ECE 329 – Fall 2022

Prof. Ravaioli – Office: 2062 ECEB

Lecture 29

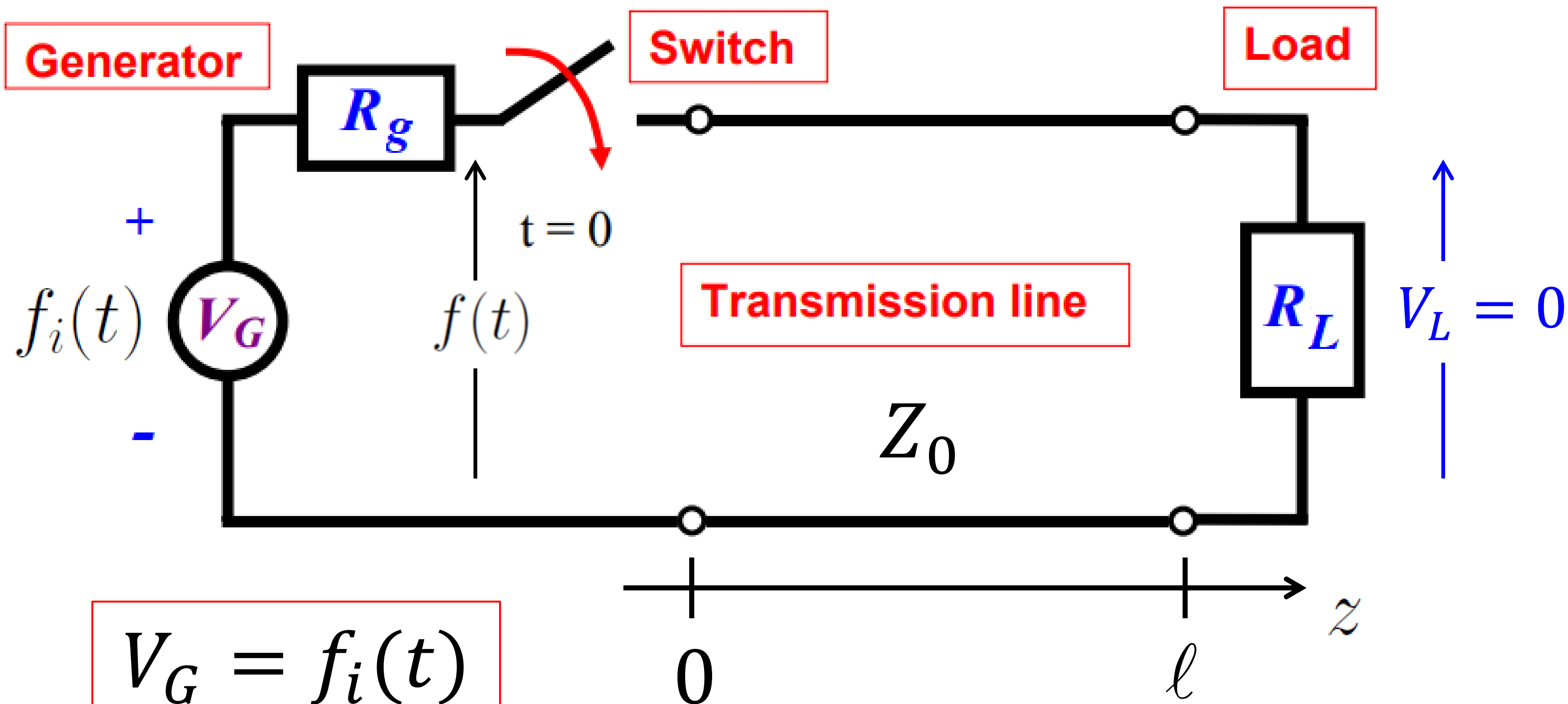
Lecture 29 – Outline

- **Transient on a transmission line**
- **Reflection coefficient**
- **Impulse response**
- **Bounce diagram**
- **Examples**

Reading assignment
Prof. Kudeki's ECE 329 Lecture Notes on Fields and Waves:
29) Bounce Diagram examples

Transient in a Transmission Line

We look for the voltage and current, $V(z,t)$ and $I(z,t)$ after the switch is closed and a certain input signal $f_i(t)$ is injected

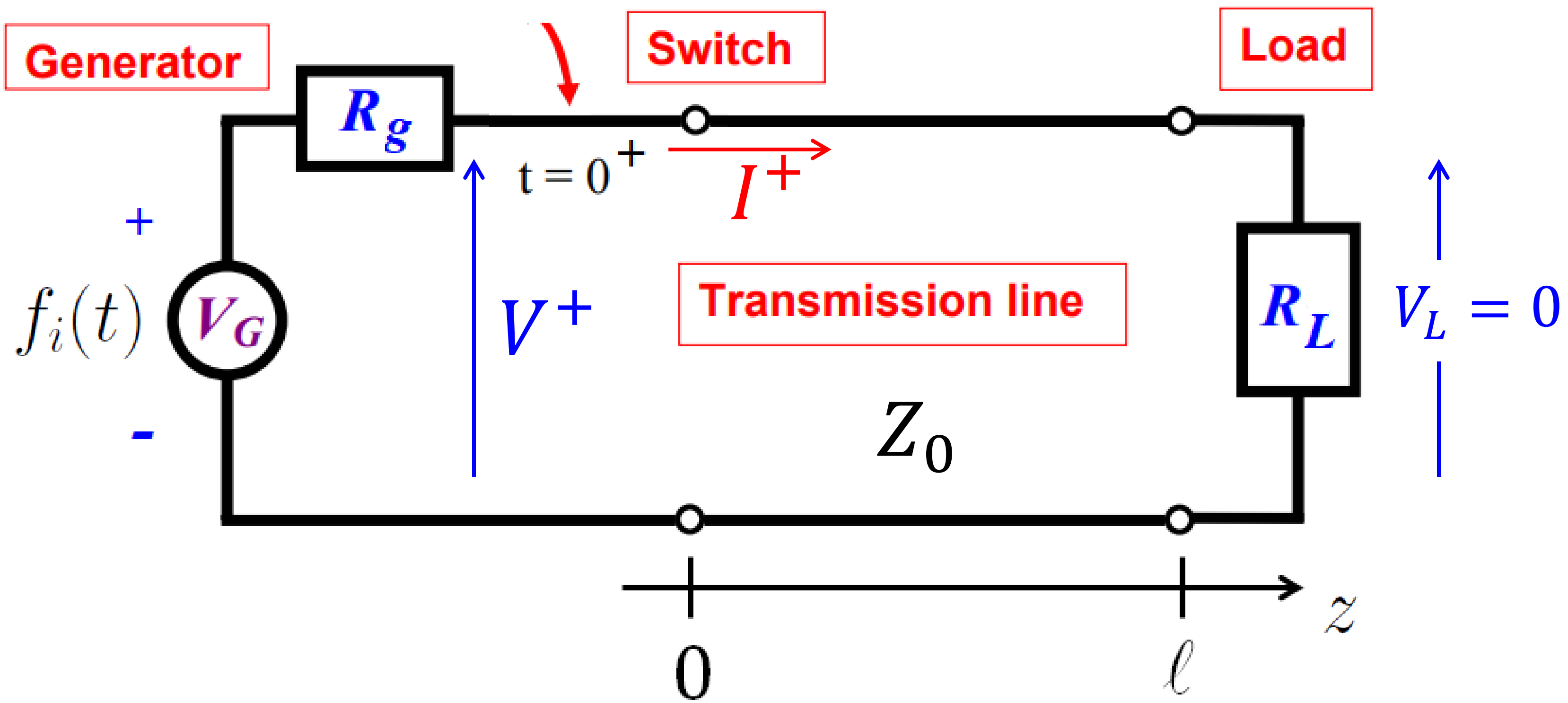


$$V_G = f_i(t)$$

Remember: for a lossless line, the characteristic impedance Z_0 is Real

Close the Switch

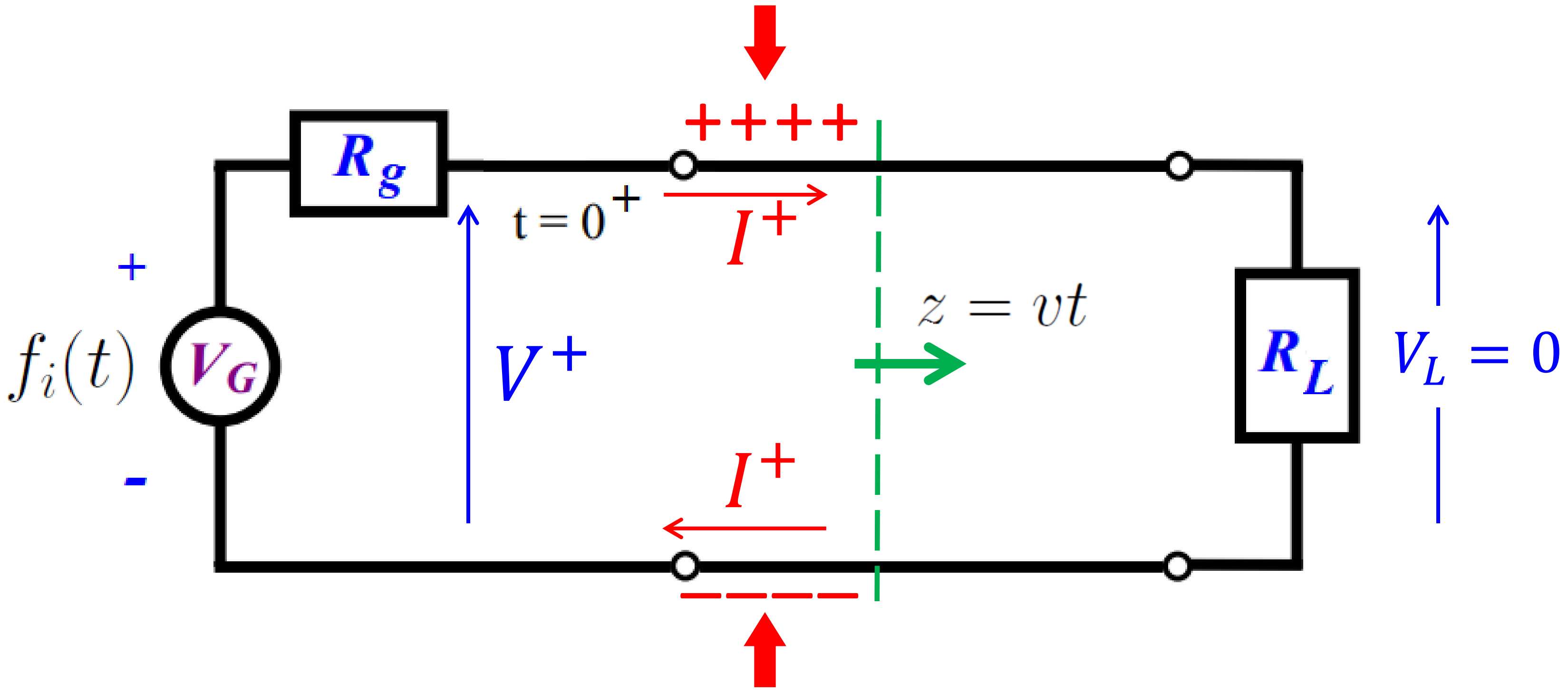
After the switch is closed, the voltage at the input of the TL varies to a value V^+ and a current I^+ begins to flow into the line.



The load voltage remains zero until the wavefront reaches the end of the line

Current Flow

After the switch is closed, positive charges start flowing into the top wire (that is, electrons are being pulled in by the generator).



Electrons are pushed into the bottom wire (as if positive charges are entering the generator), so that the same current flows.

