Lecture 18 Outline

• Multiple Quantum Wells Lasers
• Scaling Law for Multiple Quantum Wells
• Strain Effects
Quantum Well (QW) Lasers
Types of QW Lasers

(a) Single-Quantum-Well Separate-Confinement Heterostructure

(c) Graded-Index Separate-Confinement Heterostructure (GRINSCH)

(b) Multiple-Quantum-Well Separate-Confinement Heterostructure
Peak Gain versus Current Density

For a given carrier concentration $n$

$$J = J_{rad} + J_{Aug} + J_{leak}$$

$$J_{rad} = qL_z R_{sp} (n) Bn^2$$  
$$J_{Aug} = qL_z R_{Aug} (n) Cn^3$$

Commonly used empirical formula

$$g_p(J) = g_0 \left[ 1 + \ln \frac{J}{J_0} \right] = g_0 \ln \frac{J}{J_{tr}}$$

transparency at $J = J_{tr} = J_0 e^{-1}$
Let's define

\[ J_w = \eta J_{\text{applied}} = \text{injected current density for SQW} \]

\[ g_w = g_0 \left[ \ln \left( \frac{J_w}{J_0} \right) + 1 \right] = \text{peak gain coefficient for SQW} \]

where usually \( g_w \propto L_z^{-1} \)

\[ J_{tr} = J_0 e^{-1} = \text{transparency current density} \]

\[ \Gamma_w = \text{optical confinement factor per well} \]

\[ \Gamma_w = \Gamma_{op} \frac{L_z}{W_{\text{mode}}} \]

full-width at half maximum of optical mode
Modal gain for a MSQW at threshold

\[ G_{th} = n_w \Gamma_w g_w = \alpha_{tot} = \alpha_i + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]
Scaling Laws for Multiple Quantum Well (MQW) Lasers

Modal gain for a MSQW at threshold

\[ G_{th} = n_w \Gamma_w g_w = \alpha_{tot} = \alpha_i + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]

General Expression of Modal gain for a SQW

\[ G = \Gamma_w g_w = \Gamma_w g_{max} \left[ f_c (h\omega) - f_v (h\omega) \right] = \frac{\Gamma_{op} L_z}{W_{mode}} g_{max} \]

\[ h\omega = E_{h1}^{e1} (0) \]

\[ g_{max} = C_0 |\hat{e} \cdot M_{ch}|^2 \frac{m^*}{\pi \hbar^2 L_z} \delta_{nm} \]

Modal gain for a MQW

\[ G = n_w \Gamma_w g_w \]
Threshold Current Density

- Injected Current Density per QW at Threshold
  \[ J_{w,th} = \frac{\eta J_{th}}{n_w} \]

- Peak gain
  \[ g_w = g_0 \left\lfloor \ln \left( \frac{J_w}{J_0} \right) + 1 \right\rfloor \]
  \[ n_w g_w = n_w g_0 \left\lfloor \ln \left( \frac{J_w}{J_0} \right) + 1 \right\rfloor = n_w g_0 \left\lfloor \ln \left( \frac{n_w J_w}{n_w J_0} \right) + 1 \right\rfloor \]

\( n_w J_w = \text{total injected current density} \)

\( n_w J_0 e^{-1} = \text{total injected current density for transparency} \)
Threshold Current Density for MQW

\[ n_w g_w = n_w g_0 \left[ \ln \left( \frac{J_w}{J_0} \right) + 1 \right] = n_w g_0 \left[ \ln \left( \frac{n_w J_w}{n_w J_0} \right) + 1 \right] \]

\[ \frac{n_w J_w}{n_w J_0} = e^{\left( \frac{n_w g_w}{n_w g_0} \right) - 1} \Rightarrow \eta J_{th} = n_w J_w = n_w J_0 \exp \left[ \left( \frac{g_w}{g_0} \right) - 1 \right] \]

At threshold Gain = Loss

\[ n_w \Gamma_w g_w = \alpha_{tot} \]

\[ J_{th} = \frac{n_w J_0}{\eta} \exp \left[ \left( \frac{g_w}{g_0} \right) - 1 \right] \]

\[ J_{th} = \frac{n_w J_0}{\eta} \exp \left[ \left( \frac{\alpha_{tot}}{n_w \Gamma_w g_0} \right) - 1 \right] \]

\[ \ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) - 1 \]
Threshold Current Density for MQW

\[ \ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0 \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right)} - 1 \]
We have two main parameters for device optimization:

