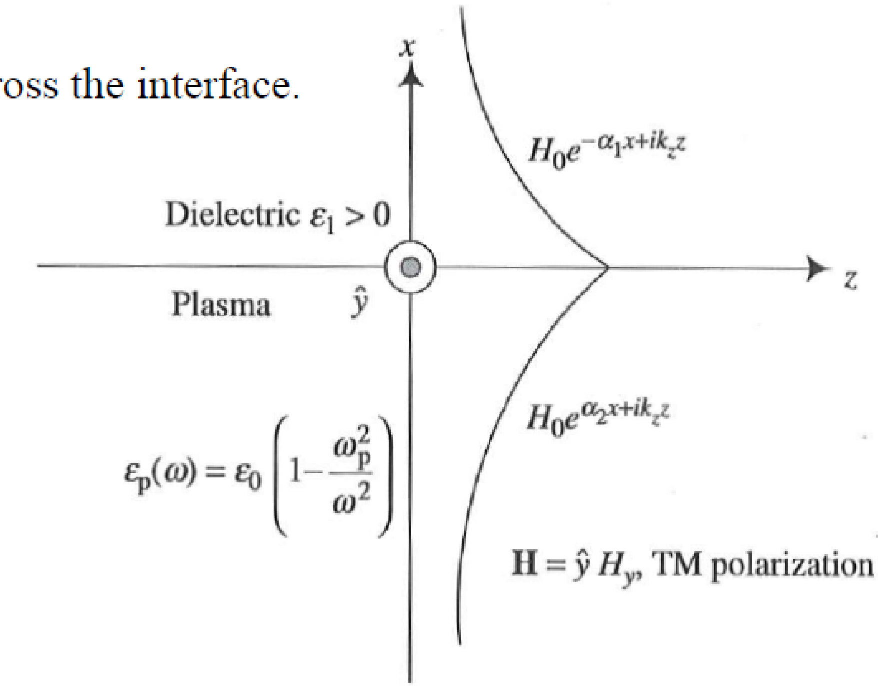
This explains why $\mathrm{Re}[\varepsilon]$ needs to change sign across the interface.

Overall, if $\varepsilon_p < -\varepsilon_1 < 0$, we obtain real solutions:

$$\alpha_1 = \omega\sqrt{\frac{-\mu_0\varepsilon_1^2}{\varepsilon_1+\varepsilon_p}}$$

$$\alpha_2 = \omega\sqrt{\frac{-\mu_0\varepsilon_p^2}{\varepsilon_1+\varepsilon_p}}$$

$$k_z = \omega\sqrt{\frac{\mu_0\varepsilon_1\varepsilon_p}{\varepsilon_1+\varepsilon_p}}$$

