ECE 329 – Fall 2022

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Lecture 38

Lecture 38 – Outline

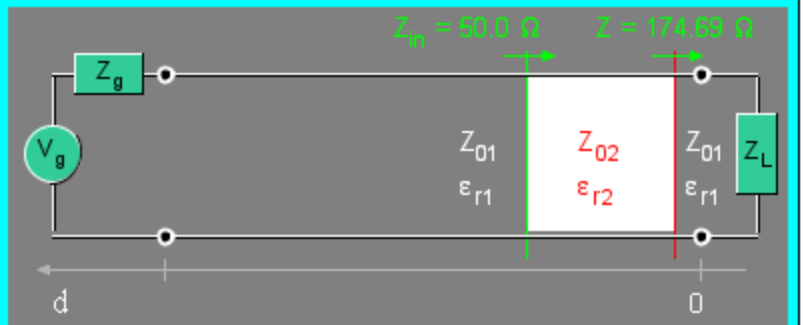
- **More on Impedance matching**
- **Applications involving combination of transmission lines for distribution networks**

Reading assignment

Prof. Kudeki's ECE 329 Lecture Notes on Fields and Waves:

38) Distribution networks

Module 2.7 Tutorial **Quarter-Wavelength Transformer**



f = 1.0 GHz
Z_{01} = 50.0 Ω
ε_{r1} = 1.0

Z_L = 100.0 + j 80.0 Ω
Z_{02} = 93.4582 Ω
ε_{r2} = 1.0

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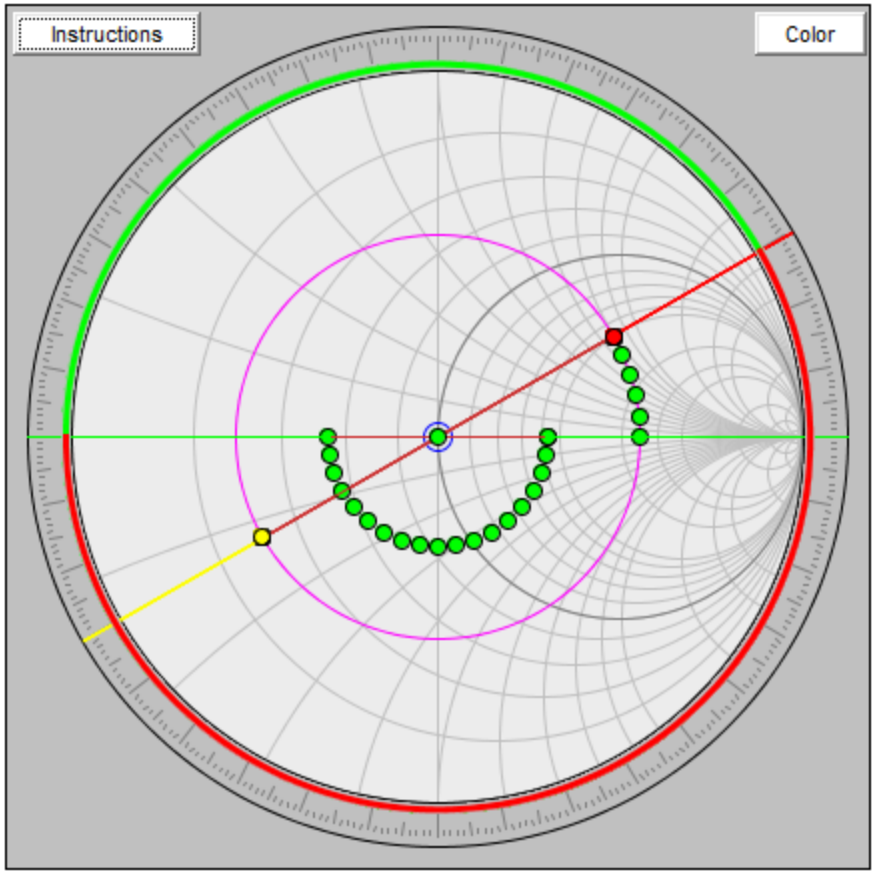
Step 6 - First Solution

Since we are going back to the original transmission line, the line impedance on the chart is re-normalized with Z_{01} :

$z_{in} = 0.535 * Z_{02} / Z_{01}$

$z_{in} = 0.535 * 93.4582 / 50.0 = 1.0$

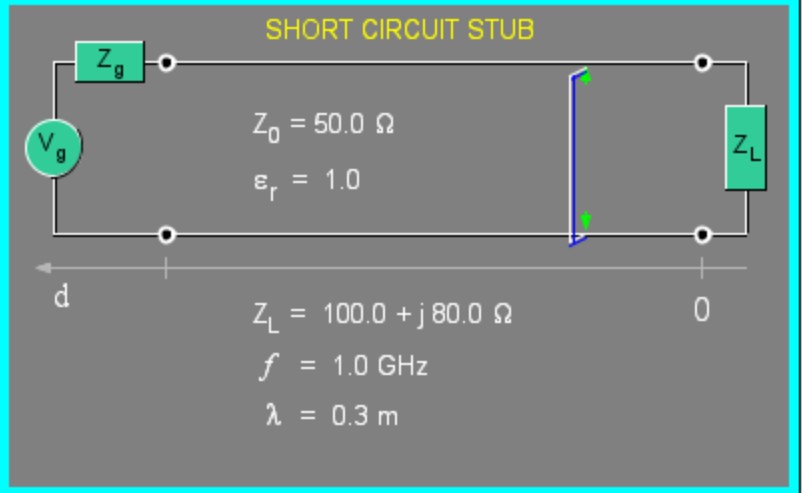


z_L = 2.0 + j 1.6
Γ_L = 0.55494 ∠ 29.92213 °
y_L = 0.30488 - j 0.2439
z (d) = 1.0 + j 0.0
Γ (d) = 0.0 ∠ 0.0 °
y (d) = 1.0 + j 0.0

