ECE 329 – Fall 2021

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Lecture 34-35

Lecture 34-35 – Outline

- **General properties of transmission lines**
- **Relationship between reflection coefficient and impedance**
- **Graphical representation of impedance in the domain of the reflection coefficient: Smith chart**
- **Fundamental operations on the Smith chart**
- **Examples of Smith chart applications**

Reading assignment

Prof. Kudeki's ECE 329 Lecture Notes on Fields and Waves:

34) Line impedance, generalized reflection coefficient, Smith Chart

35) Smith Chart examples

SUMMARY – λ/2 transmission line

$$V_{in} = -V_L$$

$$I_{in} = -I_L$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{-V_L}{-I_L} = Z_L$$

SUMMARY – λ/4 transmission line

$$V_{in} = j\,I_L Z_0 \;\longrightarrow\; I_L = -j\frac{V_{in}}{Z_0}$$

$$I_{in} = j\frac{V_L}{Z_0}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{j\,I_L Z_0}{jV_L/Z_0} = \frac{Z_0^2}{V_L/I_L} = \frac{Z_0^2}{Z_L}$$

impedance transformation of a λ/4 transformer

Example

$$Z_L = 100\,\Omega \qquad l = 0.75\lambda$$

$$Z_o = 50\,\Omega \qquad V_g = j10\ \text{V} \qquad Z_g = 25\,\Omega$$

Determine V_L and I_L

$$Z_{in} = \frac{Z_o^2}{Z(0.5\lambda)} = \frac{50^2}{100} = 25\,\Omega$$

same as load

Voltage divider

$$V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}} = j10 \frac{25}{25+25} = j5\,\text{V}$$

Example

$$Z_L = 100\,\Omega \qquad l = 0.75\lambda$$

$$Z_o = 50\,\Omega \qquad V_g = j10\ \text{V} \qquad Z_g = 25\,\Omega$$

half-wave transformer rule

$$V(0.25\lambda) = -V_{in} = -j5\,\text{V}$$

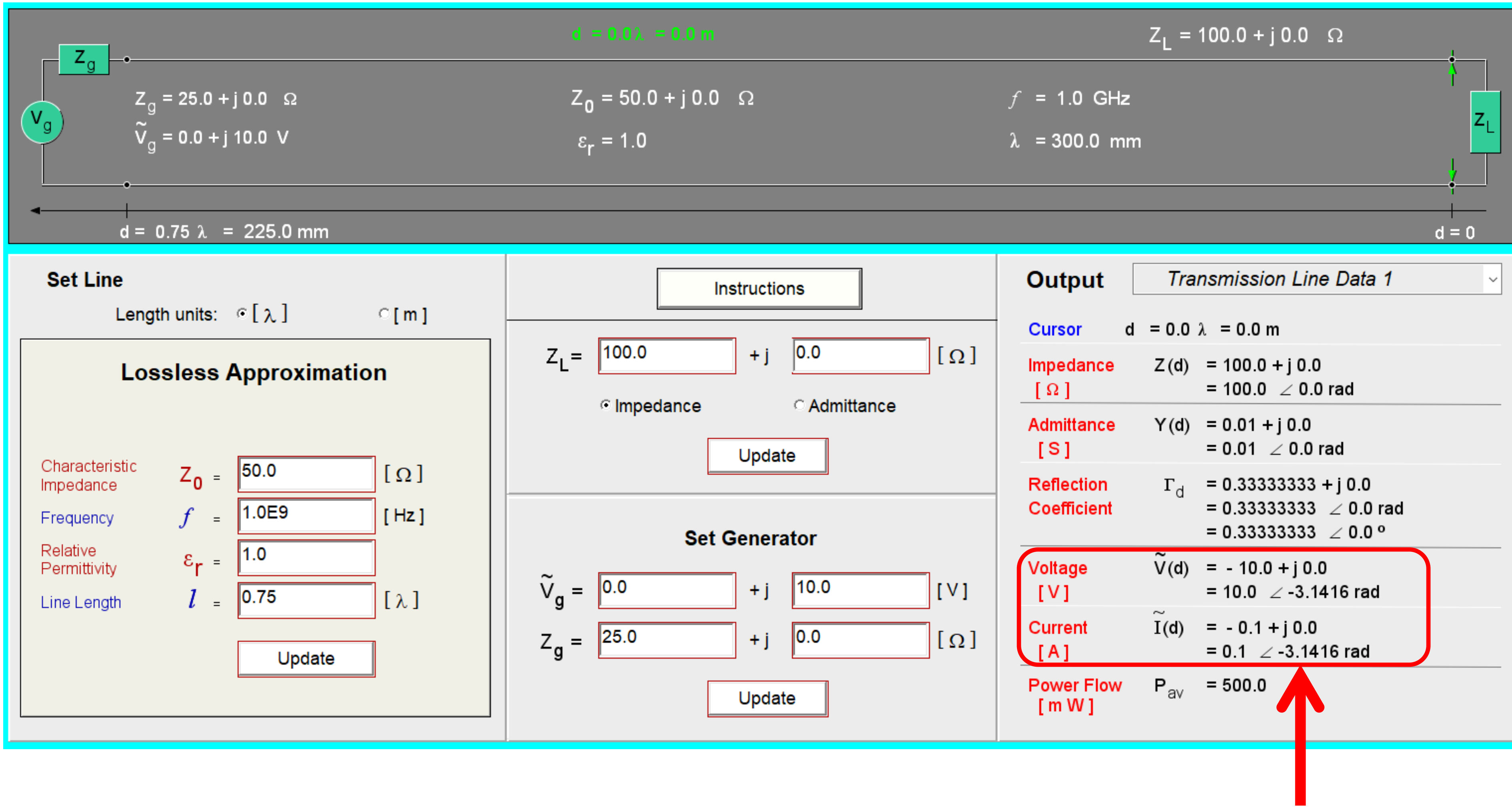
which input voltage for final quarter-wave section, and

$$I_L = -j\frac{V(0.25\lambda)}{Z_o} = -j\frac{-j5}{50} = -0.1\,\text{A}$$

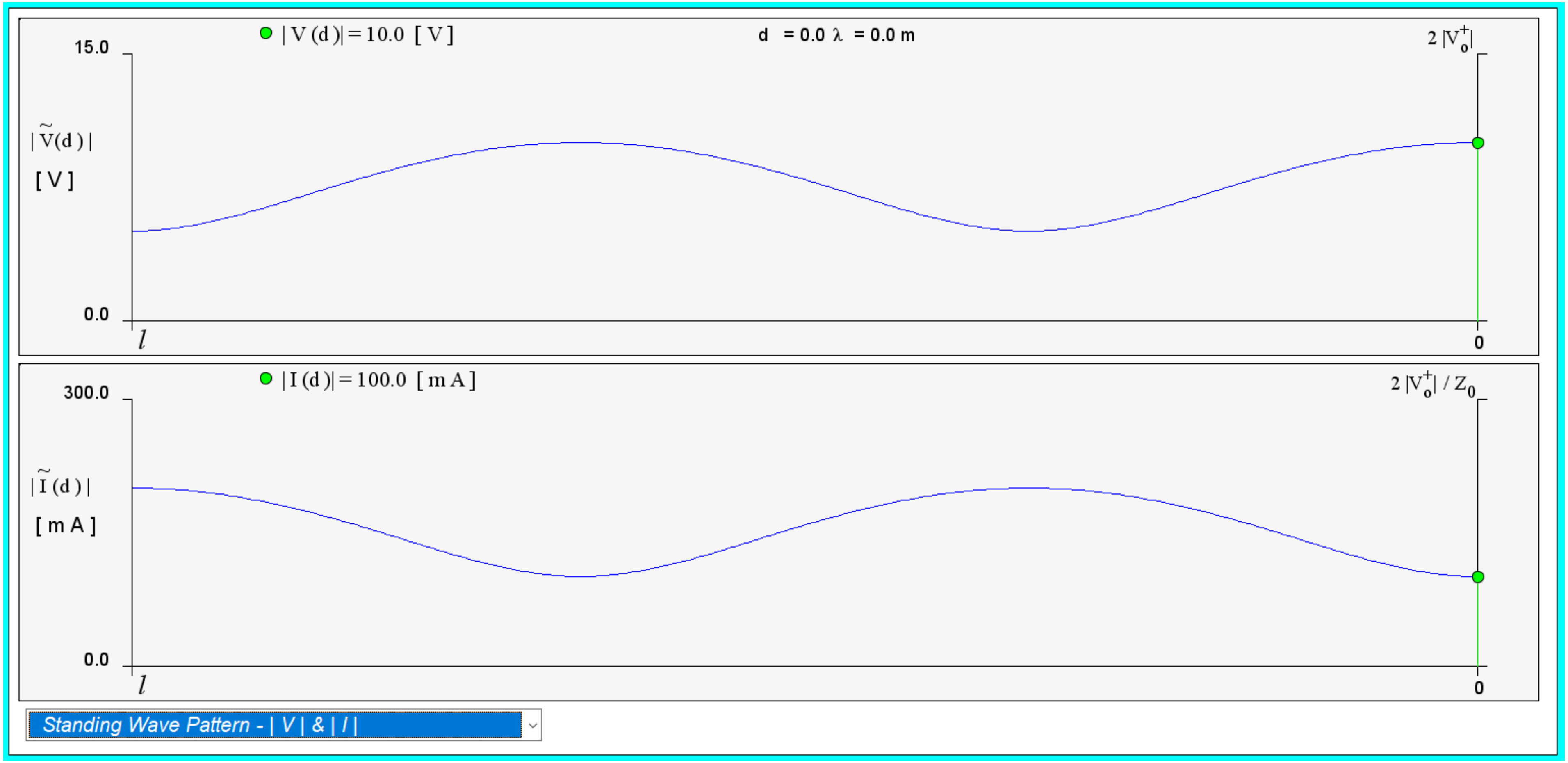
$$\boxed{I_L = -j\frac{V_{in}}{Z_0}}$$ from Lecture 33

load voltage

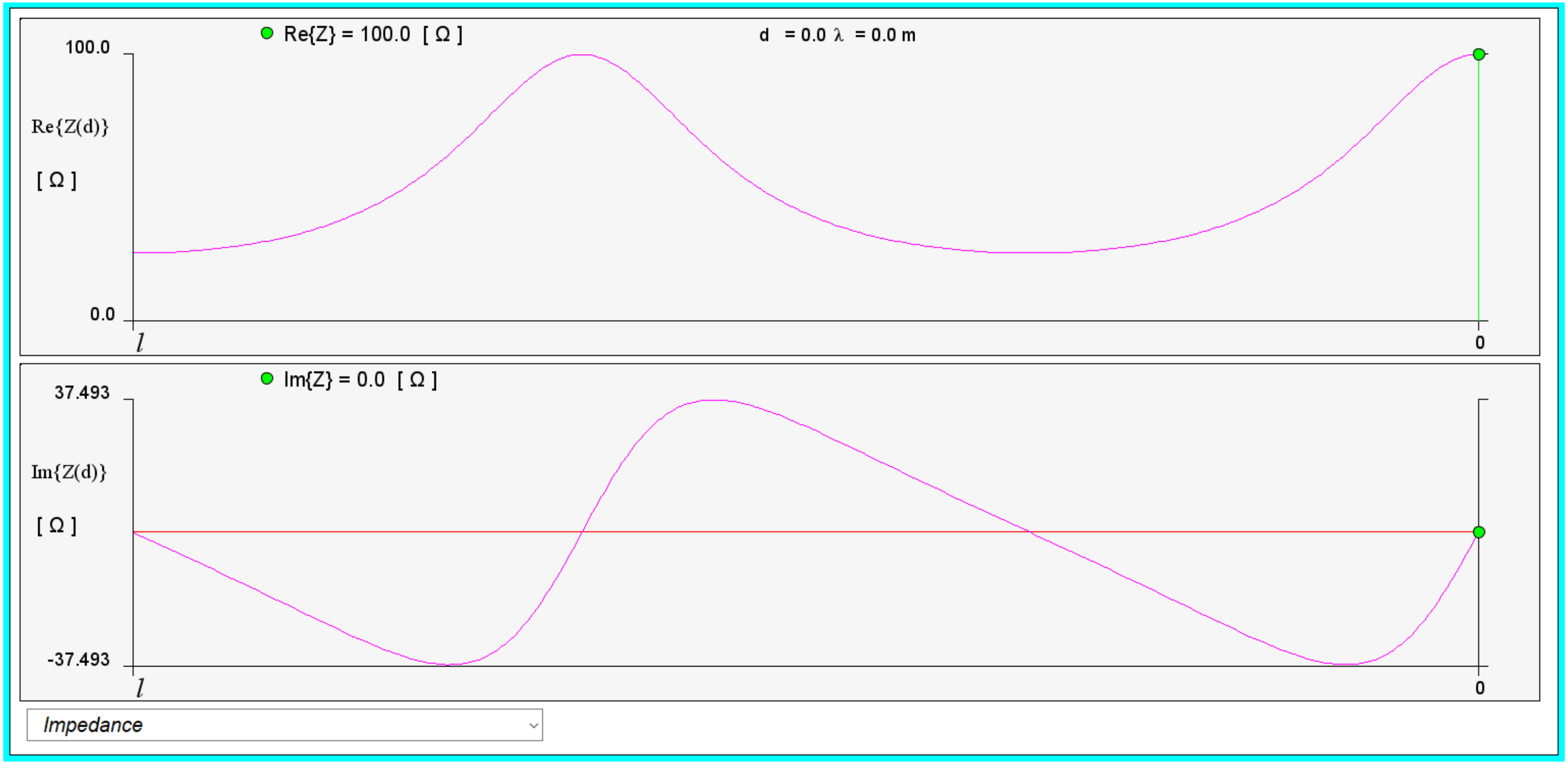
$$V_L = Z_L I_L = (100\,\Omega)(-0.1\,\text{A}) = -10\,\text{V}$$



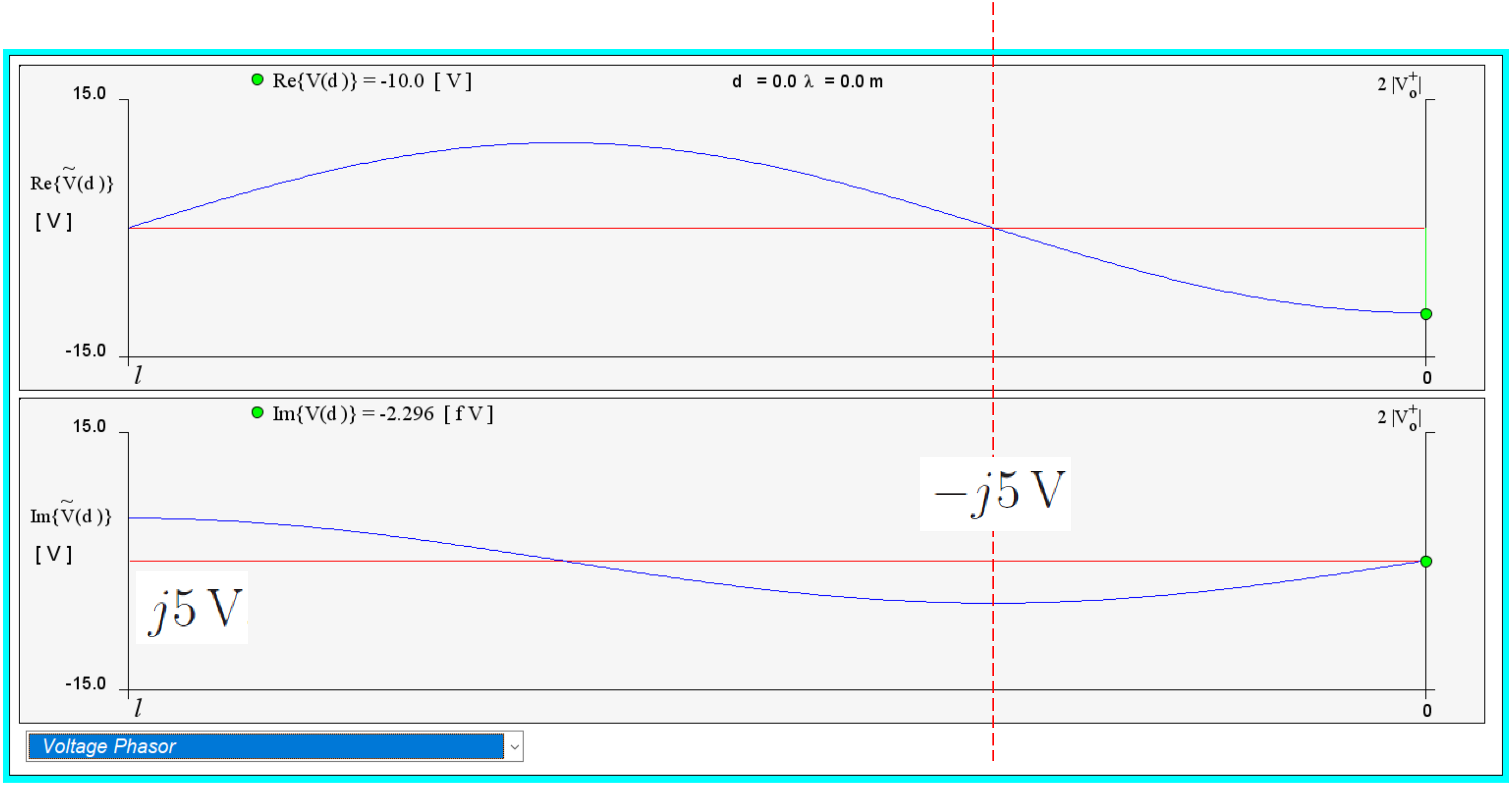
d = 0.0λ = 0.0 m
Z_L = 100.0 + j 0.0 Ω
Z_g
V_g
Z_g = 25.0 + j 0.0 Ω
Ṽ_g = 0.0 + j 10.0 V
Z_0 = 50.0 + j 0.0 Ω
ε_r = 1.0
f = 1.0 GHz
λ = 300.0 mm
Z_L
d = 0.75 λ = 225.0 mm
d = 0
Set Line
Length units: [λ] [m]
Lossless Approximation
Characteristic Impedance Z_0 = 50.0 [Ω]
Frequency f = 1.0E9 [Hz]
Relative Permittivity ε_r = 1.0
Line Length l = 0.75 [λ]
Update
Instructions
Z_L = 100.0 + j 0.0 [Ω]
Impedance
Admittance
Update
Set Generator
Ṽ_g = 0.0 + j 10.0 [V]
Z_g = 25.0 + j 0.0 [Ω]
Update
Output
Transmission Line Data 1
Cursor d = 0.0 λ = 0.0 m
Impedance [Ω] Z(d) = 100.0 + j 0.0
= 100.0 ∠ 0.0 rad
Admittance [S] Y(d) = 0.01 + j 0.0
= 0.01 ∠ 0.0 rad
Reflection Coefficient Γ_d = 0.33333333 + j 0.0
= 0.33333333 ∠ 0.0 rad
= 0.33333333 ∠ 0.0 º
Voltage [V] Ṽ(d) = - 10.0 + j 0.0
= 10.0 ∠ -3.1416 rad
Current [A] Ĩ(d) = - 0.1 + j 0.0
= 0.1 ∠ -3.1416 rad
Power Flow [m W] P_av = 500.0



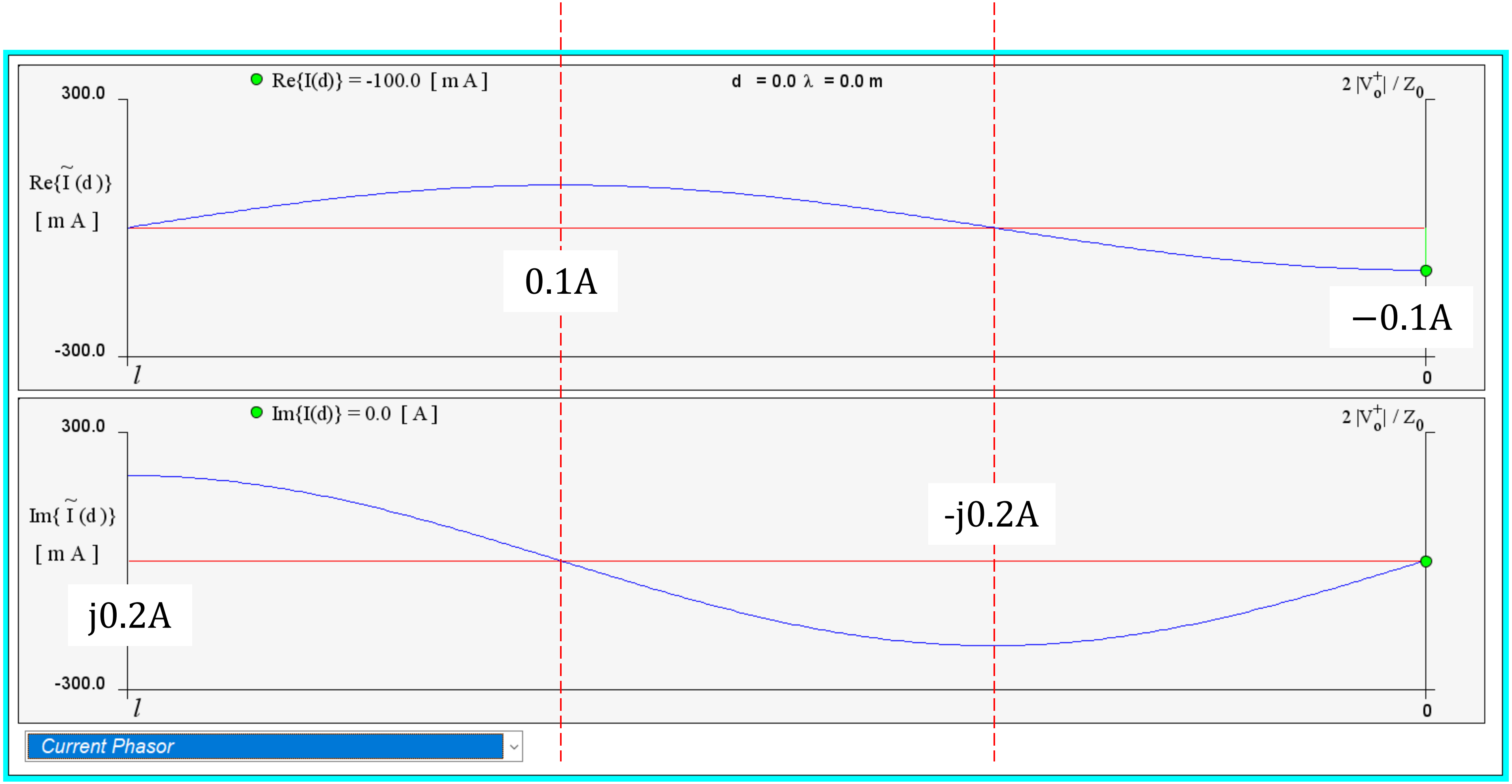
|V(d)| = 10.0 [V]
d = 0.0 λ = 0.0 m
2 |V$_o^+$|
15.0
|Ṽ(d)|
[V]
0.0
l
0
|I(d)| = 100.0 [m A]
2 |V$_o^+$| / Z$_0$
300.0
|Ĩ(d)|
[m A]
0.0
l
0
Standing Wave Pattern - | V | & | I |


Re{Z} = 100.0 [Ω]
d = 0.0 λ = 0.0 m
100.0
Re{Z(d)}
[Ω]
0.0
l
0
Im{Z} = 0.0 [Ω]
37.493
Im{Z(d)}
[Ω]
-37.493
l
0
Impedance

$d = 0.75\,\lambda$ $d = 0.25\,\lambda$



$d = 0.75\,\lambda$ $d = 0.5\,\lambda$ $d = 0.25\,\lambda$



Introduction to the Smith Chart

Once again, we have obtained the following expressions for a TL:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

load reflection coefficient

$$\Gamma(d) = \Gamma_L e^{-2j\beta d}$$

generalized reflection coefficient

$$Z(d) = \frac{V(d)}{I(d)} = Z_0 \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

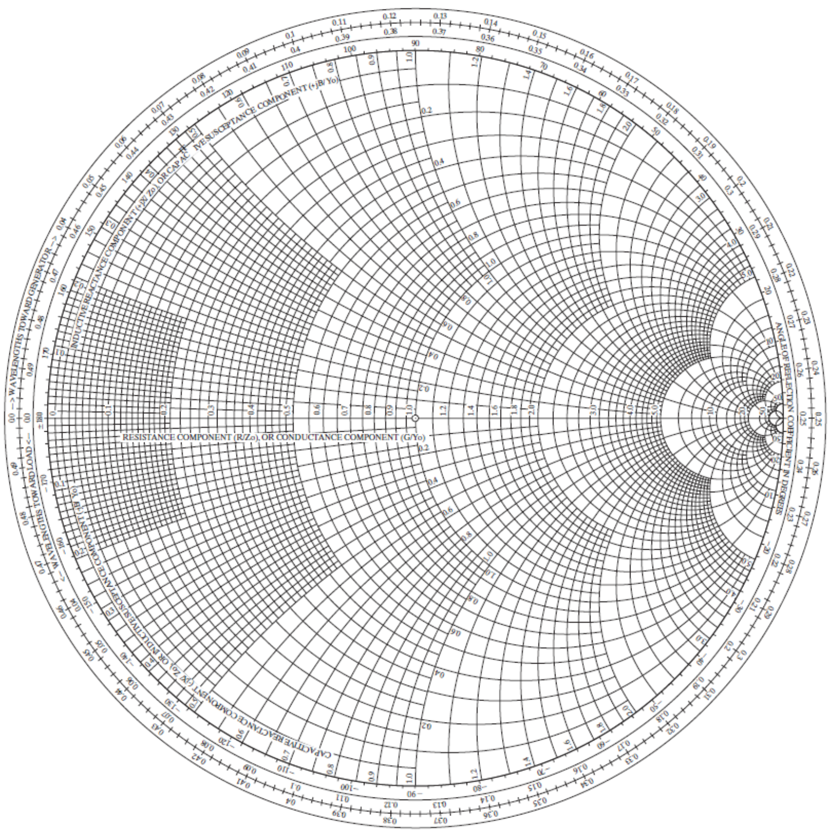
line impedance

Can we plot these quantities?

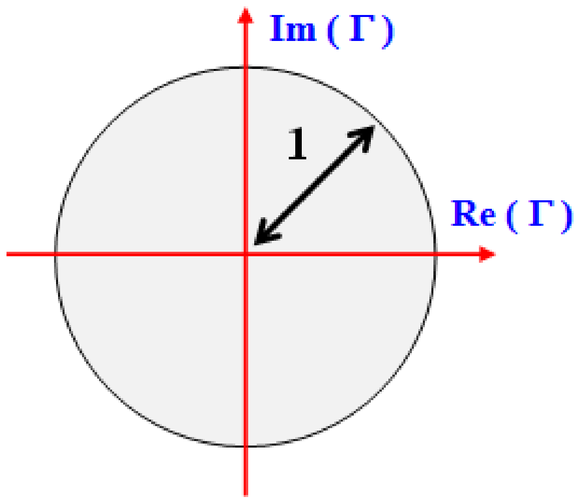
Reflection coefficient is complex (2 coordinates) and impedance is also complex (2 additional coordinates) = this is a 4-dimensional problem.