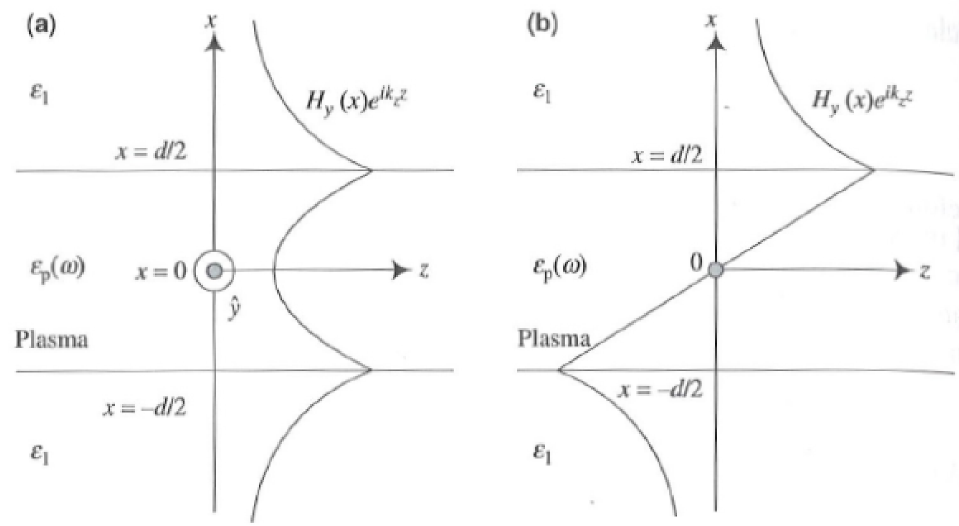


The Poynting vector is given by: $P = \frac{1}{2}\mathrm{Re}[\mathbf{E}\times\mathbf{H}] = \hat{z}\,\frac{k_z}{2\omega}\left|H_0\right|^2 \begin{cases} \frac{1}{\varepsilon_1} e^{-2\alpha_1 x} & \text{for } x \ge 0 \\ \frac{1}{\varepsilon_p} e^{2\alpha_2 x} & \text{for } x \le 0 \end{cases}$

Surface Plasmon Mode for a Double Interface

Even and odd TM modes are guided:

$$\mathbf{H}_{even} = \hat{y}e^{ik_z z}\begin{cases} C_0 e^{-\alpha_1 (x-d/2)} & x \geq d/2 \\ C_1 \cosh \alpha_2 x & |x| \leq d/2 \\ C_0 e^{\alpha_1 (x+d/2)} & x \leq -d/2 \end{cases}$$



The wave equation in each region results in:

$$-\alpha_1^2 + k_z^2 = \omega^2 \mu_0 \varepsilon_1$$

$$-\alpha_2^2 + k_z^2 = \omega^2 \mu_0 \varepsilon_p(\omega)$$

We need H_y and E_z to be continuous at $x = \pm\dfrac{d}{2}$ $\left[\text{Use: } \mathbf{E} = \nabla \times \mathbf{H} / (-i\omega\varepsilon)\right]$

$C_0 = C_1 \cosh \alpha_2 \dfrac{d}{2}$ and $-\dfrac{\alpha_1}{\varepsilon_1} C_0 = C_1 \dfrac{\alpha_2}{\varepsilon_p} \sinh \alpha_2 \dfrac{d}{2}$. Dividing these two gives:

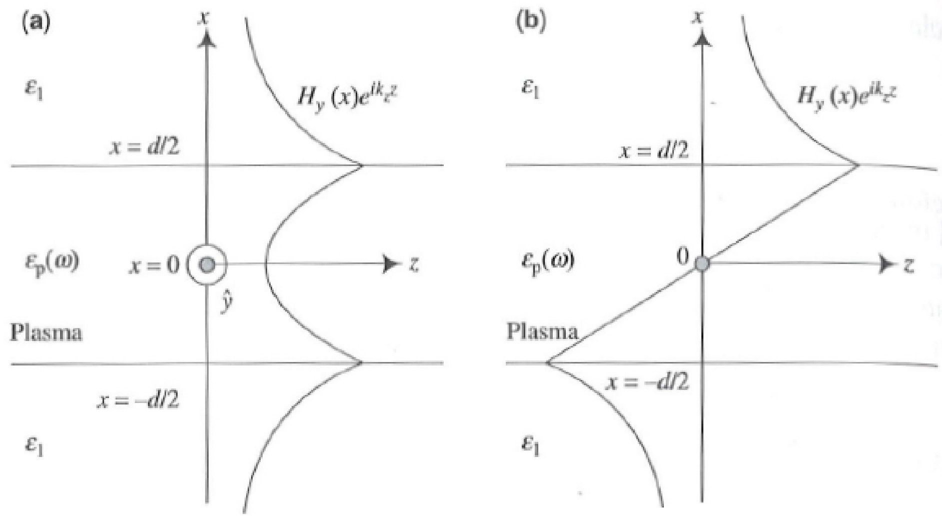
$$\alpha_1 = -\frac{\varepsilon_1}{\varepsilon_p}\alpha_2 \tanh \alpha_2 \frac{d}{2} \qquad \text{(boundary condition)}$$

$$\alpha_2^2 - \alpha_1^2 = \omega^2 \mu_0 \left(\varepsilon_1 - \varepsilon_p\right) \qquad \text{(wave equation)}$$

Surface Plasmon Mode for a Double Interface

Even and odd TM modes are guided:

$$\mathbf{H}_{odd} = \hat{y} e^{ik_z z} C_1 \begin{cases} \sinh \alpha_2 \frac{d}{2} e^{-\alpha_1 (x - \frac{d}{2})} & x \geq d/2 \\ \sinh \alpha_2 x & |x| \leq d/2 \\ -\sinh \alpha_2 \frac{d}{2} e^{\alpha_1 (x + \frac{d}{2})} & x \leq -d/2 \end{cases}$$