d = 0.2916 λ 2β d = 3.6638 rad = 209.9221°
0.5 λ - d = 0.2084 λ 2β (0.5 λ - d) = 2.6194 rad = 150.0779°

Module 2.9 Tutorial

Single-Stub Tuning



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Step 4 - First Solution

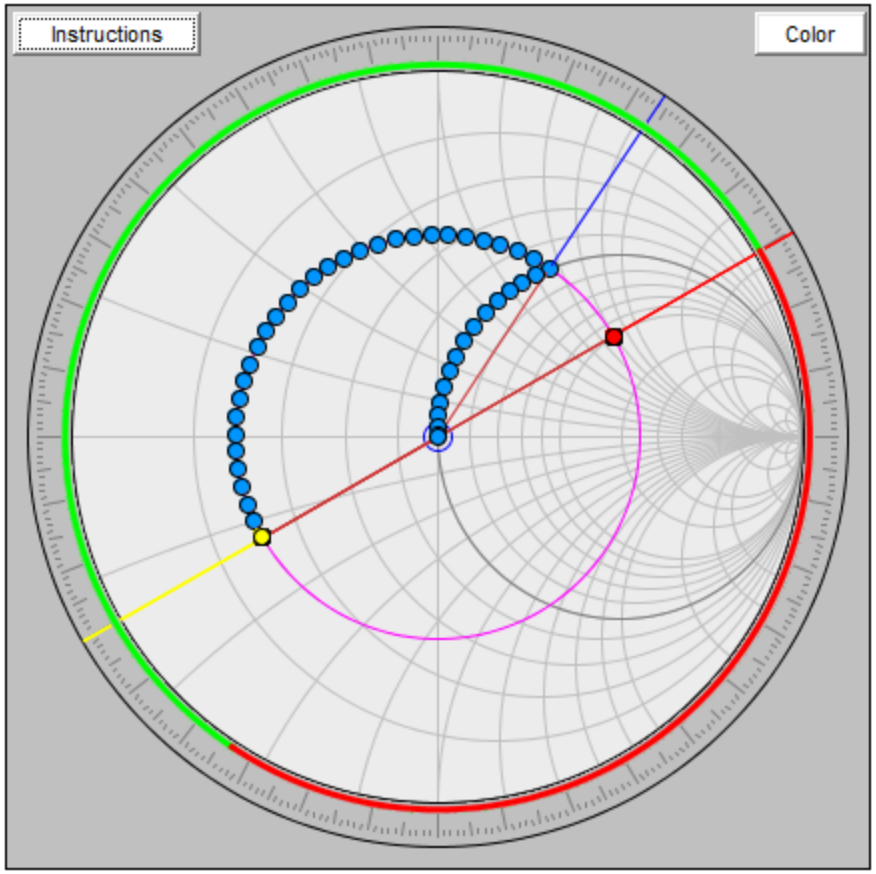
To cancel the imaginary part of the line admittance we add a stub with:

Length: l_1 = 0.1024 λ

Admittance:

Y_1 = - j 0.02668 [S] (actual)

y_1 = - j 1.33417 (normalized)