Propagation toward the load

Until the wavefront reaches the load, the input impedance of the transmission line appears to be the same as the characteristic impedance Z_0 because the current cannot yet sense the load.

The voltage front V^+ propagates with current I^+ where

$$V^+ = Z_0 I^+ = V_G \frac{Z_0}{R_g + Z_0}$$

$$I^+ = \frac{V^+}{Z_0} = \frac{f_i(0)}{R_g + Z_0}$$

The wave fronts travel with a phase velocity equal to the speed of light for the dielectric medium surrounding the wires.

The wavefront has reached the load

If the load does not match exactly the characteristic impedance of the line, the voltage V^+ and the current I^+ are not compatible with the load R_L because

$$V^+ \neq R_L I^+$$

Voltage and current adjust themselves to the load by reflecting back a wavefront with voltage V^- and current I^- such that

$$V^+ + V^- = (I^+ + I^-)R_L$$

Since also the reflected front encounters an impedance Z_0, we have

$$V^+ = Z_0\, I^+$$

$$V^- = -Z_0\, I^-$$

$$V^+ + V^- = (I^+ + I^-)R_L$$

$$V^+ + V^- = \left(\frac{V^+}{Z_0} - \frac{V^-}{Z_0}\right)R_L$$

$$\Rightarrow \quad V^- = V^+\,\frac{R_L - Z_0}{R_L + Z_0}$$

Solve for the unknown V^-

where we have the Load Reflection Coefficient Γ_L

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

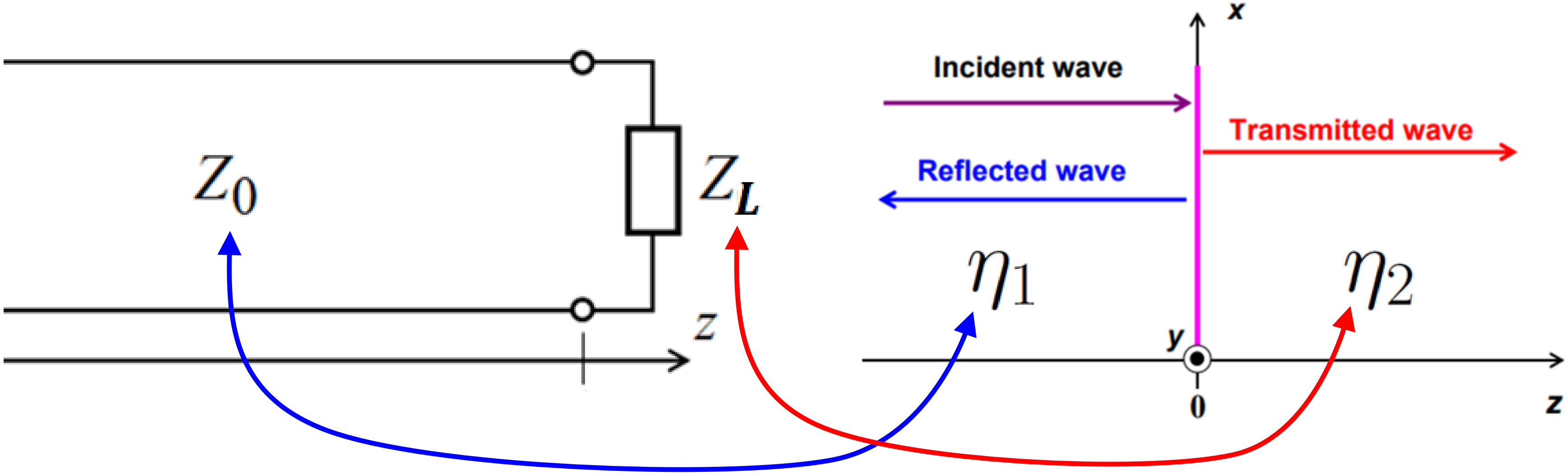
Load Reflection Coefficient Γ – Analogy with EM waves

Transmission Line

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

EM wave at an interface

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$



Reflected wavefront

The wave reflected by the load propagates in the negative direction and interferes with voltage and current found along the transmission line, which continue to be injected by the generator.

When the reflected wave reaches the input of the transmission line, it terminates on the generator impedance R_g.

If R_g does not match the line characteristic impedance Z_0, reflection back into the line occurs, generating an additional forward wave

$$V_2^+ = V^- \frac{R_G - Z_0}{R_G + Z_0}$$

and the cycle repeats, while the generator may continue to inject a forward wave…

Remember, the ideal voltage source part of the generator behaves simply as a short for the reflected wave attempting to exit the line from the input.

Special case: Load Matched to the Transmission Line

Assume that the load is matched to the TL: $R_L = Z_0$

At the load $z = \ell$

$$\frac{V(\ell,t)}{I(\ell,t)} = \frac{V_L}{I_L} = R_L = Z_o$$

$$V(\ell,t) = f(t - \tfrac{\ell}{v}) + g(t + \tfrac{\ell}{v})$$

$$I(\ell,t) = \frac{f(t - \frac{\ell}{v}) - g(t + \frac{\ell}{v})}{Z_o}$$

$$\frac{V(\ell,t)}{I(\ell,t)} = Z_o$$ **only if** $$g(t + \tfrac{\ell}{v}) = 0$$ **No Reflection**

$$V(z,t) = f(t - \frac{z}{v})$$

$$I(z,t) = \frac{1}{Z_o} f(t - \frac{z}{v})$$

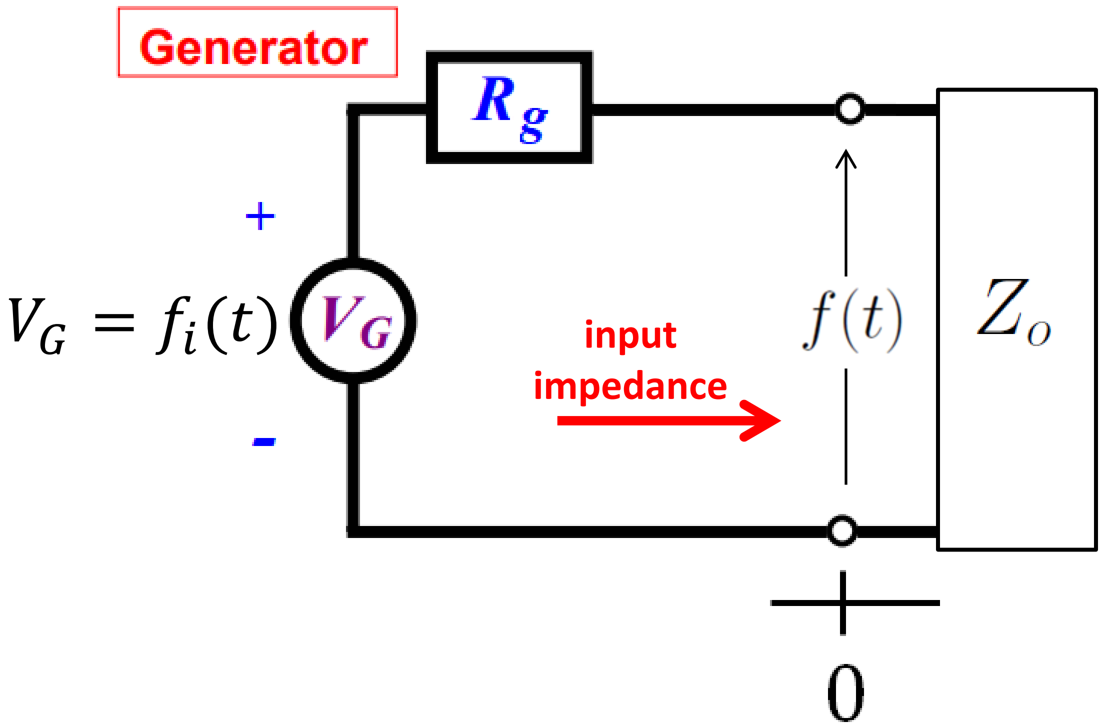
Transient in a Transmission Line

At the input $z = 0$

$$V(0,t) = f(t)$$

$$I(0,t) = \frac{1}{Z_o} f(t)$$

$$Z_o = \frac{V(0,t)}{I(0,t)}$$



Voltage Divider

$$f(t) = \underbrace{\frac{Z_o}{R_g + Z_o}}_{\tau_g} f_i(t)$$

injection coefficient τ_g

$$f(t) = \tau_g f_i(t)$$

Transient in a Transmission Line

Therefore, the distribution of voltage and current along a transmission line circuit terminated by a matched load is

$$V(z,t) = \tau_g f_i(t - \frac{z}{v})$$

$$I(z,t) = \frac{\tau_g}{Z_o} f_i(t - \frac{z}{v})$$

There are no interference patterns caused by load reflections, because there is only one forward wave propagating.

Impulse response for the matched case

A delta function input

$$f_i(t) = \delta(t)$$

generates the impulse response

$$V(z,t) = \tau_g \delta(t - \frac{z}{v}) \equiv h_z(t)$$

(From ECE 210) Convolution of a system's impulse response with any input function provides the system's response to that function

$$Y(t) = \int_{-\infty}^{+\infty} X(t) h(t-\tau)\, d\tau$$

Arbitrary Load R_L

The impulse response needs to be obtained again when the load is changed, with the new constraint

$$\frac{V(\ell,t)}{I(\ell,t)} = \frac{V_L}{I_L} = R_L$$

from which a new reflection coefficient is obtained each time

$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o}$$

In principle, one can construct the impulse response by adding the forward and reflected pulses, going back and forth in a series.

Arbitrary Load R_L

For the first roundtrip $0 < t < \frac{2\ell}{v}$
the load voltage and current expressions are

$$V(\ell,t) = \tau_g[\delta(t-\frac{\ell}{v}) + \Gamma_L\delta(t+\underbrace{\frac{\ell}{v}-\frac{2\ell}{v}}_{-\frac{\ell}{v}})]$$

$$= \tau_g\delta(t-\frac{\ell}{v})[1+\Gamma_L]$$

Written as for generic z for convenience

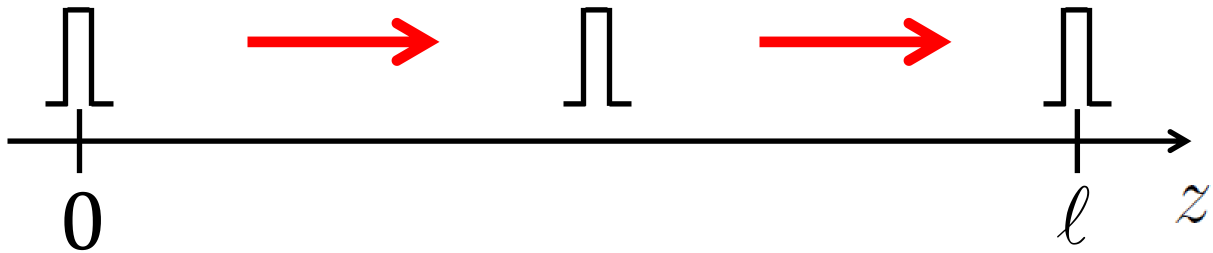
$$I(\ell,t) = \frac{\tau_g}{Z_o}[\delta(t-\frac{\ell}{v}) - \Gamma_L\delta(t+\frac{\ell}{v}-\frac{2\ell}{v})]$$

$$= \frac{\tau_g}{Z_o}\delta(t-\frac{\ell}{v})[1-\Gamma_L]$$

Bookkeeping of pulses

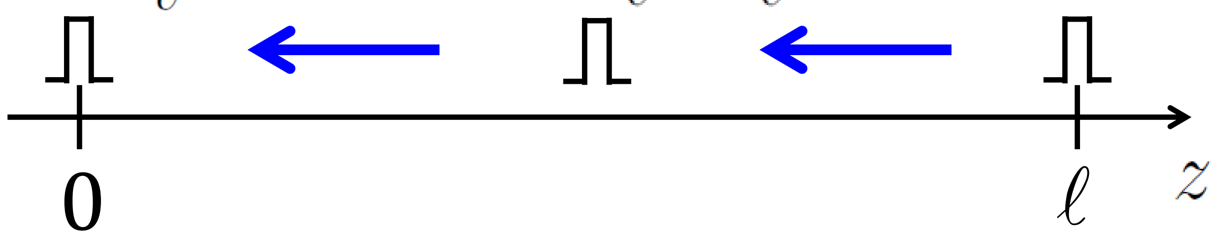
The first pulse starts at $t = 0$ from the input an it arrives to the load after a time ℓ/v

