ECE 329 – Fall 2022

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Lecture 39

Lecture 39 – Outline

- **Lossy Transmission Lines (material not included in final exam)**
- **Class wrap-up**

Reading assignment

Prof. Kudeki's ECE 329 Lecture Notes on Fields and Waves:

39) Lossy Lines

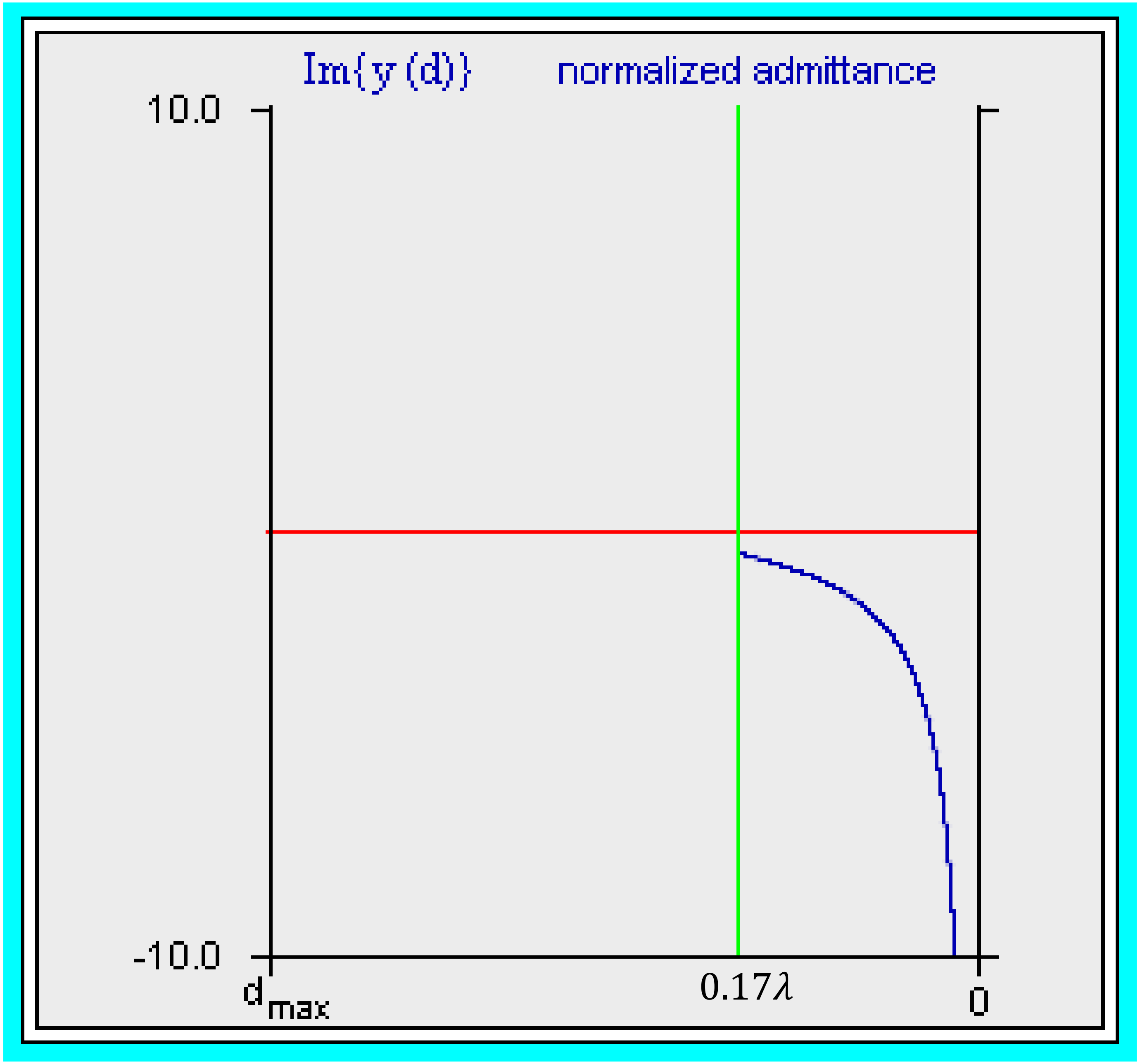
Exercise:

Design a stub line with a given normalized input admittance using the Smith Chart

