Lecture 14 Outline

• Power in Lasers
• LED vs. Laser
• Measurements
Light Output Intensity

\[ P_{out} = \left[ \text{Photon Energy} \right] \times \left[ \text{Photon Density} \right] \times \left[ \text{Effective Volume of Optical Mode} \right] \times \left[ \text{Photon Escape Rate} \right] \]

\[ = \left[ \hbar \omega \right] \times \left[ S \right] \times \left[ wL \, d_{op} \right] \times \left[ v_g \, \alpha_m \right] \]

Since

\[ d_{op} = \frac{d}{\Gamma} \]

\[ I = w \, L \, J \]

\[ P_{out} = \eta_i \left( \frac{\hbar \omega}{q} \frac{\alpha_m}{\alpha_i + \alpha_m} \right) (I - I_{th}) \]
**External Quantum Efficiency**

\[
\eta_e = \frac{dP_{out}/dI}{\hbar \omega / q} = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m} = \eta_i \frac{\ln(1/R)}{\alpha_i L + \ln(1/R)}
\]

also

\[
\eta_e^{-1} = \eta_i^{-1} \left[ 1 + \frac{\alpha_i L}{\ln(1/R)} \right]
\]

Plotting \( \eta_i^{-1} \) versus L is a line with a y-intercept of \( \eta_i^{-1} \).

The slope divided by the y-intercept is \( \frac{\alpha_i}{\ln(1/R)} \) and can determine \( \alpha_i \).
External Quantum Efficiency

\[
\eta_e^{-1} \quad \text{Inverse external quantum efficiency}
\]

\[
\eta_e^{-1} = \eta_i^{-1} \left[ 1 + \frac{\alpha_i L}{\ln \left( \frac{1}{R} \right)} \right]
\]
Leakage Current

\[ I = I_A + I_L = JwL + I_L \]

\[ I_{th} = J_{th} wL + I_{L@th} = \frac{qn_{th} (wLd)}{\eta_i \tau_e (n_{th})} + I_{L@th} \]

Revised Expression for \( P_{out} \)

\[ P_{out} = \eta_i \frac{\hbar \omega}{q} \frac{\alpha_m}{\alpha_m + \alpha_i} \left( I - I_{th} - \Delta I_L \right) \]
Temperature Dependence

Laser threshold and efficiency vary with temperature since $g(\hbar \omega)$, Auger recombination, and other processes are temperature dependent.

\[
\uparrow T \quad \uparrow I_{th}(T) \quad \downarrow \eta_e(T)
\]

\[
I_{th}(T) = \{\text{constant}\} \, e^{T/T_0} = I_{th}(T_a) \, e^{(T-T_a)/T_0}
\]

\[
\eta_e(T) = \{\text{constant}\} \, e^{-T/T_1} = \eta_e(T_a) \, e^{-(T-T_a)/T_1}
\]
Saturation of Laser Output Power

Possible causes are:
- increasing leakage current
- Junction heating
- Increasing internal absorption $\alpha_i$

In continuous wave (CW) regime there is typically a maximum power $P_{max}(T)$
Saturation of Laser Output Power

\[
\begin{align*}
P_{in} &= IV \\
\eta_{wp} &= \frac{P_{out}}{P_{in}} = \frac{\frac{\hbar \omega}{q} \frac{\alpha_m}{\alpha_i + \alpha_m} \eta_i (I - I_{th})}{IV}
\end{align*}
\]

Power conversion efficiency is high in lasers when compared to other light emitter, but it typically does not exceed 60%. The power which is not converted into light is dissipated as heat, which needs to be removed by a heatsink.

The heatsink is characterized by a \textit{thermal resistance} \(R_T\)

\[
R_T = \frac{\Delta T}{P_{in} - P_{out}}
\]

\[
\Delta T = R_T (P_{in} - P_{out}) = R_T (1 - \eta_{wp}) P_{in}
\]

\(\Delta T = \text{laser overheating}\)
Improving $P_{max}$

To decrease overheating $\Delta T$
- Develop better heatsink and laser mounting to reduce $R_T$
- Improve laser “wall-plug efficiency”
  - Reduce voltage drop across heterostructure
  - Reduce internal loss $\alpha_i$
  - Increase injection efficiency $\eta_i$
  - Reduce threshold current $I_{th}$

To decrease temperature sensitivity
- Suppress carrier leakage which affects external quantum efficiency
- Minimize non-radiative recombination which reduces the concentration at threshold
Improving $P_{max}$

Laser substrate is thick and has high thermal resistance. High power lasers are mounted upside down with the $p$-layer in contact with the heatsink.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$-cladding</td>
<td>$1 - 2 \mu m$</td>
</tr>
<tr>
<td>Active layer</td>
<td>$1 \mu m$</td>
</tr>
<tr>
<td>$n$-cladding</td>
<td>$1 - 2 \mu m$</td>
</tr>
<tr>
<td>Substrate</td>
<td>$100 \mu m$</td>
</tr>
</tbody>
</table>
Improving $P_{\text{max}}$

The thermal resistance is inversely proportional to the cavity length. For high power operation, a long cavity is better as long as laser efficiency is not affected too much.

$$\alpha_m = \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

$$\eta_e = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m}$$

As mirror losses are lower with increasing cavity length, the relative role of internal loss increases. This causes quantum efficiency to drop.
Internal loss $\alpha_i$

Much of the internal loss is due to free carrier absorption. Transitions between HH and SH valence bands, known as “intervalance band absorption” are an important component.