- Cavity length: $L$
- Number of quantum wells: $n_W$
Optimal Cavity Length to Minimize Threshold

\[
\ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) - 1
\]

\[
= \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1
\]

\[
L_{opt} = \frac{1}{2} \frac{1}{n_w \Gamma_w g_0} \ln \frac{1}{R_1 R_2}
\]
Minimum Threshold Current for $L_{opt}$

\[
\ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1
\]

\[
I_{th} = \frac{w L n_w J_0}{\eta} \exp \left[ -\frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1 \right] = \text{const} \cdot L \exp \left( \frac{L_{opt}}{L} \right)
\]

Minimum Threshold Current at Optimal Cavity Length

\[
\frac{\partial}{\partial L} I_{th} = I_{th} \left( \frac{1}{L} - \frac{L_{opt}}{L^2} \right) = 0
\]

\[
\frac{1}{L} = \frac{L_{opt}}{L^2} \Rightarrow L = L_{opt}
\]

\[
I_{th}^{\text{min}} = \frac{w L_{opt} n_w J_0}{\eta} \exp \left[ \frac{\alpha_i}{n_w \Gamma_w g_0} \right]
\]
Optimal Number of Wells for Fixed Cavity Length

\[ I_{th} = \frac{wL n_w J_0}{\eta} \exp \left[ \left( \frac{\alpha_{tot}}{n_w \Gamma_w g_0} \right) - 1 \right] \]

\[ = \text{const} \cdot n_w \exp \left( \frac{\alpha_{tot}}{n_w \Gamma_w g_0} \right) = \text{const} \cdot n_w \exp \left( \frac{n_{opt}}{n_w} \right) \]

\[ \frac{\partial}{\partial n_w} I_{th} = I_{th} \left( \frac{1}{n_w} - \frac{n_{opt}}{n_w^2} \right) = 0 \]

(implicit assumption of no coupling between wells)

\[ n_w = n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0} = \frac{1}{\Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) \]

\[ I_{th}^{\text{min}} = \frac{wL n_{opt} J_0}{\eta} \]

Take the closest integer!
Threshold Current Optimization

It is not possible to optimize simultaneously for cavity length and number of quantum wells.

If

\[ L = L_{opt} \]

\[ n_w = \frac{\alpha_m}{\Gamma_w g_0} \neq n_{opt} \]

(Unless \( \alpha_i = 0 \) which is unphysical)
Some representative results

Strain Effects
Definitions

Biaxial Compression
The strained material has a larger lattice constant resulting in compressive strain in the plane of the wafer and tension in the direction perpendicular to the surface.
Definitions

Biaxial Tension
The strained material has a smaller lattice constant resulting in tensile strain in the plane of the wafer and compression in the direction perpendicular to the surface.
Definitions

Critical Layer Thickness
Thickness beyond which dislocations form to accommodate mismatch.

edge dislocation
Some Key Points

• Strain may modify significantly the band structure of the valence band.
• Both bandgap energy and carrier effective mass may change (e.g., heavy hole effective mass becomes lighter and light hole effective mass becomes heavier in the direction of strain.
• Degeneracy of hh/lh bands is broken.
• Strain may change the threshold current of lasers.
• Strain may change polarization of emitted light.
• Reduction in threshold current density may reduce the importance of non-radiative processes such as Auger recombination.
Effect of strain on the band structure of In$_{1-x}$Ga$_x$As

Energy band gap of In$_{1-x}$Ga$_x$As bulk and as grown pseudomorphically on InP
Energy band diagram of $\text{In}_{1-x}\text{Ga}_x\text{As}$ quantum well grown on $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$

Hole bands energy isosurfaces for unstrained $\text{In}_{1-x}\text{Ga}_x\text{As}$ lattice matched to $\text{InP}$

Hole bands energy isosurfaces for In$_{1-x}$Ga$_x$As under strain
Effective mass effect on Quasi-Fermi levels

• Reducing the effective mass of holes reduces the density of states in the valence band

• Quasi-Fermi levels become more symmetrical with respect to the band edges, when effective masses are similar