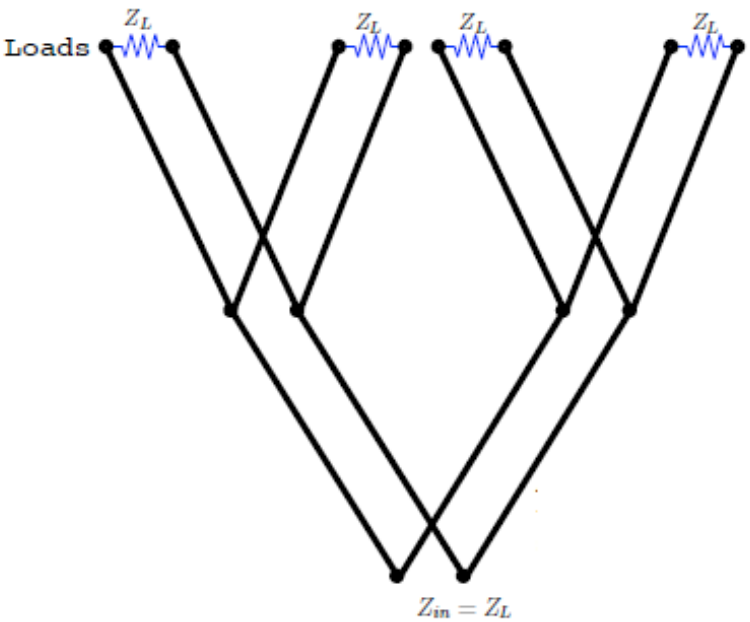


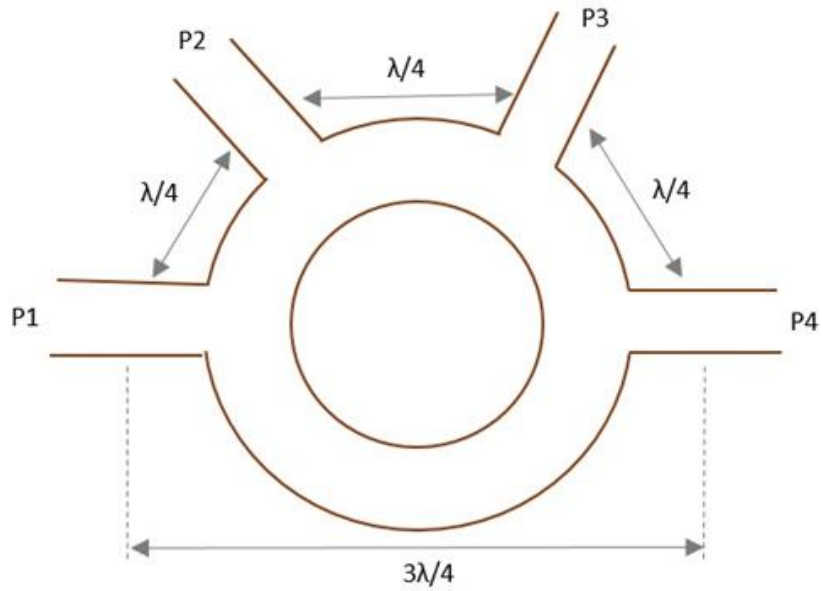
z_L = 2.0 + j 1.6
Γ_L = 0.55494 ∠ 29.92213 °
y_L = 0.30488 - j 0.2439
z(d) = 1.0 + j 0.0
Γ(d) = 0.0 ∠ 0.0 °
y(d) = 1.0 + j 0.0

d = 0.2134 λ 2β d = 2.6813 rad = 153.6287°
0.5 λ - d = 0.2866 λ 2β (0.5 λ - d) = 3.6019 rad = 206.3713°

Examples of distribution networks

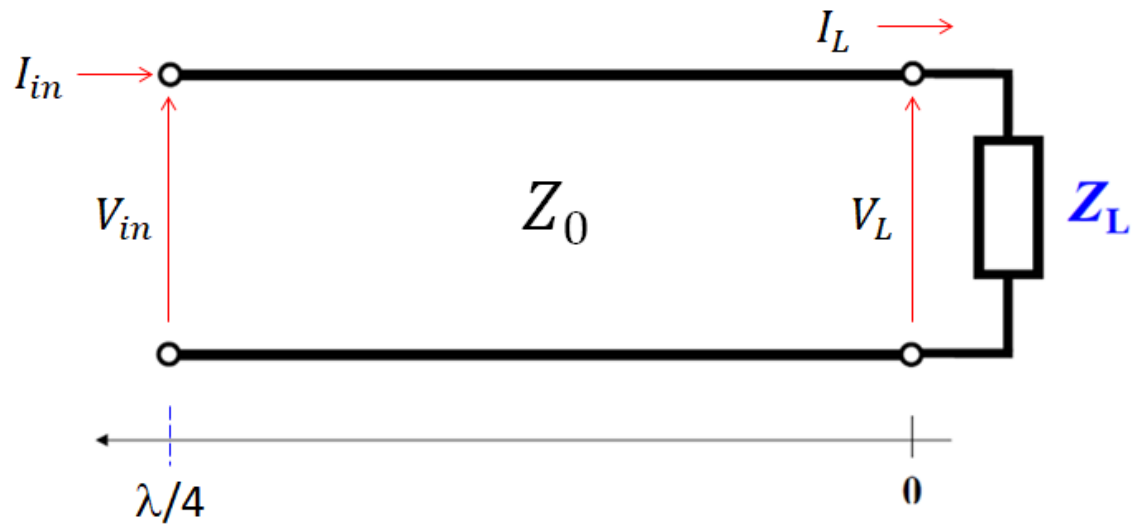
- **Lines of length λ/4 are very useful to create networks with special properties.**
- **Examples are**
  - **The corporate ladder network**
  - **The hybrid rat-race coupler**