Introduction to the Smith Chart

The Smith chart is one of the most useful graphical tools for high frequency circuit applications. The chart provides a clever way to visualize complex functions and it continues to endure popularity, decades after its original conception.



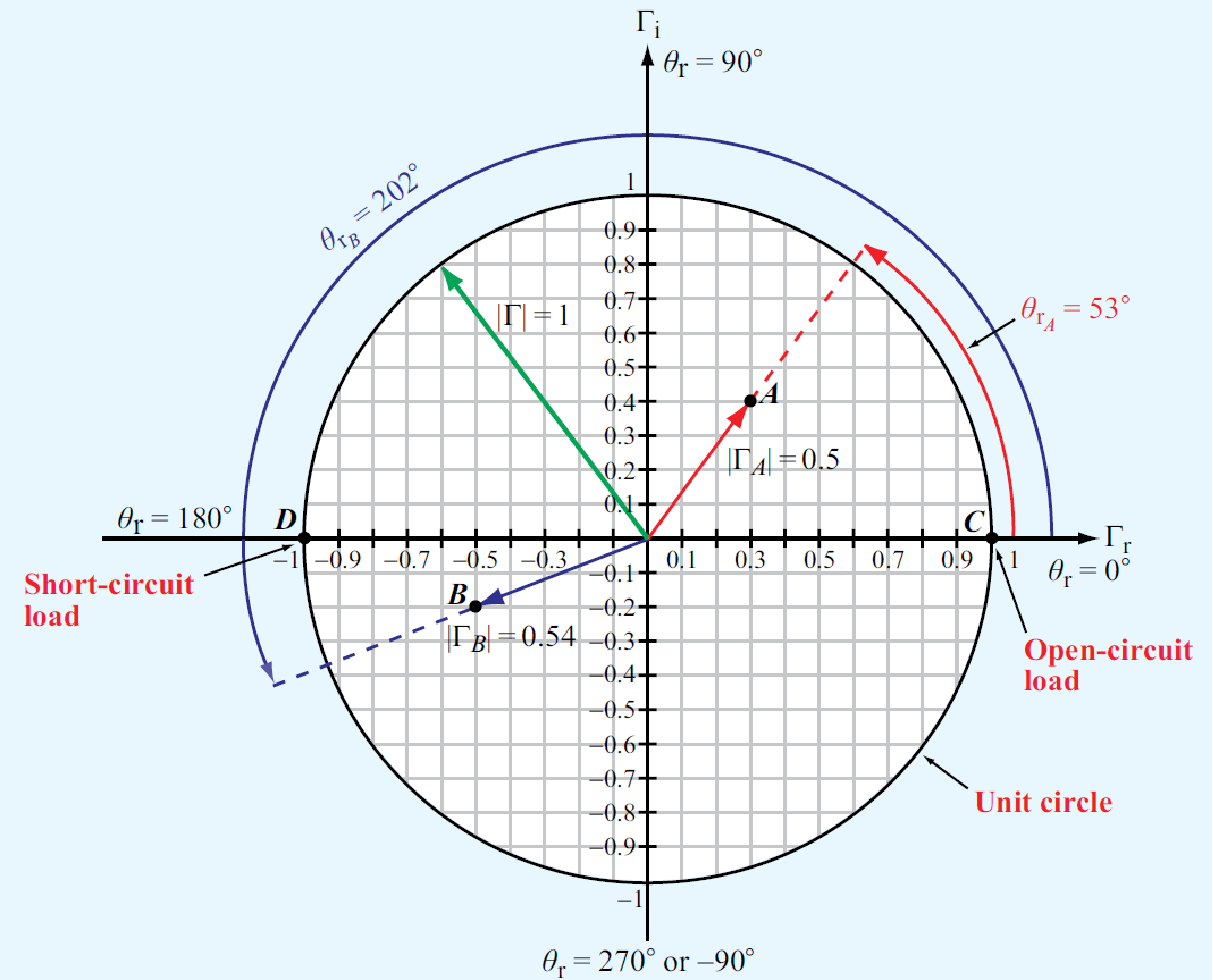
From a mathematical point of view, the Smith chart is a 4-D representation of all possible complex impedances with respect to coordinates defined by the complex reflection coefficient.



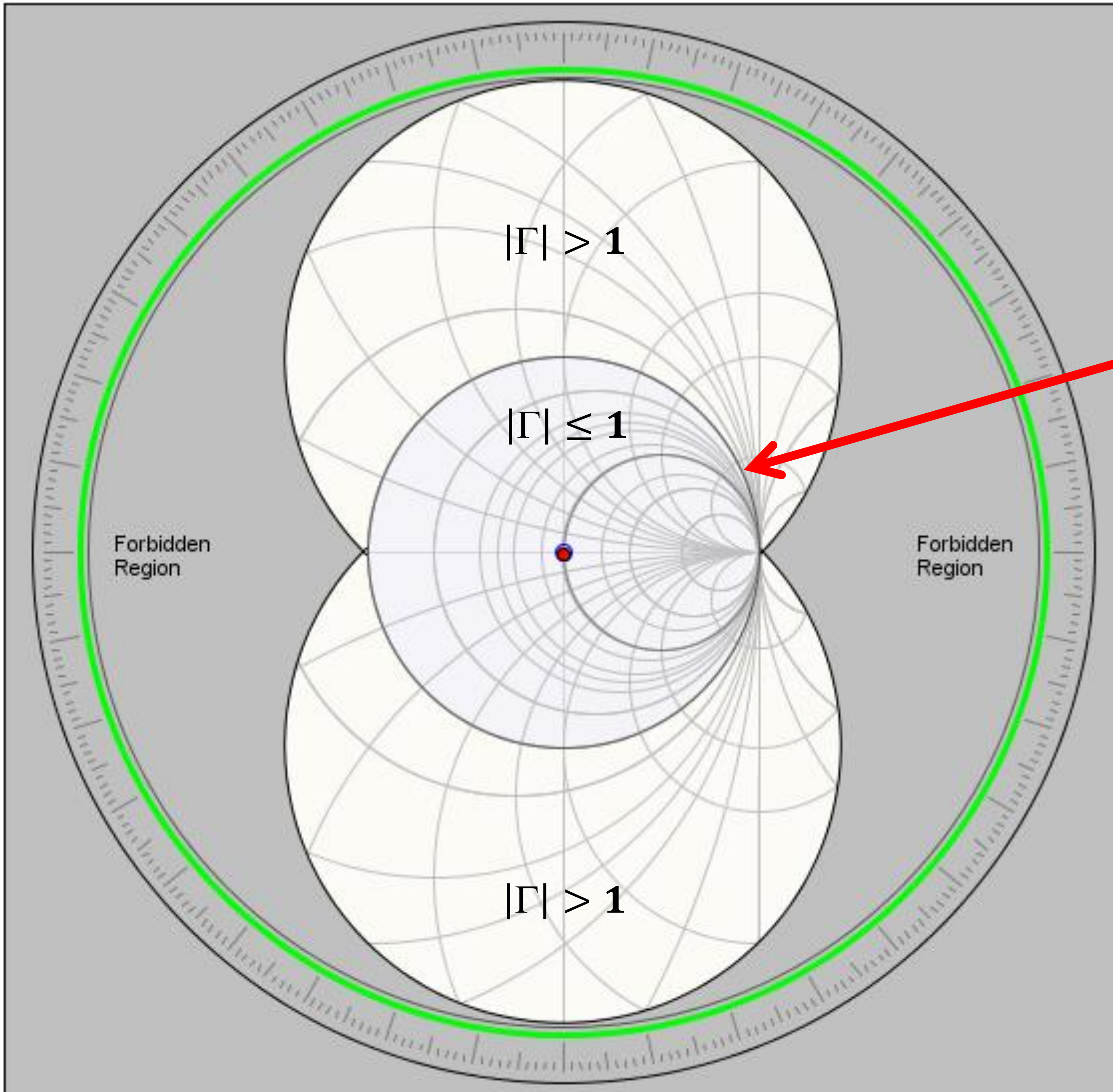
The domain of definition of the reflection coefficient for a loss-less line is a circle of unitary radius in the complex plane. This is also the domain of the Smith chart.

For a general lossy line we may have $|\Gamma| > 1$, *due to the complex characteristic impedance, thus requiring an extended Smith chart.*

Domain of the Reflection Coefficient in a loss-less line



EXTRA – This is what the extended Smith chart for lossy lines would look like



Lossless line Smith Chart

Use of this extended chart is actually rare

The goal of the Smith chart is to identify all possible impedances on the domain of existence of the reflection coefficient. To do so, we start from the general definition of line impedance

$$Z(d) = \frac{V(d)}{I(d)} = Z_0 \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

This provides the complex function $Z(d) = f\{\mathrm{Re}(\Gamma), \mathrm{Im}(\Gamma)\}$ *we wish to plot but the result would apply only to a specific value of* Z_0.

To obtain universal curves we introduce the normalized impedance

$$z_n(d) = \frac{Z(d)}{Z_0} = \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

The normalized impedance is represented on the Smith chart by using families of curves that identify the normalized resistance r (real part) and the normalized reactance x (imaginary part)

$$z_n(d) = \mathrm{Re}(z_n) + j\,\mathrm{Im}(z_n) = r + jx$$

Let's represent the reflection coefficient in terms of its coordinates

$$\Gamma(d) = \mathrm{Re}(\Gamma) + j\,\mathrm{Im}(\Gamma)$$

Now we can write

$$r + jx = \frac{1 + \mathrm{Re}(\Gamma) + j\,\mathrm{Im}(\Gamma)}{1 - \mathrm{Re}(\Gamma) - j\,\mathrm{Im}(\Gamma)} = \frac{1 - \mathrm{Re}^2(\Gamma) - \mathrm{Im}^2(\Gamma) + j2\,\mathrm{Im}(\Gamma)}{(1 - \mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)}$$

*The **real part** gives*

$$r = \frac{1 - \mathrm{Re}^2(\Gamma) - \mathrm{Im}^2(\Gamma)}{\left(1 - \mathrm{Re}(\Gamma)\right)^2 + \mathrm{Im}^2(\Gamma)}$$

Add a quantity equal to zero

$$r\left(\mathrm{Re}(\Gamma) - 1\right)^2 + \left(\mathrm{Re}^2(\Gamma) - 1\right) + r\,\mathrm{Im}^2(\Gamma) + \mathrm{Im}^2(\Gamma) + \underbrace{\frac{1}{1+r} - \frac{1}{1+r}}_{=\,0} = 0$$

$$(1+r)\left[\mathrm{Re}^2(\Gamma) - 2\,\mathrm{Re}(\Gamma)\frac{r}{1+r} + \frac{r^2}{(1+r)^2}\right] + (1+r)\,\mathrm{Im}^2(\Gamma) = \frac{1}{1+r}$$

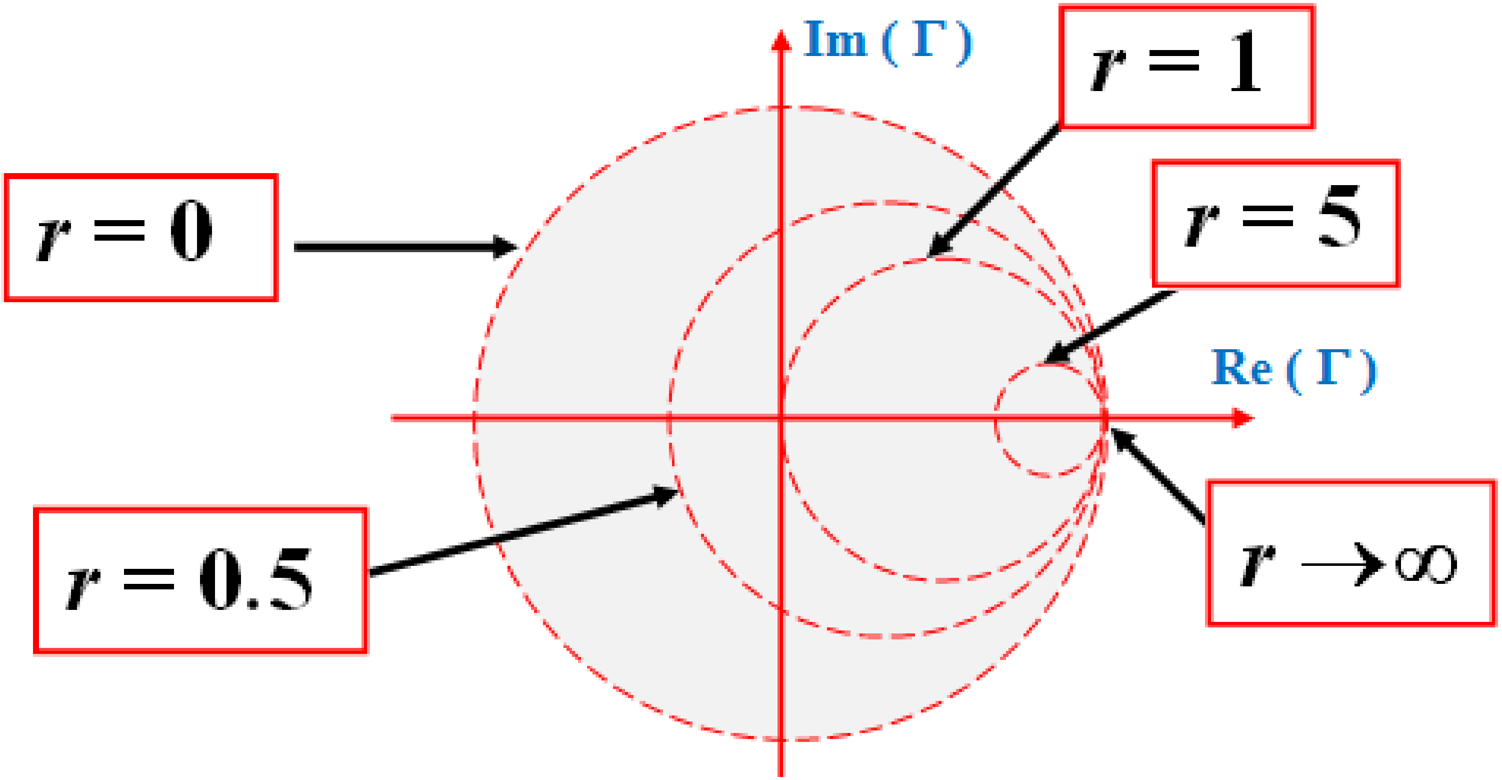
$$\Rightarrow \qquad \left[\mathrm{Re}(\Gamma) - \frac{r}{1+r}\right]^2 + \mathrm{Im}^2(\Gamma) = \left(\frac{1}{1+r}\right)^2$$

Equation of a circle

*The result for the **real part** shows that all the possible impedances with a given normalized resistance r are found on a **circle** with*

Center = $\left\{ \dfrac{r}{1+r}, 0 \right\}$ **Radius =** $\dfrac{1}{1+r}$

defining a family of circles in the domain $|\Gamma| \leq 1$.



The imaginary part gives

$$x = \frac{2\operatorname{Im}(\Gamma)}{(1-\operatorname{Re}(\Gamma))^2 + \operatorname{Im}^2(\Gamma)}$$

Multiply by x and add a quantity equal to zero

$$x^2\left[(1-\operatorname{Re}(\Gamma))^2 + \operatorname{Im}^2(\Gamma)\right] - 2x\operatorname{Im}(\Gamma) + \overbrace{1-1}^{=0} = 0$$

$$(1-\operatorname{Re}(\Gamma))^2 + \left[\operatorname{Im}^2(\Gamma) - \frac{2}{x}\operatorname{Im}(\Gamma) + \frac{1}{x^2}\right] = \frac{1}{x^2}$$

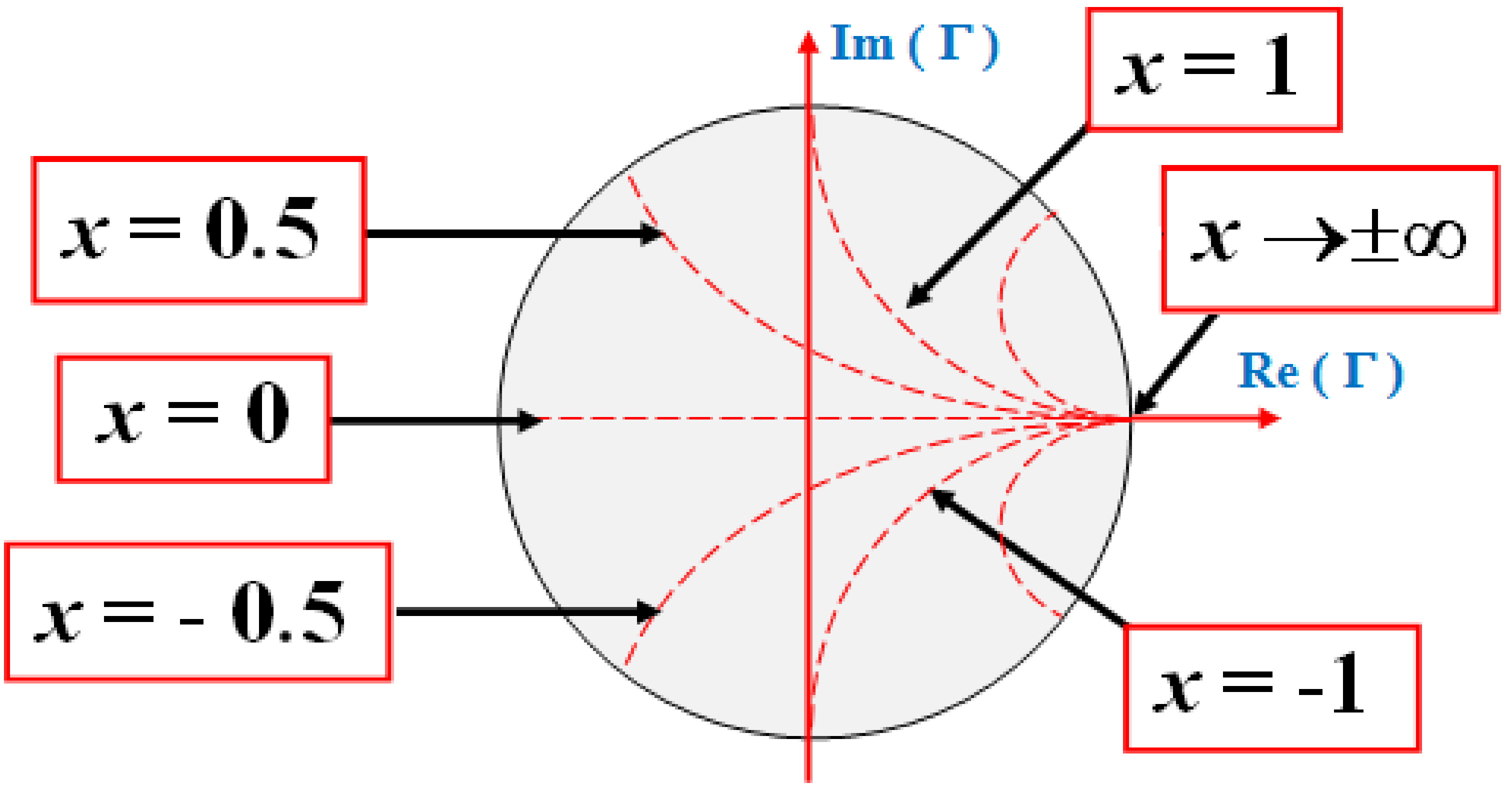
$$\Rightarrow \qquad (\operatorname{Re}(\Gamma)-1)^2 + \left[\operatorname{Im}(\Gamma) - \frac{1}{x}\right]^2 = \frac{1}{x^2}$$

Equation of a circle

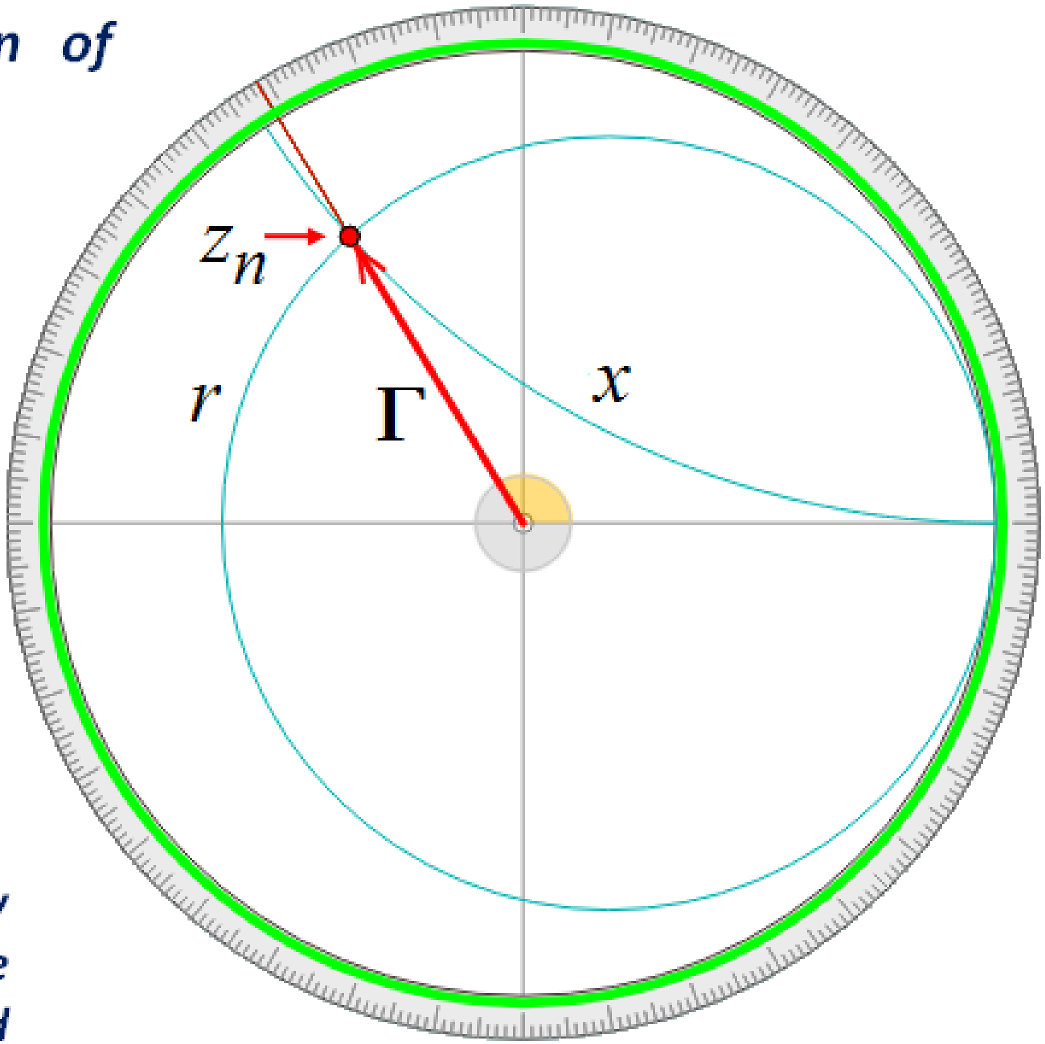
The result for the imaginary part shows that all the possible impedances with a given normalized reactance x are on a circle with

$$\text{Center} = \left\{ 1\ ,\ \frac{1}{x} \right\} \qquad \text{Radius} = \frac{1}{x}$$

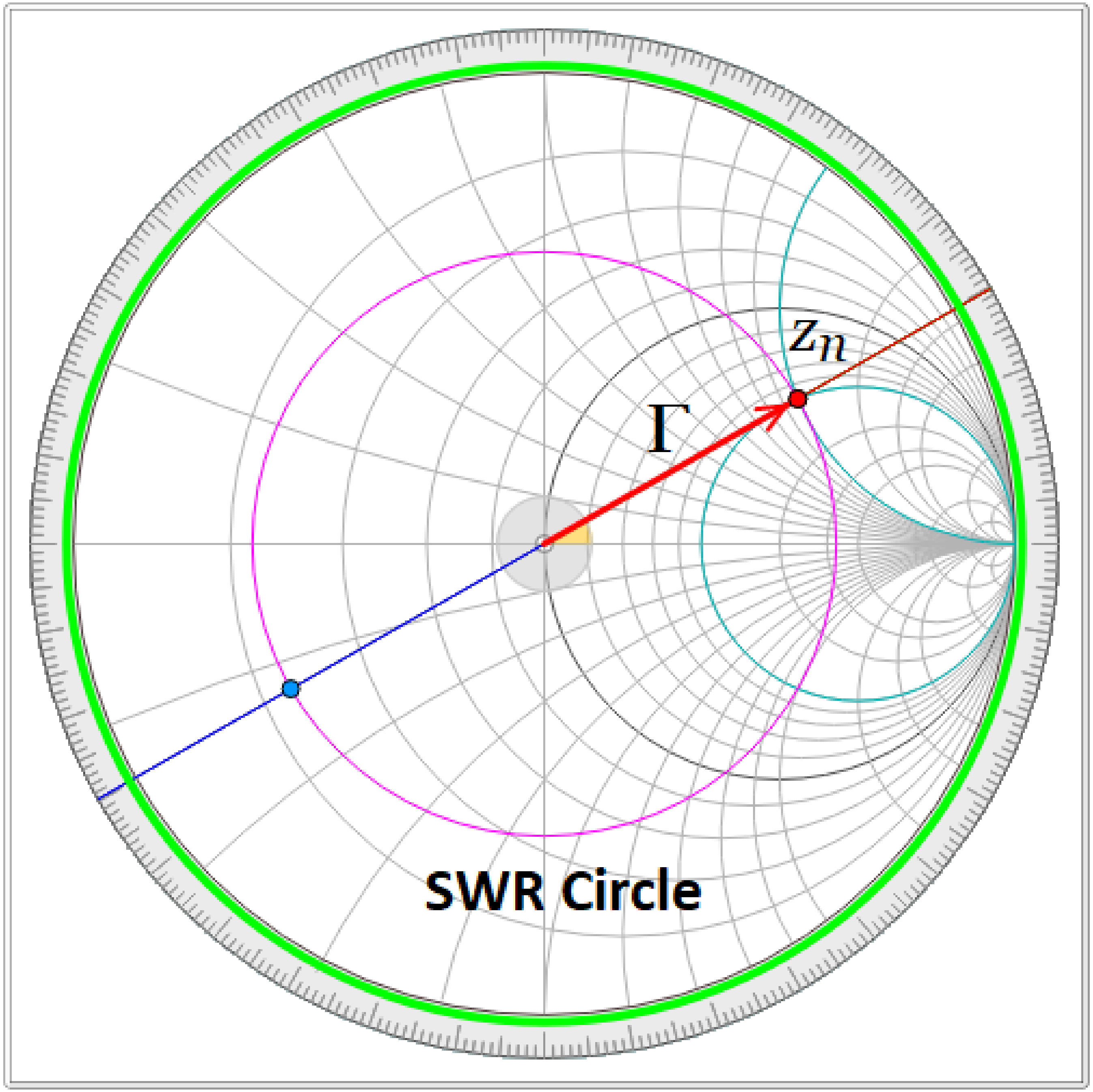
defining a family of arcs in the domain $|\,\Gamma\,| \leq 1$.