The same analysis yields:

$$\alpha_1 = -\frac{\varepsilon_1}{\varepsilon_p} \alpha_2 \coth \alpha_2 \frac{d}{2} \quad \text{(boundary condition)}$$

$$\alpha_2^2 - \alpha_1^2 = \omega^2 \mu_0 \left(\varepsilon_1 - \varepsilon_p \right) \quad \text{(wave equation)}$$

Surface Plasmon Mode for a Double Interface

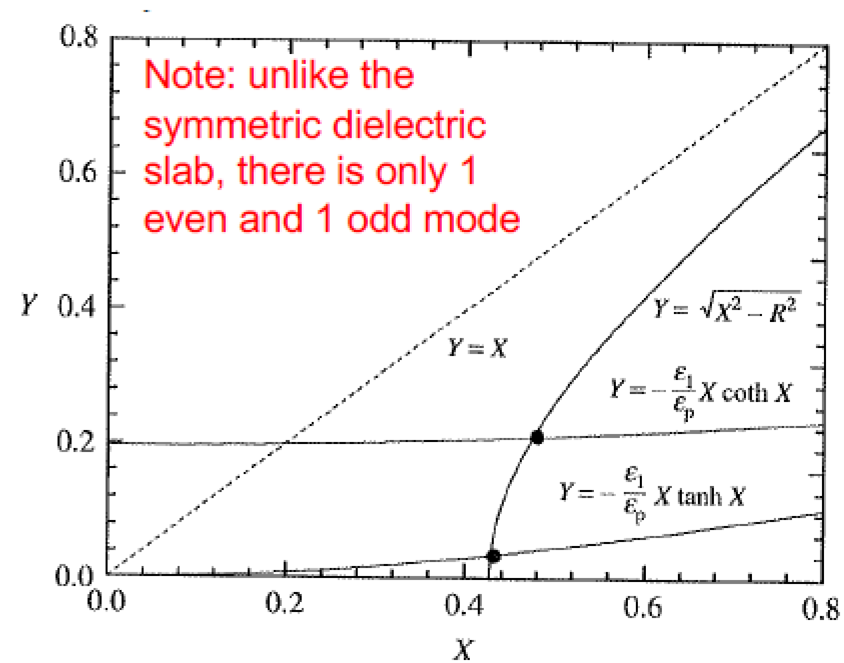
For both even and odd modes, we define:

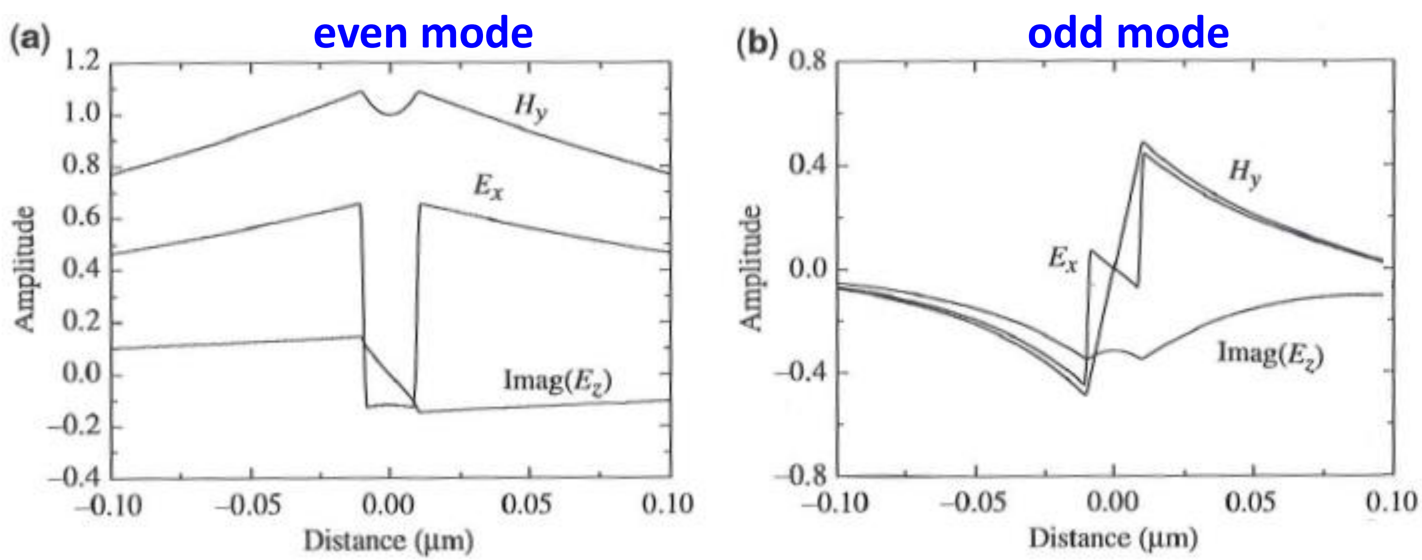
$$X = \alpha_2 \frac{d}{2}, Y = \alpha_1 \frac{d}{2}, R = \omega\sqrt{\mu_0\left(\varepsilon_1 - \varepsilon_p\right)}\frac{d}{2},$$