$\tau_g\delta(t)$ $\quad$ $\tau_g\delta(t-\frac{z}{v})$ $\quad$ $\tau_g\delta(t-\frac{\ell}{v})$



The first reflected pulse starts at $t = \ell/v$ from the load and returns to the input at time $2\ell/v$

$\tau_g\Gamma_L\delta(t-\frac{2\ell}{v})$ $\quad$ $\tau_g\Gamma_L\delta(t+\frac{z}{v}-\frac{2\ell}{v})$ $\quad$ $\tau_g\Gamma_L\delta(t-\frac{\ell}{v})$



and then

$$\tau_g\Gamma_L\Gamma_g\delta(t-\frac{z}{v}-\frac{2\ell}{v})$$

and so on…

Bookkeeping of pulses

The complete process can be written formally with summations. In many practical cases the series converges rapidly.

$$V(z,t) = \tau_g \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v})$$
$$+\tau_g \Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v})$$

$$I(z,t) = \frac{\tau_g}{Z_o} \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v})$$
$$-\frac{\tau_g}{Z_o}\Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v})$$

Bookkeeping of pulses

For $n = 0$ we have the first forward wave

$$\tau_g \delta(t - \frac{z}{v})$$

reflected by generator

$$V(z,t) = \tau_g \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v})$$

reflected by load

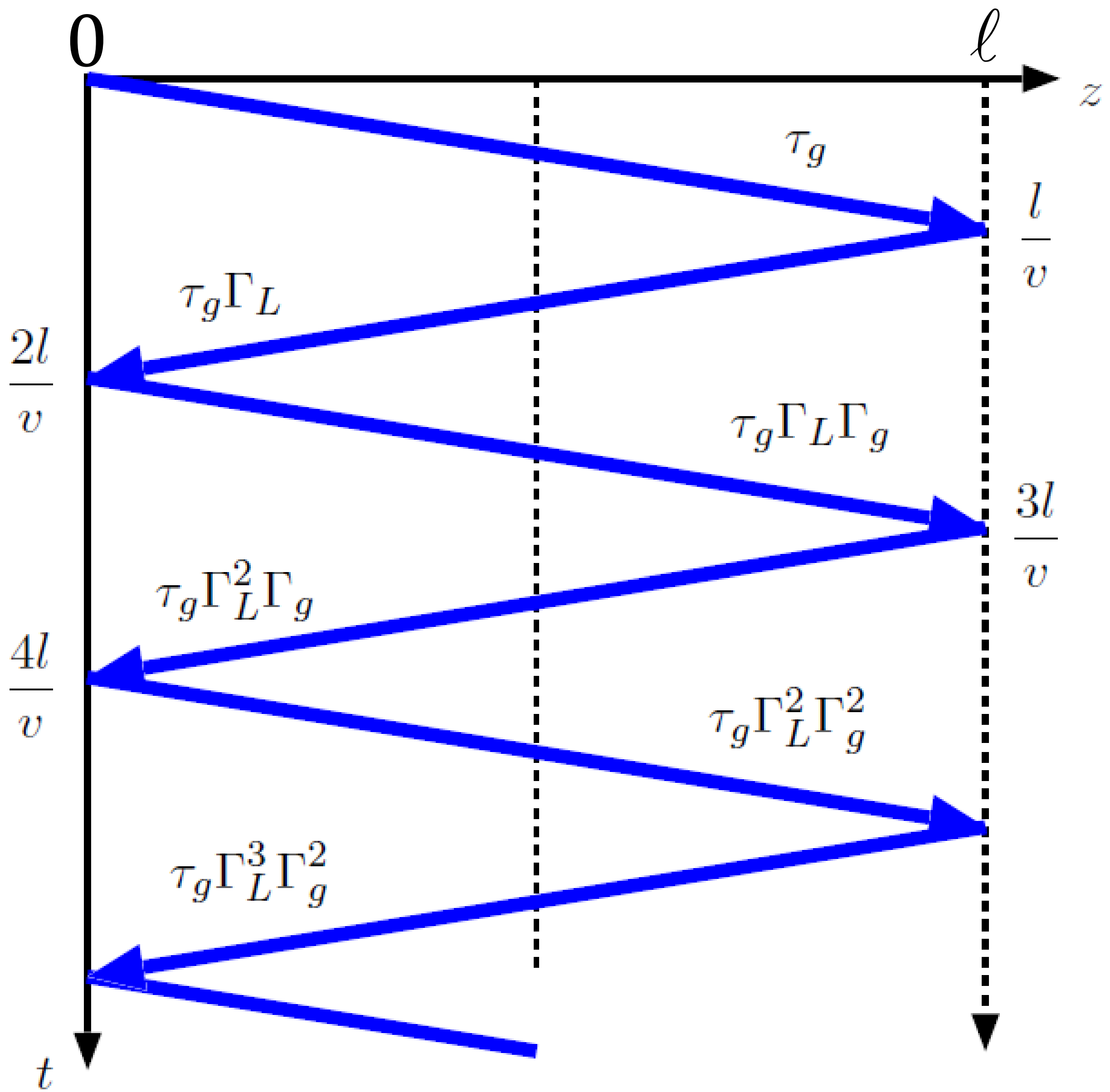
$$+\tau_g \Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v})$$

For $n = 0$ we have the first reflected wave

$$\tau_g \Gamma_L \, \delta(t + \frac{z}{v} - \frac{2\ell}{v})$$

Bounce diagram

Analysis is often done in graphical form. Each bounce adds a Γ.



Example 1

Transmission Line

Length = 900 m

phase velocity $v = c$

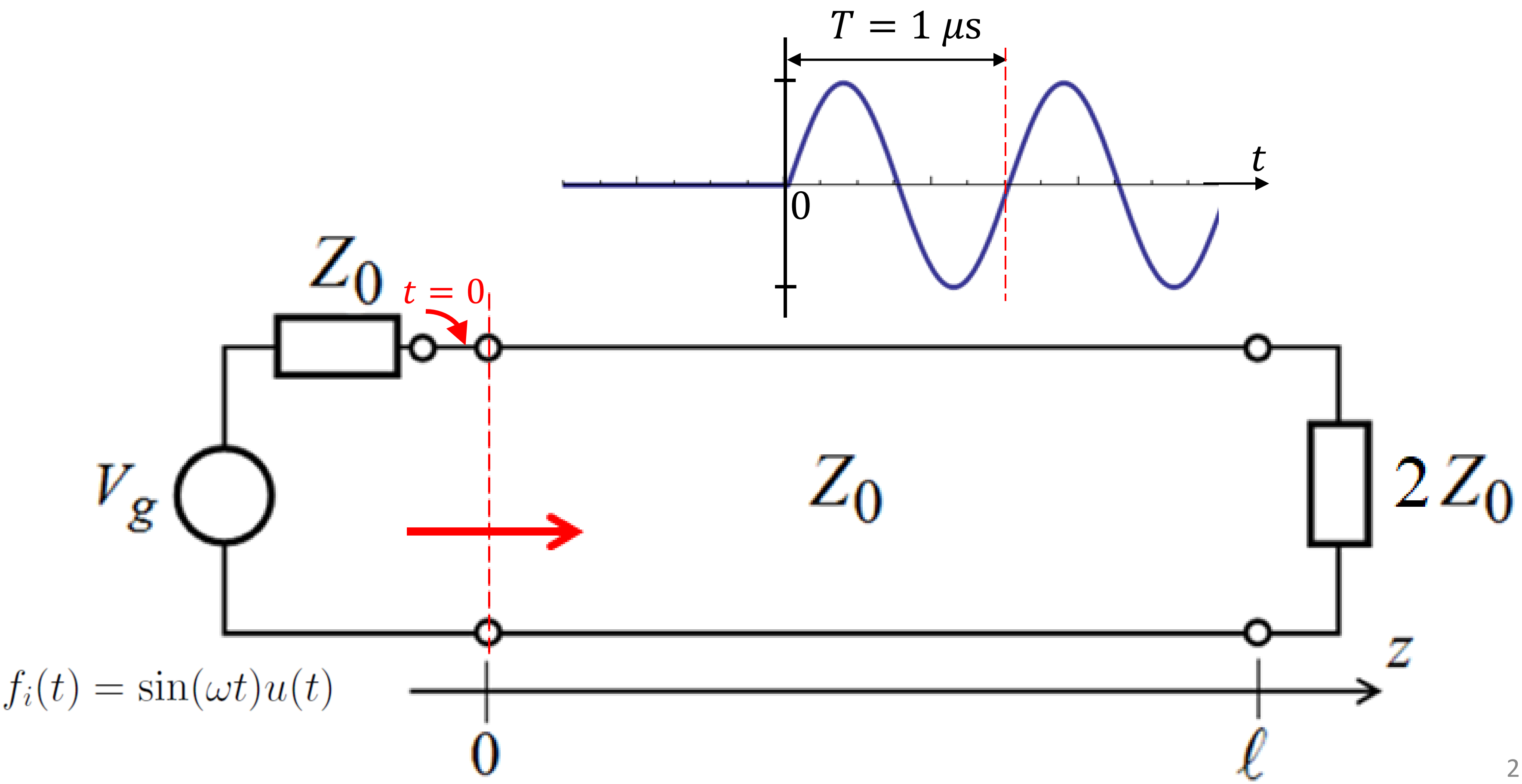
Load $R_L = 2Z_0$

Generator

$f_i(t) = \sin(\omega t)u(t)$

$R_g = Z_0$

Frequency = 1 MHz



Example 1

Transmission Line

Length = 900 m

phase velocity $v = c$

Load $R_L = 2Z_0$

Generator

$f_i(t) = \sin(\omega t)u(t)$

$R_g = Z_0$

Frequency = 1 MHz

Injection coefficient

$$\tau_g = \frac{Z_o}{R_g + Z_o} = \frac{Z_o}{Z_o + Z_o} = \frac{1}{2}$$

Reflection coefficients

$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{2Z_o - Z_o}{2Z_o + Z_o} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$\Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = \frac{Z_o - Z_o}{Z_o + Z_o} = 0$$

time-delay (roundtrip)