There is one-to-one correspondence between reflection coefficient and impedance. Any value of z_n is identified by the intersection of two curves.



The Smith Chart is assembled by drawing a number of representative curves for normalized resistance and normalized reactance.



Smith Chart for Impedances

Positive (inductive) reactance

Negative (capacitive) reactance

The five basic techniques of the Smith Chart

1) Given a reflection coefficient, obtain the corresponding impedance

2) Given the impedance (reflection coefficient) at one point on the line, find the impedance (reflection coefficient) at any other point on the line

3) Find the locations of maximum and minimum of the Standing Wave Pattern

4) Find the Voltage Standing Wave Ratio (VSWR)

5) Given an impedance find the corresponding admittance

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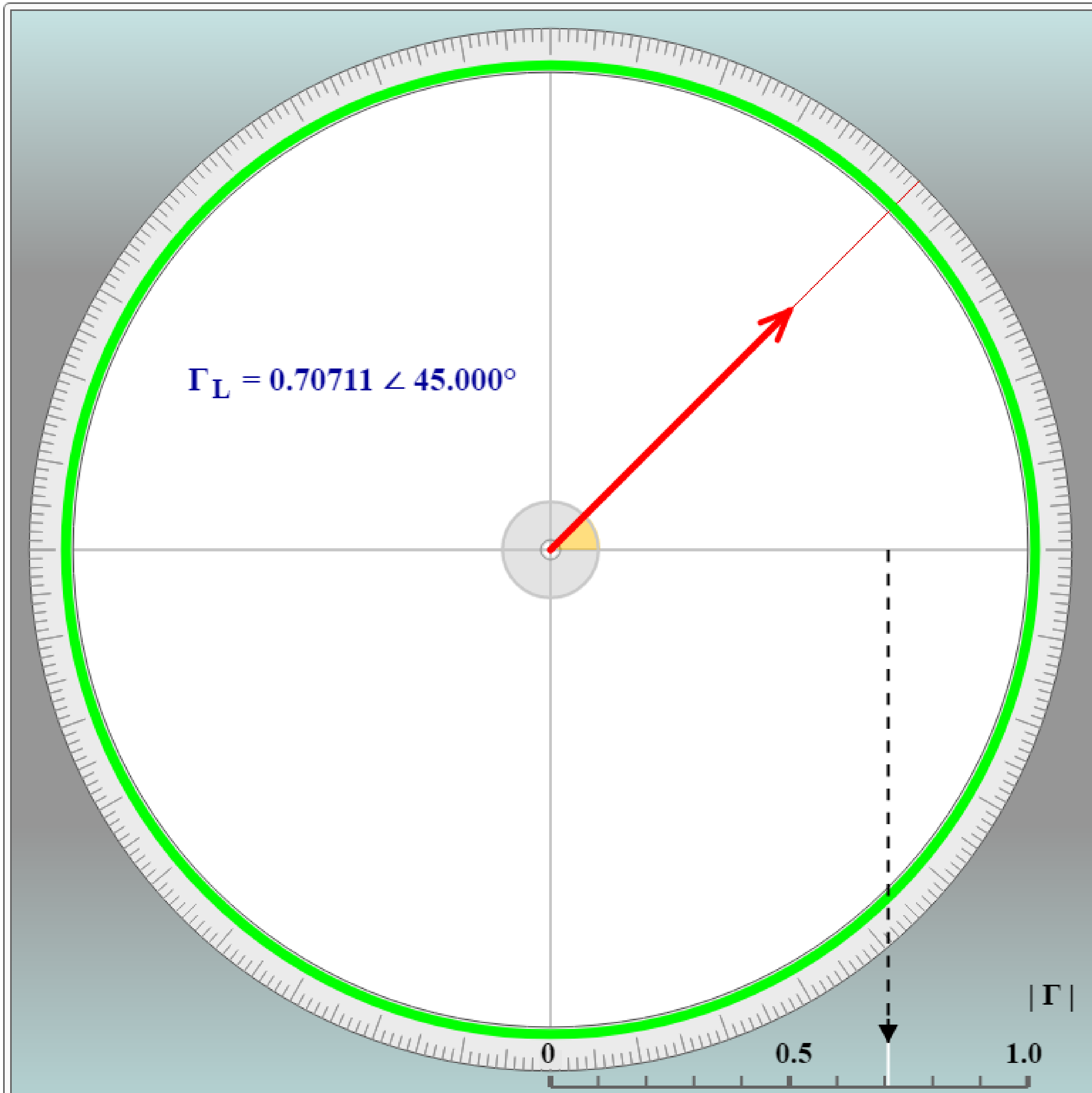
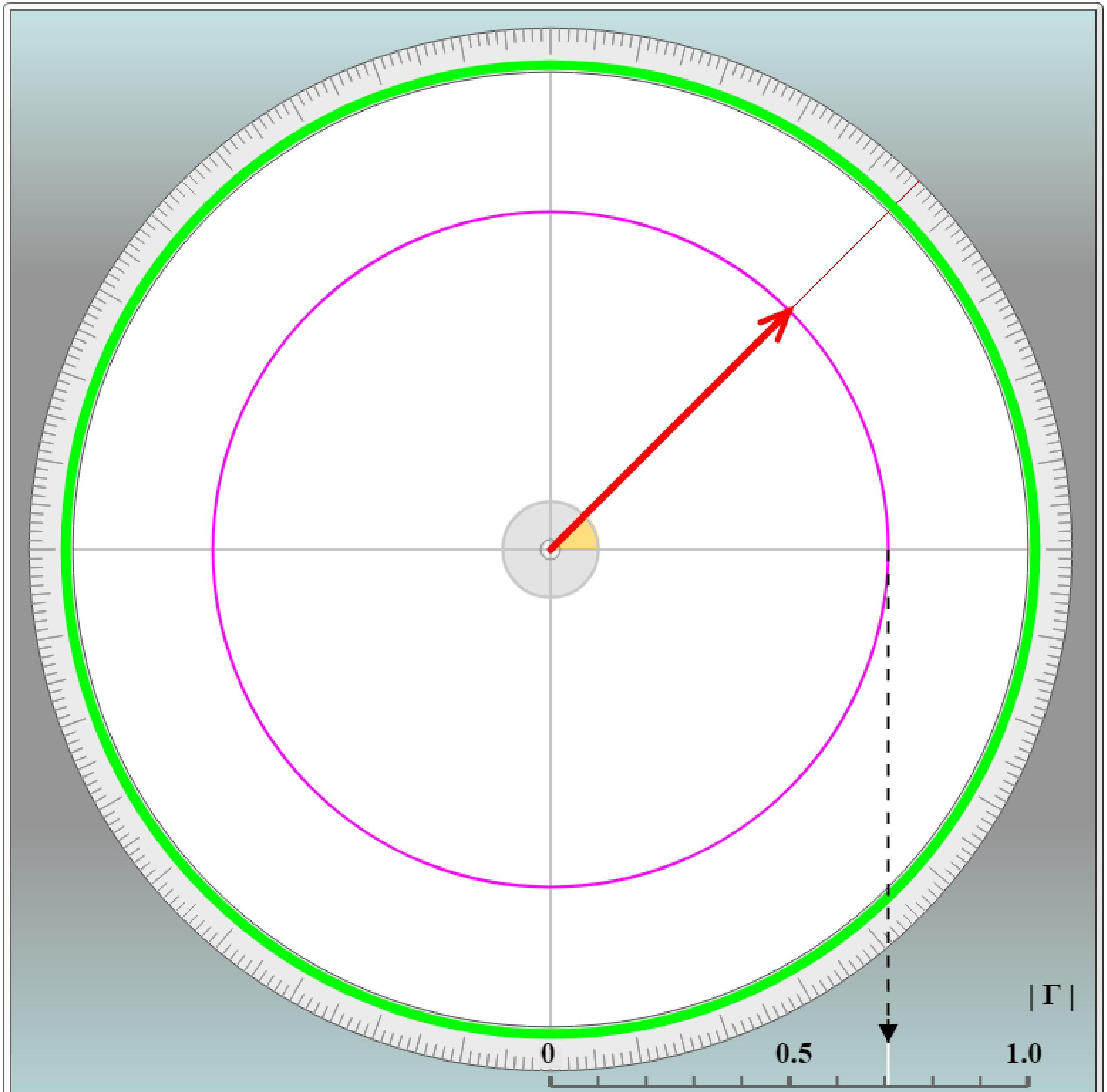
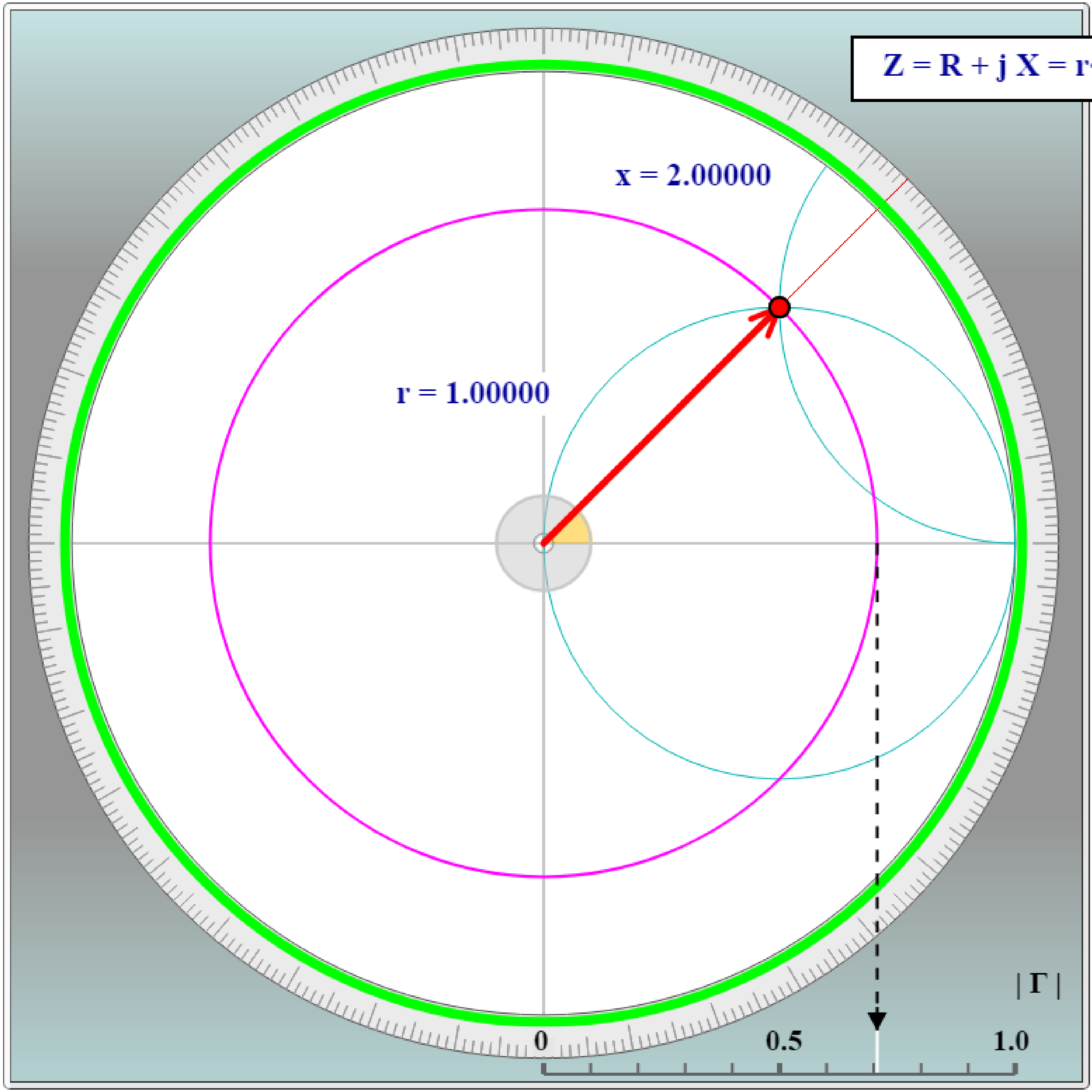

$\Gamma_L = 0.70711 \angle 45.000°$
$|\Gamma|$
0
0.5
1.0


|Γ|
0
0.5
1.0

$Z = R + j\,X = r{\cdot}Z_0 + j\,x{\cdot}Z_0$



The five basic techniques of the Smith Chart

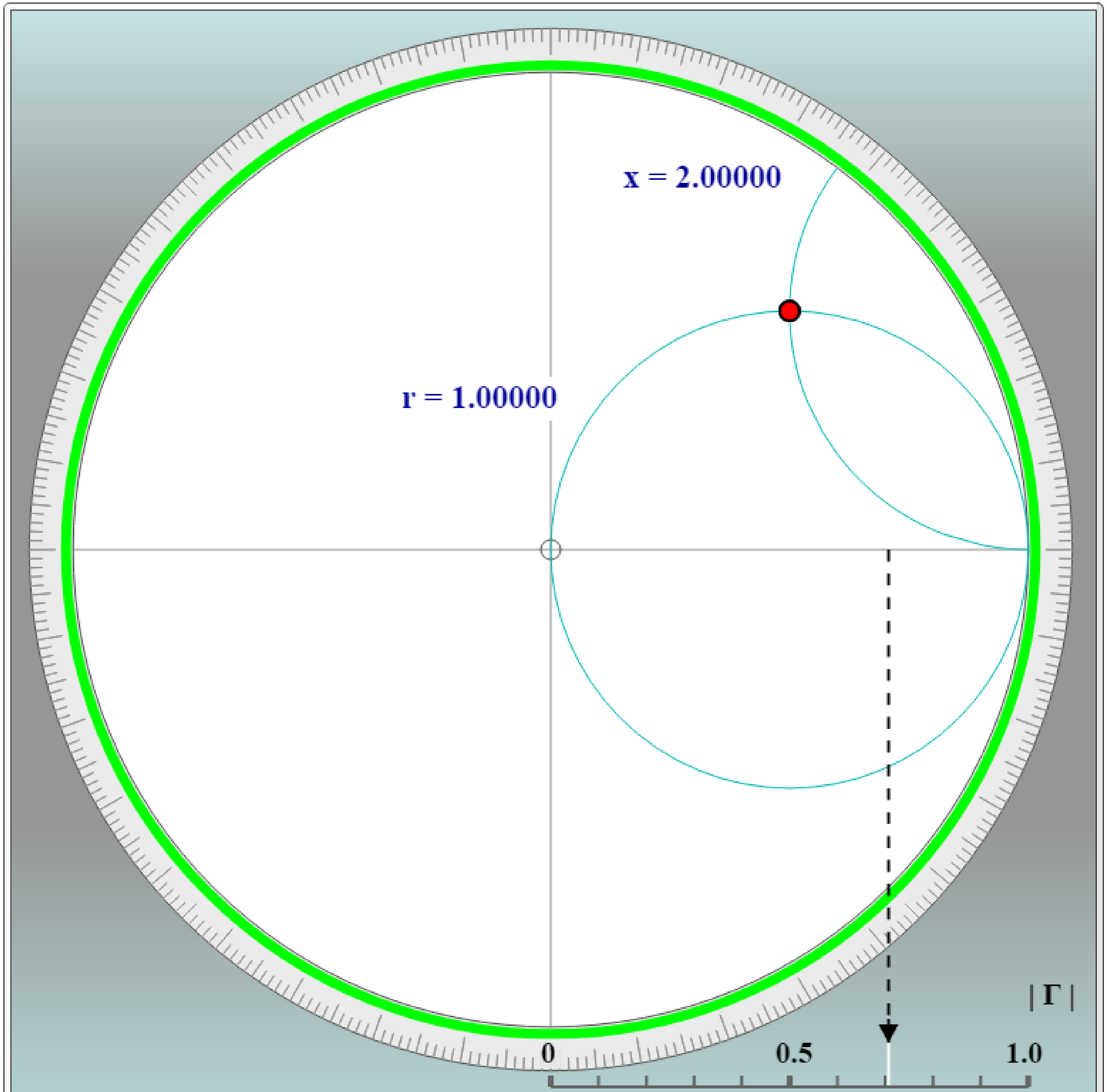
1) Given an impedance, obtain the corresponding reflection coefficient

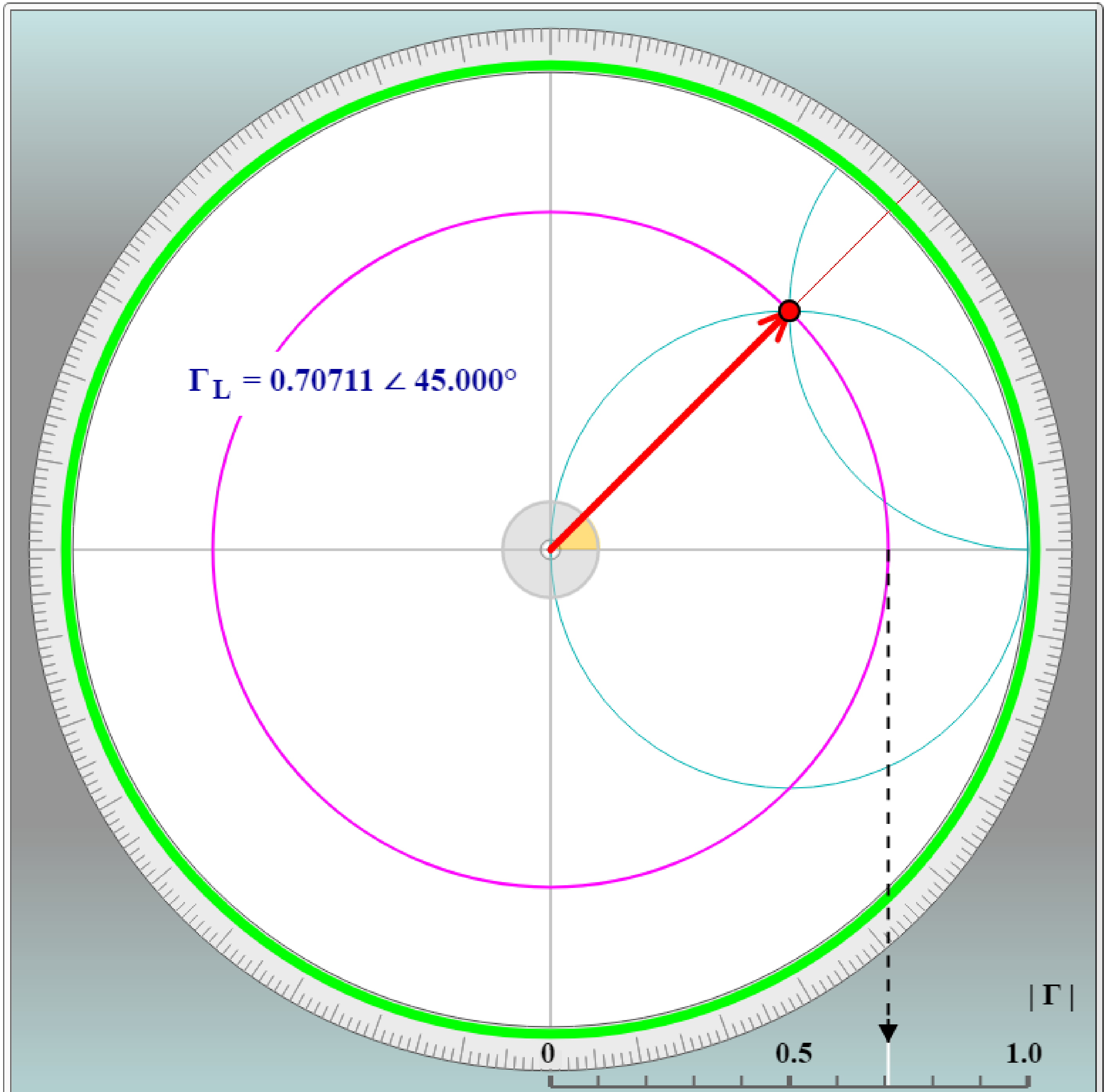
2) Given the impedance (reflection coefficient) at one point on the line, find the impedance (reflection coefficient) at any other point on the line

3) Find the locations of maximum and minimum of the Standing Wave Pattern

4) Find the Voltage Standing Wave Ratio (VSWR)

5) Given an impedance find the corresponding admittance


x = 2.00000
r = 1.00000
| Γ |
0
0.5
1.0



Γ_L = 0.70711 ∠ 45.000°
| Γ |
0
0.5
1.0

The five basic techniques of the Smith Chart

1) Given a reflection coefficient, obtain the corresponding impedance

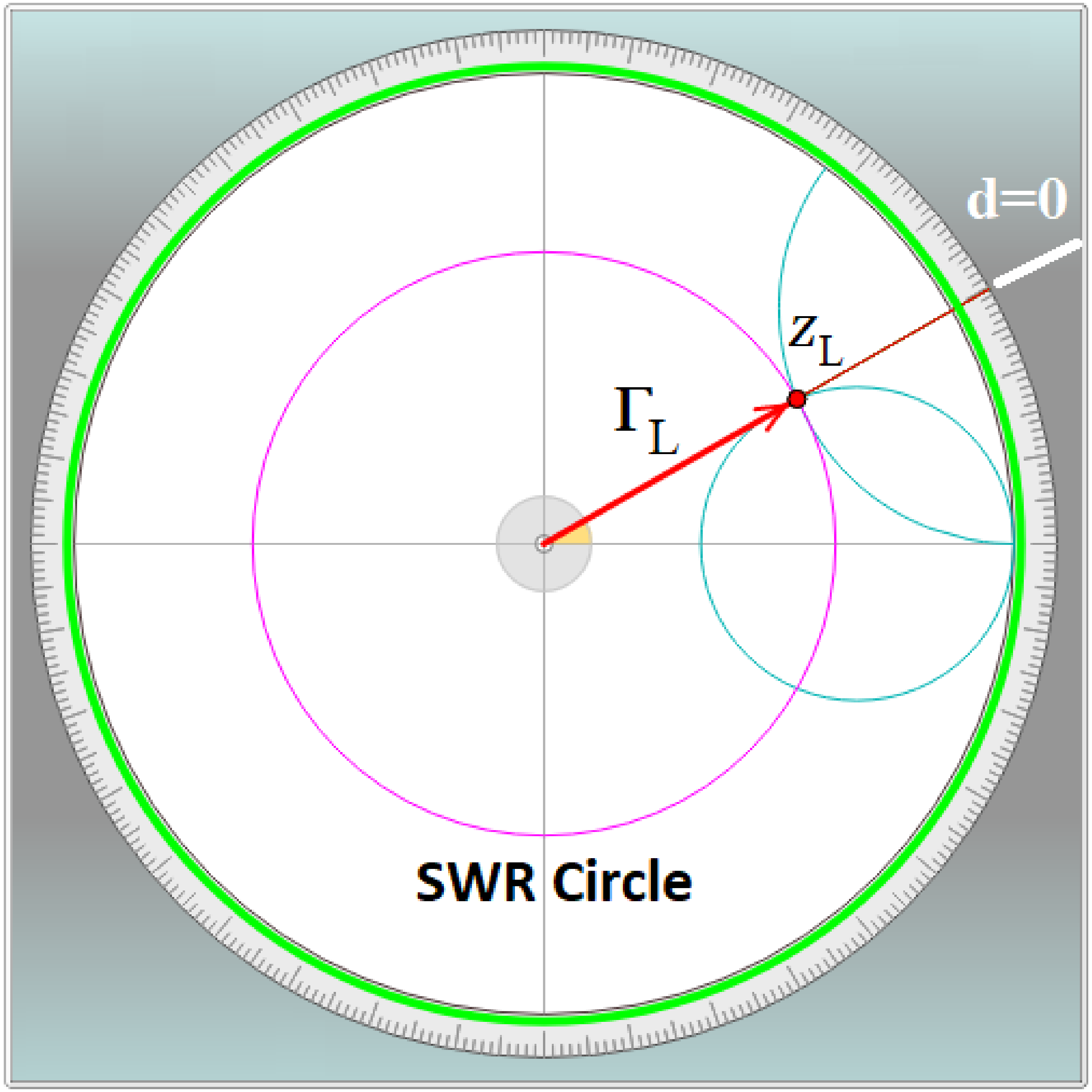
2) Given the impedance (reflection coefficient) at one point on the line, find the impedance (reflection coefficient) at any other point on the line

3) Find the locations of maximum and minimum of the Standing Wave Pattern

4) Find the Voltage Standing Wave Ratio (VSWR)