We need to solve:

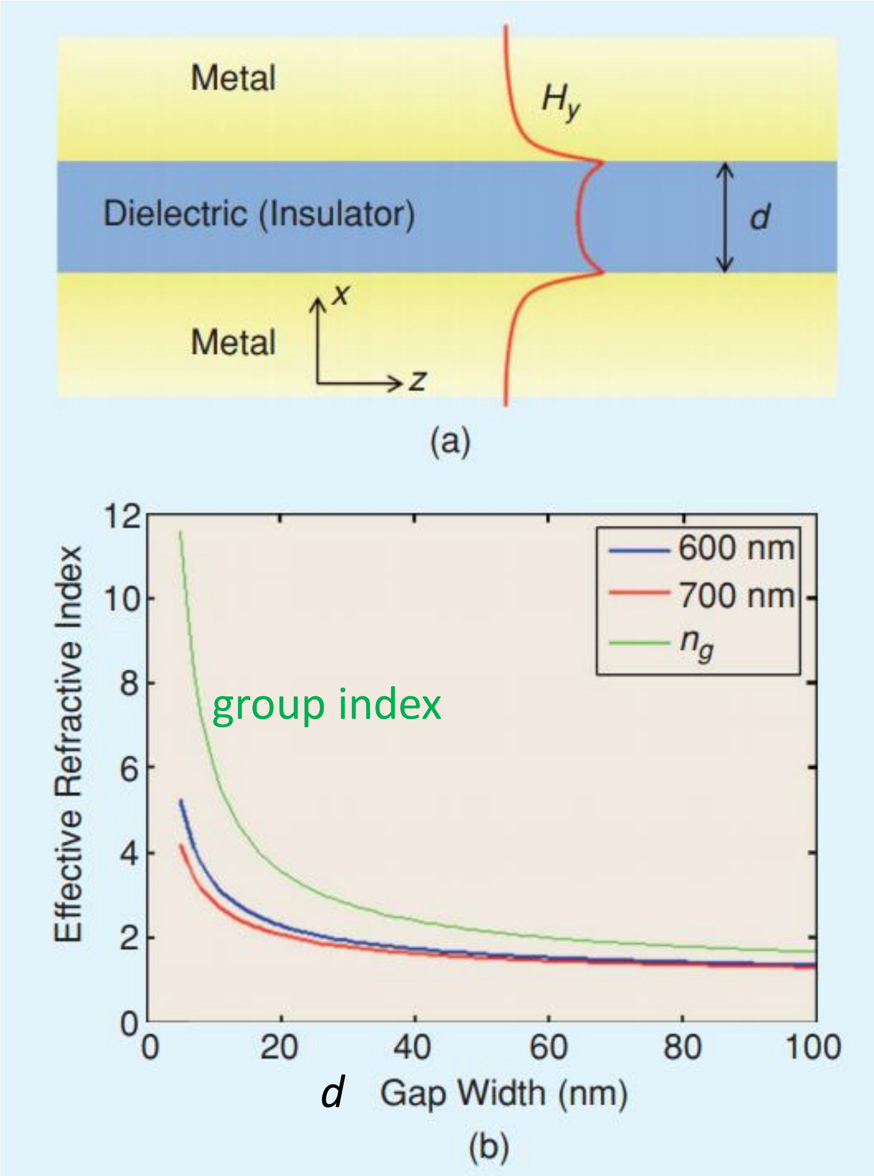
$$X^2 - Y^2 = R^2 \quad \text{(wave equation)}$$

