Lecture 38 – Outline

• More on Impedance matching
• Applications involving combination of transmission lines for distribution networks

Reading assignment
Prof. Kudeki’s ECE 329 Lecture Notes on Fields and Waves:
38) Distribution networks
Step 6 - First Solution

Since we are going back to the original transmission line, the line impedance on the chart is re-normalized with $Z_{01}$:

$$z_{\text{in}} = \frac{0.535 \times Z_{02}}{Z_{01}}$$

$$z_{\text{in}} = \frac{0.535 \times 93.4582}{50.0} = 1.0$$
**Step 4 - First Solution**

To cancel the imaginary part of the line admittance we add a stub with:

- Length: \( l_1 = 0.1024 \lambda \)
- Admittance:
  - \( Y_1 = -j \, 0.02668 \, \text{[S]} \) (actual)
  - \( y_1 = -j \, 1.33417 \) (normalized)
Examples of distribution networks

- Lines of length $\lambda/4$ are very useful to create networks with special properties.
- Examples are
  - The corporate ladder network
  - The hybrid rat-race coupler
Quick review – $\lambda/4$ transmission line

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{Z_0^2}{Z_L}$$

$$V_{in} = j I_L Z_0$$

$$I_{in} = j \frac{V_L}{Z_0}$$

$$I_L = -j \frac{V_{in}}{Z_0}$$

$$V_L = -j I_{in} Z_0$$

$$V_L = Z_L I_L$$
Corporate ladder network

All line sections are of length $\lambda/4$ and have characteristic impedance $Z_0$.
\[ Z_{in} = Z_{in1} \parallel Z_{in2} = \left( \frac{Z_L}{Z_0^2} + \frac{Z_L}{Z_0^2} \right)^{-1} = \frac{Z_0^2}{2Z_L} \]

Same result for the lines 3 & 4
\[
Z_{in5} = Z_{in6} = \frac{Z_0^2}{2Z_L} = 2Z_L
\]

\[
Z_{in} = Z_{in5} \parallel Z_{in6} = \left(\frac{1}{2Z_L} + \frac{1}{2Z_L}\right)^{-1}
\]

\[
= \left(\frac{2}{2Z_L}\right)^{-1} = Z_L
\]
If the load $Z_L$ is the same as $Z_0$ then there is no longer a constraint on the length of the individual transmission lines. The input impedance is also $Z_{in} = Z_0$ regardless of the lengths of the lines.

We have a new degree of freedom, whereby the length of each line can be varied to introduce delays. This is exploited in the development of antenna arrays to synthesize arbitrary beam patterns.
Hybrid rat-race combiner

\[ Z_0 = \sqrt{2}R \]
Hybrid rat-race combiner

Typical microstrip implementation

Each side branch has characteristic impedance $Z'_0 = R$ and is assumed to be terminated by a match impedance.

Output proportional to $(V_A - V_B)$

Output proportional to $(V_A + V_B)$

$Z_0 = \sqrt{2}R$
Input applied at port P1 – signal get split into two parts. One part travels in clockwise direction, the other travels in counterclockwise direction.

Signals reach in phase port P2 ($\lambda/4$ and $\lambda+\lambda/2$) and port P4.

Port P3 has no output because signals arrive in opposition of phase ($\lambda/2$ and $\lambda$)

Port P1 and P3 are isolated from each other. A signal “A” applied to P3 and a signal “B” applied to P1 produce a signal at port P2 proportional to their sum and a signal at port P4 proportional to their difference.

Effectively, each generator only sees a parallel of two impedances $2R // 2R = R$. 
\[ R I_L = -j \frac{V_{in}}{Z_0} R = -j \frac{V_A}{2} \frac{1}{\sqrt{2}R} R = -j \frac{V_A}{2\sqrt{2}} \]

\[ R I_L = -j \frac{V_{in}}{Z_0} R = -j \frac{V_B}{2} \frac{1}{\sqrt{2}R} R = -j \frac{V_B}{2\sqrt{2}} \]

\[ -j \frac{V_A + V_B}{2\sqrt{2}} \]

\[ V_L = Z_L I_L \]

\[ I_L = -j \frac{V_{in}}{Z_0} \]
\[ R I_L = -j \frac{V_{in}}{Z_0} R = -j \frac{V_A}{2} \frac{1}{\sqrt{2}R} R = -j \frac{V_A}{2\sqrt{2}} \]

\[ R I_L = -j \frac{V_A - V_B}{2\sqrt{2}} \]

\[ V_L = Z_L I_L \]

\[ I_L = -j \frac{V_{in}}{Z_0} \]

\[ I_L = j \frac{V_{in}}{Z_0} \]

Additional \( \lambda/2 \) length changes the sign