5) Given an impedance find the corresponding admittance

Z_L = 100.000 + j 100.000 Ω Z_0 = 50.00 Ω



Space reference for load location

$Z(0)$ = 100.000 + j 100.000 Ω

$z(0)$ = 2.000 + j 2.000

$\Gamma(0)$ = 0.62017 ∠ 29.745°

d = 0.10000 λ

$\Delta\theta = -2\beta d = -4\pi d / \lambda = -72.000°$

Z_L = 100.000 + j 100.000 Ω

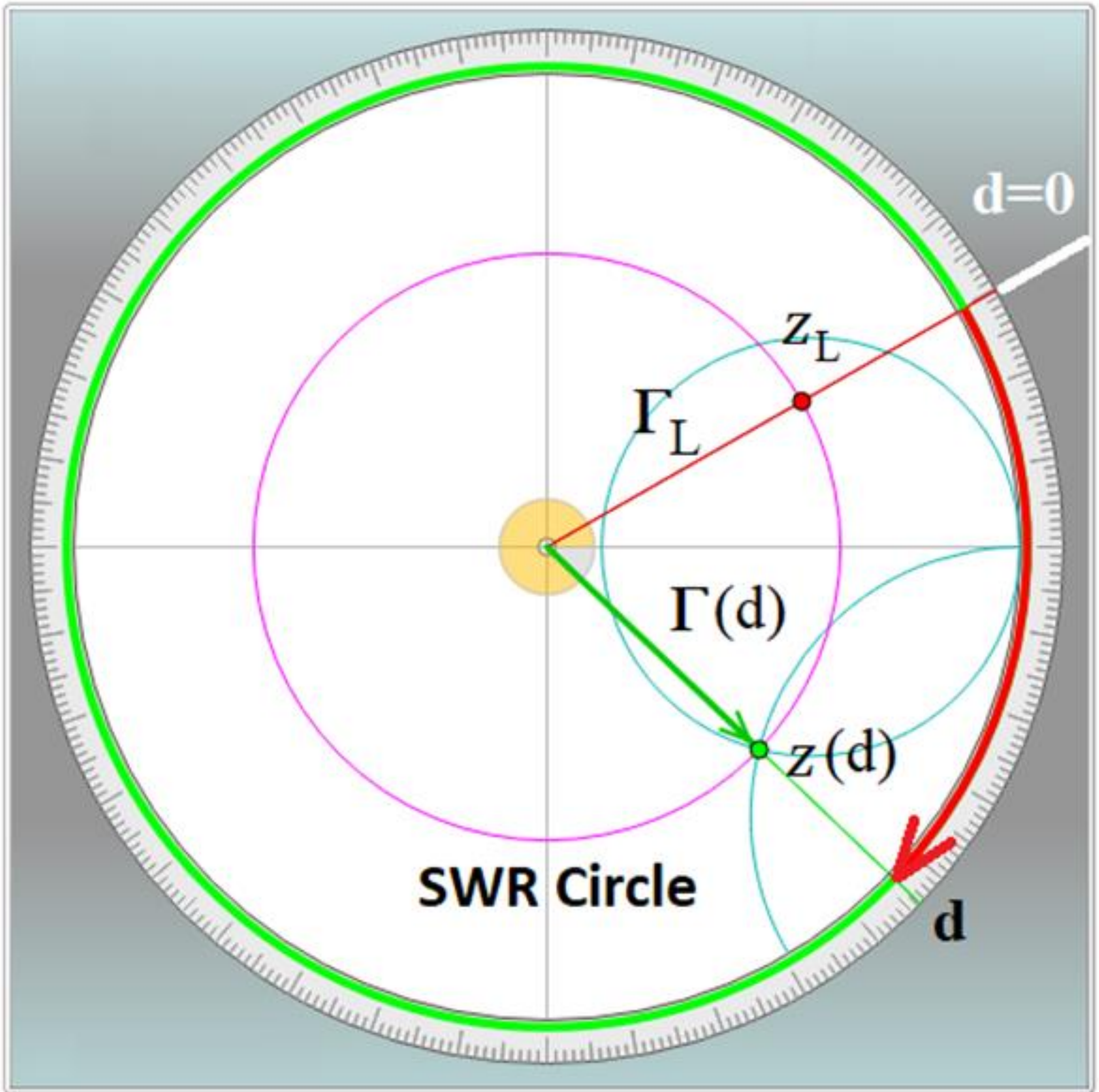
Z_0 = 50.00 Ω

Z(d) = 65.949 - j 89.383 Ω

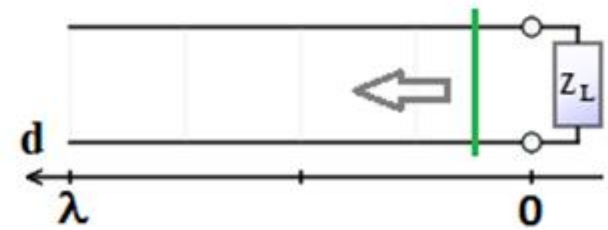
z(d) = 1.319 - j 1.788

Γ(d) = 0.62017 ∠ 317.745°

d = 0.10000 λ



when **d** *moves from load to generator*



Γ ***rotates clockwise on SWR Circle***

angle of rotation

cw

$$\Delta\theta = \mp \frac{4\pi}{\lambda} d$$

ccw

Δ θ = - 2 β d = - 4π d / λ = -72.000°

The five basic techniques of the Smith Chart

1) Given a reflection coefficient, obtain the corresponding impedance

2) Given the impedance (reflection coefficient) at one point on the line, find the impedance (reflection coefficient) at any other point on the line

3) **Find the locations of maximum and minimum of the Standing Wave Pattern**

4) Find the Voltage Standing Wave Ratio (VSWR)

5) Given an impedance find the corresponding admittance

Standing Wave Patterns

In practical applications it is very convenient to plot the magnitude of phasor voltage and phasor current along the transmission line. These are the standing wave patterns:

$$\text{Loss-less line}\begin{cases} |V(\mathrm{d})| = |V^+| \cdot |(1+\Gamma(\mathrm{d}))| \\ |I(\mathrm{d})| = \left|\dfrac{V^+}{Z_0}\right| \cdot |(1-\Gamma(\mathrm{d}))| \end{cases}$$

The voltage standing wave pattern has a maximum at locations where the generalized reflection coefficient is real and positive

$$\Gamma(\mathrm{d}) = |\Gamma_{\mathrm{L}}|$$

$$\exp(j\phi)\exp(-j2\beta\,\mathrm{d}) = 1 \qquad \Rightarrow \qquad |\phi - 2\beta\,\mathrm{d}| = 2n\pi$$

At these locations we have

$$|1 + \Gamma(\mathrm{d})| = 1 + |\Gamma_{\mathrm{L}}|$$

$$\Rightarrow V_{\max} = |V(\mathrm{d}_{\max})| = |V^{+}| \cdot (1 + |\Gamma_{\mathrm{L}}|)$$

The phase angle ϕ - 2β d changes by an amount 2π, when moving from one maximum to the next. This corresponds to a distance between successive maxima of $\lambda/2$.

The voltage standing wave pattern has a minimum at locations where the generalized reflection coefficient is real and negative

$$\Gamma(\mathrm{d}) = -\left|\Gamma_{\mathrm{L}}\right|$$

$$\exp(j\phi)\exp(-j2\beta\,\mathrm{d}) = -1 \qquad \Rightarrow \qquad \left|\phi - 2\beta\,\mathrm{d}\right| = (2n+1)\pi$$

At these locations we have

$$\left|1 + \Gamma(\mathrm{d})\right| = 1 - \left|\Gamma_{\mathrm{L}}\right|$$

$$\Rightarrow V_{\mathrm{min}} = \left|V(\mathrm{d}_{\mathrm{min}})\right| = \left|V^{+}\right| \cdot \left(1 - \left|\Gamma_{\mathrm{L}}\right|\right)$$

Also when moving from one minimum to the next, the phase angle ϕ - 2β d changes by an amount 2π. This again corresponds to a distance between successive minima of $\lambda/2$.

The voltage standing wave pattern provides immediate information on the transmission line circuit

- If the load is matched to the transmission line ($Z_L = Z_0$) the voltage standing wave pattern is flat, with value $|V^+|$.
- If the load is real and $Z_L > Z_0$, the voltage standing wave pattern starts with a maximum at the load.
- If the load is real and $Z_L < Z_0$, the voltage standing wave pattern starts with a minimum at the load.
- If the load is complex and $\mathrm{Im}(Z_L) > 0$ (inductive reactance), the voltage standing wave pattern initially increases when moving from load to generator and reaches a maximum first.
- If the load is complex and $\mathrm{Im}(Z_L) < 0$ (capacitive reactance), the voltage standing wave pattern initially decreases when moving from load to generator and reaches a minimum first.

Since in all possible cases

$$|\Gamma(d)| \leq 1$$

the voltage standing wave pattern

$$|V(d)| = |V^+| \cdot |(1 + \Gamma(d))|$$

cannot exceed the value $2\,|\,V+\,|$ in a loss-less transmission line. If the load is a short circuit, an open circuit, or a pure reactance, there is total reflection with

$$|\Gamma(d)| = 1$$

since the load cannot consume any power. The voltage standing wave pattern in these cases is characterized by

$V_{max} = 2|V^+|$ and $V_{min} = 0$.

The voltage standing wave ratio (VSWR) is an indicator of load matching which is widely used in engineering applications

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

When the load is perfectly matched to the transmission line

$$\Gamma_L = 0 \qquad \Rightarrow \qquad VSWR = 1$$

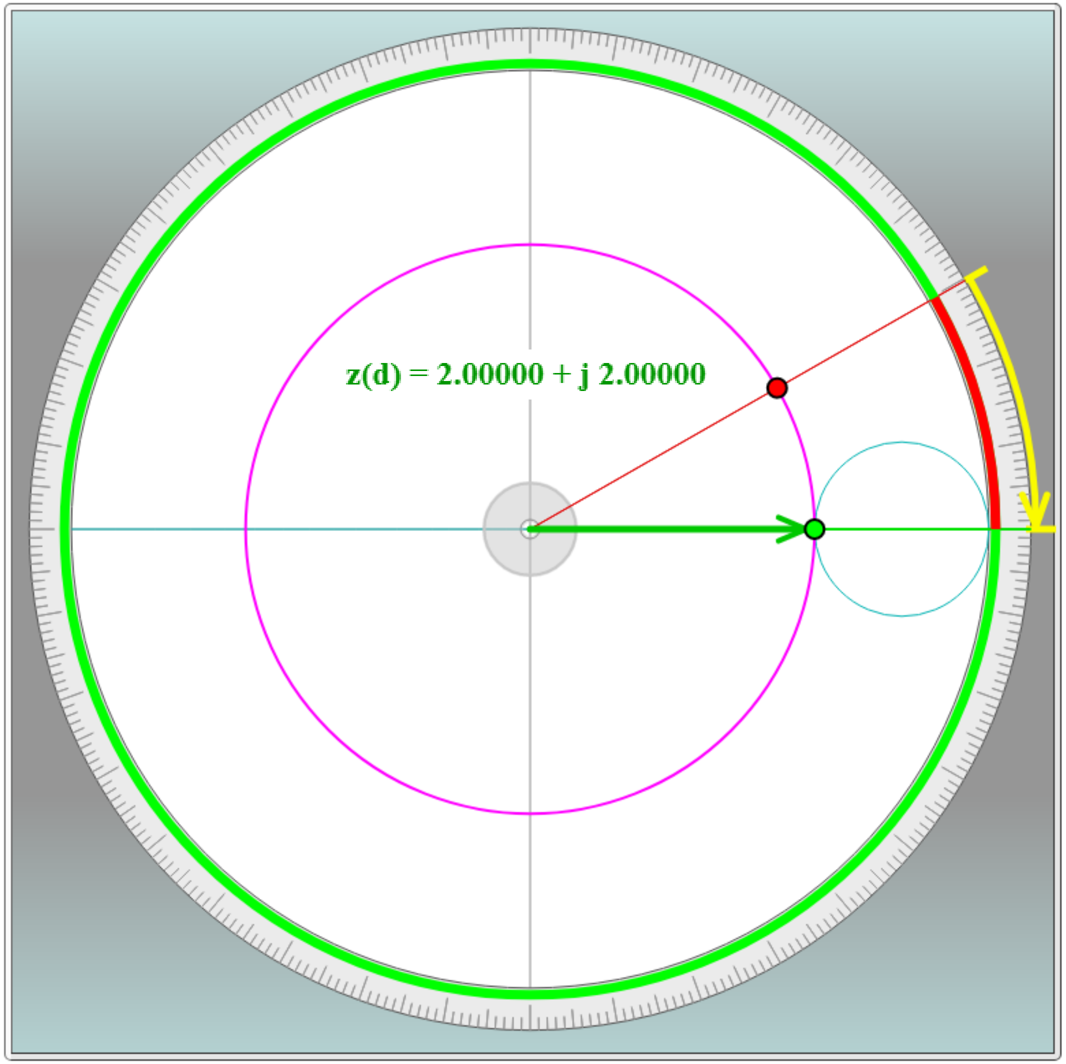
When the load is a short circuit, an open circuit or a pure reactance

$$|\Gamma_L| = 1 \qquad \Rightarrow \qquad VSWR \to \infty$$

We have the following useful relation

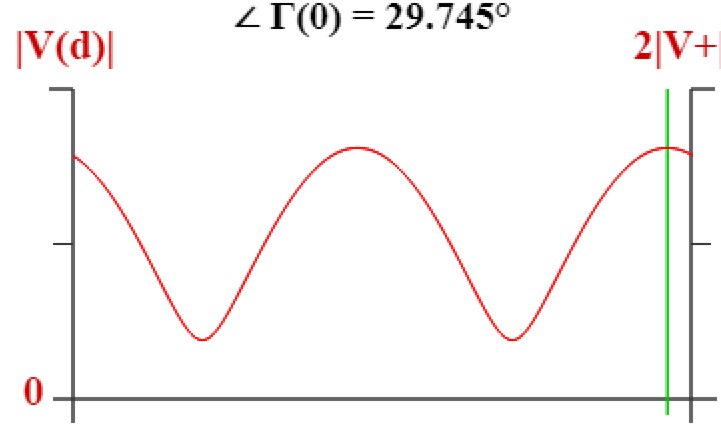
$$|\Gamma_L| = \frac{VSWR-1}{VSWR+1}$$

Z_L = 100.000 + j 100.000 Ω $\qquad$ Z_0 = 50.00 Ω



When the load reactance is positive (inductive) a maximum of voltage standing wave pattern is closest to the load.

$\angle\,\Gamma(0) = 29.745°$

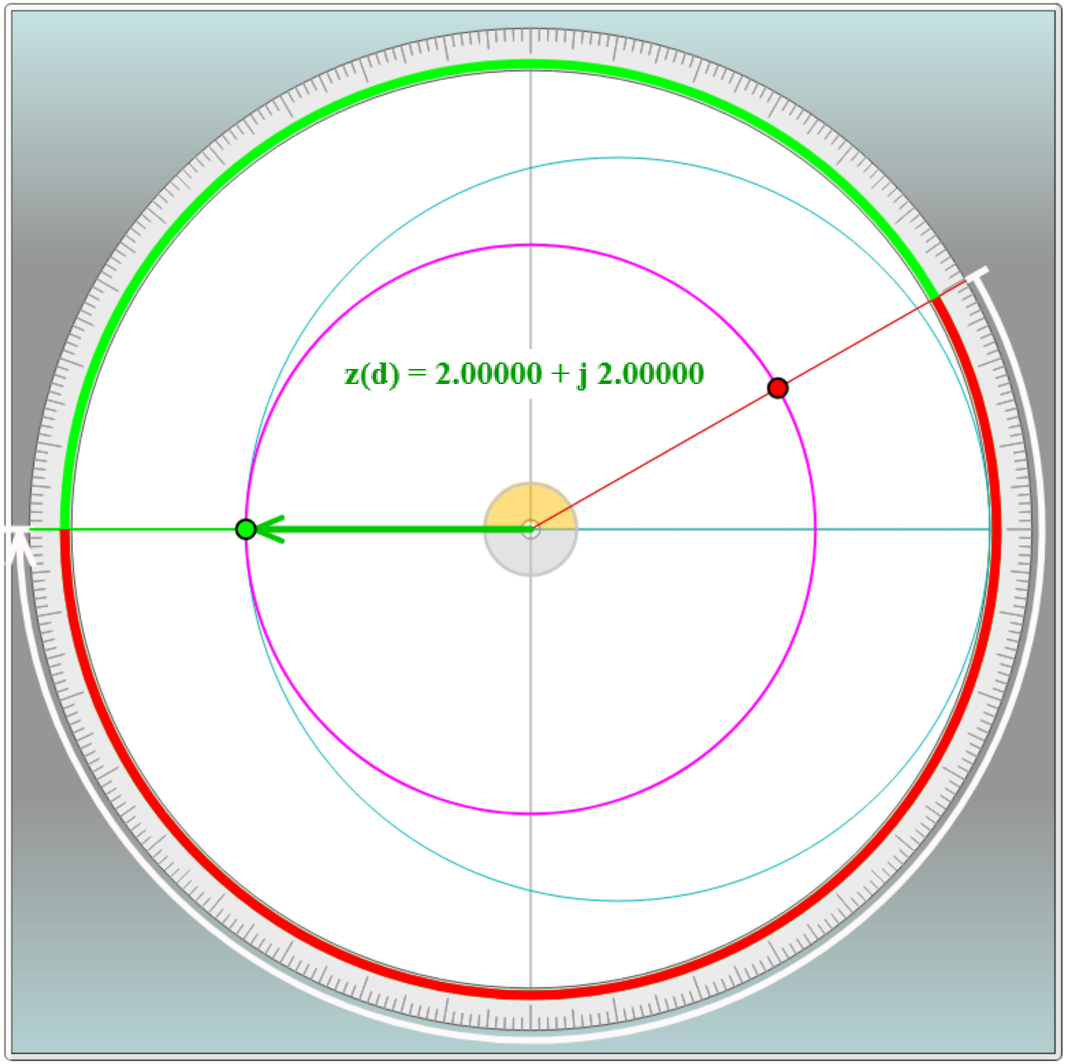


Γ [d(max)] is real and positive.

Location of the first maximum is

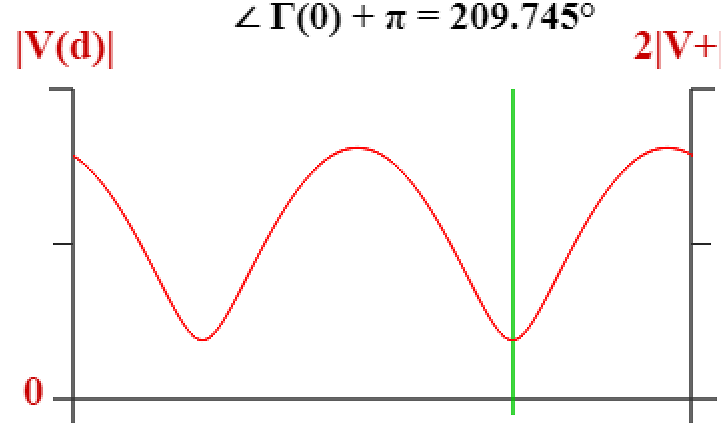
$d(max) = \angle\,\Gamma(0) \cdot \lambda/4\pi = 0.04131\ \lambda$

$Z_L = 100.000 + j\,100.000\ \Omega$ $Z_0 = 50.00\ \Omega$



When the load reactance is positive (inductive) a maximum of voltage standing wave pattern is closest to the load.

$\angle\Gamma(0) + \pi = 209.745°$

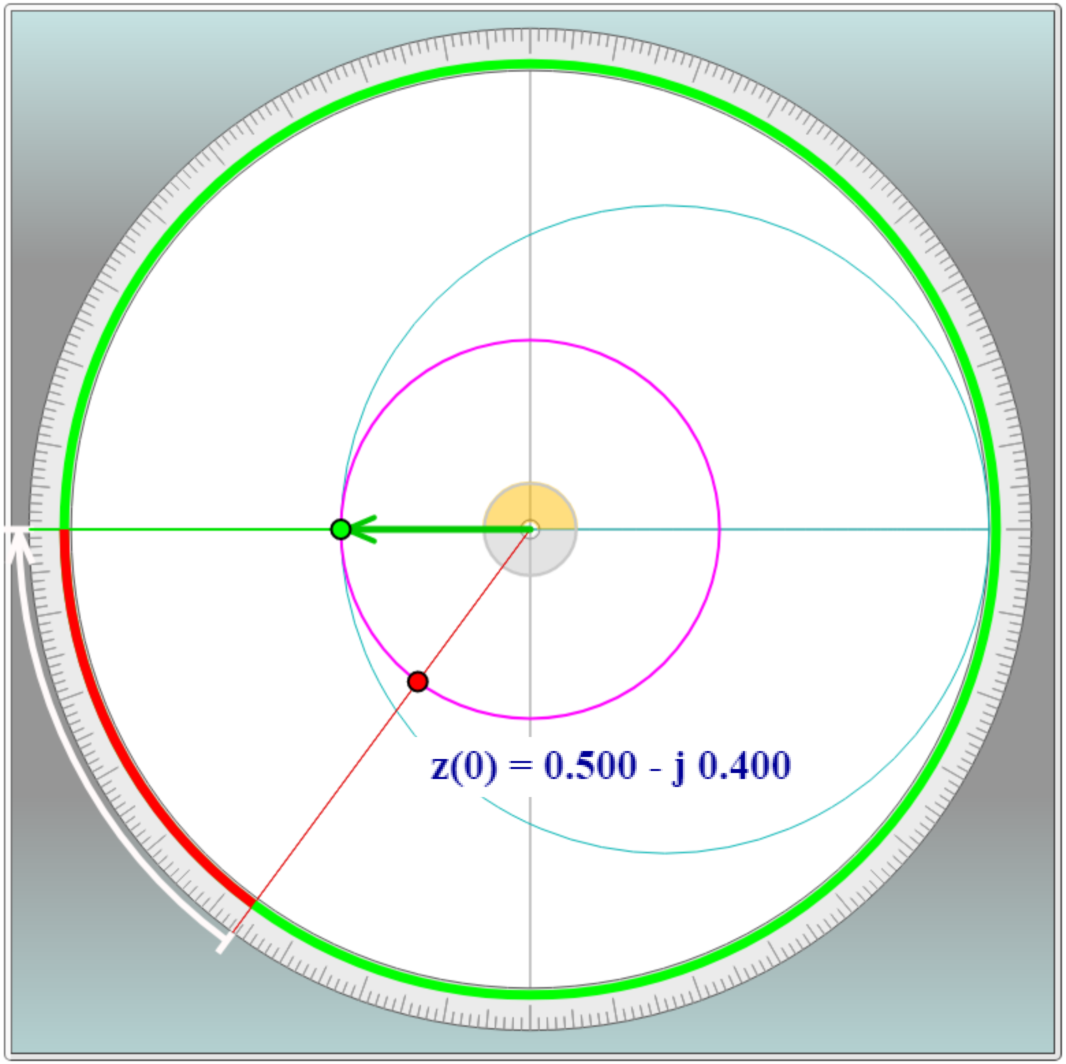


Γ [d(min)] is real and negative.

Location of the first minimum is

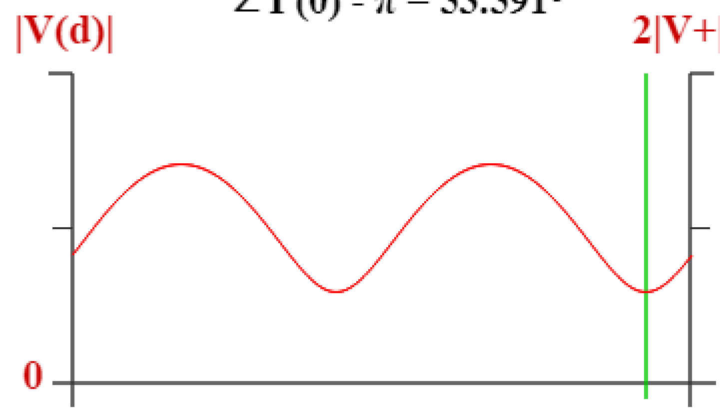
$d(min) = (\angle\Gamma(0) + \pi) \cdot \lambda/4\pi = 0.29131\ \lambda$

$Z_L = 25.000 - j\,20.000\ \Omega$ $Z_0 = 50.00\ \Omega$



When the load reactance is negative (capacitive) a minimum of voltage standing wave pattern is closest to the load.

$\angle\Gamma(0) - \pi = 53.591°$

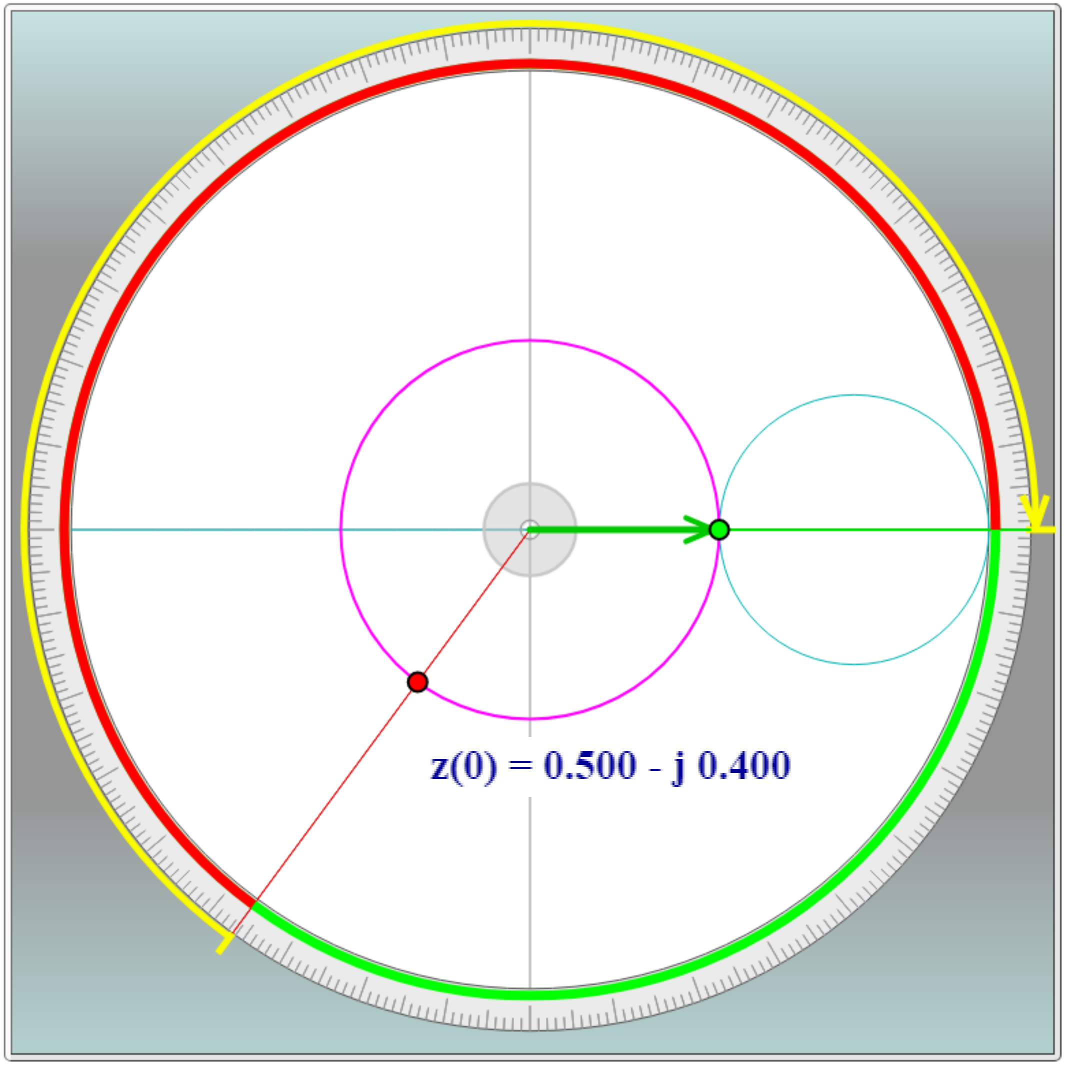


Γ [d(min)] is real and negative.

The location of the first minimum is

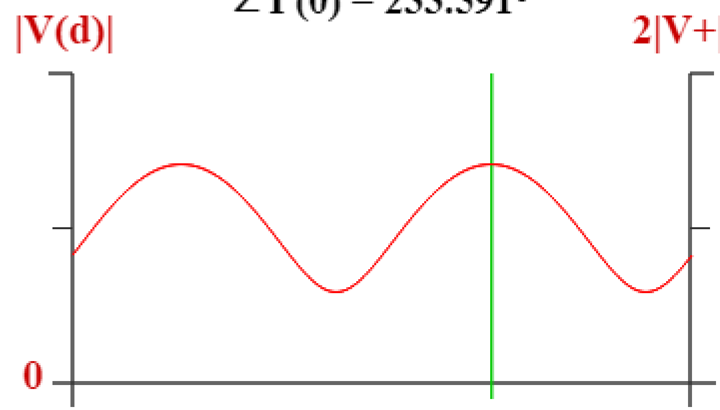
$d(min) = (\angle\Gamma(0) - \pi) \cdot \lambda/4\pi = 0.07443\ \lambda$

$Z_L = 25.000 - j\,20.000\ \Omega$ $Z_0 = 50.00\ \Omega$



When the load reactance is negative (capacitive) a minimum of voltage standing wave pattern is closest to the load.