Example

$y_{stub} \approx -j\, 0.55$

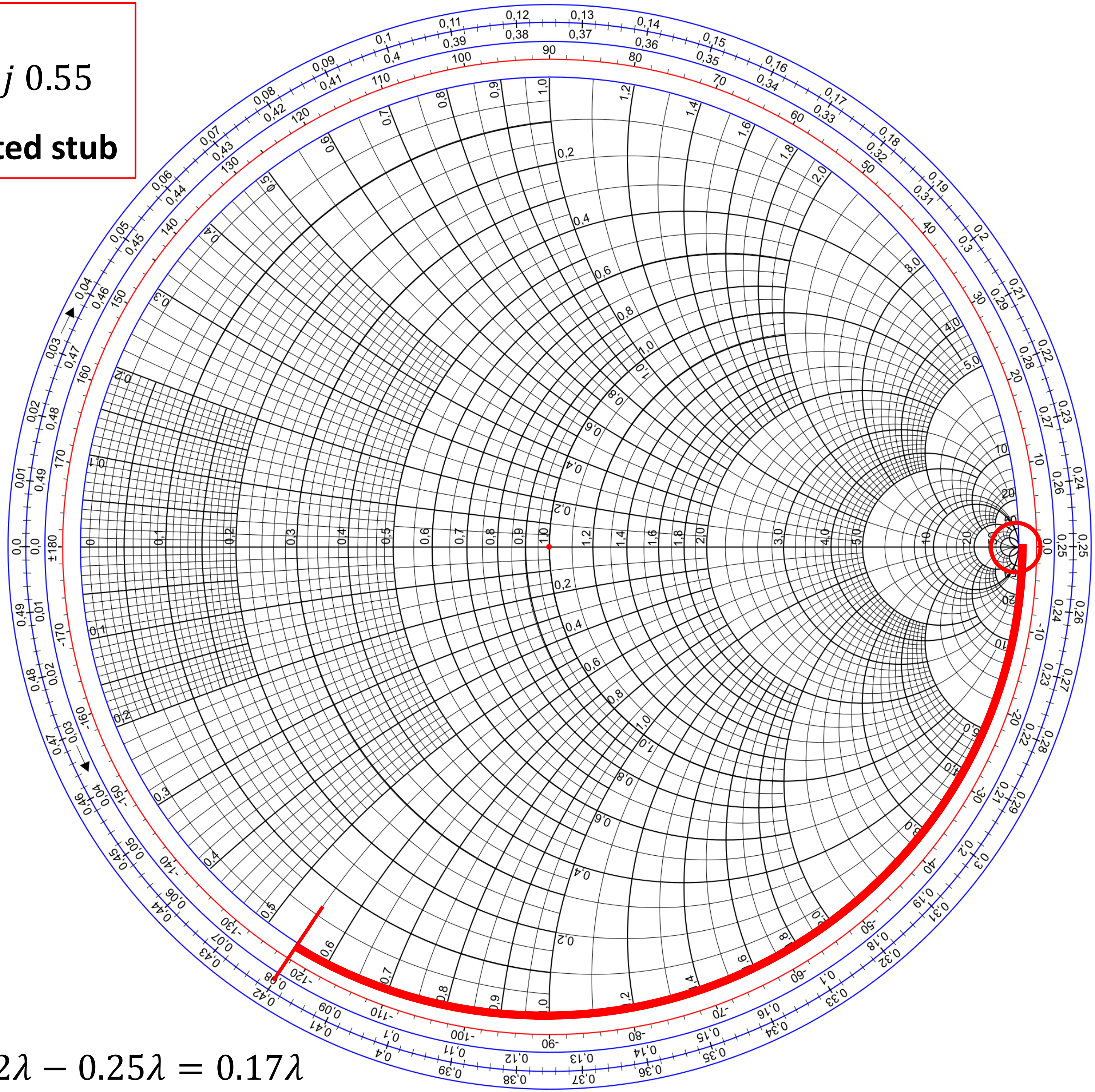
Short circuited stub



Example

$y_{stub} \approx -j\,0.55$

Short circuited stub



$y \to \infty$

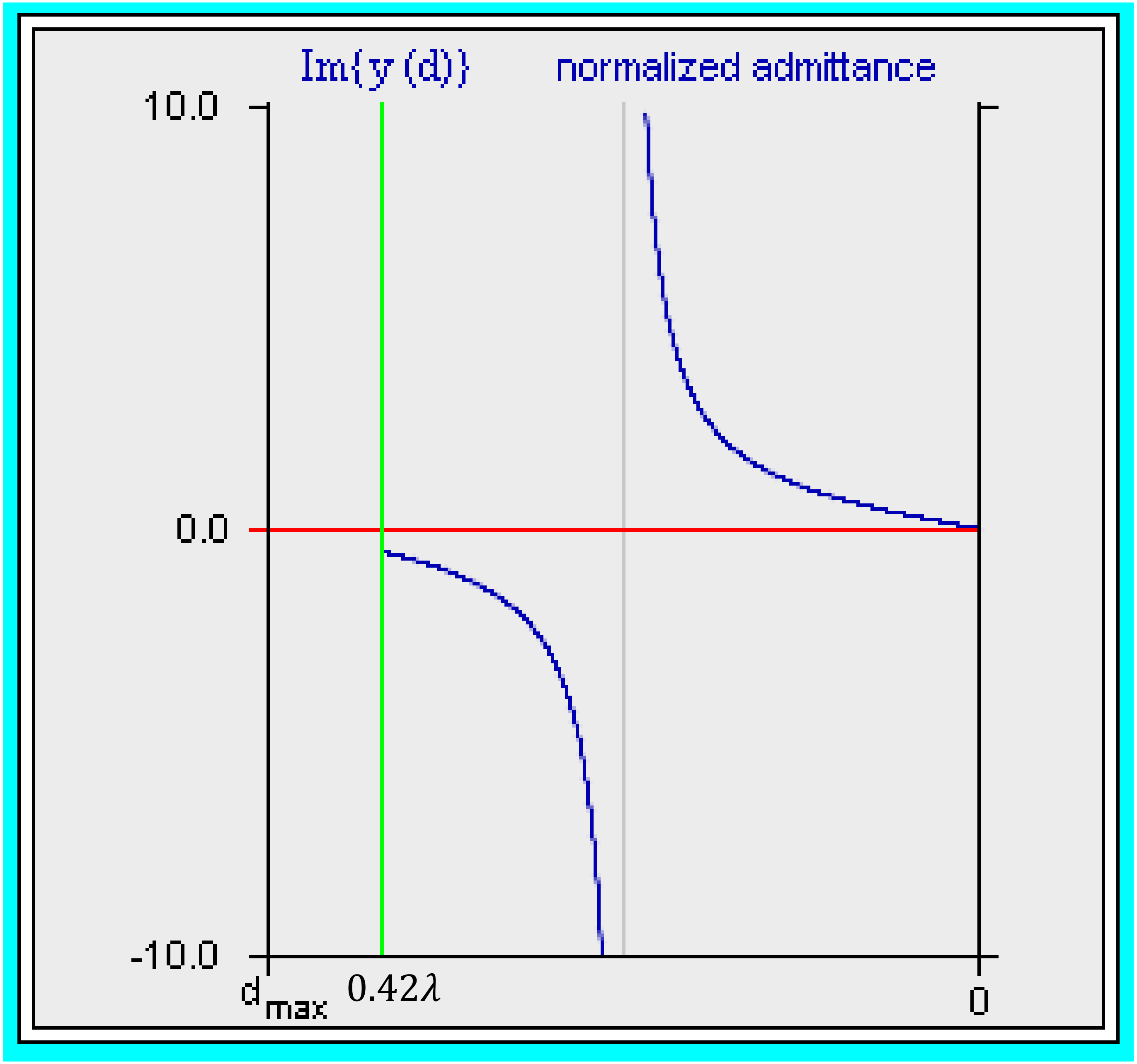
SHORT CIRCUIT