• With increased symmetry, the quasi-Fermi level has to penetrates less into the conduction band to reach density for population inversion → Less degenerate

• Also, the carrier density necessary to reach the transparency condition is reduced
population inversion condition

\[ F_c - F_v + E_g > \hbar \omega > E_g + E_{e1} - E_{h1} \]
population inversion condition

\[ F_c - F_v + E_g > \hbar \omega > E_g + E_{e1} - E_{h1} \]
Transparency Density and Peak Gain

- Remember our simple model for Peak Gain

\[ g_p(n) = g_{\text{max}} \left(f_c - f_v\right) \approx g_{\text{max}} \left(1 - e^{-n/n_c} - e^{-n/Rn_c}\right) \]

with \( R = \frac{m_h}{m_e^*} \)

Transparency density decreases with \( R \) but maximum achievable gain decreases → Design trade-off
Strain Effects on Band-Edge Energies

- Compressive strain generally increases the bandgap.
- Tensile strain generally decreases the bandgap.
- The LH band follows these trends but the HH band goes against.
- Strain affects also the conduction band structure of the conduction band. Mainly, the $\Gamma$, $L$, and $X$ valleys shift in energy at different rates.
Strain Effects in Quantum Wells

• For unstrained material, bandgap is the same for HH and LH. The energy levels in a quantum well, corresponding to HH and LH, differ because of unequal effective masses.
Strain Effects in Quantum Wells

• For strained materials HH and LH bandgaps and the energy offsets in CB and VB are different. HH and LH are in different potential wells.

• QW under tensile strain brings HH and LH quantum level closer to each other (LH has deeper well but higher energy levels)
Gain Spectrum of Strained Quantum Wells

Recall that

- C1-HH1 transition is mostly favored by TE
- C1-LH1 transition is mostly favored by TM

- Tensile strain can improve balance between TE and TM gain
- Trade-off is maximum gain linked to joint density of states through the ratio of effective masses
Recall the matrix element depends on transverse wave vector $k_t$

For $k_t = 0$,

**TE Polarization:**

$$|\hat{x} \cdot M_{c-hh}|^2 = |\hat{y} \cdot M_{c-hh}|^2 = \frac{3}{2} M_b^2$$

$$|\hat{x} \cdot M_{c-lh}|^2 = |\hat{y} \cdot M_{c-lh}|^2 = \frac{1}{2} M_b^2$$

**TM Polarization:**

$$|\hat{z} \cdot M_{c-hh}|^2 = 0$$

$$|\hat{z} \cdot M_{c-lh}|^2 = 2 M_b^2$$

Gain depends on surface (sheet) carrier concentration

$$n_S = n \times L_z$$
Momentum Matrix Element

- Normalized as $2|M_{nm}(k_t)|^2/M_b^2$ with $n = C1$, $m = HH1$ for compressive strain and $m = LH1$ for tensile strain.
Modal gain versus sheet concentration

For $\text{In}_{1-x}\text{Ga}_x\text{As} / \text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ quantum well laser working near 1.55$\mu$m

$$n_s = n \times L_z$$
Band Distortion in Strained Quantum Wells

compressive strain

lattice matched

small tensile strain

large tensile strain

Some comments

- LH can become heavier, HH can become lighter
- Compressively-strained materials can have lower valence band density of states (because HH at the top of the VB can have effective mass lower than LH at the top of the VB in the case of tensile strain)

\[ n_s = n \times L_z \]
Modal Gain versus Current Density

• Modal Gain

$$\Gamma g \propto \frac{L_z}{W_{\text{mode}}} g$$

• Empirical relationship

$$G = n_w \Gamma \Gamma_w g_w = n_w \Gamma \Gamma_0 g_0 \left[ \ln \left( \frac{n_w J_w}{n_w J_0} \right) + 1 \right]$$

Compressive strain
Smaller transparency carrier density but saturates faster.

Tensile strain
Larger transparency carrier density but increases faster (higher differential gain).

Loss mechanisms (Auger, recombination, intervalence band absorption) need to be factored in when considering current density.
Reading Assignments:

• Sections 10.3 and 10.4 of Chuang’s book
• Section 8.2.5, Appendices 1,2,3,9 (supplemental) in Coldren, Corzine and Mašanović
• Section 4.5 of Chuang’s book