$\angle \Gamma(0) = 233.591°$



Γ [d(max)] is real and positive.

The location of the first maximum is $d(max) = \angle \Gamma(0) \cdot \lambda/4\pi = 0.32443\ \lambda$

The five basic techniques of the Smith Chart

1) Given a reflection coefficient, obtain the corresponding impedance

2) Given the impedance (reflection coefficient) at one point on the line, find the impedance (reflection coefficient) at any other point on the line

3) Find the locations of maximum and minimum of the Standing Wave Pattern

4) **Find the Voltage Standing Wave Ration (VSWR)**

5) Given an impedance find the corresponding admittance

The reflection coefficient is real and positive at a maximum of the voltage standing wave pattern.

The normalized line impedance is also real and can be written as

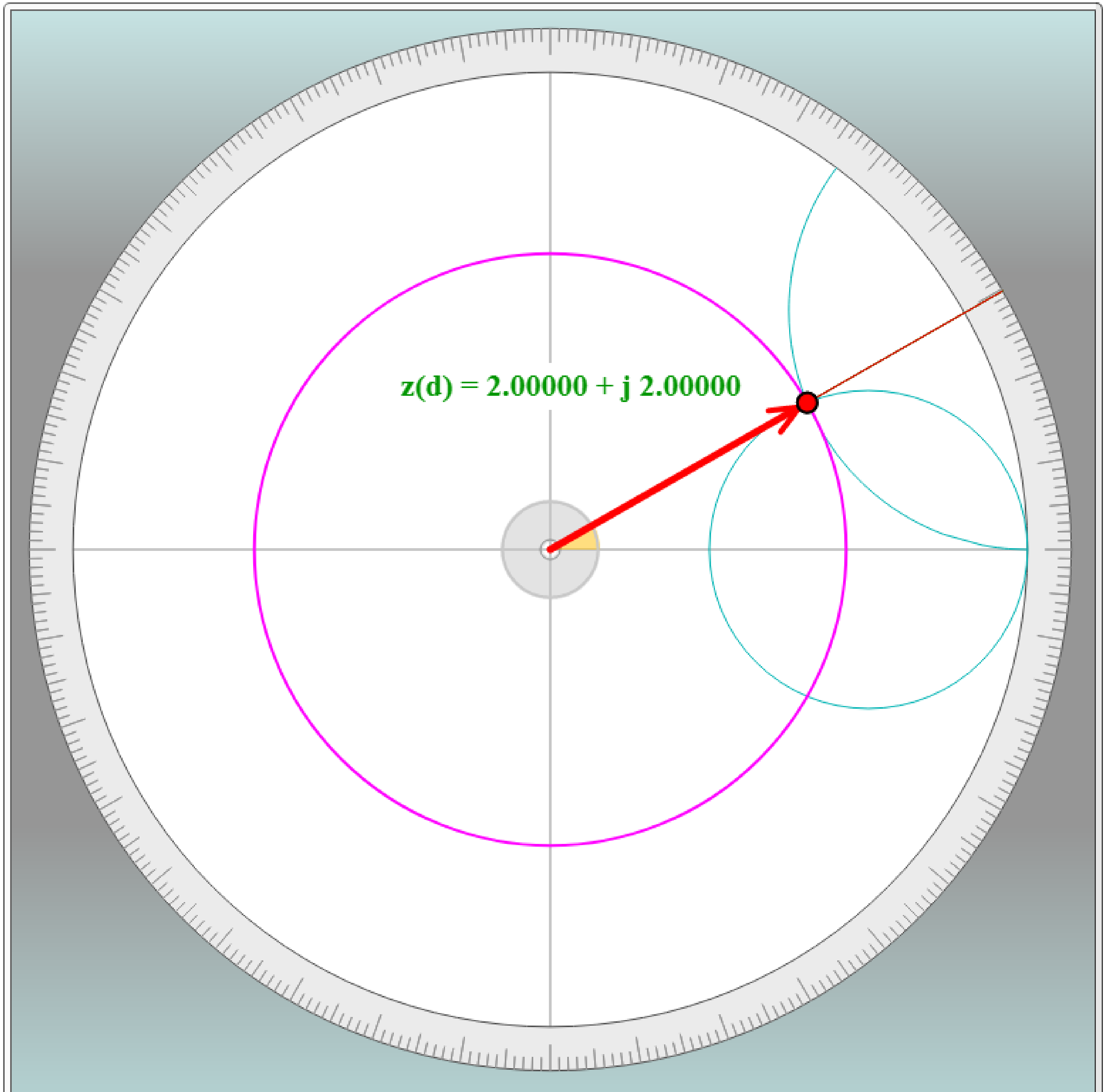
$$z[\ d(max)\ ] = (1 + |\Gamma|) / (1 - |\Gamma|)$$

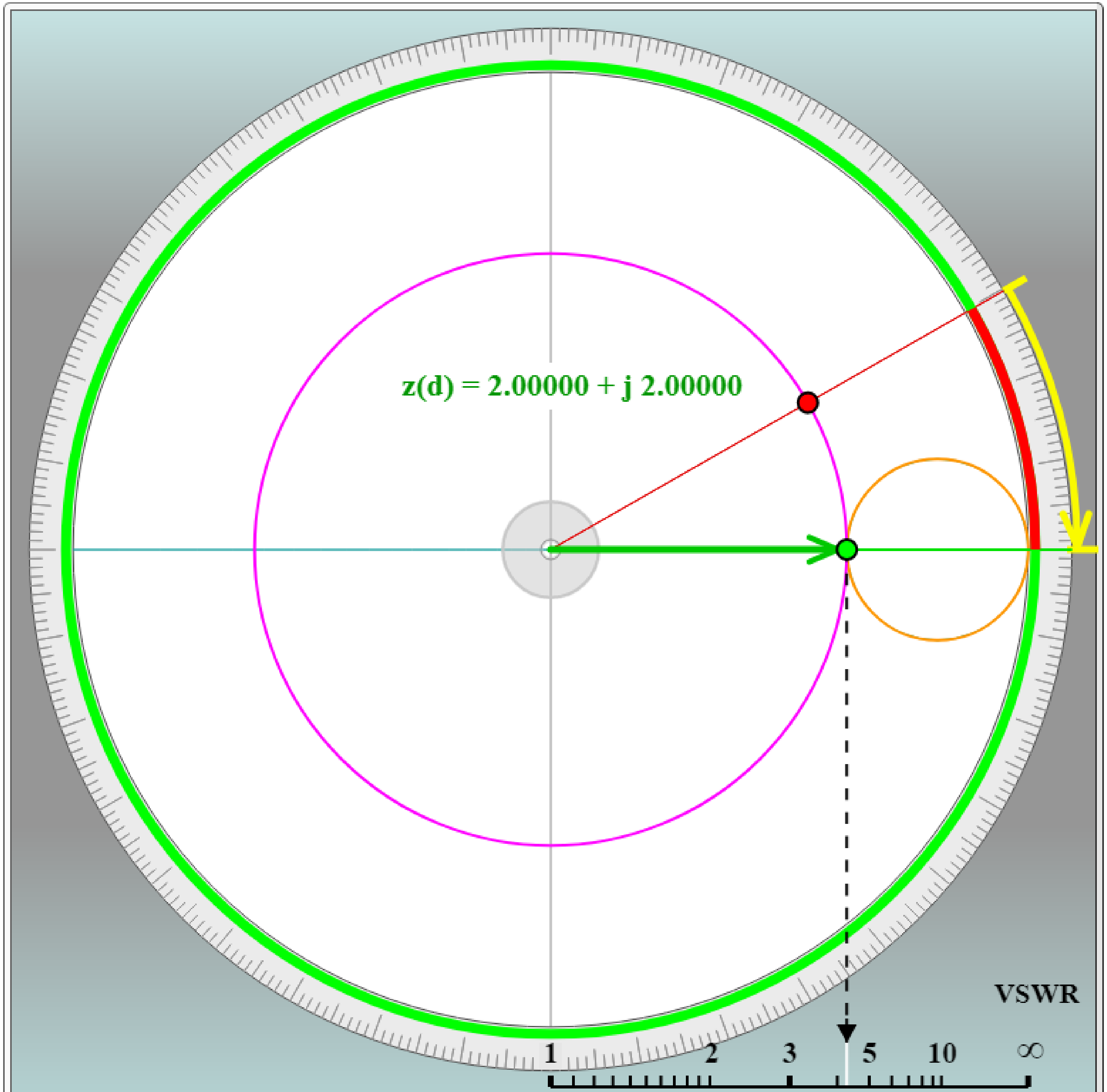
which is the same as the VSWR

$$VSWR = (1 + |\Gamma|) / (1 - |\Gamma|)$$

The circle of normalized resistance tangent to the SWR circle, therefore, identifies also the VSWR:

$$r = z[\ d(max)\ ] = VSWR = 4.266$$


z(d) = 2.00000 + j 2.00000


z(d) = 2.00000 + j 2.00000
VSWR
1
2
3
5
10
∞

The five basic techniques of the Smith Chart

1) Given a reflection coefficient, obtain the corresponding impedance

2) Given the impedance (reflection coefficient) at one point on the line, find the impedance (reflection coefficient) at any other point on the line

3) Find the locations of maximum and minimum of the Standing Wave Pattern

4) Find the Voltage Standing Wave Ration (VSWR)

5) Given an impedance find the corresponding admittance

***Note**: The normalized impedance and admittance on a lossless transmission line are defined as*

$$z_n(d) = \frac{1+\Gamma(d)}{1-\Gamma(d)} \qquad y_n(d) = \frac{1-\Gamma(d)}{1+\Gamma(d)}$$

Since $$\Gamma\left(d+\frac{\lambda}{4}\right) = -\Gamma(d)$$

we have

$$\Rightarrow \quad z_n\left(d+\frac{\lambda}{4}\right) = \frac{1+\Gamma\left(d+\frac{\lambda}{4}\right)}{1-\Gamma\left(d+\frac{\lambda}{4}\right)} = \frac{1-\Gamma(d)}{1+\Gamma(d)} = y_n(d)$$

Keep in mind that the equality $z_n\left(d+\frac{\lambda}{4}\right)=y_n(d)$

is only valid for ***normalized*** *impedance and admittance. The* ***actual*** *values are given by*

$$Z\left(d+\frac{\lambda}{4}\right)=Z_0\cdot z_n\left(d+\frac{\lambda}{4}\right)$$

$$Y(d)=Y_0\cdot y_n(d)=\frac{y_n(d)}{Z_0}$$

$$\Rightarrow \quad Z\left(d+\frac{\lambda}{4}\right)=Z_0^2\,Y(d)$$

where $Y_0=1/Z_0$ *is the* ***characteristic admittance*** *of the line.*

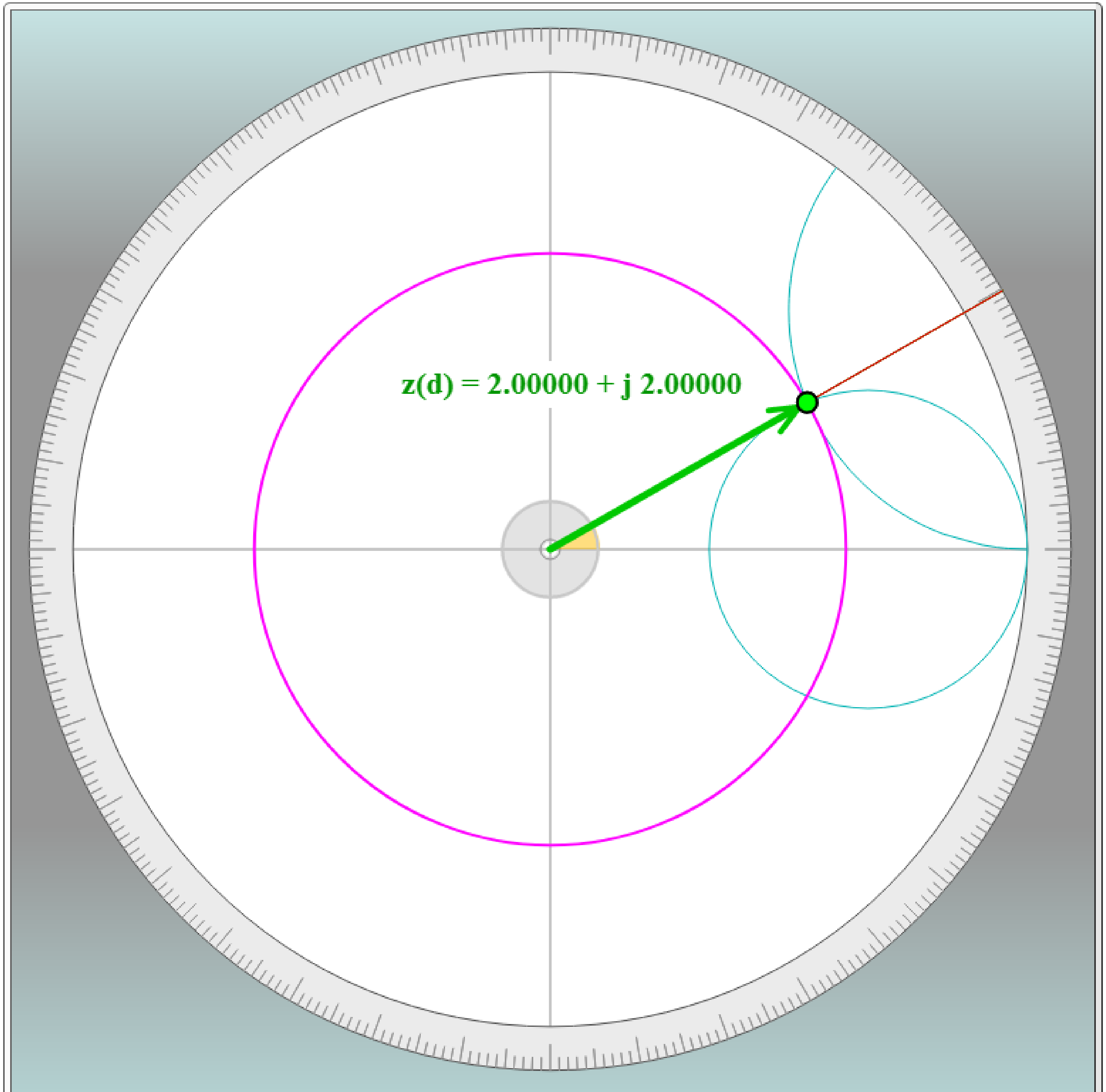
Let's review the *impedance*-*admittance* terminology:

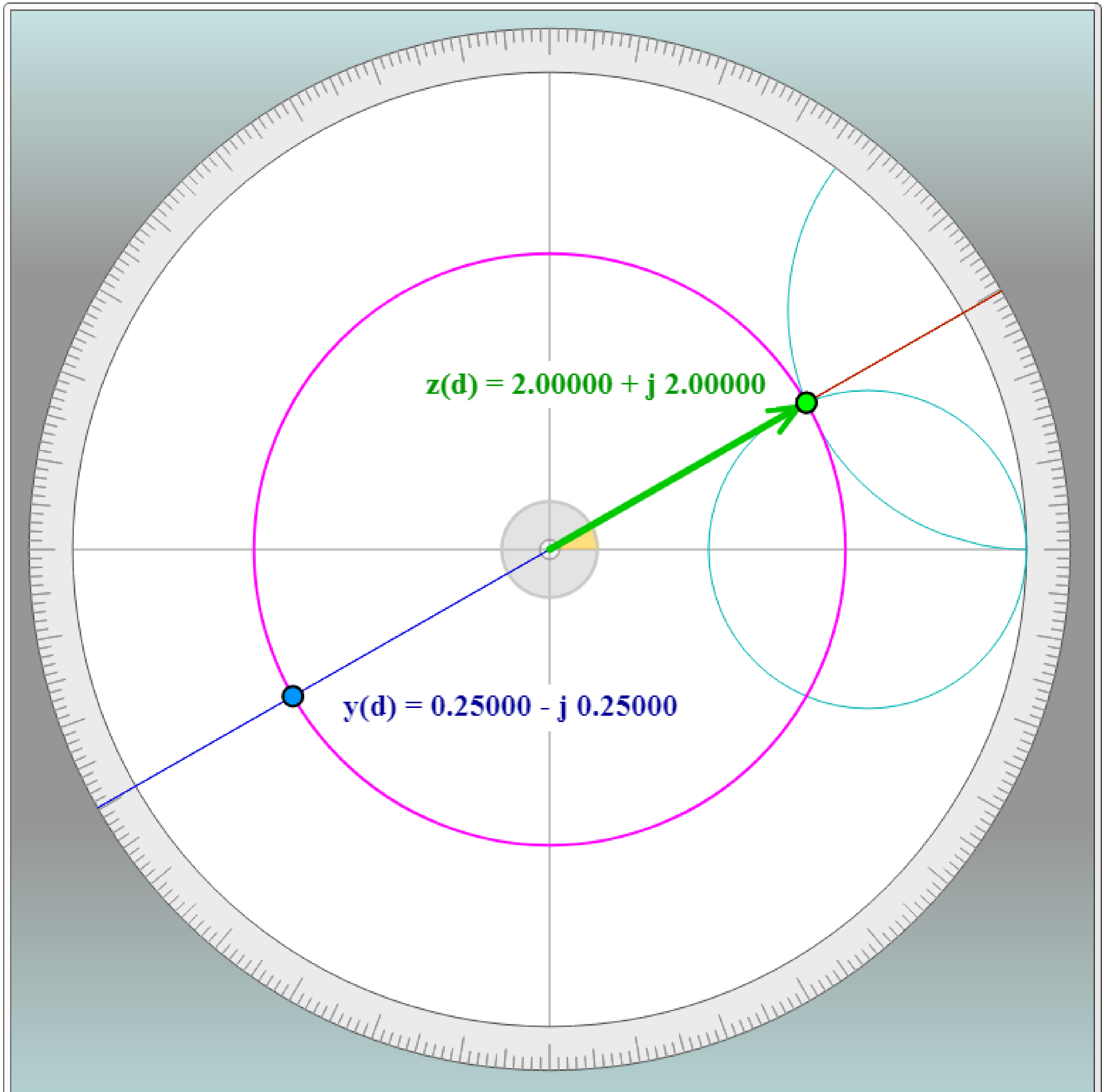
$$\text{Impedance} = \text{Resistance} + j\,\text{Reactance}$$

$$Z = R + jX$$

$$\text{Admittance} = \text{Conductance} + j\,\text{Susceptance}$$

$$Y = G + jB$$


z(d) = 2.00000 + j 2.00000


z(d) = 2.00000 + j 2.00000
y(d) = 0.25000 - j 0.25000

Normalized values

$$z(d) = (1 + \Gamma(d)) / (1 - \Gamma(d))$$

$$y(d) = (1 - \Gamma(d)) / (1 + \Gamma(d))$$

Since $\Gamma(d + \lambda/4) = -\Gamma(d)$, we have

$$z(d + \lambda/4) = y(d)$$

$\lambda/4$ corresponds to 180° rotation.

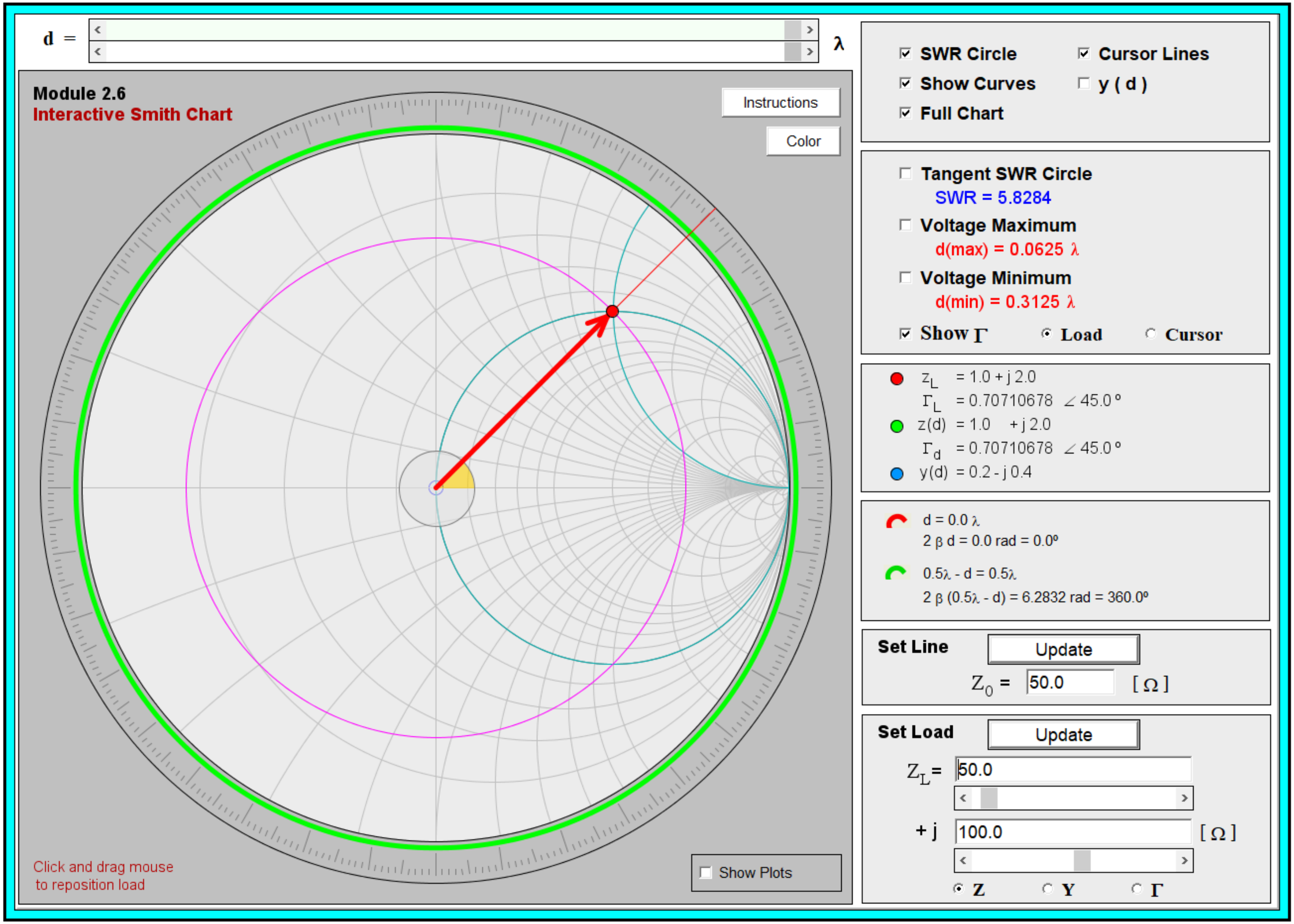
$$z(d) = 2.00000 + j\,2.00000$$

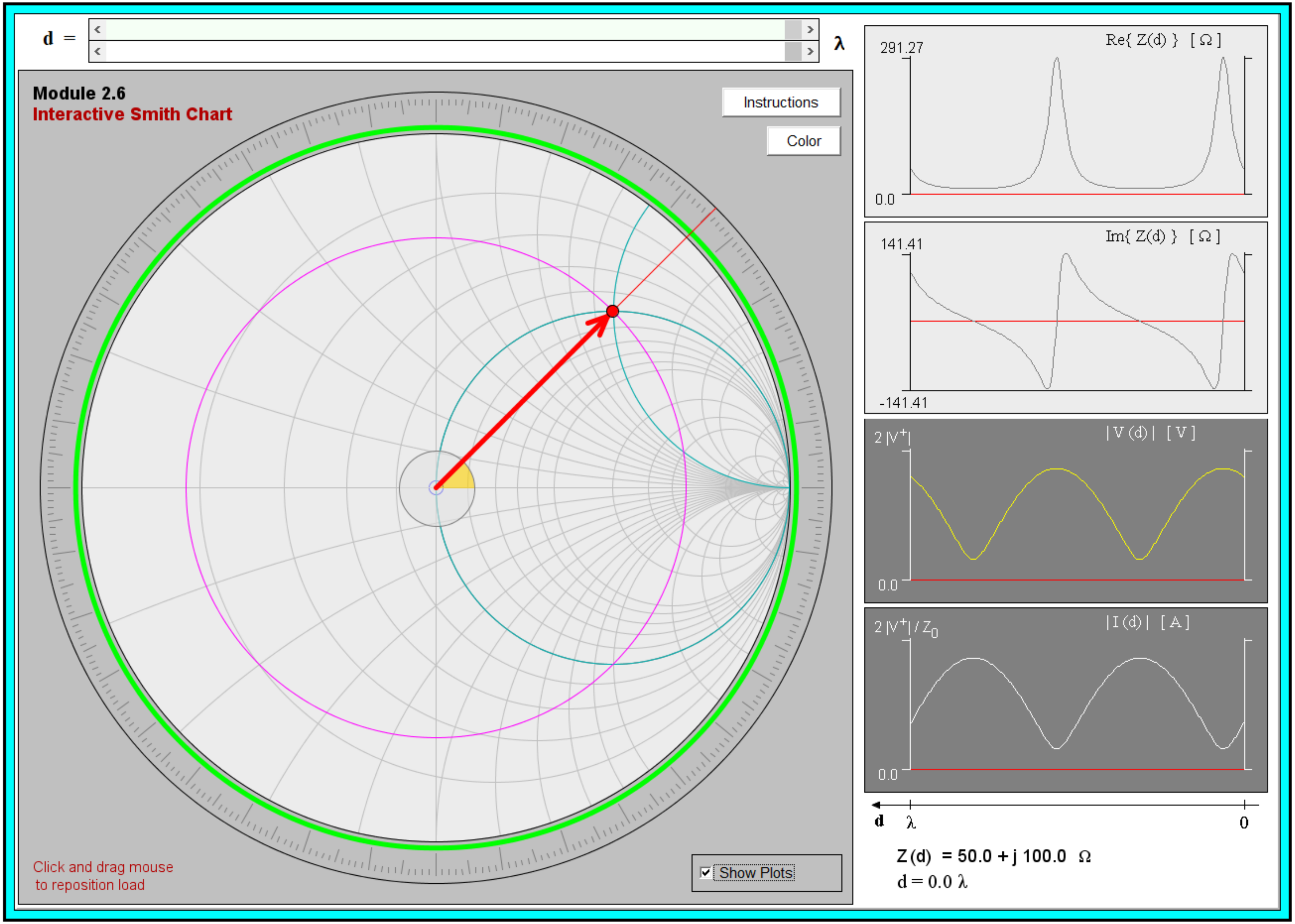
$$y(d) = 0.25000 - j\,0.25000$$

Equality $z(d + \lambda/4) = y(d)$ holds for normalized quantities, but

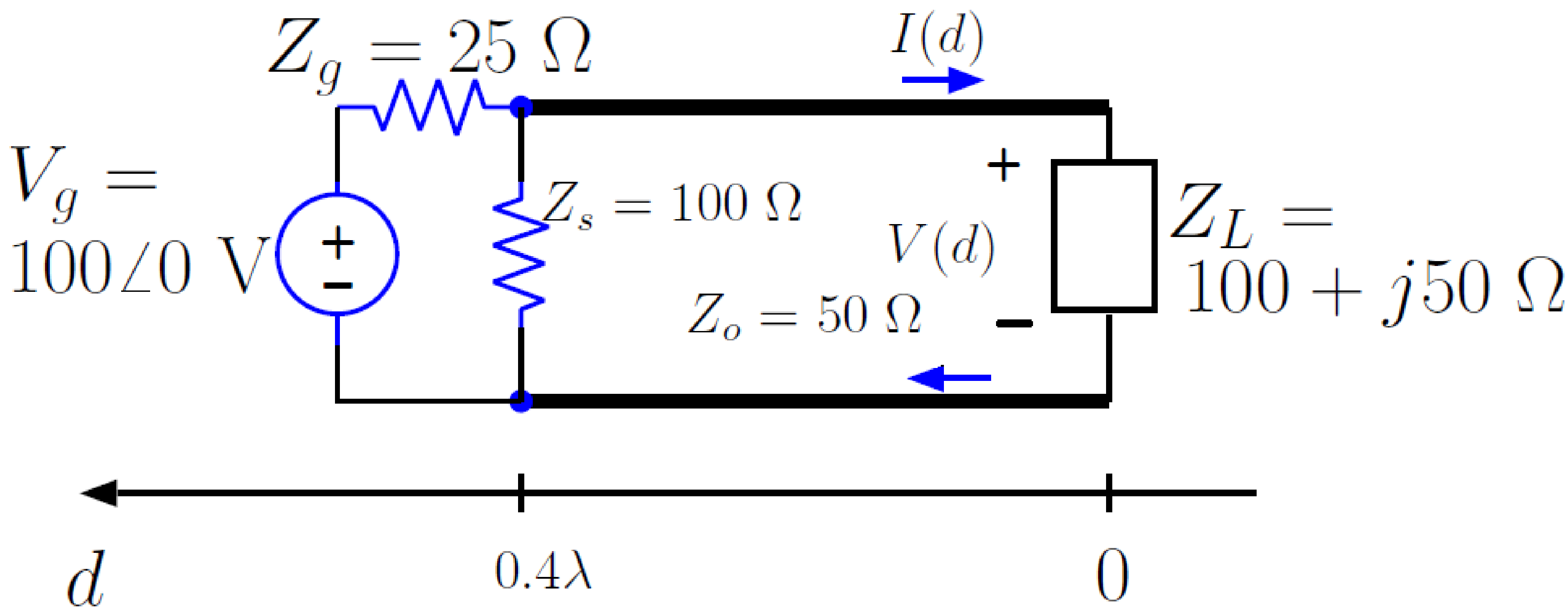
$$Z(d + \lambda/4) = z(d + \lambda/4) \cdot Z_0$$

$$Y(d) = y(d) / Z_0$$


d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse
to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 5.8284
Voltage Maximum
d(max) = 0.0625 λ
Voltage Minimum
d(min) = 0.3125 λ
Show Γ
Load
Cursor
z_L = 1.0 + j 2.0
Γ_L = 0.70710678 ∠ 45.0 °
z(d) = 1.0 + j 2.0
Γ_d = 0.70710678 ∠ 45.0 °
y(d) = 0.2 - j 0.4
d = 0.0 λ
2 β d = 0.0 rad = 0.0º
0.5λ - d = 0.5λ
2 β (0.5λ - d) = 6.2832 rad = 360.0º
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Z_L = 50.0
+ j 100.0 [Ω]
Z
Y
Γ