$L_{stub} \approx 0.42\lambda - 0.25\lambda = 0.17\lambda$

Example

$y_{stub} \approx -j\ 0.55$

Open circuited stub



Example

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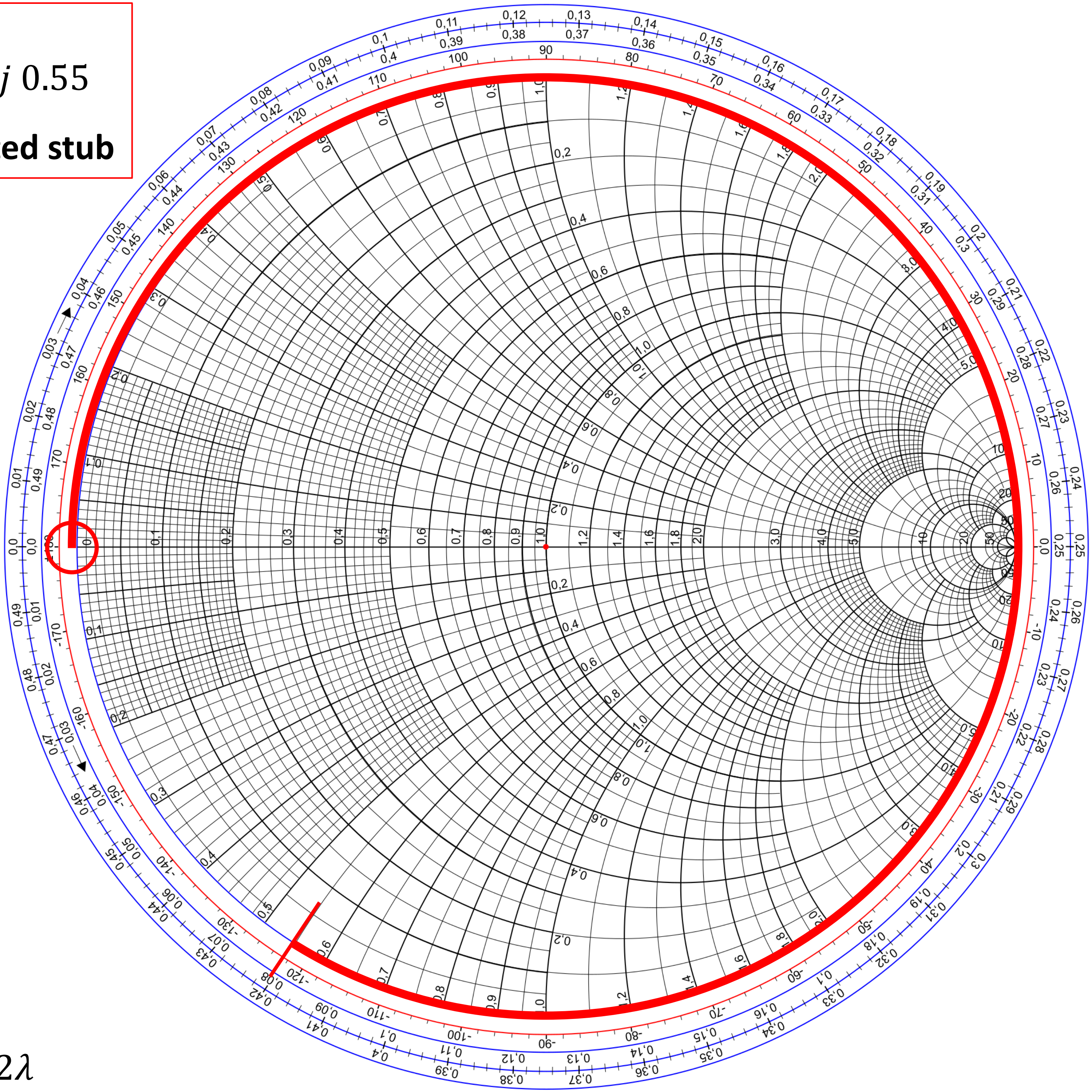
Open circuited stub