Quick review – λ/4 transmission line

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{Z_0^2}{Z_L}$$



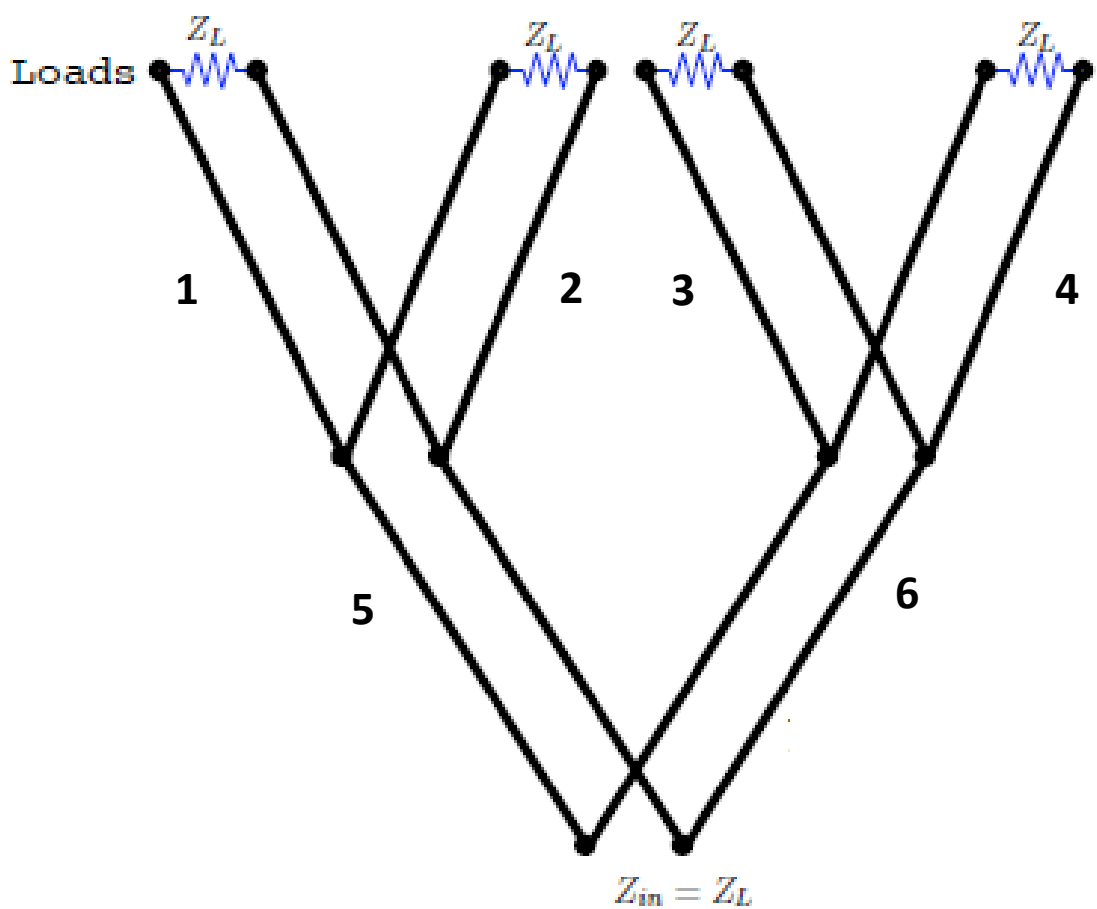
$$V_{in} = j\, I_L Z_0 \quad \longrightarrow \quad I_L = -j\frac{V_{in}}{Z_0}$$

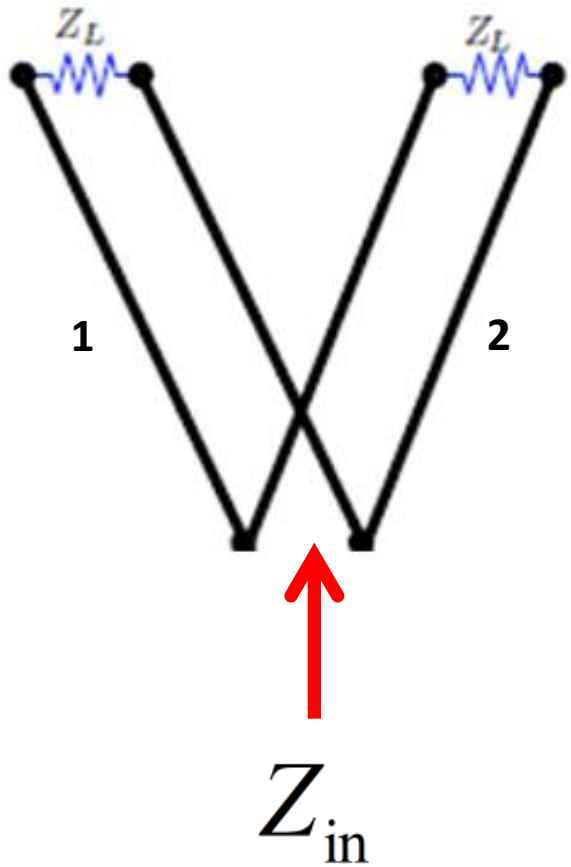
$$I_{in} = j\frac{V_L}{Z_0} \quad \longrightarrow \quad V_L = -j\, I_{in} Z_0$$