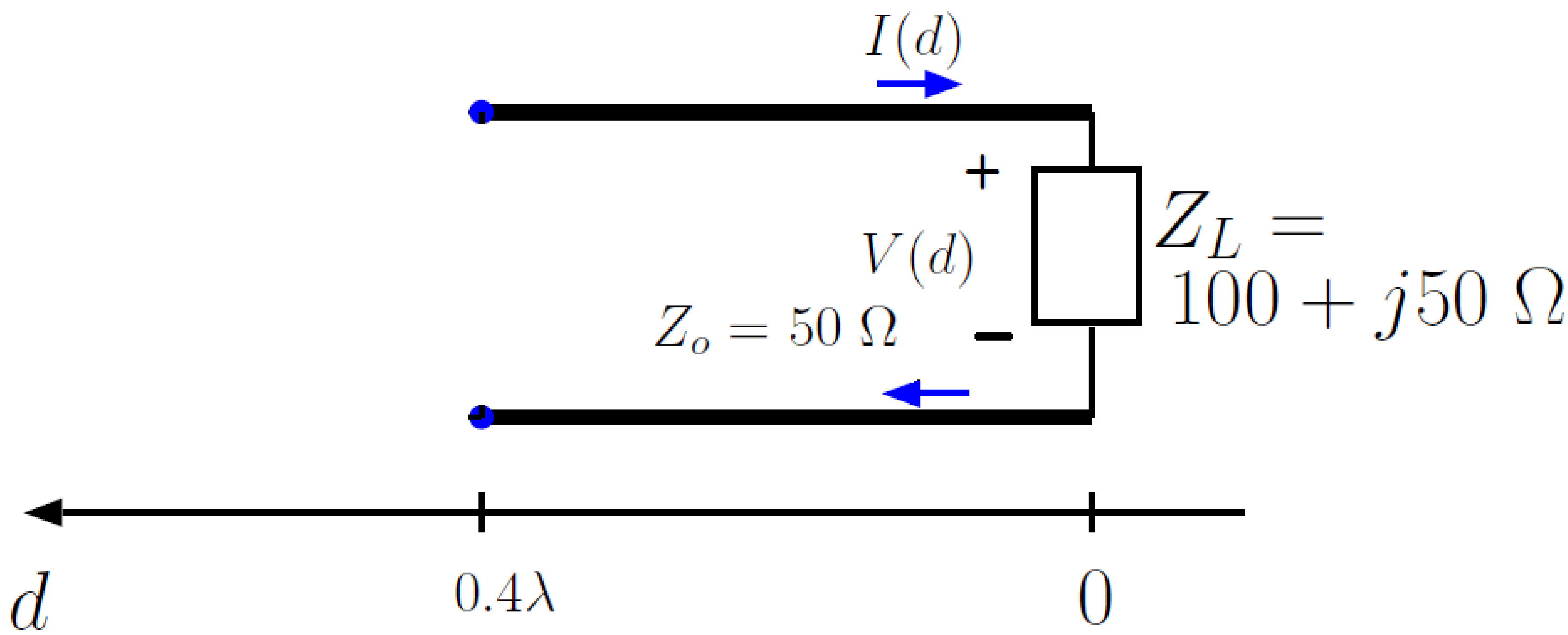

d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse
to reposition load
Show Plots
291.27
Re{ Z(d) } [Ω]
0.0
141.41
Im{ Z(d) } [Ω]
-141.41
2 |V+|
|V(d)| [V]
0.0
2 |V+| / Z0
|I(d)| [A]
0.0
d
λ
0
Z(d) = 50.0 + j 100.0 Ω
d = 0.0 λ

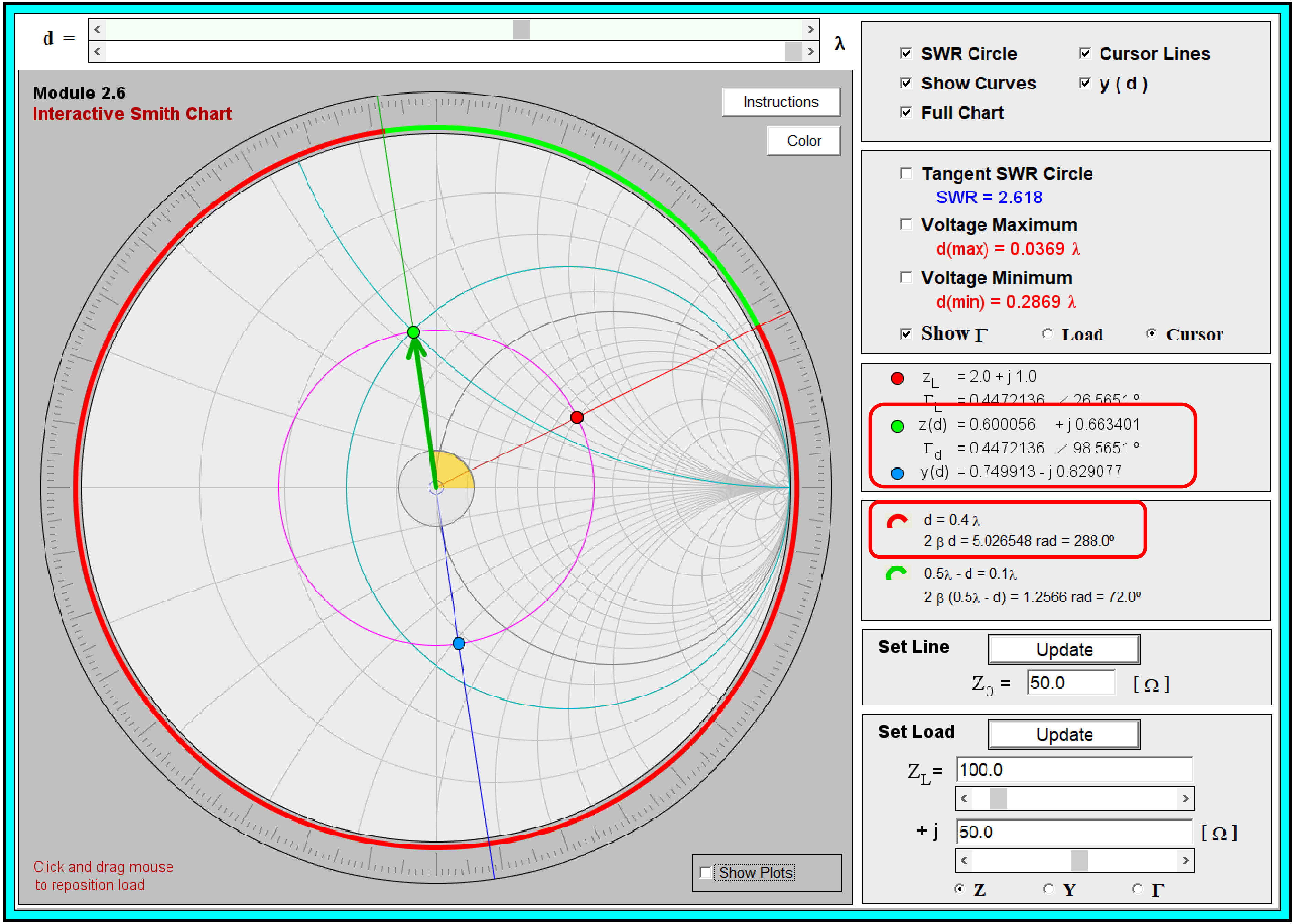
Example 1: A load $Z_L = 100 + j50\,\Omega$ is connected across a TL with $Z_o = 50\,\Omega$ and $l = 0.4\lambda$. At the generator end, $d = l$, the line is shunted by an impedance $Z_s = 100\,\Omega$. What are the input impedance Z_{in} and admittance Y_{in} of the line, including the shunt connected element.



Example 1: A load $Z_L = 100 + j50\,\Omega$ is connected across a TL with $Z_o = 50\,\Omega$ and $l = 0.4\lambda$. At the generator end, $d = l$, the line is shunted by an impedance $Z_s = 100\,\Omega$. What are the input impedance Z_{in} and admittance Y_{in} of the line, including the shunt connected element.

First find the input impedance/admittance of the line



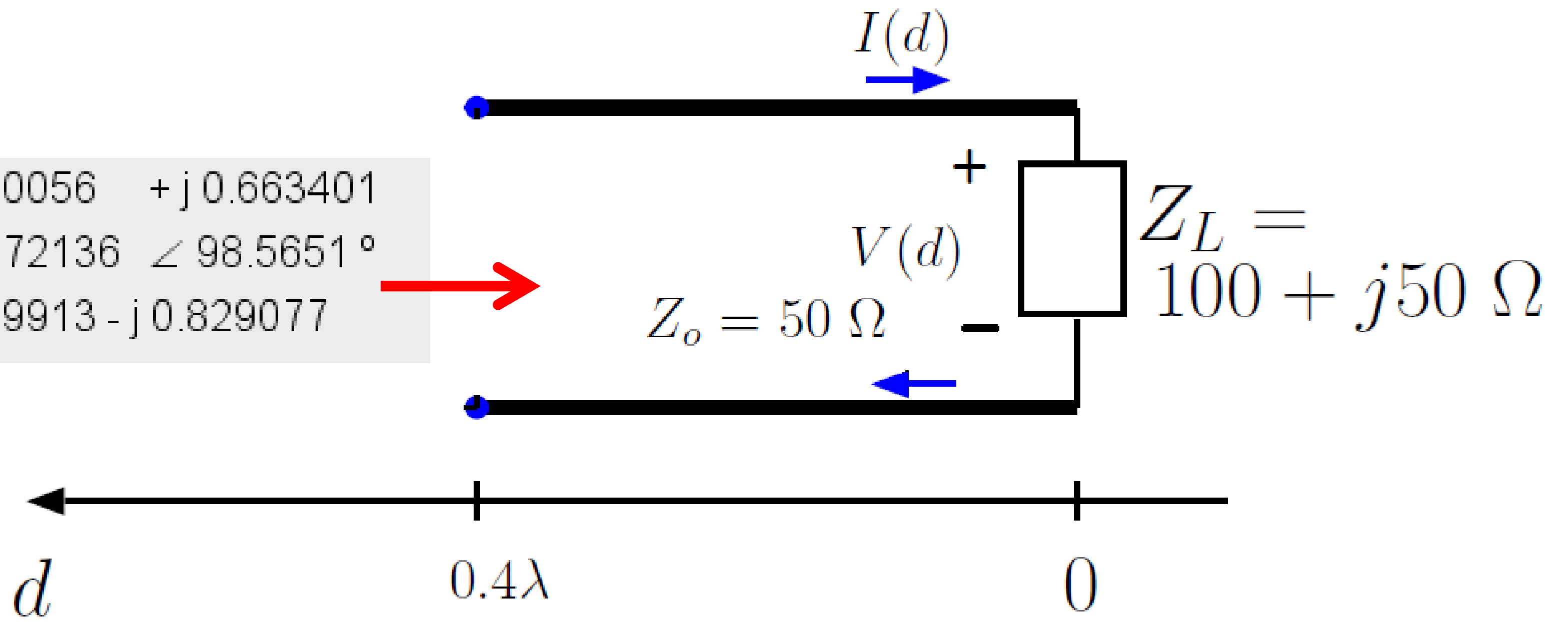


d = λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse
to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 2.618
Voltage Maximum
d(max) = 0.0369 λ
Voltage Minimum
d(min) = 0.2869 λ
Show Γ
Load
Cursor
z_L = 2.0 + j 1.0
Γ_L = 0.4472136 ∠ 26.5651 °
z(d) = 0.600056 + j 0.663401
Γ_d = 0.4472136 ∠ 98.5651 °
y(d) = 0.749913 - j 0.829077
d = 0.4 λ
2 β d = 5.026548 rad = 288.0°
0.5λ - d = 0.1λ
2 β (0.5λ - d) = 1.2566 rad = 72.0°
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Z_L = 100.0
+ j 50.0 [Ω]
Z
Y
Γ

● $z(d) = 0.600056 + j\,0.663401$

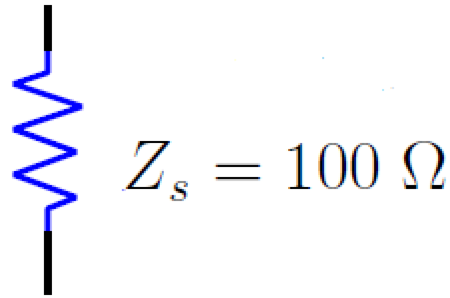
$\Gamma_d = 0.4472136 \angle 98.5651°$

● $y(d) = 0.749913 - j\,0.829077$



$$Y_s = 0.01\ \text{S}$$

Shunt resistance at input terminals



normalized admittance

$$y_s = 0.5$$

$$y_{in} = y(l) + y_{si} \approx 1.25 - j0.83$$

$$\boxed{Y_{in} = Y_o y_{in} \approx 0.025 - j0.017\,\text{S}}$$

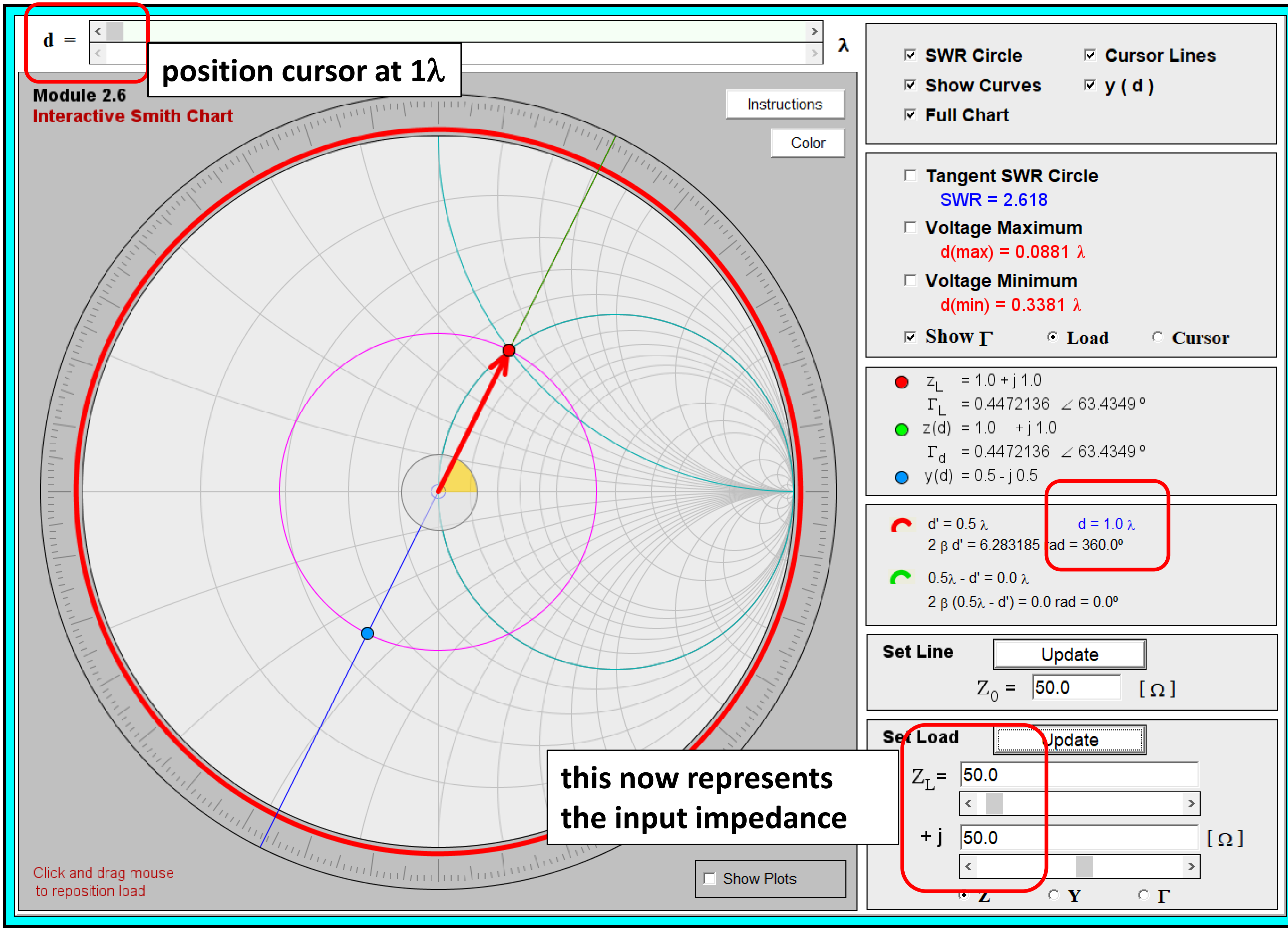
$$z_{in} = \frac{1}{y_i} \approx 0.56 + j0.37$$

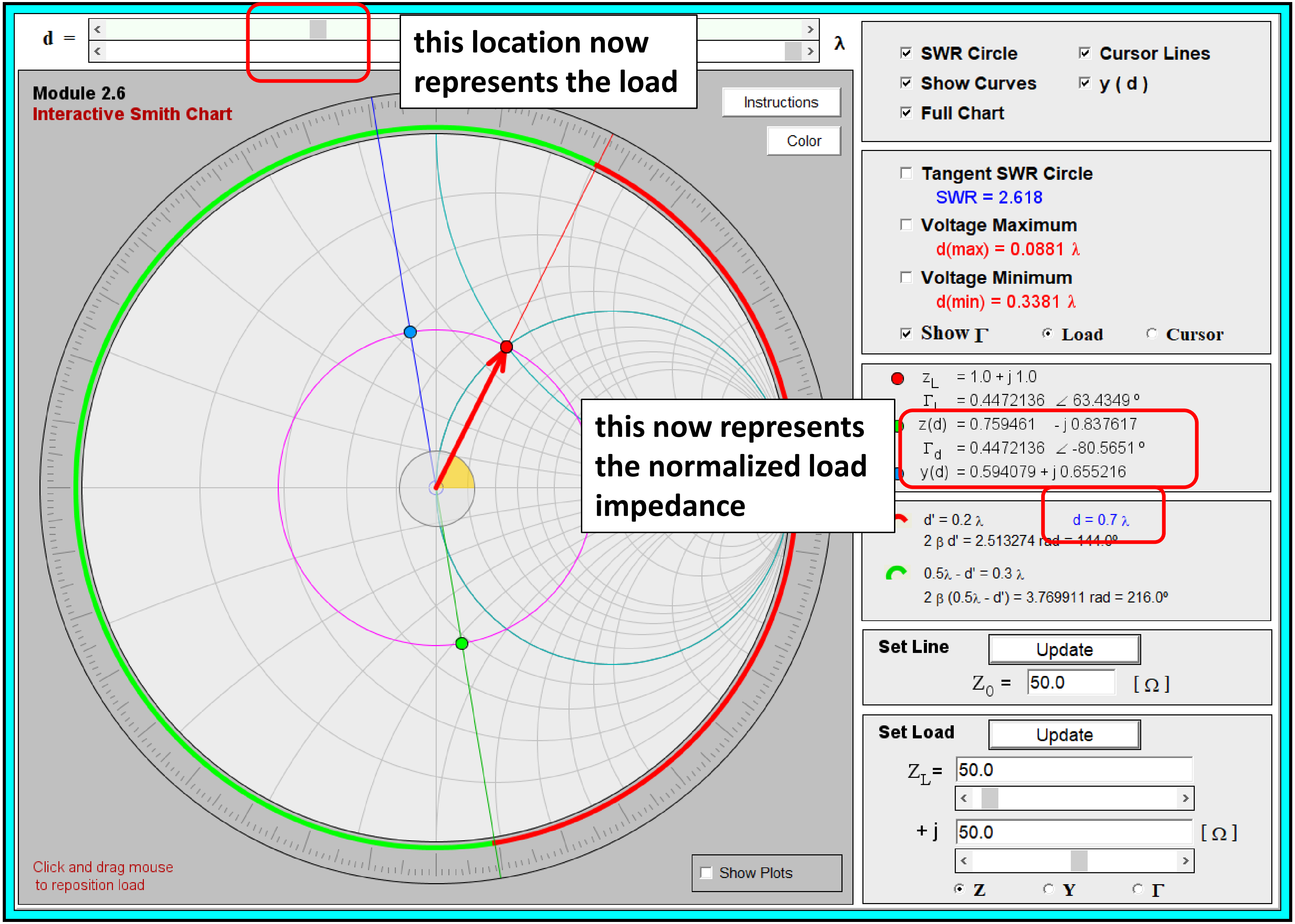
$$\boxed{Z_{in} = Z_o z_{in} \approx 27.8 + j18.4\,\Omega}$$

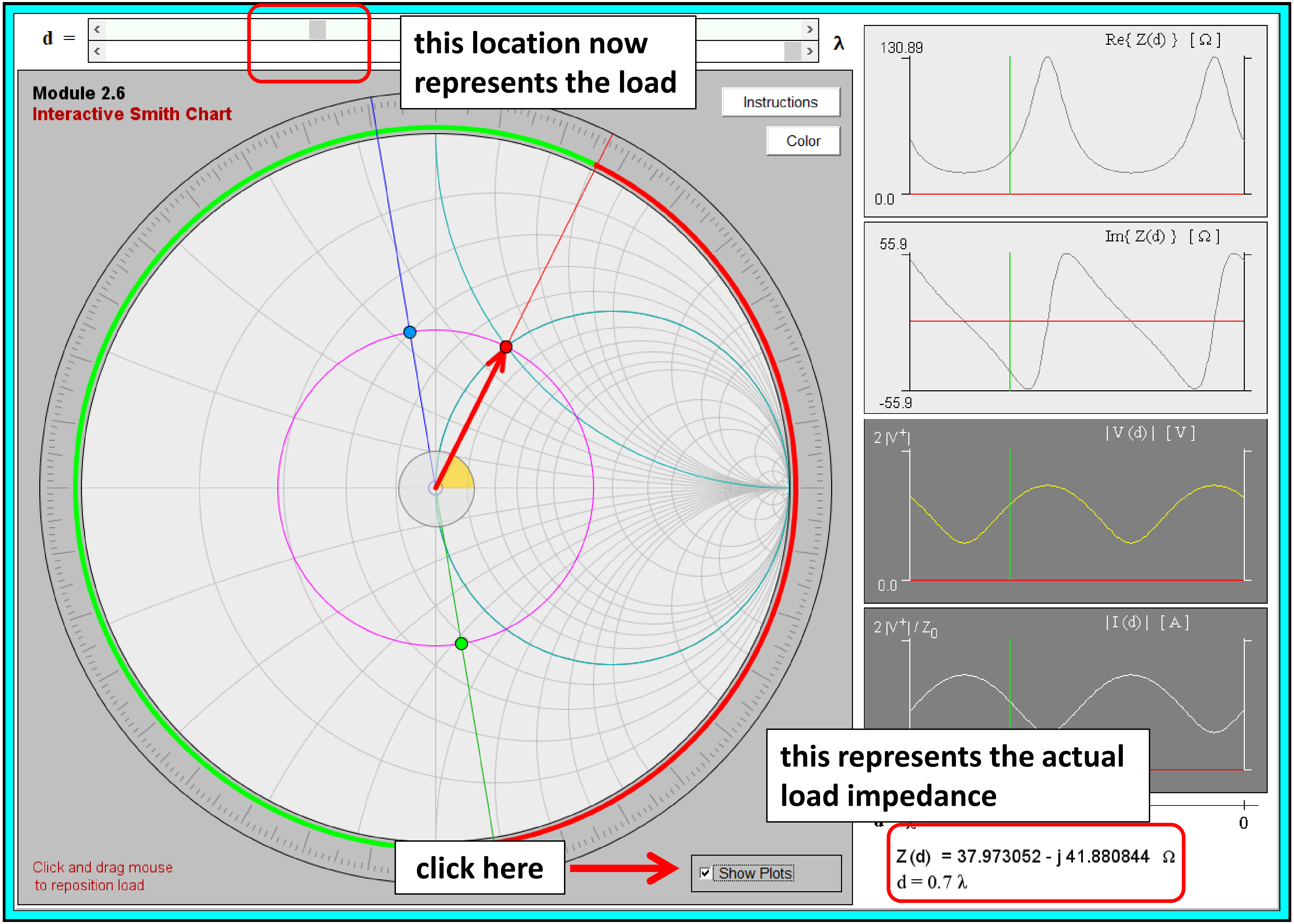
Example 3: A TL of length $l = 0.3\lambda$ has an input impedance $Z_{in} = 50 + j50\,\Omega$. Determine the load impedance $Z_L = Z(0)$ and $Y_L = Y(0)$ given that $Z_o = 50\,\Omega$ for the line.