$y = 0$

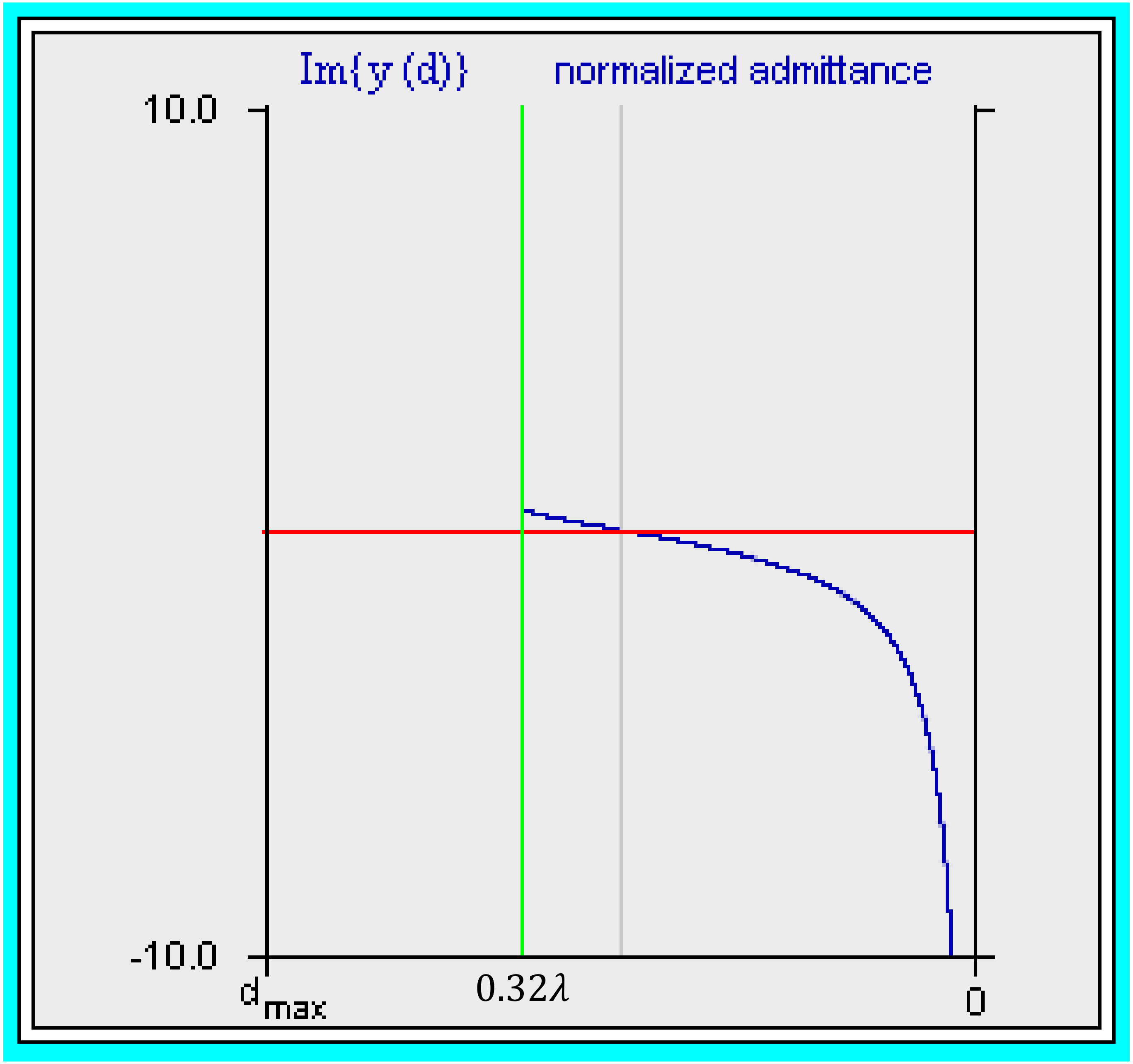
OPEN CIRCUIT

$L_{stub} \approx 0.42\lambda$

Example

$y_{stub} \approx j\,0.55$

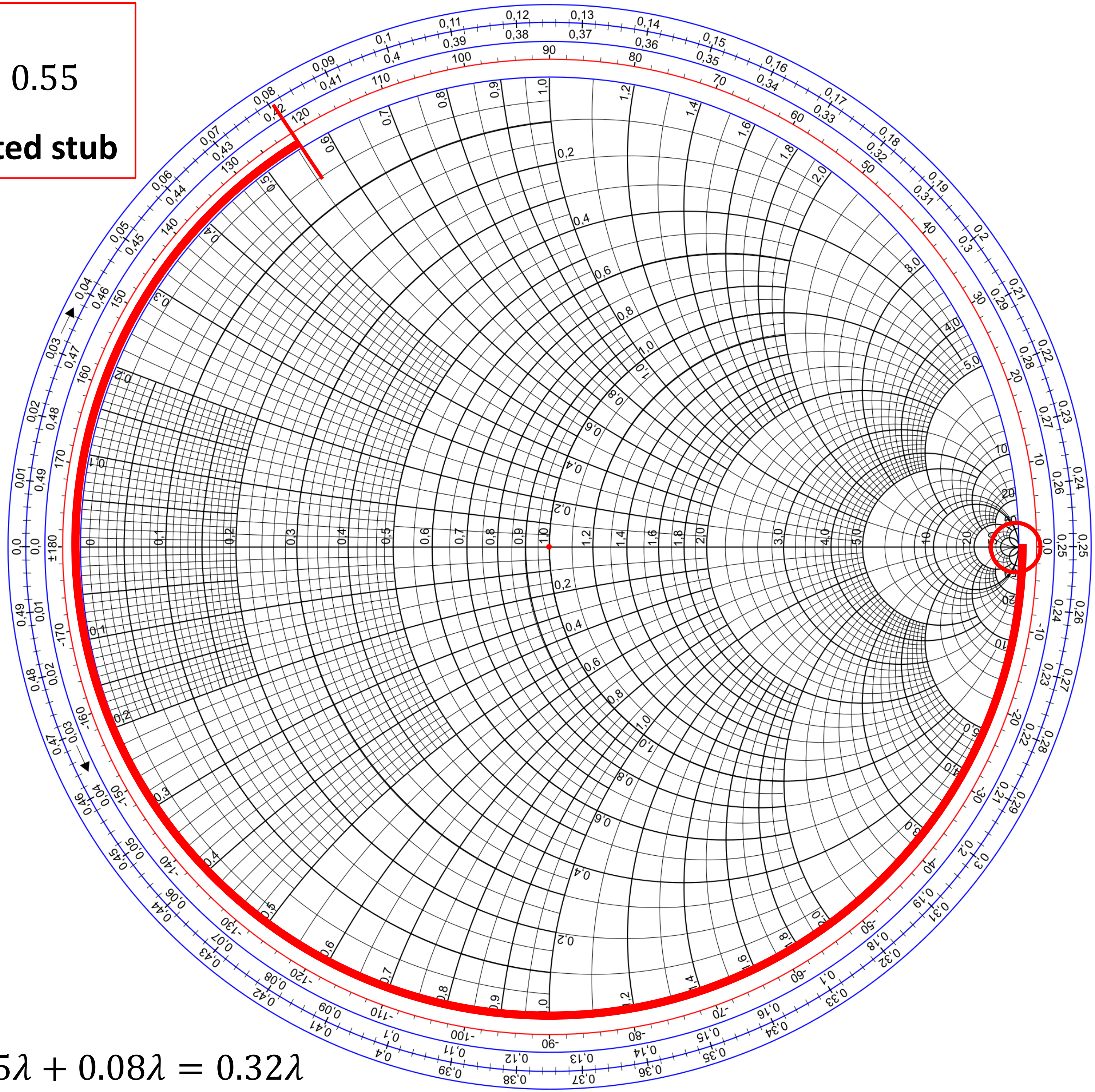
Short circuited stub



Example

$y_{stub} \approx j\,0.55$

Short circuited stub

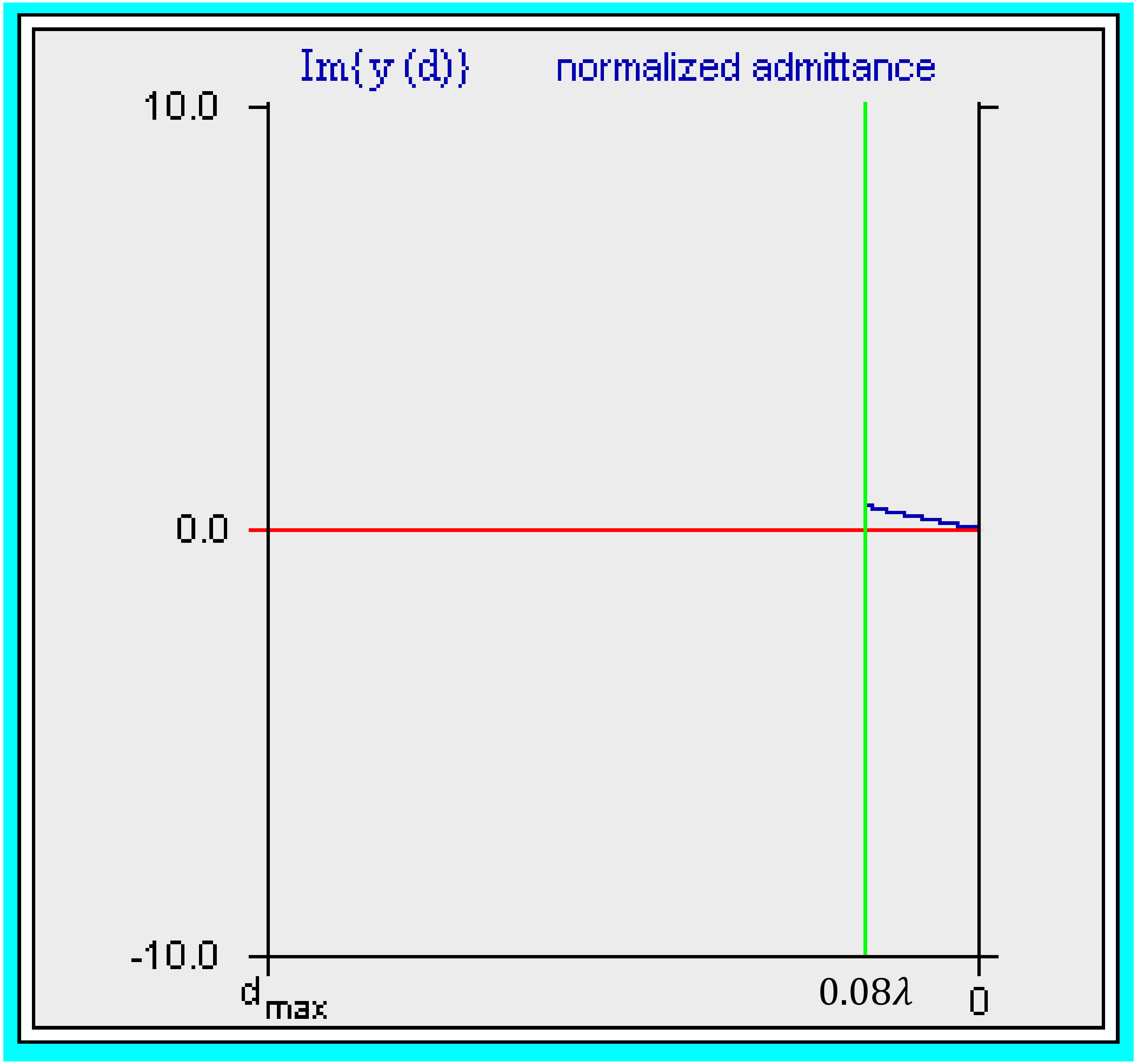