$$Y = \begin{cases} -\dfrac{\varepsilon_1}{\varepsilon_p} X \tanh X & \text{(even modes)} \\ -\dfrac{\varepsilon_1}{\varepsilon_p} X \coth X & \text{(odd modes)} \end{cases}$$





MIM structure



Calculate effective index from this relation

$$\tanh\left(\frac{d}{2}\sqrt{\beta^2-\left(\frac{\omega}{c}\right)^2\varepsilon_d}\right)=$$

$$-\frac{\varepsilon_d}{\varepsilon_m}\sqrt{\frac{\beta^2-\left(\frac{\omega}{c}\right)^2\varepsilon_m}{\beta^2-\left(\frac{\omega}{c}\right)^2\varepsilon_d}}$$

$$\boxed{\beta\equiv k_z}$$

Light slows down as *d* is made smaller

Plasmonics

A simple modulator may consist of a single metal stripe embedded in dielectric, with the same stripe being used to guide and control the optical radiation propagating in the form of surface plasmon polaritons (SPPs).

Various hybrid modulator structures have been proposed for integration

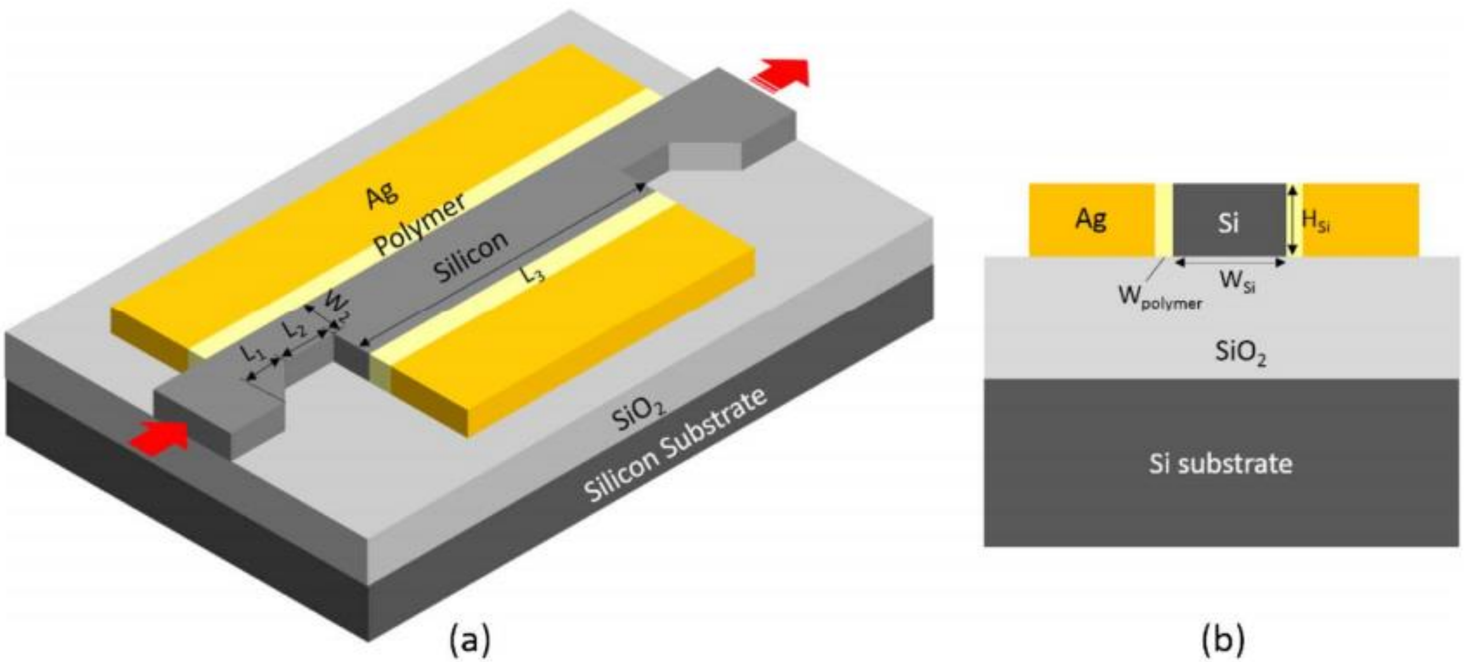


Fig. 1. (a) Schematic perspective view of the proposed optical intensity modulator. L_1, L_2, and L_3 are taper, SSHW, and DSHW lengths, respectively. (b) Cross-sectional view of the DSHW.

Sun et al., IEEE Photonics J.; Vol. 6, n. 3, pp. 1–10 (2014)