Trick: To consider an input impedance in the Java app, enter it as a load but move the cursor to 1λ. This will be now your input location corresponding to an input impedance equal to what you have entered (we exploit periodicity of the impedance).

Then, move back the cursor by the length of the line (in this case 0.3λ). This means that the location $d = 0.7\lambda$ now represents a location equivalent to the load and you can read there the corresponding impedance.

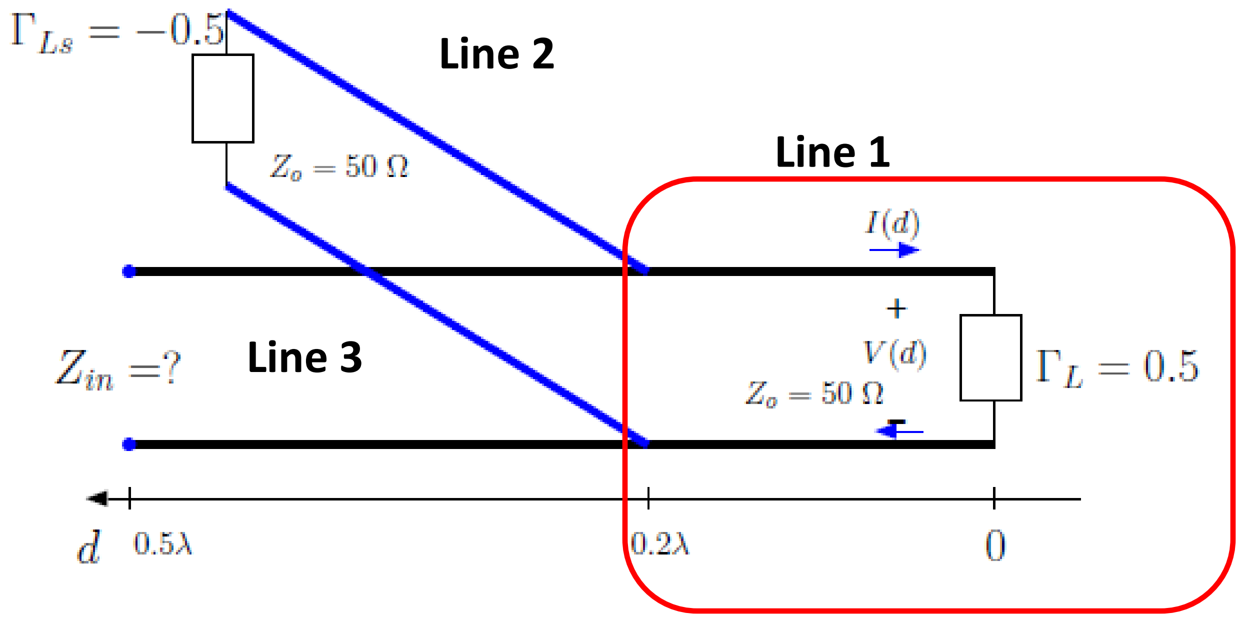

d =
λ
position cursor at 1λ
Module 2.6
Interactive Smith Chart
Instructions
Color
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 2.618
Voltage Maximum
d(max) = 0.0881 λ
Voltage Minimum
d(min) = 0.3381 λ
Show Γ
Load
Cursor
z_L = 1.0 + j 1.0
Γ_L = 0.4472136 ∠ 63.4349 °
z(d) = 1.0 + j 1.0
Γ_d = 0.4472136 ∠ 63.4349 °
y(d) = 0.5 - j 0.5
d' = 0.5 λ
d = 1.0 λ
2 β d' = 6.283185 rad = 360.0º
0.5λ - d' = 0.0 λ
2 β (0.5λ - d') = 0.0 rad = 0.0º
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Z_L = 50.0
+ j 50.0 [Ω]
Z
Y
Γ
this now represents the input impedance
Show Plots
Click and drag mouse to reposition load

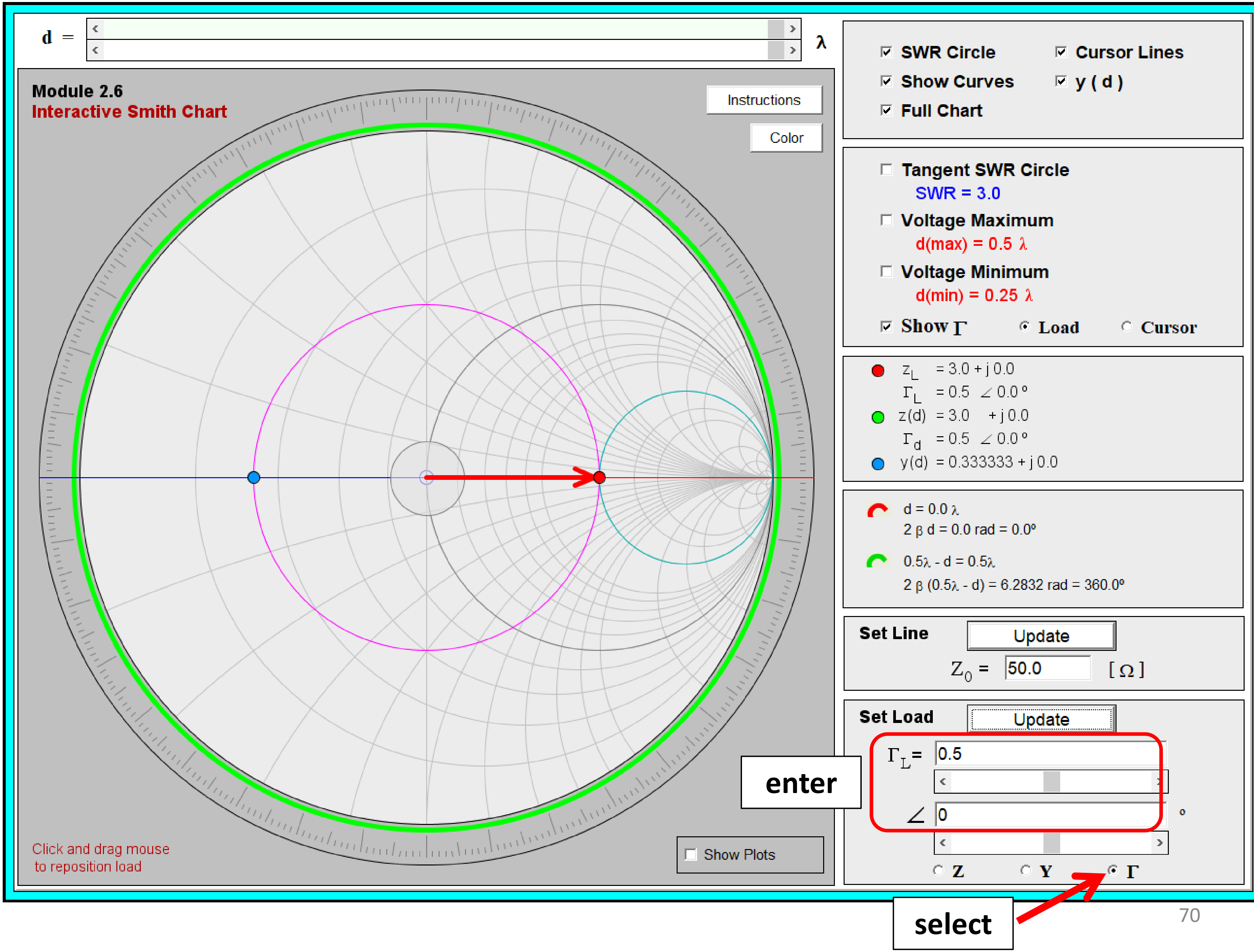

d =
λ
this location now represents the load
this now represents the normalized load impedance
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 2.618
Voltage Maximum
d(max) = 0.0881 λ
Voltage Minimum
d(min) = 0.3381 λ
Show Γ
Load
Cursor
z_L = 1.0 + j 1.0
Γ_L = 0.4472136 ∠ 63.4349 °
z(d) = 0.759461 - j 0.837617
Γ_d = 0.4472136 ∠ -80.5651 °
y(d) = 0.594079 + j 0.655216
d' = 0.2 λ
d = 0.7 λ
2 β d' = 2.513274 rad = 144.0º
0.5λ - d' = 0.3 λ
2 β (0.5λ - d') = 3.769911 rad = 216.0º
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Z_L = 50.0
+ j 50.0 [Ω]
Z
Y
Γ



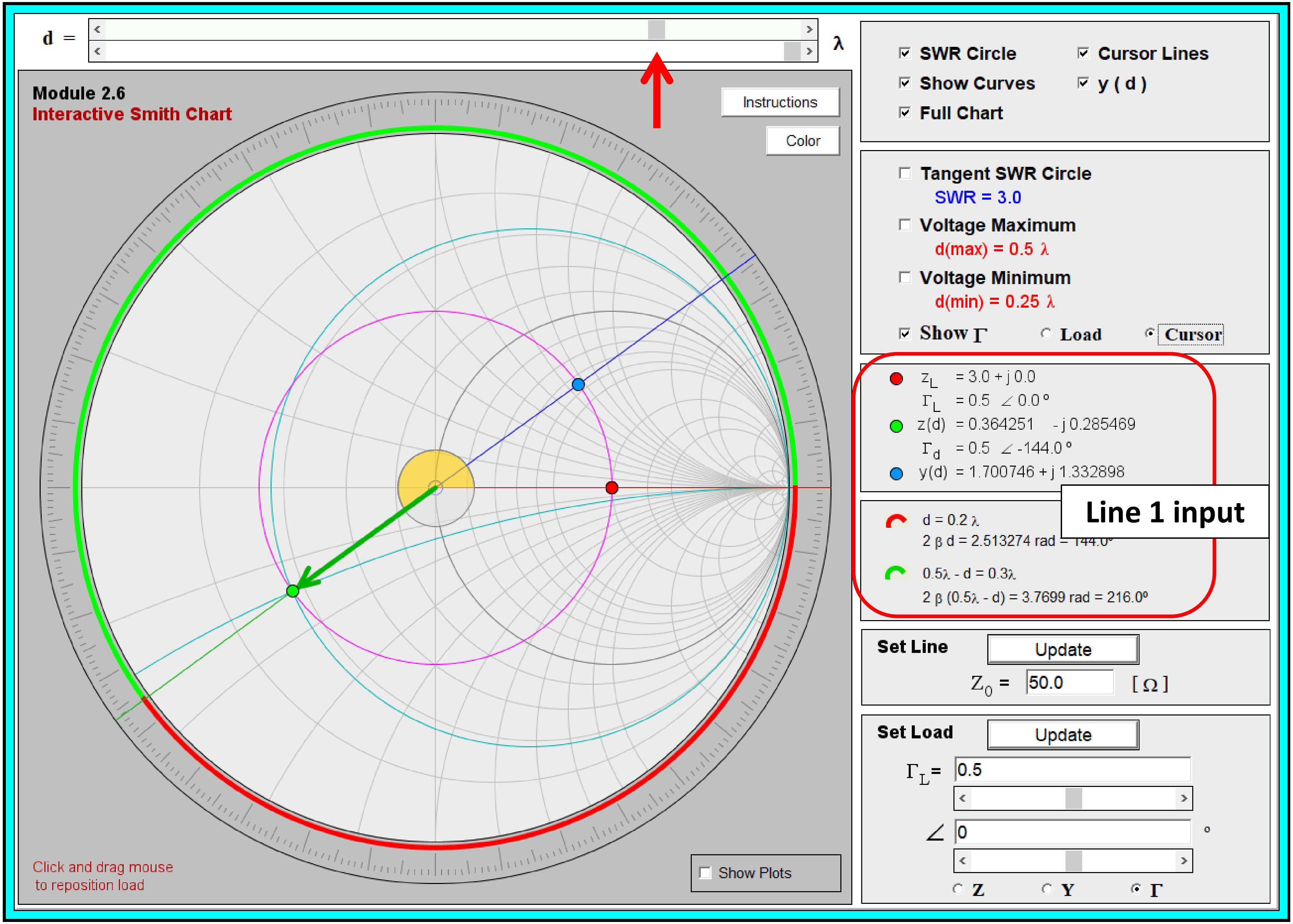
d =
λ
this location now represents the load
Module 2.6
Interactive Smith Chart
Instructions
Color
Re{ Z(d) } [Ω]
130.89
0.0
Im{ Z(d) } [Ω]
55.9
-55.9
|V(d)| [V]
2 |V+|
0.0
|I(d)| [A]
2 |V+| / Z0
this represents the actual load impedance
0
Z(d) = 37.973052 - j 41.880844 Ω
d = 0.7 λ
click here
Show Plots
Click and drag mouse to reposition load

Example 4: A TL of length $l = 0.5\lambda$ and $Z_o = 50\,\Omega$ has a load reflection coefficient $\Gamma_L = 0.5$ and and a shunt connected TL at $d = 0.2\lambda$. The shunt connected TL has $l = 0.3\lambda$, $Z_o = 50\,\Omega$, and a load reflection coefficient $\Gamma_L = -0.5$. Determine the input impedance of the line.

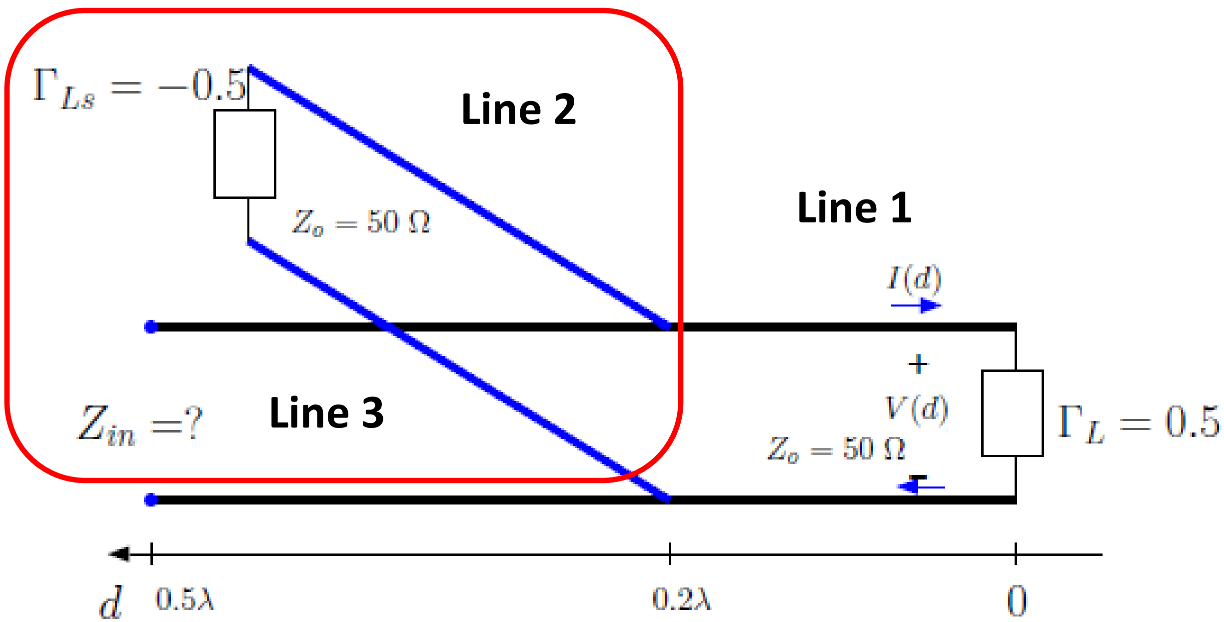


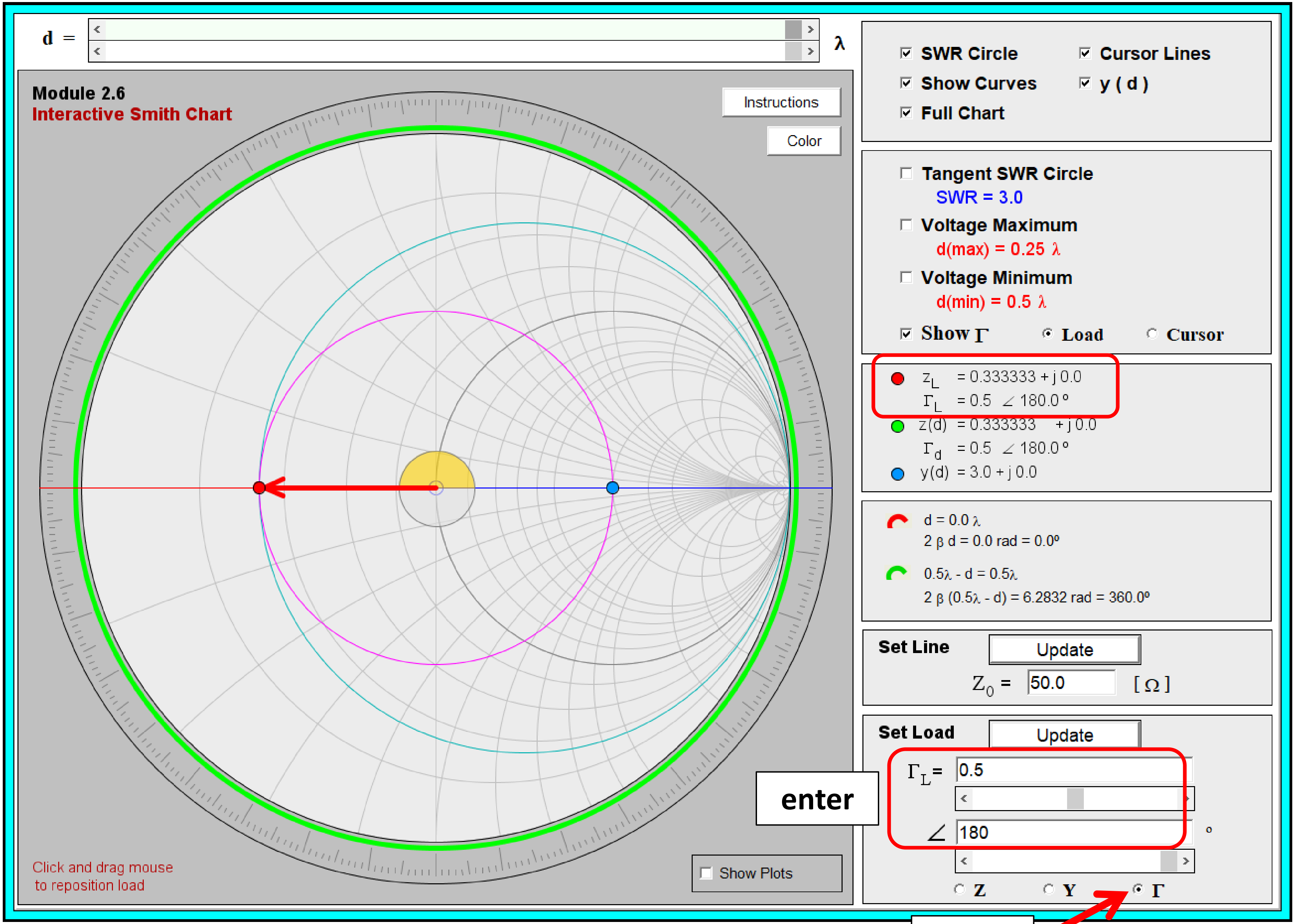


d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 3.0
Voltage Maximum
d(max) = 0.5 λ
Voltage Minimum
d(min) = 0.25 λ
Show Γ
Load
Cursor
z_L = 3.0 + j 0.0
Γ_L = 0.5 ∠ 0.0 º
z(d) = 3.0 + j 0.0
Γ_d = 0.5 ∠ 0.0 º
y(d) = 0.333333 + j 0.0
d = 0.0 λ
2 β d = 0.0 rad = 0.0º
0.5λ - d = 0.5λ
2 β (0.5λ - d) = 6.2832 rad = 360.0º
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Γ_L = 0.5
∠ 0 º
Z
Y
Γ
enter
select
Click and drag mouse
to reposition load
Show Plots


d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse
to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 3.0
Voltage Maximum
d(max) = 0.5 λ
Voltage Minimum
d(min) = 0.25 λ
Show Γ
Load
Cursor
z_L = 3.0 + j 0.0
Γ_L = 0.5 ∠ 0.0 °
z(d) = 0.364251 - j 0.285469
Γ_d = 0.5 ∠ -144.0 °
y(d) = 1.700746 + j 1.332898
d = 0.2 λ
2 β d = 2.513274 rad = 144.0°
0.5λ - d = 0.3λ
2 β (0.5λ - d) = 3.7699 rad = 216.0°
Line 1 input
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Γ_L = 0.5
∠ 0 °
Z
Y
Γ

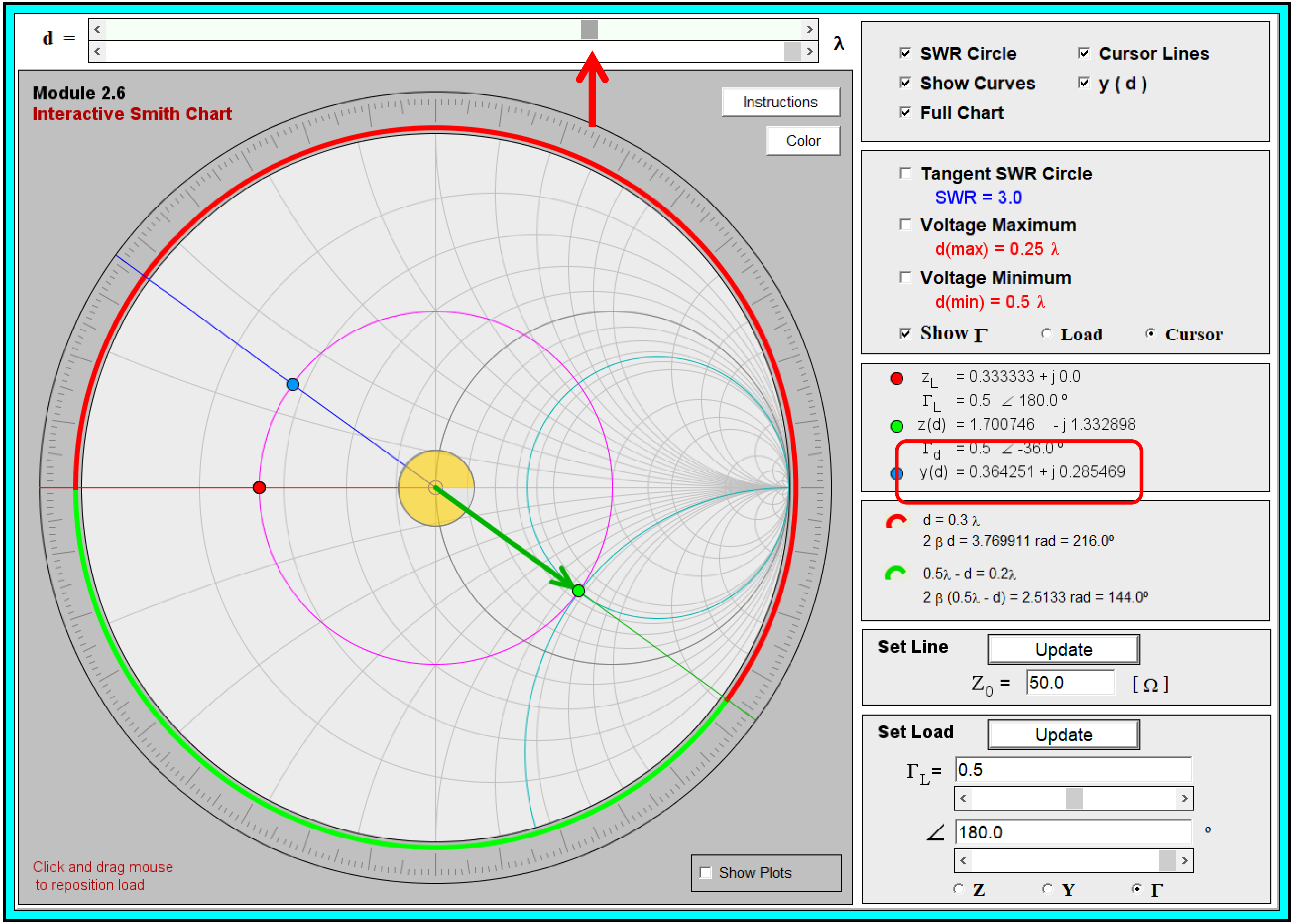
Example 4: A TL of length $l = 0.5\lambda$ and $Z_o = 50\,\Omega$ has a load reflection coefficient $\Gamma_L = 0.5$ and and a shunt connected TL at $d = 0.2\lambda$. The shunt connected TL has $l = 0.3\lambda$, $Z_o = 50\,\Omega$, and a load reflection coefficient $\Gamma_L = -0.5$. Determine the input impedance of the line.