$$L_{stub} \approx 0.25\lambda + 0.08\lambda = 0.32\lambda$$

Example

$y_{stub} \approx j\,0.55$

Open circuited stub



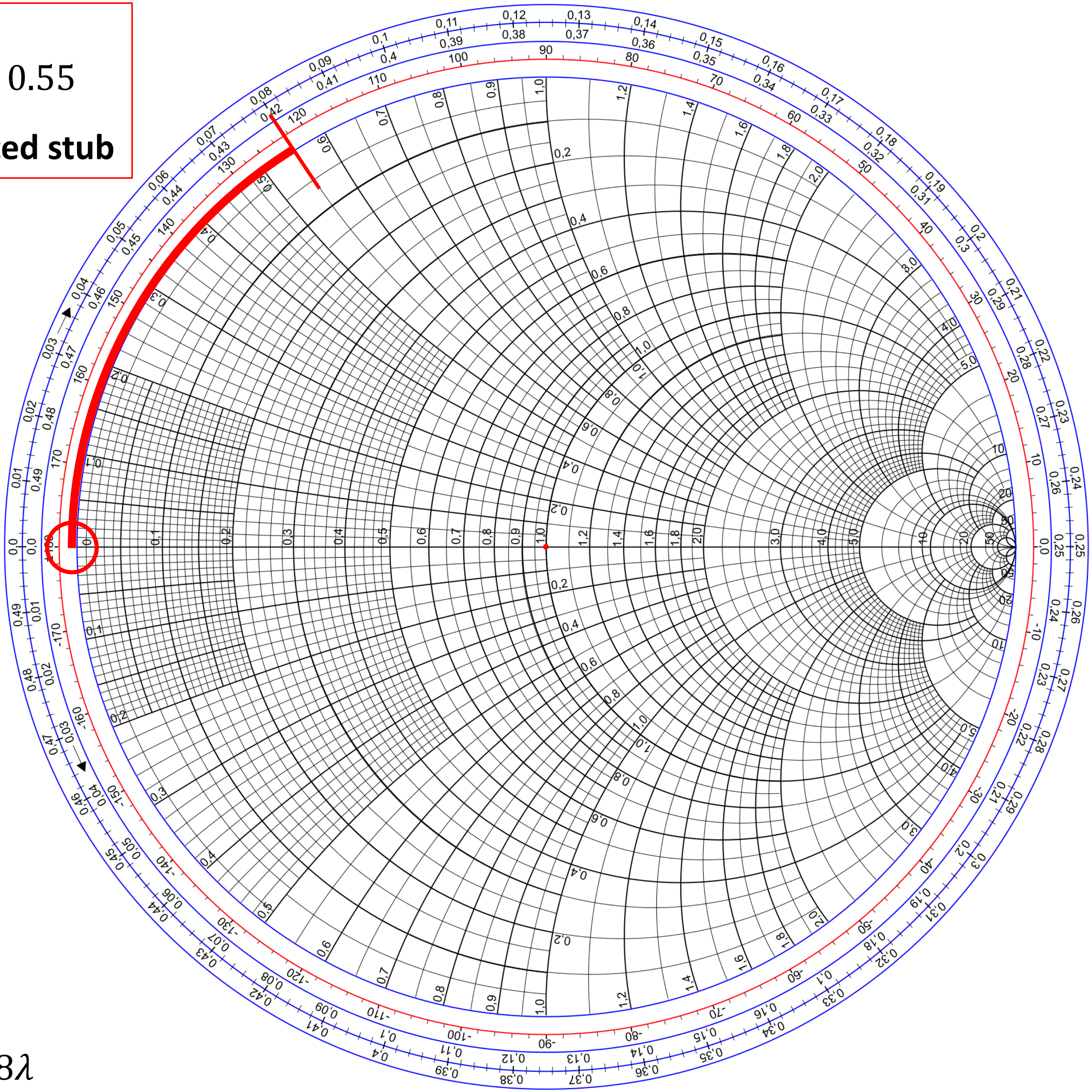
Example

$$y_{stub} \approx j\,0.55$$

Open circuited stub

$y = 0$

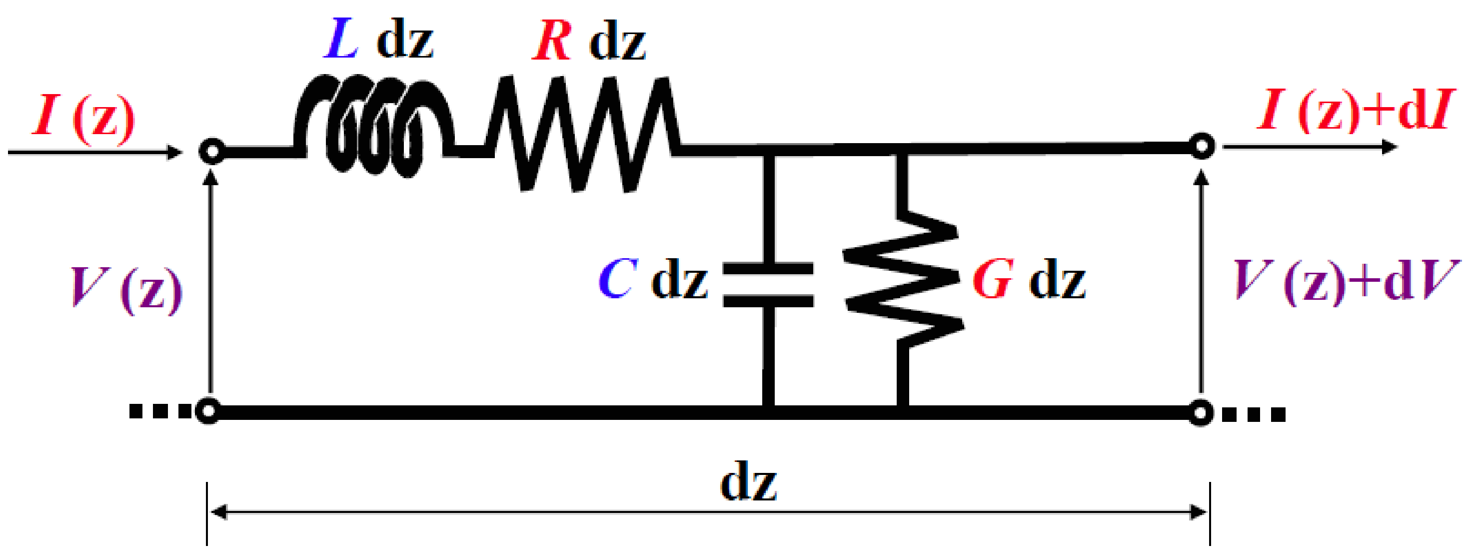
OPEN CIRCUIT



$L_{stub} \approx 0.08\lambda$

Lossy Lines (not on final exam)

Lossy Transmission Lines – Distributed impedance model



The impedance parameters L, R, C, and G represent:

L = series inductance per unit length
R = series resistance per unit length
C = shunt capacitance per unit length
G = shunt conductance per unit length.