The main contribution to net internal loss is absorption in highly doped cladding regions.

The confinement factor is somewhat reduced in a broadened waveguide but losses in the cladding regions are greatly reduced, leading to a reduced threshold current density.

Efficiency of the broadened waveguide is less sensitive to cavity length, so this structure is suitable for power applications, increasing maximum CW power.
Minimize threshold current to increase power output

\[ P_{out} = \eta_i \frac{\hbar}{\omega} \frac{\alpha_m}{\alpha_i + \alpha_m} (I - I_{th}) \]

\[ I_{th} = I_{tr} + I_{loss} \]

To minimize \( I_{tr} \)
- Reduce number of quantum wells to a minimum
- Compressive strain in QW reduces difference in effective mass between C and HH bands

To minimize \( I_{loss} \)
- Compressive strain in QW reduces difference in effective masses and increases differential gain
Minimize threshold current to increase power output

$$m_e \ll m_{hh}$$

$$m_e \approx m_{hh}$$
LED versus Lasers

Spontaneous Emission and
Amplified Spontaneous Emission (ASE)
LEDs for Display

Critical Angle
LEDs for Communications
At transparency

\[ L_W (\hbar \omega_{tr}) = L_F (\hbar \omega_{tr}) \]
**LED versus Laser**

**Window Light**

\[ L_w (h\omega) = h\omega \ r_{spon} (h\omega) \ w dL \]

**Facet Light (ASE)**

\[
L_F (h\omega) = h\omega wd \int_{z=0}^{L} r_{spon} (h\omega) e^{G_n(h\omega)z} \ dz
\]

\[
= h\omega wd \ r_{spon} (h\omega) \left[ \frac{e^{G_n(h\omega)L}}{G_n(h\omega)} - 1 \right]
\]

**net modal gain**

\[ G_n (\hbar\omega) = \Gamma g - \alpha_i \]

**LASER**

\[ r_{spon} (\hbar\omega) \equiv \text{Spontaneous Emission Rate per Unit Volume} \]
Definition: Net Modal Gain

This term typically indicates the net gain in the laser cavity structure, adjusted to take into account the partial overlap between the optical field and the active region occupied by carriers, through the confinement factor $\Gamma$. 

$$G_n (\hbar \omega) = \Gamma g - \alpha_i$$
Chuang, O’Gorman, Levi, IEEE J. Quantum Electronics, vol. 29, no. 6, p. 1631, 1993. Active region is an InGaAsP layer, lattice matched and sandwiched by InP layers in a buried bulk double heterostructure device ($d = 0.14 \text{ \mu m}$, $L = 260 \text{ \mu m}$, $W = 1 \text{ \mu m}$, $E_g = 968.4 \text{ meV}$).
Carrier Pinning

ASE and Optical Gain Measurements
Experimental Determination of Optical Gain

Measurements of gain continue to be an important component to validate and refine the design of a laser in order to meet desired performance specifications.

**Optical stripe-length method**
An external laser source is used to excite the sample under investigation. The laser beam is focused as a stripe with a cylindrical lens. The *amplified spontaneous emission* (ASE) is measured as a function of the stripe length and the gain is extracted from a fit of the data. This method is applied to a material which has not yet been processed into a complete laser structure.

**Hakki-Paoli method** – the laser is operated below threshold. If the length of the device and the facet reflectivity are known, the gain can be evaluated from maxima and minima of the Fabry-Pérot spectrum recorded with a spectrometer of sufficient resolution.

**Transmission method** – Requires a broadband weak light source, covering the spectral region of interest, transmitted through the device (which should operate in the fundamental mode with suppression of the Fabry-Pérot modes by deposition of anti-reflective coatings on the facets). The amplification of this broadband probe light in the diode is measured and it provides directly the gain spectra.
Review again: effective index $n_e$

\[ k_z = \frac{2\pi}{\lambda_z} = n_e k = n_e \frac{2\pi}{\lambda} \]

\[ n_e = \frac{2\pi}{\lambda_z} \frac{\lambda}{2\pi} = \frac{\lambda}{\lambda_z} = \frac{v_p}{f v_p z} = \frac{v_p}{v_p z} \]

$n < n_e < n_1$

Strictly, each mode is associated with a specific effective index at a given frequency
Optical Fields in Cavity with Gain

**Fabry-Perot Cavity**

Initial Field: $E_{sp}(\lambda)$

Field After Single Pass: $E'_{sp}(\lambda) = E_{sp}(\lambda) + E_{sp}(\lambda) r_1 r_2 e^{i2kL}$