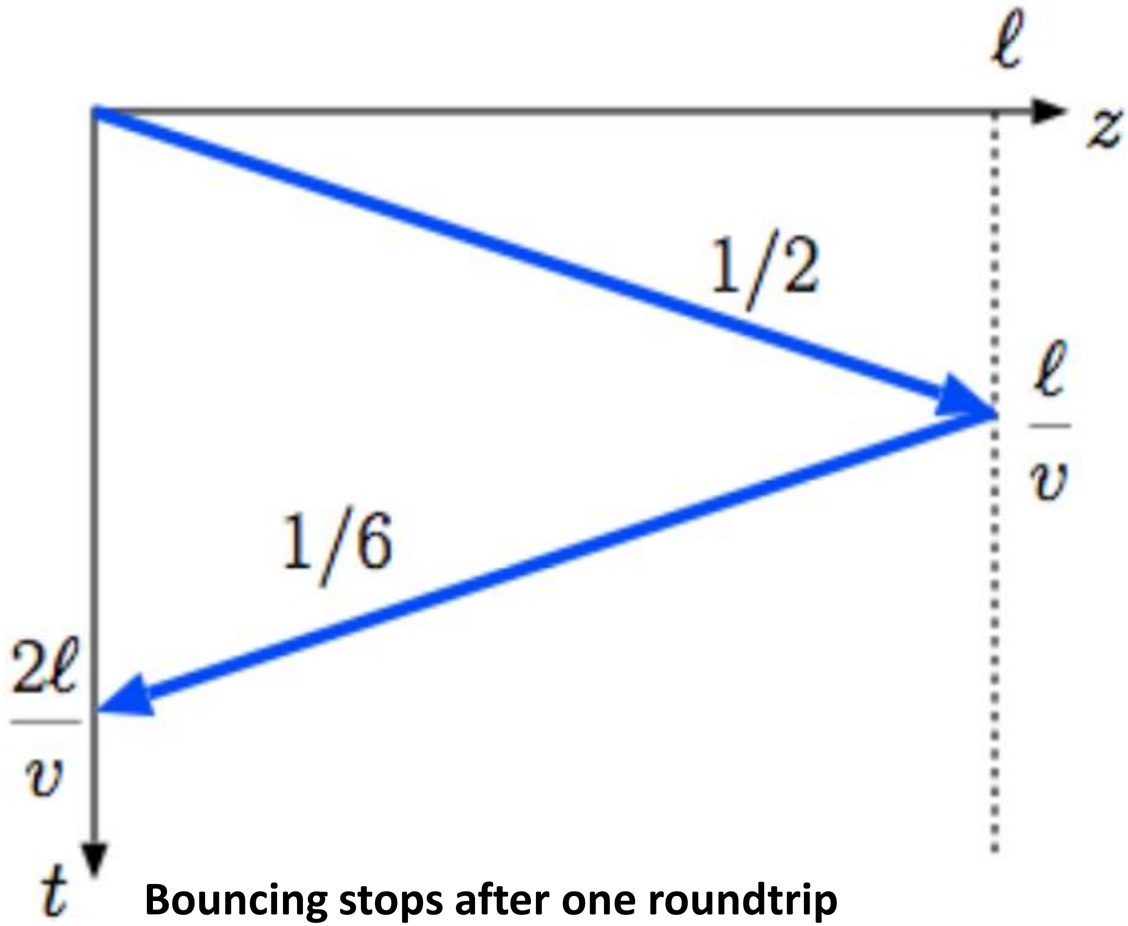
$$\frac{2\ell}{v} = \frac{2 \cdot 900\,\mathrm{m}}{300\,\mathrm{m}/\mu\mathrm{s}} = 6\,\mu\mathrm{s}$$

Example 1

$$\tau_g = \frac{1}{2}$$

$$\Gamma_L = \frac{1}{3}$$

$$\Gamma_g = 0$$



Bouncing stops after one roundtrip

impulse response

$$h_z(t) \;=\; \frac{1}{2}\delta(t - \frac{z}{c}) + \frac{1}{6}\delta(t + \frac{z}{v} - 6\mu)$$

Example 1

input signal $$f_i(t) = \sin(\omega t)u(t)$$

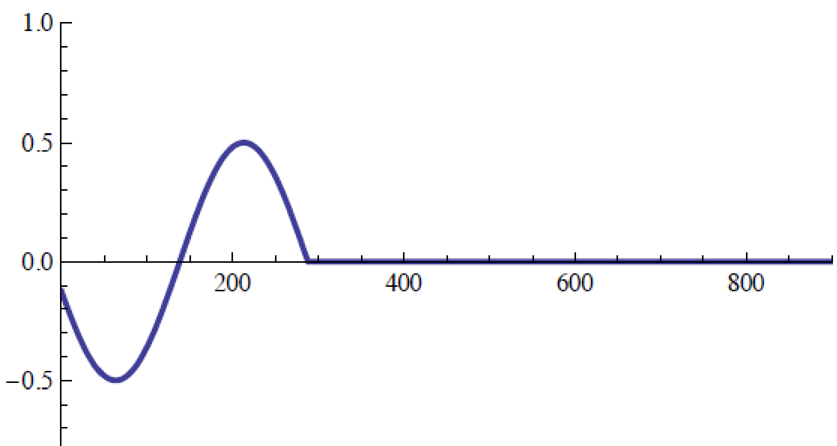
impulse response

$$h_z(t) = \frac{1}{2}\delta(t - \frac{z}{v}) + \frac{1}{6}\delta(t + \frac{z}{v} - 6\mu)$$

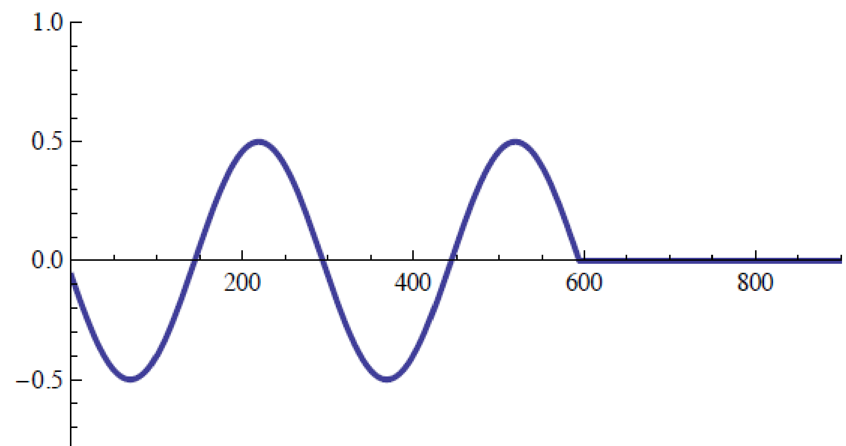
voltage wave from convolution

$$V(z,t) = h_z(t) * \sin(\omega t)u(t)$$

$$= \frac{1}{2}\sin\omega(t - \frac{z}{v})u(t - \frac{z}{v}) + \frac{1}{6}\sin\omega(t + \frac{z}{v} - 6)u(t + \frac{z}{v} - 6)$$



$t = 1\,\mu s$



$t = 2\,\mu s$

Example 2

Transmission Line

Length = 2400 m

phase velocity $v = c$

Load $R_L = 100\ \Omega$

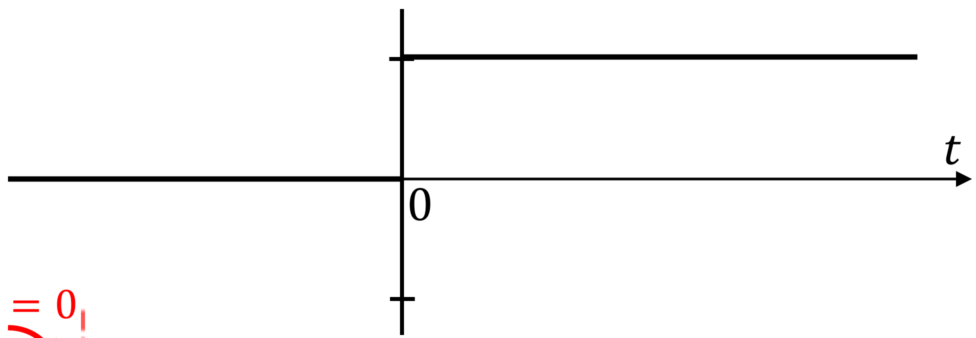
$Z_0 = 50\ \Omega$

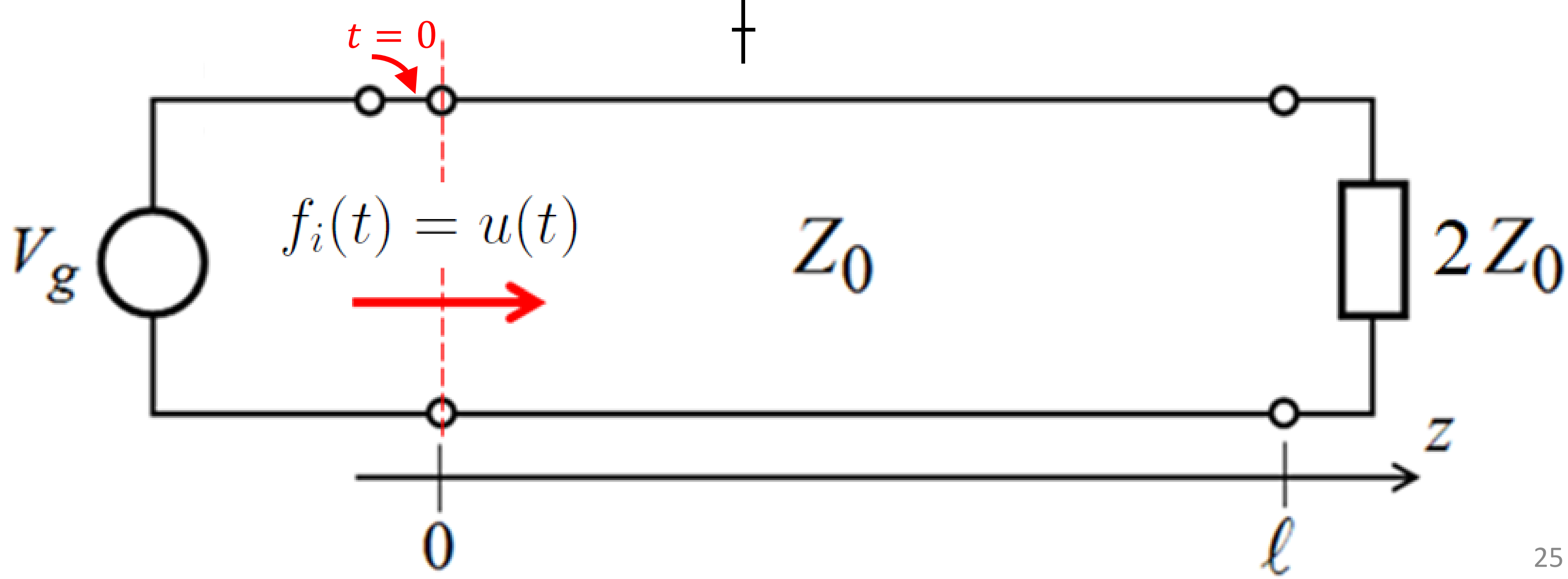
Generator

$f_i(t) = u(t)$ (unit step function)

$R_g = 0$

Determine and plot $V(1200, t)$





Example 2

Transmission Line

Length = 2400 m

phase velocity $v = c$

Load $R_L = 100\ \Omega$

Generator

$f_i(t) = u(t)$ (unit step function)

$R_g = 0$

Determine and plot $V(1200, t)$

Injection coefficient

$$\tau_g = \frac{Z_o}{R_g + Z_o} = 1$$ (ideal source)

