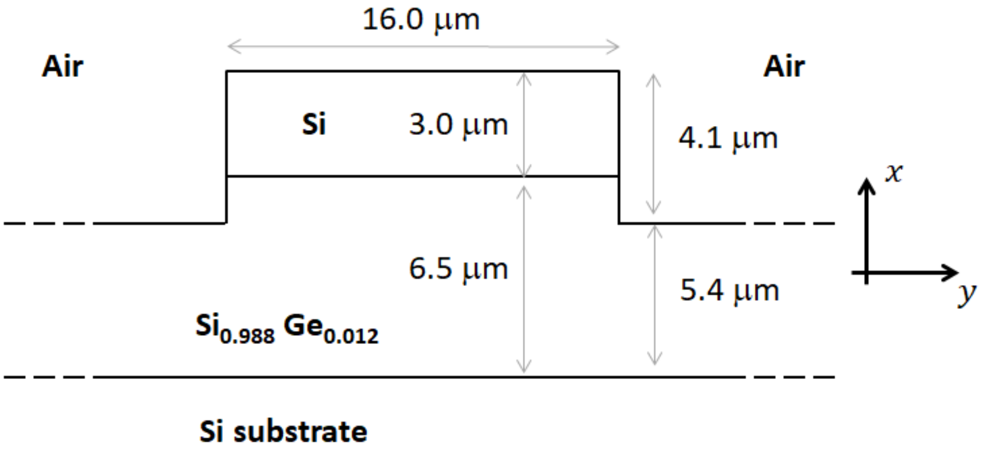
Take-Home Exam – ECE 536 (Spring 2022)

Open book, open anything. Work individually, no discussing or sharing. Due Date: May 12, 2022 (Final Exam Day). Complete four of five parts of your choice (you may complete all five, too).

Part 1 – Effective Index Solution of a Ridge Waveguide

Consider a Silicon-Germanium Ridge Waveguide operating at λ = 1.32 μm. The guided light is polarized with electric field along x.



Assume indices of refraction

$$n_{\mathrm{Si}} = 3.5 \qquad n_{air} = 1.0 \qquad n_{\mathrm{Si}_{1-x}\mathrm{Ge}_x} = n_{\mathrm{Si}} + 0.104x$$

Use the effective index method to determine the eigenvalue β (z-component of the wave vector) for the fundamental mode of the waveguide at the wavelength of operation. Outline all the steps taken and specify all the parameters obtained (β, transverse wave vector, attenuation constant in the cladding) for every step considered to arrive at the final solution.

Bonus activity – This is not required but if you have time/interest consider generating the full curve for the normalized propagation coefficient vs. the normalized frequency (V-parameter)

Notes and hints:

- Since the evanescent field penetrating the Si cap of the ridge decays fairly rapidly, assume that the Si cap is practically infinite in the x-direction.
- For steps where the structure considered has different cladding materials (air and Si) the characteristic equation for asymmetric waveguide should be used.
- Consider the appropriate polarization (TE or TM) at each step.

Part 2 – III-V Compounds for emission at 1.2 μm

For each case make use of the E_g-*a* diagram (you can use the blank chart provided or an equivalent one), to illustrate the useful domains of semiconductor alloys and the working points determined. Make also use of any available published information to go beyond simple linear Vegard's law when possible, for instance, using information from the textbooks or the review by Adachi (2017) posted on the class website, etc.

(a) One possible material is the ternary alloy $In_xGa_{1-x}As$. **Estimate the concentration x necessary for emission at 1.2 μm and the corresponding lattice constant. Explain why it is problematic to grow this material directly on available binary substrates (GaAs, InP, InAs).**