Lossy Transmission Lines – Equation

Telegraphers' equations

$$\begin{cases} \dfrac{\mathrm{d}V}{\mathrm{d}z} = -(j\omega L + R)I \\ \dfrac{\mathrm{d}I}{\mathrm{d}z} = -(j\omega C + G)V \end{cases}$$

Telephonists' equations

$$\begin{cases} \dfrac{\mathrm{d}^2 V}{\mathrm{d}z^2} = (j\omega L + R)(j\omega C + G)V \\ \dfrac{\mathrm{d}^2 I}{\mathrm{d}z^2} = (j\omega C + G)(j\omega L + R)I \end{cases}$$

Propagation constant

$$\gamma = \sqrt{(j\omega L + R)(j\omega C + G)} = \alpha + j\beta$$

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$$\boxed{V(z) = V^{+} e^{-\gamma z} + V^{-} e^{\gamma z}}$$

$$\boxed{\begin{aligned} I(z) &= \sqrt{\frac{(j\omega C + G)}{(j\omega L + R)}}\,(V^{+} e^{-\gamma z} - V^{-} e^{\gamma z}) \\ &= \frac{1}{Z_0}(V^{+} e^{-\gamma z} - V^{-} e^{\gamma z}) \end{aligned}}$$

Characteristic impedance (complex)

$$Z_0 = \sqrt{\frac{(j\omega L + R)}{(j\omega C + G)}}$$

$$\gamma = \sqrt{(j\omega L + R)(j\omega C + G)} = \alpha + j\beta$$

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Characteristic impedance (complex)

$$Z_0 = \sqrt{\frac{(j\omega L + R)}{(j\omega C + G)}}$$

$$V(\mathrm{d}) = V^+ e^{\gamma \mathrm{d}} + V^- e^{-\gamma \mathrm{d}}$$

$$I(\mathrm{d}) = \frac{1}{Z_0}\left(V^+ e^{\gamma \mathrm{d}} - V^- e^{-\gamma \mathrm{d}}\right)$$

$$\boxed{\begin{aligned} V(\mathrm{d}) &= V^+ e^{\gamma \mathrm{d}}\left(1 + \Gamma_R e^{-2\gamma \mathrm{d}}\right) \\ I(\mathrm{d}) &= \frac{V^+ e^{\gamma \mathrm{d}}}{Z_0}\left(1 - \Gamma_R e^{-2\gamma \mathrm{d}}\right) \end{aligned}}$$

$$\Gamma(\mathrm{d}) = \Gamma_R \, e^{-2\gamma \mathrm{d}}$$

$$\boxed{\begin{aligned} V(\mathrm{d}) &= V^{+} e^{\gamma \mathrm{d}} \left(1 + \Gamma(\mathrm{d})\right) \\ I(\mathrm{d}) &= \frac{V^{+} e^{\gamma \mathrm{d}}}{Z_0} \left(1 - \Gamma(\mathrm{d})\right) \end{aligned}}$$

For the general lossy line

$$Y_0 = \frac{1}{Z_0} = \frac{Z_0^*}{Z_0 \, Z_0^*} = \frac{R_0 - jX_0}{\left|Z_0\right|^2} = \frac{R_0 - jX_0}{R_0^2 + X_0^2} = G_0 + jB_0$$

$$G_0 = \frac{R_0}{R_0^2 + X_0^2} \qquad B_0 = \frac{-X_0}{R_0^2 + X_0^2}$$

Let's review the *impedance-admittance* terminology:

Impedance	=	**Resistance**	+	***j* Reactance**
Z	$=$	R	$+$	jX

Admittance	=	**Conductance**	+	***j* Susceptance**
Y	$=$	G	$+$	jB

An important practical case is the low-loss transmission line, where the reactive elements still dominate but R and G cannot be neglected as in a loss-less line. We have the following conditions:

$$\omega L >> R \qquad \omega C >> G$$

so that

$$\begin{aligned}\gamma &= \sqrt{(j\omega L + R)(j\omega C + G)} \\ &= \sqrt{j\omega L\ j\omega C\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)} \\ &\approx j\omega\sqrt{LC}\sqrt{1 + \frac{R}{j\omega L} + \frac{G}{j\omega C} - \frac{RG}{\omega^2 LC}}\end{aligned}$$

The last term under the square root can be neglected, because it is the product of two very small quantities.

The low-loss line is analogous to EM wave propagation in imperfect dielectric.
The standard Smith chart can still be used without modifications in this case.

What remains of the square root can be expanded into a truncated Taylor series

$$\gamma \approx j\omega\sqrt{LC}\left[1+\frac{1}{2}\left(\frac{R}{j\omega L}+\frac{G}{j\omega C}\right)\right]$$

$$=\frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)+j\omega\sqrt{LC}$$

so that

$$\alpha=\frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right) \qquad \beta=\omega\sqrt{LC}$$

The characteristic impedance of the low-loss line is a real quantity for all practical purposes and it is approximately the same as in a corresponding loss-less line

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}}$$

and the phase velocity associated to the wave propagation is

$$v_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}$$

BUT NOTE:

In the case of the low-loss line, the equations for voltage and current retain the same form obtained for general lossy lines.

The characteristic impedance of the loss-less line is real and we can express the power flow, anywhere on the line, as

$$\langle P(\mathrm{d},t)\rangle = \frac{1}{2}\mathrm{Re}\{V(\mathrm{d})\,I^{*}(\mathrm{d})\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{V^{+}e^{j\beta\mathrm{d}}\left(1+\Gamma_R e^{-j2\beta\mathrm{d}}\right)\right.$$

$$\left.\frac{1}{Z_0}(V^{+})^{*}e^{-j\beta\mathrm{d}}\left(1-\Gamma_R e^{-j2\beta\mathrm{d}}\right)^{*}\right\}$$

$$= \underbrace{\frac{1}{2Z_0}\left|V^{+}\right|^2}_{\boxed{\text{Incident wave}}} - \underbrace{\frac{1}{2Z_0}\left|V^{+}\right|^2\left|\Gamma_R\right|^2}_{\boxed{\text{Reflected wave}}}$$

This result is valid for any location, including the input and the load, since the transmission line does not absorb any power.