$$V_L = Z_L I_L$$

Corporate ladder network

All line sections are of length λ/4 and have characteristic impedance Z_0

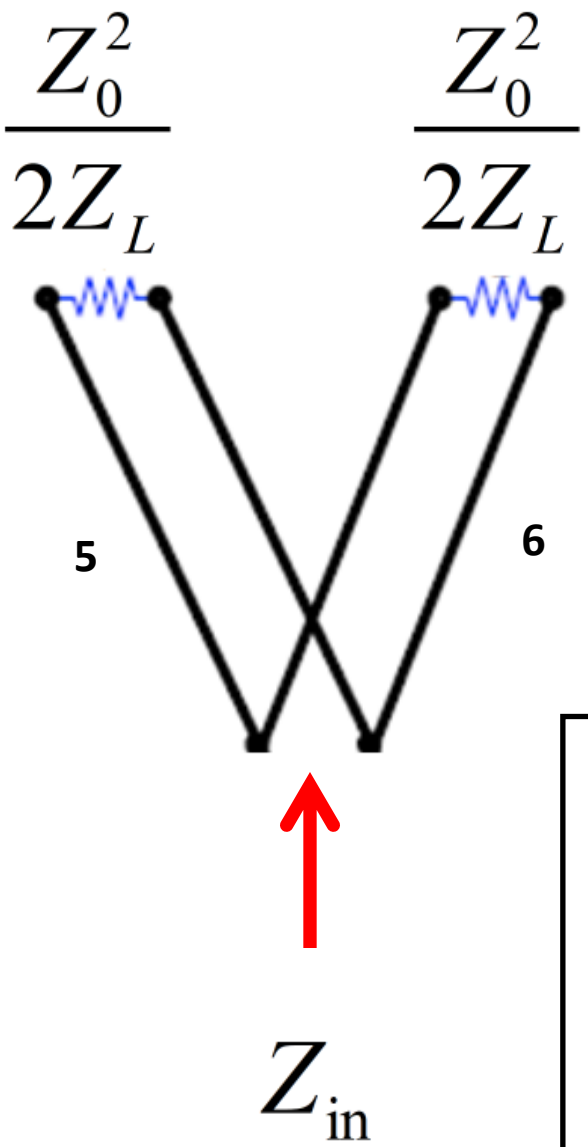




$$Z_{in1} = Z_{in2} = \frac{Z_0^2}{Z_L}$$

$$\boxed{Z_{\text{in}} = Z_{in1} \parallel Z_{in2} = \left(\frac{Z_L}{Z_0^2} + \frac{Z_L}{Z_0^2} \right)^{-1} = \frac{Z_0^2}{2Z_L}}$$

Same result for the lines 3 & 4



$$Z_{in5} = Z_{in6} = \frac{Z_0^2}{\frac{Z_0^2}{2Z_L}} = 2Z_L$$

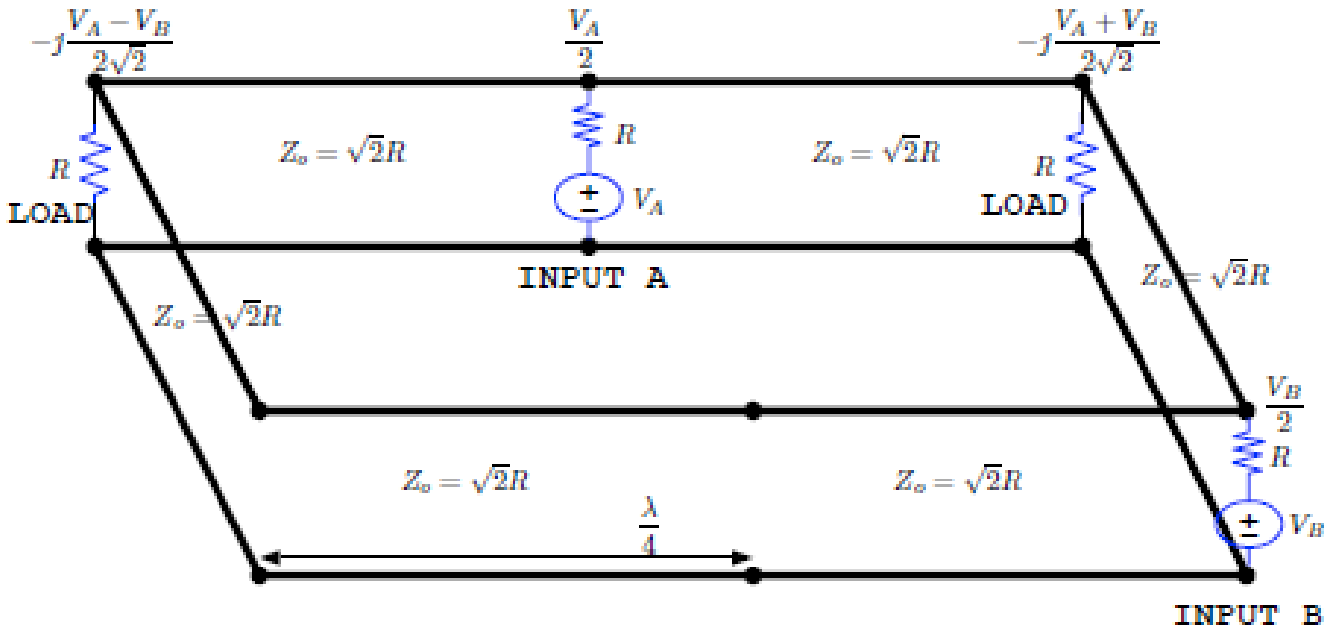
$$Z_{\text{in}} = Z_{in5} \parallel Z_{in6} = \left(\frac{1}{2Z_L} + \frac{1}{2Z_L} \right)^{-1}$$

$$= \left(\frac{2}{2Z_L} \right)^{-1} = Z_L$$

If the load Z_L is the same as Z_0 then there is no longer a constraint on the length of the individual transmission lines. The input impedance is also $Z_{in} = Z_0$ regardless of the lengths of the lines.