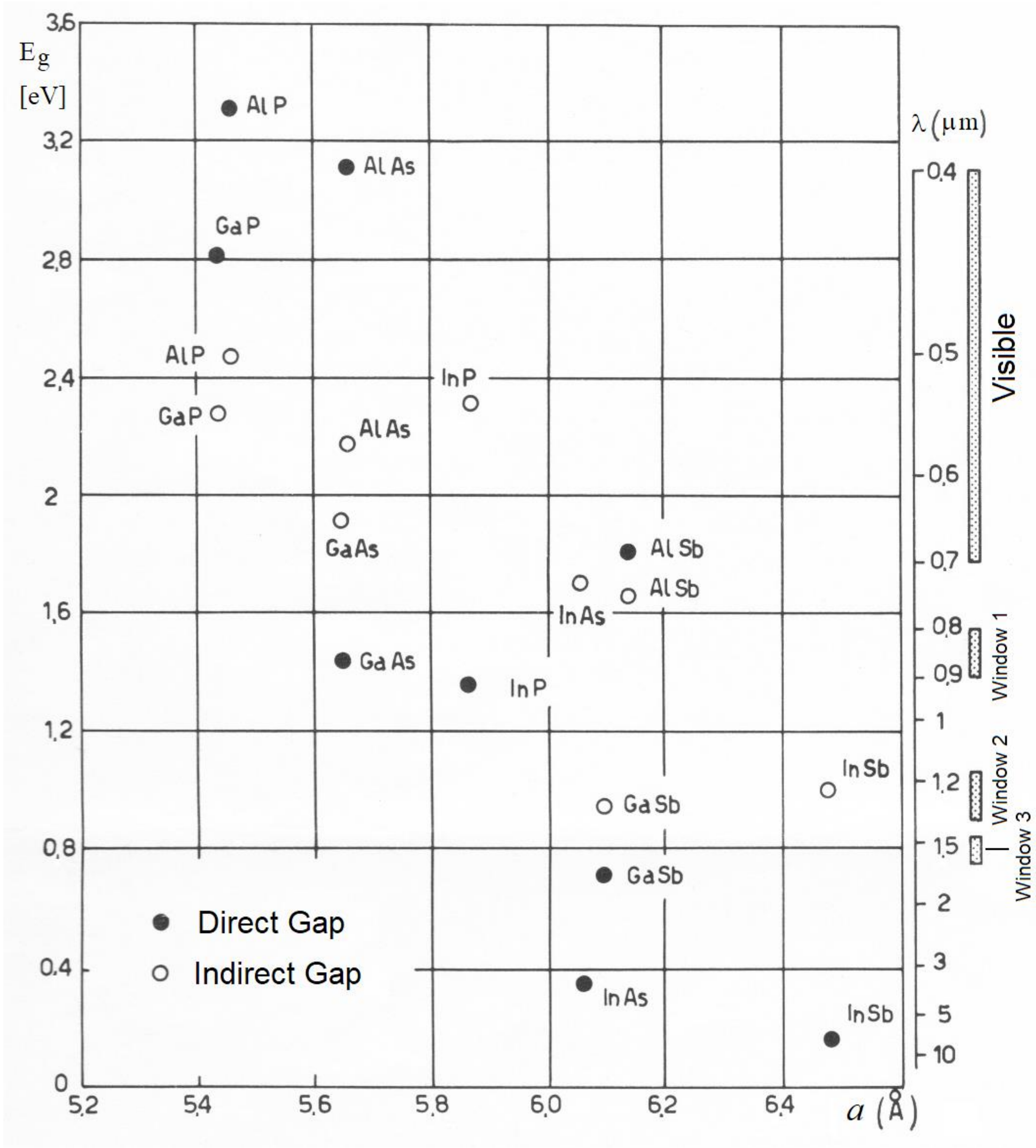
(b) A commonly used material in optoelectronics is $Ga_xIn_{1-x}As_yP_{1-y}$ which is a 2:2 quaternary alloy. **Estimate the concentrations x and y necessary to obtain emission at 1.2 μm while satisfying lattice matched condition to InP.**

(c) Another possible candidate is the ternary alloy $GaAs_zSb_{1-z}$. In principle, growth on InP could be favorable in terms of lattice match at the desired wavelength. **Estimate the corresponding concentration *z* for emission at 1.2 μm. Would a simple sandwich of InP/GaAsSb/InP be sufficient to create both optical and electrical confinement structure? Explain briefly.**

We have to notice that unfortunately it is difficult to grow approximately balanced mixtures of arsenides and antimonides with cheaper liquid epitaxy, due to discontinuity in the phase diagram at the appropriate growth temperature, though growth is easier with MBE or MOCVD at relatively low temperatures. Additionally, growth of $GaAs_zSb_{1-z}$ (or $Al_yGa_{1-y}As_zSb_{1-z}$ which we are going to consider next) is characterized by some corrosivity on InP since there are no atoms in common and thermodynamic thermal equilibrium is hard to establish.

We could consider instead the 2:2 quaternary alloy $Al_y\,Ga_{1-y}\,As_z\,Sb_{1-z}$ grown on top of GaSb. **Would the material system consisting of a sandwich GaSb/$Al_y\,Ga_{1-y}\,As_z\,Sb_{1-z}$/GaSb realize a confinement structure? If not, could you suggest a possible growth alternative using only this material system? Discuss and also illustrate on the diagram.**

(d) The 3:1 quaternary alloy $Al_x\,Ga_y\,In_{1-x-y}\,As$ is also a possible material for growth on an InP substrate. **Estimate the concentrations x and y necessary to realize a lattice matched emitter at 1.2 μm.**