In the case of low-loss lines, the characteristic impedance is again real, but the time-average power flow is position dependent because the line absorbs power.

$$
\begin{aligned}
\langle P(\mathrm{d}, t) \rangle &= \frac{1}{2}\,\mathrm{Re}\left\{ V(\mathrm{d})\, I^*(\mathrm{d}) \right\} \\
&= \frac{1}{2}\,\mathrm{Re}\Bigg\{ V^+ e^{\alpha\,\mathrm{d}} e^{j\beta\,\mathrm{d}} \left(1+\Gamma_R e^{-2\gamma\,\mathrm{d}}\right) \\
&\qquad\qquad \frac{1}{Z_0}(V^+)^* e^{\alpha\,\mathrm{d}} e^{-j\beta\,\mathrm{d}} \left(1-\Gamma_R e^{-2\gamma\,\mathrm{d}}\right)^* \Bigg\} \\
&= \underbrace{\frac{1}{2Z_0}\left|V^+\right|^2\; e^{2\alpha\,\mathrm{d}}}_{\boxed{\text{Incident wave}}} - \underbrace{\frac{1}{2Z_0}\left|V^+\right|^2\; e^{-2\alpha\,\mathrm{d}}\left|\Gamma_R\right|^2}_{\boxed{\text{Reflected wave}}}
\end{aligned}
$$

The formalism and the physical interpretation are more complicated for the case of the general lossy line, due to the complex characteristic impedance

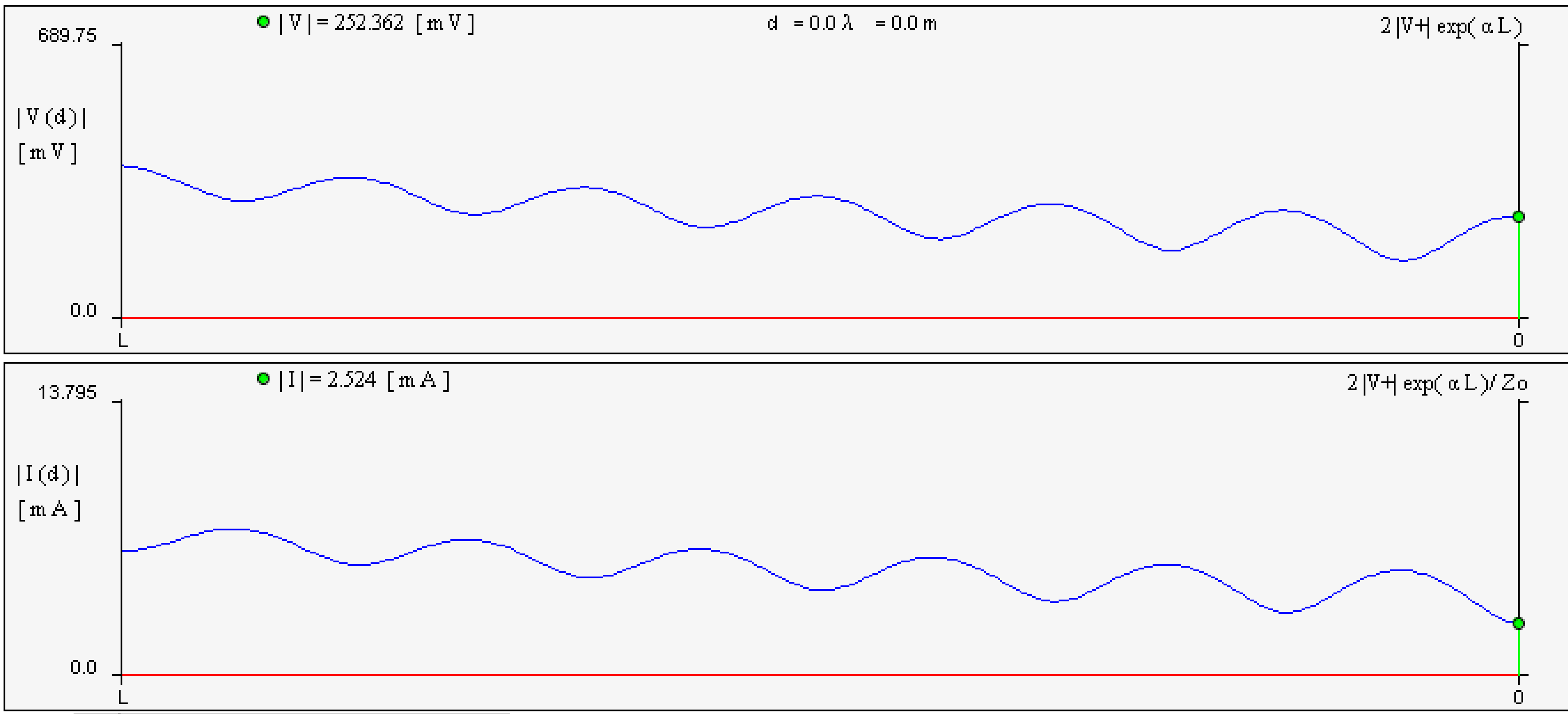
Example – Standing Wave Patterns of a low-loss transmission line

LOW-LOSS APPROXIMATION
α = 0.66713 [Ne/m] = 0.2 [Ne/ λ]

Z_g = 100.0 + j 0.0 Ω
V_g = 1.0 + j 0.0 V

Z_0 = 50.0 + j 0.0 Ω f_o = 1.0 GHz
ε_r = 1.0 λ = 299.7925 mm

Z_L = 100.0 + j 0.0 Ω



L = 3.0 λ = 899.3774 mm

Class wrap-up

Review of Transmission Line topics

Behavior of loss-less lines
Characteristic impedance
Line impedance
Reflection coefficient
Short and open circuited lines
Standing wave patterns
Smith Chart
Impedance matching