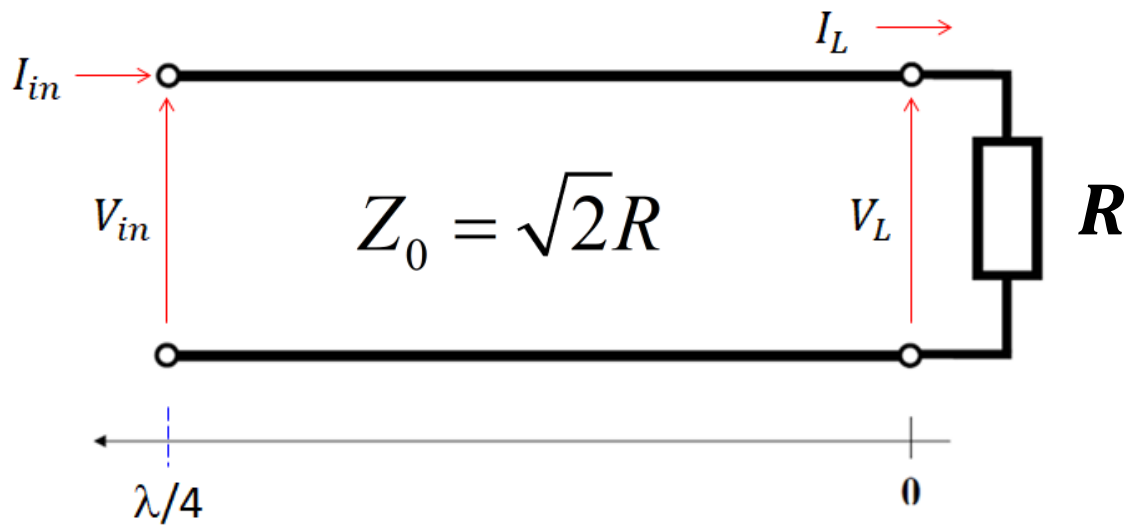
We have a new degree of freedom, whereby the length of each line can be varied to introduce delays. This is exploited in the development of antenna arrays to synthesize arbitrary beam patterns.

Hybrid rat-race combiner



$$Z_0 = \sqrt{2}R$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{Z_0^2}{Z_L}$$