E_g [eV]
3,6
3,2
2,8
2,4
2
1,6
1,2
0,8
0.4
0
5,2
5,4
5,6
5,8
6,0
6,2
6,4
a (Å)
Al P
Al As
Ga P
AlP
GaP
In P
Al As
Ga As
Al Sb
In As
Al Sb
Ga As
In P
In Sb
Ga Sb
Ga Sb
In As
In Sb
● Direct Gap
○ Indirect Gap
λ (μm)
0,4
0,5
0,6
0,7
0,8
0,9
1
1,2
1,5
2
3
5
10
Visible
Window 1
Window 2
Window 3

Part 3 – Distributed Bragg Reflector

We want to design a distributed Bragg reflector grown on a substrate with refractive index $n_{sub} = 1.5$, for use in the complete optical range. The top layer of the stack is in direct contact with air ($n_{air} = 1.0$). The structure is realized alternating ZnS layers of higher refractive index ($n_H = 2.32$) with MgF_2 layers of lower refractive index ($n_L = 1.38$). The layers have thicknesses t_H and t_L such that

$$n_H \, t_H = n_L \, t_L = \frac{\lambda_0}{4}$$

where λ_0 is the central wavelength of the band (Bragg wavelength). For this application, $\lambda_0 = 550$ nm. The grating is realized so that the first and the last layers are ZnS. In this way, the total number of individual layers is always odd. If the number of complete pairs of layers is *N*, the number of pairs used is *N* + ½ and the total number of layers is 2N+1. So, a typical complete structure would include:

- an interface between substrate and ZnS, followed by
- N complete pairs of dielectric layers ZnS/MgF_2 terminated by
- a pair of layers ZnS/Air

(a) Draw a diagram to scale in space of the index of refraction variation along the axis of the layered structure for the case *N* = 3, inclusive of substrate and air sides, to get a good feeling about the typical dimensions of these devices. Label all lengths as appropriate. List the physical wavelengths associated with the different materials.

(b) Write down the specific formula for the reflectivity of the complete structure at the Bragg wavelength, in terms of indices of refraction including substrate and air layer, and plot the reflectance *R* at the Bragg wavelength as a function of the odd number 2N+1 of layers in the grating stack.

(c) From the estimations above determine the number of layers necessary to obtain a Reflectance closest to *R* = 0.995 at the Bragg wavelength, then plot the reflectance for this grating stack in the spectrum from 400 to 800 nm.

(d) Estimate the stop-band directly from the graph and compare with the commonly used design formula below. Comment as you see fit.

$$\Delta \lambda_0 = \frac{4\lambda_0}{\pi} \sin^{-1}\left(\frac{n_H - n_L}{n_H + n_L} \right)$$

Part 4 – VCSEL Threshold

Consider a vertical-cavity surface-emitting laser (VCSEL) consisting of an active layer sandwiched between two Bragg reflectors. The active layer consists of a multiple quantum well structure with effective thickness d_a = 30 nm with a relative confinement factor $\Gamma_r = 1.8$. Assume that

- The cavity length (including spacing layers) is L = 1.6 μm.
- The two mirrors have identical reflectance R = 99%.
- The internal loss coefficient is α_i = 18 cm^{-1}.
- The differential gain is g' = 5×10^{-16} cm^2.
- The carrier density at transparency is N_{tr}= $1.2 \times l0^{18}$ cm^{-3}.
- The radiative lifetime is τ_e = 3 ns.
- The active area is circular with diameter *D* = 10 μm.
- Because of the geometry, current leakage tends to be very small, therefore, $\eta_i \approx 1$.

Estimate the carrier density and the current at threshold for this laser using the simple model introduced in class.

Part 5 – VCSEL Design

Consider a VCSEL designed for emission at 980nm with a threshold gain of 750 cm^{-1} consisting of two InGaAs quantum wells, each 10nm, and an alternating stack of GaAs/AlAs quarter wave layers (for this problem, assume the GaAs layer is adjacent to the cavity).

a) How would the designs of the gain region differ for a half-wave and full-wave cavity?

b) Assuming that the top and bottom DBR stacks provide the same reflectivity and that the gain region is placed as in (a), estimate the number of DBR pairs required for lasing (assume $n_0 = n_t = n_{GaAs}$).

c) Using the matrix method described in 5.8-9 in the text (and 3.1-5 of Corzine and Coldren) plot the reflection spectra for

   (i) DBR stacks of GaAs/AlAs designed for 980nm, one with 10 pairs and one with 20 pairs. Comment on the differences.

   (ii) DBR stacks of GaAs/$Al_{0.85}Ga_{0.15}As$ designed for 980nm with 20 pairs and plot together with the result for 20 pairs obtained at (i) above. Comment on the differences.