Reflection coefficients

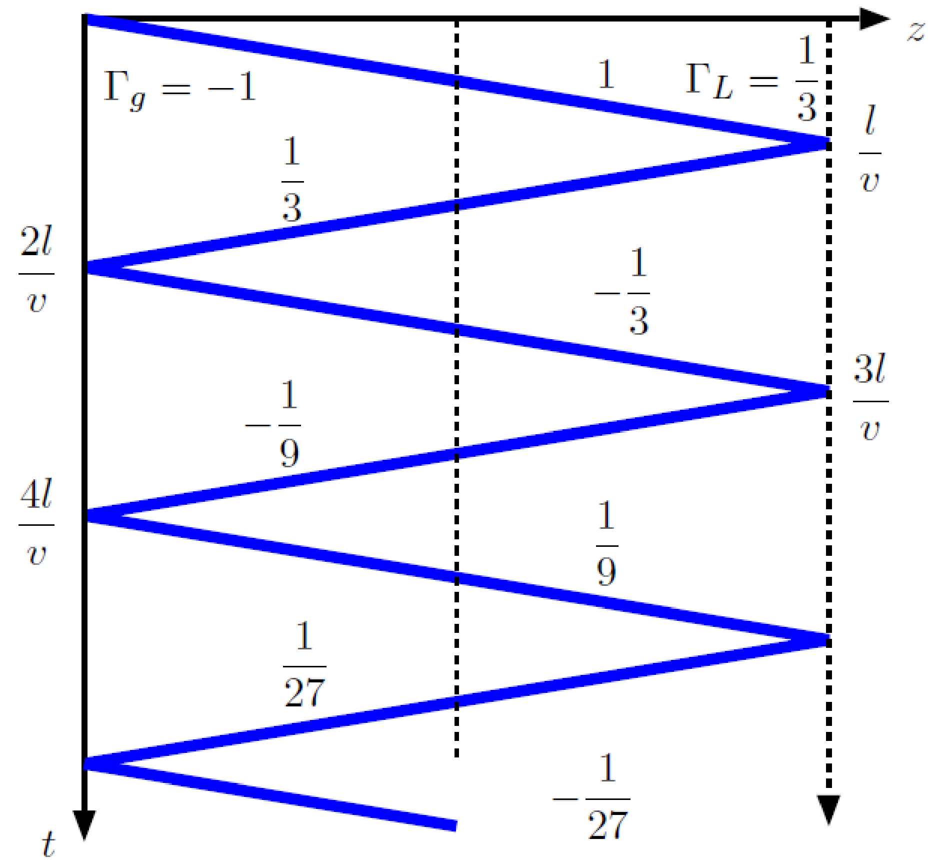
$$\Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = -1$$ (like a short circuit)

$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1}{3}$$

time-delay (one way)

$$\frac{\ell}{v} = \frac{2400\ \mathrm{m}}{300 \times 10^6\ \mathrm{m/s}} = 8\ \mu\mathrm{s}$$

Example 2

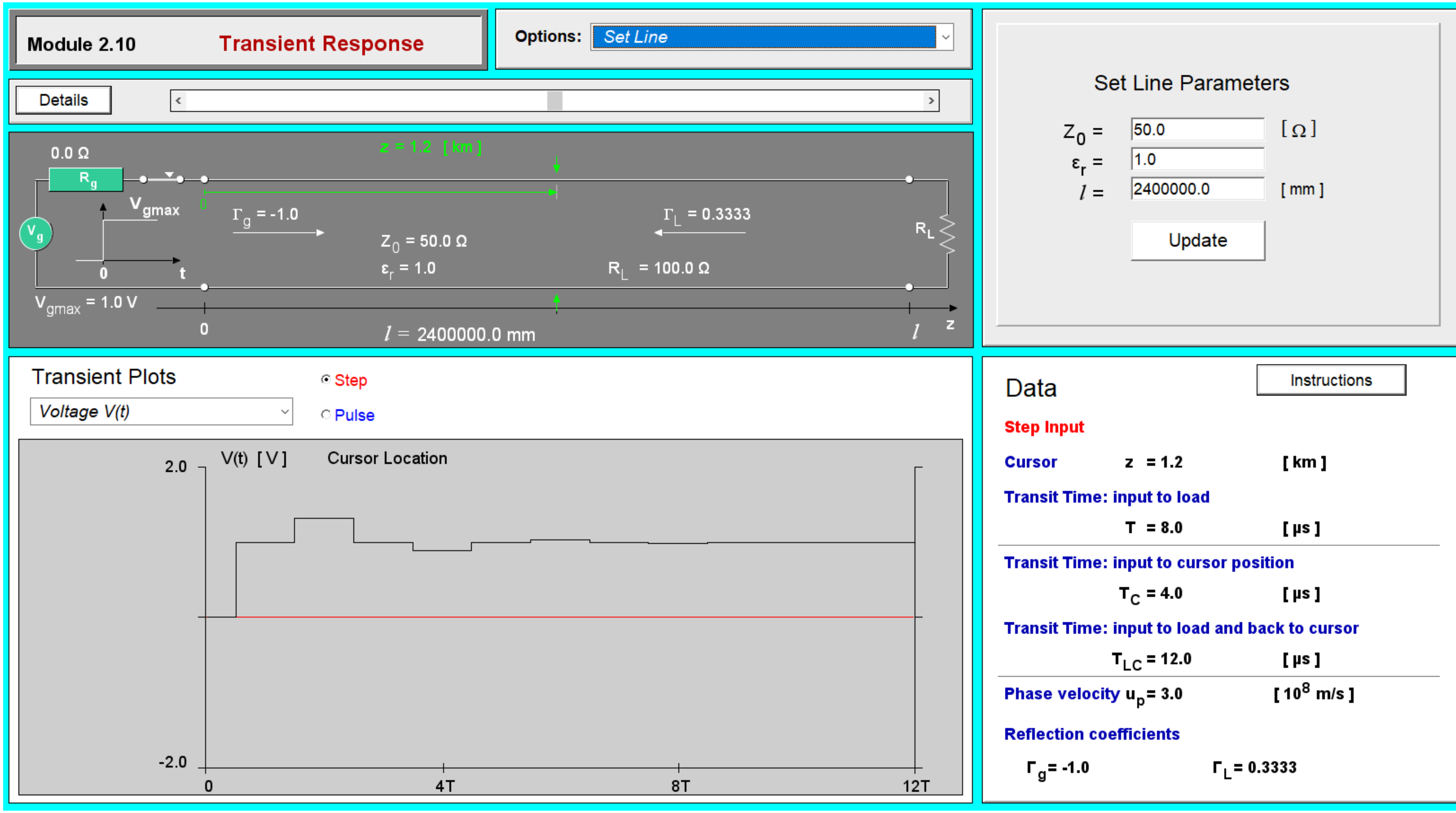


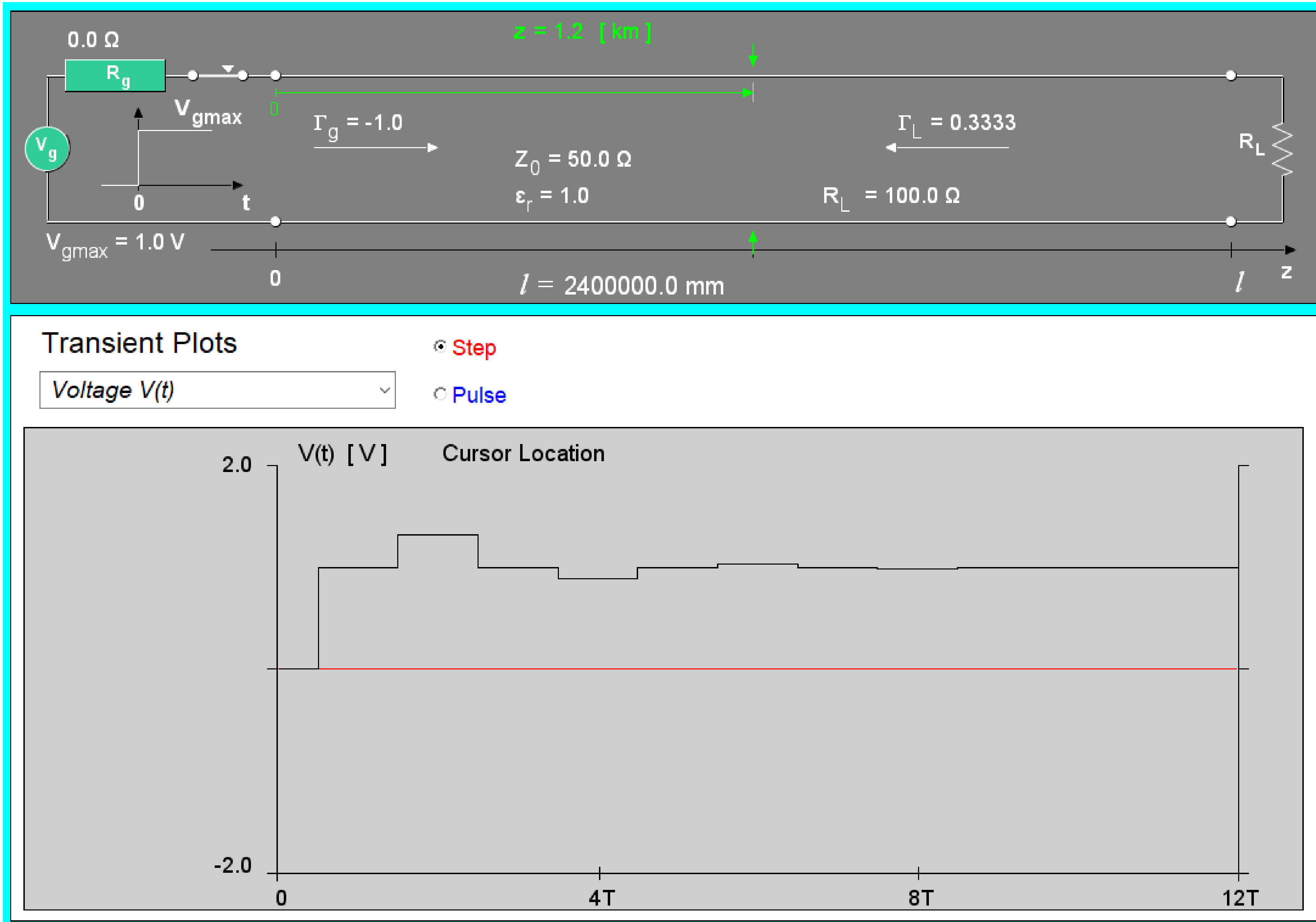
Impulse response

$$V(1200,t) = \delta(t-4) + \frac{1}{3}\delta(t-12) - \frac{1}{3}\delta(t-20) - \frac{1}{9}\delta(t-28) + \frac{1}{9}\delta(t-36) + \cdots$$