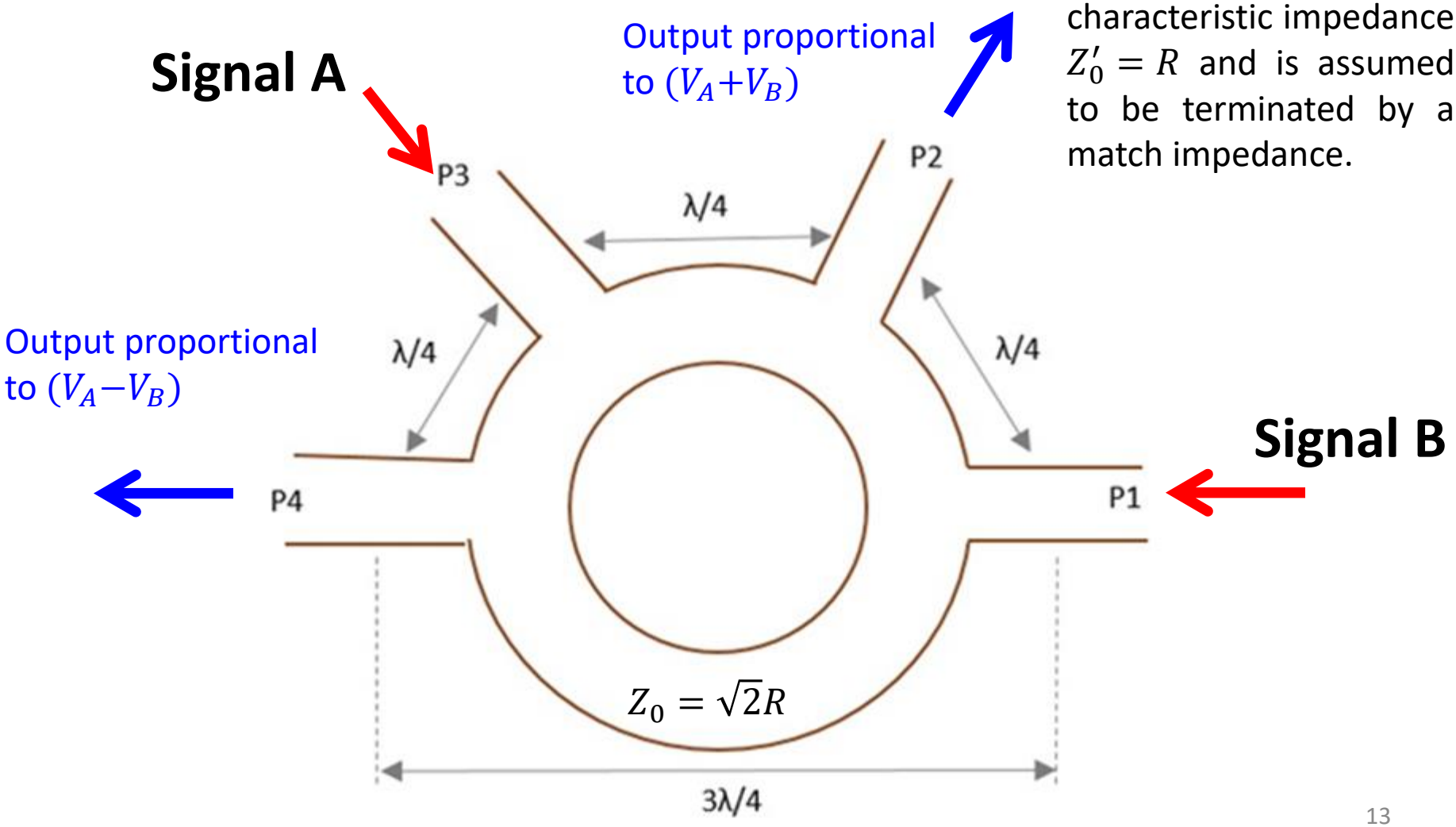
or 3λ/4

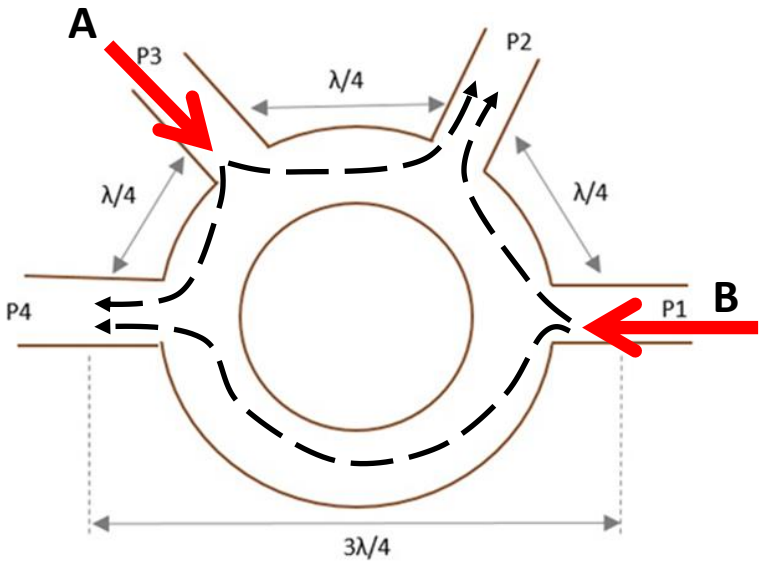
$$\boxed{Z_{in} = \frac{\left(\sqrt{2}R\right)^2}{R} = 2R}$$

Typical microstrip implementation

Hybrid rat-race combiner

Each side branch has characteristic impedance $Z_0' = R$ and is assumed to be terminated by a match impedance.





Input applied at port P1 – signal get split into two parts. One part travels in clockwise direction, the other travels in counterclockwise direction.

Signals reach in phase port P2 ($\lambda/4$ and $\lambda+\lambda/2$) and port P4.

Port P3 has no output because signals arrive in opposition of phase ($\lambda/2$ and λ)

Input applied at port P3 – Power is divided between port P2 ($\lambda/4$) and P4 ($\lambda/4$), but there is no output at port P1 because signals arrive in opposition of phase ($\lambda/2$ and λ)

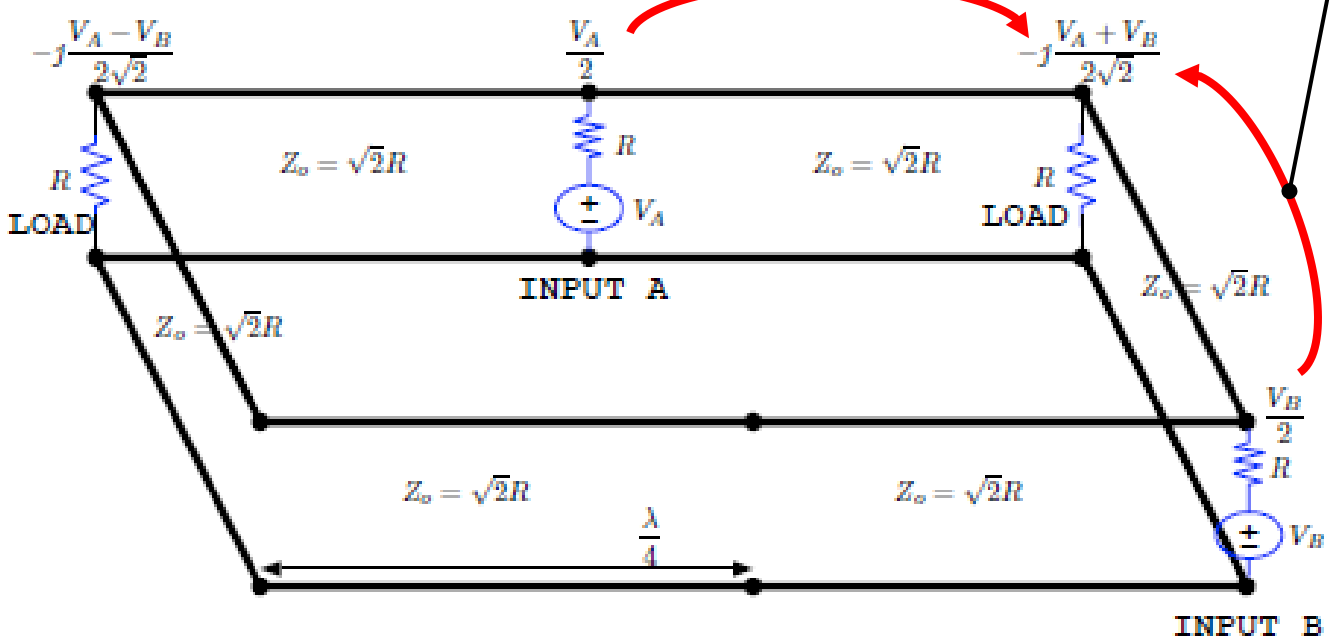
Port P1 and P3 are isolated from each other. A signal "A" applied to P3 and a signal "B" applied to P1 produce a signal at port P2 proportional to their sum and a signal at port P4 proportional to their difference.