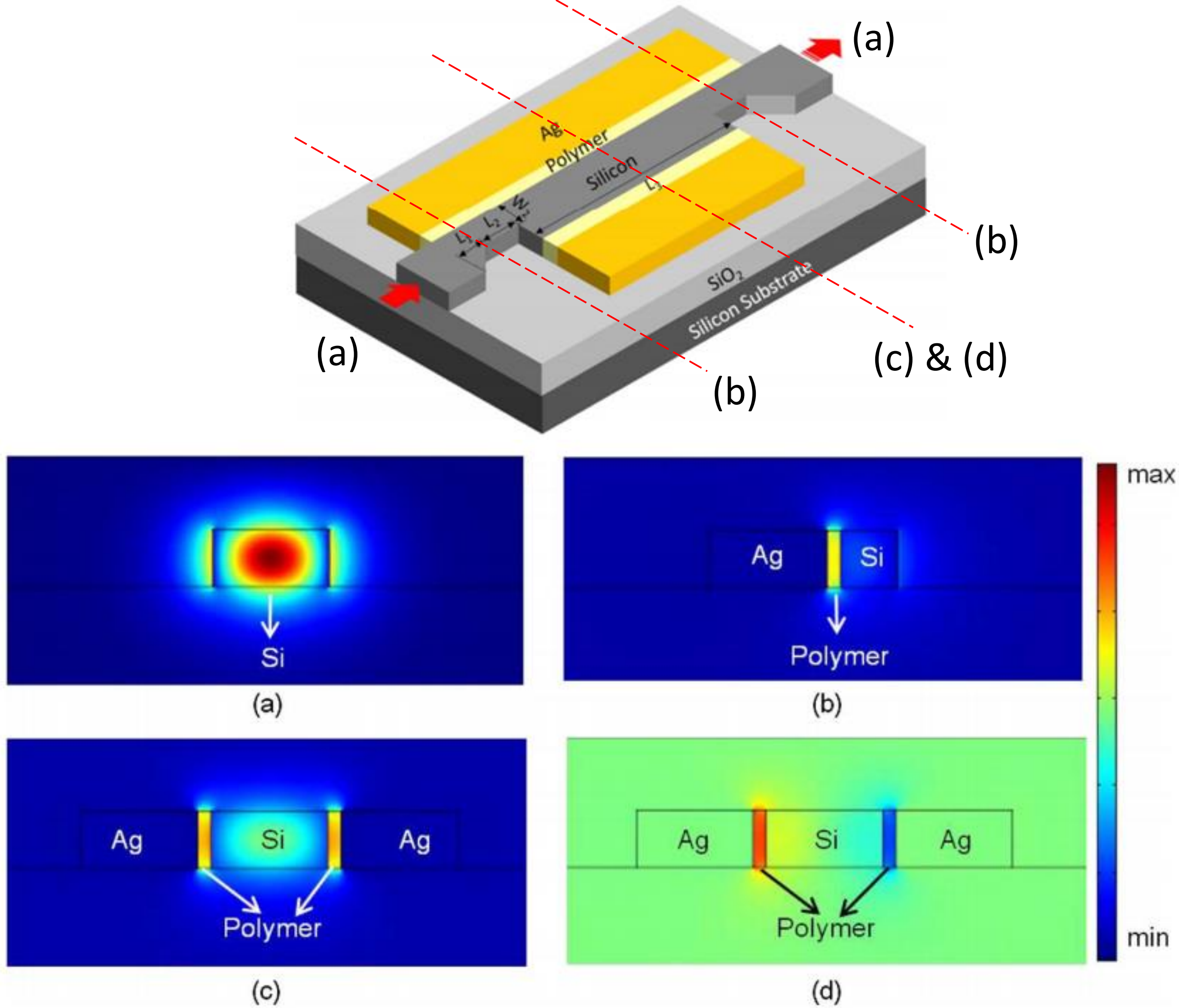


Fig. 2. Transverse electric field mode patterns for various waveguide modes: (a) Input/output silicon waveguide mode, (b) SSHW mode, (c) DSHW quasi-even mode, and (d) DSHW quasi-odd mode. $W_{Si} = 450$ nm, $H_{Si} = 250$ nm, and $W_{polymer} = 50$ nm.

Sun et al., IEEE Photonics J.; Vol. 6, n. 3, pp. 1–10 (2014)

Plasmonics

Metal based plasmonics have limitation, such as fixed plasma frequency in the material (limited range of operation frequencies) and integration challenges. Recent investigations look at semiconductor-based plasmonic devices.

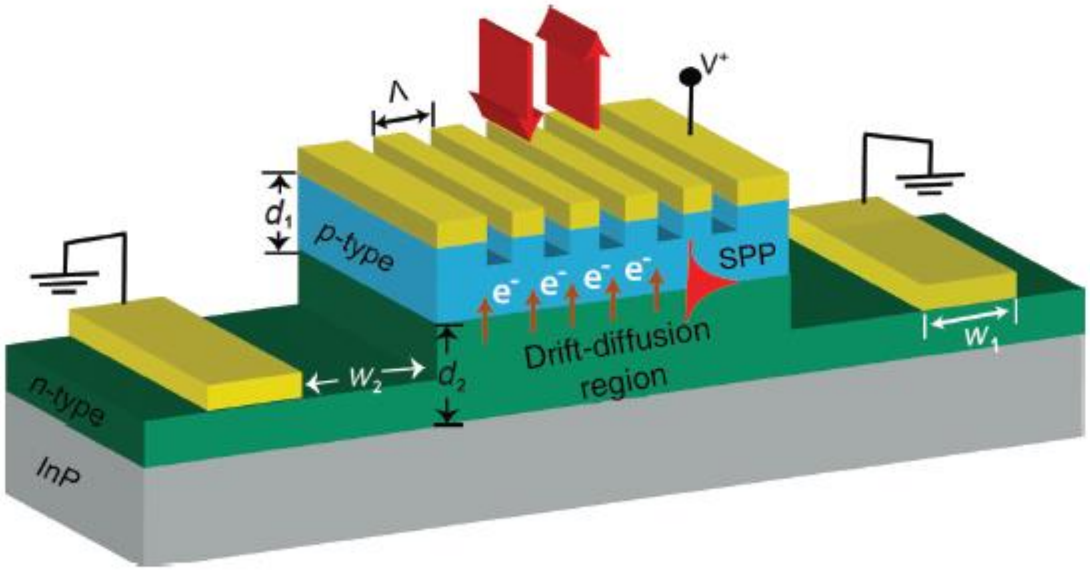


Figure 1: Basic schematic of the surface plasmon polariton diode (SPPD).
The SPPD consists of a lattice-matched indium gallium arsenide ($In_{0.53}Ga_{0.47}As$) pn++ junction grown epitaxially on an indium phosphide (InP) substrate. A grating with period $\Lambda = 2.4$ µm is used to couple mid-IR incident light to the surface plasmon polariton (SPP) modes propagating the junction interface. The relevant device sizes are as follows; $d_1 = 0.75$ µm, $d_2 = 1$ µm, $w_1 = 100$ µm, and $w_2 = 50$ µm.

Vinnakota et al., Nanophotonics 2020; 9(5): 1105–1113

Reading Assignments:

- Section 12.1 of Chuang’s book
- Chapter 8 of Chuang’s book
- Chapter 13 of Chuang’s book