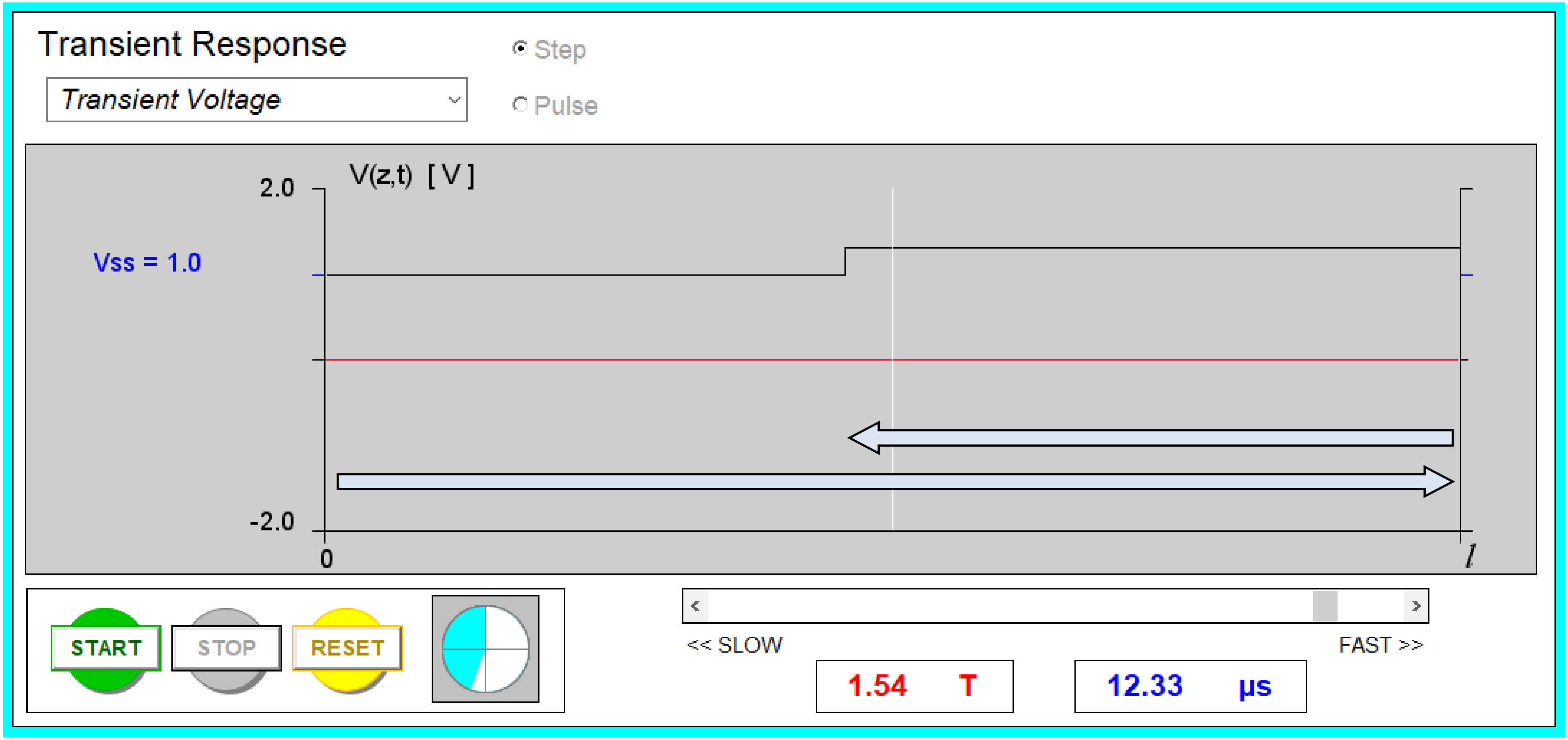
Unit-step response

$$V(1200,t) = u(t-4) + \frac{1}{3}u(t-12) - \frac{1}{3}u(t-20) - \frac{1}{9}u(t-28) + \frac{1}{9}u(t-36) + \cdots$$

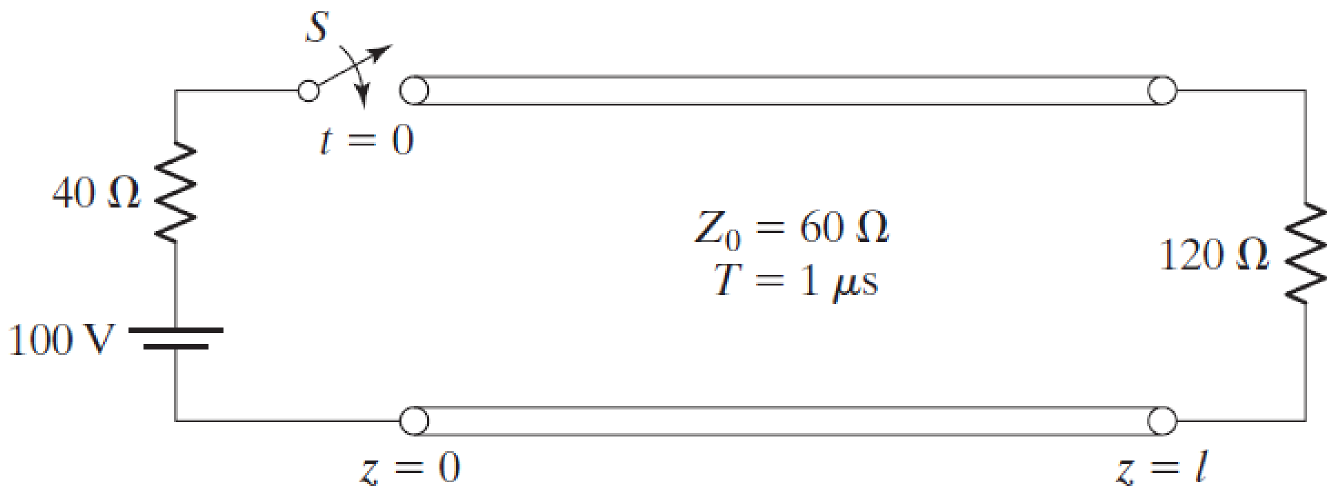

Module 2.10
Transient Response
Options:
Set Line
Details
0.0 Ω
R_g
z = 1.2 [km]
V_gmax
V_g
Γ_g = -1.0
Γ_L = 0.3333
Z_0 = 50.0 Ω
ε_r = 1.0
R_L = 100.0 Ω
R_L
0
t
V_gmax = 1.0 V
0
l = 2400000.0 mm
l
z
Set Line Parameters
Z_0 =
50.0
[Ω]
ε_r =
1.0
l =
2400000.0
[mm]
Update
Transient Plots
Step
Pulse
Voltage V(t)
2.0
V(t) [V]
Cursor Location
-2.0
0
4T
8T
12T
Data
Instructions
Step Input
Cursor
z = 1.2
[km]
Transit Time: input to load
T = 8.0
[μs]
Transit Time: input to cursor position
T_C = 4.0
[μs]
Transit Time: input to load and back to cursor
T_LC = 12.0
[μs]
Phase velocity u_p = 3.0
[10^8 m/s]
Reflection coefficients
Γ_g = -1.0
Γ_L = 0.3333


0.0 Ω
R_g
V_g
V_{gmax}
0
t
z = 1.2 [km]
0
Γ_g = -1.0
Γ_L = 0.3333
Z_0 = 50.0 Ω
ε_r = 1.0
R_L = 100.0 Ω
R_L
V_{gmax} = 1.0 V
0
l = 2400000.0 mm
l
z
Transient Plots
Voltage V(t)
Step
Pulse
V(t) [V]
Cursor Location
2.0
-2.0
0
4T
8T
12T

Transient Animation Snapshot



Example 3



Current

Injection coefficient

$$\tau_g = \frac{Z_o}{R_g + Z_o} = 0.6$$

$$I(t=0) = 1\text{A}$$

Reflection coefficients

$$\Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = -\frac{1}{5}$$

$$\Gamma_{g\,c} = \frac{1}{5}$$

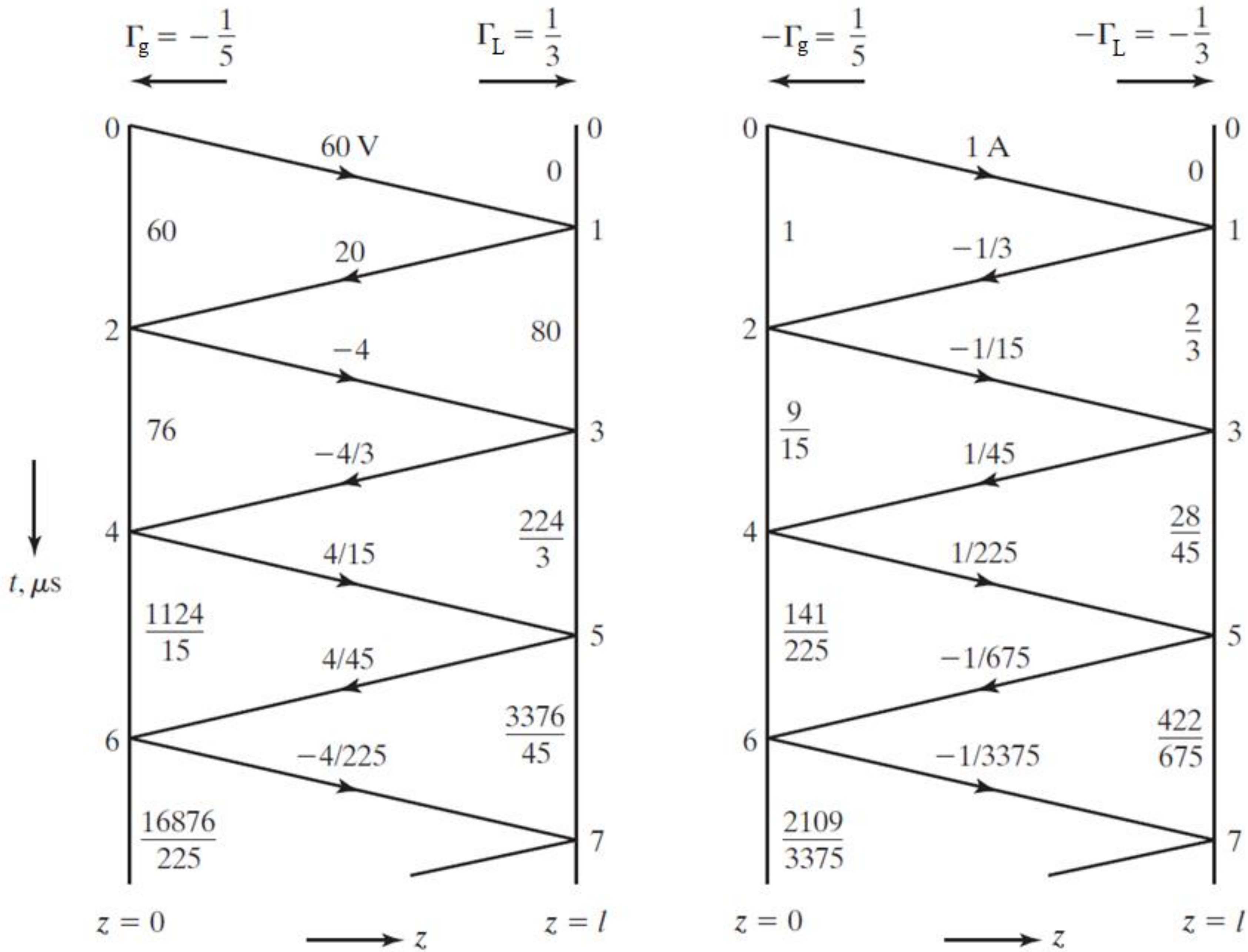
$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1}{3}$$

$$\Gamma_{L\,c} = -\frac{1}{3}$$

time-delay (one way)