General Expression: $E_{ASE}(\lambda) = E_{sp}(\lambda) \left[ 1 + r_1 r_2 e^{i2kL} + (r_1 r_2 e^{i2kL})^2 + (r_1 r_2 e^{i2kL})^3 + \ldots \right]$ \[= \frac{1}{1 - a} \]

$E_{ASE}(\lambda) = \frac{E_{sp}(\lambda)}{1 - r_1 r_2 e^{i2kL}}$

$n_e$ is the effective index

$G_n = \Gamma g - \alpha_i$ is the net modal gain

$k = k' - i \frac{G_n}{2} = \frac{2\pi}{\lambda} n_e - i \frac{G_n}{2}$

$G_n = \Gamma g - \alpha_i$
ASE Behavior below threshold

$I = 6 \, mA$

$I = 8 \, mA$
ASE Behavior below threshold

I = 13 mA

I = 15 mA
ASE Power Spectrum

\[ P(\lambda) \propto |E_{ASE}(\lambda)|^2 = \frac{|E_{sp}(\lambda)|^2}{|1 - r_1 r_2 e^{i2kL}|^2} = \frac{|E_{sp}(\lambda)|^2}{(1 - A)^2 + 4A\sin^2(k'L)} \]

\[ A = \sqrt{R_1 R_2 e^{G_n L}} \]

\[ R_1 = |r_1|^2 \]

\[ R_2 = |r_2|^2 \]

FSR = Free Spectral Range
FWHM = Full Width Half Maximum
Modal Gain from ASE Spectra – 1

\[ P(\lambda) \propto I(\lambda) = \left| E_{ASE}(\lambda) \right|^2 = \frac{\left| E_{sp}(\lambda) \right|^2}{\left| 1 - r_1 r_2 e^{i2kL} \right|^2} = \frac{\left| E_{sp}(\lambda) \right|^2}{(1 - A)^2 + 4A \sin^2(k'L)} \]

Maximum of ASE Spectrum: \( k'L = m\pi \) and \( I_{\text{max}} = \frac{\left| E_{sp}(\lambda_{\text{max}}) \right|^2}{(1 - A)^2} \)

Minimum of ASE Spectrum: \( k'L = \left( m + \frac{1}{2} \right)\pi \) and \( I_{\text{min}} = \frac{\left| E_{sp}(\lambda_{\text{min}}) \right|^2}{(1 + A)^2} \)

Taking the ratio (nearby ASE peaks): \( \frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(1 + A)^2}{(1 - A)^2} \)
Modal Gain from ASE Spectra – 2

Taking the ratio (nearby ASE peaks):
\[
\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(1 + A)^2}{(1 - A)^2}
\]

Solving for A:
\[
A = \sqrt{\frac{I_{\text{max}}}{I_{\text{min}}}} - 1 = \sqrt{R_1 R_2 e^{G_n L}}
\]

Solving for G:
\[
G_n = \frac{1}{L} \ln \frac{\sqrt{I_{\text{max}}/I_{\text{min}}} - 1}{\sqrt{I_{\text{max}}/I_{\text{min}}} + 1} + \frac{1}{2L} \ln \frac{1}{R_1 R_2}
\]

G is the net modal gain.

Note: \[ G_n(\lambda) = \Gamma g(\lambda) - \alpha_i = Q_r + \alpha_m \] where \[ Q_r = \frac{1}{L} \ln \frac{\sqrt{I_{\text{max}}/I_{\text{min}}} - 1}{\sqrt{I_{\text{max}}/I_{\text{min}}} + 1} \]
Hakki-Paoli Method for Gain Measurement

- Measure ASE spectrum
- Take the ratio of the magnitude of $I_{max}$ and $I_{min}$ for the peaks near the $m^{th}$ mode $\lambda_m$
- Calculate the mirror reflectivity and either measure the cavity length or calculate it from the measured mode spacing and the effective index $n_e$
- Gain is calculated using

$$G_n(\lambda) = Q_r + \alpha_m$$
Hakki-Paoli Method for Gain Measurement

I = 6 mA

Signal (V)

Wavelength (nm)

spectrum range enlarged in next slide
Hakki-Paoli Method for Gain Measurement
Cassidy’s Method – Variant of Hakki-Paoli Method

Based on the Hakki-Paoli method, except that the ratio: \( I_{\text{avg}} / I_{\text{min}} \) is computed.

\[
I_{\text{avg}} = \sqrt{I_{\text{max}} \cdot I_{\text{min}}}
\]

\( I_{\text{avg}} \) is more accurate to measure than \( I_{\text{max}} \) which depends on resolution of the instrument.

In alternative one can use instead \( I_{\text{max}} / I_{\text{avg}} \) if data is noisy or \( I_{\text{max}} \) is more accurate than \( I_{\text{min}} \).

JAP 56, 3096 (1984)
JQE 41, 532 (2005)
Left: ASE spectra for a 1.3 μm laser with a bulk active region and uncoated mirror facets

Right: Modal optical gain measurements with Hakki-Paoli method (continuous lines) and with Cassidy method (dotted lines)
Reading Assignments:

Sections 10.1 and 10.2 of Chuang’s book
Section 7.6 of Chuang’s book (waves in lossy media)