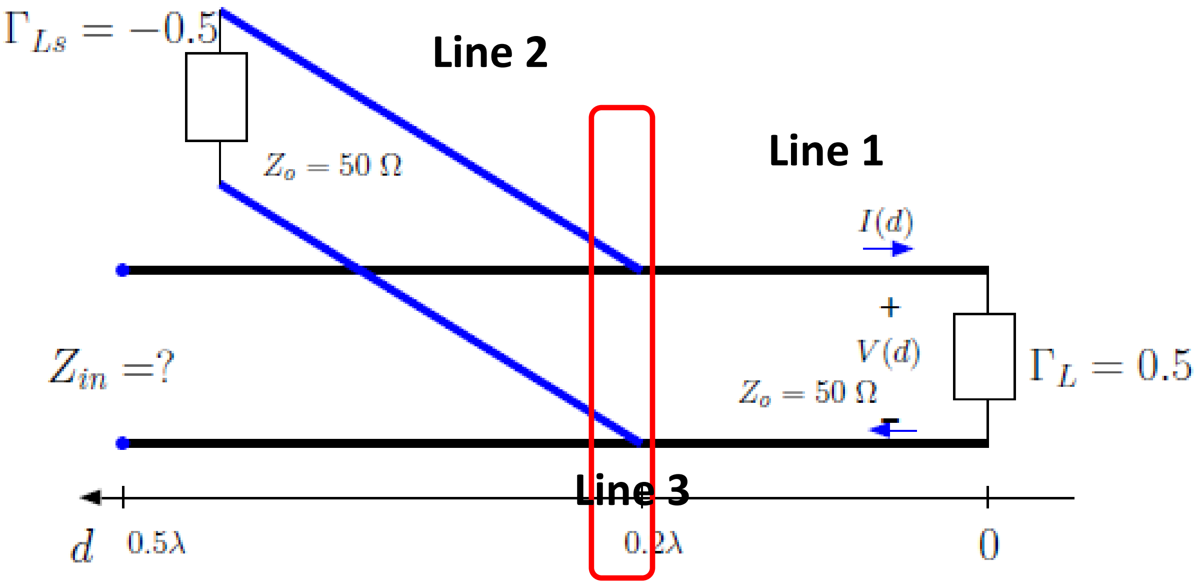


enter

select


d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse
to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 3.0
Voltage Maximum
d(max) = 0.25 λ
Voltage Minimum
d(min) = 0.5 λ
Show Γ
Load
Cursor
z_L = 0.333333 + j 0.0
Γ_L = 0.5 ∠ 180.0 °
z(d) = 1.700746 - j 1.332898
Γ_d = 0.5 ∠ -36.0 °
y(d) = 0.364251 + j 0.285469
d = 0.3 λ
2 β d = 3.769911 rad = 216.0º
0.5λ - d = 0.2λ
2 β (0.5λ - d) = 2.5133 rad = 144.0º
Set Line
Update
Z_0 =
50.0
[Ω]
Set Load
Update
Γ_L =
0.5
180.0
°
Z
Y
Γ

Example 4: A TL of length $l = 0.5\lambda$ and $Z_o = 50\,\Omega$ has a load reflection coefficient $\Gamma_L = 0.5$ and and a shunt connected TL at $d = 0.2\lambda$. The shunt connected TL has $l = 0.3\lambda$, $Z_o = 50\,\Omega$, and a load reflection coefficient $\Gamma_L = -0.5$. Determine the input impedance of the line.



input admittance Line 1

y(d) = 1.700746 + j 1.332898

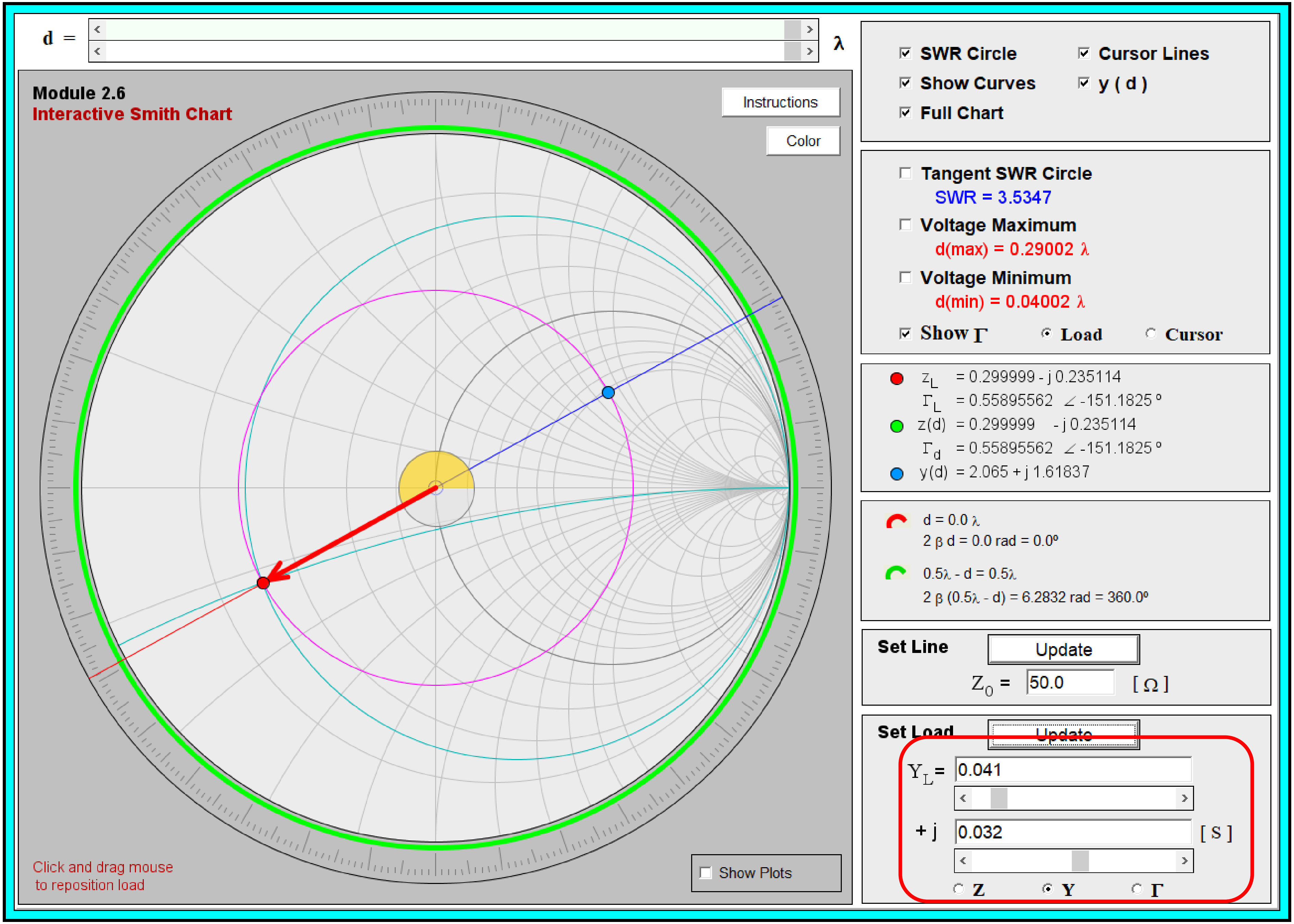
input admittance Line 2

y(d) = 0.364251 + j 0.285469

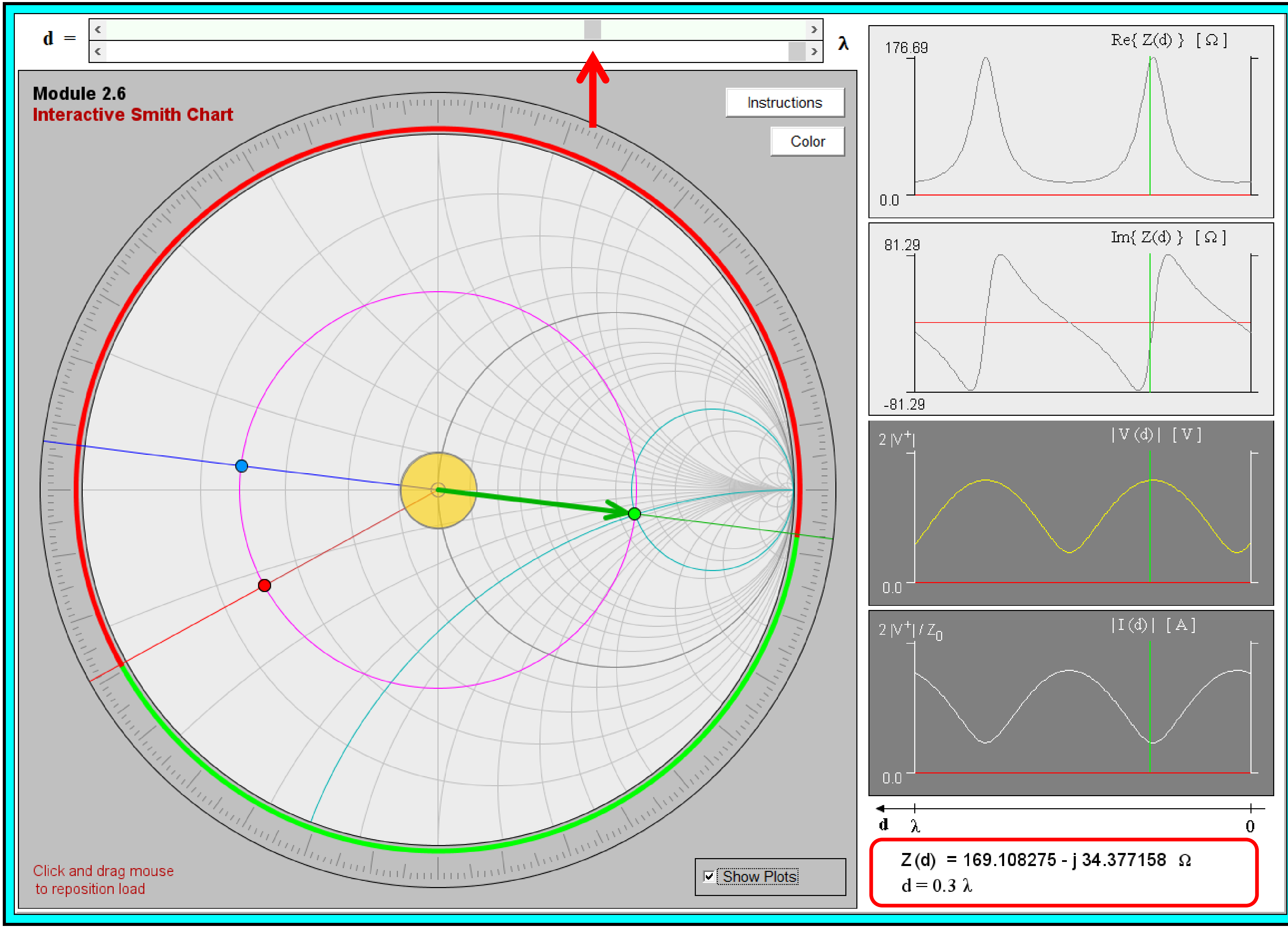
parallel of Line 1 and Line 2 input admittances is simply the sum

$$y_c = y(0.2\lambda) + y_s(0.3\lambda) \approx (1.7 + j1.33) + (0.36 + j0.29) = 2.065 + j1.61837$$

(actual admittance is obtained dividing by $Z_0 = 50\ \Omega$ or multiplying by $Y_0 = 1/50$ S)



d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 3.5347
Voltage Maximum
d(max) = 0.29002 λ
Voltage Minimum
d(min) = 0.04002 λ
Show Γ
Load
Cursor
z_L = 0.299999 - j 0.235114
Γ_L = 0.55895562 ∠ -151.1825 º
z(d) = 0.299999 - j 0.235114
Γ_d = 0.55895562 ∠ -151.1825 º
y(d) = 2.065 + j 1.61837
d = 0.0 λ
2 β d = 0.0 rad = 0.0º
0.5λ - d = 0.5λ
2 β (0.5λ - d) = 6.2832 rad = 360.0º
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Y_L = 0.041
+ j 0.032 [S]
Z
Y
Γ


d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Click and drag mouse
to reposition load
Show Plots
Re{ Z(d) } [Ω]
176.69
0.0
Im{ Z(d) } [Ω]
81.29
-81.29
2 |V+|
|V(d)| [V]
0.0
2 |V+| / Z0
|I(d)| [A]
0.0
d
λ
0
Z(d) = 169.108275 - j 34.377158 Ω
d = 0.3 λ

Example 5: What is the load impedance Z_{Ls} terminating the shunt connected stub in Example 4?

Solution: Given that the corresponding reflection coefficient is

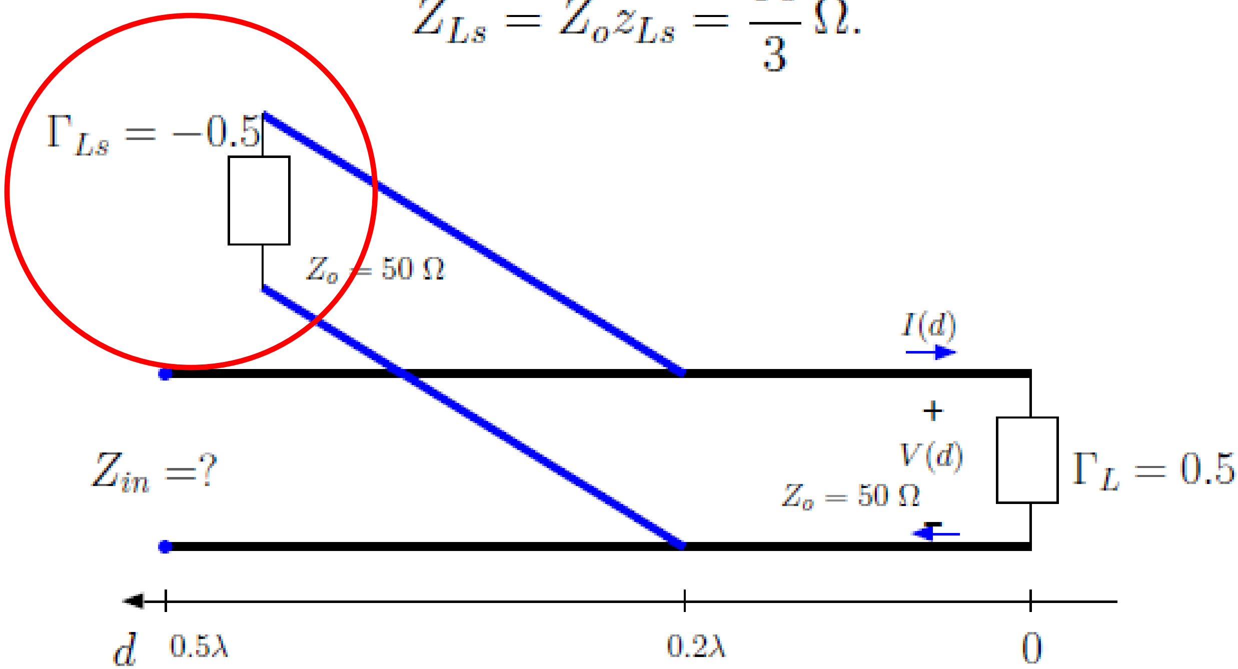
$$\Gamma_{Ls} = -0.5,$$

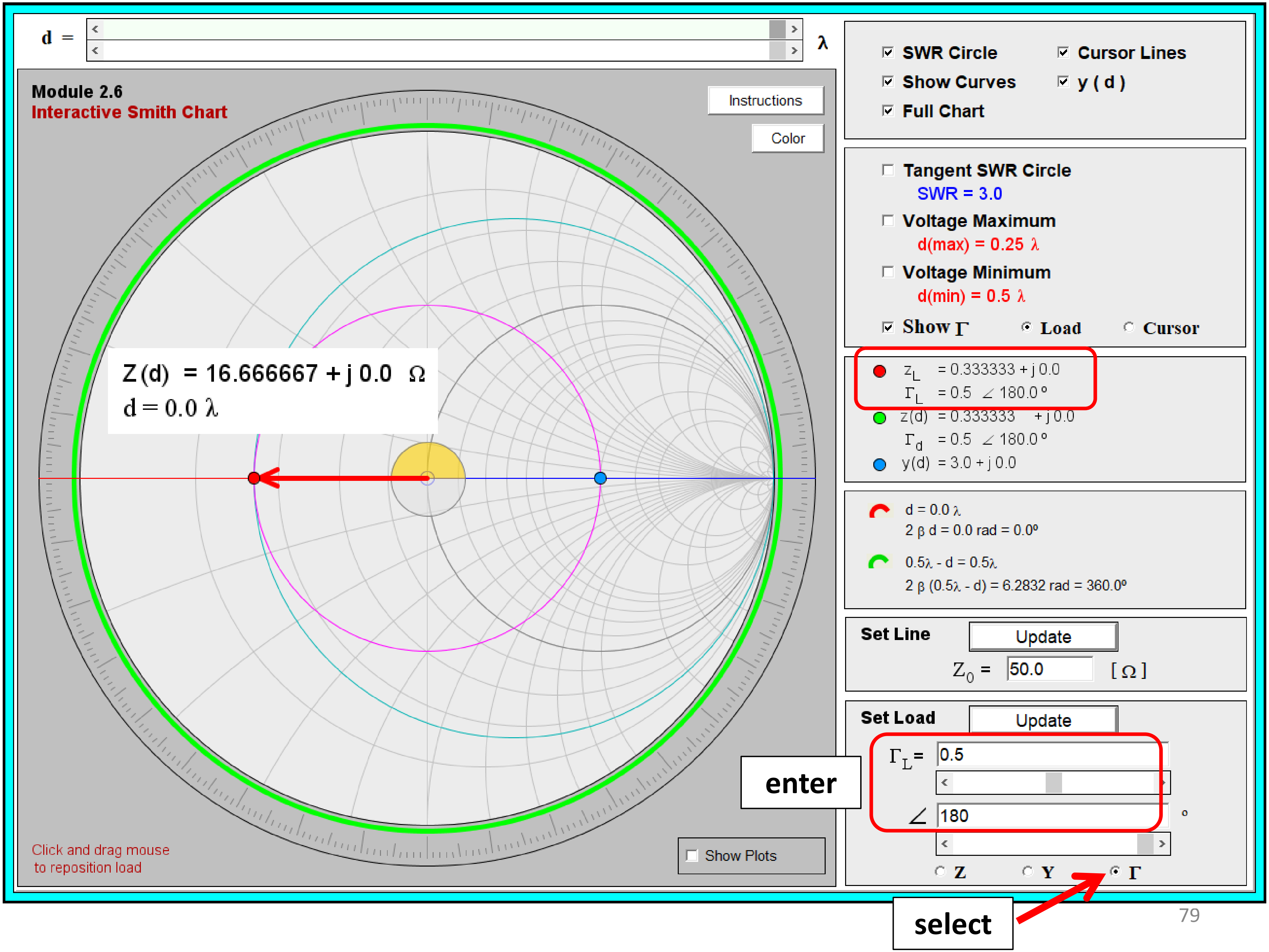
it follows from the bilinear transformation linking z_{Ls} and Γ_{Ls} that

$$z_{Ls} = \frac{1+\Gamma_{Ls}}{1-\Gamma_{Ls}} = \frac{1-0.5}{1+0.5} = \frac{1}{3}.$$

Hence, the impedance is

$$Z_{Ls} = Z_o z_{Ls} = \frac{50}{3}\,\Omega.$$




d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 3.0
Voltage Maximum
d(max) = 0.25 λ
Voltage Minimum
d(min) = 0.5 λ
Show Γ
Load
Cursor
Z (d) = 16.666667 + j 0.0 Ω
d = 0.0 λ
z_L = 0.333333 + j 0.0
Γ_L = 0.5 ∠ 180.0 º
z(d) = 0.333333 + j 0.0
Γ_d = 0.5 ∠ 180.0 º
y(d) = 3.0 + j 0.0
d = 0.0 λ
2 β d = 0.0 rad = 0.0º
0.5λ - d = 0.5λ
2 β (0.5λ - d) = 6.2832 rad = 360.0º
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Γ_L = 0.5
∠ 180 º
Z
Y
Γ
enter
select
Show Plots
Click and drag mouse
to reposition load

Example 6: What is the load impedance Z_L in Example 4?

Solution: This is similar to Example 5. Given that the load reflection coefficient is

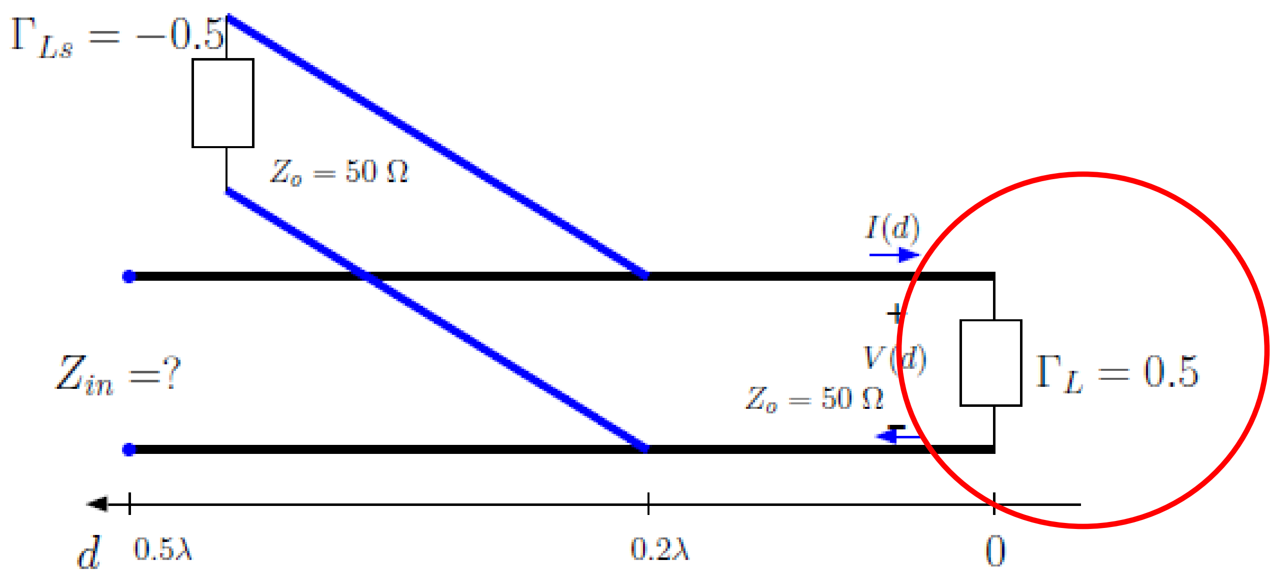
$$\Gamma_L = 0.5,$$

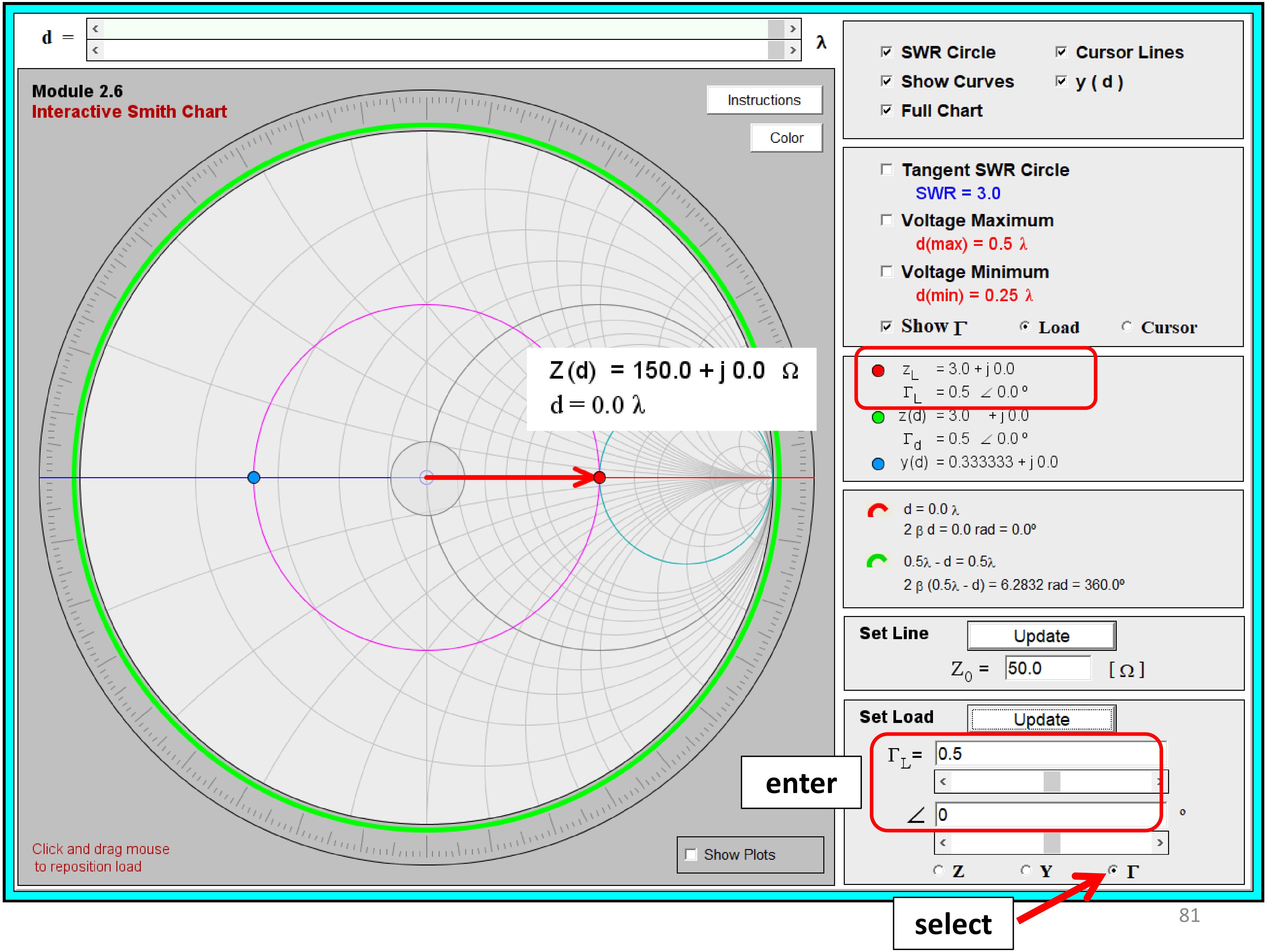
it follows from the bilinear transformation linking z_L and Γ_L that

$$z_L = \frac{1+\Gamma_L}{1-\Gamma_L} = \frac{1+0.5}{1-0.5} = 3.$$

Hence, the impedance is

$$Z_L = Z_o z_L = 150\,\Omega.$$





d =
λ
Module 2.6
Interactive Smith Chart
Instructions
Color
Z(d) = 150.0 + j 0.0 Ω
d = 0.0 λ
enter
Click and drag mouse
to reposition load
Show Plots
SWR Circle
Cursor Lines
Show Curves
y (d)
Full Chart
Tangent SWR Circle
SWR = 3.0
Voltage Maximum
d(max) = 0.5 λ
Voltage Minimum
d(min) = 0.25 λ
Show Γ
Load
Cursor
z_L = 3.0 + j 0.0
Γ_L = 0.5 ∠ 0.0 °
z(d) = 3.0 + j 0.0
Γ_d = 0.5 ∠ 0.0 °
y(d) = 0.333333 + j 0.0
d = 0.0 λ
2 β d = 0.0 rad = 0.0°
0.5λ - d = 0.5λ
2 β (0.5λ - d) = 6.2832 rad = 360.0°
Set Line
Update
Z_0 = 50.0 [Ω]
Set Load
Update
Γ_L = 0.5
∠ 0 °
Z
Y
Γ
select