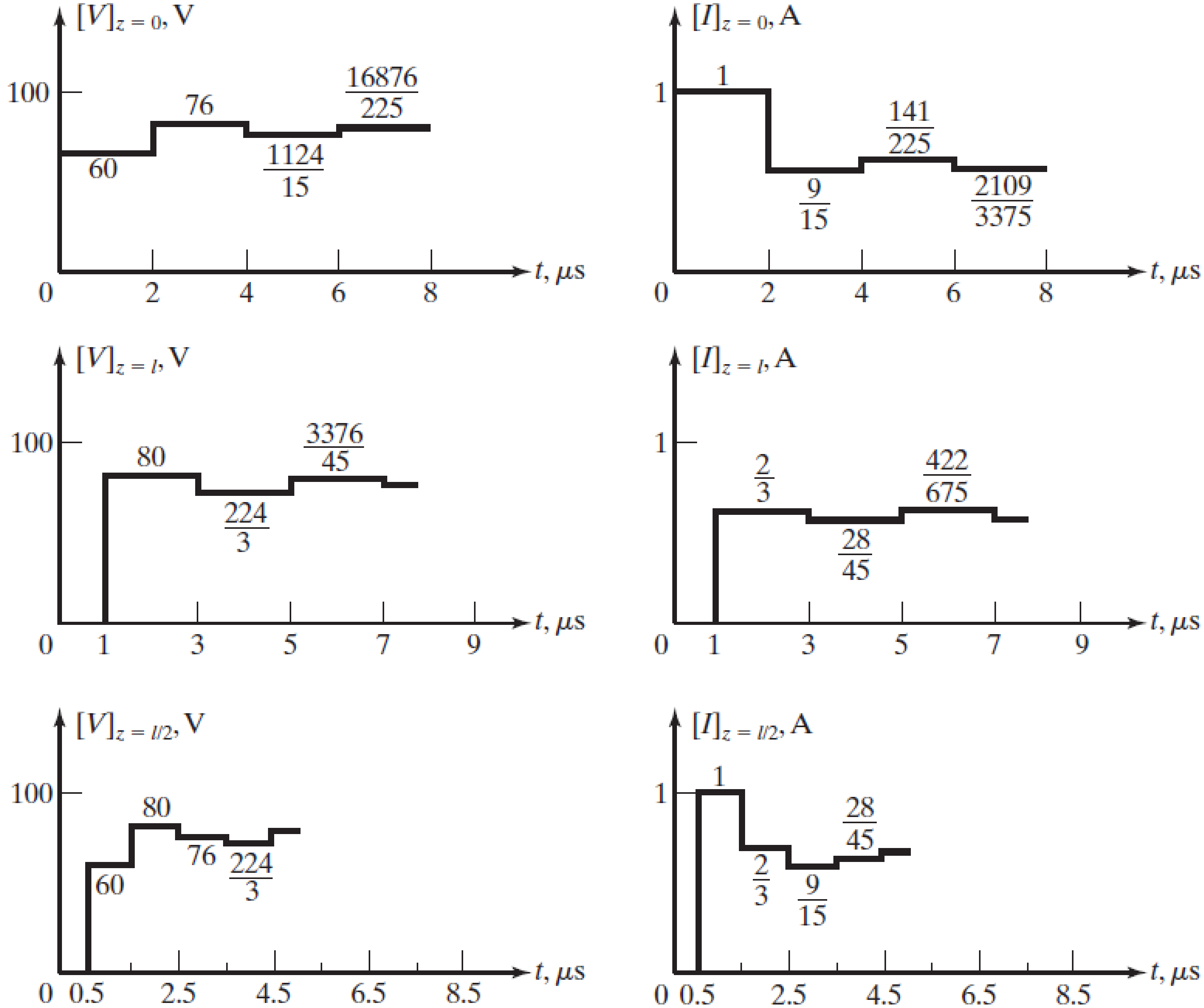
$$\frac{\ell}{v} = 1\ \mu\text{s}$$

Example 3



$\Gamma_g = -\frac{1}{5}$
$\Gamma_L = \frac{1}{3}$
0
60 V
0
0
60
1
20
2
80
−4
76
3
−4/3
4
224/3
4/15
1124/15
5
4/45
6
3376/45
−4/225
16876/225
7
z = 0
z
z = l
$-\Gamma_g = \frac{1}{5}$
$-\Gamma_L = -\frac{1}{3}$
0
1 A
0
0
1
1
−1/3
2
2/3
−1/15
9/15
3
1/45
4
28/45
1/225
141/225
5
−1/675
6
422/675
−1/3375
2109/3375
7
z = 0
z
z = l
t, μs

Example 3



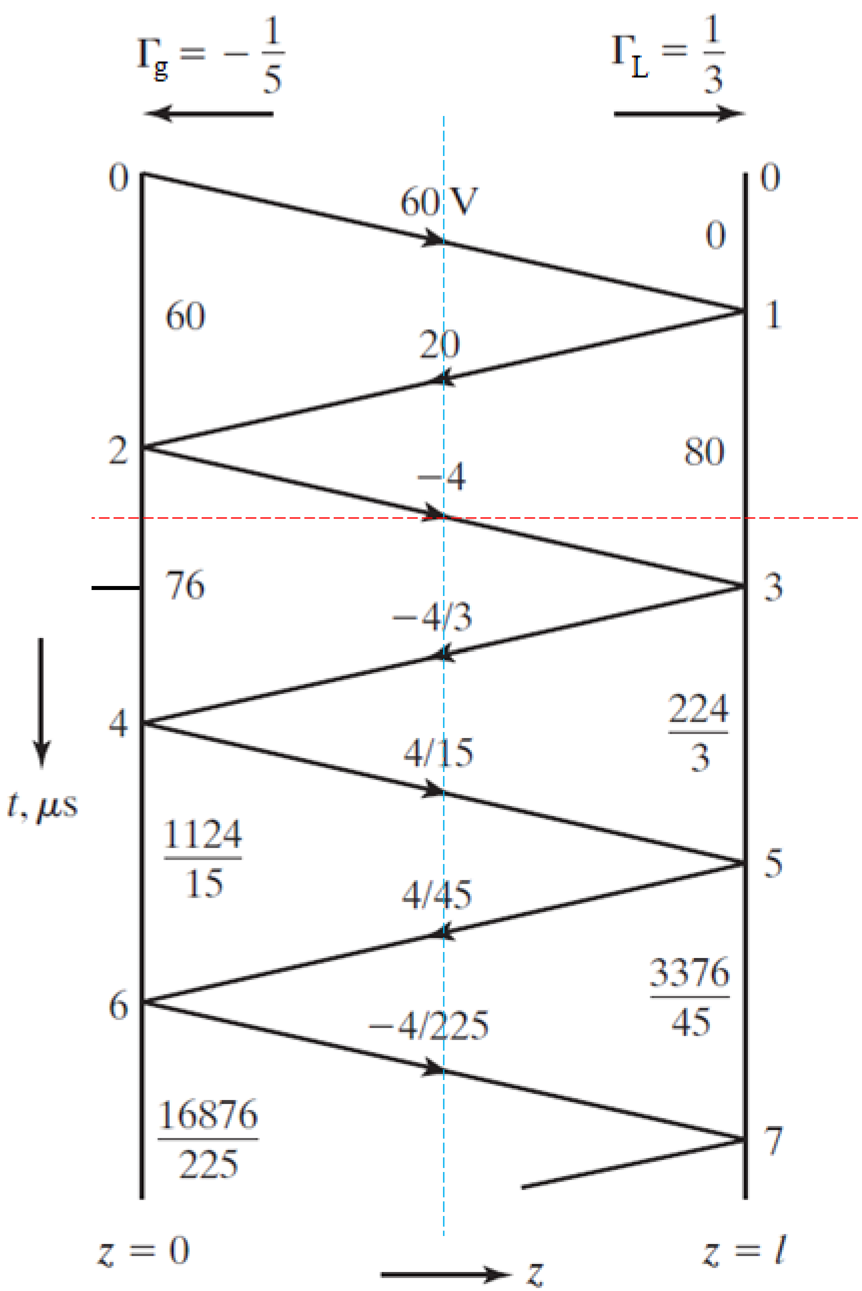
$z = 0$

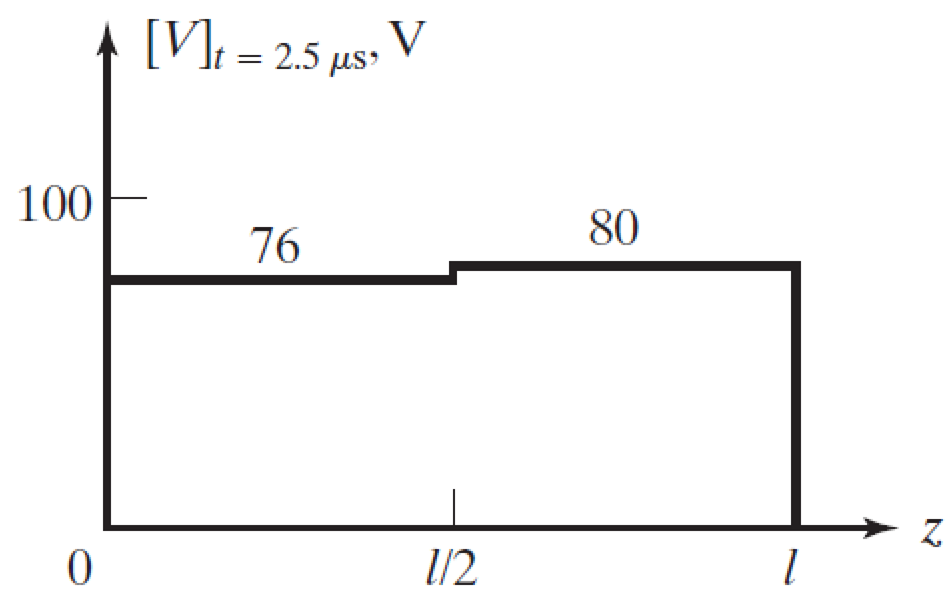
$z = l$

$z = \frac{l}{2}$

Example 3

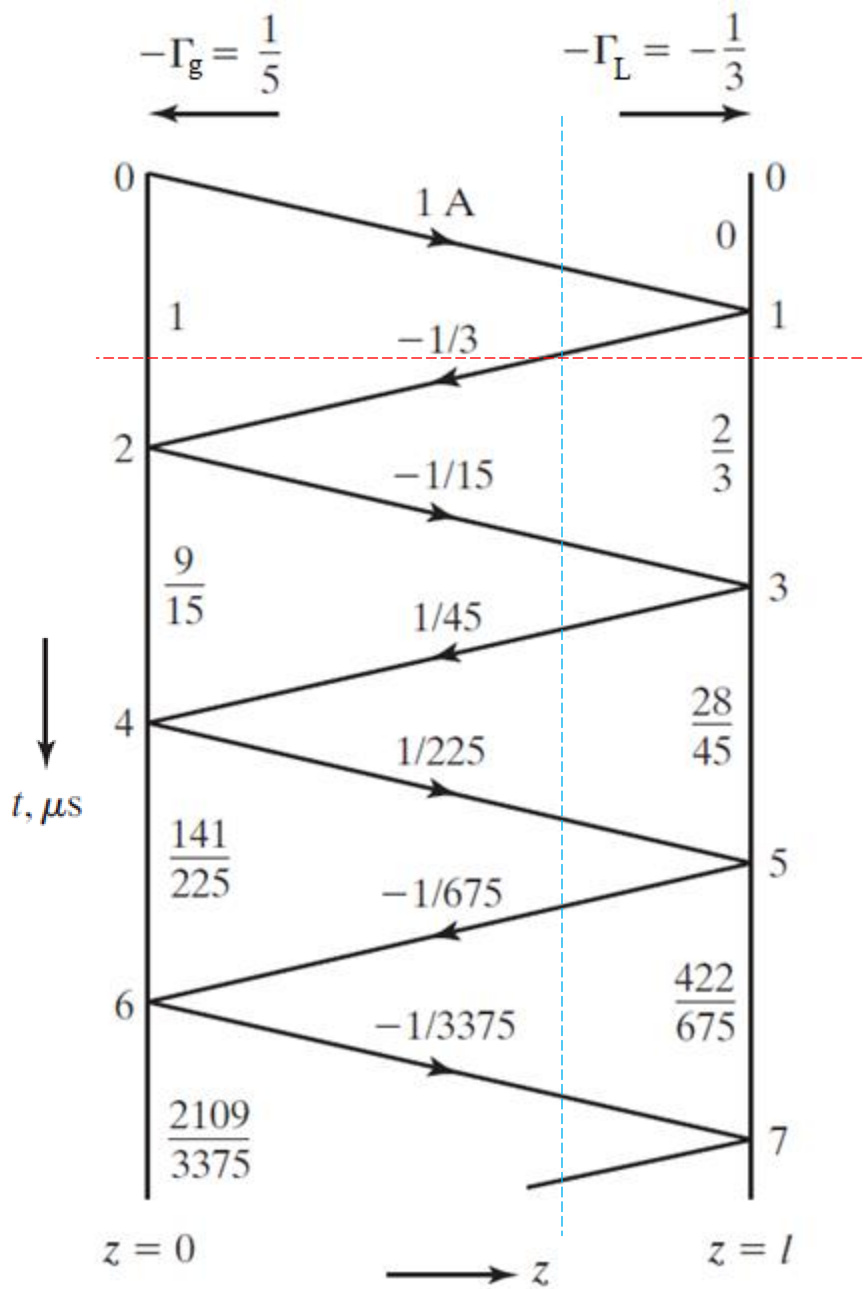
Find the voltage as a function of z at $t = 2.5\ \mu s$