Effectively, each generator only sees a parallel of two impedances $2R \,//\, 2R = R$**.**

$$R I_L = -j\frac{V_{in}}{Z_0}R = -j\frac{V_A}{2}\frac{1}{\sqrt{2}R}R = -j\frac{V_A}{2\sqrt{2}}$$

$$R I_L = -j\frac{V_{in}}{Z_0}R = -j\frac{V_B}{2}\frac{1}{\sqrt{2}R}R = -j\frac{V_B}{2\sqrt{2}}$$

$$-j\frac{V_A + V_B}{2\sqrt{2}}$$



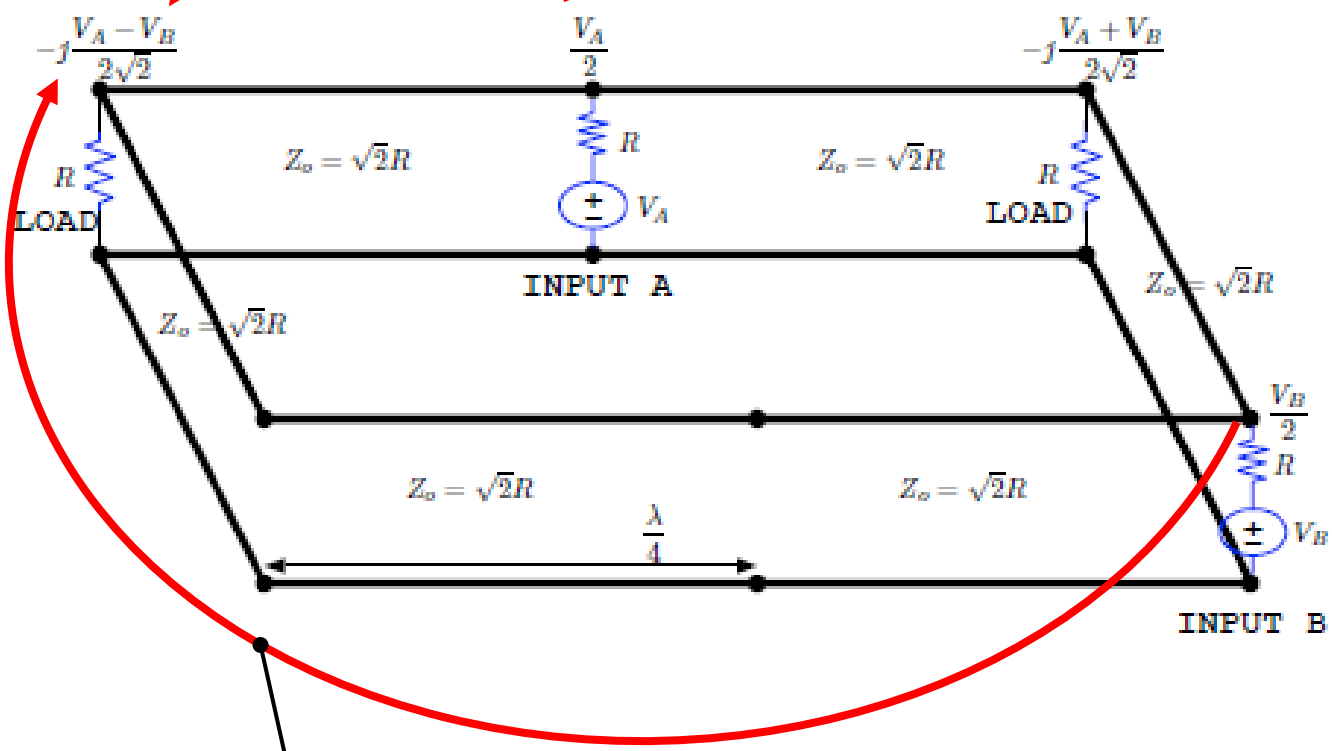
λ/4 line

$$V_L = Z_L I_L$$

$$I_L = -j\frac{V_{in}}{Z_0}$$

$$R I_L = -j\frac{V_{in}}{Z_0} R = -j\frac{V_A}{2}\frac{1}{\sqrt{2}R} R = -j\frac{V_A}{2\sqrt{2}}$$

$$-j\frac{V_A - V_B}{2\sqrt{2}}$$



$$V_L = Z_L I_L$$

λ/4 line

$$I_L = -j\frac{V_{in}}{Z_0}$$

3λ/4 line

$$I_L = j\frac{V_{in}}{Z_0}$$

$$R I_L = j\frac{V_{in}}{Z_0} R = j\frac{V_B}{2}\frac{1}{\sqrt{2}R} R = j\frac{V_B}{2\sqrt{2}}$$

Additional λ/2 length changes the sign