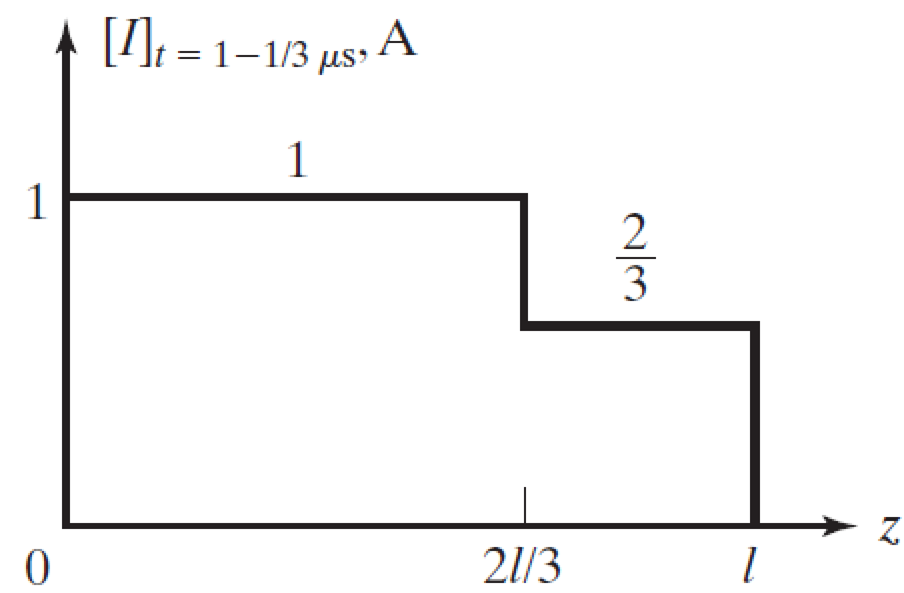




Example 3

Find the current as a function of z at $t = 1.\overline{3}\ \mu s$





Example 3

Steady State Analysis

$$\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{15}\right)^n = \frac{1}{1+1/15} = 0.9375$$

geometric series

Forward Path

$$V_{\mathrm{SS}}^{+} = 60 - 4 + \frac{4}{15} - \cdots = 60\left(1 - \frac{1}{15} + \frac{1}{15^2} - \cdots\right) = 56.25\ \mathrm{V}$$

$$I_{\mathrm{SS}}^{+} = 1 - \frac{1}{15} + \frac{1}{225} - \cdots = 1 - \frac{1}{15} + \frac{1}{15^2} - \cdots = 0.9375\ \mathrm{A}$$

Backward Path

$$V_{\mathrm{SS}}^{-} = 20 - \frac{4}{3} + \frac{4}{45} - \cdots = 20\left(1 - \frac{1}{15} + \frac{1}{15^2} - \cdots\right) = 18.75\ \mathrm{V}$$

$$I_{\mathrm{SS}}^{-} = -\frac{1}{3} + \frac{1}{45} - \frac{1}{675} + \cdots = -\frac{1}{3}\left(1 - \frac{1}{15} + \frac{1}{15^2} - \cdots\right) = -0.3125\ \mathrm{A}$$

$$V_{\mathrm{SS}} = V_{\mathrm{SS}}^{+} + V_{\mathrm{SS}}^{-} = 75\ \mathrm{V}$$

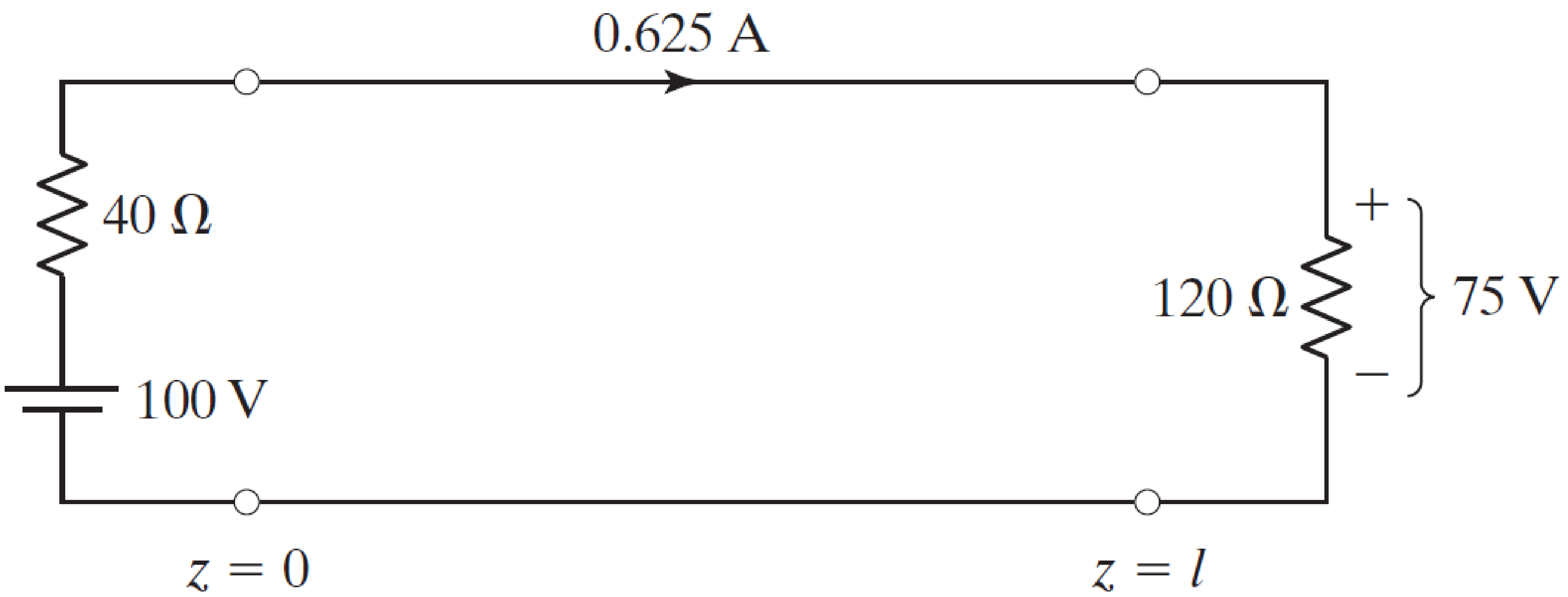
$$I_{\mathrm{SS}} = I_{\mathrm{SS}}^{+} + I_{\mathrm{SS}}^{-} = 0.625\ \mathrm{A}$$

Example 3

Steady State Analysis

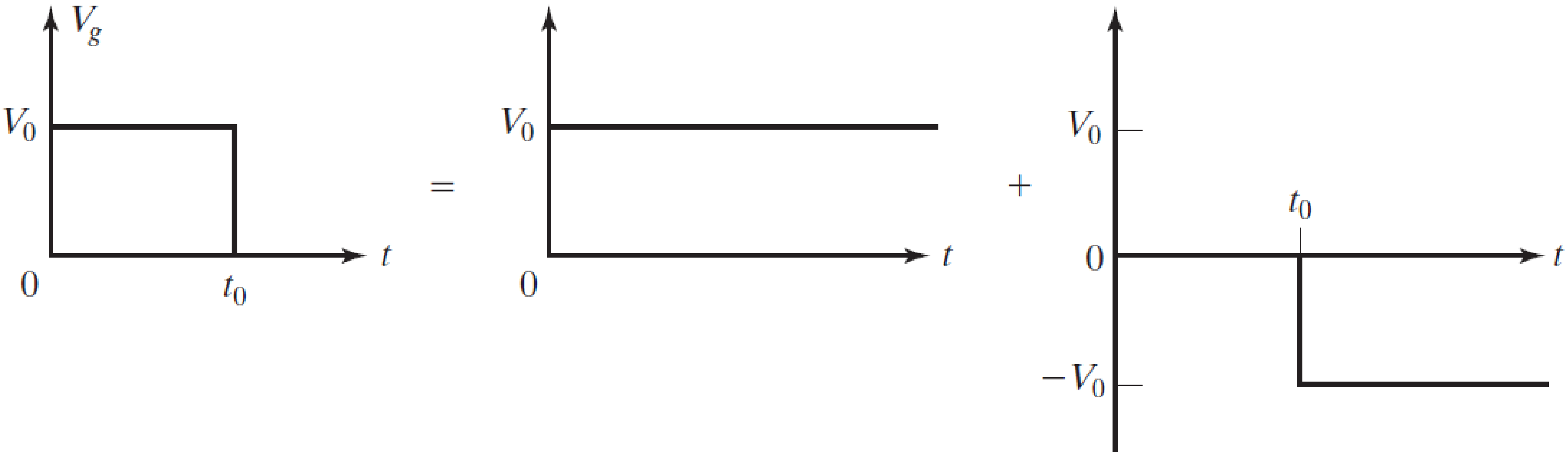
$$V_{SS} = V_{SS}^{+} + V_{SS}^{-} = 75 \text{ V}$$

$$I_{SS} = I_{SS}^{+} + I_{SS}^{-} = 0.625 \text{ A}$$



How can we analyze a pulse signal?

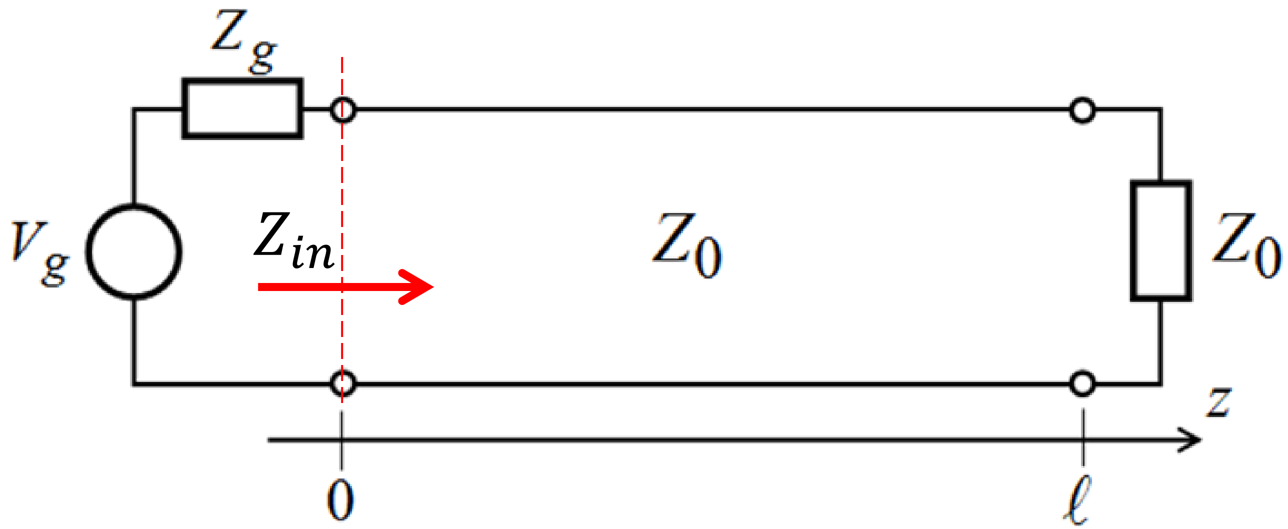
Rectangular pulse as superposition of two step function signals



Questions

What is the input impedance Z_{in} of these transmission lines?

(a)



(b)